Adaptive Fuzzy Control for State-Constrained Nonlinear Cyber-Physical Systems With Unmodeled Dynamics Against Malicious Attacks

Zhaoyang Cuan, Da-Wei Ding, *Senior Member, IEEE*, Xiaoping Liu, and Youyi Wang

***Abstract*—This article investigated the tracking control problem for a category of state-constrained high-order uncertain nonlinear cyber-physical systems (CPSs) with unmodeled dynamics, which are subject to malicious attacks launched in controller-actuator (C-A) channel. Combining the backstepping method, dynamic surface control (DSC) approach and fuzzy logic systems (FLSs), a novel adaptive fuzzy tracking control strategy is developed, where a one to one mapping is brought in to transform the state-constrained CPSs to be the ones without state constraints and a dynamical signal is introduced to cope with the unmodeled dynamics. The proposed control strategy is not only independent of the exact system model, but also accommodates the external disturbances, unmodeled dynamics and malicious attacks. In particular, the developed control strategy can make sure that the output tracking error enters a predefined small neighborhood of the origin, and guarantee all the signals of the resulting closed-loop system satisfy the corresponding state constraints during the total operation, where the tracking accuracy level are known and can be preconfigured by selecting the design parameters appropriately. Ultimately, a representative simulation is provided to illustrate the validity and superiority of the proposed control scheme.**

***Index Terms*—Cyber-physical systems, external disturbances, malicious attacks, state constraints, unmodeled dynamics.**

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Zhaoyang Cuan is with the School of Automation and Electrical Engineering, University of Science and Technology Beijing, Beijing 100083, China, and with the Key Laboratory of Knowledge Automation for Industrial Processes, Ministry of Education, University of Science and Technology Beijing, Beijing 100083, China, and also with the Shunde Graduate School, University of Science and Technology Beijing, Foshan 528000, China (e-mail: 18813059600@163.com).

Da-Wei Ding is with the School of Automation and Electrical Engineering, University of Science and Technology Beijing, Beijing 100083, China, and also with the Key Laboratory of Knowledge Automation for Industrial Processes, Ministry of Education, University of Science and Technology Beijing, Beijing 100083, China (e-mail: dingdawei@ustb.edu.cn).

Xiaoping Liu is with the Department of Electrical Engineering, Lakehead University, Thunder Bay, ON P7B 5E1, Canada (e-mail: xliu2@lakeheadu.ca).

Youyi Wang is with the School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore 639798 (e-mail: eyywang@ntu.edu.sg).

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I. Introduction

Over the past couple of decades, adaptive control for cyber-physical systems (CPSs) has gained considerable attention. Accordingly, a series of significant results about nonlinear CPSs have been completed in [1], [2], [3], [4], [5], [6], [7]. However, the aforementioned results only discussed the control problem for nonlinear CPSs when there do not exist cyber attacks. As a matter of fact, the practical CPSs are often confronted with various malicious attacks. Therefore, the control problem for nonlinear CPSs under cyber attacks becomes more realistic and critical. Therewith, concentrating on coping with this problem, some scholars have proposed different control strategies in recent years. In [8], a high-performance adaptive control strategy for hypersonic flight vehicles (HFVs) against malicious attacks has been developed. A self-triggered control approach for CPSs under actuator and sensor attacks was designed in [9]. In [10], an adaptive control architecture has been developed for multiagent systems with stochastic disturbances against sensor and actuator attacks. In [11], a novel control approach has been proposed, where an adaptive neural network estimator works to do online estimation for malicious attacks. Nevertheless, it is noteworthy that there exist diverse constraints in practical CPSs, such as state constraints and output constraints. In this case, the aforementioned control approaches are not capable of dealing with this issue.

In recent years, it has turned into a particularly momentous issue that the barrier Lyapunov functions (BLFs) approach [12] was widely utilized in the control design for state-constrained nonlinear CPSs. A host of important adaptive control strategies have been developed for state-constrained nonlinear CPSs by means of the BLFs technique [13], [14], [15], [16], [17], [18]. It is worth noting that there exists an assumption in the considered systems that the upper and lower boundaries of the control gains are known in [13], and the relative design parameters include unknown constants, which is unreasonable. Conservative BLFs based control schemes for full state-constrained systems must depend on the feasibility condition for the virtual control law. In particular, a new control approach has been developed to remove the strict feasibility condition in [16]. What should be pointed out is that the abovementioned studies only investigated the stability control problem of the closed-loop CPSs when there are not malicious attacks. Admittedly, in loads of practical engineering applications, such as flight control, chemical reaction process and electronic circuit, there exist inevitably malicious attacks in the systems. On the other hand, due to certain factors including modelling simplifications, measurement noises and modelling uncertainties, external disturbances and unmodeled dynamics also exist in the systems, which could cause severe destruction to the resulting closed-loop system performance. Worse than that, the coexistence of malicious attacks, external disturbances and unmodeled dynamics will make it more difficult to guarantee the stability of the state-constrained high-order nonlinear CPSs. In essence, it is a challenging issue to accomplish the stability of the state-constrained high-order nonlinear CPSs with unmodeled dynamics and external disturbances against malicious attacks. The critical difficulties lie in how to develop an adaptive control strategy to assure the security and stabilization of the state-constrained nonlinear CPSs. To the best of the authors' knowledge, there have been few results involving this problem. Practically the existing control schemes for state-constrained nonlinear CPSs can not tackle malicious attacks, external disturbances and unmodeled dynamics simultaneously, which inspired our present study.

In the light of a class of state-constrained high-order uncertain nonlinear CPSs with external disturbances and unmodeled dynamics against malicious attacks, this article concentrates on developing a novel adaptive fuzzy tracking control strategy with the help of the backstepping approach, the fuzzy logic systems (FLSs) technology and the dynamic surface control (DSC) technique. In contrast with the most existing results, the distinguished contribution of this article can be exhibited as follows:

1) By designing a one to one mapping, the state-constrained CPSs are trasformed to be the ones without state constraints. Besides, a dynamical signal is introduced to resolve the unmodeled dynamics. Therewith, a novel adaptive fuzzy tracking control strategy is developed to guarantee the uniform ultimate stability for a category of state-constrained high-order uncertain nonlinear CPSs with unmodeled dynamics against malicious attacks, which is not only independent of the exact system model, but also accommodates external disturbances, unmodeled dynamics and malicious attacks.
2) Compared with the control schemes in [13], [14], [15], [16], [17] by means of BLFs approach, our developed control strategy is simpler by introducing an asymmetric one to one mapping. In particular, the developed control strategy can make sure that the output tracking error converges to a predefined small neighborhood of the origin, and guarantee that all the signals of the resulting closed-loop system satisfy the corresponding state constraints. In particular, the tracking accuracy level can be preconfigured by selecting the design parameters appropriately.

The rest of this article is arranged as follows. In Section II, the basic knowledge is introduced, and the problem to be resolved is formulated. Section III presents the detailed analysis for the design of the adaptive fuzzy control scheme and stability analysis of the resulting closed-loop system. In Section IV, a representative example is presented to verify the effectiveness and validity of our proposed approach. Finally, Section V summarizes conclusions and discussions of the future scope.

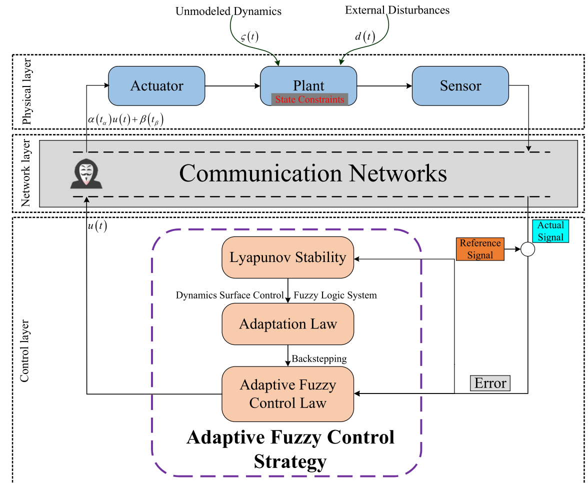

Fig. 1. The schematic diagram of the CPSs.

