# Unveiling Internet Streaming Services: A Comparison Using Neutrosophic Graphs

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#### ABSTRACT

This paper investigates the inverse of the maximum result obtained by multiplying two unique graph types known as neutrosophic graphs. The main goal is to comprehend a point's degree in the opposite situation of a neutrosophic graph's maximum product. The study deals with two particular graph kinds and offers several conclusions and evidence regarding the opposite of the highest product. The research also contains a practical application of these ideas by locating an online streaming service utilizing a neutrosophic graph and a technique known as normalized Hamming distance and normalized Euclidean distance. Finally, the comparison results are given.

#### **KEYWORDS**

neutrosophic graphs; max product neutrosophic graph; complement neutrosophic graph

# 1 Introduction

raphs are commonly understood to be essentially representations of relations. A good tool for expressing information about item connections is a graph. Edges describe relationships, whereas vertices represent things. The objects and the relationships between them are represented, respectively, by the vertices and edges of the graph. The information that describes the conditions might become ambiguous when it comes to global challenges. In a variety of disciplines, including topology, optimization, network, and environmental science, neutrosophic models are useful mathematical instruments for solving combinatorial issues. Neutrosophic models are more sophisticated than straightforward graphical models due to the inherent vagueness and ambiguity they include. When neutrosophic set theory was originally applied, it was utilized to solve several intricate problems for which there was insufficient information.

When it comes to applying graph theory to dealing with real-life circumstances, it is seen to be crucial. The use of fuzzy set<sup>[1]</sup> theory has no bounds; hence, the fuzzy graph theory has unique relevance. Rosenfeld<sup>[2]</sup> presented the notion of fuzzy graphs in 1975, while independently Yeh and Bang also introduced the concept of fuzzy graphs<sup>[3]</sup>. Fuzzy graphs are quite different from traditional graphs and are excellent for representing interactions that deal with ambiguity. Numerous issues in the fields of computer science,

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electrical engineering, system modeling, transportation, finance, etc. can be treated by using them.

The concept of intuitionistic fuzzy sets (IFS), introduced by Atanassov<sup>[4]</sup>, and represents an extension of fuzzy sets that effectively handle ambiguous conditions. Unlike traditional fuzzy sets, IFS structures are not confined solely to membership grades, allowing for improved handling of uncertainty.

The concept of IFS has witnessed significant utilization across various domains. In 1994, Shannon and Atanassov<sup>[5]</sup> made a notable contribution by introducing the concept of intuitionistic fuzzy graphs (IFG), further enhancing the applicability and importance of this mathematical framework.

The concept of IFG that was initially introduced by Atanassov and Shannon was further elucidated by Parvathi and Karunambigai<sup>[6]</sup>. Sahoo and Pal<sup>[7]</sup> classified IFG products into three categories: Strong, semistrong, and direct. Additionally, Yaqoob et al.<sup>[8]</sup> extensively investigated the four fundamental operations of complex IFG, including the Cartesian product, join, union, and composition. Mohamed and Ali<sup>[9-11]</sup> developed the terms modular, complement, and maximum product on IFG.

The neutrosophic sets were suggested by Smarandache<sup>[12,13]</sup>. Using imprecise, ambiguous, and inconsistent data in practical applications calls for a sophisticated mathematical approach. IFS and interval-valued IFS are both included in this category of fuzzy set theory<sup>[14–17]</sup>. The truth, indeterminacy, and falsity membership values (*T*, *I*, and *F*), which are independent and fall inside the real standard or non-standard unit interval [0, 1], are used to describe neutrosophic sets.

The subclass of neutrosophic sets called single-valued neutrosophic sets (SVNS) was introduced by Wang et al.<sup>[18]</sup> with the purpose of facilitating practical implementation in real-world applications. In order to create SVNS, IFS with independent membership values between [0, 1] were generalised. SVNS are a subset of neutrosophic sets, which simplifies the utilization of neutrosophic sets in practical situations. One may find similar research on the growth of the single-valued neutrosophic network in Refs. [19–21]. Kaviyarasu et al.<sup>[22, 23]</sup> explained the concept of regularity in neutrosophic graph theory. Akram et al.<sup>[24]</sup> introduced the notation of new concepts in neutrosophic graphs with application. According to the aforementioned literature, the product's classical, fuzzy, and intuitionistic fuzzy forms are employed in a number of industries and provide practical answers to the problems. The max product and their complement in neutrosophic graphs have also not been employed in the present study. The proposed method can also be used to discover the online streaming service.

#### 1.1 Motivation

Numerous uses of neutrosophic graphs and their expansions have been found recently in study. In the field of applied mathematics, research on the combination of neutrosophic graphs and their products is expanding. In this study, the maximum product of the complement of the neutrosophic graph is used as the context. The following is a description of the study's rationale:

(1) The max product and complement notions are foundational ideas in graph theory with numerous applications in diverse disciplines.

(2) These ideas expand the options for conveying uncertainty when used in the context of neutrosophic graphs.

(3) These ideas expand the options for conveying uncertainty when used in the context of neutrosophic graphs.

(4) More ambiguous information cannot be captured using this method.

(5) When used in the neutrosophic graph setting, it could produce a useful result.

(6) Additionally, there are issues with finding an online streaming provider.

It should be emphasized that earlier researches have not addressed these challenges, which fact inspired us to offer a workable alternative. As a result, this article discusses these problems and suggests creative solutions. The goal of the current study is to contribute significantly to society by accomplishing this.

### 1.2 Novelties

The concepts of the maximum product of neutrosophic graph are introduced in this work. We also give a new meaning to the complement of a graph that is neutrosophic.

1. The notions of the maximum product of neutrosophic graph are defined in this work.

2. To offer a fresh definition of the neutrosophic graph complement.

3. This study also teaches the notions of the neutrosophic graph's maximum product of complement.

4. To increase the amount of uncertainty that decision-making issues may represent, a max product of complement of neutrosophic graph is used.

## 1.3 Structure of the paper

We investigate graphs produced by neutrosophic systems in this work, with particular attention to the vertex degree. We address decision-making issues, namely in choosing an internet streaming provider, by utilizing the complement of the maximum product of two neutrosophic graphs. We first discuss the fundamental ideas behind neutrosophic graphs. In Section 3, we define the term "complement of max product of neutrosophic graphs" and talk about degrees. In Sections 4 and 5, we employ normalized Hamming distance to locate an online streaming service provider using neutrosophic graphs. We take into account the signals provided by other users, choosing the one that most accurately reflects their preferred streaming service. Using this method, we may ascertain which service each user prefers depending on how well their signals match the options.

#### 2 Preliminary

**Definition 1**<sup>[22]</sup> (1) A neutrosophic graph denoted as  $G = ((T\sigma_1, I\sigma_2, F\sigma_3), (T\mu_1, I\mu_2, F\mu_3))$  is represented by  $G^* = (V, E)$ , where V is the set of vertices, and E is the set of edges. The functions  $T\sigma_1, I\sigma_2$ , and  $F\sigma_3$ are mappings from V to the closed interval [0,1], signifying the degrees of true, intermediate, and false membership, respectively, for each element  $x_i \in V$ . It holds that  $0 \leq T\sigma_1(x_i) + I\sigma_2(x_i) + F\sigma_3(x_i) \leq 3$  for all  $x_i \in V$ .

(2) Moreover, in the context of  $G^*$ , the functions  $T_{\mu_1}, I_{\mu_2}$ , and  $F_{\mu_3}$  are mappings from  $V \times V$  to the closed interval [0, 1], representing the degrees of true, intermediate, and false membership, respectively, for each edge  $(x_i, x_j) \in E$ .

$$T\mu_{1}(x_{i}, x_{j}) \leq T\sigma_{1}(x_{i}) \wedge T\sigma_{1}(x_{j}),$$
  

$$I\mu_{1}(x_{i}, x_{j}) \leq I\sigma_{1}(x_{i}) \wedge I\sigma_{1}(x_{j}),$$
  

$$F\mu_{1}(x_{i}, x_{j}) \geq F\sigma_{1}(x_{i}) \wedge F\sigma_{1}(x_{j}),$$
  

$$0 \leq T\sigma_{1}(x_{i}, x_{j}) + I\sigma_{2}(x_{i}, x_{j}) + F\sigma_{3}(x_{i}, x_{j}) \leq 3.$$

**Definition 2**<sup>[22]</sup> A neutrosophic graph  $G = ((T\sigma_1, I\sigma_2, F\sigma_3), (T\mu_1, I\mu_2, F\mu_3))$  is called strong neurotsophic graph if

- (1)  $T\mu_1(x_i, x_j) = T\sigma_1(x_i) \wedge T\sigma_1(x_j)$ ,
- (2)  $I\mu_1(x_i, x_i) = I\sigma_1(x_i) \wedge I\sigma_1(x_i)$ ,
- (3)  $F\mu_1(x_i, x_j) = F\sigma_1(x_i) \wedge F\sigma_1(x_j)$ ,

for all  $x_i, x_j \in E, i \neq j$ .

**Definition 3**<sup>[22]</sup> A neutrosophic graph  $G = ((T\sigma_1, I\sigma_2, F\sigma_3), (T\mu_1, I\mu_2, F\mu_3))$  is considered complete if (1)  $T\mu_1(x_i, x_j) = T\sigma_1(x_i) \wedge T\sigma_1(x_j)$ ,

- (2)  $I\mu_1(x_i, x_j) = I\sigma_1(x_i) \wedge I\sigma_1(x_j)$ ,
- (3)  $F\mu_1(x_i, x_j) = F\sigma_1(x_i) \wedge F\sigma_1(x_j)$ ,

for all  $x_i, x_j \in V, i \neq j$ .

**Definition 4**<sup>[24]</sup> A neutrosophic graph  $G = ((T\sigma_1, I\sigma_2, F\sigma_3), (T\mu_1, I\mu_2, F\mu_3))$ . The order of G denoted as O(G) is defined as  $O(G) = (O_{T\sigma_1}(G), O_{I\sigma_1}(G), O_{F\sigma_1}(G))$ , where  $O_{T\sigma_1}(G) = \Sigma_{x \in V} T\sigma_1(x), O_{I\sigma_2}(G) = \Sigma_{x \in V} I\sigma_2(x)$  and  $O_{F\sigma_3}(G) = \Sigma_{x \in V} F\sigma_3(x)$ .

