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LETTER

Multiple-wave source localization using UAVs in NLOS environments

Shinichi Murata[1, a)], **Takahiro Matsuda**[1, b)], **and Takefumi Hiraguri**[2, c)]

Abstract Localization techniques for unknown radio wave sources are crucial from the perspective of efficient utilization of frequency resources. The authors have studied methods for localizing a single wave source using unmanned aerial vehicles (UAVs) in non-line-of-sight (NLOS) environments based on maximum likelihood estimation. In this study, we propose a localization method for multiple wave sources by extending the single-wave source localization method. In the proposed method, the direction of arrivals (DoAs) at UAVs is modeled with a mixture of von-Mises distributions, and the wave sources are estimated by superimposing the DoA distributions estimated at the UAVs. The proposed method is validated with a simple simulation experiment with two wave sources.

Keywords: wave source localization, UAV, non-line-of-sight, maximum likelihood estimation

Classification: Wireless communication technologies

1. Introduction

Frequency resources are becoming scarce owing to the proliferation of wireless communication systems. To address this issue, frequency sharing techniques have been studied to share bandwidth among different wireless systems. In cognitive radio networks [1], secondary systems utilize unoccupied frequency bands of the primary system. Here, it is important for the secondary system to localize the wave source of the primary system to avoid interference with the primary system. For example, dynamic interference avoidance for cognitive radio networks using a radio environment map (REM) [2], which represents the spatial distribution of the power spectrum, has been proposed to effectively estimate white spaces and permissible interference power. The REM is constructed using information such as RSS measured at multiple locations. And it is crucial to accurately interpolate data at unmeasured locations in the REM construction. As a representative method for this interpolation, the location estimation based (LIvE) REM construction technique, which involves localizing the positions of wave sources, has been studied in [3].

In non-line-of-sight (NLOS) environments, a wave source localization method using unmanned aerial vehicles (UAVs) was proposed [4, 5]. In this method, UAVs equipped with array antennas are placed at multiple points, and each UAV estimates the direction of arrivals (DoAs) of received signals with array signal processing. Under the assumption that only one single-wave source exists in the target area, it is localized with a maximum likelihood estimator, where the likelihood function is formulated with the DoAs obtained at the UAVs. In this study, under the assumption that there may be multiple wave sources and the number of wave sources is unknown, we propose a wave source localization method.

The remainder of this paper is organized as follows. Section 2 described the single-wave source localization method proposed in [4, 5]. In Section 3, the wave source localization method for multiple wave sources is described. In Section 4, the performance of the proposed method is evaluated with simulation experiments. In Section 5, we summarize this study.

2. Single-wave source localization

2.1 System model

Figure 1 illustrates the system model considered in this study, in which some UAVs and unknown wave sources exist. We assume that all the UAVs are placed at the same height greater than the average height of the surrounding structures, and all wave sources are placed near the ground. We also assume that propagation environments between UAVs and wave sources are NLOS due to the presence of structures. Let $u_m (m = 1, 2, \ldots, M)$ and v_n $(n = 1, 2, \cdots, N)$ denote the wave sources and UAVs, respectively, where M and N represent the numbers of wave sources and UAVs, respectively. Each UAV can estimate the DoAs of received signals using array antenna signal processing. The wave source localization problem can be formulated in a three-dimensional space since the wave sources and UAVs are placed at different heights. However, we simply consider the two-dimensional problem by projecting the locations of wave sources and UAVs onto the two-dimensional plane. We represent the positions of u_m $(m = 1, 2, \ldots, M)$ and v_n $(n = 1, 2, \ldots, N)$ with $\boldsymbol{q}_m \in \mathbb{R}^2$ and $\boldsymbol{r}_n \in \mathbb{R}^2$, respectively.

