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A Layout Method of Space-Based Pseudolite System Based on GDOP Geometry

YANG Xin, WANG Feixue, LIU Wenxiang, XIAO Wei, and YE Xiaozhou

(*National University of Defense Science and Technology, Changsha 410073, China*)

Abstract — The pseudolite system can be used to provide positioning and timing service for users in a specific area. In order to provide better positioning and timing service, a good geometric configuration needs to be formed for the pseudolite system. For the problem of pseudolite system deployment, the average and mean square values of geometric dilution of precision (GDOP) are optimized in this paper for users in a target area. We proposed a space-based pseudolite deployment method based on GDOP geometry, starting from the minimum GDOP value for users in the central area. From the simulation results, it can be seen that, compared with the empirical method and the NSGA-II method, the method in this paper has the smallest average GDOP, a better robustness, and a higher positioning accuracy in the target area, through which a pseudolite deployment proposal can be quickly obtained.

Key words — Pseudolite, Positioning accuracy, Geometric dilution of precision, Geometry structure.

I. Introduction

After the Beidou-3 global positioning system was launched in 2020 [1], users around the world can obtain positioning and navigation timing information through the Beidou satellite navigation system. However, due to terrain occlusion and other issues, users in some areas, such as river valleys and mining areas, still cannot receive satellite signals. On the other hand, since the satellite is far from the ground, the landing power of its signals is weak which are easily interfered, resulting in the loss of the satellite signals for users. In order to avoid the loss of life and properties of users in these situations, a pseudolite system needs to be established to provide a spatial and temporal reference for area navigation. Pseudolites are used to increase the accuracy, precision, and the redundancy of satellite navigation in the areas with a limited line of sight to navigation satellites. Moreover, pseudolites can also be used as anti-jamming systems as the power of their signals can be changed at any time [2], [3].

Pseudolites can be divided into ground-based and space-based pseudolites according to their placement position. Ground-based pseudolites are easily blocked by terrain because of their placement height on the ground. The positions of pseudolites are stationary, and the service range of their signal coverage is relatively small. Compared with ground-based pseudolites, a space-based pseudolite system can be used to overcome the above shortcomings to a certain extent. A space-based pseudolite platform can be carried on UAVs or airships. The stratosphere is considered as an ideal area for the operation of space-based flight platforms, because of its stable meteorological conditions and low wind speeds. Although the aircraft in this space has the characteristics of better survivability, longer working time, and wider coverage than others [4], [5], this space is also considered to be the "dead zone" of the UAVs: the air in this space is very thin, so it is difficult to generate a driving force, and the temperature here is extremely low, which is easy to cause hardware failures.

The deployment of pseudolite systems is very flexible, but the geometric distribution of independent-networked pseudolite systems will directly affect the users' positioning accuracy [6]. Deploying multiple pseudolites is the first step to meet the requirements on positioning accuracy of users in a specific area, and the advantages and disadvantages of the deployment proposal will be directly mapped to the users' positioning results.

At present, the research on the deployment of pseudolite systems can be roughly divided into two types. One of which is the layout proposal based on ex-

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perience. Layout methods are given based on experience in most of the research, and there is no specific theoretical derivation process. Therefore, these empirical layout methods may lead to good results in specific application scenarios, but they lack universality. Another type of deployment method is the search layout method based on the optimization algorithm, which are used to obtain a deployment proposal that satisfies the requirements of users, but the defects of this type of method are also obvious: one is that it takes a long time to get results, the other is that the output deployment schemes of these studies are random.

