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PAPER

Uplink Overloaded MIMO Spatial Multiplexing With Iterative Colored Noise Cancellation in Massive MIMO Systems

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SUMMARY This paper proposes overloaded MIMO spatial multiplexing for uplinks with the massive MIMO where the number of the transmit antennas is less than that of the receive antennas. The proposed uplink overloaded MIMO spatial multiplexing makes it possible to increase the uplink user throughput by raising the number of the signal streams to that of the receive antennas. Moreover, we propose a novel idea called as "iterative colored noise cancellation" to improve the transmission performance of the proposed overloaded MIMO spatial multiplexing. This paper analyses the transmission performance that the proposed spatial multiplexing is degraded by the colored noise. Iterative colored noise cancellation is proposed for mitigating the performance degradation. The performance of the proposed spatial multiplexing is evaluated in an overloaded MIMO system where 2 antennas and 4 antennas are placed on the transmitter and the receiver respectively. The proposed colored noise cancellation achieves a gain of about 10 dB. The proposed uplink overloaded spatial multiplexing attains a diversity order of about 6 even when the number of the spatially multiplexed signal streams is increased to 6.

key words: *Overloaded MIMO, spatial multiplexing, non-linear precoding, QR-decomposition, colored noise, massive MIMO.*

1. Introduction

Communication speed has been raised to about Gbps by using many cutting-edge techniques even in wireless communication systems. Among them, multiple-input-multiple-output (MIMO) spatial multiplexing has been playing a crucial role in enhancing communication speed [1]–[4]. For the enhancement, many MIMO techniques have been proposed such as space time codes, precoding techniques, iterative decoding, and so on [5]–[8]. To multiply the throughput enhancement, lots of antennas are installed on base stations in the fifth generation (5G) cellular system, which is called "Massive MIMO" [9]–[12]. Many techniques have been proposed for Massive MIMO [13]–[15]. While those techniques achieve superior transmission performance, The number of the spatially multiplexed signal streams is basically limited by the minimum number of transmit antennas and receive antennas in those techniques. For the user throughput enhancement, some techniques have been proposed to increase the number of the signal streams, such as non-orthogonal multiple access [16]–[21], faster-than-Nyquist (FTN) [22], and overloaded MIMO spatial multiplexing [23]. Especially, overloaded MIMO spatial multiplexing can increase the downlink throughput in massive MIMO systems by making use of the system configuration where lots of antennas are equipped on the base stations, because the overloaded MIMO spatial multiplexing can raise the number of spatially multiplexed signal streams to that of transmit antennas. The use of massive MIMO in the latest wireless systems such as the 5G cellular system has driven researchers to have an interest in overloaded MIMO spatial multiplexing. They have intensively considered non-linear receivers [24], [25], because the non-linear receivers achieve superior transmission performance even in overloaded MIMO systems. Since such non linear receives impose prohibitive high computational load on terminals, computational complexity reduction has also been considered [26]–[31]. On the other hand, uplink transmission speed is desired to raise in the beyond 5th generation cellular system (B5G) or the 6th generation cellular system (6G). Although multi-user MIMO (MU-MIMO) is useful in uplinks for increasing the network throughput, the MU-MIMO can't increase the user throughput. While overloaded MIMO spatial multiplexing techniques have been proposed, they are only able to increase the number of spatially multiplexed signal streams to that of transmit antennas at most, even when the number of transmit antennas is less than that of receive antennas. The number of signal streams is limited by that of transmit antennas as far as the conventional overloaded MIMO techniques are utilized.

This paper proposes uplink overloaded MIMO spatial multiplexing with iterative colored noise cancellation, which increases the user throughput through raising the spatially multiplexed signal streams in uplinks with the massive MIMO. The proposed overloaded spatial multiplexing achieves higher user throughput by making use of the system configuration in the uplinks with the massive MIMO where the number of transmit antennas is less than that of receive antennas. The proposed overloaded MIMO spatial multiplexing can raise the number of spatially multiplexed signal streams to that of receive antennas in spite of the number of transmit antennas. For such spatial multiplexing, we employ a unitary transformed non-linear precoding based on the minimum mean square error (MMSE). The unitary transform is obtained through the channel matrix triangulation based on the equal gain transform [32]. Moreover, we propose iterative colored noise cancellation that is employed at the receiver in order to improve the transmission performance.

Throughout the paper, j, $\mathfrak{R}[c]$ and $\mathfrak{I}[c]$ represent the imaginary unit, a real part and an imaginary part of a com-

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plex number c, respectively. Superscript T and H indicate transpose and Hermitian transpose of a matrix or a vector, respectively. $\mathbf{A}^{-1}$, tr[$\mathbf{A}$], and $\mathbf{A}_{m,n}$ indicate inversion, a trace, and an (m, n)-entry of a matrix $\mathbf{A}$. E $[c]$ and $\lfloor r \rfloor$ denote the ensemble average of c and the biggest integer less than r.

2. System Model

Many antennas are installed at a base station in systems with the massive MIMO for higher throughput. We investigate an uplink in such systems where the number of the receive antennas N_R is more than that of the transmit antennas N_T. The information bit stream is encoded and the encoder output bit stream is fed to a modulator via a bit interleaver. The modulator output signals are provided to a precoder that is proposed in the following section. The precoder output signal streams are fed to the N_T antennas for the signal transmission. Let $\mathbf{X} \in \mathbb{C}^{N_\mathrm{T}}$ and $\mathbf{W} \in \mathbb{C}^{N_\mathrm{T} \times N_\mathrm{S}}$ denote a transmission signal vector and a precoding weight where N_S indicates the number of the spatially multiplexed signal streams, the transmission signal vector $\mathbf{X}$ is written as,

$$\mathbf{X} = \mathbf{WS}. \tag{1}$$

In (1), $\mathbf{S} \in \mathbb{C}^{N_\mathrm{S} \times 1}$ represents an input signal vector for the precoder. In this paper, we apply a precoding based on the minimum mean square error (MMSE) as follows [7].