As in [5], we assume that DoAs of signals transmitted from a wave source follow the von-Mises distribution [6], which is a unimodal distribution and can be approximated with the Gaussian distribution when the distance between the wave source and the UAV is sufficiently large. Let Θ denote the random variable for DoAs. The von-Mises distribution of DoA has two parameters: mean ϕ and concentration parameter κ, and the probability density function $p_\Theta(\theta \mid \phi, \kappa)$ is given by the following equation:

$$p_\Theta(\theta \mid \phi, \kappa) = \Pr(\Theta = \theta \mid \phi, \kappa)$$

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[1] Graduate School of Systems Design, Tokyo Metropolitan University, Hino-shi, Tokyo 191–0065, Japan

[2] Faculty of Fundamental Engineering, Nippon Institute of Technology, Minamisaitama-gun, Saitama 345–8501, Japan

a) murata-shinichi@ed.tmu.ac.jp

b) takahiro.m@tmu.ac.jp

c) hira@nit.ac.jp

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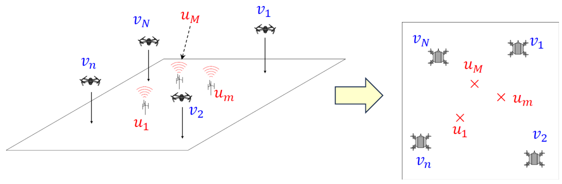

Fig. 1 System model.

$$= \frac{1}{2\pi I_0(\kappa)} \exp\left(\kappa \cos(\theta - \phi)\right)$$

where $I_\nu(\kappa)$ represents the ν-th order modified Bessel function of the first kind. Note that the von-Mises distribution becomes the uniform distribution $p_\Theta(\theta \mid \phi, \kappa) = 1/(2\pi)$ $(0 \leq \theta < 2\pi)$ when $\kappa = 0$, and satisfies the periodic condition, i.e., $p_\Theta(\theta \mid \phi, \kappa) = p_\Theta(\theta + 2\pi \mid \phi, \kappa)$.

2.2 DoA estimation method

In the single-wave source localization method, the wave source is estimated with DoAs obtained at the UAVs. We employ the DoA estimation technique based on compressed sensing (CS) [7]. CS [8] is known as a method to solve underdetermined linear inverse problems. Under the assumption that the unknown vector is a sparse vector, CS estimates it from a small number of observations. The number of array elements might be limited because of the payload limit of UAVs. Therefore, CS is suitable for DoAs estimation using UAVs.

As illustrated in Fig. 2, we adopt a uniform circular array (UCA), which is capable of estimating the DoAs of incident signals from all directions. On the circumference of a circle with radius R, there are U antenna elements spaced at the interval of d. Let (α_l, β_l) $(l = 1, 2, \ldots, L)$, and ψ_k $(k = 1, 2, \ldots, K)$ denote the position of the l -th element and the DoA of the k-th incident signal, respectively, where K represents the number of DoAs. The steering vector $\boldsymbol{a}(\psi_k)$ can be expressed as:

$$\boldsymbol{a}(\psi_k) = \left(a_1(\psi_k)\ a_2(\psi_k)\ \cdots\ a_U(\psi_k)\right)^\top,$$

$$a_l(\psi_k) = \exp\left(-j\frac{2\pi}{\lambda_0}(\alpha_l \cos\psi_k + \beta_l \sin\psi_k)\right),$$

where λ_0 represents the carrier wavelength.

Let $\boldsymbol{y} = (y_1\ y_2\ \cdots\ y_U)^\top$ and $\boldsymbol{x} = (x_1\ x_2\ \cdots\ x_K)^\top$ denote the received signal vector and the complex amplitude vector consisting of incident signals, where y_u and x_k represent the received signal at the u-th antenna element and the complex amplitude of the k-th incident signal, respectively. The relationship between $\boldsymbol{x}$ and $\boldsymbol{y}$ can be expressed as:

$$\boldsymbol{y} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{w}, \quad \boldsymbol{A} = \left(\boldsymbol{a}(\psi_1)\ \boldsymbol{a}(\psi_2)\ \cdots\ \boldsymbol{a}(\psi_K)\right)$$

where $\boldsymbol{w}$ represents the noise vector and $\boldsymbol{x}$ can be estimated by solving the optimization problem:

$$\min_{\boldsymbol{x}} \frac{1}{2}\|\boldsymbol{y} - \boldsymbol{A}\boldsymbol{x}\|_2^2 - \mu\|\boldsymbol{x}\|_1, \tag{1}$$

where $\|z\|_p (p = 1, 2)$ represents the ℓ_p norm of vector z defined as $\|z\|_p = (\sum_{n=1}^{N} |z_n|^p)^{1/p}$. In this study, the first iterative shrinkage threshold algorithm (FISTA) [9] is adopted to obtain the optimal solution $\boldsymbol{x} = (\hat{x}_1\ \hat{x}_2\ \cdots\ \hat{x}_K)^\top$ for (1). Suppose that there are K_n non-zero elements $\{\hat{x}_{i1}, \hat{x}_{i2}, \cdots, \hat{x}_{i_{K_{\mathrm{nz}}}},$

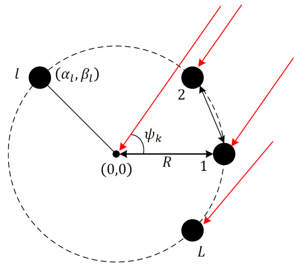

Fig. 2 Geometry of UCA.