II. Problem Formulation and Basic Knowledge

Consider a category of nonlinear CPSs described in Fig. 1, where the physical plant is depicted by the following n-order uncertain nonlinear continuous-time systems with unmodeled dynamics in strict-feedback form:

$$\begin{cases} \dot{\varsigma} = p(\varsigma, \xi, t) \\ \dot{\xi}_i = \varphi_i(\bar{\xi}_i) + \psi_i(\bar{\xi}_i)\xi_{i+1} + d_i(t) + \varrho_i(\varsigma, \xi, t), i = 1, \ldots, n-1, \\ \dot{\xi}_n = \varphi_n(\bar{\xi}_n) + \psi_n(\bar{\xi}_n)\left(\alpha(t_m)u(t) + \beta(t_a)\right) + d_n(t) \\ \quad + \varrho_n(\varsigma, \xi, t), \\ y = \xi_1, \end{cases} \tag{1}$$

where $\bar{\xi}_i = [\xi_1, \xi_2, \ldots, \xi_i]^T \in R^i$, for $1 \leq i \leq$ n and $\xi = [\xi_1, \xi_2, \ldots, \xi_n]$ are the state vectors; $\varsigma \in R^{n_0}$ stands for the unmodeled dynamics; $y \in R$ is the output vector and $u \in R$ is the control input; $d_i(t)$ is any possible bounded external disturbance and meets $|d_i(t)| \leq \bar{d}_i$ with $\bar{d}_i$ being an unknown nonegative constant; $\varrho_i(\varsigma, \xi, t), i = 1, \ldots, n$ denote the unknown uncertain disturbances; $\varphi(\cdot)$ and $\psi(\cdot)$ denote unknown smooth nonlinear continuous-time functions of states, which represent the dynamics functions of the nonlinear CPSs; All the states must keep in the sets $\Upsilon_{\xi_i} = \{\xi_i : -c_{ui1} < \xi_i < c_{ui2}\}$, where c_{ui1} and c_{ui2} are known positive design parameters. Attempting to put the multiplicative actuation attacks $\alpha(t_m) \in (0, \infty)$ and additive attack signal $\beta(t_a)$ to the control input $u(t)$, the malicious attacks threaten the considered CPSs all the time. Here, t_m represents a time instant when the multiplicative actuation attacks will happen, and t_a denotes the instant when additive attack signal happens.

Hence, in light of the CPSs presented in Fig. 1, the control goal is to develop a control law to guarantee that the system output $y(t)$ can track the reference signal $y_d(t)$, in this sense, the tracking error between $y(t)$ and $y_d(t)$ converges to an arbitrary small neighborhood of the origin. In the meantime, all the states $\xi_i \in \Upsilon_{\xi_i}$ satisfy for $i = 1, 2, \ldots, n$ in the total operation.

Remark 1: In particular, the fruitful results about tracking control approach for state-constrained nonlinear CPSs have been completed in [19], [20]. However, the aforementioned results only discussed the control problem for nonlinear CPSs when there do not exist cyber attacks, unmodeled dynamics and external disturbances. In practice, the coexistence of malicious attacks, unmodeled dynamics and external disturbances could increase the difficulty of solving the considered problem. Therefore, the control strategy developed in this article differs from the tracking control approach in [19], [20].

Definition 1 ([21]): If the unmodeled dynamics ς is exponentially input-state-pracitcally stable (exp-ISpS), i.e., for $\dot{\varsigma} = p(\varsigma, \xi, t)$, then there exist a class of K_∞ functions $\overline{\beta}_1$, $\overline{\beta}_2$ and a Lyapunov function $V(\varsigma)$ such that

$$\overline{\beta}_1 (\|\varsigma\|) \leq V(\varsigma) \leq \overline{\beta}_2 (\|\varsigma\|), \tag{2}$$

and there exist positive design constants $e > 0$, $f \geq 0$ and a class of K_∞ function $\alpha(\cdot)$ such that

$$\frac{\partial V(\varsigma)}{\partial \varsigma} p(\varsigma, \xi, t) \leq -eV(\varsigma) + \alpha(|\xi_1|) + f, \quad \forall t \geq 0. \tag{3}$$

Assumption 1 ([16]): The sign of $\psi_i(\overline{\xi}_i)$, $1 \leq i \leq n$ is known and it is assumed that there exist unknown constants $\psi_{i,0} > 0$ such that

$$\psi_{i,0} \leq |\psi_i(\overline{\xi}_i)| < \infty. \tag{4}$$

Assumption 2 ([21]): The unmodeled dynamics ς is exp-ISpS.

Assumption 3 ([21]): There exist unknown nonnegative continuous functions $\varrho_{i1}(\cdot)$ and nondecreasing continuous functions $\varrho_{i2}(\cdot)$ such that

$$|\varrho_i(\varsigma, \xi, t)| \leq \varrho_{i1}(\|\overline{\xi}_i\|) + \varrho_{i2}(\|\varsigma\|), \forall (\varsigma, \xi, t) \in R^{n_0} \times R^n \times R_+, \tag{5}$$

where $\varrho_{i2}(0) = 0$, $i = 1, 2, \ldots, n$.

Assumption 4 ([22]): The ideal reference trajectory vector $[y_d, \dot{y}_d, \ddot{y}_d]^T \in \Upsilon_d$ is continuous and available on known compact set $\Upsilon_d = \{[y_d, \dot{y}_d, \ddot{y}_d]^T : y_d^2 + \dot{y}_d^2 + \ddot{y}_d^2 \leq D_0\} \subset R^3$ and $|y_d| < D_1 < \min\{c_{u11}, c_{u12}\}$, where $D_0 > 0$ and $D_1 > 0$ are known scalars.

Assumption 5 ([10]): Both $\alpha(t_m)$ and $\beta(t_a)$ are unknown time-varying and bounded functions, in the sense that, there exist known positive scalars $\alpha_{\min}$ and $\beta_{\max}$ such that $\alpha_{\min} < \alpha(t_m)$ and $|\beta(t_a)| \leq \beta_{\max}$.

Lemma 1 ([21]): If a Lyapunov function V is exp-ISpS for a system $\dot{\varsigma} = p(\varsigma, \xi, t)$, i.e. (2) and (3) are true, then, for any constant $\overline{e} \in (0, e)$, any initial instant $t_0 > 0$, any initial condition $\varsigma_0 = \varsigma(t_0)$, $b_0 > 0$, for any continuous function $\overline{\alpha}$ such that $\overline{\alpha}(|\xi_1|) \geq \alpha(|\xi_1|)$, there exist a finite $T_0 = \max\left\{0, \log\left[\frac{V(\xi_0)}{b_0}\right] / (e - \overline{e})\right\} \geq 0$, a nonnegative function $B(t_0, t)$, defined for all $t \geq t_0$ and a signal depicted by

$$\dot{b} = -\overline{e}b + \overline{\alpha}(|\xi_1|) + f, \quad b(t_0) = b_0, \tag{6}$$

such that $B(t_0, t) = 0$ for $t \geq t_0 + T_0$ and $V(\varsigma) \leq b(t) + B(t_0, t)$ with $B(t_0, t) = \max\{0, \exp(-e(t - t_0))V(\varsigma_0) - \exp(-\overline{e}(t - t_0))b_0\}$, where $\log(\bullet)$ denotes the natural logarithm of $\bullet$.