$$\mathbf{W} = g\mathbf{H}^\mathrm{H} \left(\mathbf{H}\mathbf{H}^\mathrm{H} + \rho\mathbf{I}\right)^{-1}, \tag{2}$$

where $g \in \mathbb{R}$, $\rho \in \mathbb{R}$, $\mathbf{I} \in \mathbb{C}^{N_\mathrm{R} \times N_\mathrm{R}}$ and $\mathbf{H} \in \mathbb{C}^{N_\mathrm{R} \times N_\mathrm{T}}$ indicate a normalization factor, a constant, the identity matrix, and a channel matrix between the transmitter and the receiver. The normalization factor and the constant are defined in the following section. If the MMSE precoding is applied, the number of the signal streams becomes identical to N_R, i.e, $N_\mathrm{S} = N_\mathrm{R}$, because of the size of the MMSE precoder weight defined in (2). When the transmission signals are emitted from the N_T antennas, the transmission signal vector is received at the receiver as,

$$\mathbf{Y} = \mathbf{HX} + \mathbf{N}. \tag{3}$$

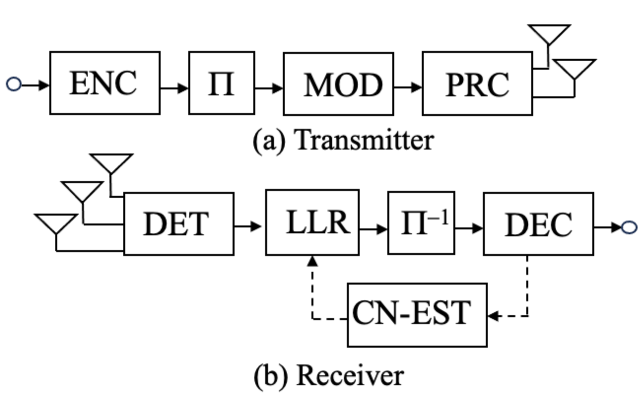

Fig. 1 System Model

In (3), $\mathbf{Y} \in \mathbb{C}^{N_\mathrm{R}}$ and $\mathbf{N} \in \mathbb{C}^{N_\mathrm{R}}$ denote a received signal vector and an additive white Gaussian noise (AWGN) vector. The received signal vector is provided to a detector and the detector output signals are fed to a decoder via a deinterleaver. The system model is illustrated in the figure 1. In the figure, LLR and CN-EST stand for the log likelihood ratio and a colored noise estimation respectively, which are explained in the following sections.

Since the number of the receive antennas N_R is more than that of the transmit antennas N_T, the number of the eigenvalues is reduced to N_T in the channel. Because the number of the streams N_S is equal to that of the receiver, the number of the streams exceeds that of the eigenvalues in the channel, which causes serious transmission performance degradation in the system with the conventional MMSE precoding.

We propose techniques to make the system achieve superior transmission performance in the following section.

3. Iterative Colored Noise Cancellation in Overloaded MIMO Systems

The system model defined in (3) is rewritten for overloaded MIMO spatial multiplexing as,

$$\mathbf{Y} = \begin{pmatrix}\mathbf{H} & \sqrt{\rho}\mathbf{I}\end{pmatrix} \begin{pmatrix}\mathbf{X} \\ \tilde{\mathbf{X}}\end{pmatrix} - \sqrt{\rho}\tilde{\mathbf{X}} + \mathbf{N}. \tag{4}$$

$\tilde{\mathbf{X}} \in \mathbb{C}^{N_\mathrm{R}}$ represents a virtual transmission signal vector. While the system model defined in (3) is an $N_\mathrm{R} \times N_\mathrm{T}$ MIMO system, the system model defined in (4) looks extended to an $N_\mathrm{R} \times (N_\mathrm{T} + N_\mathrm{R})$ MIMO system.

3.1 Non-Linear Precoding for Overloaded MIMO

If the system model is extended to that shown in (4), the MMSE precoding can be also extended as follows.

$$\overline{\mathbf{W}} = \begin{pmatrix}\mathbf{W} \\ \tilde{\mathbf{W}}\end{pmatrix} = g \begin{pmatrix}\mathbf{H}^\mathrm{H} \\ \sqrt{\rho}\mathbf{I}\end{pmatrix} \left(\mathbf{H}\mathbf{H}^\mathrm{H} + \rho\mathbf{I}\right)^{-1}, \tag{5}$$

where $\overline{\mathbf{W}} \in \mathbb{C}^{(N_\mathrm{R}+N_\mathrm{T}) \times N_\mathrm{R}}$ and $\tilde{\mathbf{W}} \in \mathbb{C}^{N_\mathrm{R} \times N_\mathrm{R}}$ represent an extended weight matrix and an additional weight matrix defined as,

$$\tilde{\mathbf{W}} = g\sqrt{\rho}\left(\mathbf{H}\mathbf{H}^\mathrm{H} + \rho\mathbf{I}\right)^{-1}. \tag{6}$$

To meet the MMSE criterion, the constant ρ should be set as $\rho = \frac{N_\mathrm{R}\sigma^2}{P_0}$ where $P_0 \in \mathbb{R}$ and $\sigma^2 \in \mathbb{R}$ represent the transmission signal power and the noise variance of the AWGN. When the conventional MMSE precoder is applied, the performance is greatly degraded even if the system model is extended. To alleviate the performance degradation, we introduce linear transformation as follows.

$$\begin{pmatrix}\mathbf{H}^\mathrm{H} \\ \sqrt{\rho}\mathbf{I}\end{pmatrix}\mathbf{T} = \mathbf{QDR} \tag{7}$$

$\mathbf{T} \in \mathbb{C}^{N_\mathrm{R} \times N_\mathrm{R}}$, $\mathbf{Q} \in \mathbb{C}^{(N_\mathrm{R}+N_\mathrm{T}) \times N_\mathrm{R}}$, $\mathbf{D} \in \mathbb{R}^{N_\mathrm{R}}$, and $\mathbf{R} \in \mathbb{C}^{N_\mathrm{R} \times N_\mathrm{R}}$

in (7) indicate a transform matrix, a semi-orthogonal matrix, i.e., $\mathbf{Q}^{\mathrm{H}}\mathbf{Q} = \mathbf{I}$, a real diagonal matrix, and an upper triangular matrix with 1 in all the diagonal positions, respectively. If the channel matrix is transformed with the transform matrix $\mathbf{T}$, the extended weight matrix $\overline{\mathbf{W}}$ can be also transformed as,

$$\overline{\mathbf{W}} = g\mathbf{Q}\mathbf{D}^{-1}\mathbf{R}^{-\mathrm{H}}\mathbf{T}^{\mathrm{H}} \tag{8}$$