In the research on pseudolite deployment in recent years, the research in [7] proposed an experience-based search algorithm for the optimal layout of pseudolite areas. The core idea of this research is that there are always better layouts around a layout, so the optimal layout is found through empirical search algorithm. Reference [8] proposed a pseudolite layout scheme based on near-space airship positioning, the research in [9] proposed a pseudolite layout method based on the geometric structure of geometric dilution precision; researches [10], [11] gives an empirical pseudolite layout scheme respectively. In addition, some scholars have applied the optimization algorithm to the optimal layout design of pseudolites, such as [12], where the linear decreasing weight particle swarm algorithm is used to optimize the deployment. Moreover, sparse A* algorithm [13], genetic algorithm [14]–[17], particle swarm optimization [18] and NSGA-II [19], [20] can be used to optimize pseudolite deployment. The pseudolite deployment scheme based on the optimization algorithm can often obtain a satisfactory result for the user, but it also has a common defect, that is, it takes a long time to obtain the results, which will limit the maneuverability of the space-based pseudolite platforms. Therefore, a pseudolite layout optimization algorithm that can be quickly deployed is proposed in this paper.

II. Analysis of the Proposed Algorithm

In order to achieve independent pseudolite system positioning, the location selection of pseudolites needs to meet several conditions: one is that users in the target area can receive more than four pseudolite signals, that is, the elevation angles of users and pseudolites are greater than the receiver elevation cutoff angle, and the other is that the height of the pseudolites has to meet the requirements on flight height of a carrier platform. On the other hand, in order to provide higher-precision positioning service, the GDOP value of users in this area should be optimized.

When providing positioning and timing service for users in a specific area through a pseudolite system, it is necessary to ensure the positioning performance of all users in the area, that is, no matter where the users are located, they can obtain a good positioning accuracy. We assume that users are uniformly distributed in the target area, which can be shown as $P_i, i = 1, 2, \ldots, M$ in Fig.1. The mean and mean squared error of GDOP values of users in the region need to be minimized. Therefore, the specific optimization objectives of the pseudolite system can be expressed as follows:

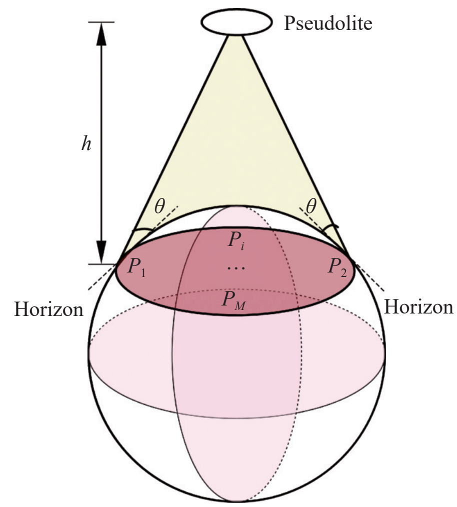

Fig. 1. Schematic diagram of space-based pseudolite signal coverage.

1) The average GDOP of users in the area is the smallest, which is

$$\min \frac{\sum_{i=1}^{M} GDOP_i}{M} \tag{1}$$

$GDOP_i$ indicates the GDOP value of the ith user in the target region.

2) The mean square error of the GDOP value is the smallest, which is

$$\min S = \sqrt{\frac{1}{M}\sum_{i=1}^{M}\left(GDOP_i - \overline{GDOP}\right)^2} \tag{2}$$

$\overline{GDOP}$ represents the average GDOP of M users in the target area.

1. Pseudolites deployment space

In order to meet the signal coverage of the target area, the pseudolite position can only take values within a certain range. We assume that the elevation cutoff angle of the received signal is θ which is also marked in Fig.1, h means the height of the pseudolite, the target area to cover is a circle of radius r. Firstly, we create a coordinate system with the center of the circle as the origin, which is shown as Fig.2, and the pseudolite coordinates are (x, y, z) respectively, so the layout space $\boldsymbol{V}$ of pseudolites is

$$\begin{cases} x^2+y^2 \le \left(\dfrac{z}{\tan(\theta)}-r\right)^2 \\ h_{\min} \le z \le h_{\max} \end{cases} \tag{3}$$

Among them, $h_{\min}$ is the lowest altitude at which the pseudolite can be deployed, $h_{\max}$ indicates the maximum altitude. When the pseudolite signal can cover the target area, the pseudolite position $SatPos$ must belong to the space $\boldsymbol{V}$, which can be written as $SatPos \in \boldsymbol{V}$. Specifically, the space $\boldsymbol{V}$ is shown in the Fig.2.