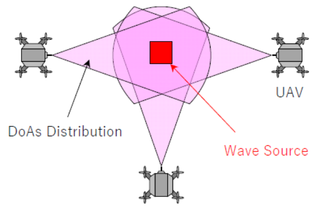

Fig. 3 Concept of wave source localization.

where $K_{\mathrm{nz}}(K_{\mathrm{nz}} < K)\}$ represents the number of non-zero elements. The set $\mathcal{T} = \{\hat{\theta}_n^{(k)} \mid k = 1, 2, \cdots, K_{\mathrm{nz}}\}$ of estimated DoAs is expressed as:

$$\hat{\theta}_n^{(k)} = \psi_{i_k}\ (k = 1, 2, \ldots, K_{\mathrm{nz}}, n = 1, 2, \ldots, N).$$

2.3 Maximum likelihood estimation for single-wave source localization

In this section, we set $M = 1$, and $\boldsymbol{u}_1$ and $\boldsymbol{q}_1$ are simply denoted as $\boldsymbol{u}$ and $\boldsymbol{q}$. Figure 3 illustrates the concept of the single-wave source localization proposed in [4, 5]. In this method, a wave source is localized simply by superimposing the DoA distributions obtained by multiple UAVs placed at different measurement points. The DoA distribution at $\boldsymbol{r}_n$ is estimated by the following procedure. Let the $\hat{\boldsymbol{\theta}}_n = \left(\hat{\theta}_n^{(1)}, \hat{\theta}_n^{(2)}, \cdots, \hat{\theta}_n^{(K_n)}\right)$ denote the DoAs estimated at v_n, where $\hat{\theta}_n^{(k)}$ and K_n denote the k-th estimated DoA and the number of DoAs at $\boldsymbol{r}_n$, respectively. Assume that the mean value of the DoA distribution coincides with the direction of the wave source. The direction of a wave source ϕ_n is then obtained by

$$\phi_n = \frac{\sum_{k=1}^{K_n} \sin\hat{\theta}_n^{(k)}}{\sum_{k=1}^{K_n} \cos\hat{\theta}_n^{(k)}}.$$

ϕ_n can be formulated as a function $\phi_n(\boldsymbol{q})$ of position $\boldsymbol{q}$ [5]:

$$\phi_n(\boldsymbol{q}) = \cos^{-1}\left(\frac{(\boldsymbol{q} - \boldsymbol{r}_n)^\top \boldsymbol{e}_n}{\|\boldsymbol{q} - \boldsymbol{r}_n\|_2}\right),$$

where $\boldsymbol{e}_n$ denotes the reference vector of v_n. The likelihood function $L_n(\phi_n(\boldsymbol{q}), \kappa_n)$ of ϕ_n and the concentration parameter κ_n for the DoA distribution at $\boldsymbol{r}_n$ is given by

$$L_n(\phi_n(\boldsymbol{q}), \kappa_n) = \prod_{k=1}^{K_n} \frac{1}{2\pi I_0(\kappa)} \exp\left(\kappa_n \cos\left(\hat{\theta}_n^{(k)} - \phi_n(\boldsymbol{q})\right)\right).$$

