

Optical nonlinearity mitigation using reservoir computing with tapped-delay lines

Kai Ikuta^{1, a)}, Yuta Ito¹, and Moriya Nakamura¹

Abstract We investigated a nonlinear equalizer based on reservoir computing (RC) with tapped-delay lines for compensating for linear and nonlinear waveform distortion caused by, e.g., chromatic dispersion and self-phase modulation. We evaluated and compared the equalization performance of RC systems with and without tapped-delay lines by numerical simulation, while varying the parameters and the transmission distance. The proposed equalizer construction enables the use of RC with a shorter time-constant than that used for conventional RC-based nonlinear equalizers.

Keywords: optical fiber communication, digital coherent transmission, optical nonlinearity compensation, digital signal processing, reservoir computing

Classification: Fiber-optic transmission for communications

1. Introduction

Digital coherent communication systems have been developed to meet the high demand for communication traffic in optical communication systems. Additionally, these systems use digital signal processing (DSP) to mitigate waveform distortion such as chromatic dispersion (CD) and nonlinear distortion due to Kerr nonlinear effects. Compensation for CD is achieved using frequency domain equalizers (FDEs) and time-domain finite impulse response (FIR) filters, both implemented in DSP with appropriate complexity. As for nonlinear compensation techniques, digital backpropagation (DBP) and nonlinear equalizers based on the Volterra series transfer function (VSTF) have been studied [1, 2, 3, 4]. However, these techniques require enormous computational resources. To reduce computational complexity, nonlinear equalizers based on machine learning (ML) are extremely attractive [5, 6, 7, 8, 9]. We investigated the optimal size of an artificial neural network (ANN) for compensating for CD and self-phase modulation (SPM) in 16-ary quadrature amplitude modulation (16QAM) signals [6]. Recently some studies have been conducted to facilitate the implementation of ML-based nonlinear equalizers on field-programmable gate arrays (FPGAs) [8, 9]. Reservoir computing (RC) has attracted significant attention for its low learning costs and ease of implementation in physical systems [10, 11, 12, 13]. An RC system includes a hidden layer composed of a fixed recurrent neural network (RNN), known as a reservoir layer,

which is capable of storing signals. These signals can be utilized to compensate for waveform distortion. Photonic RC, implemented on a silicon chip, has been experimentally employed to compensate for waveform distortion in on-off keying (OOK) signals transmitted over 25 km of standard single-mode fiber (SSMF) [12]. This technology enables high-speed signal processing within the optical domain. Additionally, RC systems configured in DSP have been extensively researched for their parameter adjustment flexibility [14, 15, 16, 17, 18]. For instance, a DSP-based RC system was applied to a 32-GBaud OOK transmission system to evaluate its nonlinear compensation capabilities [14]. We have also explored DSP-based RC for polarization demultiplexing and nonlinear waveform equalization in polarization multiplexed 16QAM signals over 50 km of SSMF [15]. In other work, it has been noted that photonic RC systems have difficulty to realize a sufficiently long time constant [13]. The short time-constant of RC poses a major challenge in addressing waveform distortion resulting from substantial CD. To overcome this, alternative methods employing feed-forward tapped-delay lines instead of extended time constants in RC have been introduced [16, 17, 18]. In one study, DSP-based RC with tapped-delay lines was implemented for 32-GBaud 4-level pulse amplitude modulation (PAM4) signals to mitigate CD [17]. We previously determined that a nonlinear equalizer using RC with tapped-delay lines outperforms those without such configurations [18]. In this paper, we assess the nonlinearity compensation efficacy of an RC system with tapped-delay lines across various transmission distances and compare it with RC systems without tapped-delay lines. Additionally, we explore the optimal size of the reservoir layer for use as a nonlinear equalizer.