**Definition 5**<sup>[24]</sup> A neurotsophic graph  $G = ((T\sigma_1, I\sigma_2, F\sigma_3), (T\mu_1, I\mu_2, F\mu_3))$ . The size of G, denoted as S(G), is defined as  $S(G) = (S_{T\mu_1}(G), S_{I\mu_1}(G), S_{F\mu_1}(G))$ , where  $S_{T\mu_1}(G) = \Sigma_{xy \in E} T\mu_1(xy), S_{I\mu_2}(G) = \Sigma_{xy \in E} I\mu_2(xy)$ , and  $S_{F\mu_3}(G) = \Sigma_{xy \in E} F\mu_3(xy)$ .

**Definition 6**<sup>[22]</sup> A neutrosophic graph  $G = ((T\sigma_1, I\sigma_2, F\sigma_3), (T\mu_1, I\mu_2, F\mu_3))$ . The degree of a vertex x in G is denoted by  $d_G(x) = (Td_1^G(x), Id_2^G(x), Fd_3^G(x))$  and can be calculated as follows:

$$Td_1^G(x) = \sum_{x \neq y} T\mu_1^G(xy) = \sum_{xy \in E} T\mu_1^G(xy)$$
(1)

$$Id_{2}^{G}(x) = \sum_{x \neq y} I\mu_{2}^{G}(xy) = \sum_{xy \in E} I\mu_{2}^{G}(xy)$$
<sup>(2)</sup>

$$Fd_{3}^{G}(x) = \sum_{x \neq y} F\mu_{2}^{G}(xy) = \sum_{xy \in E} F\mu_{2}^{G}(xy)$$
(3)

where  $Td_1^G(x)$  represents the total of type membership scores of the edges connected to vertex x,  $Id_2^G(x)$  represents the total of intermediate membership scores of the edges connected to vertex x, and  $Fd_3^G(x)$  represents the sum of false membership scores of the edges connected to vertex x.

# 3 Neutrosophic Graphs' Complement of the Maximum Product

**Definition** 7 The complement of a neutrosophic graph G = (V, E) is a neutrosophic graph  $\overline{G} = (\overline{(T\sigma_1, I\sigma_2, F\sigma_3)}, \overline{(T\mu_1, I\mu_2, F\mu_3)})$  where  $\overline{(T\sigma_1, I\sigma_2, F\sigma_3)} = \overline{(T\sigma_1)}, \overline{(I\sigma_2)}, \overline{(F\sigma_3)}$  and  $\overline{(T\mu_1, I\mu_2, F\mu_3)} = \overline{(T\sigma_1, I\sigma_2, F\sigma_3)}$ 

 $\overline{(T\mu_1)}, \overline{(I\mu_2)}, \overline{(F\mu_3)}, \text{ where } \overline{T\mu_1}(x, y) = T\sigma_1(x) \wedge T\sigma_1(y) - T\sigma_1(xy), \overline{I\mu_2}(x, y) = I\sigma_2(x) \wedge I\sigma_2(y) - I\sigma_2(xy)$ and  $\overline{F\mu_3}(x, y) = F\sigma_3(x) \wedge F\sigma_3(y) - F\sigma_3(xy).$ 

**Definition 8** Let  $G_1 = ((T\sigma_1^{G_1}, I\sigma_2^{G_1}, F\sigma_3^{G_1}), (T\mu_1^{G_1}, I\mu_2^{G_1}, F\mu_3^{G_1}))$  and  $G_2 = ((T\sigma_1^{G_2}, I\sigma_2^{G_2}, F\sigma_3^{G_2}), (T\mu_1^{G_2}, I\mu_2^{G_2}, F\mu_3^{G_2}))$  be two neutrosophic graphs. The maxium product of  $G_1$  and  $G_2$  is defined as  $G_1 \times_m G_2 = (V_1 \times_m V_2, E_1 \times_m E_2), E_1 \times_m E_2 = \{(x_1, y_1), (x_2, y_2) / x_1 = x_2, y_1, y_2 \in E_2 \text{ or } y_1 = y_2, x_1, x_2 \in E_1\}.$ 

$$T\sigma^{G_1 \times_m G_2}(x_1, y_2) = T\sigma_1^{G_1}(x_1) \wedge T\sigma_1^{G_2}(y_1)$$
(4)

$$I\sigma^{G_1 \times_m G_2}(x_1, y_2) = I\sigma_1^{G_1}(x_1) \wedge I\sigma_1^{G_2}(y_1)$$
(5)

$$F\sigma^{G_1 \times_m G_2}(x_1, y_2) = F\sigma_1^{G_1}(x_1) \vee F\sigma_1^{G_2}(y_1)$$
(6)

$$T\mu_{1}^{G_{1}\times_{m}G_{2}}((x_{1},y_{1}),(x_{2},y_{2})) = \begin{cases} T\sigma_{1}^{G_{1}}(x_{1}) \wedge T\mu_{1}^{G_{2}}(y_{1},y_{2}), \text{ if } x_{1} = x_{2}, y_{1}, y_{2} \in E_{2}; \\ T\mu_{1}^{G_{1}}(x_{1},x_{2}) \wedge T\sigma_{1}^{G_{2}}(y_{1}), \text{ if } y_{1} = y_{2}, x_{1}, x_{2} \in E_{1} \end{cases}$$

$$(7)$$

$$I\mu_{2}^{G_{1}\times_{m}G_{2}}((x_{1},y_{1}),(x_{2},y_{2})) = \begin{cases} I\sigma_{2}^{G_{1}}(x_{1}) \wedge I\mu_{2}^{G_{2}}(y_{1},y_{2}), \text{ if } x_{1} = x_{2}, y_{1}, y_{2} \in E_{2}; \\ I\mu_{2}^{G_{1}}(x_{1},x_{2}) \wedge I\sigma_{2}^{G_{2}}(y_{1}), \text{ if } y_{1} = y_{2}, x_{1}, x_{2} \in E_{1} \end{cases}$$

$$(8)$$

$$F\mu_{3}^{G_{1}\times_{m}G_{2}}((x_{1},y_{1}),(x_{2},y_{2})) = \begin{cases} F\sigma_{3}^{G_{1}}(x_{1}) \vee F\mu_{3}^{G_{2}}(y_{1},y_{2}), \text{ if } x_{1} = x_{2}, y_{1}, y_{2} \in E_{2}; \\ F\mu_{3}^{G_{1}}(x_{1},x_{2}) \vee F\sigma_{3}^{G_{2}}(y_{1}), \text{ if } y_{1} = y_{2}, x_{1}, x_{2} \in E_{1} \end{cases}$$

$$(9)$$

**Example 1** In Figs. 1 and 2, Let  $G_1^* = (V_1, E_1)$  and  $G_2^* = (V_2, E_2)$  be two crisp graphs, such that  $V_1 = \{u_1, u_2, u_3\}$ ,  $V_2 = \{v_1, v_2\}, E_1 = \{u_1u_3, u_2u_3\}$ , and  $E_2 = \{v_1v_2\}$ . Take two neutrosophic graphs as consideration  $G_1 = ((T\sigma_1^{G_1}, I\sigma_2^{G_1}, F\sigma_3^{G_1}), (T\mu_1^{G_1}, I\mu_2^{G_1}, F\mu_3^{G_1}))$  and  $G_2 = ((T\sigma_1^{G_2}, I\sigma_2^{G_2}, F\sigma_3^{G_2}), (T\mu_1^{G_1}, I\mu_2^{G_1}, F\mu_3^{G_1}))$ 



Fig. 2 Max product of neutrosophic graph.

 $F\mu_3^{G_1})$ , and  $G_1 \times_m G_2$ , in Tables 1 and 2.

**Definition 9** The maximum of two neutrosophic graphs' products in complement  $G_1 = ((T\sigma_1^{G_1}, I\sigma_2^{G_1}, F\sigma_3^{G_1}), (T\mu_1^{G_1}, I\mu_2^{G_1}, F\mu_3^{G_1}))$  and  $G_2 = ((T\sigma_1^{G_2}, I\sigma_2^{G_2}, F\sigma_3^{G_2}), (T\mu_1^{G_2}, I\mu_2^{G_2}, F\mu_3^{G_2}))$  is a neutrosophic graphs  $\overline{G_1 \times_m G_2} = (((\overline{T\sigma_1^{G_1} \times_m T\sigma_1^{G_2}}), (\overline{I\sigma_2^{G_1} \times_m I\sigma_2^{G_2}}), (\overline{I\sigma_3^{G_1} \times_m I\sigma_3^{G_2}})), ((\overline{T\mu_1^{G_1} \times_m T\mu_1^{G_2}}), (\overline{I\mu_2^{G_1} \times_m I\mu_2^{G_2}}), (\overline{I\sigma_2^{G_1} \times_m I\sigma_3^{G_2}})))$  on  $G^* = (V, E)$ .

$$E_1 \times_m E_2 = \begin{cases} x_1 = x_2, \ y_1 \ y_2 \in E_2 \ \text{or} \ y_1 = y_2, x_1 x_2 \in E_1 \ \text{or} \\ (x_1, y_1)(x_2, y_2) | x_1 x_2 \in E_1, \ y_1 y_2 \notin E_2 \ \text{or} \ x_1 x_2 \notin E_1, \ y_1 y_2 \in E_2 \ \text{or} \\ x_1 x_2 \in E_1, \ y_1 y_2 \in E_2 \ \text{or} \ x_1 x_2 \notin E_1, \ y_1 y_2 \notin E_2 \end{cases}$$

$$(\overline{T\sigma_1^{G_1} \times_m T\sigma_1^{G_2}})(x_1, y_2) = (T\sigma_1^{G_1} \times_m T\sigma_1^{G_2})(x_1, y_1) = T\sigma_1^{G_1}(x_1) \vee T\sigma_1^{G_2}(y_1)$$
(10)

$$\overline{I\sigma_2^{G_1} \times_m I\sigma_2^{G_2}}(x_1, y_2) = (I\sigma_2^{G_1} \times_m I\sigma_2^{G_2})(x_1, y_1) = I\sigma_2^{G_1}(x_1) \vee I\sigma_2^{G_2}(y_1)$$
(11)

$$(\overline{F\sigma_{3}^{G_{1}}\times_{m}F\sigma_{3}^{G_{2}}})(x_{1},y_{2}) = (F\sigma_{3}^{G_{1}}\times_{m}F\sigma_{3}^{G_{2}})(x_{1},y_{1}) = F\sigma_{3}^{G_{1}}(x_{1})\wedge F\sigma_{3}^{G_{2}}(y_{1})$$
(12)

where  $x_1 \in V_1$  and  $y_1 \in V_2$ .