If the above extended weight matrix is substituted for (4), the extended transmission signal vector $\overline{\mathbf{X}} \in \mathbb{C}^{(N_{\mathrm{T}}+N_{\mathrm{R}})\times 1}$ is written in the following equation.

$$\overline{\mathbf{X}} = \begin{pmatrix} \mathbf{X} \\ \tilde{\mathbf{X}} \end{pmatrix} = \overline{\mathbf{W}}\mathbf{S} = g\mathbf{Q}\mathbf{D}^{-1}\mathbf{R}^{-\mathrm{H}}\mathbf{T}^{\mathrm{H}}\mathbf{S} \tag{9}$$

The signal vector $\mathbf{S}$ is obtained through a linear transformation as $\mathbf{S} = \mathbf{T}^{-\mathrm{H}}\dot{\mathbf{A}}$, where the vector $\dot{\mathbf{A}} \in \mathbb{C}^{N_{\mathrm{R}}}$ is provided by a feedback filtering with non-linear signal processing, which is explained below. When the signal vector $\mathbf{S}$ is substituted for (9), the extended transmission signal vector can be rewritten as,

$$\overline{\mathbf{X}} = g\mathbf{Q}\mathbf{D}^{-1}\mathbf{R}^{-\mathrm{H}}\dot{\mathbf{A}} = g\mathbf{Q}\mathbf{D}^{-1}\mathbf{v}, \tag{10}$$

where $\mathbf{v}$ is defined as $\mathbf{v} = \mathbf{R}^{-\mathrm{H}}\dot{\mathbf{A}}$. If the semi-orthogonal matrix $\mathbf{Q}$ is further decomposed as $\mathbf{Q} = \begin{pmatrix}\mathbf{Q}_1^{\mathrm{T}} & \mathbf{Q}_2^{\mathrm{T}}\end{pmatrix}^{\mathrm{T}}$ where $\mathbf{Q}_1 \in \mathbb{C}^{N_{\mathrm{T}}\times N_{\mathrm{R}}}$ and $\mathbf{Q}_2 \in \mathbb{C}^{N_{\mathrm{R}}\times N_{\mathrm{R}}}$ represent an upper part and a lower part of the semi-orthogonal matrix $\mathbf{Q}$, the transmission signal vector $\mathbf{X}$ and the virtual transmission signal vector $\tilde{\mathbf{X}}$ can be expressed as,

$$\mathbf{X} = g\mathbf{Q}_1\mathbf{D}^{-1}\mathbf{v}, \quad \tilde{\mathbf{X}} = g\mathbf{Q}_2\mathbf{D}^{-1}\mathbf{v} \tag{11}$$

On the other hand, let the upper triangular matrix $\mathbf{R}$ be decomposed as $\mathbf{R}^{\mathrm{H}} = \mathbf{I} + \mathbf{B}$ where $\mathbf{B} \in \mathbb{C}^{N_{\mathrm{R}}\times N_{\mathrm{R}}}$ indicates a lower triangular matrix with "0" in all the diagonal positions, the vector $\mathbf{v}$ can be obtained with a non-linear filter used in the Tomlingson-Harashima precoding (THP) as follows.

$$\begin{aligned}\mathbf{v} &= \mathbf{R}^{-\mathrm{H}}\dot{\mathbf{A}} \\ &= \dot{\mathbf{A}} - \mathbf{B}\mathbf{v} = \mathbf{A} + \mathbf{K}M - \mathbf{B}\mathbf{v},\end{aligned} \tag{12}$$

where

$$\dot{\mathbf{A}} = \mathbf{A} + \mathbf{K}M. \tag{13}$$

In (12), $\mathbf{A} \in \mathbb{C}^{N_{\mathrm{R}}}$, $\mathbf{K} \in \mathbb{C}^{N_{\mathrm{R}}}$, and $M \in \mathbb{Z}$ indicate a modulation signal vector, a Gaussian integer vector, and a modulus used in the non-linear filter. As a result, the normalization factor g can be defined as,

$$g = \sqrt{\frac{P_0}{\mathrm{tr}\left[\mathbf{Q}_1^{\mathrm{H}}\mathbf{Q}_1\mathbf{D}\Psi_{\mathrm{v}}\mathbf{D}^{\mathrm{H}}\right]}}. \tag{14}$$

$\Psi_{\mathrm{v}} \in \mathbb{C}^{N_{\mathrm{R}}\times N_{\mathrm{R}}}$ in (14) represents a correlation matrix of the vector $\mathbf{v}$, i.e., $\Psi_{\mathrm{v}} = \mathrm{E}\left[\mathbf{v}\mathbf{v}^{\mathrm{H}}\right]$.

3.2 Iterative Colored Noise Cancellation

3.2.1 Signal Detection

Because the MMSE precoding is employed at the transmitter, signal detection at the receiver becomes simple even in the overloaded MIMO systems. However, since the modulation signal vector $\dot{\mathbf{A}}$ is transformed at the transmitter, the received signal has to be inverse-transformed to detect the modulation signals as follows.

$$\begin{aligned}\mathbf{Z} &= g^{-1}\mathbf{T}^{\mathrm{H}}\mathbf{Y} = \mathbf{A} + \mathbf{K}M + g^{-1}\mathbf{T}^{\mathrm{H}}\left(\mathbf{N} - \sqrt{\rho}\tilde{\mathbf{X}}\right) \\ &= \mathbf{A} + \overline{\mathbf{N}} + g^{-1}\mathbf{T}^{\mathrm{H}}\mathbf{N}\end{aligned} \tag{15}$$

$\mathbf{Z} \in \mathbb{C}^{N_{\mathrm{R}}}$ and $\overline{\mathbf{N}} \in \mathbb{C}^{N_{\mathrm{R}}}$ in (15) indicate a soft demodulated signal vector and a colored noise vector defined as $\overline{\mathbf{N}} = \mathbf{K}M - g^{-1}\sqrt{\rho}\mathbf{T}^{\mathrm{H}}\tilde{\mathbf{X}}$. If the colored noise is removed from the soft demodulated signal vector, the transmission performance will be improved. Actually, although the Gaussian integer multiples vector $\mathbf{K}M$ can be removed with the modulo function from the colored noise vector, the residual colored noise vector is fed to the decoder, which causes the performance degradation if the conventional receiver is straightforwardly applied. Since the residual colored noise vector consists of the virtual transmission signal vector $\tilde{\mathbf{X}}$, the colored noise $\overline{\mathbf{N}}$ can be possibly estimated if the modulation signal vector $\mathbf{A}$ is successfully estimated.