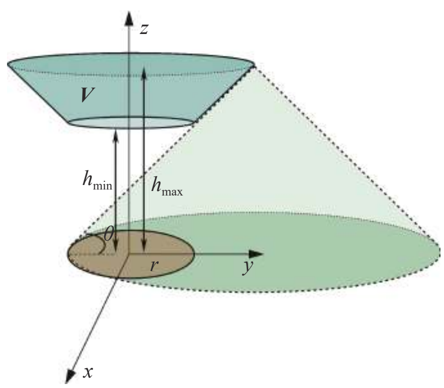

Fig. 2. Schematic diagram of the space $\boldsymbol{V}$ where pseudolites can be deployed.

2. Geometric dilution of precision (GDOP)

After receiving the pseudolite signal, receiver uses the pseudolite measurements to solve the positioning equation, and the positioning error covariance matrix is [21]

$$\boldsymbol{G}=\left(\boldsymbol{H}^{\mathrm{T}}\boldsymbol{H}\right)^{-1}=\begin{bmatrix} g_{11} & g_{12} & g_{13} & g_{14} \\ g_{21} & g_{22} & g_{23} & g_{24} \\ g_{31} & g_{32} & g_{33} & g_{34} \\ g_{41} & g_{42} & g_{43} & g_{44} \end{bmatrix} \tag{4}$$

where g_{ij} means the element of the ith row and jth column of the $\boldsymbol{G}$, $\boldsymbol{H}$ is the observation matrix in the navigation solution equation as follows:

$$\boldsymbol{H}=\begin{bmatrix} \vec{\boldsymbol{e}}_1^{\,T} & -1 \\ \vec{\boldsymbol{e}}_2^{\,T} & -1 \\ \vdots & \vdots \\ \vec{\boldsymbol{e}}_n^{\,T} & -1 \end{bmatrix}=\begin{bmatrix} a_1 & b_1 & c_1 & -1 \\ a_2 & b_2 & c_2 & -1 \\ \vdots & \vdots & \vdots & \vdots \\ a_n & b_n & c_n & -1 \end{bmatrix} \tag{5}$$

where $\vec{\boldsymbol{e}}_i$ represents the line-of-sight unit direction vector from the user to the ith pseudolite.

The GDOP value is expressed as

$$\begin{aligned} GDOP &= \sqrt{g_{11}+g_{22}+g_{33}+g_{44}}=\sqrt{\operatorname{Tr}\left(\boldsymbol{H}^{\mathrm{T}}\boldsymbol{H}\right)^{-1}} \\ &= \sqrt{\frac{\operatorname{Tr}[\operatorname{adj}(\boldsymbol{H}^{\mathrm{T}}\boldsymbol{H})]}{\det(\boldsymbol{H}^{\mathrm{T}}\boldsymbol{H})}} \end{aligned} \tag{6}$$

$(\cdot)^{\mathrm{T}}$ represents the conjugate transpose of a matrix, $\operatorname{adj}(\cdot)$ represents the conjugate of a matrix, $\det(\cdot)$ represents the determinant of a matrix.

3. Geometry of GDOP

The premise that a pseudolite system can provide positioning service for the target area is that it can achieve quadruple coverage of a specific area, that is, receivers in a specific area can receive at least four or more pseudolite signals. The geometry of quadruple coverage, for example, is shown as Fig.3.

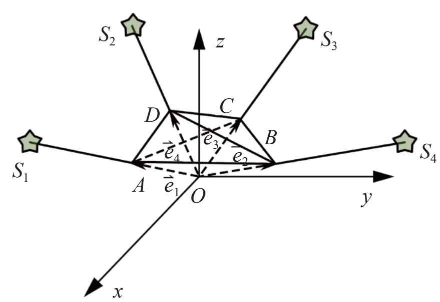

Fig. 3. Geometry of GDOP Values.

According to [22], [23], GDOP is related to a ratio, whose denominator is the volume of simplex enclosed by each vertex.