In [4], $\boldsymbol{q}$ is estimated by obtaining $\boldsymbol{q}$ to maximize the joint likelihood function $L(\boldsymbol{\phi}(\boldsymbol{q}), \boldsymbol{\kappa})$

$$L(\boldsymbol{\phi}(\boldsymbol{q}), \boldsymbol{\kappa}) = \sum_{n=1}^{N} L_n(\phi_n(\boldsymbol{q}), \kappa_n),$$

where $\boldsymbol{\phi}(\boldsymbol{q}) = (\phi_1(\boldsymbol{q})\ \phi_2(\boldsymbol{q})\ \cdots\ \phi_N(\boldsymbol{q}))$ and $\boldsymbol{\kappa} = (\kappa_1\ \kappa_2\ \cdots\ \kappa_N)$. In [5], a simpler method is adopted. In this method, ϕ_n and κ_n $(n = 1, 2, \ldots, N)$ are estimated by

$$(\hat{\phi}_n, \hat{\kappa}_n) = \arg\max L_n(\phi_n, \kappa_n),$$

and $\boldsymbol{q}$ is estimated by the superimposed probability density functions:

$$\hat{\boldsymbol{q}} = \arg\max_{\boldsymbol{q}} \prod_{n=1}^{N} p_\Theta(\phi_n(\boldsymbol{q}) \mid \hat{\phi}_n, \hat{\kappa}_n).$$

3. Multiple-wave source localization method

The single-wave source localization method described in the previous section cannot be applied to cases of multiple wave sources because the von-Mises distribution is a unimodal distribution. In this study, we extend the single-wave source localization method to the multiple-wave source localization method by adopting a mixture of von-Mises distributions.

3.1 Multimodal distribution for multiple-wave source localization

Let $p_{m,n}(\theta \mid \phi_{m,n}, \kappa_{m,n})$ $(n = 1, 2, \ldots, N, m = 1, 2, \cdots, M)$ denotes the DoAs distribution at v_n generated by wave source u_m. We assume that DoAs of incident signals from each wave source follow the von-Mises distribution with mean $\phi_{m,n}$ and concentration parameter $\kappa_{m,n}$. Then, $p_{m,n}(\theta)$ can be expressed as:

$$p_{m,n}(\theta \mid \phi_{m,n}, \kappa_{m,n}) = \frac{1}{2\pi I_0(\kappa_{m,n})} \exp\left(\kappa_{m,n} \cos\left(\theta - \phi_{m,n}\right)\right).$$

By mixing M von-Mises distributions with mixing coefficients $\pi_{m,n}$, the DoA distribution $p_n(\theta)$ at v_n is expressed as:

$$p_n^{(M)}(\theta \mid \boldsymbol{\phi}_n, \boldsymbol{\kappa}_n, \boldsymbol{\pi}_n) = \sum_{m=1}^{M} \pi_{m,n} p_{m,n}(\theta \mid \phi_{m,n}, \kappa_{m,n}),$$

where $\pi_{m,n}$ satisfies $\sum_{m=1}^{M} \pi_{m,n} = 1$, and $\boldsymbol{\phi_n} = (\phi_{1,n}\ \phi_{2,n}\ \cdots\ \phi_{M,n})$, $\boldsymbol{\kappa}_n = (\kappa_{1,n}\ \kappa_{2,n}\ \cdots\ \kappa_{M,n})$, $\boldsymbol{\pi}_n = (\pi_{1,n}\ \pi_{2,n}\ \cdots\ \pi_{M,n})$

The likelihood function $L_n^{(M)}(\boldsymbol{\phi}_n, \boldsymbol{\kappa}_n, \boldsymbol{\pi}_n)$ of DoAs at v_n is given by

$$L_n^{(M)}(\boldsymbol{\phi}_n, \boldsymbol{\kappa}_n, \boldsymbol{\pi}_n) = \prod_{k=1}^{K_n} p_n^{(M)}(\theta^{(k)} \mid \boldsymbol{\phi}_n, \boldsymbol{\kappa}_n, \boldsymbol{\pi}_n).$$

For given M, $\hat{\boldsymbol{\phi}}_n$, $\hat{\boldsymbol{\kappa}}_n$, and $\hat{\boldsymbol{\pi}}_n$ are estimated by

$$\left(\hat{\boldsymbol{\phi}}_n^{(M)}, \hat{\boldsymbol{\kappa}}_n^{(M)}, \hat{\boldsymbol{\pi}}_n^{(M)}\right) = \arg\max_{\boldsymbol{\phi}_n, \boldsymbol{\kappa}_n, \boldsymbol{\pi}_n} L_n^{(M)}\left(\boldsymbol{\phi}_n, \boldsymbol{\kappa}_n, \boldsymbol{\pi}_n\right). \quad (2)$$

3.2 Multiple-wave source localization

M must be adequately determined because the number of wave sources is unknown. M is estimated individually at each UAV. Let $\hat{M}_n$ denote M estimated at v_n. The proposed method obtains $\hat{M}_n$ using the Akaike information criterion (AIC) as follows:

