Event-Triggered Adaptive Finite-Time Control for Switched Cyberphysical Systems With Uncertain Deception Attacks

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***Abstract*—This article is concerned with the event-triggered adaptive finite-time control problem for a class of switched cyber-physical systems (CPSs) subject to uncertain deception attacks. We deploy the finite time command filter to alleviate the possible explosion of complexity rendered by the virtual control signal's duplicative differentiation. To stabilize the target system with a guaranteed disturbance rejection performance index, a disturbance observer is constructed by using fuzzy logic systems (FLS) with the hope of estimating the unknown external disturbance. Furthermore, the Nussbaum gain is utilized to resist the unknown control efficiency generated by uncertain deception attacks. Based on the common Lyapunov function (CLF) method, sufficient conditions are derived to ensure that all signals in the closed-loop system are uniformly bounded at a finite time. Finally, the simulation example shows the validity and efficacy of the developed control method.**

***Index Terms*—Backlash-like hysteresis, event-triggered strategy, finite time control, nussbaum gain, uncertain deception attacks.**

I. Introduction

THE classical view of cyber-physical systems (CPSs) arises from the seamless integration of computational, communication, and physical components, showing better performance in deploying, maintaining, and saving systems resources [1]. Recently, the controller design problem of cyber-physical systems has drawn expansive research interest because of its strong capability in many realistic scenarios, including smart grids, military defense, and home automation [2], [3]. Resilient control is a fundamental investigation issue for the dynamic analysis of CPSs since acquiring an accurate state target is often crucial for many practical assignments, such as computing and communication. However, some common distractions pose tremendous resilient control challenges, such as time delay, external disturbance, and cyberattacks. Generally speaking, there are three primary sorts of cyberattacks against CPS, including replay attacks [4], denial-of-service (DoS) attacks [5], and deception attacks [6]. Among them, deception attacks act on the control system by injecting false dates into the actual system states, and it will cause the sensors and actuators to receive false signals. Therefore, under deception attacks, there is no doubt that the system performance will be seriously influenced, even devastated [7], [8], [9], [10], [11], [12]. For linear CPSs, Xu et al. in [8] proposed an adaptive control architecture to tackle actuator and sensor attacks, in which the whole system signals are uniformly ultimately bounded. As for the nonlinear CPSs, an adaptive resilient control scheme was introduced in [9] where the unknown state-delay was addressed by using the Lyapunov-krasovskii analysis method. Furthermore, for cases where only the output is available, an adaptive output feedback control was presented in [10] for the high-order CPSs with sensor and actuator attacks, in which k-filter was utilized to construct the state observer.

In most existing outcomes concerning the resilient control problem of CPSs, the adaptive controller has been synthesized based on the desired communication channel from all the network nodes of cyber-physical systems. However, such a set of availability is sometimes unrealistic when the bandwidth of the channel is limited, especially when the size of the CPSs becomes enormous [13], [14]. Therefore, it is rational to consider the resilient control problem for CPSs with switching topology. In the past few decades, a great deal of remarkable research have been carried out about switched single-input and single-output (SISO) systems [15], switched multiple-input and multiple-output (MIMO) systems [16], switched pure-feedback systems [17] and switched multiagent systems [18]. It is worth mentioning that FLSs and neural networks are the common approximators, which are wildly applied in control systems [19], [20], [21], [22]. In [23], an adaptive neural fault-tolerant tracking control method was presented for a class of switched nonlinear

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systems where the unmodeled dynamics are identified by radial basis function neural networks (RBFNNs). So as to construct the state and fault observer, Cheng et al. in [24] proposed an adaptive fuzzy approach for a class of switched systems subject to actuator and sensor faults where the switching signal must satisfy the limitation of the average dwell time. More notable is that the above achievements pertain to the sort of nonfinite time control.

Most available consequences of resilient control of CPSs have focused on the asymptotic or exponential stability as a steady-state manner described over an infinite time horizon. Nevertheless, it is often more practical for a controlled system to retain preferred transient properties (e.g., quick reaction, high tracking accuracy, and finite-time convergence) over a finite time interval in many practical applications. Thus, the finite-time control problem has recently become a vital investigation issue attracting particular interest [25], [26], [27], [28]. In [26], an observer-based adaptive finite time containment control method was presented for nonlinear input-delay MASs, in which the finite-time convergence was accomplished. For pure-feedback switched systems, Zhou et al. in [27] proposed a prescribed-performance-based adaptive finite-time control scheme where the dynamic surface control (DSC) technique was utilized to eliminate the negative impact of the explosion of the differentiation. Similar to DSC, the command filtering method is also an effective tool to cope with the complexity caused by the reduplicative differentiation of the virtual controller. A command-filter-based adaptive finite time tracking control was introduced in [28], in which all system states are unmeasurable.

Traditionally, the resilient control algorithms for CPSs are performed occasionally with the time-triggered strategy. However, while notably lessening the complexity in controller synthesis, the time-triggered mechanism may lead to excessive consumption of the limited resource (e.g., network bandwidth) because of some needless signal transmissions. Therefore, for the resource-saving purpose, an alternative is to utilize the so-called event-triggered mechanism, whose idea is to trigger the signal transmission only when certain events occur, thereby mitigating the network communication burden. In the past few years, the event-triggered control mechanism has attracted boosting research interests and has widely utilized in the controller design of various systems, including stochastic systems [29], [30], constrained systems [31], pure-feedback systems [32] and large scale systems [33]. As such, it is of both theoretical significance and practically necessary to explore the adaptive finite-time control algorithms for switched cyber-physical systems with uncertain deception attacks with an event-triggered mechanism.

Inspired by the above discussions, there is a practical necessity to explore the finite-time stability problem for a class of switched cyber-physical systems subject to uncertain deception attacks and backlash-like hysteresis. Specifically, the event-triggered adaptive fuzzy finite time control strategy is presented such that all state signals of the CPSs are uniformly bounded over finite time. In contrast with the extant relative works, the main contribution contains three folds:

- Distinguished from the works in [9] and [34], we introduce the Nussbaum gain function to replace the unknown control coefficients induced by the uncertain deception attacks. Moreover, the finite-time command filter is utilized to avoid the explosion of complexity, and the filtering error is counteracted by designing the compensating signal systems.
- Unlike the previous contributions [35], this article considers a general case of CPSs with the personal deception attack of each subsystem. In addition, the compounded disturbance observer based on the compromised states is developed to tackle the input hysteresis.
- To lessen the communication burden of CPSs, we introduce a switching threshold event-triggered mechanism, providing more flexibility in balancing the communication burden and control performance. Furthermore, based on the Lyapunov function method, all state signals in switched CPSs are uniformly bounded over finite time.

II. Problem Formulation and Preliminaries

A. System Model

Consider the following nonlinear cyberphysical system subject to uncertain deception attacks and backlash-like hysteresis:

$$\begin{cases}\dot{x}_1 = f_{1,\sigma(t)}(x_1) + x_2\\ \dot{x}_i = f_{i,\sigma(t)}(\bar{x}_i) + x_{i+1}\\ \dot{x}_n = f_{n,\sigma(t)}(\bar{x}_n) + u_s(v) + \varrho_u(\bar{x}_n, t) + d(\bar{x}_n, t)\\ \breve{x}_k = x_k + \varrho_x(x_k, t)\\ y = x_1\end{cases} \tag{1}$$

where $\bar{x}_i = [x_1, \ldots, x_i]^{\mathrm{T}}$ denotes the state variable with $i = 2, \ldots, n-1$, u_s, y, ϱ_u are the control input, system output and actuator attack respectively. $d(\bar{x}_n, t)$ represents the unknown but bounded external disturbance. $\breve{x}_k$ refers to as the state variable compromised by the sensor attacks, $\varrho_x(x_k, t)$ represents the uncertain sensor deception attack with $k = 1, \ldots, n$. $f_{k,\sigma(t)}$ denotes the unknown but smooth nonlinear function, and $\sigma(t) : [0, +\infty) \to \mathbb{M}_i = 1, \ldots, M_i$ is the switching signal. We claim that the sth switched subsystem is activated when $\sigma(t) = s$. $u_s(v)$ indicates the backlash-like hysteresis type of nonlinear impacts on actuator, which can be depicted as $\frac{du_s}{dt} = \omega|\frac{dv}{dt}|(\varphi v(t) - u_s) + \xi\frac{dv}{dt}$ with v being the input of backlash-like hysteresis, and ω, φ, ξ being unknown positive constants, φ satisfies $\varphi \geq \xi$. Based on the results in [36], it yields

$$u_s(v) = \varphi v(t) + h(v), \tag{2}$$

where $h(v) = (u_s(0) - \varphi v(0))e^{-\omega(v - v(0))\mathrm{sign}\dot{v}} + e^{-\omega v \mathrm{sign}\dot{v}}\int_{v(0)}^{v}(\xi - \varphi)e^{-\omega\tau\mathrm{sign}\dot{v}}d\tau$ is bounded, such that $h(v) \leq \bar{h}$, $\bar{h}$ is unknown positive constant.

Therefore, model (1) can be recast as

$$\begin{cases}\dot{x}_i = f_{i,\sigma(t)}(\bar{x}_i) + x_{i+1}\\ \dot{x}_n = f_{n,\sigma(t)}(\bar{x}_n) + \varphi_0 v(t) + g(v) + d(\bar{x}_n, t)\\ y = x_1\end{cases} \tag{3}$$

where $g(v) = (\varphi - \varphi_0)v(t) + \varrho_u(\bar{x}_n, t) + h(v)$, φ_0 is the known constant.

Control Objective: The primary objective of this article is to establish an event-triggered adaptive fuzzy finite-time control strategy for a class of nonlinear switched CPSs subject to unknown deception attacks and backlash-like hysteresis such that all signals converge to the neighbor of the origin at a finite time.

To achieve this objective, some necessary lemmas and assumptions are listed.

Aussmption 1: Suppose that $\varrho_x(x_i,t)$ and $\varrho_u(\bar{x}_n,t)$ satisfy $\varrho_x(x_i,t)=a_i(t)x_i(t)$, and $\varrho_u(t,\bar{x}_n(t))=\beta(t)\Xi(\bar{x}_n(t))$ with $i=1,2,\ldots,n$, respectively, where $a_i(t)$, $\beta(t)$ are the unknown time-varying signals, and $\Xi(\bar{x}_n)$ is the unknown continuous nonlinear function. In addition, the variable a_i should satisfy $1+a_i(t)\neq 0$. Without loss of generality, we assume that the sign of $1+a_i(t)$ is positive. Thus, there exists two unknown positive constants $\bar{a}_i$, $\underline{a}_i$ and $\bar{\beta}_i$ satisfying $\underline{a}_i\leq a_i(t)\leq \bar{a}_i$ and $\beta(t)\leq\bar{\beta}$.