III. Location Layout Algorithm

The core idea of the algorithm proposed in this paper is to ensure that the GDOP value of the users in the center is the smallest, and because the changes in the direction of the receiver and the satellite in surrounding areas are continuous, the GDOP value of users around the center will be gradually increase, so that the GDOP takes a minimum value in the center. Based on the above thinking, it is necessary to find out what conditions $\boldsymbol{H}$ should satisfy to make the GDOP can obtain the minimum value.

The problem of finding the minimum value of GDOP is essentially the problem of finding the extreme value of a function. From the previous section, we can see that the value of GDOP is related to the volume of simplex. Therefore, the problem of finding the minimum value of GDOP can be equivalent to finding the maximum volume of simplex [22]–[25].

1. Quadruple coverage

To find the largest volume of simplex is to find the maximum volume of the tetrahedron enclosed by the endpoints of each unit line-of-sight vector. Moreover, each endpoint is located on the spherical surface of the unit sphere. However, due to the limitations of the users' receiving elevation angle, the placement position of pseudolites $SatPos \in \boldsymbol{V}$ must be satisfied. According to the formula (3), only when the triangular pyramid ABCD is distributed in the upper half of the unit sphere $z \geq z_0$, it can be ensured that no matter where the users are in the target area, the pseudolite signals can be received.

When the volume of the triangular pyramid ABCD is the largest, the area of the base ΔABC and the distance from it to the vertex D are the largest, so that the base ΔABC is the inscribed triangle of the unit sphere section circle and ΔABC is an equilateral triangle. When OD is perpendicular to the base ΔABC, the height of the triangular pyramid ABCD is the largest, which can be shown in the Fig.4.

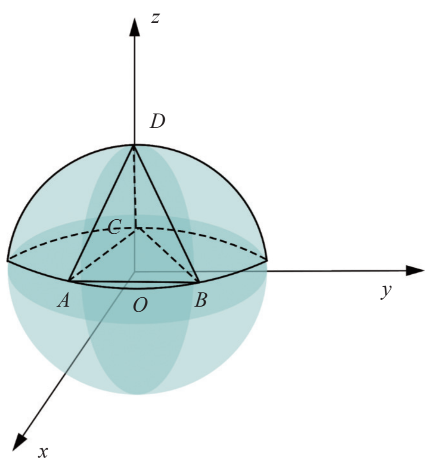

Fig. 4. GDOP geometry considering elevation cut-off angle.

The value of z_0 specifically is

$$\vartheta = \tan^{-1}\left(\frac{h_{\max}}{\dfrac{h_{\max}}{\tan(\theta)} - r}\right) \tag{7}$$

$$z_0 = \sin(\vartheta) \tag{8}$$

The inscribed triangular pyramid with the largest volume in the sphere is a regular triangular pyramid, and the matrix $\boldsymbol{H}$ of the user at the origin is $\boldsymbol{H}_0$, which is

$$\boldsymbol{H}_0 = \begin{bmatrix} 0 & 0 & 1 & -1 \\ 0 & -\sqrt{1-z_0^2} & z_0 & -1 \\ -\dfrac{\sqrt{3}}{2}\sqrt{1-z_0^2} & \dfrac{1}{2}\sqrt{1-z_0^2} & z_0 & -1 \\ \dfrac{\sqrt{3}}{2}\sqrt{1-z_0^2} & \dfrac{1}{2}\sqrt{1-z_0^2} & z_0 & -1 \end{bmatrix} \tag{9}$$

The unit vector of each pseudolite to the origin can be obtained from the matrix $\boldsymbol{H}_0$, and the pseudolite position at this time can be supposed as

$$\boldsymbol{Pos} = \begin{bmatrix} 0 & 0 & l_1 \\ 0 & -\sqrt{1-z_0^2}\,l_2 & z_0 l_2 \\ -\dfrac{\sqrt{3}}{2}\sqrt{1-z_0^2}\,l_3 & \dfrac{1}{2}\sqrt{1-z_0^2}\,l_3 & z_0 l_3 \\ -\dfrac{\sqrt{3}}{2}\sqrt{1-z_0^2}\,l_4 & \dfrac{1}{2}\sqrt{1-z_0^2}\,l_4 & z_0 l_4 \end{bmatrix} \tag{10}$$

where l_i denotes the distance between the ith pseudolite to the origin, so the locations of pseudolites can be shown in detail as Fig.5.