1. Set $M_n := 1$.
2. Estimate $\hat{\boldsymbol{\phi}}_n^{(M_n)}$, $\hat{\boldsymbol{\kappa}}_n^{(M_n)}$, and $\hat{\boldsymbol{\pi}}_n^{(M_n)}$ by (2), and calculate $L_n^{(M_n)}(\hat{\boldsymbol{\phi}}_n^{(M_n)}, \hat{\boldsymbol{\kappa}}_n^{(M_n)}, \hat{\boldsymbol{\pi}}_n^{(M_n)})$.
3. Calculate AIC
$$\mathrm{AIC}_n^{(M_n)} = -2 \ln L_n^{(M_n)}(\hat{\boldsymbol{\phi}}_n^{(M_n)}, \hat{\boldsymbol{\kappa}}_n^{(M_n)}, \hat{\boldsymbol{\pi}}_n^{(M_n)}) + 2\eta,$$
where $\eta = 3M_n - 1$ represents the number of parameters.
4. If $M_n < M_{\max}$, set $M_n := M_n + 1$ and go to step 2.
5. M_n is estimated by finding M_n which minimizes $AIC_n^{(M_n)}$:
$$\hat{M}_n = \arg\max_{M_n} AIC_n^{(M_n)}.$$

As in the single-wave source localization in [5], the positions of the wave sources are estimated by the superimposed DoAs distribution $p(\boldsymbol{r})$:

$$p(\boldsymbol{q}) = \prod_{n=1}^{N} p_n(\phi_n(\boldsymbol{q}) | \hat{\boldsymbol{\phi}}_n^{(\hat{M}_n)}, \hat{\boldsymbol{\kappa}}_n^{(\hat{M}_n)}, \hat{\boldsymbol{\pi}}_n^{(\hat{M}_n)}).$$

Note that the numbers of wave sources at different UAVs may differ, i.e., $M_n \neq M_{n'}$ $(n \neq n')$; this is because if different wave sources are placed in the same direction from the perspective of a UAV, they appear to be a single wave source and the number of wave sources observed at the UAV is set to a smaller value than the actual number. The number $\hat{M}$ of wave sources is finally determined by the mode in the set $\{\hat{M}_1, \hat{M}_2, \ldots, \hat{M}_N\}$ of the estimated number of wave sources:

$$\hat{M} = \text{mode}\left\{\hat{M}_1, \hat{M}_2, \cdots, \hat{M}_N\right\}$$

The positions of the $\hat{M}$ wave sources are estimated by the set $Q = \{\boldsymbol{q}_m \mid m = 1, 2, \ldots, \hat{M}\}$ of positions where the $\hat{M}$ largest peak values of $p(\boldsymbol{q})$ exist.

4. Simulation experiment

4.1 Simulation environment

Simulation experiments are conducted to evaluate the performance of the proposed method. The simulation area is set to 900 [m] × 900 [m], and the UAVs are placed at the circumference with a radius of 450[m] centered around the middle of the area. M wave sources are randomly placed within a 300 [m] × 300 [m] sub-area at the center of the simulation area. We assume that each UAV obtains 10 DoAs from each wave source (i.e., $K_n = 10M$).

As mentioned in Section 2., as a DoAs distribution in NLOS environment, we assume that DoAs follow the von-Mises distribution. As in the probabilistic model in [5], the DoAs obtained at each UAV are generated according to a von-Mises distribution. For UAV v_n, we set $\phi_{m,n}$ $(m = 1, 2, \ldots, M)$ in the direction of the wave source u_m. $\kappa_{m,n}$ is defined as a cubic polynomial of the distance between u_m and v_n with the same coefficients as in [5]. In this study, we set $M = 2$ and define the distance between u_1 and u_2 as $d_{1,2} = \|\boldsymbol{q}_1 - \boldsymbol{q}_2\|_2$. In the proposed method, the ability to estimate the number of wave sources with AIC depends on the distance between the wave sources because closer wave sources are regarded as one wave source. Therefore, in the subsequent subsection, we evaluate the performance of the proposed method against $d_{1,2}$.