Lemma 2 ([23]): Given any continuous function $h(\xi)$ defined on a compact set Λ and any arbitrary accuracy $\varepsilon > 0$, there exists a FLS $K^T \Delta(\xi)$ such that

$$\sup_{\xi \in \Lambda} |h(\xi) - K^T \Delta(\xi)| \leq \varepsilon, \tag{7}$$

where $K = [\kappa_1, \kappa_2, \ldots, \kappa_n]^T$ denotes the ideal constant weighting vector. $\Delta(\xi)$ and $\delta_j(\xi)$ represent the basis function vector and Gaussian function, respectively, which can be obtained as follows:

$$\Delta(\xi) = \frac{[\delta_1(\xi), \delta_2(\xi), \ldots, \delta_n(\xi)]^T}{\sum_{j=1}^n \delta_j(\xi)}, \tag{8}$$

$$\delta_j(\xi) = \exp\left[\frac{-(\xi - c_j)^T(\xi - c_j)}{\mu_j^T \mu_j}\right], \tag{9}$$

where $j = 1, 2, \ldots, n$, $c_j = [c_{j1}, c_{j2}, \ldots, c_{jn}]^T$, indicates the center vector, $\mu_j = [\mu_{j1}, \mu_{j2}, \ldots, \mu_{jn}]^T$ denotes the width of the Gaussian functions, and $n > 1$ represents the number of the fuzzy rules. By means of Lemma 2, the FLSs can be applied to identify any smooth functions on a compact space.

Accordingly, in the next section, we will be devoted to combining the backstepping approach, the FLSs technique and the DSC technology to develop a novel adaptive fuzzy tracking control strategy to achieve the control objective.

III. Main Results

This section contributes to the development of the adaptive fuzzy tracking control strategy and stability analysis of the resulting closed-loop system. Notably, by combining the backstepping approach, the FLSs technique and DSC technology, we are devoted to constructing the virtual control signal and the control law to accomplish the control objective elaborated above. Ultimately, the main results are summarized in Theorem 1.

Focusing on coping with the full state constraints, a one to one nonlinear mapping (NM) is introduced as follows:

$$k_i = \log \frac{c_{ui1} + \xi_i}{c_{ui2} - \xi_i}, i = 1, 2, \ldots, n. \tag{10}$$

As stated above, since all the states must keep in the sets $\Upsilon_{\xi_i} = \{\xi_i : -c_{ui1} < \xi_i < c_{ui2}\}$, where c_{ui1} and c_{ui2} are known positive design parameters, it is apparent that (10) is well defined.

We can infer from (10) that the inverse mapping can be calculated as

$$\xi_i = c_{ui2} - \frac{c_{ui2} + c_{ui1}}{e^{k_i} + 1}. \tag{11}$$

Accordingly, we can get

$$\dot{k}_i = \frac{e^{k_i} + e^{-k_i} + 2}{c_{ui1} + c_{ui2}} \dot{\xi}_i. \tag{12}$$

Then, (1) can be reconstructed as

$$\dot{\varsigma} = p(\varsigma, \xi, t),$$
$$\dot{k}_i = \Phi_i(\overline{k}_{i+1}) + k_{i+1} + \Gamma_i(\varsigma, \overline{k}_n, t), \quad i = 1, \ldots, n-1, \tag{13}$$
$$\dot{k}_n = \Phi_n(\overline{k}_n) + \Psi_n(\overline{k}_n)\left(\alpha(t_m)u(t) + \beta(t_a)\right) + \Gamma_n(\varsigma, \overline{k}_n, t),$$

where $\overline{k}_i = [k_1, k_2, \ldots, k_i]$, $i = 1, \ldots, n$,

$$\Phi_i(\overline{k}_{i+1}) = g_i(k_i)(\varphi_i(\overline{\xi}_i) + \psi_i(\overline{\xi}_i)\xi_{i+1} + d_i) - k_{i+1}, \quad i = 1, \ldots, n-1, \tag{14}$$

$$\Phi_n(\overline{k}_n) = g_n(k_n)(\varphi_n(\overline{\xi}_n) + d_n),$$
$$\Psi_n(\overline{k}_n) = g_n(k_n)\psi_n(\overline{\xi}_n), \tag{15}$$
$$\Gamma_i(\varsigma, \overline{k}_n, t) = g_i(k_i)\varrho_i(\varsigma, \xi, t), i = 1, \ldots, n. \tag{16}$$
$$g_i(k_i) = \frac{e^{k_i} + e^{-k_i} + 2}{c_{ui1} + c_{ui2}}, i = 1, \ldots, n. \tag{17}$$

It can be inferred from (15), (16) and Assumption 3 that $\Psi_n(\overline{k}_n) \geq \frac{2\psi_{n,0}}{c_{un1} + c_{un2}} > 0$ and $|\Gamma_i(\overline{k}_n, t)| \leq g_i(k_i)[\varrho_{i1}(\|\overline{\xi}_i\|) + \varrho_{i2}(\|\varsigma\|)]$.

Remark 2: (2) implies that $\|\varsigma\| \leq \overline{\beta}_1^{-1}(V(\varsigma))$. In accordance with Lemma 2, it can be inferred that there must exist a positive scalar B_0 such that $\|\varsigma\| \leq \overline{\beta}_1^{-1}(b + B_0), \forall t \geq 0$. This inequality can be utilized to deal with the uncertain terms in the following procedure of controller design.

Set $\theta_1 = \log \frac{c_{u11} + y_d}{c_{u12} - y_d}$. In the backstepping, the design of adaptive fuzzy controller is based on the transformation of coordinates: $s_i = k_i - \theta_i, i = 1, 2, \ldots, n$, where θ_i denotes the output of a first-order filter with ζ_{i-1} being the input. In particular, ζ_{i-1} stands for an intermediate parameter which will be designed for the $(i-1)$-th subsystem. For the sake of brevity and readability, the following notations are defined as

$$\overline{s}_i = [s_1, s_2, \ldots, s_i]^T,$$
$$\overline{q}_j = [q_2, q_3, \ldots, q_j]^T, \tag{18}$$
$$\overline{\hat{\mu}}_i = [\hat{\mu}_1, \hat{\mu}_2, \ldots, \hat{\mu}_i]^T,$$

where $\hat{\mu}$ is the estimation of μ and $\tilde{\mu} = \mu - \hat{\mu}$, and it will be explained in detail later. $q_j = \theta_j - \zeta_{j-1}, j = 2, 3, \ldots, n$.

In what follows, concrete details about the iterative design procedure based on the backstepping method will be supplied to demonstrate how to obtain the primary results.

Step 1 : Take the following Lyapunov function candidate:

$$V_{s_1} = \frac{1}{2}s_1^2. \tag{19}$$

The time derivative of s_1 is

$$\dot{s}_1 = \Phi_1(\overline{k}_2) + k_2 + \Gamma_1(\varsigma, \overline{k}_n, t) - \dot{\theta}_1, \tag{20}$$

accordingly, with the help of Lemma 2, the time derivative of V_{s_1} along (13) can be calculated as

$$\dot{V}_{s_1} = s_1\left[s_2 + q_2 + \zeta_1 + \Phi_1(\overline{k}_2) + \Gamma_1(\varsigma, \overline{k}_n, t) - \dot{\theta}_1\right]$$
$$\leq s_1[s_2 + q_2 + \zeta_1 + h_1(\Sigma_1) + g_1(k_1)d_1] + \frac{1}{4}$$
$$\leq s_1[s_2 + q_2 + \zeta_1 + g_1(k_1)d_1 + K_1^T\Delta_1(\Sigma_1) + \epsilon_1(\Sigma_1)] + \frac{1}{4}$$
$$\leq s_1[s_2 + q_2 + \zeta_1 + g_1(k_1)d_1] + \frac{1}{2x_1^2}s_1^2\mu_1\|\Delta_1(\Sigma_1)\|^2 + \frac{1}{2}x_1^2$$
$$+ s_1\epsilon_1(\Sigma_1) + \frac{1}{4}, \tag{21}$$

where x_1 is a positive design parameter.

$$h_1(\Sigma_1) = \Phi_1(\overline{k}_2) - g_1(k_1)d_1 + s_1g_1^2(k_1)\Big[\varrho_{11}(\|\overline{\xi}_i\|) + \varrho_{12}(\overline{\beta}_1^{-1}(b + B_0))\Big]^2 - \dot{\theta}_1 \tag{22}$$

with $\Sigma_1 = [\overline{k}_2, s_1, \dot{\theta}_1, b]^T \in R^5$.