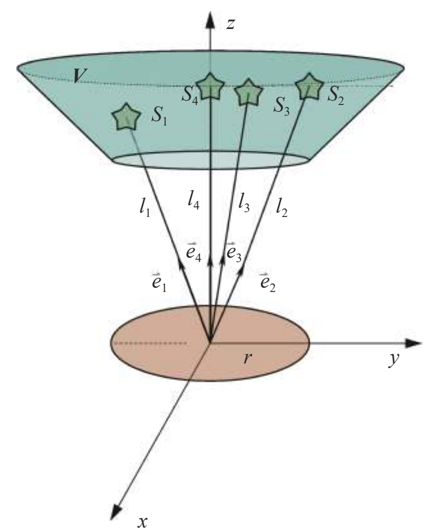

Fig. 5. Schematic diagram of pseudolite location.

It can be easily obtained that the lines whose unit direction vectors are $\vec{e}_i (i = 2, 3, 4)$ are tangent to space $\boldsymbol{V}$, and each has only one intersection. So, there has

$$l_2 = l_3 = l_4 = \frac{h_{\max}}{z_0} \tag{11}$$

Then the pseudolite position at this time can be expressed as

$$\boldsymbol{Pos} = \begin{bmatrix} 0 & 0 & \zeta \\ 0 & -R' & h_{\max} \\ -\dfrac{\sqrt{3}}{2}R' & \dfrac{1}{2}R' & h_{\max} \\ \dfrac{\sqrt{3}}{2}R' & \dfrac{1}{2}R' & h_{\max} \end{bmatrix} \tag{12}$$

$$R' = \frac{h_{\max}}{\tan(\vartheta)} \tag{13}$$

where ζ in this matrix means the altitude of the first pseudolite, which represents an unknown quantity that we have to resolve.

The matrix $\boldsymbol{H}$ of the user whose coordinate is

$(r\cos\varphi, r\sin\varphi, 0)$ can be expressed as

$$\boldsymbol{H}=\begin{bmatrix} \dfrac{-r\cos\varphi}{q_1} & \dfrac{-r\sin\varphi}{q_1} & \dfrac{\zeta}{q_1} & -1 \\ \dfrac{-r\cos\varphi}{q_2} & \dfrac{R'-r\sin\varphi}{q_2} & \dfrac{h_{\max}}{q_2} & -1 \\ \dfrac{-\frac{\sqrt{3}}{2}R'-r\cos\varphi}{q_3} & \dfrac{-\frac{1}{2}R'-r\sin\varphi}{q_3} & \dfrac{h_{\max}}{q_3} & -1 \\ \dfrac{\frac{\sqrt{3}}{2}R'-r\cos\varphi}{q_4} & \dfrac{-\frac{1}{2}R'-r\sin\varphi}{q_4} & \dfrac{h_{\max}}{q_4} & -1 \end{bmatrix} \tag{14}$$

$$q_1=\sqrt{r^2+\zeta^2} \tag{15}$$

$$q_2=\sqrt{(-r\cos\varphi)^2+(R'-r\sin\varphi)^2+h_{\max^2}} \tag{16}$$

$$q_3=\sqrt{\left(-\frac{\sqrt{3}}{2}R'-r\cos\varphi\right)^2+\left(-\frac{1}{2}R'-r\sin\varphi\right)^2+h_{\max}^2} \tag{17}$$

$$q_4=\sqrt{\left(\frac{\sqrt{3}}{2}R'-r\cos\varphi\right)^2+\left(-\frac{1}{2}R'-r\sin\varphi\right)^2+h_{\max}^2} \tag{18}$$

Assuming that the function of $f(\zeta,\varphi)$ is

$$f(\zeta,\varphi)=\det(\boldsymbol{H}^{\mathrm{T}}\boldsymbol{H}) \tag{19}$$

Taking the partial derivative of $f(\zeta,\varphi)$ with the respect to φ, we have

$$f'_{\varphi}(\zeta,\varphi)=\frac{\partial f(\zeta,\varphi)}{\partial\varphi} \tag{20}$$