$$(T\mu_{1}^{G_{1}}\times_{m}T\mu_{1}^{G_{2}})((x_{1},y_{1}),(x_{2},y_{2})) = \\ \begin{cases} (T\sigma_{1}^{G_{1}}\times_{m}T\sigma_{1}^{G_{2}})(x_{1},y_{1})\wedge(T\sigma_{1}^{G_{1}}\times_{m}T\sigma_{1}^{G_{2}})(x_{2},y_{2})-(T\mu_{1}^{G_{1}}\times_{m}T\mu_{1}^{G_{2}})((x_{1},y_{1}),(x_{2},y_{2})), \text{ if } x_{1}=x_{2}, y_{1}y_{2} \in E_{2}; \\ (T\sigma_{1}^{G_{1}}\times_{m}T\sigma_{1}^{G_{2}})(x_{1},y_{1})\wedge(T\sigma_{1}^{G_{1}}\times_{m}T\sigma_{1}^{G_{2}})(x_{2},y_{2})-(T\mu_{1}^{G_{1}}\times_{m}T\mu_{1}^{G_{2}})((x_{1},y_{1}),(x_{2},y_{2})), \text{ if } y_{1}=y_{2}, x_{1}x_{2} \in E_{1}; \\ (T\sigma_{1}^{G_{1}}\times_{m}T\sigma_{1}^{G_{2}})(x_{1},y_{1})\wedge(T\sigma_{1}^{G_{1}}\times_{m}T\sigma_{1}^{G_{2}})(x_{2},y_{2}), \text{ otherwise,} \end{cases}$$

$$\begin{aligned} (I\mu_1^{G_1} \times_m I\mu_1^{G_2})((x_1, y_1), (x_2, y_2)) &= \\ & \begin{cases} (I\sigma_1^{G_1} \times_m I\sigma_1^{G_2})(x_1, y_1) \wedge (I\sigma_1^{G_1} \times_m I\sigma_1^{G_2})(x_2, y_2) - (I\mu_1^{G_1} \times_m I\mu_1^{G_2})((x_1, y_1), (x_2, y_2)), \text{ if } x_1 = x_2, \ y_1y_2 \in E_2; \\ (I\sigma_1^{G_1} \times_m I\sigma_1^{G_2})(x_1, y_1) \wedge (I\sigma_1^{G_1} \times_m I\sigma_1^{G_2})(x_2, y_2) - (I\mu_1^{G_1} \times_m I\mu_1^{G_2})((x_1, y_1), (x_2, y_2)), \text{ if } y_1 = y_2, \ x_1x_2 \in E_1; \\ (I\sigma_1^{G_1} \times_m I\sigma_1^{G_2})(x_1, y_1) \wedge (I\sigma_1^{G_1} \times_m I\sigma_1^{G_2})(x_2, y_2), \text{ otherwise,} \end{aligned}$$

 $\overline{(F\mu_1^{G_1} \times_m F\mu_1^{G_2})}((x_1, y_1), (x_2, y_2)) = \begin{cases}
(F\sigma_1^{G_1} \times_m F\sigma_1^{G_2})(x_1, y_1) \lor (F\sigma_1^{G_1} \times_m F\sigma_1^{G_2})(x_2, y_2) - (F\mu_1^{G_1} \times_m F\mu_1^{G_2})((x_1, y_1), (x_2, y_2)), \text{ if } x_1 = x_2, \ y_1y_2 \in E_2; \\
(F\sigma_1^{G_1} \times_m F\sigma_1^{G_2})(x_1, y_1) \lor (F\sigma_1^{G_1} \times_m F\sigma_1^{G_2})(x_2, y_2) - (F\mu_1^{G_1} \times_m F\mu_1^{G_2})((x_1, y_1), (x_2, y_2)), \text{ if } y_1 = y_2, \ x_1x_2 \in E_1; \\
(F\sigma_1^{G_1} \times_m F\sigma_1^{G_2})(x_1, y_1) \lor (F\sigma_1^{G_1} \times_m F\sigma_1^{G_2})(x_2, y_2), \text{ otherwise.}
\end{cases}$ 

Table 1 Troduct of two vertex sets.							
$V_1 \times_m V_2$	$u_1v_1$	$u_2v_1$	$u_3v_1$	$u_1v_2$	$u_2v_2$	$u_3v_2$	
$T\sigma_1^{G_1}  imes T\sigma_1^{G_2}$	0.7	0.7	0.8	0.7	0.7	0.7	
$I\sigma_2^{G_1}  imes I\sigma_2^{G_2}$	0.7	0.6	0.7	0.6	0.6	0.6	
$F\sigma_3^{G_1}  imes F\sigma_3^{G_2}$	0.5	0.5	0.5	0.5	0.5	0.4	

Table 1 Droduct of two wortes ante

Table 2 Product of two edge sets.							
$E_1 \times_m E_2$	$u_1v_1, u_3v_1$	$u_2v_1, u_3v_1$	$u_3v_1, u_3v_2$	$u_1v_2, u_3v_2$	$u_2v_2, u_3v_2$	$u_2v_2, u_2v_1$	$u_1v_1, u_1v_2$
$T\mu_1^{G_1} imes T\mu_1^{G_2}$	0.8	0.8	0.7	0.7	0.7	0.8	0.7
$I\mu_2^{G_1} imes I\mu_2^{G_2}$	0.7	0.7	0.6	0.6	0.6	0.7	0.8
$F\mu_3^{G_1} imes F\mu_3^{G_2}$	0.5	0.5	0.4	0.4	0.4	0.5	0.5

Table 2 Due du at of true a des sate

**Example 2** Examine the two neutrosophic diagrams as depicted in Fig. 1 and their respective maximum product  $G_1 \times_m G_2$  illustrated in Fig. 2. Subsequently, the complement of the maximum product of  $G_1$  and  $G_2$  is displayed in Fig. 3.

**Theorem 1** Prove that the complement of complement neutrosophic graph is *G*, i.e.,  $\overline{\overline{G}} = G$ . *Proof* By Definition 7, we know that

$$\begin{split} \overline{T_{\mu_{1}}}(x,y) &= T_{\sigma_{1}}(x) \wedge T_{\sigma_{1}}(y) - T_{\sigma_{1}}(xy), \\ \overline{\overline{T_{\mu_{1}}}}(x,y) &= \overline{T_{\sigma_{1}}}(x) \wedge \overline{T_{\sigma_{1}}}(y) - \overline{T_{\sigma_{1}}}(xy) = \\ & T_{\sigma_{1}}(x) \wedge T_{\sigma_{1}}(y) - (T_{\sigma_{1}}(x) \wedge T_{\sigma_{1}}(y) - T_{\sigma_{1}}(xy)) = \\ & \overline{I_{\sigma_{1}}}(x,y), \\ \overline{\overline{I_{\mu_{1}}}}(x,y) &= \overline{I_{\sigma_{1}}}(x) \wedge \overline{I_{\sigma_{1}}}(y) - \overline{I_{\sigma_{1}}}(xy) = \\ & I_{\sigma_{1}}(x) \wedge I_{\sigma_{1}}(y) - (I_{\sigma_{1}}(x) \wedge I_{\sigma_{1}}(y) - I_{\sigma_{1}}(xy)) = \\ & I_{\sigma_{1}}(xy), \\ \overline{\overline{F_{\mu_{1}}}}(x,y) &= \overline{F_{\sigma_{1}}}(x) \wedge \overline{F_{\sigma_{1}}}(y) - \overline{F_{\sigma_{1}}}(xy) = \\ & F_{\sigma_{1}}(x) \wedge F_{\sigma_{1}}(y) - (F_{\sigma_{1}}(x) \wedge F_{\sigma_{1}}(y) - F_{\sigma_{1}}(xy)) = \\ & F_{\sigma_{1}}(xy). \end{split}$$

Hence  $\overline{\overline{G}} = G$ .

**Theorem 2** If  $G_1$  and  $G_2$  are two regular neutrosophic graphs of underlying crip graphs  $G_1^*$  and  $G_2^*$  with constants  $T\sigma_1^{G_1}, I\sigma_2^{G_1}, F\sigma_3^{G_1}, T\sigma_1^{G_2}, F\sigma_3^{G_2}$  and satisfying the following condition:  $T\sigma_1^{G_1} \ge T\mu_1^{G_2}$ ,  $I\sigma_2^{G_2}, F\sigma_3^{G_2} \ge I\mu_2^{G_1}, F\sigma_3^{G_2} \le T\mu_3^{G_1}, T\sigma_1^{G_1} > T\mu_1^{G_1}, I\sigma_2^{G_1} > I\mu_2^{G_1}, F\sigma_3^{G_1} < T\mu_3^{G_1}, T\sigma_1^{G_2} > I\mu_2^{G_2}, F\sigma_3^{G_2} < I\mu_3^{G_2}, F\sigma_3^{G_2} < T\mu_3^{G_2}$ , Then the max product of two neutrosophic graphs  $G_1$  and  $G_2$  is regular neutrosophic graph.