2. RC-based nonlinear equalizer

Figure 1 shows the structure of a conventional RC-based nonlinear equalizer, which does not employ any tapped-delay line. The RC system includes an input layer, a reservoir

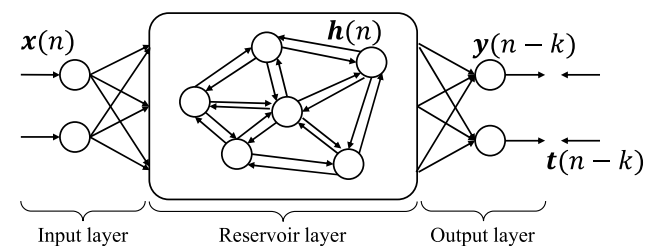


Fig. 1 RC-based nonlinear equalizer w/o tapped-delay lines.

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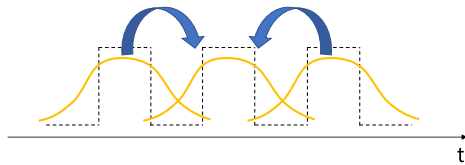


Fig. 2 Dispersion in optical fiber.

layer, and an output layer. The input and output layers each consist of two units designed to process the in-phase (I) and quadrature (Q) components of optical signals. The input weights, w^{in} , are randomly configured to follow a uniform distribution within the interval $[-1, 1]$, forming a $2 \times M$ weight matrix, where M represents the number of reservoir units. These weights are initially set and remain fixed. The reservoir layer consists of an RNN. The reservoir weights, w^{res} , are initialized randomly from a standard normal distribution with a mean of 0 and a variance of 1, forming an $M \times M$ weight matrix. Both w^{in} and w^{res} are initially randomized and fixed throughout the learning process. The output of the reservoir units, \mathbf{h} , is described as

$$\mathbf{h}(n) = f(\mathbf{w}^{in} \mathbf{x}(n) + \mathbf{w}^{res} \mathbf{h}(n-1)), \quad (1)$$

where $f(\cdot)$ is the activate function of the reservoir units $f(u) = \tanh(u)$, \mathbf{x} is the input signal to input-layer units, and n is the time index of the sampled signal [10]. The output from the output-layer unit, \mathbf{y} , is described as

$$\mathbf{y}(n) = \mathbf{w}^{out} \mathbf{h}(n), \quad (2)$$

where \mathbf{w}^{out} is the output weights generated randomly from a uniform distribution within the interval $[0, 1]$, forming an $M \times 2$ weight matrix. In this model, only the output weights are trained using the Least Mean Squares (LMS) algorithm to minimize the error, e , which is expressed as

$$e = \frac{1}{2}(\mathbf{y}(n-k) - \mathbf{t}(n-k))^2, \quad (3)$$

where \mathbf{t} represents the training signal and k denotes the time shift between the received signal, which is input to the input-layer units, and the training signal. The time shift parameter is critically important for equalizers to compensate for waveform distortion. Figure 2 shows waveform distortion caused by CD. Effective compensation requires appropriately delaying the training signal relative to the received signal. This time shift enables the equalizer to accurately compensate for the target signal, while also incorporating both the preceding and following signals into its processing. Here, we consider the time constant of an RC-based equalizer. Although the reservoir weights, w^{res} , are randomly generated and initially set, these weights are scaled to satisfy the echo state property [10]. The condition is scaled by

$$\max(|\lambda^{res}|) < 1, \quad (4)$$

where λ^{res} represents the eigenvalue of the weight matrix, w^{res} , and the left side of Eq. (4) is called the spectral radius. The spectral radius is crucial as it determines the time constant of the RC. The quality of the signals stored in the reservoir layer deteriorates over time. It has been noted that a photonic RC system faced challenges in realizing a long