Design the virtual control signal ζ_1 as follows:

$$\zeta_1 = -f_1s_1 - \frac{1}{2x_1^2}s_1\hat{\mu}_1\|\Delta_1(\Sigma_1)\|^2, \tag{23}$$

where $f_1 > 0$ denotes a design constant, $\hat{\mu}_1$ represents the estimation of μ_1.

Then, the adaptation law of the unknown parameter μ_i is designed as

$$\dot{\hat{\mu}}_i = \lambda_i\left[\frac{1}{2x_i^2}s_i^2\|\Delta_i(\Sigma_i)\|^2 - \gamma_i\hat{\mu}_i\right], \tag{24}$$

where the design parameters λ_i and γ_i are strictly positive.

Subsequently, we define θ_2 as

$$\iota_2\dot{\theta}_2 + \theta_2 = \zeta_1, \quad \theta_2(0) = \zeta_1(0), \tag{25}$$

where $\iota_2 > 0$ denotes a design parameter.

In the light of (25), we can obtain $\dot{\theta}_2 = -\frac{q_2}{\iota_2}$. Since $k_2 = s_2 + q_2 + \zeta_1 = s_2 + q_2 - f_1s_1 - s_1\hat{\mu}_1\|\Delta_1(\Sigma_1)\|^2/(2x_1^2)$, utilizing (21) and Young's inequality, we can obtain

$$\dot{V}_{s_1} \leq s_1[s_2 + q_2 - f_1s_1] - \frac{s_1^2}{2x_1^2}\hat{\mu}_1\|\Delta_1(\Sigma_1)\|^2$$
$$+ \frac{1}{2x_1^2}s_1^2\mu_1\|\Delta_1(\Sigma_1)\|^2 + \frac{1}{2}x_1^2 + s_1\epsilon_1(\Sigma_1)$$
$$+ s_1g_1(k_1)d_1 + \frac{1}{4}$$
$$\leq (-f_1 + 3)s_1^2 + \frac{1}{4}s_2^2 + \frac{1}{4}q_2^2 + \frac{1}{4}g_1^2(k_1)d_1^2$$
$$+ \frac{s_1^2}{2x_1^2}\tilde{\mu}_1\|\Delta_1(\Sigma_1)\|^2 + \frac{1}{2}x_1^2 + \frac{1}{4}$$
$$+ |s_1|\eta_1(\overline{s}_2, q_2, \hat{\mu}_1, b, y_d, \dot{y}_d), \tag{26}$$

where the continuous function $\eta_1(\overline{s}_2, q_2, \hat{\mu}_1, b, y_d, \dot{y}_d)$ satisfies the following condition

$$|\epsilon_1(\Sigma_1)| \leq \eta_1(\overline{s}_2, q_2, \hat{\mu}_1, b, y_d, \dot{y}_d). \tag{27}$$

Utilizing Young's inequality, we can obtain

$$|s_1|\eta_1 \leq s_1^2 + \frac{1}{4}\eta_1^2. \tag{28}$$

Thus, the following inequality can be obtained

$$\dot{V}_{s_1} \leq (-f_1+4)s_1^2 + \frac{1}{4}s_2^2 + \frac{1}{4}q_2^2 + \frac{1}{4}g_1^2(k_1)d_1^2 + \frac{s_1^2}{2x_1^2}\tilde{\mu}_1\|\Delta_1(\varSigma_1)\|^2 + \frac{1}{2}x_1^2 + \frac{1}{4} + \frac{1}{4}\eta_1^2. \tag{29}$$

In accordance with Assumption 3, we have

$$\dot{q}_2 = -\frac{q_2}{\iota_2} + \Bigg[f_1\dot{s}_1 + \frac{\dot{s}_1}{2x_1^2}\hat{\mu}_1\|\Delta_1(\varSigma_1)\|^2 + \frac{s_1}{2x_1^2}\dot{\hat{\mu}}_1\|\Delta_1(\varSigma_1)\|^2 + \frac{s_1\hat{\mu}_1}{2x_1^2}\frac{d\|\Delta_1(\varSigma_1)\|^2}{dt}\Bigg], \tag{30}$$

$$|\dot{q}_2 + \frac{q_2}{\iota_2}| \leq \tau_2\left(\bar{s}_3, \bar{q}_3, \bar{\hat{\mu}}_2, b, y_d, \dot{y}_d, \ddot{y}_d\right), \tag{31}$$

where the function $\tau_2(\bar{s}_3, \bar{q}_3, \bar{\hat{\mu}}_2, b, y_d, \dot{y}_d, \ddot{y}_d)$ is continuous.

Combining (30) and (31), one has

$$\begin{aligned} q_2\dot{q}_2 &\leq -\frac{q_2^2}{\iota_2} + |q_2|\tau_2\left(\bar{s}_3, \bar{q}_3, \bar{\hat{\mu}}_2, b, y_d, \dot{y}_d, \ddot{y}_d\right) \\ &\leq -\frac{q_2^2}{\iota_2} + q_2^2 + \frac{1}{4}\tau_2^2. \end{aligned} \tag{32}$$

Step i $(2 \leq i \leq n-1)$: Consider a Lyapunov function candidate in the following form of:

$$V_{s_i} = \frac{1}{2}s_i^2. \tag{33}$$

Owing to $s_i = k_i - \theta_i$, differentiating s_i produces

$$\dot{s}_i = \varPhi_i(\bar{k}_{i+1}) + k_{i+1} + \varGamma_i(\varsigma, \bar{k}_n, t) - \dot{\theta}_i. \tag{34}$$

Consequently, with the aid of Lemma 2, the time derivative of V_{s_i} along (13) can be calculated as follows:

$$\begin{aligned} \dot{V}_{s_i} &= s_i\left[s_{i+1} + q_{i+1} + \zeta_i + \varPhi_i(\bar{k}_{i+1}) + \varGamma_i(\varsigma, \bar{k}_n, t) - \dot{\theta}_i\right] \\ &\leq s_i[s_{i+1} + q_{i+1} + \zeta_i + h_i(\varSigma_i) + g_i(k_i)d_i] + \frac{1}{4} \\ &\leq s_i[s_{i+1}+q_{i+1}+\zeta_i+g_i(k_i)d_i+K_i^T\Delta_i(\varSigma_i)+\epsilon_i(\varSigma_i)]+\frac{1}{4} \\ &\leq s_i[s_{i+1}+q_{i+1}+\zeta_i + g_i(k_i)d_i]+\frac{s_i^2}{2x_i^2}\mu_i\|\Delta_i(\varSigma_i)\|^2+\frac{1}{2}x_i^2 \\ &\quad + s_i\epsilon_i(\varSigma_i) + \frac{1}{4}, \end{aligned} \tag{35}$$

where x_i is a positive design parameter.

$$h_i(\varSigma_i) = \varPhi_i(\bar{k}_{i+1}) - g_i(k_i)d_i + s_i g_i^2(k_i)\Big[\varrho_{i1}(\|\bar{\xi}_i\|) + \varrho_{i2}(\bar{\beta}_1^{-1}(b + B_0))\Big]^2 - \dot{\theta}_i \tag{36}$$

with $\varSigma_i = [\bar{k}_{i+1}, s_i, \dot{\theta}_i, b]^T \in R^{i+4}$.

Design the virtual control signal ζ_i as follows:

$$\zeta_i = -f_i s_i - \frac{1}{2x_i^2}s_i\hat{\mu}_i\|\Delta_i(\varSigma_i)\|^2, \tag{37}$$

where $f_i > 0$ is a design constant, $\hat{\mu}_i$ stands for the estimation of μ_i.