Next section proposes a technique to remove the colored noise. Hereafter, we assume that the quaternary phase shift keying (QPSK) is used as a modulation scheme to simplify the following explanation, though any modulation scheme can be applied to the proposed cancellation.

3.2.2 Colored Noise Cancellation

We apply a soft-input-soft-output decoder, because it can output a log likelihood ratio (LLR) of the coded bit. Let $b \in \mathbb{R}$ and $\mathrm{P}(b) \in \mathbb{R}$ represent a coded bit and probability of a coded bit b, the LLR of the coded bit $\gamma_{\mathrm{c}}(b) \in \mathbb{R}$ can be defined as $\gamma_{\mathrm{c}}(b) = \log\frac{\mathrm{P}(b=1)}{\mathrm{P}(b=0)} \in \mathbb{R}$. In a word, the decoder is able to estimates the probability of an encoded bit b being transmitted, $\mathrm{P}(b)$. For instance, let a modulation signal vector $\mathbf{A}_k \in \mathbb{C}^{N_{\mathrm{R}}}$ be defined as $\mathbf{A}_k = (a_k(1)\cdots a_k(N_{\mathrm{R}}))^{\mathrm{T}}$ where $k \in \mathbb{Z}$ and $a_k(i) \in \mathbb{C}$ indicate an index of the modulation signal vectors and ith entry of the vector $\mathbf{A}_k$, the decoder estimates a probability of the vector $\mathbf{A}_k$ being transmitted, i.e., $\prod_{i=1}^{N_{\mathrm{T}}}\mathrm{P}(\Re[a_k(i)])\mathrm{P}(\Im[a_k(i)])$. However, the correlation between the coded bits is neglected in the derivation of the probability. On the assumption of the modulation signal vector $\mathbf{A}_k$ being transmitted, a tentative virtual transmission signal vector $\tilde{\mathbf{X}}_k \in \mathbb{C}^{N_{\mathrm{R}}}$ can be defined as follows.

$$\mathbf{v}_k = \mathbf{R}^{-\mathrm{H}}\dot{\mathbf{A}}_k \tag{16}$$

$$\tilde{\mathbf{X}}_k = g\mathbf{Q}_2\mathbf{D}^{-1}\mathbf{v}_k \tag{17}$$

$\mathbf{v}_k \in \mathbb{C}^{N_{\mathrm{R}}}$ and $\dot{\mathbf{A}}_k \in \mathbb{C}^{N_{\mathrm{R}}}$ denote a feedback filter output vector and a Gaussian integer multiples added modulation signal vector when the vector $\mathbf{A}_k$ is assumed to be transmitted. If the tentative virtual transmission signal vector $\tilde{\mathbf{X}}_k$ is assumed, the colored noise vector can be removed in conjunction with the modulo function as follows.

$$\bar{\mathbf{Z}}_k = \mathrm{Mod}\left[\mathbf{Z}, M\right] + g^{-1}\sqrt{\rho}\mathbf{T}^{\mathrm{H}}\tilde{\mathbf{X}}_k$$

$$= \mathbf{A} + g^{-1}\mathbf{T}^{\mathrm{H}}\left(\mathbf{N} + \sqrt{\rho}\left(\tilde{\mathbf{X}}_k - \tilde{\mathbf{X}}\right)\right) \qquad (18)$$

Mod $[\mathbf{V}, M]$ and $\bar{\mathbf{Z}}_k \in \mathbb{C}^{N_\mathrm{R}}$ in (18) denote a modulo function where $\mathbf{V} \in \mathbb{C}^{N_\mathrm{R}}$ represents a vector † and a colored-noise-removed soft demodulation vector. As is known, on the other hand, a real part of a QPSK symbol conveys one bit as well as an imaginary part. When we assume that the modulation signal vector $\mathbf{A}_k$ is transmitted with the probability of $\prod_{i=1}^{N_\mathrm{T}} \mathrm{P}(\Re[a_k(i)])\mathrm{P}(\Im[a_k(i)])$, *a posteriori* bit LLR of the real part of the signal $a(m)$, $\gamma\left(\Re\left[a(m)\right]|\mathbf{Z}\right) \in \mathbb{R}$, can be estimated as follows.

$$\gamma\left(\Re\left[a(m)\right]|\mathbf{Z}\right) = \log\frac{\mathrm{P}\left(\Re\left[a(m)\right] = 1|\mathbf{Z}\right)}{\mathrm{P}\left(\Re\left[a(m)\right] = -1|\mathbf{Z}\right)} = \log\frac{\max\limits_{k \in B_{\Re[a(m)]=1}}\left[\mathrm{P}\left(\mathbf{Z}|\mathbf{A}_k\right)\mathrm{P}\left(\mathbf{A}_k\right)\right]}{\max\limits_{k \in B_{\Re[a(m)]=-1}}\left[\mathrm{P}\left(\mathbf{Z}|\mathbf{A}_k\right)\mathrm{P}\left(\mathbf{A}_k\right)\right]} \qquad (19)$$

In (19), $\mathrm{P}\left(a|b\right) \in \mathbb{R}$, $\gamma\left(b|\mathbf{Z}\right) \in \mathbb{R}$, and $B_{\Re[a(m)]=b}$ represent a conditional probability that an event a happens after an event b occurred, *a posteriori* LLR of a bit b, and a set of the indexes of the modulation signal vectors containing $\Re[a(m)] = b$. The *a posteriori* bit LLR is further manipulated in (20). After the bit LLR of $\Re[a(m)]$ is subtracted from the *a posteriori* bit LLR as shown in (21), its output $\gamma_\mathrm{e}\left(\Re[a(m)]\right) \in \mathbb{R}$ is obtained as,

$$\gamma_\mathrm{e}\left(\Re[a(m)]\right) = \gamma(\Re[a(m)]\,|\mathbf{Z}) - \gamma_\mathrm{c}(\Re[a(m)])\,. \quad (21)$$

$\gamma_\mathrm{e}\left(\Re[a(m)]\right) \in \mathbb{R}$ is named as an extrinsic LLR in this paper. If the extrinsic LLR is provided, the decoder outputs the bit LLRs of the coded bits again, which are regarded as updates of the bit LLR. The LLR is provided to (20) for updating the *a posteriori* bit LLR. In a word, the bit LLR and the *a posteriori* bit LLR are updated by turns, which is named "iterative colored noise cancellation" in this paper. Especially, the *a posteriori* bit LLR estimation based on (18) and (19) is named as Gaussian integer multiples removal with modulo functions (GMR).