Remark 1: In this article, we make the following coordinate transformation: $\lambda_i=1/(1+a_i(t))$. Then, it is obvious that the sign of λ_i is always positive and there exists unknown positive constants $\underline{\lambda}_i$, $\bar{\lambda}_i$ and $\bar{\lambda}_{i,d}$ such that $\underline{\lambda}_i\leq\lambda_i\leq\bar{\lambda}_i$, and $|\dot{\lambda}_i|\leq\bar{\lambda}_{i,d}$. In addition, Assumption 1 roots in the previous results [8], [12] about linear CPSs, it plays a important role in maintaining the controllability of the systems subject to the unknown deception attacks. To be specific, the aim of Assumption 1 is to exclude the situation $1+a_i(t)=0$, which means that the compromised state $\breve{x}_i=(1+a_i(t))x_i=0$ such that the designed controller will lose efficacy.

Lemma 1: [20] For $\varpi>0$, $\psi>0$ and $\gamma>0$,

$$|A|^{\varpi}|B|^{\psi}\leq\frac{\varpi}{\varpi+\psi}\gamma|A|^{\varpi+\psi}+\frac{\psi}{\varpi+\psi}\gamma^{-\frac{\varpi}{\psi}}|B|^{\varpi+\psi},$$

where A and B are any real variables.

Lemma 2: [37] There are positive constant δ and variable μ, such that

$$0\leq|\mu|-\mu\tanh\left(\frac{\mu}{\delta}\right)\leq\kappa\delta,$$

where $\kappa=0.2785$.

Lemma 3: [25] If the smooth positive-definite function $W(t)$ conforms to

$$\dot{W}(t)\leq -aW(t)-bW^q(t),\forall t\geq t_0,$$

where $0<q<1$, a, b are positive constants, then, $W(t)$ will converge to the equilibrium point over finite time T, in which $T\leq t_0+(1/(a(1-q)))\ln(aV^{1-q}(t_0)+b)/b$.

Lemma 4: [21] Based on the approximation property of FLSs, it is widely applied in the design of adaptive controller. A FLS mainly includes fuzzy inference engine, knowledge base, defuzzifier, and fuzzifier. For any consecutive function $g(\omega)$, which is defined on the compact set Ξ, FLSs can achieve the following performance:

$$g(\omega)=\theta^T\psi(\omega)+\varepsilon,$$

where ε denotes the bounded approximation error satisfying $|\varepsilon|\leq\epsilon$, and ϵ is an unknown constant.

B. Nussbaum Gain

In this article, the unknown control coefficients caused by the deception attacks is addressed with using the Nussbaum gains. A continuous function $N(\Psi):\mathbb{R}\to\mathbb{R}$ can be declared to be a function of Nussbaum type, when it satisfies

$$\lim_{s\to\infty}\sup\frac{1}{s}\int_0^s N(\Psi)d\Psi=+\infty,$$

$$\lim_{s\to\infty}\inf\frac{1}{s}\int_0^s N(\Psi)d\Psi=-\infty.$$

Such as, the continuous functions $N(\Psi)=\Psi^2\cos(\Psi)$ and $e^{\Psi^2}\cos(\pi/2\Psi)$ are applied in [22], [38].

Lemma 5: [22] Suppose $V(\cdot)\geq 0$ and Ψ are smooth functions defined on $[0,t_f)$, $N(\cdot)$ is a smooth Nussbaum-type function. When the undermentioned condition is met

$$V(t)\leq c_0+e^{-c_1 t}\int_0^t[h(\cdot)N(\Psi)+1]\dot{\Psi}e^{c_1\xi}d\xi,$$

where $h(\cdot)$ is a time-varying bounded parameter which takes values in the unknown closed intervals $L=[p-,p+]$ with $0\notin I$, and c_0 denotes a suitable constant, c_1 is a nonnegative constant. Then $V(t)$, $\int_0^t h(\cdot)N(\Psi)\dot{\Psi}e^{c_1\xi}d\xi$ must be bounded on $[0,t_f)$.

III. Main Results

In this part, an adaptive fuzzy event-triggered-based finite-time control strategy will be proposed to resist the uncertain deception attacks. To start with, the following coordinate transformation is defined:

$$\begin{cases}z_1=x_1\\ z_i=x_i-\lambda_i\alpha_{i-1}^f, i=2,\ldots,n,\end{cases}\tag{4}$$

where α_{i-1}^f denotes the output of filter with the virtual controller α_{i-1} as the input. Because of the unknown deception attacks, the real state variable x_i is not available such that z_i can not be employed to design the controllers and adaptive laws. Thus, relying on available state and virtual control input, we construct a new coordinate transformation as follows:

$$\begin{cases}\breve{z}_1=\breve{x}_1\\ \breve{z}_i=\breve{x}_i-\alpha_{i-1}^f,\end{cases}\tag{5}$$

where $\breve{z}_i$ plays a vital role during the design of control and adaptive laws.

To prevent the burst of differentiation, an finite-time command filtering technique will be employed

$$\begin{cases}\dot{\phi}_{i0}=l_{i0}\\ l_{i0}=-\eta_{i0}|\phi_{i0}-\alpha_i|^{\frac{1}{2}}\mathrm{sign}(\phi_{i0}-\alpha_i)+\phi_{i1}\\ \dot{\phi}_{i1}=-\eta_{i1}\mathrm{sign}(\phi_{i1}-l_{i0}),\end{cases}\tag{6}$$

where η_{i0},η_{i1} are the positive design parameters, α_i is the virtual control signal. From [39], the terms $|\phi_{i0}-\alpha_i|^{\frac{1}{2}}\mathrm{sign}(\phi_{i0}-\alpha_i)$ and $\mathrm{sign}(\phi_{i1}-l_{i0})$ in (6) can ensure that the filter can remain finite-time stable, when the parameters η_{i0} and η_{i1} are chosen properly.

As the order of system dynamic increasing, the error caused by the filter will bring serious impact on control performance. In order to eliminate the filtering error, let $e_i=\alpha_i^f-\alpha_i$, we define the following filtering error compensating signal Λ:

$$\begin{cases}\dot{\Lambda}_i=r_{i1}\Lambda_i-r_{i2}\Lambda_i^{2q-1}-W_i\Lambda_i+e_i\\ \dot{\Lambda}_n=r_{n1}\Lambda_n-r_{n2}\Lambda_n^{2q-1}, i=1,\ldots,n-1\end{cases}\tag{7}$$

where $q = (2\bar{p}-1)/(2\bar{p}+1)$ and r_{ij} are positive design parameters with $j = 1,2,3$, $\bar{p} \geq 2$; $W_i = (|\breve{z}_i \Lambda_i| + r_{i3} e_i^2)/(||\Lambda_i||^2)$.

Next, the recursive procedure for the event-triggered adaptive fuzzy control will be given.

Step 1: Construct the first Lyapunov candidate function:

$$V_1 = \frac{1}{2} z_1^2 + \frac{1}{2}\tilde{\Theta}_1^2 + \frac{1}{2}\Lambda_1^2, \tag{8}$$

where $\tilde{\Theta}_1$ denotes the estimation error of Θ_1, and it will be given later.

According to the (4), the derivative of z_i can be obtained:

$$\begin{aligned} \dot{z}_i &= f_{1,s}(x_1) + x_2 \\ &= f_{1,s}(x_1) + z_2 + \lambda_2 \alpha_1^f. \end{aligned} \tag{9}$$

Therefore, the first time derivative of V_1 has

$$\begin{aligned} \dot{V}_1 &= z_1(f_{1,s}(x_1) + z_2 + \lambda_2 \alpha_1^f) + \tilde{\Theta}_1 \dot{\tilde{\Theta}}_1 + \Lambda_1 \dot{\Lambda}_1 \\ &= z_1(z_2 + F_{1,s}(Z_1) + \lambda_2 \alpha_1) + \tilde{\Theta}_1 \dot{\tilde{\Theta}}_1 + \Lambda_1 \dot{\Lambda}_1 - r_{11} z_1^{2q} \\ &\quad - \left(c_1 + \frac{3}{2}\right) z_1^2 + c_1 \breve{z}_1^2, \end{aligned} \tag{10}$$

where $F_{1,s} = (c_1 + 3/2) z_1 + \lambda_2 e_1 + r_{12} z_1^{2q-1} - c_1 z_1/\lambda_1^2 + f_{1,s}(x_1)$, $Z_1 = [z_1, x_1, \lambda_1]^T$, $\breve{Z}_1 = [\breve{z}_1, \breve{x}_1]^T$. Draw support from the approximation property of FLSs, we can obtain

$$\begin{aligned} F_{1,s} &= \theta_{1,s}^T \psi_1(Z_1) + \varepsilon_{1,s}(Z_1) \\ &= \theta_{1,s}^T \psi_1(\breve{Z}_1) + \varepsilon_{1,s}(Z_1) + \chi_{1,s}, \end{aligned} \tag{11}$$

where $\chi_{1,s} = \theta_{1,s}^T(\psi_1(Z_1) - \psi_1(\breve{Z}_1))$ is bounded according to Lemma 4, which satisfies $|\chi_1| \leq \bar{\chi}_1$.

Based on the Young's inequality, we have

$$\begin{aligned} z_1 \theta_{1,s}^T \psi_1(\breve{Z}_1) &\leq \frac{||\theta_{1,s}||^2 z_1^2 \psi_1^T(\breve{Z}_1)\psi_1(\breve{Z}_1)}{2\,k_1^2} + \frac{1}{2}k_1^2 \\ &\leq \frac{\Theta_1 \breve{z}_1^2 \psi_1^T(\breve{Z}_1)\psi_1(\breve{Z}_1)}{2\,k_1^2} + \frac{1}{2}k_1^2, \end{aligned} \tag{12}$$

$$z_1(\varepsilon_{1,s} + \chi_{1,s}) \leq z_1^2 + \frac{1}{2}\bar{\varepsilon}_1^2 + \frac{1}{2}\bar{\chi}_1^2, \tag{13}$$

$$-W_1 \Lambda_1 \leq -|\breve{z}_1 \Lambda_1| - r_{13} e_1^2, \tag{14}$$

$$\Lambda_1 e_1 \leq \frac{1}{2}\Lambda_1^2 + \frac{1}{2}e_1^2, \tag{15}$$

$$z_1 z_2 \leq \frac{1}{2} z_1^2 + \frac{1}{2} z_2^2, \tag{16}$$

where k_1 is a constant, $\Theta_1 = \max_{s\in\sigma(t)} \bar{\lambda}_1^2 ||\theta_{1,s}||^2$, $\hat{\Theta}_1 = \Theta_1 - \tilde{\Theta}_1$ is its estimation value, $\bar{\varepsilon}_1^2 = \max_{s\in\sigma(t)} ||\bar{\varepsilon}_{1,s}||^2$, and $\bar{\chi}_1^2 = \max_{s\in\sigma(t)} ||\chi_{1,s}||^2$.

