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LETTER

Resonant harmonic reaction in twin- and quadruplet-diode rectifiers

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Abstract This study answers the basic question of whether a simple lumped-constant (LC) resonator works for harmonic reactions in diode rectifiers. To explore how the LC resonator generally reacts to harmonic waves, we present three typical diode rectifier topologies: double-voltage, double-current, and full-bridge. Through an algebraic circuit analysis of the diode's time-domain switching operation, we find that the second- and higher-order harmonic waves generated by the diodes are fully reflected by the LC resonator, return to the diodes, and are recycled for effective dc power generation. This outcome offers a simpler alternative solution for high-efficiency RF rectification than usual class-E and -F implementation.

Keywords: Nonlinear circuit, lumped-constant resonator, duality theorem, Fourier series, harmonic balance, power conversion efficiency

Classification: Transmission systems and transmission equipment for communications

1. Introduction

Following broadcasting and telecommunication services, the wireless engineering now finds its third ubiquitous commercial application: power transfer. Whereas the two predecessor services are intended to deliver information signals such as voice and data, the mission of wireless power transfer (WPT) is to deliver energy through the air. In a typical WPT system, the dc source power is upconverted into a radio frequency (RF) to pass through an air segment. The received RF power is then reconverted back to dc. The component indispensable for this reconversion is called the RF rectifier, which may appear similar to but in fact differs from the signal detector used in communication receivers. This is because the RF rectifier prioritizes its power conversion efficiency over its broadband flatness or low noise figure. To enhance the power conversion efficiency, we focus on a crucial technique: *harmonic reaction*. The history of harmonic reactions dates back to the 1960s when they were first applied to an RF power amplifier, now known as class-E [1], and then extended to diode rectifiers in the 1980s [2]. Although the design theory of class-E rectifiers is well established [3], a more basic question remains as to whether an alternative solution exists for harmonic reactions that is simpler than class-E. Based on this background, this study considers the utmost simple reactor topology: lumped-constant (LC) resonator, as in Fig. 1. We theoretically explore how diode rectifiers behave when an LC resonator is added in front of three typical rectifier topologies: double-voltage, double-current, and full-bridge.

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2. Series LC resonator

We begin with an LC resonator added in series to a double-voltage rectifier employing twin diodes, as shown in Fig. 2. The rectifier is excited using a sinusoidal voltage source

$$v_s(t) = V_s \sin \omega t \qquad (1)$$

at the RF input port. The series LC elements are tuned exactly to resonate at the above RF angular frequency ω. The resonance is formulated as

$$\omega L = \frac{1}{\omega C} = QZ_{in} \qquad (2)$$

where Q stands for the quality factor. We assume a high Q that brings the resonator a sufficient reactance against the second- and higher-order harmonics. The RF input impedance Z_{in} is derived later.

Thanks to the resonance, the RF input voltage passes straight through the LC and reaches the diodes. Pumped by this RF voltage, the diodes generate dc voltage, which is smoothed by the large capacitor C_∞ and delivered to the output port. The diodes generate not only dc but also multiple harmonics due to strong nonlinearity. The point is that the harmonics are reflected by the LC resonator back to the diodes, and are effectively recycled for dc power generation. If not for the LC resonator, the harmonics would go out of the circuit, causing energy loss and possibly undesirable electromagnetic interference with other sensitive electronics. That is the strong motivation to add the LC resonator as a harmonic reflector.

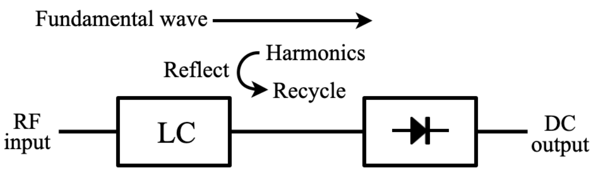

Fig. 1 Basic idea of simple harmonic reaction rectifier

3. Double-voltage rectifier

We now formulate in detail how the rectifier shown in Fig. 2 operates. Synchronized with the input sinusoid (1), the twin diodes D_1 and D_2 periodically turn on and off. For the sinusoid's positive half cycle (PHC), D_1 stays off and D_2 stays on. For the negative half cycle (NHC), they precisely alternate. They are both off only at the very moment between PHC and NHC. They never turn on simultaneously. If they were to do so, the smoothing capacitor C_∞ would be short-circuited. Remember that C_∞ has a sufficiently large capacitance to maintain a constant output voltage without ripples. We assume, in this study, an ideal model with zero on-voltage and zero parasitic capacitance for all diodes. Overall, the voltage observed across D_1 creates a 50% duty rectangular waveform, which is simply formulated as