Then, the adaptation law of the unknown parameter μ_i is designed as same as (24).

Subsequently, we define θ_{i+1} as

$$\iota_{i+1}\dot{\theta}_{i+1} + \theta_{i+1} = \zeta_i, \quad \theta_{i+1}(0) = \zeta_i(0), \tag{38}$$

where ι_{i+1} is a positive design parameter.

Since $q_{i+1} = \theta_{i+1} - \zeta_i$, for $i = 2, 3, \ldots, n-1$, we can obtain $\dot{\theta}_{i+1} = -\frac{q_{i+1}}{\iota_{i+1}}$. Noting that $s_{i+1} = k_{i+1} - \theta_{i+1}$, one has

$$k_{i+1} = s_{i+1} + q_{i+1} - f_i s_i - \frac{1}{2x_i^2}s_i\hat{\mu}_i\|\Delta_i(\varSigma_i)\|^2. \tag{39}$$

By means of Young's inequality and (35), the following inequality can be derived

$$\begin{aligned} \dot{V}_{s_i} &\leq (-f_i+3)s_i^2 + \frac{1}{4}s_{i+1}^2 + \frac{1}{4}q_{i+1}^2 + \frac{1}{4}g_i^2(k_i)d_i^2 \\ &\quad + \frac{s_i^2}{2x_i^2}\tilde{\mu}_i\|\Delta_i(\varSigma_i)\|^2 + \frac{1}{2}x_i^2 + \frac{1}{4} \\ &\quad + |s_i|\eta_i(\bar{s}_{i+1}, q_{i+1}, \hat{\mu}_i, b, y_d, \dot{y}_d), \end{aligned} \tag{40}$$

where the continuous function $\eta_i(\bar{s}_{i+1}, q_{i+1}, \hat{\mu}_i, b, y_d, \dot{y}_d)$ satisfies the following condition

$$|\epsilon_i(\varSigma_i)| \leq \eta_i(\bar{s}_{i+1}, q_{i+1}, \hat{\mu}_i, b, y_d, \dot{y}_d). \tag{41}$$

According to Young's inequality, we have

$$|s_i|\eta_i \leq s_i^2 + \frac{1}{4}\eta_i^2. \tag{42}$$

Therefore, the following expression can be obtained

$$\begin{aligned} \dot{V}_{s_i} &\leq (-f_1+4)s_i^2 + \frac{1}{4}s_{i+1}^2 + \frac{1}{4}q_{i+1}^2 + \frac{1}{4}g_i^2(k_i)d_i^2 \\ &\quad + \frac{s_i^2}{2x_i^2}\tilde{\mu}_i\|\Delta_i(\varSigma_i)\|^2 + \frac{1}{2}x_i^2 + \frac{1}{4} + \frac{1}{4}\eta_i^2. \end{aligned} \tag{43}$$

Noting Assumption 3, we have

$$\begin{aligned} \dot{q}_{i+1} &= -\frac{q_{i+1}}{\iota_{i+1}} + \Bigg[f_i\dot{s}_i + \frac{\dot{s}_i}{2x_i^2}\hat{\mu}_i\|\Delta_i(\varSigma_i)\|^2 \\ &\quad + \frac{s_i}{2x_i^2}\dot{\hat{\mu}}_i\|\Delta_i(\varSigma_i)\|^2 + \frac{s_i\hat{\mu}_i}{2x_i^2}\frac{d\|\Delta_i(\varSigma_i)\|^2}{dt}\Bigg], \end{aligned} \tag{44}$$

$$\left|\dot{q}_{i+1} + \frac{q_{i+1}}{\iota_{i+1}}\right| \leq \tau_{i+1}\left(\bar{s}_{i+2}, \bar{q}_{i+2}, \bar{\hat{\mu}}_{i+1}, b, y_d, \dot{y}_d, \ddot{y}_d\right), \tag{45}$$

where the function $\tau_{i+1}(\bar{s}_{i+2}, \bar{q}_{i+2}, \bar{\hat{\mu}}_{i+1}, b, y_d, \dot{y}_d, \ddot{y}_d)$ is continuous.

In terms of (44) and (45), we can obtain

$$\begin{aligned} q_{i+1}\dot{q}_{i+1} &\leq -\frac{q_{i+1}^2}{\iota_{i+1}} + |q_{i+1}|\tau_{i+1}\left(\bar{s}_{i+2}, \bar{q}_{i+2}, \bar{\hat{\mu}}_{i+1}, b, y_d, \dot{y}_d, \ddot{y}_d\right) \\ &\leq -\frac{q_{i+1}^2}{\iota_{i+1}} + q_{i+1}^2 + \frac{1}{4}\tau_{i+1}^2. \end{aligned} \tag{46}$$

Step n: At this step, the control law u will be designed. According to $s_n = k_n - \theta_n$, we have

$$\begin{aligned} \dot{s}_n &= \varPhi_n(\bar{k}_n) + \varPsi_n(\bar{k}_n)\left(\alpha(t_m)u(t) + \beta(t_a)\right) \\ &\quad + \varGamma_n(\varsigma, \bar{k}_n, t) - \dot{\theta}_n. \end{aligned} \tag{47}$$

Define the following smooth function

$$V_{s_n} = \int_0^{s_n} \frac{\chi}{\Psi_n(\bar{k}_{n-1}, \chi + \theta_n)} d\chi. \tag{48}$$

With the help of the second mean value theorem for integral, V_{s_n} can be reconstructed as $V_{s_n} = s_n^2/2\Psi_n(\bar{k}_{n-1}, \vartheta_{s_n} s_n + \theta_n)$ with $\vartheta_{s_n} \in (0,1)$. Since $0 < 2\psi_{n,0}/(c_{un1} + c_{un2}) < \Psi_n(\bar{k}_n)$, it can be easily inferred that V_{s_n} is positive with regard to s_n.

Differentiating V_{s_n} and applying (47) result in

$$\begin{aligned}\dot{V}_{s_n} &= \frac{s_n}{\Psi_n(\bar{k}_n)}\dot{s}_n + \int_0^{s_n} \chi \left[\sum_{m=1}^{n-1} \frac{\partial \Psi_n^{-1}(\bar{k}_{n-1}, \chi + \theta_n)}{\partial k_m}\right. \\ &\quad \times \left(\Phi_m(\bar{k}_{m+1}) + k_{m+1} + \Gamma_m(\varsigma, \bar{k}_n, t)\right) \\ &\quad \left. + \frac{\partial \Psi_n^{-1}(\bar{k}_{n-1}, \chi + \theta_n)}{\partial \theta_n}\dot{\theta}_n\right] d\chi \\ &= \frac{s_n}{\Psi_n(\bar{k}_n)}\dot{s}_n + \sum_{m=1}^{n-1} \Gamma_m(\varsigma, \bar{k}_n, t) \\ &\quad \times s_n^2 \int_0^1 \omega \frac{\partial \Psi_n^{-1}(\bar{k}_{n-1}, \omega s_n + \theta_n)}{\partial k_m} d\omega \\ &\quad + s_n^2 \int_0^1 \omega \left\{\sum_{m=1}^{n-1} \frac{\partial \Psi_n^{-1}(\bar{k}_{n-1}, \omega s_n + \theta_n)}{\partial k_m}\right. \\ &\quad \left. \times \left[\Phi_m(\bar{k}_{m+1}) + k_{m+1}\right]\right\} d\omega + \frac{\dot{\theta}_n s_n}{\Psi_n(\bar{k}_n)} \\ &\quad - \dot{\theta}_n s_n \int_0^1 \frac{1}{\Psi_n^{-1}(\bar{k}_{n-1}, \omega s_n + \theta_n)} d\omega. \end{aligned} \tag{49}$$

By means of Young's inequality, the following expression can be gained

$$\begin{aligned}&\left|s_n^2 \Gamma_m(\varsigma, \bar{k}_n, t) \int_0^1 \omega \frac{\partial \Psi_n^{-1}(\bar{k}_{n-1}, \omega s_n + \theta_n)}{\partial k_m}\right| \\ &\le s_n^4 g_m^2(k_m) \upsilon_m^2(\bar{\xi}_m, b) \left[\int_0^1 \omega \frac{\partial \Psi_n^{-1}(\bar{k}_{n-1}, \omega s_n + \theta_n)}{\partial k_m} d\omega\right]^2 + \frac{1}{4}, \end{aligned} \tag{50}$$

where $\upsilon_m(\bar{\xi}_m, b) = \varrho_{m1}(\|\bar{\xi}_m\|) + \varrho_{m2}(\bar{\beta}_1^{-1}(b + B_0)), m = 1, 2, \ldots, n$.