Substituting (12)–(16) into (10), we have

$$\begin{aligned} \dot{V}_1 \leq &-r_{12} z_1^{2q} - c_1 z_1^2 + c_1 \breve{z}_1^2 + \lambda_2 z_1 \alpha_1 + \frac{1}{2} z_2^2 - |\breve{z}_1 \Lambda_1| + \Delta_1' \\ &+ \frac{\hat{\Theta}_1 \breve{z}_1^2 \psi_1^T(\breve{Z}_1)\psi_1(\breve{Z}_1)}{2\,k_1^2} - r_{11}\Lambda_1^2 - r_{12}\Lambda_1^{2q} - \bar{r}_{13} e_1^2 \\ &+ \tilde{\Theta}_1 \left(\frac{\breve{z}_1^2 \psi_1^T(\breve{Z}_1)\psi_1(\breve{Z}_1)}{2\,k_1^2} - \dot{\hat{\Theta}}_1 \right), \end{aligned} \tag{17}$$

where $\bar{r}_{13} = r_{13} - 1/2$, $\Delta_1' = (1/2)k_1^2 + (1/2)\bar{\varepsilon}_1^2 + (1/2)\bar{\chi}_1^2$.

Design the first controller α_1, adaptive law $\hat{\Theta}_1$, and auxiliary variable Φ_1 as

$$\alpha_1 = N(\Phi_1)\left(c_1 \breve{z}_1 + \frac{\hat{\Theta}_1 \breve{z}_1 \psi_1^T(\breve{Z}_1)\psi_1(\breve{Z}_1)}{2\,k_1^2} - \Lambda_1 \right), \tag{18}$$

$$\dot{\Phi}_1 = c_1 \breve{z}_1^2 + \frac{\hat{\Theta}_1 \breve{z}_1^2 \psi_1^T(\breve{Z}_1)\psi_1(\breve{Z}_1)}{2\,k_1^2} - \breve{z}_1 \Lambda_1, \tag{19}$$

$$\dot{\hat{\Theta}}_1 = \frac{\breve{z}_1^2 \psi_1^T(\breve{Z}_1)\psi_1(\breve{Z}_1)}{2\,k_1^2} - \sigma_1 \hat{\Theta}_1, \tag{20}$$

where c_1, σ_1 and k_1 are positive constant to be determined.

According to the above design, (17) can be rewritten as

$$\begin{aligned} \dot{V}_1 \leq &-r_{12} z_1^{2q} - c_1 z_1^2 - r_{11}\Lambda_1^2 - r_{12}\Lambda_1^{2q} - \bar{r}_{13} e_1^2 + \frac{1}{2} z_2^2 \\ &+ (g_1 N(\Phi_1) + 1)\dot{\Phi}_1 + \sigma_1 \hat{\Theta}_1 \tilde{\Theta}_1 + \Delta_1', \end{aligned} \tag{21}$$

where $g_1 = \lambda_1(t)\lambda_2(t)$.

Applying Lemma 1 and the Young's inequality, it yields

$$\sigma_1 \hat{\Theta}_1 \tilde{\Theta}_1 \leq \bar{\sigma}_1 \Theta_1^2 - \bar{\sigma}_1 \tilde{\Theta}_1^2, \tag{22}$$

$$\bar{\sigma}_1 \left(\frac{1}{2}\tilde{\Theta}_1^2\right)^q \leq \frac{\bar{\sigma}_1}{2}\tilde{\Theta}_1^2 + \bar{\sigma}_1 (1-q) q^{1-q}, \tag{23}$$

where $\bar{\sigma}_1 = (1/2)\sigma_1$.

Substituting (22) and (23) into (21), we can obtain

$$\begin{aligned} \dot{V}_1 \leq &-r_{12} z_1^{2q} - c_1 z_1^2 - r_{11}\Lambda_1^2 - r_{12}\Lambda_1^{2q} - \bar{r}_{13} e_1^2 + \frac{1}{2} z_2^2 \\ &- \frac{\bar{\sigma}_1}{2}\tilde{\Theta}_1^2 - \bar{\sigma}_1\left(\frac{1}{2}\tilde{\Theta}_1^2\right)^q + (g_1 N(\Phi_1) + 1)\dot{\Phi}_1 + \Delta_1, \end{aligned} \tag{24}$$

where $\Delta_1 = \Delta_1' + \bar{\sigma}_1 \Theta_1^2 + \bar{\sigma}_1 (1-q) q^{\frac{q}{1-q}}$.

Step 2: Choosing the 2th Lyapunov candidate function

$$V_2 = V_1 + \frac{1}{2} z_2^2 + \frac{1}{2}\tilde{\Theta}_2^2 + \frac{1}{2}\Lambda_2^2, \tag{25}$$

where $\tilde{\Theta}_2$ denotes the estimation error of Θ_2, which will be defined later.

Based on (4), $\dot{z}_2$ can be represented as

$$\begin{aligned} \dot{z}_2 &= \dot{x}_2 - \dot{\lambda}_2 \alpha_1^f - \lambda_2 \dot{\alpha}_1^f \\ &= z_3 + \lambda_3 \alpha_2^f + f_{2,s}(\bar{x}_2) - \dot{\lambda}_2 \alpha_1^f - \lambda \dot{\alpha}_1^f. \end{aligned} \tag{26}$$

Therefore, $\dot{V}_2$ can be obtained

$$\begin{aligned} \dot{V}_2 &= \dot{V}_1 + z_2(z_3 + \lambda_3 \alpha_2^f + f_{2,s}(\bar{x}_2) - \dot{\lambda}_2 \alpha_1^f - \lambda \dot{\alpha}_1^f) \\ &\quad + \tilde{\Theta}_2 \dot{\tilde{\Theta}}_2 + \Lambda_2 \dot{\Lambda}_2 \\ &= \dot{V}_1 - r_{22} z_2^{2q-1} - c_2 z_2^2 + z_2(z_3 + \lambda_3 \alpha_2 + F_{2,s}(Z_2)) \\ &\quad + \tilde{\Theta}_2 \dot{\tilde{\Theta}}_2 + \Lambda_2 \dot{\Lambda}_2 + c_1 \breve{z}_2^2 - 2 z_2^2, \end{aligned} \tag{27}$$

where $F_{2,s}(Z_2) = r_{22}z_2^{2q-1} + c_2z_2 + \lambda_3 e_2 + f_{2,s}(\bar{x}_2) - \dot{\lambda}_2\alpha_1^f - \lambda\dot{\alpha}_1^f + 2z_2 - c_2z_2/\lambda_2^2$, c_2 is a positive constant.

Similar with Step 1, we have

$$F_{2,s}(Z_2) = \theta_{2,s}^T\psi_2(\breve{Z}_2) + \varepsilon_{2,s} + \chi_{2,s}, \tag{28}$$

where $\chi_{2,s} = \theta_{2,s}(\psi_2(Z_2) - \psi_2(\breve{Z}_2))$, $Z_2 = [z_2, x_2, x_1, \lambda_2, \hat{\Theta}_1]^T$ is not available, $\breve{Z}_2 = [\breve{z}_2, \breve{x}_2, \breve{x}_1, \hat{\Theta}_1]^T$ can be obtained.

By utilizing Young's inequality, it yields

$$\begin{aligned} z_2(F_{2,s}(\breve{Z}_2) + \varepsilon_{2,s} + \chi_{2,s}) &\le \frac{||\theta_{2,s}||^2 z_2^2\psi_2^T(\breve{Z}_2)\psi_2(\breve{Z}_2)}{2\,k_2^2} \\ &+ \frac{1}{2}k_2^2 + \frac{1}{2}\bar{\varepsilon}_{2,s}^2 + \frac{1}{2}\bar{\chi}_{2,s}^2 + z_2^2 \\ &\le \frac{\Theta_2\breve{z}_2^2\psi_2^T(\breve{Z}_2)\psi_2(\breve{Z}_2)}{2\,k_2^2} + \frac{1}{2}k_2^2 \\ &+ \frac{1}{2}\bar{\varepsilon}_2^2 + \frac{1}{2}\bar{\chi}_2^2 + z_2^2, \end{aligned} \tag{29}$$

$$-W_2\Lambda_2 \le -|\breve{z}_2\Lambda_2| - r_{23}e_2^2, \tag{30}$$

$$\Lambda_2 e_2 \le \frac{1}{2}\Lambda_2^2 + \frac{1}{2}e_2^2, \tag{31}$$

$$z_2z_3 \le \frac{1}{2}z_2^2 + \frac{1}{2}z_3^2, \tag{32}$$

where $\Theta_2 = \max_{s\in\sigma(t)} \bar{\lambda}_2^2||\theta_{2,s}||^2$, $\hat{\Theta}_2 = \Theta_2 - \tilde{\Theta}_2$ denotes the estimation value of Θ_2, $\bar{\varepsilon}_2^2 = \max_{s\in\sigma(t)} ||\bar{\varepsilon}_{2,s}||^2$, $\bar{\chi}_2^2 = \max_{s\in\sigma(t)} ||\chi_{2,s}||^2$, and $k_2 > 0$ is a constant.

Substituting (29)–(32) into (27), it yields

$$\begin{aligned} \dot{V}_2 \le\; & \dot{V}_1 - r_{22}z_2^{2q-1} - c_2z_2^2 + c_2\breve{z}_2^2 + \lambda_3 z_2\alpha_2 + \frac{1}{2}z_3^2 - |\breve{z}_2\Lambda_2| \\ &+ \frac{\hat{\Theta}_2\breve{z}_2^2\psi_2^T(\breve{Z}_2)\psi_2(\breve{Z}_2)}{2\,k_2^2} - r_{21}\Lambda_2^2 - r_{22}\Lambda_2^{2q} - \bar{r}_{23}e_2^2 \\ &+ \tilde{\Theta}_2\left(\frac{\breve{z}_2^2\psi_2^T(\breve{Z}_2)\psi_1(\breve{Z}_2)}{2\,k_2^2} - \dot{\hat{\Theta}}_2\right) + \frac{1}{2}k_2^2 + \frac{1}{2}\bar{\varepsilon}_{2,s}^2 \\ &+ \frac{1}{2}\bar{\chi}_{2,s}^2 - \frac{1}{2}z_2^2, \end{aligned} \tag{33}$$

where $\bar{r}_{23} = r_{23} - (1/2)$.