It can be obtained that when φ equals to $\frac{\pi}{6}$, $\frac{\pi}{2}$, $\frac{5\pi}{6}$, $\frac{7\pi}{6}$, $\frac{3\pi}{2}$, $\frac{11\pi}{6}$, there has

$$f'_{\varphi}(\zeta,\varphi)=0 \tag{21}$$

That is, when φ is at these points, regardless of the value of ζ, $f'_{\varphi}(\zeta,\varphi)$ is always equal to 0, and there also has

$$f\left(\zeta,\frac{\pi}{6}\right)=f\left(\zeta,\frac{5\pi}{6}\right)=f\left(\zeta,\frac{3\pi}{2}\right) \tag{22}$$

$$f\left(\zeta,\frac{\pi}{2}\right)=f\left(\zeta,\frac{7\pi}{6}\right)=f\left(\zeta,\frac{11\pi}{6}\right) \tag{23}$$

Suppose the functions $g_1(\zeta)$, $g_2(\zeta)$ are respectively

$$g_1(\zeta)=f\left(\zeta,\frac{\pi}{6}\right) \tag{24}$$

$$g_2(\zeta)=f\left(\zeta,\frac{\pi}{2}\right) \tag{25}$$

the derivatives of ζ for these two functions have the following characteristics

$$g'_1(\zeta)=g'_2(\zeta) \tag{26}$$

The derivatives of these two functions are equal, so we assume that

$$g'(\zeta)=g'_1(\zeta)=g'_2(\zeta) \tag{27}$$

When the equation (27) is equal to 0,

$$g'(\zeta)=0 \tag{28}$$

there has the following equation:

$$\zeta=\delta_0=\frac{1}{h_{\max}}(R'r-r^2+\Theta) \tag{29}$$

where

$$\Theta=\frac{\frac{3}{2}R'r}{\sqrt{h_{\max^2}+R'^2+R'r+r^2}-\sqrt{h_{\max^2}+R'^2-2R'r+r^2}} \tag{30}$$

Moreover, there has

$$\begin{cases}\zeta\le\delta_0, & g'(\zeta)\le 0\\ \zeta>\delta_0, & g'(\zeta)>0\end{cases} \tag{31}$$

According to formula (31), it can be seen that the function change trend is decreasing first and then increasing.

From the previous analysis, it can be obtained that the value range of ζ is $h_{\min}\le\zeta\le h_{\max}$. Hence, when $h_{\min}\ge\delta_0$, $\det(\boldsymbol{H})$ increases with increasing ζ, and when $\zeta_0=h_{\max}$, $\det(\boldsymbol{H})$ has the max value; When $h_{\max}\le\delta_0$, $\det(\boldsymbol{H})$ decreases as ζ increase, and when $\zeta_0=h_{\min}$, it has the max value of $\det(\boldsymbol{H})$; When $h_{\min}<\delta_0<h_{\max}$, $f(\zeta_0,\frac{\pi}{6})=\max\limits_{\zeta_0\in[h_{\min},h_{\max}]}[f(h_{\min},\frac{\pi}{6}),f(h_{\max},\frac{\pi}{6})]$.

From the above analysis, it is clear that when φ is $\pi/6$, either $h_{\min}$ maximizes $\det(\boldsymbol{H}')$ or $h_{\max}$, so for ζ we take the value of one with the smallest GDOP value among these two

$$GDOP\left(\zeta_0,\frac{\pi}{6}\right)=\min_{\zeta_0\in[h_{\min},h_{\max}]}\left[GDOP\left(h_{\min},\frac{\pi}{6}\right),GDOP\left(h_{\max},\frac{\pi}{6}\right)\right] \tag{32}$$

It means that ζ_0 takes the value in the binary set $[h_{\min},h_{\max}]$ with smallest GDOP.