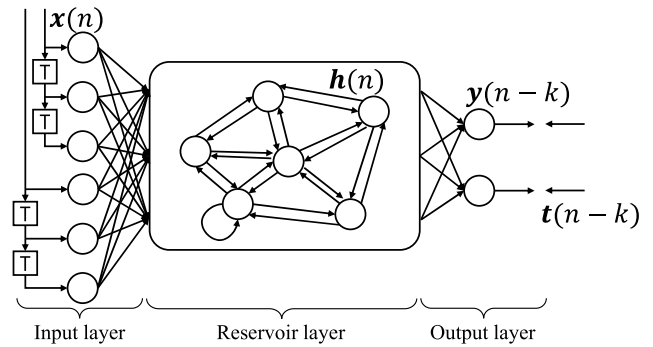


Fig. 3 RC-based nonlinear equalizer w/ tapped-delay lines.

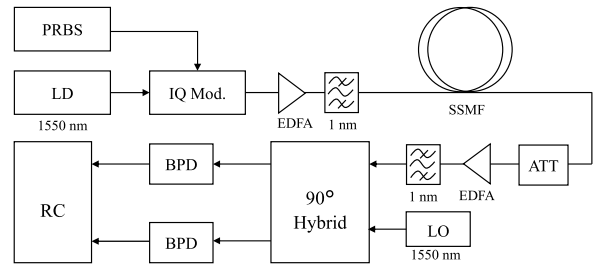


Fig. 4 16QAM transmission system setup.

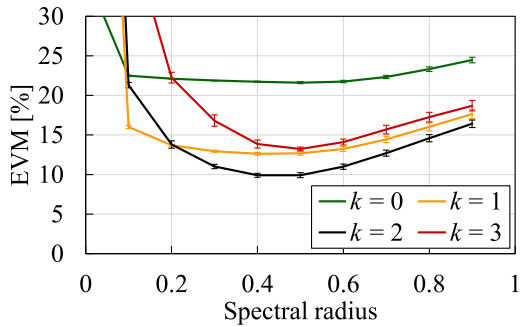
time constant [13]. Figure 3 shows our proposed structure of an RC-based nonlinear equalizer with two feedforward tapped-delay lines, one for the I-component and the other one for the Q-component of the received signal. In this case, the matrix size of the input weights, w^{in} , is $2L \times M$, where L is the length of the two tapped-delay lines. This structure allows for an adjustable time constant by varying the length of the feedforward tapped-delay lines. In this study, we employed tapped-delay lines with lengths of 3 and 5. We did not utilize sparse connections among the reservoir units.

3. System setup

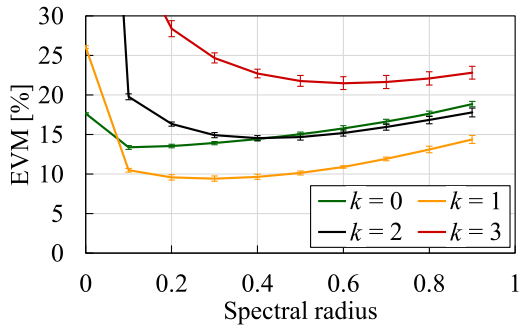
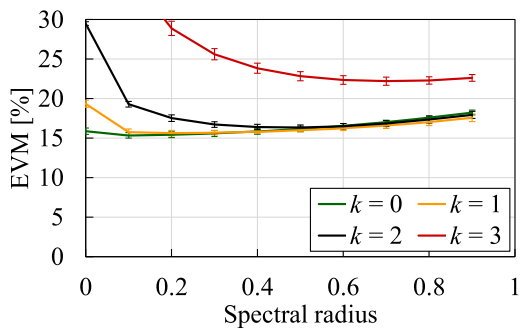
Figure 4 shows the 16QAM optical-fiber transmission system used in our numerical simulations. A 10-Gsymbol/s 16QAM optical signal, modulated by a pseudo-random binary sequence (PRBS) of PRBS $2^{15}-1$, was transmitted over SSMFs. Various transmission lengths were used: 20 km, 50 km, 70 km, and 100 km. The wavelength of the laser diode (LD) was 1550 nm. The input signal power to the SSMF was as large as 10 dBm, sufficient to induce fiber nonlinearity. The noise figure of the Er-doped fiber amplifier (EDFA) was about 5 dB. The optical signal was received using optical homodyne detection, assuming an ideally phase-locked local oscillator (LO). The optical power of the LO was 10 dBm. After transmission, the distorted signals were compensated using the RC-based nonlinear equalizers in the DSP. The signal quality after compensation was evaluated using the error vector magnitude (EVM). In the training process of the RC, we confirmed the training results using some different data sets to ensure that no overfitting occurred.