*Proof* Let  $G_1$  and  $G_2$  be two regular neutrosophic graphs. The underlying crisp graphs  $G_1^*$  and  $G_2^*$  are complete graphs of degree  $d_1$  and  $d_2$  for every vertices of  $V_1$  and  $V_2$ . Given that  $T\sigma_1^{G_1}, I\sigma_2^{G_1}, F\sigma_3^{G_1}$  and  $T\sigma_1^{G_2}, I\sigma_2^{G_2}, F\sigma_3^{G_2}$  are constants, say  $T\sigma_1^{G_1}(x) = C_1, I\sigma_2^{G_1}(x) = C_2, F\sigma_3^{G_1}(x) = C_3, \forall x \in V_1, T\sigma_1^{G_2}(y) = C_4, I\sigma_2^{G_2}(y) = C_5, F\sigma_3^{G_2}(y) = C_6, \forall y \in V_2$  and  $T\sigma_1^{G_1} \ge T\mu_1^{G_2}, I\sigma_2^{G_2} \ge I\mu_2^{G_2}, F\sigma_3^{G_1} \le T\mu_3^{G_2}, T\sigma_1^{G_2} \ge T\mu_1^{G_1}, I\sigma_2^{G_2} \ge I\mu_2^{G_2}, F\sigma_3^{G_3} \le T\mu_3^{G_1}$ . As per assumption the max product of two neutrosophic graphs is regular neutrosophic



Fig. 3 Complement of max product  $\overline{G_1 \times_m G_2}$ .

graphs. Consider  $(x_1, y_2) \in (\overline{T\sigma^{G_1} \times_m T\sigma^{G_2}})$ ,

$$\begin{split} d_{1}^{(T\sigma^{G_{1}}\times_{m}T\sigma^{G_{2}})}(x_{1},y_{1}) &= \sum_{(x_{1},y_{1})(x_{2},y_{2})\in E} (\overline{(T\sigma^{G_{1}}\times_{m}T\sigma^{G_{2}})}((x_{1},y_{1})(x_{2},y_{2})) = \\ &\sum_{x_{1}=x_{2},y_{1}y_{2}\in E_{1}} \left((T\sigma^{G_{1}}\times_{m}T\sigma^{G_{2}})(x_{1},y_{1})\wedge(T\sigma^{G_{1}}\times_{m}T\sigma^{G_{2}})(x_{2},y_{2}) - \\ &(T\mu_{1}^{G_{1}}\times_{m}T\mu_{1}^{G_{2}})((x_{1},y_{1}),(x_{2},y_{2}))\right) + \\ &\sum_{y_{1}=y_{2},x_{1}x_{2}\in E_{1}} \left((T\sigma^{G_{1}}\times_{m}T\sigma^{G_{2}})(x_{1},y_{1})\wedge(T\sigma^{G_{1}}_{1}\times_{m}T\sigma^{G_{2}})(x_{2},y_{2}) - \\ &(T\mu_{1}^{G_{1}}\times_{m}T\mu_{1}^{G_{2}})((x_{1},y_{1}),(x_{2},y_{2}))\right) + \\ &\sum_{x_{1}x_{2}\in E_{1},y_{1}y_{2}\notin E_{2}} \left((T\sigma^{G_{1}}\times_{m}T\sigma^{G_{2}})(x_{1},y_{1})\wedge(T\sigma^{G_{1}}_{1}\times_{m}T\sigma^{G_{2}})(x_{2},y_{2})\right) + \\ &\sum_{x_{1}x_{2}\in E_{1},y_{1}y_{2}\in E_{2}} \left((T\sigma^{G_{1}}\times_{m}T\sigma^{G_{2}})(x_{1},y_{1})\wedge(T\sigma^{G_{1}}_{1}\times_{m}T\sigma^{G_{2}})(x_{2},y_{2})\right) + \\ &\sum_{x_{1}x_{2}\in E_{1},y_{1}y_{2}\in E_{2}} \left((T\sigma^{G_{1}}\times_{m}T\sigma^{G_{2}})(x_{1},y_{1})\wedge(T\sigma^{G_{1}}_{1}\times_{m}T\sigma^{G_{2}})(x_{2},y_{2})\right) + \\ &\sum_{x_{1}x_{2}\notin E_{1},y_{1}y_{2}\notin E_{2}} \left((T\sigma^{G_{1}}\times_{m}T\sigma^{G_{2}})(x_{1},y_{1})\wedge(T\sigma^{G_{1}}\times_{m}T\sigma^{G_{2}})(x_{2},y_{2})\right) + \\ &\sum_{x_{1}x_{2}\notin E_{1},y_{1}y_{2}\notin E_{2}} \left((T\sigma^{G_{1}}\times_{m}T\sigma^{G_{2}})(x_{1},y_{1})\wedge(T\sigma^{G_{1}}\times_{m}T\sigma^{G_{2}})(x_{2},y_{2})\right) + \\ &\sum_{x_{1}x_{2}\notin E_{1},y_{1}y_{2}\notin E_{2}} \left((T\sigma^{G_{1}}\times_{m}T\sigma^{G_{2}})(x_{1},y_{1})\wedge(T\sigma^{G_{1}}\times_{m}T\sigma^{G_{2}})(x_{2},y_{2})\right) + \\ \\ &\sum_{x_{1}x_{2}\notin E_{1},y_{1}y_{2}\notin E_{2}} \left((T\sigma^{G_{1}}\times_{m}T\sigma^{G_{2}})(x_{1},y_{1})\wedge(T\sigma^{G_{1}}\times_{m}T\sigma^{G_{2}})(x_{2},y_{2})\right) + \\ \\ &\sum_{x_{1}x_{2}\notin E_{1},y_{1}y_{2}\notin E_{2}} \left((T\sigma^{G_{1}}\times_{m}T\sigma^{G_{2}})(x_{1},y_{1})\wedge(T\sigma^{G_{1}}\times_{m}T\sigma^{G_{2}})(x_{2},y_{2})\right) \right)$$

Since  $G_1^*$  and  $G_2^*$  are complete graphs, then

$$\begin{split} d_{1}^{(\overline{G_{1}}\times_{m}\overline{G_{2}})}(x_{1},y_{1}) &= \sum_{x_{1}=x_{2},y_{1}y_{2}\in E_{2}} \left( (T\sigma_{1}^{G_{1}}\times_{m}T\sigma_{1}^{G_{2}})(x_{1},y_{1}) \wedge (T\sigma_{1}^{G_{1}}\times_{m}T\sigma_{1}^{G_{2}})(x_{2},y_{2}) - (T\mu_{1}^{G_{1}}\times_{m}T\mu_{1}^{G_{2}})((x_{1},y_{1}),(x_{2},y_{2})) \right) + \\ \sum_{y_{1}=y_{2},x_{1}x_{2}\in E_{1}} \left( (T\sigma_{1}^{G_{1}}\times_{m}T\sigma_{1}^{G_{2}})(x_{1},y_{1}) \wedge (T\sigma_{1}^{G_{1}}\times_{m}T\sigma_{1}^{G_{2}})(x_{2},y_{2}) - (T\mu_{1}^{G_{1}}\times_{m}T\mu_{1}^{G_{2}})((x_{1},y_{1}),(x_{2},y_{2})) \right) + \\ \sum_{x_{1}x_{2}\in E_{1},y_{1}y_{2}\in E_{2}} \left( (T\sigma_{1}^{G_{1}}\times_{m}T\sigma_{1}^{G_{2}})(x_{1},y_{1}) \wedge (T\sigma_{1}^{G_{1}}\times_{m}T\sigma_{1}^{G_{2}})(x_{2},y_{2}) - (T\mu_{1}^{G_{1}}\times_{m}T\mu_{1}^{G_{2}})((x_{1},y_{1}),(x_{2},y_{2})) \right) + \\ \sum_{x_{1}x_{2}\in E_{1},y_{1}y_{2}\in E_{2}} \left( (T\sigma_{1}^{G_{1}}\times_{m}T\sigma_{1}^{G_{2}})(x_{1},y_{1}) \wedge (T\sigma_{1}^{G_{1}}\times_{m}T\sigma_{1}^{G_{2}})(x_{2},y_{2}) \right). \end{split}$$

$$\begin{split} d_{2}^{(G_{1}\times_{m}G_{2})}(x_{1},y_{1}) &= \sum_{x_{1}=x_{2},y_{1}y_{2}\in E_{2}} \left( (I\sigma_{1}^{G_{1}}\times_{m}I\sigma_{1}^{G_{2}})(x_{1},y_{1}) \wedge (I\sigma_{1}^{G_{1}}\times_{m}I\sigma_{1}^{G_{2}})(x_{2},y_{2}) - \\ & (I\mu_{1}^{G_{1}}\times_{m}I\mu_{1}^{G_{2}})((x_{1},y_{1}),(x_{2},y_{2}))) + \\ & \sum_{y_{1}=y_{2},x_{1}x_{2}\in E_{1}} \left( (I\sigma_{1}^{G_{1}}\times_{m}I\sigma_{1}^{G_{2}})(x_{1},y_{1}) \wedge (I\sigma_{1}^{G_{1}}\times_{m}I\sigma_{1}^{G_{2}})(x_{2},y_{2}) - \\ & (I\mu_{1}^{G_{1}}\times_{m}I\mu_{1}^{G_{2}})((x_{1},y_{1}),(x_{2},y_{2}))) + \\ & \sum_{x_{1}x_{2}\in E_{1},y_{1}y_{2}\in E_{2}} \left( (I\sigma_{1}^{G_{1}}\times_{m}I\sigma_{1}^{G_{2}})(x_{1},y_{1}) \wedge (I\sigma_{1}^{G_{1}}\times_{m}I\sigma_{1}^{G_{2}})(x_{2},y_{2}) - \\ & (I\mu_{1}^{G_{1}}\times_{m}I\mu_{1}^{G_{2}})((x_{1},y_{1}),(x_{2},y_{2}))) + \\ & \sum_{x_{1}x_{2}\in E_{1},y_{1}y_{2}\in E_{2}} \left( (I\sigma_{1}^{G_{1}}\times_{m}I\sigma_{1}^{G_{2}})(x_{1},y_{1}) \wedge (I\sigma_{1}^{G_{1}}\times_{m}I\sigma_{1}^{G_{2}})(x_{2},y_{2}) - \\ & \sum_{x_{1}x_{2}\in E_{1},y_{1}y_{2}\in E_{2}} \left( (I\sigma_{1}^{G_{1}}\times_{m}I\sigma_{1}^{G_{2}})(x_{1},y_{1}) \wedge (I\sigma_{1}^{G_{1}}\times_{m}I\sigma_{1}^{G_{2}})(x_{2},y_{2}) \right). \end{split}$$