Based on the above conclusions, the pseudolite positions are optimized as

$$\boldsymbol{Pos} = \begin{bmatrix} 0 & 0 & \zeta_0 \\ 0 & -R' & h_{\max} \\ -\dfrac{\sqrt{3}}{2}R' & \dfrac{1}{2}R' & h_{\max} \\ \dfrac{\sqrt{3}}{2}R' & \dfrac{1}{2}R' & h_{\max} \end{bmatrix} \tag{33}$$

2. Simulation

Assuming that a quadruple coverage pseudolite system needs to be deployed, the space height range of the pseudolites is limited from 15 km to 20 km above the ground, the radius of the target area is 100 km, and the number of pseudolites is 4, and the elevation cutoff angle of the receiver is 5°. The empirical layout method 1 given in [11], the empirical layout method 2 refers to [8], the method based on the NSGA-II algorithm [19], and the proposed method are used to obtain deployment proposals.

After using the proposed algorithm, the GDOP value of the receiver in the target area is shown in Fig.6.

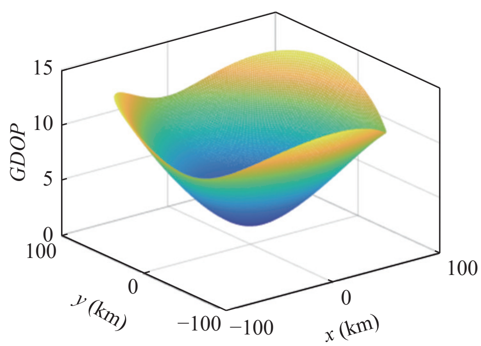

Fig. 6. The GDOP value of users in the target area under the method of this paper.

Under the pseudolite deployment scheme obtained through the empirical layout method 1, the GDOP value distribution of each user in the target area is shown in Fig.7.

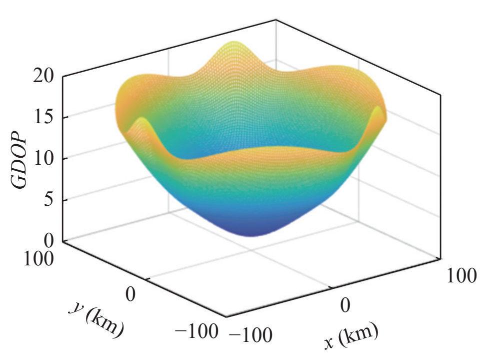

Fig. 7. GDOP value of users in target area under empirical method 1.

The GDOP value of each user through the empirical layout method 2 is shown in Fig.8.

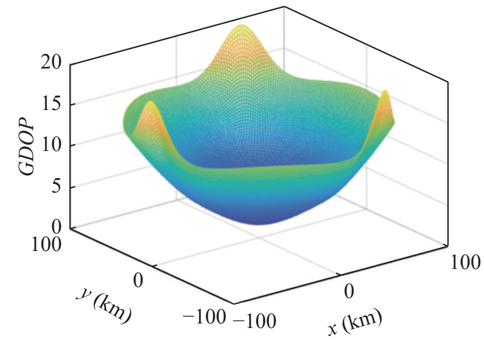

Fig. 8. GDOP value of users in target area under empirical method 2.

The user GDOP value distribution of users in the pseudolite deployment scheme obtained through the NSGA-II algorithm is shown in Fig.9.

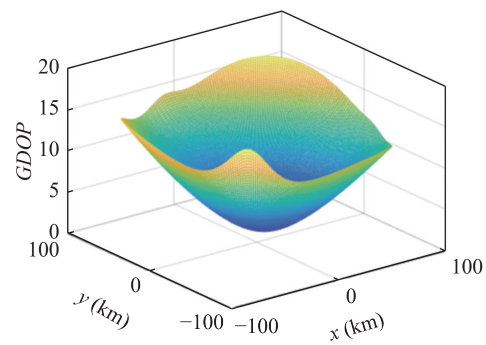

Fig. 9. GDOP value of users in target area under NSGA-II method.

We have counted the GDOP value of users in the target area, and the results are shown in Table 1. The average GDOP value, the mean square error value, and the maximum as well as the minimum value of the users are counted respectively. The minimum GDOP represents the upper limit of users' positioning accuracy in the target area, and the maximum GDOP value represents the deterioration degree of the users' positioning accuracy in this area, that is, the worst positioning performance.