$$v_{D1}(t) = \begin{cases} V_\mathrm{o} & \text{for PHC} \\ 0 & \text{for NHC} \end{cases} \qquad (3)$$

where V_o denotes the dc output voltage across C_∞. Subsequently, Kirchhoff's voltage law along the loop of D_1, D_2, and C_∞ finds that the voltage across D_2 also produces the same rectangular waveform but in exactly the opposite phase. The twin diodes, getting together, alternately supply their voltage V_o in total to the dc output port.

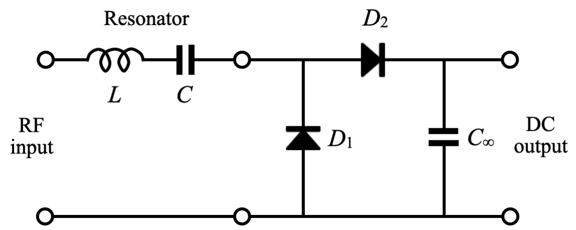

Fig. 2 Series resonator and double-voltage rectifier

Since the LC resonator is tuned at ω, the two waveforms $v_s(t)$ and $v_{D1}(t)$ must balance in their fundamental harmonic terms, extracted by the Fourier theorem as

$$V_s = \frac{2}{T}\int_0^T v_{D1}(t)\sin\omega t\, dt \qquad (4)$$

where T stands for the time period to meet $\omega T = 2\pi$. Adopting (3) into the integral, the right-hand side results in

$$= \frac{2}{\pi}V_\mathrm{o} \qquad (5)$$

Despite the strong nonlinearity of the diodes, the dc output voltage is found directly proportional to the RF input voltage amplitude.

We next focus on the RF current flowing through the LC resonator. Although the rectangular voltage (3) contains multiple harmonics, the resonator admits only the fundamental waves to flow. In other words, the current exhibits a purely sinusoidal waveform as

$$i_s(t) = I_s \sin\omega t \qquad (6)$$

Note that (6) is completely in phase with (1) because (6) must satisfy $i_s(0) = 0$ at the moment when the two diodes are both off.

We are now ready to calculate the current waveforms of the two diodes $i_{D1}(t)$ and $i_{D2}(t)$, smoothing capacitor $i_{C\infty}(t)$, and dc output current I_o. Kirchhoff's current law leads to

$$i_s(t) + i_{D1}(t) = i_{D2}(t) \qquad (7)$$

$$i_{D2}(t) = i_{C\infty}(t) + I_\mathrm{o} \qquad (8)$$

at the two nodes before and after D_2, respectively. Adopting (6) into (7) and (8) solves them as

$$i_{D1}(t) = \begin{cases} 0 \\ -I_s\sin\omega t \end{cases} \qquad (9)$$

$$i_{D2}(t) = \begin{cases} I_s\sin\omega t \\ 0 \end{cases} \qquad (10)$$

$$i_{C\infty}(t) = \begin{cases} I_s\sin\omega t - I_\mathrm{o} \\ -I_\mathrm{o} \end{cases} \qquad (11)$$

On each right-hand side, the upper and lower cases apply to PHC and NHC, respectively. It is needless for (9) and (10) to explain why no current flows when the diode stays off.