Substituting (47) and (50) into (50) results in

$$\begin{aligned}\dot{V}_{s_n} &\le s_n \left[\alpha(t_m)u(t) + \beta(t_a) + \frac{g_n(k_n)d_n}{\Psi_n(\bar{k}_n)} + h_n(\Sigma_n)\right] + \frac{n}{4} \\ &= s_n \left[\alpha(t_m)u(t) + \beta(t_a) + \frac{g_n(k_n)d_n}{\Psi_n(\bar{k}_n)}\right] \\ &\quad + s_n \left[K_n^T \Delta_n(\Sigma_n) + \epsilon_n(\Sigma_n)\right] + \frac{n}{4}, \end{aligned} \tag{51}$$

where

$$\begin{aligned}h_n(\Sigma_n) &= \frac{\Phi_n(\bar{k}_n) - g_n(k_n)d_n}{\Psi_n(\bar{k}_n)} + s_n^3 \sum_{m=1}^{n-1} g_m^2(k_m) \upsilon_m^2(\bar{\xi}_m, b) \\ &\quad \times \left[\int_0^1 \omega \frac{\partial \Psi_n^{-1}(\bar{k}_{n-1}, \omega s_n + \theta_n)}{\partial k_m} d\omega\right]^2 \\ &\quad + s_n \int_0^1 \omega \left\{\sum_{m=1}^{n-1} \frac{\partial \Psi_n^{-1}(\bar{k}_{n-1}, \omega s_n + \theta_n)}{\partial k_m}\right. \\ &\quad \left. \times \left[\Phi_m(\bar{k}_{m+1}) + k_{m+1}\right]\right\} d\omega \\ &\quad - \dot{\theta}_n \int_0^1 \frac{1}{\Psi_n^{-1}(\bar{k}_{n-1}, \omega s_n + \theta_n)} d\omega \\ &\quad + \frac{s_n g_n^2(\bar{k}_n) \upsilon_n^2(\bar{\xi}_n, b)}{\Psi_n^2(\bar{k}_n)} + \frac{n}{4} \end{aligned} \tag{52}$$

with

$$\Sigma_n = \left[\bar{k}_n, s_n, \dot{\theta}_n, b\right]^T \in R^{n+3}. \tag{53}$$

Then, we design the following control law u

$$u = -\frac{f_n}{\alpha_{\min}} s_n - \frac{1}{2\alpha_{\min} x_n^2} s_n \hat{\mu}_n \|\Delta_n(\Sigma_n)\|^2, \tag{54}$$

where $f_n > 0$ is the design parameter, $\hat{\mu}_n$ denotes the estimation of μ_n.

Akin to the procedure at i-th step, we can get

$$\begin{aligned}\dot{V}_{s_n} &\le (-f_n + 3)\, s_n^2 + \frac{g_n^2(k_n)d_n^2}{4\Psi_n^2(\bar{k}_n)} + \frac{\beta_{\max}^2}{4} \\ &\quad + \frac{s_n^2}{2x_n^2} \tilde{\mu}_n \|\Delta_n(\Sigma_n)\|^2 + \frac{1}{2}x_n^2 + \frac{n}{4} + \frac{1}{4}\eta_n^2 \end{aligned} \tag{55}$$

with the continuous function $\eta_n(\bar{s}_n, q_n, \bar{\hat{\mu}}_{n-1}, b, y_d, \dot{y}_d)$ satisfiing $|\epsilon_n(\Sigma_n)| \le \eta_n(\bar{s}_n, q_n, \bar{\hat{\mu}}_{n-1}, b, y_d, \dot{y}_d)$.

Now, we define the following set

$$\Upsilon_n = \left\{\left[\bar{s}_n^T, \bar{q}_n^T, \bar{\hat{\mu}}_n^T\right]^T : V_n < \nu\right\} \in R^{\nu_n}, \tag{56}$$

where $\nu > 0$ is the design parameter, $\nu_n = 3n - 1$, and

$$V_n = \sum_{m=1}^{n}\left[V_{s_m} + \frac{1}{2\varpi_m}\tilde{\mu}_m^2\right] + \frac{1}{2}\sum_{m=2}^{n} q_m^2. \tag{57}$$

If $V_n \in \nu$, then we can obtain s_m, $\tilde{\mu}_m$ and $q_p \in L_\infty$, $m = 1, 2, \ldots, n$, $p = 2, 3, \ldots, n$. Since $y_d \in L_\infty$, $s_1 = y - \theta_1$, $y = s_1 + \theta_1 \in L_\infty$, which implies $b \in L_\infty$. As a consequence, η_i and τ_i have a maximum W_i and H_i over the compact set $\Upsilon_d \times \Upsilon_n$.

Thus far, the following theorem can be utilized to summarize the above results.

Theorem 1: Consider the high-order uncertain nonlinear CPSs (1) with unmodeled dynamics under malicious attacks and suppose Assumptions 1–4 are true. For the bounded initial conditions, satisfying $V_n(0) < \nu$, $c_{u11}, c_{u12} < D_1$ and $\xi_m(0) \in \Upsilon_{\xi_m}$, the designed controller (54), together with the virtual controller (37) for $1 \le m \le n-1$ and adaptation law (24) for $m = 1, 2, \ldots, n$, can make sure that overall signals of the resulting closed-loop system are practically globally stable, in the sense that all of the signals in the closed-loop system are bounded and $\xi_m \in \Upsilon_{\xi_m}$, $\forall t \ge 0$, i.e., all the state constraints are always satisfied, besides, f_m and ι_m meet the following conditions

$$\begin{aligned} f_m &\geq \frac{17}{4} + \frac{\chi_0}{2}, m = 1, 2, \ldots, n, \\ f_n &\geq \frac{17}{4} + \frac{\chi_0 (c_{un1} + c_{un2})}{2\psi_{n,0}}, \\ \frac{1}{\iota_m} &\geq \frac{5}{4} + \chi_0, m = 2, 3, \ldots, n, \\ \chi_0 &= \min\{\varpi_1\gamma_1, \varpi_2\gamma_2, \ldots, \varpi_n\gamma_n\}. \end{aligned} \tag{58}$$

Proof: Consider the following overall Lyapunov function

$$V = V_n = \sum_{m=1}^{n} \left[V_{s_m} + \frac{1}{2\varpi_m}\tilde{\mu}_m^2 \right] + \frac{1}{2}\sum_{m=2}^{n} q_m^2. \tag{59}$$

Differentiating (59) yields

$$\dot{V} = \sum_{m=1}^{n} \left[\dot{V}_{s_m} - \frac{1}{\varpi_m}\tilde{\mu}_m\dot{\hat{\mu}}_m \right] + \sum_{m=1}^{n-1} q_{m+1}\dot{q}_{m+1}. \tag{60}$$