Next, the virtual controller α_2, auxiliary variable Φ_2, and adaptive law $\hat{\Theta}_2$ can be designed as:

$$\alpha_2 = N(\Phi_2)(c_2\breve{z}_2 + \frac{\hat{\Theta}_2\breve{z}_2\psi_2^T(\breve{Z}_2)\psi_2(\breve{Z}_2)}{2\,k_2^2} - \Lambda_2), \tag{34}$$

$$\dot{\Phi}_2 = c_2\breve{z}_2^2 + \frac{\hat{\Theta}_2\breve{z}_2^2\psi_2^T(\breve{Z}_2)\psi_2(\breve{Z}_2)}{2\,k_1^2} - \breve{z}_2\Lambda_2, \tag{35}$$

$$\dot{\hat{\Theta}}_2 = \frac{\breve{z}_2^2\psi_2^T(\breve{Z}_2)\psi_2(\breve{Z}_2)}{2\,k_2^2} - \sigma_2\hat{\Theta}_2, \tag{36}$$

where c_2, σ_2 and k_2 are positive design parameters.

Substituting (34)–(36) into (33), we have

$$\begin{aligned} \dot{V}_2 \le\; & \dot{V}_1 - r_{22}z_2^{2q} - c_2z_2^2 - r_{21}\Lambda_2^2 - r_{22}\Lambda_2^{2q} - \bar{r}_{23}e_2^2 + \frac{1}{2}z_3^2 \\ &+ (g_2N(\Phi_2)+1)\dot{\Phi}_2 + \sigma_2\hat{\Theta}_2\tilde{\Theta}_2 + \frac{1}{2}k_2^2 + \frac{1}{2}\bar{\varepsilon}_{2,s}^2 \\ &+ \frac{1}{2}\bar{\chi}_{2,s}^2 - \frac{1}{2}z_2^2, \end{aligned} \tag{37}$$

where $g_2 = \lambda_2(t)\lambda_3(t)$.

For the same reason with (22) and (23), it yields

$$\sigma_2\hat{\Theta}_2\tilde{\Theta}_2 \le \bar{\sigma}_2\Theta_2^2 - \bar{\sigma}_2\tilde{\Theta}_2^2, \tag{38}$$

$$\bar{\sigma}_2\left(\frac{1}{2}\tilde{\Theta}_2^2\right)^q \le \bar{\sigma}_2(1-q)q^{1-q} + \frac{\bar{\sigma}_2}{2}\tilde{\Theta}_2^2, \tag{39}$$

where $\bar{\sigma}_2 = (1/2)\sigma_2$.

Substituting (21), (38), (39) into (37), we can obtain

$$\begin{aligned} \dot{V}_2 \le \sum_{l=1}^{2}&(-r_{21}z_l^{2q} - c_lz_l^2 - r_{l1}\Lambda_l^2 - r_{l2}\Lambda_l^{2q} - \bar{r}_{l3}e_l^2 - \frac{\bar{\sigma}_l}{2}\tilde{\Theta}_l^2 \\ &- \bar{\sigma}_l\left(\frac{1}{2}\tilde{\Theta}_l^2\right)^q + (g_lN(\Phi_l)+1)\dot{\Phi}_l) + \frac{1}{2}z_2^3 + \Delta_2, \end{aligned} \tag{40}$$

where $\Delta_2 = \Delta_1 + (1/2)k_2^2 + (1/2)\varepsilon_2^2 + (1/2)\bar{\chi}_2^2 + \bar{\sigma}_1\Theta_2^2 + \bar{\sigma}_2(1-q)q^{\frac{q}{1-q}}$.

Step i: Define the ith Lyapunov candidate function as

$$V_i = V_{i-1} + \frac{1}{2}z_i^2 + \frac{1}{2}\tilde{\Theta}_i^2 + \frac{1}{2}\Lambda_i^2, \tag{41}$$

where $\tilde{\Theta}_i$ is the estimation error of Θ_i.

Similar with the above steps, we design the ith virtual control law α_i, auxiliary variable Φ_i, and adaptive law $\hat{\Theta}_i$ as follows:

$$\alpha_i = N(\Phi_i)\left(c_i\breve{z}_i + \frac{\hat{\Theta}_i\breve{z}_i\psi_i^T(\breve{Z}_i)\psi_i(\breve{Z}_i)}{2\,k_i^2} - \Lambda_i\right), \tag{42}$$

$$\dot{\Phi}_i = c_i\breve{z}_i^2 + \frac{\hat{\Theta}_i\breve{z}_i^2\psi_i^T(\breve{Z}_i)\psi_i(\breve{Z}_i)}{2\,k_i^2} - \breve{z}_i\Lambda_i, \tag{43}$$

$$\dot{\hat{\Theta}}_i = \frac{\breve{z}_i^2\psi_i^T(\breve{Z}_i)\psi_i(\breve{Z}_i)}{2\,k_i^2} - \sigma_i\hat{\Theta}_i, \tag{44}$$

where c_i, k_i and σ_i are positive constants to be designed.

According to the above equation, we have

$$\begin{aligned} \dot{V}_i \le \sum_{l=1}^{i}&\Big(-r_{l2}z_l^{2q} - c_lz_l^2 - r_{l1}\Lambda_l^2 - r_{l2}\Lambda_l^{2q} - \bar{r}_{l3}e_l^2 - \frac{\bar{\sigma}_l}{2}\tilde{\Theta}_l^2 \\ &- \bar{\sigma}_l\left(\frac{1}{2}\tilde{\Theta}_l^2\right)^q + (g_lN(\Phi_l)+1)\dot{\Phi}_l\Big) + \frac{1}{2}z_{i+1}^2 + \Delta_i, \end{aligned} \tag{45}$$

where $\Delta_i = \Delta_{i-1} + (1/2)k_i^2 + (1/2)\varepsilon_i^2 + (1/2)\bar{\chi}_i^2 + \bar{\sigma}_i\Theta_i^2 + \bar{\sigma}_i(1-q)q^{\frac{q}{1-q}}$, $\bar{\sigma}_i = \sigma_i/2$, and $g_i = \lambda_i(t)\lambda_{i+1}(t)$.

Step n: In view of (4), the first-order time derivative of z_n can be obtained

$$\begin{aligned}\dot{z}_n &= \dot{x}_n - \dot{\lambda}_n\alpha_n^f - \lambda_n\dot{\alpha}_n^f \\ &= f_{n,\sigma(t)}(\bar{x}_n) + \varphi_0 v(t) + \Upsilon_1 - \dot{\lambda}_n\alpha_n^f - \lambda_n\dot{\alpha}_n^f, \end{aligned} \tag{46}$$

where $\Upsilon_1 = g(v) + d(\bar{x}_n, t)$ can be viewed as the unknown and bounded disturbance, and there exists two positive constant satisfies $|\Upsilon_1| \leq \bar{\Upsilon}_1, |\dot{\Upsilon}_1| \leq \bar{\Upsilon}_{1,d}$. Inspired by [19], we will construct an disturbance observer, and (46) can be rewritten as

$$\begin{aligned}\dot{z}_n &= m_1^{-1}F_{n1,s}(Z_n) + \Upsilon_1 \\ &= m_1^{-1}(\theta_{n1,s}^T\psi_{n1}(\breve{Z}_n) + \varepsilon_{n1,s} + \chi_{n1,s}) + \Upsilon_1, \end{aligned} \tag{47}$$

where $F_{n1,s} = f_{n,\sigma(t)}(\bar{x}_n) + \varphi_0 v(t) - \dot{\lambda}_n\alpha_n^f - \lambda_n\dot{\alpha}_n^f$, $\chi_{n1,s} = \theta_{n1,s}(\psi_{n1}(Z_2) - \psi_{n1}(\breve{Z}_n))$, $Z_n = [z_n, x_1, \ldots, x_n, \hat{\Theta}_{n-1}, \lambda_n]^T$, $\breve{Z}_n = [\breve{z}_n, \breve{x}_1, \ldots, \breve{x}_n, \hat{\Theta}_{n-1}]^T$, and m_1^{-1} is a designed constant.

Next, we introduce a auxiliary variable $\hbar_1$:

$$\hbar_1 = \breve{z}_n - \xi_1 = \frac{z_n}{\lambda_n} - \xi_1, \tag{48}$$

the intermediate variable ξ is designed as

$$\dot{\xi}_1 = b_1\hbar_1 + \frac{\hat{\phi}_1\hbar_1\psi_{n1}^T(\breve{Z}_{n1})\psi_{n1}(\breve{Z}_{n1})}{2m_1k_{n1}^2}, \tag{49}$$

where b_1, k_{n1} are positive constant, $\hat{\phi}_1$ will be introduced later, and $\hat{\phi}_1 = \phi_1 - \tilde{\phi}_1$.

Therefore, the derivative of $\hbar_1$ can be denoted as

$$\begin{aligned}\dot{\hbar}_1 = &\frac{1}{\lambda_n}(m_1^{-1}(\theta_{n1,s}^T\psi_{n1}(\breve{Z}_n) + \varepsilon_{n1,s} + \chi_{n1,s}) + \Upsilon_1) \\ &- \frac{\dot{\lambda}_n}{\lambda_n^2}z_n - b_1\hbar_1 - \frac{\hat{\phi}_1\hbar_1\psi_{n1}^T(\breve{Z}_{n1})\psi_{n1}(\breve{Z}_{n1})}{2m_1k_{n1}^2}. \end{aligned} \tag{50}$$

Draw upon Young's inequality, the following formulas holds:

$$\begin{aligned}&\frac{\hbar_1}{m_1\lambda_n}(\theta_{n1,s}^T\psi_{n1}(\breve{Z}_n) + \varepsilon_{n1,s} + \chi_{n1,s}) \leq \hbar_1^2 + \frac{\bar{\chi}_{n1}^2}{2\,m_1^2\underline{\lambda}_n^2} \\ &+ \frac{\bar{\varepsilon}_{n1}^2}{2\,m_1^2\underline{\lambda}_n^2} + \frac{k_{n1}^2}{2m_1} + \frac{\phi_1\hbar_1^2\psi_{n1}^T(\breve{Z}_n)\psi_{n1}(\breve{Z}_n)}{2m_1k_{n1}^2}, \end{aligned} \tag{51}$$

$$\frac{\hbar_1\Upsilon_1}{\lambda_n} \leq \frac{1}{2}\hbar_1^2 + \frac{\bar{\Upsilon}_1^2}{2\underline{\lambda}_n^2}, \tag{52}$$

$$-\frac{\dot{\lambda}_n}{\lambda_n^2}z_n\hbar_1 \leq \frac{1}{2}\hbar_1^2 + \frac{\bar{\lambda}_{n,d}^2}{\underline{\lambda}_n^4}z_n^2, \tag{53}$$

where $\phi_1 = \max_{s\in\sigma(t)}\frac{||\theta_{n1,s}||^2}{\underline{\lambda}_n^2}$, $\bar{\varepsilon}_{n1}^2 = \max_{s\in\sigma(t)}||\bar{\chi}_{n1,s}||^2$, $\bar{\chi}_{n1}^2 = \max_{s\in\sigma(t)}||\bar{\chi}_{n1,s}||^2$.