In general, a capacitor does not allow a dc current to pass through. This rule applies to C_∞ as

$$\int_0^T i_{C\infty}(t)dt = 0 \qquad (12)$$

Substituting (11) into (12), we obtain

$$I_s = \pi I_\mathrm{o} \qquad (13)$$

To satisfy (5) and (13) at once, we can finally deduce the RF input impedance and dc output power as

$$Z_\mathrm{in} = \frac{V_s}{I_s} = \frac{2}{\pi^2}R_\mathrm{o} \qquad (14)$$

$$V_\mathrm{o}I_\mathrm{o} = \frac{1}{2}V_sI_s \qquad (15)$$

where $R_\mathrm{o} = V_\mathrm{o}/I_\mathrm{o}$ denotes the assumed dc load resistance. Because the right-hand side of (15) signifies the effective RF input power, we can conclude that the theoretical power conversion efficiency is 100%. Referring to the same rectifier performing at 92.3% without harmonic reaction [4], a loss of 7.7% is now fully retrieved by the simple LC resonator.

To visualize the analytical sequence from (1) to (15), Fig. 3 illustrates the flowchart along with the equation numbers. With minor changes, this flow also applies to the two rectifiers described in the following sections.

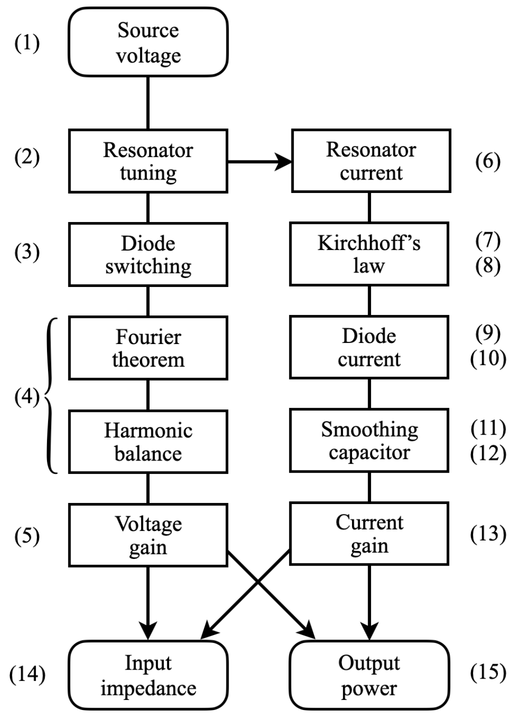

Fig. 3 Flowchart for resonant rectifier circuit analysis

4. Parallel LC resonator

Our next reactor is an LC resonator added in parallel to the double-current rectifier as shown in Fig. 4. This circuit is the counterpart of that in Fig. 2. Therefore, the formulation can be expedited by the duality theorem. The excitation voltage (1) alters to the sinusoidal current

$$i_s(t) = I_s \sin \omega t \qquad (16)$$

at the RF input port. The parallel LC elements are tuned in the same manner as (2), but Q moves to the denominator as

$$\omega L = \frac{1}{\omega C} = \frac{Z_{in}}{Q} \qquad (17)$$

Therefore, in turn, a higher Q brings the resonator a sufficiently low reactance. Although the circuit topology differs from that in Fig. 2, it applies again such that the higher harmonics are reflected by the resonator, return to the diodes, and are recycled for dc power generation.

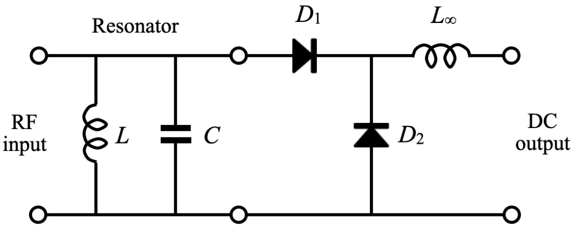

Fig. 4 Parallel resonator and double-current rectifier

5. Double-current rectifier

We continue following the duality theorem to explain the circuit operation in Fig. 4. The twin diodes D_1 and D_2 periodically turn on and off. For PHC of the input sinusoid, D_1 stays on and D_2 stays off. For NHC, they precisely alternate. They are both on only at the moment between PHC and NHC. The choke coil L_∞ has a sufficiently large inductance to maintain a constant output current. Overall, the current flowing through D_1 creates a 50% duty rectangular waveform, which is formulated as

$$i_{D1}(t) = \begin{cases} I_o & \text{for PHC} \\ 0 & \text{for NHC} \end{cases} \qquad (18)$$

where I_o denotes the dc output current. Kirchhoff's current law at the node that connects D_1, D_2, and L_∞ then determines that the current through D_2 also produces the same 50% duty rectangular waveform in the opposite phase. Thus, the twin diodes alternately supply their current I_o yielding 100% duty in total to the dc output port. This operation is called double-current rectification [4].