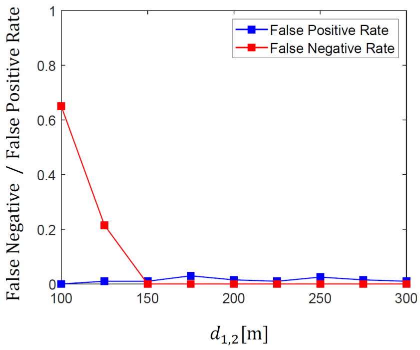

Fig. 4 Simulation result.

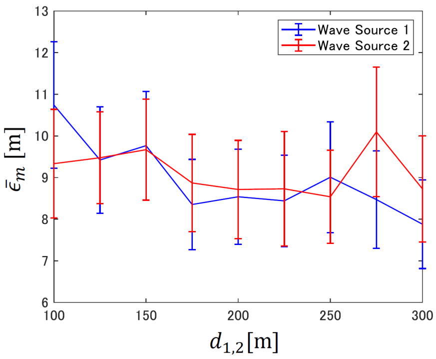

Fig. 5 Localization error and 95% confidence interval.

4.2 Simulation results

We set $d_{1,2} \in \{100, 125, \ldots 300\}$, and conduct 100 simulation experiments for each $d_{1,2}$. Let $\hat{\boldsymbol{q}}_m^{(i)}$ $(m = 1, 2)$ denote the estimated position of $\boldsymbol{q}_m$ in the i-th simulation experiment $(i = 1, 2, \ldots, 100)$. The localization error $\epsilon_m^{(i)}$ and average localization error $\bar{\epsilon}_m$ are defined as:

$$\epsilon_m^{(i)} = \|\boldsymbol{q}_m - \hat{\boldsymbol{q}}_m^{(i)}\|_2, \quad \bar{\epsilon}_m = \frac{1}{100}\sum_{i=1}^{100} \epsilon_m^{(i)},$$

respectively. We consider that u_m is detected if $\epsilon_m^{(i)} \leq \epsilon_{\text{th}}$ and the wave sources are correctly localized only if both the wave sources are detected, that is, $\epsilon_1^{(i)} \leq \epsilon_{\text{th}}$ and $\epsilon_2^{(i)} \leq \epsilon_{\text{th}}$. If only one peak value or no peak value is found in $p(\boldsymbol{q})$, a *false negative* error occurs. If either or both $\epsilon_1^{(i)}$ and $\epsilon_2^{(i)}$ exceed ϵ_{th}, or more than three wave sources are detected, a *false positive* error occurs. For given $d_{1,2}$, false negative rate $\eta_{\text{FN}}(i)$ and false positive rate $\eta_{\text{FP}}(i)$ are defined as:

$$\eta_{\text{FN}}(d_{1,2}) = \frac{\rho_{\text{FN}}(d_{1,2})}{100}, \quad \eta_{\text{FP}}(d_{1,2}) = \frac{\rho_{\text{FP}}(d_{1,2})}{100},$$

where $\rho_{\text{FN}}(d_{1,2})$ and $\rho_{\text{FP}}(d_{1,2})$ denote the number of false negative errors and false positive errors.

Figure 4 shows $\eta_{\text{FN}}(d_{1,2})$ and $\eta_{\text{FP}}(d_{1,2})$ vs. $d_{1,2}$, where $\epsilon_{\text{th}} = 50$ [m]. We observe that $\eta_{\text{FN}}(d_{1,2})$ decreases as $d_{1,2}$ increases; this is because a larger $d_{1,2}$ makes it easy to distinguish the wave sources. Conversely, $\eta_{\text{FP}}(d_{1,2})$ for every $d_{1,2}$ is lower than 0.03. Therefore, the proposed method can localize sufficiently separated two wave sources.

Figure 5 shows $\bar{\epsilon}_m$ $(m = 1, 2)$ vs. $d_{1,2}$. From the figure, we observe that the proposed method can estimate the positions of wave sources 1 and 2 within the average estimation error of less than 11[m].

5. Conclusion

In this study, we proposed a wave source localization method for multiple wave sources. In the proposed method, DoA distributions at UAVs are modeled with mixtures of von-Mises distributions. From the DoA distributions estimated at the UAVs, the positions of the wave sources are estimated with the superimposed DoA distributions. The simulation results showed that the proposed method can accurately localize the wave sources in a situation where the wave sources are sufficiently separated. In future research, we will investigate the performance of the proposed method for more than three wave sources.

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