4. Results and discussion

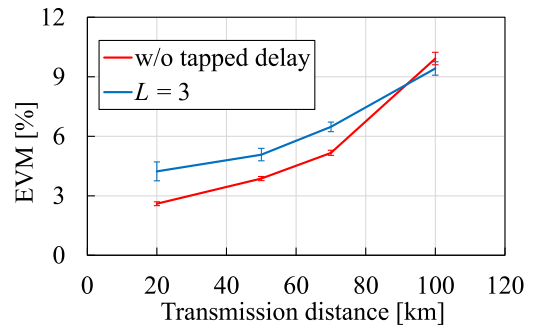
Figure 5(a) shows the EVM versus spectral radius when applying nonlinear compensation to the signals transmitted



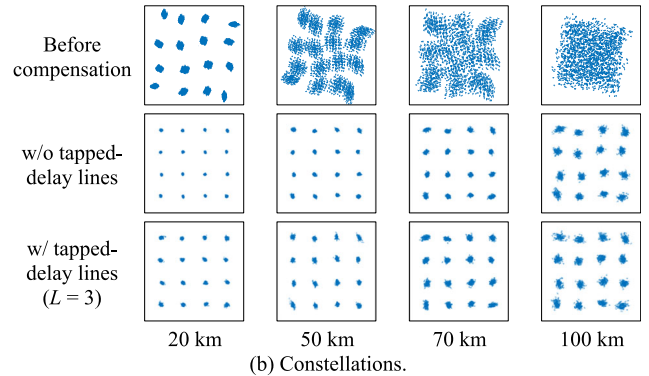
(a) w/o tapped delay lines.

(b) w/ tapped-delay lines ($L = 3$).(c) w/ tapped-delay lines ($L = 5$).**Fig. 5** EVM versus spectral radius.

over a 100-km SSMF using the conventional RC-based nonlinear equalizer without tapped-delay lines. The number of reservoir units was $M = 200$. We plotted the averages from 10 trials under each condition, while varying the weights randomly. The error bars indicate the standard deviation. At a spectral radius of as small as 0.1, the RC failed to achieve low EVM due to an insufficiently long time constant to compensate for the CD accumulated over a 100-km SSMF. When the spectral radius was larger than 0.5, the RC systems also failed to achieve low EVMs, as excessively long time-constants retained unnecessary signals in the reservoir layer, thus degrading compensation performance. The optimal EVM of 9.9% was achieved with a spectral radius of approximately 0.4, with an optimal time shift of 2. Figure 5(b) shows the EVM versus the spectral radius for our proposed RC-based nonlinear equalizer using the tapped-delay lines with a tap-length of 3. In this case, the RC systems achieved almost the same EVMs even when the spectral radius was as small as 0.1. Due to the tapped-delay lines, we do not need to use the deteriorated signals stored in the reservoir layer, utilizing instead the signals fed through the tapped-delay lines. Therefore, the equalizer does not need a long time constant of the reservoir layer. An optimal



(a) EVM versus transmission distance.



(b) Constellations.

Fig. 6 Compensation performance over transmission distance ($M = 200$).