Because the LC resonator is tuned at ω, the input current must balance with D_1 in their fundamental harmonic components, extracted as

$$I_s = \frac{2}{T}\int_0^T i_{D1}(t) \sin \omega t \, dt \qquad (19)$$

Adopting (18) into the integral, the right-hand side results in

$$= \frac{2}{\pi} I_o \qquad (20)$$

of the same proportion as seen in (5).

We next focus on the RF voltage across the LC resonator. Although the rectangular current (18) contains multiple harmonics, the resonator enhances only the fundamental wave for build-up. In other words, the voltage exhibits a sinusoidal waveform as

$$v_s(t) = V_s \sin \omega t \qquad (21)$$

Note that (21) is completely in phase with (16) because (21) must satisfy $v_s(0) = 0$ at the moment when the two diodes are both on.

We can now calculate the voltage waveforms across the two diodes $v_{D1}(t)$ and $v_{D2}(t)$, choke coil $v_{L\infty}(t)$, and dc output voltage V_o. The Kirchhoff's voltage law leads to

$$v_s(t) + v_{D1}(t) = v_{D2}(t) \qquad (22)$$

$$v_{D2}(t) = v_{L\infty}(t) + V_o \qquad (23)$$

along the two loops before and after D_2, respectively. Adopting (21) into (22) and (23) solves them as

$$v_{D1}(t) = \begin{cases} 0 \\ -V_s \sin \omega t \end{cases} \qquad (24)$$

$$v_{D2}(t) = \begin{cases} V_s \sin \omega t \\ 0 \end{cases} \qquad (25)$$

$$v_{L\infty}(t) = \begin{cases} V_s \sin \omega t - V_o \\ -V_o \end{cases} \qquad (26)$$

On each right-hand side, the upper and lower cases apply to PHC and NHC, respectively. For (24) and (25) to explain the zero voltage when the diode stays on is unnecessary.

Generally, the inductor does not allow a dc voltage to build across. This rule applies to L_∞ as

$$\int_0^T v_{L\infty}(t) dt = 0 \qquad (27)$$

Substituting (26) into (27), we obtain

$$V_s = \pi V_o \qquad (28)$$

To satisfy (20) and (28) at once, we can finally deduce the RF input impedance and dc output power as

$$Z_{\text{in}} = \frac{V_s}{I_s} = \frac{\pi^2}{2} R_o \qquad (29)$$

$$V_o I_o = \frac{1}{2} V_s I_s \qquad (30)$$

where $R_o = V_o/I_o$ designates the dc load resistance. Because (30) is identical to (15), we conclude again that the LC resonator works for a complete harmonic reaction, despite of its simple structure.

6. Full-bridge rectifier

The series LC resonator used in Fig. 2 appears now again to work for the full-bridge rectifier. This rectifier employs quadruplet diodes as shown in Fig. 5. Topological symmetry makes the quadruplets operate as two pairs of twins. We assume the sinusoidal RF input voltage same as in (1). For PHC, D_1 and D_4 stay off, whereas D_2 and D_3 stay on. For NHC, the two pairs precisely alternate. The capacitor C_∞ always maintains a constant output voltage. Overall, the voltage observed across each diode creates a 50% duty rectangular waveform, formulated as

$$v_{D1}(t) = v_{D4}(t) = \begin{cases} V_o & \text{for PHC} \\ 0 & \text{for NHC} \end{cases} \qquad (31)$$

$$v_{D2}(t) = v_{D3}(t) = \begin{cases} 0 & \text{for PHC} \\ -V_o & \text{for NHC} \end{cases} \qquad (32)$$

where V_o denotes the dc output voltage.

Because the LC resonator is tuned at ω, the input voltage must balance with D_1 plus D_3 in their fundamental harmonic components, extracted as

$$V_s = \frac{2}{T}\int_0^T \{v_{D1}(t)+v_{D3}(t)\}\sin\omega t\, dt \qquad (33)$$

from the Fourier series. Adopting (31) and (32) into the integral, the right-hand side results in

$$= \frac{4}{\pi}V_o \qquad (34)$$

We notice that (34) is exactly twice the voltage described in (5). This is to say, the double-voltage rectifier needs only half the RF input voltage to produce the same dc output voltage as the full bridge does.