According to the above analysis, we have

$$\begin{aligned}\hbar_1\dot{\hbar}_1 \leq &-(b_1 - 2)\hbar_1^2 + \frac{\tilde{\phi}_1\hbar_1^2\psi_{n1}^T(\breve{Z}_n)\psi_{n1}(\breve{Z}_n)}{2m_1k_{n1}^2} \\ &+ \frac{\bar{\lambda}_{n,d}^2}{\underline{\lambda}_n^4}z_n^2 + \Delta_{\hbar_1}, \end{aligned} \tag{54}$$

where $\Delta_{\hbar_1} = \bar{\chi}_{n1}^2/(2\,m_1^2\underline{\lambda}_n^2) + \bar{\varepsilon}_{n1}^2/(2\,m_1^2\underline{\lambda}_n^2) + k_{n1}^2/(2\,m_1^2) + \bar{\Upsilon}_1^2/(2\underline{\lambda}_n^2)$.

Design the parameter adaptive law $\hat{\phi}_1$ as

$$\dot{\hat{\phi}}_1 = \begin{cases}\frac{\hbar_1^2\psi_{n1}^T(\breve{Z}_n)\psi_{n1}(\breve{Z}_n)}{2m_1k_{n1}^2} - \sigma_{n1}\hat{\phi}_1, & \iota_1 \leq 0 \\ 0, & \iota_1 \geq 0\end{cases} \tag{55}$$

where $\iota_1 = (\hbar_1^2\psi_{n1}^T(\breve{Z}_n)\psi_{n1}(\breve{Z}_n)/2m_1k_{n1}^2) - \sigma_{n1}\hat{\phi}_1$, such that $\hat{\phi}_1$ is nonincreasing function, and $0 < \hat{\phi}_1 \leq \bar{\phi}_1 = \hat{\phi}_1(0)$, the initial condition of $\hat{\phi}_1(0)$ can be set manually.

Construct the disturbance observer $\hat{\Upsilon}_1$ as

$$\hat{\Upsilon}_1 = m_1(\hbar_1 - \eta_1), \tag{56}$$

where $\dot{\eta}_1 = -b_1\hbar_1 + \hat{\Upsilon}_1$.

Therefore, the derivative of $\hat{\Upsilon}_1$ can be represented as

$$\begin{aligned}\dot{\hat{\Upsilon}}_1 &= m_1\dot{\hbar}_1 - m_1\dot{\eta}_1 \\ &= \frac{1}{\lambda_n}((\theta_{n1,s}^T\psi_{n1}(\breve{Z}_n) + \varepsilon_{n1,s} + \chi_{n1,s}) + m_1\Upsilon_1)) \\ &\quad - \frac{m_1\dot{\lambda}_n}{\lambda_n^2}z_n - m_1\hat{\Upsilon}_1 - \frac{\hat{\phi}_1\hbar_1\psi_{n1}^T(\breve{Z}_n)\psi_{n1}(\breve{Z}_n)}{2\,m_1^2k_{n1}^2}, \end{aligned} \tag{57}$$

where $\dot{\tilde{\Upsilon}}_1 = \dot{\Upsilon}_1 - \dot{\hat{\Upsilon}}_1$.

According to the above equation, we have

$$\tilde{\Upsilon}_1\dot{\tilde{\Upsilon}}_1 = \tilde{\Upsilon}_1\dot{\Upsilon}_1 - \tilde{\Upsilon}_1\dot{\hat{\Upsilon}}_1. \tag{58}$$

The following formulas can be deduced with Young's inequality theorem:

$$-\frac{\tilde{\Upsilon}_1}{\lambda_n}(\theta_{n1,s}^T\psi_{n1}(\breve{Z}_n)) \leq \frac{l_1}{2}\tilde{\Upsilon}_1^2 + \frac{\phi_1}{2l_1\underline{\lambda}_n^2}, \tag{59}$$

$$-\frac{\tilde{\Upsilon}_1}{\lambda_n}(\varepsilon_{n1,s} + \chi_{n1,s}) \leq l_1\tilde{\Upsilon}_1^2 + \frac{\bar{\varepsilon}_{n1}^2}{2\underline{\lambda}_n^2} + \frac{\bar{\chi}_{n1}^2}{2\underline{\lambda}_n^2}, \tag{60}$$

$$\begin{aligned}-\frac{m_1}{\lambda_n}\Upsilon_1\tilde{\Upsilon}_1 + m_1\hat{\Upsilon}_1\tilde{\Upsilon}_1 \leq &\frac{m_1^2}{l_1\underline{\lambda}_n^2}\bar{\Upsilon}_1 + \frac{l_1}{2}\tilde{\Upsilon}^2 \\ &+ \frac{m_1}{2}\bar{\Upsilon}_1^2 - \frac{m_1}{2}\tilde{\Upsilon}_1^2, \end{aligned} \tag{61}$$

$$\frac{m_1\dot{\lambda}_n}{\lambda_n^2}z_n\tilde{\Upsilon}_1 \leq \frac{l_1}{2}\tilde{\Upsilon}_1^2 + \frac{m_1^2\bar{\lambda}_{n,d}^2}{2l_1\underline{\lambda}_n^4}z_n^2, \tag{62}$$

$$\frac{\hat{\phi}_1\hbar_1\psi_{n1}^T(\breve{Z}_n)\psi_{n1}(\breve{Z}_n)}{2\,k_{n1}^2}\tilde{\Upsilon}_1 \leq \frac{l_1}{2}\tilde{\Upsilon}_1^2 + \frac{\bar{\phi}_1^2}{2l_1k_{n1}^4}\hbar_1^2, \tag{63}$$

$$\tilde{\Upsilon}_1\dot{\Upsilon}_1 \leq \frac{l_1}{2}\tilde{\Upsilon}_1 + \frac{1}{2l_1}\bar{\Upsilon}_{1,d}^2. \tag{64}$$

Substituting (59)–(64) into (58), we have

$$\tilde{\Upsilon}_1\dot{\tilde{\Upsilon}}_1 \leq -\left(\frac{m_1}{2} - \frac{7}{2}l_1\right)\tilde{\Upsilon}_1^2 + \frac{m_1^2\bar{\lambda}_{n,d}^2}{2l_1\underline{\lambda}_n^4}z_n^2 + \frac{\bar{\phi}_1^2}{2l_1k_{n1}^4}\hbar_1^2 + \Delta_{\Upsilon_1}, \tag{65}$$

where $\Delta_{\Upsilon_1} = \bar{\Upsilon}_{1,d}^2/(2l_1) + \phi_1/(2l_1) + m_1^2\bar{\Upsilon}_1^2/(l_1\underline{\lambda}_n^2) + m_1\bar{\Upsilon}_1^2/2 + \bar{\varepsilon}_{n1}^2/(2\underline{\lambda}_n^2) + \bar{\chi}_{n1}^2/(2\underline{\lambda}_n^2)$.

Choosing the nth Lyapunov candidate function

$$V_n = V_{n-1} + \frac{1}{2}z_n^2 + \frac{1}{2}\tilde{\Theta}_n^2 + \frac{1}{2}\hbar_1^2 + \frac{1}{2}\tilde{\phi}_1^2 + \frac{1}{2}\tilde{\Upsilon}_1^2 + \frac{1}{2}\Lambda_n^2. \tag{66}$$

where $\tilde{\Theta}_n$ represents the estimation error of Θ_n, Θ_n will be given later.

Based on (46), $\dot{V}_n$ can be denoted as

$$\dot{V}_n = \dot{V}_{n-1} + z_n(f_{n,s}(\bar{x}_n) + \varphi_0 v(t) + \Upsilon_1 - \dot{\lambda}_n\alpha_n^f - \lambda_n\dot{\alpha}_n^f) + \tilde{\Theta}_n\dot{\tilde{\Theta}}_n + \hbar_1\dot{\hbar}_1 + \tilde{\phi}_1\dot{\tilde{\phi}}_1 + \tilde{\Upsilon}_1\dot{\tilde{\Upsilon}}_1 + \Lambda_n\dot{\Lambda}_n. \tag{67}$$

Substituting (54) and (65) into (67), it yields

$$\begin{aligned}\dot{V}_n \leq{}& \dot{V}_{n-1} - r_{n2}z_n^{2q} - c_n z_n^2 + z_n(F_{n,s}(Z_n) + \varphi_0 v(t) + \Upsilon_1)\\
&- \left(b_1 - 2 - \frac{\bar{\phi}_1^2}{2l_1k_{n1}^4}\right)\hbar_1^2 - \left(\frac{m_1}{2} - \frac{7}{2}l_1\right)\tilde{\Upsilon}_1^2 + \Delta_{\Upsilon_1} + \Delta_{\hbar_1}\\
&+ \tilde{\phi}_1\left(\frac{\hbar_1^2\psi_{n1}^T(\breve{Z}_n)\psi_{n1}(\breve{Z}_n)}{2\,k_{n1}^2} - \dot{\hat{\phi}}_1\right) + \Lambda_n\dot{\Lambda}_n + c_n\breve{z}_n^2\\
&- 2z_n^2 + \tilde{\Theta}_n\dot{\tilde{\Theta}}_n\\
\leq{}& \dot{V}_{n-1} - r_{n2}z^{2q} - c_n z_n^2 + z_n(F_{n,s}(Z_n) + \varphi_0 v(t) + \Upsilon_1)\\
&- \bar{b}_1\hbar_1^2 - \bar{m}_1\tilde{\Upsilon}_1^2 + \Delta_{\Upsilon_1} + \Delta_{\hbar_1} - r_{n1}\Lambda_n^2 - r_{n2}\Lambda_1^{2q}\\
&+ \tilde{\phi}_1\left(\frac{\hbar_1^2\psi_{n1}^T(\breve{Z}_n)\psi_{n1}(\breve{Z}_n)}{2\,k_{n1}^2} - \dot{\hat{\phi}}_1\right) + c_n\breve{z}_n^2\\
&- 2z_n^2 + \tilde{\Theta}_n\dot{\tilde{\Theta}}_n,\end{aligned} \tag{68}$$

where $F_{n,s}(Z_n) = r_{n2}z_n^{2q-1} + c_n z_n - \dot{\lambda}_n\alpha_{n-1}^f - \lambda_n\dot{\alpha}_{n-1}^f + 2z_n - c_n z_n/\lambda_n^2 + \bar{\lambda}_{n,d}^2 z_n/\underline{\lambda}_n^4 + m_1^2\lambda_{n,d}^2 z_n/(2l_1\underline{\lambda}_n^4)$, $\bar{b}_1 = b_1 - 2 - \bar{\phi}_1^2/(2l_1k_{n1}^4)$, $\bar{m}_1 = m_1/2 - 7l_1/2$.