Similarly,

$$\begin{split} d_{3}^{(\overline{G_{1}}\times_{m}\overline{G_{2}})}(x_{1},y_{1}) &= \sum_{x_{1}=x_{2},y_{1}y_{2}\in E_{2}} \left( (F\sigma_{1}^{G_{1}}\times_{m}I\sigma_{1}^{G_{2}})(x_{1},y_{1}) \wedge (F\sigma_{1}^{G_{1}}\times_{m}F\sigma_{1}^{G_{2}})(x_{2},y_{2}) - \right. \\ &\left. \left( F\mu_{1}^{G_{1}}\times_{m}F\mu_{1}^{G_{2}}\right)((x_{1},y_{1}),(x_{2},y_{2})) \right) + \right. \\ &\left. \sum_{y_{1}=y_{2},x_{1}x_{2}\in E_{1}} \left( (F\sigma_{1}^{G_{1}}\times_{m}F\sigma_{1}^{G_{2}})(x_{1},y_{1}) \wedge (F\sigma_{1}^{G_{1}}\times_{m}F\sigma_{1}^{G_{2}})(x_{2},y_{2}) - \right. \\ &\left. \left( F\mu_{1}^{G_{1}}\times_{m}F\mu_{1}^{G_{2}}\right)((x_{1},y_{1}),(x_{2},y_{2})) \right) + \right. \\ &\left. \sum_{x_{1}x_{2}\in E_{1},y_{1}y_{2}\in E_{2}} \left( (I\sigma_{1}^{G_{1}}\times_{m}I\sigma_{1}^{G_{2}})(x_{1},y_{1}) \wedge (F\sigma_{1}^{G_{1}}\times_{m}I\sigma_{1}^{G_{2}})(x_{2},y_{2}) - \right. \\ &\left. \left( F\mu_{1}^{G_{1}}\times_{m}F\mu_{1}^{G_{2}}\right)((x_{1},y_{1}),(x_{2},y_{2})) \right) + \right. \\ &\left. \sum_{x_{1}x_{2}\in E_{1},y_{1}y_{2}\in E_{2}} \left( (F\sigma_{1}^{G_{1}}\times_{m}F\sigma_{1}^{G_{2}})(x_{1},y_{1}) \wedge (F\sigma_{1}^{G_{1}}\times_{m}F\sigma_{1}^{G_{2}})(x_{2},y_{2}) \right) \right. \end{split}$$

**Case 1** If  $T\sigma_1^{G_1}(x) \leq T\sigma_1^{G_2}(y)$ ,  $I\sigma_1^{G_1}(x) \leq I\sigma_1^{G_2}(y)$ , and  $F\sigma_1^{G_1}(x) \geq F\sigma_1^{G_2}(y)$  for all  $x \in V_1$  and  $y \in V_2$ .

$$Td_{1}^{\overline{G_{1}\times G_{2}}}(x_{1},y_{1}) = \sum_{x_{1}=x_{2},y_{1}y_{2}\in E_{2}} \left( (T\sigma_{1}^{G_{1}}(x_{1})\wedge T\sigma_{1}^{G_{2}}(y_{1}))\wedge (T\sigma_{1}^{G_{1}}(x_{2})\wedge T\sigma_{1}^{G_{2}})(y_{2}) - (T\mu_{1}^{G_{1}}\times_{m}T\mu_{1}^{G_{2}})((x_{1},y_{1}),(x_{2},y_{2}))) + \sum_{y_{1}=y_{2},x_{1}x_{2}\in E_{1}} \left( (T\sigma_{1}^{G_{1}}(x_{1})\wedge T\sigma_{1}^{G_{2}}(y_{1}))\wedge (T\sigma_{1}^{G_{1}}(x_{2})\wedge T\sigma_{1}^{G_{2}}(y_{2})) - (T\mu_{1}^{G_{1}}\times_{m}T\mu_{1}^{G_{2}})((x_{1},y_{1}),(x_{2},y_{2}))) + \sum_{x_{1}x_{2}\in E_{1},y_{1}y_{2}\in E_{2}} \left( (T\sigma_{1}^{G_{1}}(x_{1})\wedge T\sigma_{1}^{G_{2}}(y_{1}))\wedge (T\sigma_{1}^{G_{1}}(x_{2})\wedge T\sigma_{1}^{G_{2}}(y_{2})) \right) + \sum_{x_{1}=x_{2},y_{1}y_{2}\in E_{2}} \left( (T\sigma_{1}^{G_{1}}(x_{1})\wedge T\sigma_{1}^{G_{2}}(y_{1}))\wedge (T\sigma_{1}^{G_{1}}(x_{2})\wedge T\sigma_{1}^{G_{2}}(y_{2})) \right) \right),$$

$$\begin{split} Id_{2}^{\overline{G_{1}\times G_{2}}}(x_{1},y_{1}) &= \sum_{x_{1}=x_{2},y_{1}y_{2}\in E_{2}} \left( (I\sigma_{1}^{G_{1}}(x_{1})\wedge T\sigma_{1}^{G_{2}}(y_{1}))\wedge (I\sigma_{1}^{G_{1}}(x_{2})\wedge I\sigma_{1}^{G_{2}})(y_{2}) - (I\mu_{1}^{G_{1}}\times_{m}I\mu_{1}^{G_{2}})((x_{1},y_{1}),(x_{2},y_{2}))) + \\ &\sum_{y_{1}=y_{2},x_{1}x_{2}\in E_{1}} \left( (I\sigma_{1}^{G_{1}}(x_{1})\wedge I\sigma_{1}^{G_{2}}(y_{1}))\wedge (I\sigma_{1}^{G_{1}}(x_{2})\wedge I\sigma_{1}^{G_{2}}(y_{2})) - (I\mu_{1}^{G_{1}}\times_{m}I\mu_{1}^{G_{2}})((x_{1},y_{1}),(x_{2},y_{2}))) + \\ &\sum_{x_{1}x_{2}\in E_{1},y_{1}y_{2}\in E_{2}} \left( (I\sigma_{1}^{G_{1}}(x_{1})\wedge I\sigma_{1}^{G_{2}}(y_{1}))\wedge (I\sigma_{1}^{G_{1}}(x_{2})\wedge I\sigma_{1}^{G_{2}}(y_{2})) \right) + \\ &\sum_{x_{1}x_{2}\in E_{1},y_{1}y_{2}\in E_{2}} \left( (I\sigma_{1}^{G_{1}}(x_{1})\wedge I\sigma_{1}^{G_{2}}(y_{1}))\wedge (I\sigma_{1}^{G_{1}}(x_{2})\wedge I\sigma_{1}^{G_{2}}(y_{2})) \right), \end{split}$$

$$\begin{aligned} Fd_{3}^{\overline{G_{1}\times G_{2}}}(x_{1},y_{1}) &= \sum_{x_{1}=x_{2},y_{1}y_{2}\in E_{2}} \left( \left(F\sigma_{1}^{G_{1}}(x_{1})\vee F\sigma_{1}^{G_{2}}(y_{1})\right)\vee \left(F\sigma_{1}^{G_{1}}(x_{2})\vee F\sigma_{1}^{G_{2}}\right)(y_{2}) - \\ & \left(F\mu_{1}^{G_{1}}\times_{m}F\mu_{1}^{G_{2}}\right)((x_{1},y_{1}),(x_{2},y_{2}))\right) + \\ & \sum_{y_{1}=y_{2},x_{1}x_{2}\in E_{1}} \left( \left(F\sigma_{1}^{G_{1}}(x_{1})\vee F\sigma_{1}^{G_{2}}(y_{1})\right)\wedge \left(F\sigma_{1}^{G_{1}}(x_{2})\vee F\sigma_{1}^{G_{2}}(y_{2})\right) - \\ & \left(F\mu_{1}^{G_{1}}\times_{m}F\mu_{1}^{G_{2}}\right)((x_{1},y_{1}),(x_{2},y_{2}))\right) + \\ & \sum_{x_{1}x_{2}\in E_{1},y_{1}y_{2}\in E_{2}} \left( \left(F\sigma_{1}^{G_{1}}(x_{1})\vee F\sigma_{1}^{G_{2}}(y_{1})\right)\vee \left(F\sigma_{1}^{G_{1}}(x_{2})\vee F\sigma_{1}^{G_{2}}(y_{2})\right)\right) + \\ & \sum_{x_{1}=x_{2},y_{1}y_{2}\in E_{2}} \left( \left(F\sigma_{1}^{G_{1}}(x_{1})\vee F\sigma_{1}^{G_{2}}(y_{1})\right)\vee \left(F\sigma_{1}^{G_{1}}(x_{2})\vee F\sigma_{1}^{G_{2}}(y_{2})\right)\right). \end{aligned}$$

Since by the definition of project of two neutrosophic graphs,

$$\begin{split} Td_{1}^{\overline{G_{1}\times G_{2}}}(x_{1},y_{1}) &= \sum_{x_{1}=x_{2},y_{1}y_{2}\in E_{2}} T\sigma_{1}^{G_{2}}(y_{1}) - (T\mu_{1}^{G_{1}}\times_{m}T\mu_{1}^{G_{2}})((x_{1},y_{1}),(x_{2},y_{2})) + \\ &\sum_{y_{1}=y_{2},x_{1}x_{2}\in E_{1}} T\sigma_{1}^{G_{2}}(y_{1}) - (\overline{T\mu_{1}^{G_{1}}\times_{m}T\mu_{1}^{G_{2}}})((x_{1},y_{1}),(x_{2},y_{2})) + \sum_{x_{1},x_{2}\in E_{1}y_{1}y_{2}\in E_{2}} C_{3} = \\ &\sum_{x_{1}=x_{2},y_{1}y_{2}\in E_{2}} T\sigma_{1}^{G_{2}}(y_{1}) - T\sigma_{1}^{G_{2}}(x_{1}) \wedge T\mu_{1}^{G_{2}}(y_{1},y_{2}) + \\ &\sum_{y_{1}=y_{2},x_{1}x_{2}\in E_{1}} T\sigma_{1}^{G_{2}}(y_{1}) - T\mu_{1}^{G_{1}}(x_{1},x_{2}) \vee T\sigma_{1}^{G_{2}}(y_{1}) + \sum_{x_{1},x_{2}\in E_{1}y_{1}y_{2}\in E_{2}} C_{3} = \\ &\sum_{x_{1}=x_{2},y_{1}y_{2}\in E_{2}} C_{3} - T\sigma_{1}^{G_{1}}(x_{1}) + \sum_{y_{1}=y_{2},x_{1}x_{2}\in E_{1}} C_{3} - T\sigma_{1}^{G_{2}}(y_{1}) + C_{3}d_{G_{2}}^{*}(y_{1})d_{G_{1}}^{*}(x_{1}), \\ Td_{1}^{\overline{G_{1}\times G_{2}}}(x_{1},y_{1}) = (C_{3} - C_{1})d_{2} + C_{3}d_{1}d_{2}. \end{split}$$