3.2.3 Gaussian Integer Multiples Cancellation

Although the Gaussian integer multiples are removed by the modulo function in the GMR as shown in (18), the modulo function possibly fails in the successful removal of the Gaussian integer multiples due to the AWGN, which causes serious performance degradation. Besides, the Gaussian integer multiples can be estimated if the modulation signal vector is given, because the Gaussian integer multiples are a part of the feedback filter output signals. When the modulation signal vector $\mathbf{A}_k$ is given as described in the previous section, a tentative Gaussian integer multiples can be obtained as,

$$\mathbf{K}_k = \mathbf{R}^{\mathrm{H}}\mathbf{v}_k - \mathbf{A}_k\,. \qquad (22)$$

$\mathbf{K}_k \in \mathbb{C}^{N_\mathrm{R}}$ in (22) represents a tentative Gaussian integer multiple vector. If the tentative Gaussian integer multiples vector is assumed, the colored noise vector can be removed from the soft demodulated signal vector with the tentative Gaussian integer multiples vector and the tentative virtual transmission signal vector, as follows.

$$\begin{aligned}\bar{\mathbf{Z}}_k &= \mathbf{Z} + g^{-1}\sqrt{\rho}\mathbf{T}^{\mathrm{H}}\tilde{\mathbf{X}}_k - \mathbf{K}_k M \\ &= \mathbf{A} + g^{-1}\mathbf{T}^{\mathrm{H}}\left(\mathbf{N} + \sqrt{\rho}\left(\tilde{\mathbf{X}}_k - \tilde{\mathbf{X}}\right)\right) + \left(\mathbf{K} - \mathbf{K}_k\right)M \quad (23)\end{aligned}$$

If the colored-noise-removed soft demodulation vector $\bar{\mathbf{Z}}_k$ is fed to the *a posteriori* bit LLR estimation in (20), the *a posteriori* bit LLR could be estimated with higher accuracy. This *a posteriori* bit LLR estimation based on (19) and (23) is named as Gaussian integer multiples cancellation (GMC) ††.

3.3 Initial stage of Proposed Iterative Cancellation

The proposed iterative cancellation is initiated with providing the extrinsic LLR to the decoder. However, the probability $\mathrm{P}\left(a(m)\right)$ has not been obtained at the initial stage of the iteration. It is reasonable to set both $\mathrm{P}\left(\Re[a(m)] = +1\right)$ and $\mathrm{P}\left(\Im[a(m)] = -1\right)$ to $\frac{1}{2}$, because *a priori* information of the symbol $a(m)$ is not given. When all the probability is the same, the log likelihood ratios in the right hand side of (20) become all zero. The LLR estimation for the bit $\Re[a(m)]$ defined in (20) is reduced to the maximum likelihood estimation. Because a little performance gain can be only expected with the maximum likelihood estimation, we approximate the maximum likelihood estimation with the region detection for complexity reduction at the initial stage of the iteration only. In other words, the soft-demodulated signal vector $\mathbf{Z}$ is provided to the modulo function. The modulo function output vector Mod$[\mathbf{Z}, M]$ is fed to the region detection to get a tentative detection signal vector $\bar{\mathbf{A}}$. The tentative detection signal vector is used to get a colored-noise-removed soft demodulation vector $\bar{\mathbf{Z}}$ where the tentative detection signal vector $\bar{\mathbf{A}}$ is used as a substitute of a vector $\mathbf{A}_k$. The GMC and the GMR output the extrinsic LLR where the colored-noise-removed soft demodulation vector $\bar{\mathbf{Z}}$ is used as a substitute of $\mathbf{Z}_k$. In a word, because the modulation signal vectors are merged to the tentative detection signal vector, the maximization process in (19) and (20) is not performed at the initial stage of the iteration.

4. Simulation

The performance of the proposed uplink overloaded MIMO spatial multiplexing is evaluated by computer simulation. The modulation scheme is the quaternary phase shift keying

†Let $c \in \mathbb{C}$ and $r \in \mathbb{R}$ denote a complex number and a real number, the modulo function cmod $[c, M]$ can be defined as cmod $[c, M] = \mathrm{mod}(\Re[c], M) + \mathrm{j}\cdot\mathrm{mod}(\Im[c], M)$ where $\mathrm{mod}(r, M)$ indicates a function defined as $\mathrm{mod}(r, M) = r - M\lfloor\frac{r+\frac{M}{2}}{M}\rfloor$. Let a vector $\mathbf{V} \in \mathbb{R}^{N_\mathrm{R}}$ be defined as $\mathbf{V} = (v(1)\cdots v(N_\mathrm{R}))^{\mathrm{T}}$, Mod $[\mathbf{V}, M]$ can be defined as Mod $[\mathbf{V}, M] = (\mathrm{cmod}\,[v(1), M]\cdots\mathrm{cmod}\,[v(N_\mathrm{R}), M])^{\mathrm{T}}$.