Substituting (29), (32), (43), (46), and (55) into (60) and applying (24) produces

$$\begin{aligned} \dot{V} \leq & \sum_{m=1}^{n} \left[\left(-f_m + \frac{17}{4} \right) s_m^2 \right] \\ & + \sum_{m=1}^{n-1} \left[-\frac{q_{m+1}^2}{\iota_2} + q_{m+1}^2 + \frac{1}{4}\tau_{m+1}^2 \right] \\ & + \sum_{m=1}^{n} \left[\frac{1}{2}x_m^2 + \frac{1}{4} + \frac{1}{4}\eta_m^2 \right] \\ & + \sum_{m=1}^{n-1} \left[\frac{1}{4}g_m^2(k_m)d_m^2 \right] + \frac{g_n^2(k_n)d_n^2}{4\Psi_n^2(\bar{k}_n)} \\ & + \sum_{m=1}^{n} (\gamma_m\tilde{\mu}_m\hat{\mu}_m) + \frac{n + \beta_{\max}^2}{4}. \end{aligned} \tag{61}$$

By means of Young's inequality, we have

$$\gamma_m\tilde{\mu}_m\hat{\mu}_m \leq \gamma_m \left[-\frac{\tilde{\mu}_m^2}{2} + \frac{\mu_m^2}{2} \right]. \tag{62}$$

In the event that $V < \nu$, then $\eta_m^2 < W_m^2$ and $\tau_{m+1}^2 < H_{m+1}^2$. Denote

$$\begin{aligned} \Theta = & \frac{n}{2} + \frac{1}{4}\sum_{m=1}^{n-1} H_{m+1}^2 + \frac{1}{4}\sum_{m=1}^{n} W_m^2 + \sum_{m=1}^{n-1} \left[\frac{1}{4}g_m^2(k_m)d_m^2 \right] \\ & + \frac{g_n^2(k_n)d_n^2}{4\Psi_n^2(\bar{k}_n)} + \frac{\beta_{\max}^2}{4} + \sum_{m=1}^{n} \left[\frac{x_m^2 + \gamma_m\mu_m^2}{2} \right] \end{aligned} \tag{63}$$

Putting (58) and (62) into (61) yields

$$\dot{V} \leq -\chi_0 V + \Theta. \tag{64}$$

Consequently, we can obtain that the condition (58) is sufficient. ■

If $V = \nu$ and $\chi_0 > \frac{\Theta}{\nu}$, it is evident that $\dot{V} \leq 0$, which illustrates that $V(t) \leq \nu$, $\forall t \geq 0$ for $V(0) \leq \nu$. From (64), we can obtain

$$0 \leq V(t) \leq \frac{\Theta}{\chi_0} + \left(V(0) - \frac{\Theta}{\chi_0} \right) e^{-\chi_0 t}. \tag{65}$$

Thus, it is apparent that all the signals of the resulting closed-loop system are uniformly ultimately bounded, including s_m, q_m and $\hat{\mu}_m$. In addition, θ_{m+1} and ζ_m are also uniformly ultimately bounded. Owing to $k_m = s_m + q_m + \zeta_{m-1}$, s_m, q_m, $\zeta_{m-1} \in L_\infty$, it is obvious that $k_m \in L_\infty$, which means that $\xi_m \in \Upsilon_{\xi_m}$, i.e., all the state constraints are always satisfied. Due to $y \in L_\infty$, $b \in L_\infty$, in accordance with Remark 2 and (65), we can obtain

$$|s_1| \leq \sqrt{\frac{2\Theta}{\chi_0} + 2\left[V(0) - \frac{\Theta}{\chi_0} \right] e^{-\chi_0 t}}. \tag{66}$$

According to (58) and (63), for any given scalars D_0, ν, x_m and γ_m, we can infer that $\frac{\Theta}{\chi_0}$ can be make arbitrarily small by selecting large enough design parameter λ_m. That is to say, s_1 can be made arbitrarily small when $t \to \infty$.

Remark 3: As we can see from (11) that

$$\xi_1 = c_{u12} - \frac{c_{u12} + c_{u11}}{e^{k_1} + 1}. \tag{67}$$

In the same way, we have $y_d = c_{u12} - \frac{c_{u12} + c_{u11}}{e^{\theta_1} + 1}$. Consequently, the tracking error can be expressed as follows:

$$y - y_d = \frac{(c_{u11} + c_{u12})\, e^{k_1} \left(1 - e^{-s_1}\right)}{\left(e^{k_1 - s_1} + 1\right)\left(e^{k_1} + 1\right)}. \tag{68}$$

By means of mean value theorem, we can obtain that there exists a scalar $\sigma_{s_1} \in (0, 1)$ such that $1 - e^{-s_1} = s_1 e^{-\sigma_{s_1} s_1}$, therfore, in the light of (68), the following inequality holds

$$|y - y_d| \leq (c_{u11} + c_{u12})e^{k_1}e^{-\sigma_{s_1}s_1}|s_1|. \tag{69}$$

The aforementioned statements imply that s_1 can be made arbitrarily small by selecting the large design parameter λ_m. Therefore, owing to $k_1, s_1 \in L_\infty$, by selecting the design parameters properly, we can make the tracking error $y - y_d$ arbitrarily small.

IV. Examples

In an effort to substantiate the validity and superiority of our proposed control strategy, a representative example is presented as follows:

Consider a strict-feedback uncertain nonlinear CPSs under malicious attacks borrowed from [16], whose dynamics is described by the following differential equations:

$$\begin{cases} \dot{\varsigma} = -\varsigma + 0.5\xi_1^2 \sin(\xi_1 t), \\ \dot{\xi}_1 = \xi_1 e^{0.5\xi_1} + \left(1 + \xi_1^2\right)\xi_2 + d_1 + \varrho_1(\varsigma, \xi_1, \xi_2, t), \\ \dot{\xi}_2 = \xi_1\xi_2^2 + (3 + \cos(\xi_1))\left(\alpha(t_m)u + \beta(t_a)\right) + d_2 \\ \qquad + \varrho_2(\varsigma, \xi_1, \xi_2, t), \\ y = \xi_1, \end{cases} \tag{70}$$

where the unmodeled dynamics are $\varrho_1(\varsigma, \xi_1, \xi_2, t) = 0.2\varsigma\xi_1 \sin(\xi_2 t)$, $\varrho_2(\varsigma, \xi_1, \xi_2, t) = 0.1\varsigma\cos(0.5\xi_2 t)$, the malicious attack signals $\alpha(t_m)$ and $\beta(t_a)$ are $\alpha(t_m) = 0.003 + 0.001\exp(0.1t)$ and $\beta(t_a) = \cos^2(\xi_1)\xi_2$. The

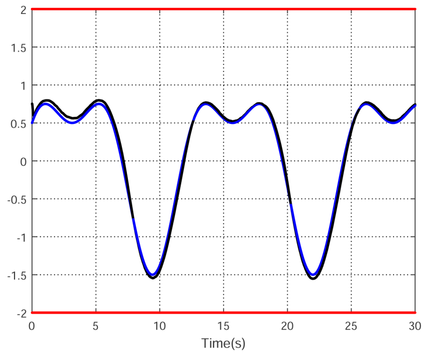

Fig. 2. Profiles of output y (black line) and reference signal y_d (blue line).

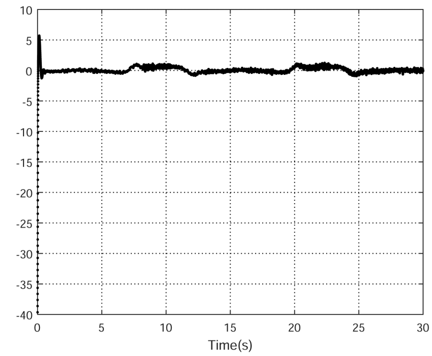

Fig. 3. Profile of control law u.

reference signal is chosen as $y_d = 0.5\cos(t) + \sin(0.5t)$. d_1 and d_2 are possible bounded external disturbances with $\overline{d}_1 = \overline{d}_2 = 1$. The dynamic signal are designed as $\dot{b} = -b + 2.5\xi_1^4 + 0.625$.