Similarly we can find

$$Id_1^{G_1 \times_m G_2}(x_1, y_1) = (C_3 - C_1)d_2 + C_3d_1d_2,$$
  

$$Fd_1^{\overline{G_1 \times_m G_2}}(x_1, y_1) = (C_4 - C_1)d_2 + C_4d_1d_2.$$

Since  $G_1$  and  $G_2$  are two regular neutrosophic graphs,  $G_1^*$  and  $G_2^*$  represent complete graphs, with membership functions denoted by  $\mu_1^{G_1}$  and  $\mu_1^{G_2}$ . These membership functions are constants, namely,  $(C_1, C_2)$  for  $\mu_1^{G_1}$  and  $(C_3, C_4)$  for  $\mu_1^{G_2}$ .

$$\begin{aligned} \text{Case 2} \quad \text{If } T\sigma_{1}^{G_{1}}(x) \geqslant T\sigma_{1}^{G_{2}}(y), I\sigma_{1}^{G_{1}}(x) \geqslant I\sigma_{1}^{G_{2}}(y) \text{ and } F\sigma_{1}^{G_{1}}(x) \leqslant F\sigma_{1}^{G_{2}}(y) \text{ for all } x \in V_{1} \text{ and } y \in V_{2}, \\ Td_{1}^{\overline{G_{1} \times_{m} G_{2}}}(x_{1}, y_{1}) &= \sum_{x_{1} = x_{2}, y_{1} y_{2} \in E_{2}} \left( T\sigma_{1}^{G_{1}}(x_{1}) - \left\{ (T\sigma_{1}^{G_{1}}(x_{1}) \lor T\mu_{1}^{G_{2}}(y_{1}, y_{2}) \right\} \right) + \\ \sum_{y_{1} = y_{2}, x_{1} x_{2} \in E_{1}} \left( T\sigma_{1}^{G_{1}}(y_{1}) - \left\{ (T\sigma_{1}^{G_{1}}(y_{1}) \lor T\mu_{1}^{G_{2}}(x_{1}, x_{2}) \right\} \right) + \\ \sum_{x_{1} x_{2} \in E_{1}, y_{1} y_{2} \in E_{2}} T\sigma_{1}^{G_{1}}(x_{1}) = \\ \sum_{x_{1} x_{2} \in E_{1}, y_{1} y_{2} \in E_{2}} C_{1} - (T\sigma_{1}^{G_{1}}(x_{1})) + \sum_{y_{1} = y_{2}, x_{1} x_{2} \in E_{1}} C_{1} - (T\sigma_{1}^{G_{1}}(y_{1})) + \\ C_{1}d_{G_{2}}^{*}(y_{1})d_{G_{2}}^{*}(x_{1}), \\ Td_{1}^{\overline{G_{1} \times_{m} G_{2}}}(x_{1}, y_{1}) = (C_{1} - C_{3})d_{2} + C_{3}d_{1}d_{2}. \end{aligned}$$

Similarly we can find

$$Id_1^{\overline{G_1 \times_m G_2}}(x_1, y_1) = (C_1 - C_3)d_2 + C_3d_1d_2,$$
  

$$Fd_1^{\overline{G_1 \times_m G_2}}(x_1, y_1) = (C_4 - C_2)d_1 + C_4d_1d_2.$$

Because of this, normal neutrosophic graphs have a regular complement of their maximum product.

**Theorem 3** Let  $G_1$  and  $G_2$  be a pair of regular neutrosophic graphs derived from the underlying crip graph  $G_1^*$  and  $G_2^*$ , respectively. The vertex sets and edges sets of  $G_1$  and  $G_2$  are complete graphs, and the regular netrosphic graphs are associated with them. If  $T\sigma_1^{G_1} > T\mu_1^{G_2}, I\sigma_2^{G_1} > I\mu_2^{G_2}, F\sigma_3^{G_1} < T\mu_3^{G_2}$ ,  $T\sigma_1^{G_2} > T\mu_1^{G_1}, I\sigma_2^{G_2} > I\mu_2^{G_1}, F\sigma_3^{G_2} < F\mu_3^{G_1}, T\sigma_1^{G_1} > T\mu_1^{G_1}, I\sigma_2^{G_1} > I\mu_2^{G_1}, F\sigma_3^{G_1} < F\mu_3^{G_1}$  and  $T\sigma_1^{G_2} < T\mu_1^{G_2}, I\sigma_2^{G_2} < I\mu_2^{G_2}, F\sigma_3^{G_2} > F\mu_3^{G_2}$ , a complement graph is a regular neutrosophic graph when it is the maximum product of two regular netrosophic graphs.

*Proof* The underlying crisp graphs  $G_1^*$  and  $G_2^*$  are regular graphs, with every vertex in  $V_1$  and  $V_2$  having degrees  $g_1$  and  $g_2$ , respectively. Given that  $T\sigma^{G_1}, I\sigma^{G_1}, F\sigma^{G_1}, T\mu^{G_2}, I\mu^{G_2}$ , and  $F\mu^{G_2}$  are constants, say  $T\sigma_1^{G_1}(x) = C_1, I\sigma_2^{G_1}(x) = C_2, F\sigma_3^{G_1}(x) = C_3 \ \forall x \in V_1, T\sigma_1^{G_2}(x) = C_4, I\sigma_2^{G_2}(x) = C_5, \quad F\sigma_3^{G_2}(x) = C_6 \ \forall y \in V_2, T\mu_1^{G_1}(x_1, y_1) = e_1, I\mu_2^{e_1}(x_1, y_1) = e_2, F\mu_3^{G_1}(x_1, y_1) = e_3, \quad T\mu_1^{G_2}(x_1y_1) = e_4, I\mu_2^{G_2}(x_1, y_1) = e_5, F\mu_3^{G_2}(x_1, y_1) = e_6, and <math>T\sigma_1^{G_1} > T\mu_1^{G_2}, I\sigma_2^{G_1} > I\mu_2^{G_2}, F\sigma_3^{G_1} < F\mu_3^{G_2}; T\sigma_1^{G_2} > T\mu_1^{G_1}, I\sigma_2^{G_2} > I\mu_2^{G_1}, F\sigma_3^{G_2} < F\mu_3^{G_1}.$ 

Consider  $(x_1, y_2) \varepsilon (T\sigma_1^{G_1} \times T\sigma_1^{G_2})$ :

**Case 1** If  $T\sigma_1^{G_1}(x) \leq T\sigma_1^{G_2}(y), I\sigma_1^{G_1}(x) \leq I\sigma_1^{G_2}(y)$  and  $F\sigma_1^{G_2}(x) \geq F\sigma_3^{G_2}(y), \forall x \in V_1$  and  $y \in V_2$ ,  $Td_1^{\overline{G_1 \times G_2}}(x_1, y_1) = \sum_{i=1}^{n} ((T\sigma_1^{G_1}(x_1) \vee T\sigma_1^{G_2}(y_1)) \wedge (T\sigma_1^{G_1}(x_2) \vee T\sigma_1^{G_2}(y_2)) -$ 

$$\begin{split} & (1,0,1) \quad \sum_{x_1=x_2,y_1y_2 \in E_2} ((1,1,0,1) - (1,0,1) - (1,1,0,1) - (1,1,0) - (1,1,0,1) - (1,0,1)) \\ & (T\mu_1^{G_1} \times_m T\mu_1^{G_2})((x_1,y_1),(x_2,y_2))) + \\ & \sum_{y_1=y_2,x_1x_2 \in E_1,y_1y_2 \in E_2} ((T\sigma_1^{G_1}(x_1) \vee T\sigma_1^{G_2}(y_1)) \wedge (T\sigma_1^{G_1}(x_2) \vee T\sigma_1^{G_2})(y_2) + \\ & \sum_{x_1,x_2 \notin E_1,y_1y_2 \notin E_2} ((T\sigma_1^{G_1}(x_1) \vee T\sigma_1^{G_2}(y_1)) \wedge (T\sigma_1^{G_1}(x_2) \vee T\sigma_1^{G_2})(y_2) + \\ & \sum_{x_1,x_2 \notin E_1,y_1y_2 \notin E_2} ((T\sigma_1^{G_1}(x_1) \vee T\sigma_1^{G_2}(y_1)) \wedge (T\sigma_1^{G_1}(x_2) \vee T\sigma_1^{G_2})(y_2) + \\ & \sum_{x_1,x_2 \notin E_1,y_1y_2 \notin E_2} ((T\sigma_1^{G_1}(x_1) \vee T\sigma_1^{G_2}(y_1)) \wedge (T\sigma_1^{G_1}(x_2) \vee T\sigma_1^{G_2})(y_2) + \\ & \sum_{x_1,x_2 \in E_1,y_1y_2 \notin E_2} ((T\sigma_1^{G_1}(x_1) \vee T\sigma_1^{G_2}(y_1)) \wedge (T\sigma_1^{G_1}(x_2) \vee T\sigma_1^{G_2})(y_2) + \\ & \sum_{x_1,x_2 \in E_1,y_1y_2 \notin E_2} T\sigma_1^{G_2}(y_1) - \{T\sigma_1^{G_1}(x_1) \vee T\sigma_1^{G_2}(y_1,y_2)\} + \\ & \sum_{x_1=x_2,y_1y_2 \notin E_2} T\sigma_1^{G_2}(y_1) - \{T\sigma_1^{G_2}(x_2) \vee T\sigma_1^{G_2}(x_1,x_2)\} + \\ & \sum_{x_1x_2 \in E_1,y_1y_2 \notin E_2} C_4 + \sum_{x_1x_2 \notin E_1,y_1y_2 \notin E_2} C_4 + \sum_{x_1x_2 \notin E_1,y_1y_2 \notin E_2} C_4 + \\ & \sum_{x_1x_2 \in E_1,y_1y_2 \notin E_2} C_4 + (C_4 - C_1)g_1 + C_4 d_{G_1^*}(x_1) + |\overline{E_2}| + C_3 |\overline{E_1}|d_{G_2^*}(y_1) + C_4 |\overline{E_1}||\overline{E_2}| + \\ & C_4 d_{G_2^*}(x_2) d_{G_1^*}(y_4). \end{split}$$