††As is described above, the GMC requires the non-linear signal processing as well as the GMR. The computational complexity issue caused by the non-linear signal processing is one of our future works.

$$\begin{aligned}\gamma\left(\Re\left[a(m)\right]|\mathbf{Z}\right) =\ &\log\frac{\mathrm{P}\left(\Re\left[a(m)\right]=1|\mathbf{Z}\right)}{\mathrm{P}\left(\Re\left[a(m)\right]=-1|\mathbf{Z}\right)}\\ &+\max_{k\in B_{\Re[a(m)]=1}}\left[-\frac{1}{2\sigma^2}\left|\bar{\mathbf{Z}}_k-\mathbf{A}_k\right|^2+\sum_{n\neq m}^{N_\mathrm{T}}\log\frac{\mathrm{P}\left(\Re\left[a(n)\right]=\Re\left[a_k(n)\right]\right)}{\mathrm{P}\left(\Re\left[a(n)\right]=-1\right)}+\sum_{n=1}^{N_\mathrm{T}}\log\frac{\mathrm{P}\left(\Im\left[a(n)\right]=\Im\left[a_k(n)\right]\right)}{\mathrm{P}\left(\Im\left[a(n)\right]=-1\right)}\right]\\ &-\max_{k\in B_{\Re[a(m)]=-1}}\left[-\frac{1}{2\sigma^2}\left|\bar{\mathbf{Z}}_k-\mathbf{A}_k\right|^2+\sum_{n\neq m}^{N_\mathrm{T}}\log\frac{\mathrm{P}\left(\Re\left[a(n)\right]=\Re\left[a_k(n)\right]\right)}{\mathrm{P}\left(\Re\left[a(n)\right]=-1\right)}+\sum_{n=1}^{N_\mathrm{T}}\log\frac{\mathrm{P}\left(\Im\left[a(n)\right]=\Im\left[a_k(n)\right]\right)}{\mathrm{P}\left(\Im\left[a(n)\right]=-1\right)}\right]\end{aligned} \tag{20}$$

Table 1 Simulation parameters

Modulation	QPSK
Precoding	Equal gain transformed non-linear
Channel model	i.i.d. Rayleigh fading
$(N_\mathrm{T}, N_\mathrm{R})$	(2, 4), (2, 6), (2, 8)
Overloading Ratio	2.0, 3.0, 4.0
Channel estimation	Perfect
Error correction code	Convolutional code ($R=\frac{1}{2}$ $K=3$)
Decoder	Soft output Viterbi algorithm

(QPSK), and the rate half convolutional code with constraint length of 3 is used [33]. As is inferred in the derivation process described above, the transform matrix $\mathbf{T}$ is required to be invertible. The equal gain transform is one of linear transforms defined in (7). Since the equal gain transform is implemented with a unitary transform matrix [32], the equal gain transform can be applied for the linear transform in (7). The independent and identically distributed (i.i.d.) Rayleigh fading channel based on Jakes' model [34] is applied to the MIMO channels. While the number of the transmit antennas is fixed to 2, the number of the receive antennas is set to 4, 6, and 8 †. Since the number of the streams is equal to that of the receive antennas, the performance is evaluated in the system with the overloading ratio of 2, 3 and 4. Simulation parameters are listed in Table. 1.

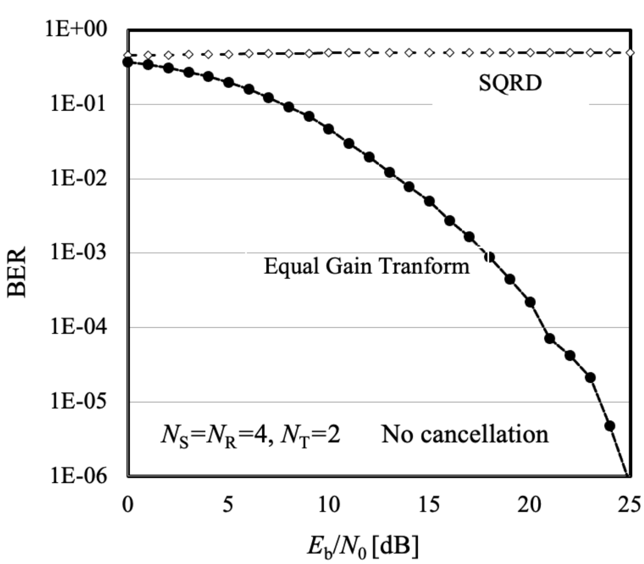

Fig. 2 BER Performance of Equal Gain Transform in Overloaded MIMO

4.1 Equal Gain Transform

Fig. 2 compares the proposed overloaded MIMO spatial multiplexing with the equal gain transform and that with the sorted QR decomposition (SQRD) [35], [36], which is one of representatives for the transform in (7) ††. The number of the spatially multiplexed signal streams is 4, whereas that of the transmit antennas is 2 in the figure. However, the iterative colored noise cancellation is not deployed in order to evaluate how the transmission performance is affected by the transform. In the performance evaluation, in fact, the modulo function output vector Mod $[\mathbf{Z}, M]$ is directly provided to the decoder as soft input signals, which is regarded as the demodulation in the conventional receiver. As is shown in the figure, the equal gain transform enables the proposed MIMO spatial multiplexing to achieve superior transmission performance. While the SQRD tries to arrange the diagonal elements of the matrix $\mathbf{D}$ in ascending order, the equal gain transform guarantees to equalize the diagonal elements, which makes difference in the performance. The detail is explained in Appendix A.

4.2 Colored Noise Cancellation

Fig. 3 compares the proposed colored noise cancellation with the GMR and that with the GMC. The number of the spatially multiplexed signal streams N_S is 4, whereas that of the transmit antennas N_R is also 2 in the figure. As is shown in the figure, the GMC achieves better BER performance that the GMR. Because the difference between the GMC and GMR is a way to estimate the Gaussian integer multiples, the estimation makes the difference in term of the BER performance. In the GMR, the Gaussian integer multiples are removed with the modulo function, which is independent

†If our proposed overloaded MIMO spatial multiplexing achieves superior transmission performance in the system with this parameter sets, we can conclude that our proposed spatial multiplexing is beneficial in uplinks of the massive MIMO systems where the number of receive antennas is much bigger than that of transmit antennas. Since our proposed spatial multiplexing comprises linear signal processing, our proposed spatial multiplexing can be applied to the massive MIMO systems with any antennas, for instance, the massive MIMO considered for the fifth generation cellular system.

††The permutation matrix plays a role of the transform matrix $\mathbf{T}$ in (7), when the SQRD is used. Since the permutation matrix is one of unitary matrices, the permutation matrix is invertible, which means that the SQRD can be applied to the proposed precoding.

of the *a posteriori* bit LLR estimation. On the other hand, since the Gaussian integer multiples vector $\mathbf{K}_k M$ is associated with the modulation signal vectors $\mathbf{A}_k$ in the GMC, the Gaussian integer multiples vector assists the *a posteriori* bit LLR estimation, which results in better estimation performance than the GMR.