For the same reason as in (6), the input RF current exhibits a sinusoidal waveform as

$$i_s(t) = I_s \sin\omega t \qquad (35)$$

In terms of each diode current, Kirchhoff's current law presents

$$i_s(t) = i_{D2}(t) - i_{D1}(t) = i_{D3}(t) - i_{D4}(t) \qquad (36)$$

$$i_{D1}(t) + i_{D3}(t) = i_{D2}(t) + i_{D4}(t) = i_{C\infty}(t) + I_o \qquad (37)$$

at the input and output ports, respectively. Adopting (35) into (36) and (37) solves them as

$$i_{D1}(t) = i_{D4}(t) = \begin{cases} 0 \\ -I_s\sin\omega t \end{cases} \qquad (38)$$

$$i_{D2}(t) = i_{D3}(t) = \begin{cases} I_s\sin\omega t \\ 0 \end{cases} \qquad (39)$$

$$i_{C\infty}(t) = \begin{cases} I_s\sin\omega t - I_o \\ -I_s\sin\omega t - I_o \end{cases} \qquad (40)$$

where the upper and lower cases apply to PHC and NHC, respectively.

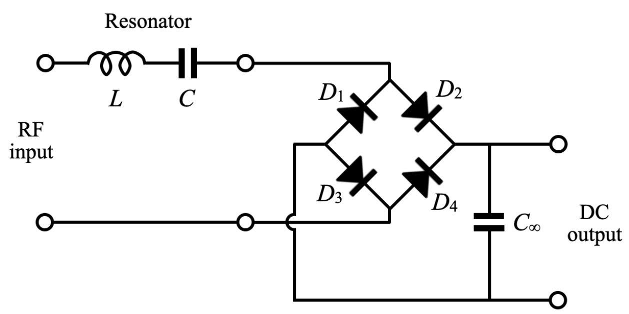

Fig. 5 Series resonator and full-bridge rectifier

The rule (12) applies again to C_∞ as

$$\int_0^T i_{C\infty}(t)dt = 0 \qquad (41)$$

Substituting (40) into (41), we obtain

$$I_s = \frac{\pi}{2}I_o \qquad (42)$$

From (34) and (42), we can finally deduce the RF input impedance and the dc output power as

$$Z_{in} = \frac{V_s}{I_s} = \frac{8}{\pi^2}R_o \qquad (43)$$

$$V_o I_o = \frac{1}{2}V_s I_s \qquad (44)$$

where $R_o = V_o/I_o$ designates the dc load resistance. Because (44) is identical to (15) and (30), we conclude once again that the LC resonator works for a complete harmonic reaction.

Table I Theoretical performance of resonant harmonic reaction rectifiers

Topology		Double voltage	Double current	Full bridge
LC resonator		Series	Parallel	Series
Input impedance	Z_{in}	$\frac{2}{\pi^2}R_o$	$\frac{\pi^2}{2}R_o$	$\frac{8}{\pi^2}R_o$
Voltage gain	$\frac{V_o}{V_s}$	$\frac{\pi}{2}$	$\frac{1}{\pi}$	$\frac{\pi}{4}$
Current gain	$\frac{I_o}{I_s}$	$\frac{1}{\pi}$	$\frac{\pi}{2}$	$\frac{2}{\pi}$
Power conversion efficiency		100%		

7. Conclusion

We have explored the behavior of LC resonators added to the front of three typical rectifiers. Time-domain circuit analysis rigorously deduced the input impedance, voltage and current gains, and power conversion efficiency of each rectifier. The resultant formulas are concisely summarized in Table I. These formulas are indispensable when we implement the rectifiers into practical RF systems with good interface matching.

Since the theoretical power conversion efficiency is found to be 100% common in the three rectifiers, we conclude that the simple LC resonator works for a full harmonic reaction. This outcome offers a simpler alternative approach to high-efficiency RF power rectification as compared with the typical class-E and -F solutions.

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