Relying on the approximation characteristic of the FLSs, we can obtain

$$F_{n,s}(Z_n) = \theta_{n,s}^T\psi_n(\breve{z}_n) + \varepsilon_{n,s} + \chi_{n,s}, \tag{69}$$

where $\chi_{n,s} = \theta_{n,s}^T(\psi_n(z_n) - \psi_n(\breve{z}_n))$.

Obviously, the following inequality holds

$$z_n(\theta_{n,s}^T\psi_n(\breve{Z}_n) + \varepsilon_{n,s} + \chi_{n,s}) \leq \frac{\Theta_n\breve{z}_n^2\psi_n^T(\breve{Z}_n)\psi_n(\breve{Z}_n)}{2\,k_n^2} + \frac{1}{2}k_n^2 + z_n^2 + \frac{1}{2}\bar{\varepsilon}_n^2 + \frac{1}{2}\bar{\chi}_n^2, \tag{70}$$

where $\Theta_n = \max_{s\in\sigma(t)}\bar{\lambda}_n^2||\theta_{n,s}||^2$, $\hat{\Theta}_n = \Theta_n - \tilde{\Theta}_n$ is its estimation value, $\bar{\varepsilon}_n = \max_{s\in\sigma(t)}||\varepsilon_{n,s}||^2$, $\bar{\chi}_n = \max_{s\in\sigma(t)}||\chi_{n,s}||^2$, and k_n represents positive parameter.

Substituting (70) into (68), (68) is expressed as

$$\begin{aligned}\dot{V}_n \leq{}& \dot{V}_{n-1} - r_{n1}z_n^{2q} - c_n z_n^2 + z_n\varphi_0 v(t) + \frac{1}{2}\bar{\varepsilon}_n^2 + \frac{1}{2}\bar{\chi}_n^2\\
&- \bar{b}_1\hbar_1^2 - \bar{m}_1\tilde{\Upsilon}_1^2 + \Delta_{\Upsilon_1} + \Delta_{\hbar_1} - r_{n1}\Lambda_n^2 - r_{n2}\Lambda_1^{2q}\\
&+ \tilde{\phi}_1\left(\frac{\hbar_1^2\psi_{n1}^T(\breve{Z}_n)\psi_{n1}(\breve{Z}_n)}{2m_1k_{n1}^2} - \dot{\hat{\phi}}_1\right) + c_n\breve{z}_n^2 - z_n^2 + \frac{1}{2}k_n^2\\
&+ \frac{\hat{\Theta}_n\breve{z}_n^2\psi_n^T(\breve{Z}_n)\psi_n(\breve{Z}_n)}{2\,k_n^2} + \tilde{\Theta}_n\left(\frac{\breve{z}_n^2\psi_n^T(\breve{Z}_n)\psi_n(\breve{Z}_n)}{2\,k_n^2}\right.\\
&\left. + z_n\Upsilon_1 - \dot{\hat{\Theta}}_n\right).\end{aligned} \tag{71}$$

Construct the switching threshold event-triggered mechanism (ETM) as follows:

$$v(t) = \nu(t_k), \quad \forall t \in [t_k, t_{k+1}), \tag{72}$$

$$t_{k+1} = \begin{cases}\inf\{t > t_k|\,|\epsilon(t)| \geq \delta_1|v(t)| + \mu_1\},\\ \text{if} \quad |u_s(t)| \leq M_1;\\ \inf\{t > t_k|\,|\epsilon(t)| \geq \mu_2\},\\ \text{if} \quad |u_s(t)| > M_1;\end{cases} \tag{73}$$

where $\epsilon(t) = \nu(t) - v(t)$ represents measurement error, t_k, t_{k+1} are the kth update and the $(k+1)$th update of the control signal, respectively. Once $|\epsilon(t)|$ satisfies the preset condition after $t > t_k$, the actual control signal changes, i.e., $v(t) = \nu(t_{k+1})$. Different from the fixed and relative threshold ETM, the switching threshold ETM is capable of balancing system performance and communication burden. When the input signal is large, i.e, the case $u_s > M_1$, the fixed threshold ETM is activated to achieve fast respond. When the input signal satisfies that the case $u_s \leq M_1$, the relative threshold ETM can achieve precision control. Thus, in this article, the proposed control strategy can provide more flexibility on system respond and control performance.

Next, we define the following parameters as:

$$\delta = \begin{cases}\delta_1, & |u_s(t)| \leq M_1;\\ 0, & |u_s(t)| > M_1;\end{cases} \qquad \mu = \begin{cases}\mu_1, & |u_s(t)| \leq M_1;\\ \mu_2, & |u_s(t)| > M_1;\end{cases} \tag{74}$$

where $0 < \delta_1 < 1$, $M_1 > 0$, $\mu_1 > 0$, $\mu_2 > 0$ denote design constants. $\forall t \in [t_k, t_{k+1})$, assume the time-varying function $o_j(t)(j = 1, 2)$ satisfies $o_j(t) \in [-1, 1]$, $o_j(t_k) = 0$ and $o_j(t_{k+1}) = \pm 1$, it yields

$$\nu(t) = (1 + \delta o_1(t))v(t) + \mu o_2(t), \tag{75}$$

$$v(t) = \frac{\nu(t)}{1 + o_1(t)\delta} - \frac{\mu o_2(t)}{1 + o_1(t)\delta}. \tag{76}$$

Design the actual controller $\nu(t)$, nth adaptive law $\hat{\Theta}_n$, and the auxiliary variable Φ_n as follows:

$$\nu(t) = -(1 + \delta)\left(\alpha_n\tanh\left(\frac{\alpha_n\breve{z}_n}{\gamma_1}\right) + \mu\tanh\left(\frac{\mu\breve{z}_n}{\gamma_1}\right)\right), \tag{77}$$

$$\alpha_n = N(\Phi_n)\left(c_n\breve{z}_n + \frac{\hat{\Theta}_n\breve{z}_n\psi_n^T(\breve{Z}_n)\psi_n(\breve{Z}_n)}{2\,k_n^2}\right) - \frac{1}{\varphi_0}\hat{\Upsilon}_1, \tag{78}$$

$$\dot{\Phi}_n = c_n\breve{z}_n^2 + \frac{\hat{\Theta}_n\breve{z}_n^2\psi_n^T(\breve{Z}_n)\psi_n(\breve{Z}_n)}{2\,k_n^2}, \tag{79}$$

$$\dot{\hat{\Theta}}_n = \frac{\breve{z}_n^2\psi_n^T(\breve{Z}_n)\psi_n(\breve{Z}_n)}{2\,k_n^2} - \sigma_n\hat{\Theta}_n, \tag{80}$$

where c_n, σ_n and $\gamma_1$are positive parameters. Thus, we have

$$\begin{aligned}\varphi_0 z_n v(t) &= \varphi_0 z_n \left(\frac{\nu(t)}{1+o_1(t)\delta} - \frac{\mu o_2(t)}{1+o_1(t)\delta}\right)\\ &\le \varphi_0 \lambda_n \left(-\alpha_n \breve{z}_n \tanh\left(\frac{\alpha_n \breve{z}_n}{\gamma_1}\right) + \alpha_n \breve{z}_n\right.\\ &\quad \left. + |\alpha_n \breve{z}_n| + 0.2785\gamma_1\right)\\ &\le \varphi_0 \lambda_n (\alpha_n \breve{z}_n + 0.557\gamma_1)\\ &\le \varphi_0 \lambda_n N(\Phi_n)\dot{\Phi}_n - z_n \hat{\Upsilon}_1 + 0.557\gamma_1 \bar{\lambda}_n. \qquad (81)\end{aligned}$$

Substituting (77)–(81) into (71), one has

$$\begin{aligned}\dot{V}_n &\le \dot{V}_{n-1} - r_{n1} z_n^{2q} - c_n z_n^2 + \frac{1}{2}\bar{\varepsilon}_n^2 + \frac{1}{2}\bar{\chi}_n^2 + z_n \tilde{\Upsilon}_1 - \bar{b}_1 \hbar_1^2\\ &\quad - \bar{m}_1 \tilde{\Upsilon}_1^2 + \Delta_{\Upsilon_1} + \Delta_{\hbar_1} - r_{n1}\Lambda_n^2 - r_{n2}\Lambda_1^{2q} + \sigma_n \tilde{\Theta}_n \hat{\Theta}_n\\ &\quad + \tilde{\phi}_1 \Big(\frac{\hbar_1^2 \psi_{n1}^T(\breve{Z}_n)\psi_{n1}(\breve{Z}_n)}{2\,k_{n1}^2} - \dot{\hat{\phi}}_1\Big) + c_n \breve{z}_n^2 - z_n^2 + \frac{1}{2}k_n^2\\ &\quad + (g_n N(\Phi_n) + 1)\dot{\Phi}_n + 0.557\gamma_1 \bar{\lambda}_n, \qquad (82)\end{aligned}$$

where $g_n = \varphi_0 \lambda_n$.

In the light of the aforementioned analysis, we summarize the main conclusions in Theorem 1.

Theorem 1: Consider the switched cyberphysical system (1) with uncertain deception attacks and backlash-like hysteresis under Assumption 1. By designing the virtual controllers ((18), (34), (42)), adaptive laws ((20), (36), (44), (80)), and actual controller (77), when these design parameters satisfy the following conditions: $\bar{r}_{i3} > 0\ (i = 1, \ldots, n)$, $\bar{b}_1 > 0$ and $\bar{m}_1 > 1/2$, the whole variables in (1) are bounded.