Because the GMC is better than the GMR as shown in Fig. 3, the proposed overloaded MIMO spatial multiplexing with the colored noise cancellation based on the GMC is hereafter evaluated only.

4.3 Iterative Cancellation

Fig. 4 shows the BER performance of the proposed uplink overloaded MIMO spatial multiplexing with the iterative colored noise cancellation based on the GMC. The number of the spatially multiplexed signal streams N_S is 4, whereas that of the transmit antennas N_T is also 2. In the figure, the cancellation is iterated twice in the figure, which is referred as "2nd iteration". "1st iteration" indicates the performance of the proposed spatial multiplexing with the proposed cancellation at the initial stage described in Sec. 3.3. In addition, the performance without the colored noise cancellation is added as a reference. While the proposed overloaded MIMO spatial multiplexing with the cancellation at the initial stage attains a gain of about 1 dB only at the BER of 10^{-5}, the increase in the iteration enables the proposed spatial multiplexing to achieve a gain of more 10 dB at the BER of 10^{-5}.

Fig. 5 shows the BER performance with respect to the number of the iterations. The number of the transmit anten-

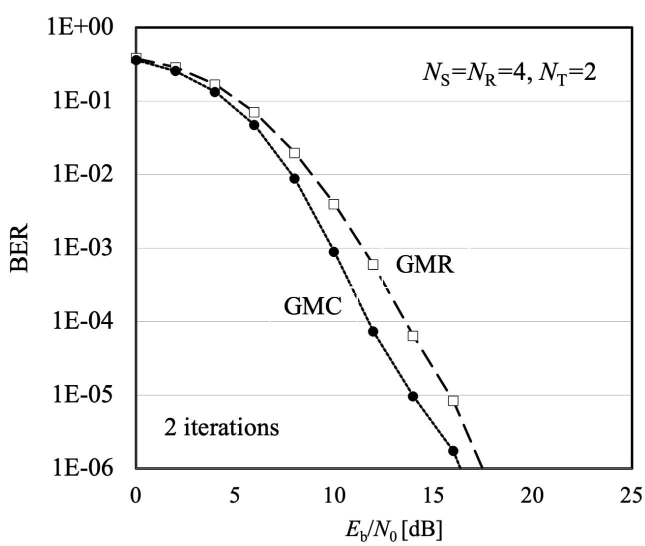

Fig. 3 Comparison of Proposed Cancellation Techniques

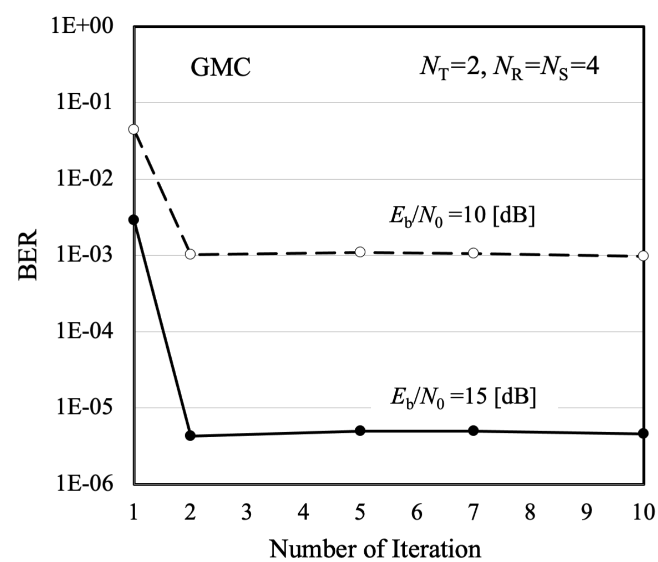

Fig. 5 BER Performance of GMC With Respect to The Number of Iterations

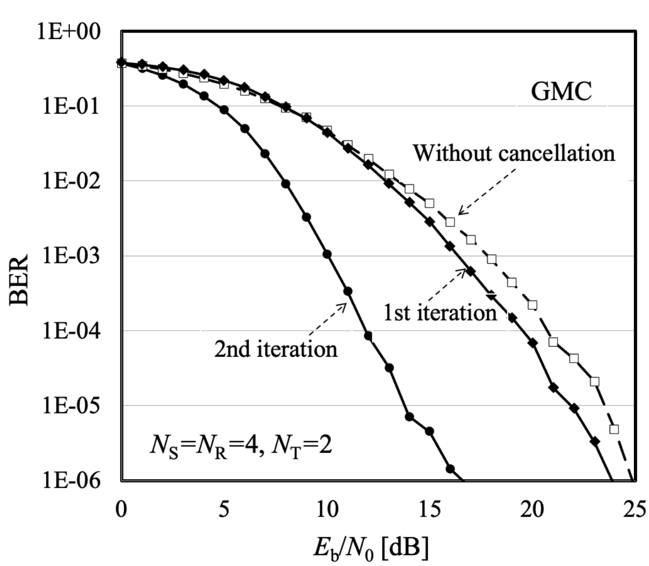

Fig. 4 BER Performance of Proposed Iterative Cancellation

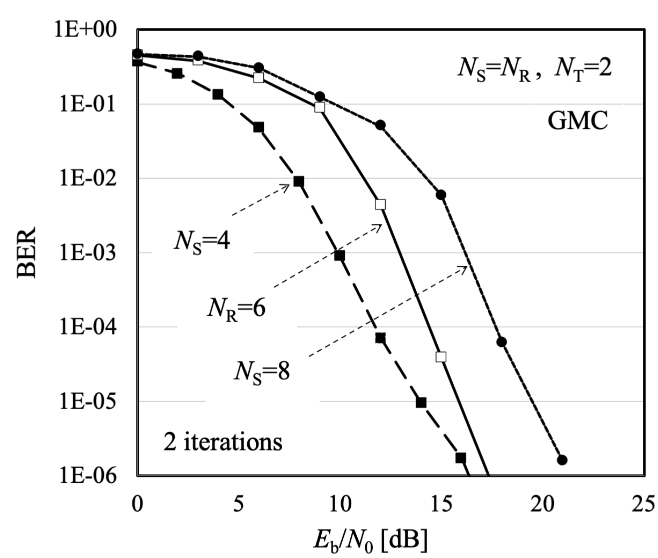

Fig. 6 BER Performance of GMC in Overloaded MIMO Channels

nas and the received antennas are the same to those in Fig. 4 as well as the number of the signal streams. Even if the E_b/N_0 is changed from 10 dB to 15 dB, the proposed spatial multiplexing achieves convergence within 2 iterations.