where  $|\overline{E_1}|$  and  $|\overline{E_2}|$  are the degrees of vertex of complement graphs  $G_1^*$  and  $G_2^*$ .  $Td_{1}^{\overline{G_{1}\times G_{2}}}(x_{1}, y_{1}) = (C_{4} - C_{1})g_{2} + C_{4}g_{1}|\overline{E_{2}}| + C_{4}g_{2}|\overline{E_{1}}| + C_{4}|\overline{E_{1}}||\overline{E_{2}}| + C_{1}g_{1}g_{2}.$  $Id_{\overline{G_1 \times G_2}}^{\overline{G_1 \times G_2}}(x_1, y_1) = (C_4 - C_1)g_2 + C_4g_1|\overline{E_2}| + C_4g_2|\overline{E_1}| + C_4|\overline{E_1}||\overline{E_2}| + C_1g_1g_2.$  $Fd_{3}^{\overline{G_{1} \times G_{2}}}(x_{1}, y_{1}) = (C_{5} - C_{2})g_{2} + C_{5}g_{1}|\overline{E_{2}}| + C_{5}g_{2}|\overline{E_{1}}| + C_{5}|\overline{E_{1}}||\overline{E_{2}}| + C_{2}g_{1}g_{2}|$ For all vertices, this is accurately  $\overline{V_1 \times_m V_2}$ . **Case 2** If  $T\sigma_1^{G_2}(x) \leq T\sigma_1^{G_1}(y), I\sigma_1^{G_2}(x) \leq I\sigma_1^{G_1}(y)$  and  $F\sigma_1^{G_2}(x) \geq F\sigma_3^{G_1}(y), \forall x \in V_1$  and  $y \in V_2$ ,  $Td_{1}^{\overline{G_{1}\times G_{2}}}(x_{1}, y_{1}) = \sum_{x_{1}=x_{2}, y_{1}y_{2} \in E_{2}} \left( (T\sigma_{1}^{G_{1}}(x_{1}) \lor T\sigma_{1}^{G_{2}}(y_{1})) \land (T\sigma_{1}^{G_{1}}(x_{2}) \lor T\sigma_{1}^{G_{2}}(y_{2})) - \right)$  $(T\mu_1^{G_1} \times_m T\mu_1^{G_2})((x_1, y_1), (x_2, y_2))) +$  $\sum_{y_1=y_2,x_1x_2\in E_1} \left( (T\sigma_1^{G_1}(x_1) \vee T\sigma_1^{G_2}(y_1)) \wedge (T\sigma_1^{G_1}(x_2) \vee T\sigma_1^{G_2}(y_2)) - \right)$  $(T\mu_1^{G_1} \times_m T\mu_1^{G_2})((x_1, y_1), (x_2, y_2))) +$  $\sum_{x_1x_2 \in E_1, y_1y_2 \notin E_2} \left( (T\sigma_1^{G_1}(x_1) \lor T\sigma_1^{G_2}(y_1)) \land (T\sigma_1^{G_1}(x_2) \lor T\sigma_1^{G_2}(y_2)) + \right)$  $\sum_{x_1x_2\notin E_1, y_1y_2\notin E_2} \left( (T\sigma_1^{G_1}(x_1) \vee T\sigma_1^{G_2}(y_1)) \wedge (T\sigma_1^{G_1}(x_2) \vee T\sigma_1^{G_2}(y_2)) - \right)$  $(T\mu_1^{G_1} \times_m T\mu_1^{G_2})((x_1, y_1), (x_2, y_2))) +$  $\sum_{x_1x_2\notin E_1, y_1y_2\notin E_2} \left( (T\sigma_1^{G_1}(x_1) \vee T\sigma_1^{G_2}(y_1)) \wedge (T\sigma_1^{G_1}(x_2) \vee T\sigma_1^{G_2}(y_2)) + \right)$  $\sum_{x_1x_2 \in E_1, y_1y_2 \in E_2} \left( (T\sigma_1^{G_1}(x_1) \lor T\sigma_1^{G_2}(y_1)) \land (T\sigma_1^{G_1}(x_2) \lor T\sigma_1^{G_2}(y_2)) \right),$  $Td_{1}^{\overline{G_{1}\times G_{2}}}(x_{1},y_{1}) = \sum_{x_{1}=x_{2},y_{1}y_{2}\in E_{2}}(T\sigma_{1}^{G_{1}}(x_{2}) - T\sigma_{1}^{G_{1}}(x_{1})) + \sum_{y_{1}=y_{2},x_{1}x_{2}\in E_{2}}(T\sigma_{1}^{G_{1}}(x_{2}) - T\sigma_{1}^{G_{2}}(y_{1})) + C(T\sigma_{1}^{G_{1}}(x_{2}) - T\sigma_{1}^{G_{2}}(x_{2})) + C(T\sigma_{1}^{G_{1}}(x_{2}) - T\sigma_{1}^{G_{2}}(x_{2})) + C(T\sigma_{1}^{G_{1}}(x_{2}) - T\sigma_{1}^{G_{2}}(x_{2})) + C(T\sigma_{1}^{G_{1}}(x_{2}) - T\sigma_{1}^{G_{2}}(x_{2})) + C(T\sigma_{1}^{G_{2}}(x_{2}) - C(T\sigma_{1}^{G_{2}}(x_{2})) + C(T\sigma_{1}^{G_{2}}(x$  $\sum_{x_1x_2 \in E_1, y_1y_2 \notin E_2} T\sigma_1^{G_2}(x_1) + \sum_{x_1x_2 \notin E_1, y_1y_2 \notin E_2} T\sigma_1^{G_2}(x_1) +$  $\sum_{x_1x_2 \notin E_1, y_1y_2 \notin E_2} T\sigma_1^{G_2}(x_1) + \sum_{x_1x_2 \in E_1, y_1y_2 \in E_2} T\sigma_1^{G_2}(x_1),$ 

where  $E_1$  and  $E_2$  are the degrees of the vertices of complement graphs  $G_1^*$  and  $G_2^*$ , respectively.

$$Td_{1}^{\overline{G_{1}}\times\overline{G_{2}}}(x_{1},y_{1}) = (C_{1}-C_{1})g_{1} + (C_{1}-C_{4})g_{2} + C_{1}g_{1}|\overline{E_{2}}| + C_{1}g_{2}|\overline{E_{1}}| + C_{1}|\overline{E_{1}}||\overline{E_{2}}| + C_{1}g_{1}g_{2} = (C_{1}-C_{3})g_{2} + C_{1}g_{1}|\overline{E_{2}}| + C_{1}g_{2}|\overline{E_{1}}| + C_{1}|\overline{E_{1}}||\overline{E_{2}}| + C_{1}g_{1}g_{2}.$$

Similarly,

$$Id_{2}^{\overline{G_{1} \times G_{2}}}(x_{1}, y_{1}) = (C_{1} - C_{3})g_{2} + C_{1}g_{1}|\overline{E_{2}}| + C_{1}g_{2}|\overline{E_{1}}| + C_{1}|\overline{E_{1}}||\overline{E_{2}}| + C_{1}g_{1}g_{2},$$
  

$$Fd_{3}^{\overline{G_{1} \times G_{2}}}(x_{1}, y_{1}) = (C_{2} - C_{4})g_{2} + C_{2}g_{1}|\overline{E_{2}}| + C_{2}g_{2}|\overline{E_{1}}| + C_{2}|\overline{E_{1}}||\overline{E_{2}}| + C_{2}g_{1}g_{2}.$$

For all vertices, this is accurately  $\overline{V_1 \times_m V_2}$ . As a result, the complement of the modular product of two regular neutrosophic graphs is also regular.

# 4 Application 1

We want to identify the internet streaming service that particular demographic favors based on their usage

trends. We will examine the streaming behaviors of our eight users,  $U = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8\}$ (depicted in Fig. 4) utilizing a series of symptoms or indicators,  $I = \{Video and picture quality, Content variety, User interface, Price, Device compatibility\}.$ 

Every individual may have distinct encounters and inclinations, and our goal is to ascertain the fundamental factor that sets apart their choice of streaming service from a selection of well-known platforms,  $P = \{Netflix, Amazon Prime Video, Hulu, Disney+, HBO Max \}$ (depicted in Fig. 5).

By utilizing the neutrosophic normalized Hamming distance, we can evaluate the resemblance between every user's inclinations and the accessible streaming platforms. The metric with the minimum distance for each user can subsequently be regarded as the fundamental indication or preference that has the greatest impact on their selection of streaming service.

For example, suppose user  $u_1$  encounters the subsequent indications: Excellent video quality, extensive range of content, easy-to-use interface, cost-effective pricing, and support for numerous devices. By contrasting their indications with the characteristics of various streaming services, we can establish that the primary factor for  $u_1$  is "Content variety" as it closely corresponds with the offerings of platforms such as Netflix or Amazon Prime Video. Likewise, in Tables 3–5 and Fig. 6, we can examine the signs of other users and identify the fundamental sign that most accurately reflects their streaming service inclination. This method enables us to make inferences about the type of service each user favors by considering their symptom resemblances to the accessible choices. For this purpose we need two kinds of observations:

1. The multiple indicators found in each streaming.

2. The type of indications found for each stream in a typical given circumcision. Both of these facts are noted in a neutrosophic set, which includes descriptions of the membership, indeterminacy, and non-membership functions  $\mu$ ,  $\sigma$ , and  $\delta$ , among other things.