Remark 2: Here are some guidelines for parameter selection. For example, based on the aforementioned stability analysis, we can design large $c_1, \cdots, c_n, r_{11}, r_{12}, \cdots, r_{n1}, r_{n2}, \sigma_1, \cdots, \sigma_n, \sigma_{n1}, m_1$, and b_1 to obtain a good control performance. Moreover, increasing $c_1, \cdots, c_n, r_{11}, r_{12}, \cdots, r_{n1}, r_{n2}, \sigma_1, \cdots, \sigma_n, \sigma_{n1}, m_1$ and b_1 can accelerate the convergence rate of system, and the corresponding control cost will augment. The designer should choose the appropriate parameters according to the specific situation in the engineering systems.

IV. Simulation Results

In this section, we construct a simulation example to verify the feasibility of the proposed control strategy.

We consider the following second-order nonlinear cyberphysical system:

$$\begin{aligned}\dot{x}_1 &= x_2 + f_{\sigma(t),1}(x_1)\\ \dot{x}_3 &= u_s + f_{\sigma(t),2}(\bar{x}_2) + \varrho_u(t, \bar{x}_2) + d_1(t, \bar{x}_2)\end{aligned}$$

with $\sigma(t) \in \mathbb{M} = \{1, 2\}$, $f_{1,1} = x_1 \sin(x_1)$, $f_{1,2} = x_2 \cos(x_1)$, $f_{2,1} = x_1 \cos(x_1)$, $f_{2,2} = x_1 x_2 + x_2 \sin(x_1)$, the actuator deception attack is set as $\varrho_u = \cos(t) x_1 \sin(x_2)$, the external disturbance is defined as $d_1 = 0.5\sin(x_1 x_2)\sin(3t)$, and the sensor deception attacks denote $\varrho_x(x_1, t) = 0.3 x_1 \cos(t)$, $\varrho_x(x_2, t) = x_2(0.2 + 0.5\sin(t))$.

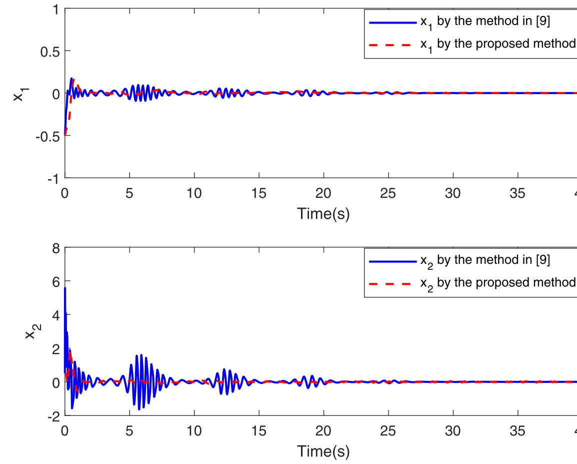

Fig. 1. Trajectories of state $x_i (i = 1, 2)$.

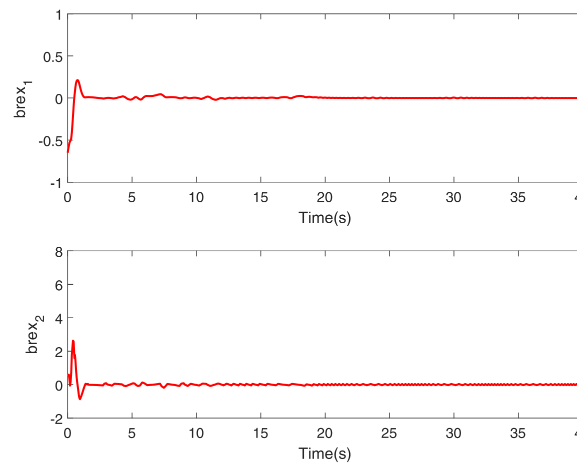

Fig. 2. Trajectories of state $\breve{x}_i$ under unknown deception attacks (i = 1, 2).

Assume that the backlash-like hysteresis of actuator is modeled as the following form:

$$\frac{du_s}{dt} = \omega\left|\frac{dv}{dt}\right|(\varphi v(t) - u_s) + \xi\frac{dv}{dt},$$

with $\omega = 2$, $\varphi = 0.5$, $\xi = 1$.

We define the following design constants: $c_1 = 22$, $c_2 = 25$; $\sigma_1 = 3, \sigma_2 = 2, \sigma_{21} = 3$; $k_1 = 6$, $k_2 = 6$, $k_{31} = 6$; $r_{11} = 2$, $r_{12} = 2$, $r_{13} = 1$; $r_{21} = 5$, $r_{22} = 3$; $m_1 = 15$, $b_1 = 20$, $\gamma_1 = 0.25$, $\varphi_0 = 5$. Besides, the filtering parameters are defined as: $\eta_{10} = 5, \eta_{11} = 1$; $\eta_{10} = 10, \eta_{11} = 3$.

The design parameters about switching event-triggered mechanism are designed as:

$$\delta = \begin{cases} 0.15, & |u_s(t)| \le 2;\\ 0, & |u_s(t)| > 2; \end{cases} \qquad \mu = \begin{cases} 0.5, & |u_s(t)| \le 2;\\ 1.5, & |u_s(t)| > 2; \end{cases}$$

In addition, the initial state variable of the closed-loop system are defined as: $x(0) = [0.5, -0.5]^T$; $\hat{\Theta}(0) = [0.8, 0.8]^T$; $\hat{\phi}_1(0) = 0.8$. From Figs. 1 and 2, it can be easily concluded that the second-order cyberphysical system is stable, and the state variables converge to the neighbor of the origin. Compared

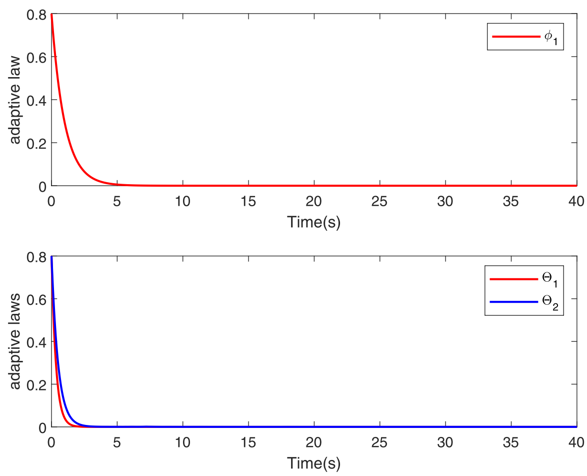

Fig. 3. Adaptive laws ϕ_1, $\Theta_i(i = 1, 2)$.

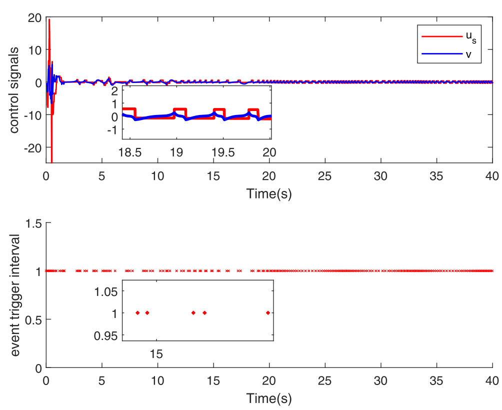

Fig. 4. Actual controller and triggered intervals.

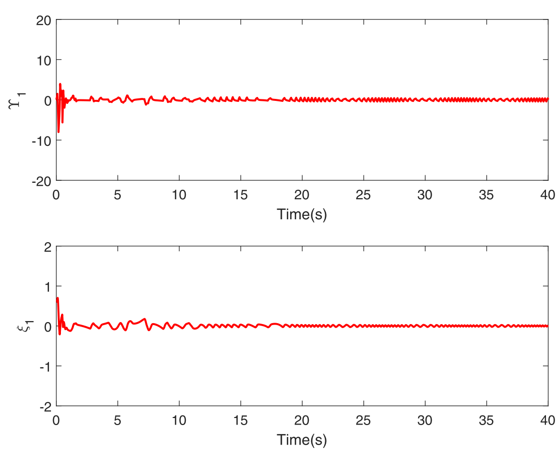

Fig. 5. Trajectories of disturbance observer Υ_1 and intermitted variable $\hbar_1$.

with the existing method in [9], our control strategy has better transient-state performance. The curves of parameter adaptive laws is shown in Fig. 3. In Fig. 4, the curves of actual control input and triggered intervals is plotted. According to the simulation results, the number of data transmission has been reduced from 40000 to 264. Fig. 5 shows the time trajectories of

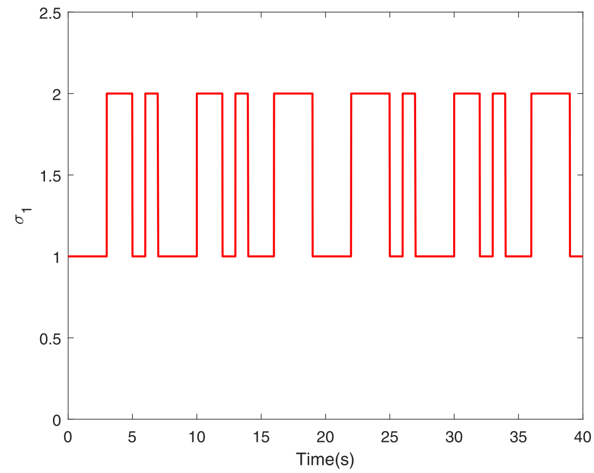

Fig. 6. Switching signal $\sigma(t)$.

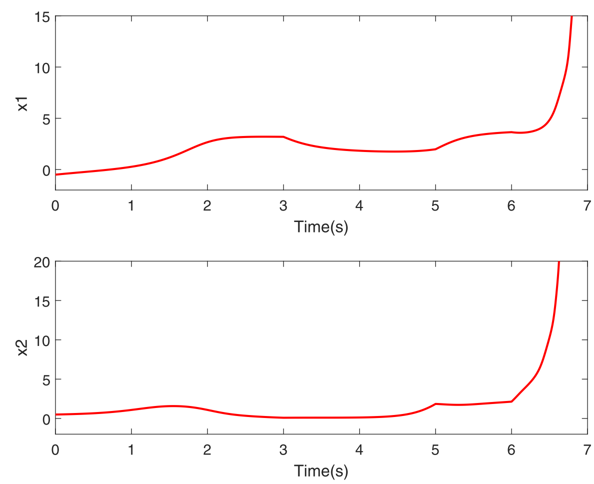

Fig. 7. Trajectories of state x_i under unknown deception attacks without resilient control strategy (i = 1, 2).

the disturbance observer and auxiliary variables. In Fig. 6, the switching rule $\sigma(t)$ is given. When the system is not equipped with the proposed controller, the curves of state x_i is shown in Fig. 7.