4.4 Overloading Ratio

Fig. 6 shows the BER performance where the number of the signal streams N_S is changed. Actually, the number of the signal streams is increased to 6 and 8, which corresponds to overloading ratio of 3 and 4, respectively. The performance is compared with that of the proposed spatial multiplexing with overloading ratio of 2. As is shown in the figure, the proposed uplink overloaded MIMO spatial multiplexing achieves a diversity order of about 6 as the number of the signal streams N_S is raised to 6, because the number of the receive antennas is increased as well. Even when the number of the signal streams is further increased to 8, i.e. the overloading ratio of 4, the proposed uplink overloaded MIMO spatial multiplexing still achieves a diversity order of 4.

5. Conclusion

This paper proposes uplink overloaded MIMO spatial multiplexing with iterative colored noise cancellation for the massive MIMO systems. The proposed overloaded spatial multiplexing achieves higher throughput by making use of the system configuration in the uplink with the massive MIMO where the number of transmit antennas is less than that of receive antennas. While the number of the spatially multiplexed signal streams is equal to that of the transmit antennas if conventional MIMO techniques are applied, the proposed overloaded MIMO spatial multiplexing can raise the number of the spatially multiplexed signal streams to that of the receive antennas in spite of the number of the transmit antennas in the uplink. In a word, the proposed MIMO spatial multiplexing achieves higher throughput than conventional MIMO spatial multiplexing techniques in the uplink of the system with the massive MIMO.

We have analyzed the performance of the proposed uplink overloaded MIMO spatial multiplexing that the performance is degraded due to the colored noise. We propose iterative colored noise cancellation where the *a posteriori* bit LLR is estimated. The colored noise cancellation performance is enhanced in conjunction with the Gaussian integer multiples estimation when non-linear precoding is applied. This paper employs the equal gain transform for the non-linear precoding for higher transmission performance.

The performance of the proposed uplink overloaded MIMO spatial multiplexing is evaluated in systems where the number of the spatially multiplexed signal streams is increased to that of the receive antennas, e.g., from 4 to 8, even though the number of the transmit antennas is fixed to 2. The proposed colored noise cancellation with 2 iterations achieves a gain of more than 10 dB at the BER of 10^{-5} when the number of the signal streams is 4, i.e, the overloading ratio of 2. Even when the overloading ratio is raised to 4, the proposed uplink overloaded MIMO spatial multiplexing achieves a diversity order of 4.

Consequently, the proposed MIMO spatial multiplexing achieves superior transmission performance in channels where the number of transmit antennas is less than that of receive antennas, even though the proposed MIMO spatial multiplexing is able to transmit more spatially multiplexed signal streams than conventional MIMO techniques for higher throughput. This is the reason why the proposed MIMO spatial multiplexing is suitable for the uplink of the system with the massive MIMO.

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Appendix A: Performance Analysis of Proposed Overloaded MIMO Spatial Multiplexing Without Cancellation

As is described in Sec. 3.3, the modulo function output vector $\mathrm{Mod}[\mathbf{Z}, M]$ is fed to the region detection to get a tentative detection signal vector $\bar{\mathbf{A}}$. This means that the modulo output vector subtracted by the modulation signal vector $\mathbf{A}$ is regarded as the noise vector $\tilde{\mathbf{N}} \in \mathbb{C}^{N_\mathrm{R}}$ at the initial stage.

$$\tilde{\mathbf{N}} = \mathrm{Mod}[\mathbf{Z}, M] - \mathbf{A} \approx g^{-1}\mathbf{T}^\mathrm{H}\left(\mathbf{N} - \sqrt{\rho}\tilde{\mathbf{X}}\right) \qquad (\mathrm{A}\cdot 1)$$

The Gaussian integer multiples are assumed to be ideally removed in the above equation. The noise power in the streams can be estimated with the correlation of the noise vector $\tilde{\mathbf{N}}$ as,

$$\mathrm{E}\left[\tilde{\mathbf{N}}\tilde{\mathbf{N}}^\mathrm{H}\right] = 2g^{-2}\sigma^2\mathbf{I} + \frac{\rho M^2}{6}\mathbf{T}^\mathrm{H}\mathbf{Q}_2\,|\mathbf{D}|^{-2}\,\mathbf{Q}_2^\mathrm{H}\mathbf{T} \qquad (\mathrm{A}\cdot 2)$$

When the SQRD or the equal gain transform is applied, the transform matrix $\mathbf{T}$ becomes unitary, which is utilized in the above derivation. In addition, we apply the approximation of $\Psi_\mathrm{v} \approx \frac{M^2}{6}\mathbf{I}$ in the derivation process, which has been introduced in some papers, for instance, [36] †. The noise power of the mth streams can be expressed as the mth diagonal value of the correlation matrix $\mathrm{E}\left[\tilde{\mathbf{N}}\tilde{\mathbf{N}}^\mathrm{H}\right]$. When the SQRD is applied, the diagonal elements of the matrix $\mathbf{D}$ are distributed. Since the number of the eigenvalues is N_T which is smaller than the number of the streams N_R, some of the diagonal elements could get very small. The small values is transformed to big values in the diagonal elements in the inverse of the matrix $|\mathbf{D}|^{-2}$, which causes the second term in the right hand side of (A·2) bigger. When the equal gain transform is applied, on the other hand, the diagonal elements of the matrix $\mathbf{D}$ is equalized, which equalizes the diagonal elements in the inverse of the matrix $|\mathbf{D}|^{-2}$, which reduces the second terms as small as possible. The difference in the second term causes the proposed multiplexing with the SQRD much inferior to that with the equal gain transform.

† This approximation is exact on the assumption that $v(m)$ is distributed uniformly between $-\frac{M}{2}$ and $\frac{M}{2}$. However, because the assumption is not always satisfied, the equation is called approximation.