In terms of our proposed control strategy, the total computation cost is relatively small. It allows us to set sampling period as 5 ms in the entire simulation process, which satisfies most practical engineering applications requirements. The related design parameters are chosen as $c_{u11} = c_{u12} = 2$, $c_{u21} = 2$, $c_{u22} = 2.5$, $f_1 = 10$, $f_2 = 15$, $\lambda_1 = \lambda_2 = 50$, $\gamma_1 = \gamma_2 = 0.01$, $x_1 = x_2 = 5$. The initial conditions are chosen as $\xi_1(0) = 0.75$, $\xi_2(0) = 0.1$, $\varsigma(0) = 0.1$, $b(0) = 0.1$, $\theta_2(0) = 0.1$, $\hat{\mu}_1(0) = 2$, $\hat{\mu}_2(0) = 0.5$. Consequently, by selecting the partitioning points $-4, -3, -2, -1, 0, 1, 2, 3, 4$, the fuzzy sets are defined over interval $[-4, 4]$ for all the variables. The corresponding fuzzy membership functions are defined as follows:

$$\delta_1(\Sigma_1) = \exp\left(-\frac{(\Sigma_1 - \Sigma_{v,1})^T(\Sigma_1 - \Sigma_{v,1})}{8}\right),$$

$$\delta_2(\Sigma_2) = \exp\left(-\frac{(\Sigma_2 - \Sigma_{v,2})^T(\Sigma_2 - \Sigma_{v,2})}{8}\right),$$

where $v = 1, 2, \ldots, 9$. $\Sigma_1, \Sigma_2, \Sigma_{v,1}$ and $\Sigma_{v,2}$ are expressed as follows:

$$\Sigma_1 = [k_1, k_2, s_1, \dot{\theta}_1, b]^T,$$

$$\Sigma_2 = [k_1, k_2, s_2, \dot{\theta}_2, b]^T,$$

$$\Sigma_{v,1} = \left[\overbrace{-5+v, -5+v, \ldots, -5+v}^{5}\right]^T,$$

$$\Sigma_{v,2} = \left[\overbrace{-5+v, -5+v, \ldots, -5+v}^{5}\right]^T.$$

In order to further verify the superiority of our proposed control strategy, a comparison between our proposed approach and [24] is completed. Figs. 2, 3, 4, 5, and 6 display the overall simulation results. Fig. 2 exhibits the trajectories of the system output y and the reference signal y_d. As we can observe from Fig. 2 that the fairly good tracking performance is guaranteed and the state constraint condition on state ξ_1 is satisfied during the

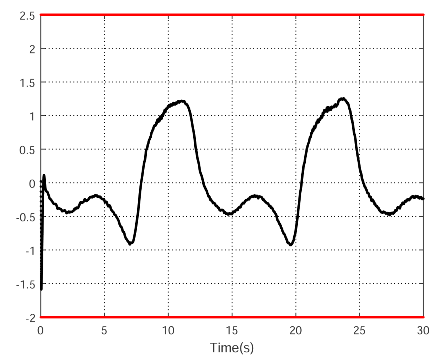

Fig. 4. Profile of stete ξ_2.

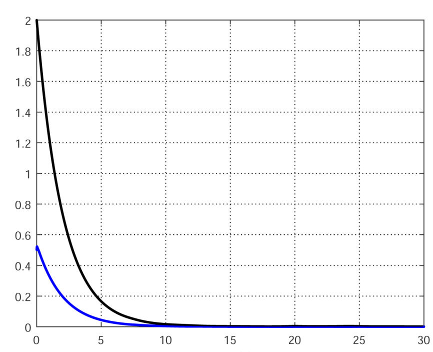

Fig. 5. Profiles of adaptation law $\hat{\mu}_1$ (black line) and $\hat{\mu}_2$ (blue line).

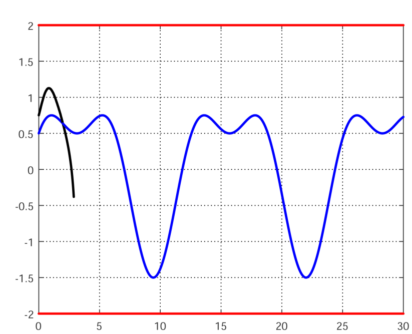

Fig. 6. Profiles of output y (black line) and reference signal y_d (blue line) using the method in [24].

total control operation. Fig. 4 shows that the state constraint on state ξ_2 is not violated all the time. Fig. 6 presents the trajectories of the system output y and the reference signal y_d utlizing the method in [24]. It is apparent that the tracking performance can not be guaranteed. All in all, the good tracking control performance is guaranteed drawing support from the developed control strategy in Theorem 1, which not only means the control objective is accomplished, but also shows the superiority in contrast with the approach in [24].

V. Conclusion

In this article, we have investigated the tracking control problem for a category of state-constrained high-order nonlinear CPSs with unmodeled dynamics and external disturbances against malicious attacks. Combining the backstepping approach, FLSs technology and DSC technique, a novel adaptive fuzzy tracking control strategy has been proposed, which is not only independent of the exact system model, but also accommodates external disturbances, unmodeled dynamics and malicious attacks. We have substantiated that the developed control strategy can guarantee that the output tracking error enters a predefined small neighborhood of the origin, and make sure that all the signals of the resulting closed-loop system satisfy the corresponding state constraints during the total operation, where the tracking accuracy level are known and can be preconfigured by selecting the design parameters appropriately. Eventually, a representative instance has been supplied to illustrate the effectiveness and superiority of our proposed control scheme. Note that in many practical engineering applications, the systems are always restricted to keep stable in finite time or fixed time. In addition, what makes the matter worse is that diverse categories of cyber attacks may exist simultaneously in the cyber-physical environment. Therefore, in our future research, we will ulteriorly consider how to develop a fixed-time adaptive fuzzy control law to the state-constrained high-order uncertain nonlinear CPSs with unmodeled dynamics under more sophisticated cyber-physical environment, where cyber threats such as DoS attacks and deception attacks might coexist.

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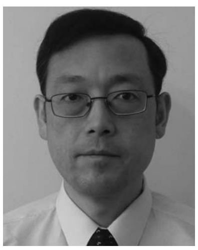
Zhaoyang Cuan received the B.E. degree from the University of Science and Technology Beijing, Beijing, China, in 2018. He is currently working toward the Ph.D. degree from the University of Science and Technology Beijing, Beijing, China. His research interests include cyber-physical systems, robust control, and safety control of unmanned ground/surface vehicles.

Da-Wei Ding (Senior Member, IEEE) received the B.E. degree from the Ocean University of China, Qingdao, China, in 2003, and the Ph.D. degree in control theory and engineering from Northeastern University, Shenyang, China, in 2010. He is currently a Professor with the School of Automation and Electrical Engineering, University of Science and Technology Beijing, Beijing, China. His research interests include robust control and filtering, multi-agent systems, and cyber-physical systems.



Xiaoping Liu received the B.E., M.E., and Ph.D. degrees in electrical engineering from Northeastern University, Shenyang, China, in 1984, 1987, and 1989, respectively. He spent more than ten years with the School of Information Science and Engineering, Northeastern University. In 2001, he joined the Department of Electrical Engineering, Lakehead University, Thunder Bay, ON, Canada. His research interests include nonlinear control systems, singular systems, and robust control. Dr. Liu is a member of the Professional Engineers of Ontario.

Youyi Wang received the B.E. degree in electrical engineering from the University of Science and Technology Beiing, Beijing, China, in 1982, the M.E. degree in electrical engineering from Tsinghua University, Beijing, in 1984, and the Ph.D. degree from the University of Newcastle, Callaghan, NSW, Australia, in 1991. In 1991, he joined the School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore. He is currently a Full Professor with the School of Electrical and Electronic Engineering. His research interests include nonlinear control, robust control, control and applications of control theory to power systems, electricdrive systems, information storage systems, fuel cell systems, and renewable energy systems.