V. Conclusion

This article explored the finite-time adaptive control issue against a class of switched CPSs suffered from uncertain deception attacks under a novel event-triggered mechanism. Specifically, we utilized a finite-time convergent command filter to deal with the blast of complexity rendered by the repetitious differentiation of the virtual control law. The measurement control signal would be transmitted to the actuator side only when the pre-established event-triggered mechanism is satisfied. Furthermore, sufficient conditions have been deduced to ensure that every state signal belonging to the CPS is uniformly finite-time bounded. Finally, we displayed the simulation result to demonstrate the effectiveness of the proposed scheme. Follow-up work will further improve the convergence rate and extend the current investigations to the delay case.

APPENDIX

Proof: Similar with (22) and (23), we have

$$z_n\tilde{\Upsilon}_1 \le \frac{1}{2}z_n^2 + \frac{1}{2}\tilde{\Upsilon}_1^2,$$

$$\sigma_n\hat{\Theta}_n\tilde{\Theta}_n \le \frac{\sigma_n}{2}\Theta_n^2 - \frac{\bar{\sigma}_n}{2}\tilde{\Theta}_n^2 - \bar{\sigma}_n\left(\frac{1}{2}\tilde{\Theta}_n^2\right)^q + \bar{\sigma}_n(1-q)q^{\frac{q}{1-q}},$$

$$-\bar{b}_1\hbar_1^2 \le -\frac{\bar{b}_1}{2}\hbar_1^2 - \bar{b}_1\left(\frac{1}{2}\hbar_1^2\right)^q + \bar{b}_1(1-q)q^{\frac{q}{1-q}},$$

$$-m_1'\tilde{\Upsilon}_1^2 \le -\frac{m_1'}{2}\tilde{\Upsilon}_1^2 - m_1'\left(\frac{1}{2}\tilde{\Upsilon}_1^2\right)^q + m_1'(1-q)q^{\frac{q}{1-q}},$$

where $\bar{\sigma}_n = \sigma_n/2$, $m_1' = \bar{m}_1 - 1/2$.

On the basis of the above steps, we can obtain the information about $\dot{V}_{n-1}$,

$$\dot{V}_{n-1} \le \sum_{l=1}^{n-1}\Bigg(-r_{l1}z_l^{2q} - c_l z_l^2 - r_{l1}\Lambda_l^2 - r_{l2}\Lambda_l^{2q} - \bar{r}_{l3}e_l^2 + \frac{1}{2}z_n^2 - \bar{\sigma}_l\left(\frac{1}{2}\tilde{\Theta}_l^2\right)^q + (g_l N(\Phi_l)+1)\dot{\Phi}_l\Bigg) - \frac{\bar{\sigma}_l}{2}\tilde{\Theta}_l^2 + \Delta_{n-1}.$$

According to the above analysis, one has

$$\begin{aligned}\dot{V}_n \le{}& \sum_{l=1}^{n}(-r_{l1}z_l^{2q} - c_l z_l^2 - r_{l1}\Lambda_l^2 - r_{l2}\Lambda_l^{2q} - \bar{\sigma}_l\left(\frac{1}{2}\tilde{\Theta}_l^2\right)^q \\ &+ (g_l N(\Phi_l)+1)\dot{\Phi}_l - \frac{\bar{b}_1}{2}\hbar_1^2 - \bar{b}_1\left(\frac{1}{2}\hbar_1^2\right)^q - \frac{m_1'}{2}\tilde{\Upsilon}_1^2 \\ &- m_1'\left(\frac{1}{2}\tilde{\Upsilon}_1^2\right)^q + \Delta_n - \sum_{k=1}^{n-1}r_{k3}e_k^2 - \frac{\bar{\sigma}_l}{2}\tilde{\Theta}_l^2 \\ &+ \tilde{\phi}_1\left(\frac{\hbar_1^2\psi_{n1}^T(\breve{Z}_n)\psi_{n1}(\breve{Z}_n)}{2k_{n1}^2} - \dot{\hat{\phi}}_1\right),\end{aligned} \tag{83}$$

where $\Delta_n = \Delta_{n-1} + \bar{\sigma}_n(1-q)q^{\frac{q}{1-q}} + 0.577\gamma_1\bar{\lambda}_n + \Delta_{\Upsilon_1} + \Delta_{\hbar_1} + (\bar{\sigma}_n/2)\Theta_n^2 + (1/2)\bar{\varepsilon}_n^2 + (1/2)\bar{\chi}_n^2 + (1/2)k_n^2 + (\bar{b}_1 + m_1')(1-q)q^{\frac{q}{1-q}}$.

Substituting (55) into (83), we have

$$\begin{aligned}\dot{V}_n \le{}& \sum_{l=1}^{n}(-r_{l1}z_l^{2q} - c_l z_l^2 - r_{l1}\Lambda_l^2 - r_{l2}\Lambda_l^{2q} - \bar{\sigma}_l(\frac{1}{2}\tilde{\Theta}_l^2)^q, \\ &+ (g_l N(\Phi_l)+1)\dot{\Phi}_l - \frac{\bar{b}_1}{2}\hbar_1^2 - \bar{b}_1\left(\frac{1}{2}\hbar_1^2\right)^q - \frac{m_1'}{2}\tilde{\Upsilon}_1^2 \\ &- m_1'\left(\frac{1}{2}\tilde{\Upsilon}_1^2\right)^q + \Delta_n + \sigma_{n1}\hat{\phi}_1\tilde{\phi}_1 - \frac{\bar{\sigma}_l}{2}\tilde{\Theta}_l^2.\end{aligned} \tag{84}$$

Similarly, the following inequality holds:

$$\sigma_{n1}\hat{\phi}_1\tilde{\phi}_1 \le \frac{\sigma_{n1}}{2}\phi_1^2 - \frac{\bar{\sigma}_{n1}}{2}\tilde{\phi}_n^2 - \bar{\sigma}_{n1}\left(\frac{1}{2}\tilde{\phi}_n^2\right)^q + \bar{\sigma}_{n1}(1-q)q^{\frac{q}{1-q}},$$

where $\bar{\sigma}_{n1} = \sigma_{n1}/2$.

Thus, it yields

$$\begin{aligned}\dot{V}_n \le{}& \sum_{l=1}^{n}(-z_l^{2q} - c_l z_l^2 - r_{l1}\Lambda_l^2 - r_{l2}\Lambda_l^{2q} - \bar{\sigma}_l\left(\frac{1}{2}\tilde{\Theta}_l^2\right)^q \\ &+ (g_l N(\Phi_l)+1)\dot{\Phi}_l) - \frac{\bar{b}_1}{2}\hbar_1^2 - \bar{b}_1\left(\frac{1}{2}\hbar_1^2\right)^q - \frac{m_1'}{2}\tilde{\Upsilon}_1^2 \\ &- m_1'\left(\frac{1}{2}\tilde{\Upsilon}_1^2\right)^q - \frac{\bar{\sigma}_{n1}}{2}\tilde{\phi}_n^2 - \bar{\sigma}_{n1}\left(\frac{1}{2}\tilde{\phi}_n^2\right)^q - \frac{\bar{\sigma}_l}{2}\tilde{\Theta}_l^2 + \Delta \\ \le{}& -\rho V_n^q - aV_n + \sum_{l=1}^{n}(g_l N(\Phi_l)+1)\dot{\Phi}_l + \Delta,\end{aligned}$$

where $\Delta = \Delta_n + \bar{\sigma}_{n1}(1-q)q^{\frac{q}{1-q}} + (\sigma_{n1}/2)\phi_1^2$, $\rho = \min\{2^q, 2^q r_{l2}, \bar{\sigma}_l, \bar{\sigma}_{n1}, \bar{b}_1, m_1'\}$, $a = \min\{c_l, \bar{\sigma}_l, \bar{\sigma}_{n1}, r_{l1}, r_{l2}, m_1', \bar{b}_1\}$.

Furthermore, from Lemma 5, we can deduce that $\sum_{l=1}^{n}(g_l N(\phi_l)+1)\dot{\Phi}_l$ is bounded over the interval $[0, t_o)$ satisfying $\Delta' = \max_{t\in t_s}\sum_{l=1}^{n}(g_l N(\Phi_l)+1)\dot{\Phi}_l$. Define $\bar{\Delta} = \Delta + \Delta'$, (85) is further expressed as

$$\dot{V}_n \le -\rho V_n^q - aV_n + \bar{\Delta}. \tag{85}$$

According to [28], we can derive that whole variables in the closed-loop system (1) are stable, and the overall Lyapunov function satisfies

$$\left\{\lim_{t\to T_f} \mid V_n \le \min\left\{\frac{\bar{\Delta}}{(1-\pi)a}, \left[\frac{\bar{\Delta}}{(1-\pi)\rho}\right]^{\frac{1}{q}}\right\}\right\},$$

where $0 < \pi < 1$. Therefore, we have

$$|z_i| \le \min\left\{\sqrt{\frac{2\bar{\Delta}}{(1-\pi)a}}, \sqrt{2\left[\frac{\bar{\Delta}}{(1-\pi)\rho}\right]^{\frac{1}{q}}}\right\},$$

under a finite time

$$T_f \le \max\left\{\frac{1}{\pi a(1-q)}\ln\frac{\pi a V_n^{1-q}(0)+\rho}{\rho}, \frac{1}{a(1-q)}\ln\frac{aV_n^{1-q}(0)+\pi\rho}{\pi\rho}\right\}.$$

To avoid the successive triggering of ETM (i.e., Zeno behavior), we will expand the analysis. Based on the equation $\epsilon(t) = \nu(t) - v(t)$, we have $d|\epsilon(t)|/dt = \dot{\epsilon}\mathrm{sign}(\dot{\epsilon}) \le |\dot{\nu}|$.

Because $\dot{\nu}$ is a function consisting of bounded variables $\breve{z}_n$, $\hat{\Theta}_n$, $\hat{\Upsilon}_1$, $|\dot{\nu}|$ must have an upper bound ν^* such that $|\dot{\nu}| \le \nu^*$. So, we can obtain

$$|\epsilon(t)| = \int_{t_k}^{t_{k+1}}|\dot{\nu}|dt \le \int_{t_k}^{t_{k+1}}\nu^* dt \le \nu^*(t_{k+1}-t_k). \tag{86}$$

From the ETM (73), one has

$$t_{k+1} - t_k \ge \frac{\max\{\delta_1|v(t)| + \mu_1, \mu_2\}}{\nu^*} > \frac{\max\{\mu_1, \mu_2\}}{\nu^*} > 0.$$

Seeing that the minimum value of the trigger interval is always greater than zero, it means the Zeno phenomenon will not occur. Eventually, the proof of Theorem 1 is finished.

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