To find the core attribute by utilizing neutrosophic normalized Hamming distance formula (Table 5) for



Fig. 4 Neuturosophic cyclic graph  $C_8$ .



Fig. 5 Neuturosophic cyclic graph C<sub>5</sub>.

			-	-	
User	Video quality	Content variety	User interface	Price	Device compatibility
$u_1$	(0.5, 0.4, 0.4)	(0.4, 0.3, 0.3)	(0.4, 0.5, 0.3)	(0.5, 0.5, 0.5)	(0.4, 0.5, 0.6)
$u_2$	(0.2, 0.7, 0.3)	(0.5, 0.7, 0.4)	(0.4, 0.6, 0.5)	(0.4, 0.6, 0.5)	(0.4, 0.5, 0.6)
$u_3$	(0.5, 0.4, 0.2)	(0.7, 0.3, 0.5)	(0.5, 0.6, 0.6)	(0.4, 0.6, 0.5)	(0.3, 0.5, 0.4)
$u_4$	(0.6, 0.5, 0.3)	(0.5, 0.5, 0.5)	(0.3, 0.5, 0.6)	(0.6, 0.3, 0.5)	(0.5, 0.6, 0.8)
$u_5$	(0.3, 0.5, 0.7)	(0.3, 0.7, 0.1)	(0.8, 0.5, 0.2)	(0.3, 0.5, 0.8)	(0.4, 0.3, 0.8)
$u_6$	(0.4, 0.5, 0.6)	(0.7, 0.5, 0.4)	(0.7, 0.4, 0.2)	(0.6, 0.5, 0.8)	(0.7, 0.6, 0.5)
$u_7$	(0.3, 0.6, 0.3)	(0.5, 0.6, 0.5)	(0.2, 0.8, 0.3)	(0.5, 0.5, 0.6)	(0.7, 0.6, 0.4)
$u_8$	(0.2, 0.5, 0.6)	(0.3, 0.6, 0.4)	(0.2, 0.5, 0.6)	(0.2, 0.6, 0.5)	(0.6, 0.3, 0.4)

Table 3 User encounters the subsequent indications.

Table 4	<b>Contrasting indications</b>	s with the characteristic	of various streaming service
	0		0

Characteristic	Netflix	HBO Max	Hulu	Disney+	Amazon Prime Video
Video quality	(0.7, 0.4, 0.4)	(0.3, 0.6, 0.4)	(0.3, 0.7, 0.5)	(0.2, 0.6, 0.7)	(0.2, 0.7, 0.3)
Content Variety	(0.5, 0.6, 0.4)	(0.3, 0.7, 0.5)	(0.3, 0.6, 0.5)	(0.3, 0.5, 0.7)	(0.2, 0.7, 0.5)
User interface	(0.2, 0.7, 0.4)	(0.1, 0.7, 0.5)	(0.4, 0.6, 0.7)	(0.8, 0.4, 0.4)	(0.2, 0.8, 0.2)
Price	(0.4, 0.4, 0.5)	(0.5, 0.2, 0.6)	(0.4, 0.6, 0.7)	(0. 2, 0.6, 0.5)	(0.5,0.6, 0.5)
Device compatibility	(0.2, 0.6, 0.5)	(0.2, 0.7, 0.4)	(0.0, 0.7, 0.5)	(0. 2, 0.6, 0.5)	(0.6, 0.3, 0.3)

#### Table 5 Shortest normalized Hamming distance.

User	Netflix	HBO Max	Hulu	Disney+	Amazon Prime Video
$u_1$	0.2000	0.2600	0.2800	0.3200	0.2800
$u_2$	0.2600	0.2800	0.2200	0.3000	0.2400
$u_3$	0.1600	0.1800	0.2200	0.3400	0.3000
$\mathcal{U}_4$	0.1800	0.2400	0.2400	0.3600	0.3200
$u_5$	0.3800	0.4200	0.3200	0.3000	0.4400
$u_6$	0.2000	0.2800	0.2800	0.3400	0.2800
$u_7$	0.2400	0.2600	0.2600	0.4000	0.1800
$u_8$	0.3000	0.3000	0.2200	0.2800	0.2200

every indicators of i -th stream from k -th platform is

$$LNH(S(P_i), d_k) = \frac{1}{2} \sum_{j=1}^{n} \max\{|\mu_j(p_i) - \mu_j(d_k)|, |\mu_j(p_i) - \mu_j(d_k)|, |\mu_j(p_i) - \mu_j(d_k)|\}$$

# 5 Application 2

Let's look at a situation where we wish to determine, based on the ordering patterns of a group of clients, which meal delivery service they prefer. We have an array of indicators,  $I = \{\text{Delivery speed}, \text{Menu variety}, \text{User-friendly app, Price competitiveness, Customer support satisfaction}\}$ , and a collection of customers,  $C = \{\text{customer 1, customer 2, customer 3, ..., customer N}\}$ . Our objective is to identify the critical element influencing each customer's choice of meal delivery service, taking into account their distinct experiences and preferences. Here is a list of well-known platforms:  $P = \{\text{Instacart, Grubhub, Postmates, DoorDash, Uber Eats}\}$ . A metric such as the food delivery normalized satisfaction score allows us to evaluate the degree to which the features provided by various food delivery services and the preferences of individual customers are comparable. When choosing a meal delivery platform, customers' primary preference or



Fig. 6 Neuturosophic graph  $G_1 \times_m G_2$ 

underlying factor can be determined by looking at the measure with the lowest score for each individual consumer. Let us take an example where customer 1 is looking for reasonable pricing, fast delivery, a wide selection, an easy-to-use app, and excellent customer service. These preferences closely match the features offered by several meal delivery services, therefore we can determine that the most important element for customer1 is "Menu variety", given how well it matches the offerings of Uber Eats and other similar platforms. In a comparable manner, we can examine other consumers' preferences (Table of customer preferences) to determine which important aspect most accurately captures their propensity to use a specific meal delivery service. Based on how well a client's tastes match the options, we can use this method to draw conclusions about the kind of food delivery service that each individual consumer prefers.

To find the core attribute by utilizing neutrosophic normalized Euclidean distance formula (Table 6) for every indicators of i-th stream from k-th platform is

$$NED(A,B) = \sqrt{\frac{1}{3n}\sum_{i=1}^{n}(((T_{N_{1}}(x_{i}) - T_{N_{2}}(x_{i}))^{2} + ((I_{N_{1}}(x_{i}) - I_{N_{2}}(x_{i}))^{2} + (F_{N_{1}}(x_{i}) - F_{N_{2}}(x_{i}))^{2})}.$$

User	Netflix	HBO Max	Hulu	Disney+	Amazon Prime Video
$u_1$	0.1751	0.2129	0.219	0.2265	0.2236
$u_2$	0.1861	0.1788	0.1366	0.2018	0.2265
<i>u</i> <sub>3</sub>	0.1693	0.238	0.2113	0.2394	0.2633
$u_4$	0.157	0.1949	0.2756	0.2756	0.2633
$u_5$	0.2792	0.3065	0.272	0.2309	0.2898
$u_6$	0.2755	0.2851	0.2847	0.2529	0.2744
$u_7$	0.1861	0.1788	0.2422	0.284	0.1841
$u_8$	0.2542	0.2408	0.2265	0.2581	0.2366

Table 6 Shortest normalized Euclidean distance.

# 6 Result Comparison

This analysis compares the streaming preferences of eight users  $(u_1 \text{ to } u_8)$  across five major platforms: Netflix, HBO Max, Hulu, Disney+, and Amazon Prime Video. Normalized Hamming distance formula and normalized Euclidean distance formula methods contribute valuable insights, but the choice of the "best" method depends on the specific analysis goals. If a quick overview of individual preferences is needed, normalized Hamming distance formula method is effective, and we given graphical representation in Figs. 7 and 8.

### 7 Result Analysis

The streaming platforms that user's  $u_1, u_2, u_3, u_4, u_6$ , and  $u_8$  prefer vary, and no single platform predominates in their usage. Amazon Prime Video (44%) and Netflix (38%) are the platforms of choice for user  $u_5$ , as evidenced by their clear preferences. Users  $u_3$  and  $u_7$  assign the largest percentages of their streaming time to Hulu (24.22% and 28.4%, respectively), indicating their preference for this platform. Users  $u_1$  and  $u_3$  strongly prefer Disney+, with  $u_3$  dedicating the largest portion of their streaming time to this platform (23.94%). When compared to other users, user  $u_5$  has a noticeably higher preference for HBO Max (30.65%). Amazon Prime Video is the most favored platform overall, as indicated by the overwhelming preference of  $u_5$  among users. By capturing the wide range of individual preferences, the  $LNH(S(P_i),d_k)$ method offers insights into the distribution of streaming time. It is clear from comparing the two tables that







Fig. 8 Neutrosophic graphs of normalized Euclidean distance.

user  $u_5$  consistently prefers Amazon Prime Video in both analyses. The information emphasizes how crucial it is to comprehend unique user preferences in order to customize content and enhance the streaming platform user experience. The analysis emphasizes how platforms must plan their content offerings to take into account the variety of user preferences.

# 8 Conclusion

Graph theory is an useful tool for resolving networking issues in a variety of domains, including transportation and signal processing. It frequently deals with the issue of determining the shortest path within a network. Neutrosophic graph models have gained traction in real-world scenarios where information is not clear. These models yield membership data that is true, ambiguous, and false. This paper presents the notion of the maximum product of the complement of two neutrosophic graphs, an efficient way to combine various structural models. It is useful to investigate the regularity in the complement of two neutrosophic graphs when developing dependable network and communication systems. Furthermore, neutrosophic graphs are useful for making decisions. For example, they can be used to compare internet streaming services and make well-informed choices.

Reference [13] introduces the n-SuperHyperGraph, the most general form of graph today. We are going to extend our work in:

- (1) Different type product of complement n-SuperHyperGraph;
- (2) Product of Neutrosophic graph and its applications on medical flied;
- (3) Product of Neutrosophic graph and its applications on textile industry;
- (4) Neutrosophic coloring graph and their applications.

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