EVM of 9.4% was achieved, representing a 0.5% improvement compared to the scenario without tapped-delay lines, at a spectral radius of 0.3 and a time shift of 1. Figure 5(c) shows the EVM versus the spectral radius for an RC-based nonlinear equalizer using the tapped-delay lines with a tap-length of 5. The RC-based equalizer was unable to achieve low EVMs at any of the observed spectral radii and time shifts. This degradation in performance occurs because the excessive length of the tapped-delay lines introduces unnecessary signals into the RC, thus impairing the performance of the compensation. Next, we compared the compensation performance of the equalizers using the tapped-delay lines with 3 taps to those without tapped-delay lines, across varying transmission distances. Here, the number of the reservoir units was $M = 200$. Figure 6(a) shows the results. For transmission distances shorter than 70 km, the RC-based nonlinear equalizer without tapped-delay lines outperformed the one with the tapped-delay lines. As previously discussed, this degradation in performance is attributable to unnecessary signals introduced by the excessively long tapped-delay lines. Conversely, at a transmission distance of 100 km, the compensation performance of the equalizer with the tapped-delay lines outperformed the one without tapped-delay lines. Figure 6(b) shows the constellations before and after the compensation using the RC-based nonlinear equalizers. The constellations show that the equalizers without the tapped-delay lines were effective for transmission distances shorter than 70 km. However, for a transmission distance of 100 km, minor waveform distortions are still observable in the constellations. We explored the performance of the RC-based nonlinear equalizers by varying the number of reservoir units. Figure 7(a) shows the EVM after compensation versus the number of reservoir units for a transmission distance of 100 km. The equalizers using the

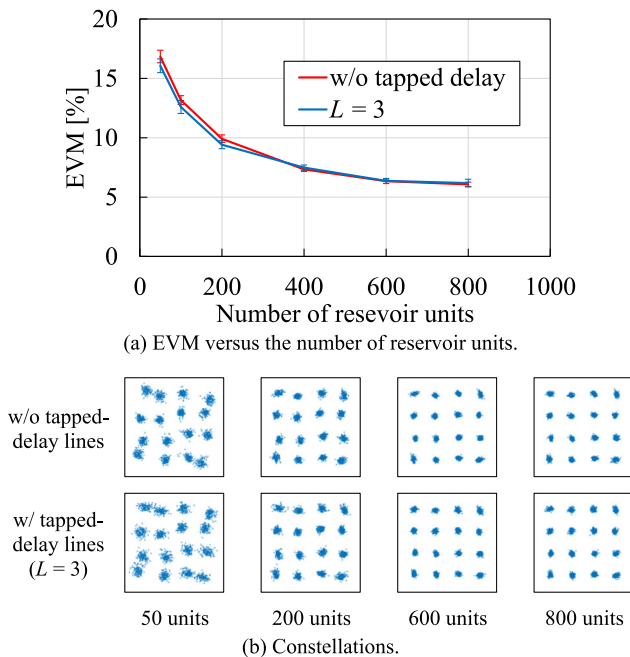


Fig. 7 Compensation performance over the number of reservoir units.

tapped-delay lines with 3 taps outperformed the one without the tapped-delay lines, when the number of reservoir units was less than or equal to $M = 200$. However, both equalizers achieved the same EVM value as the number of reservoir units increased, and the performance approached saturation at $M = 600$. Figure 7(b) shows that approximately 600 reservoir units were sufficient for both RC-based nonlinear equalizers to effectively handle the 100-km transmission. However, implementing an RC with as many as 600 reservoir units on DSP presents significant challenges due to computational complexity. One possible approach to implement such a massive reservoir is to employ an optical reservoir [11, 12, 13]. The tapped-delay lines can also be implemented using the optical domain [19].

5. Conclusion

We proposed and investigated a nonlinear equalizer based on RC using tapped-delay lines. The proposed scheme does not require a long time-constant for the reservoir, unlike conventional RC-based equalizers. The proposed equalizer showed comparable equalization performance in our numerical simulations. However, the length of the tapped-delay lines has to be adjusted depending on the transmission distance and the influence of the chromatic dispersion of the optical fiber.

Acknowledgments

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