Table 1. GDOP statistical results for each method

Method	Mean GDOP	Mean square error of GDOP	The minimum GDOP	The maximum GDOP
This paper	5.8812	3.1176	1.9301	13.4209
Empirical method 1	7.6509	4.5855	2.0246	19.5426
Empirical method 2	6.6127	3.8663	2.0246	20.0828
NSGA-II method	6.5956	3.6465	2.0483	16.9873

From the statistical results, it can be seen that the result obtained through the proposed method is smaller than that through the empirical method 1 and the empirical method 2 in terms of mean GDOP and mean square error. Compared with the NSGA-II method, the average GDOP and the mean square error of GDOP of this method are also smaller. The minimum value of GDOP through the proposed method is the smallest among results through all methods, which can make the positioning accuracy of users higher than other methods, and the maximum value of GDOP in this method is also smaller than that of other methods, which can better ensure the users' positioning performance. It takes a long time to obtain the deployment scheme through NSAG-II method, the running time of which exceeds 27 min, while the time required for the proposed method is less than 1 s.

In conclusion, the deployment proposal obtained through the proposed method can ensure that users in the center get a better positioning accuracy, and the average as well as mean square error GDOP of users is smaller. Moreover, the proposed method is much effective than NSGA-II.

3. Fivefold coverage

By analogy with quadruple coverage, the layout of the five-fold coverage is optimized. The geometry whose five vertices are inscribed in the unit sphere is shown in the Fig.10. When the volume of the geometry ABCDE is the largest, the area of its bottom surface ABCD is the largest and the vertical distance from the vertex E to the ground is the longest. Therefore, its base ABCD is a square inscribed in a circle $z = z_0$ of the cross-section of a unit hemisphere, and OE is perpendicular to the base ABCD.

The matrix $\boldsymbol{H}$ of the user at the origin is $\boldsymbol{H}_0$, and can be expressed as

$$\boldsymbol{H}_0 = \begin{bmatrix} 0 & 0 & 1 & -1 \\ 0 & -\sqrt{1-z_0^2} & z_0 & -1 \\ 0 & \sqrt{1-z_0^2} & z_0 & -1 \\ \sqrt{1-z_0^2} & 0 & z_0 & -1 \\ -\sqrt{1-z_0^2} & 0 & z_0 & -1 \end{bmatrix} \tag{34}$$

In the same way, the lines whose unit direction vector are $\overrightarrow{\boldsymbol{e}}_2, \overrightarrow{\boldsymbol{e}}_3, \overrightarrow{\boldsymbol{e}}_4, \overrightarrow{\boldsymbol{e}}_5$ are also tangent to the space $\boldsymbol{V}$, and there are only one point of intersection.

Hence, the pseudolites position can be expressed as follows:

$$\boldsymbol{Pos} = \begin{bmatrix} 0 & 0 & \zeta \\ 0 & R' & h_{\max} \\ 0 & -R' & h_{\max} \\ R' & 0 & h_{\max} \\ -R' & 0 & h_{\max} \end{bmatrix} \tag{35}$$

where ζ indicates the altitude of the first pseudolite.

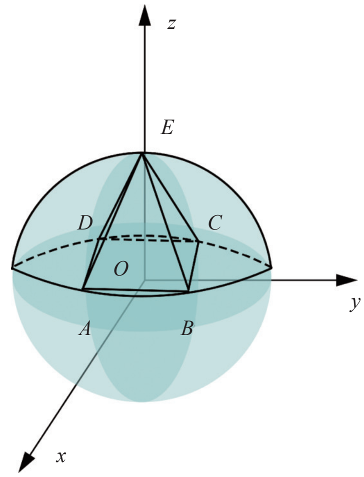

Fig. 10. Sphere inscribed geometry for fivefold coverage.

By analogy with the derivation process of quadruple coverage, we can get

$$GDOP(\zeta_0, 0) = \min_{\zeta_0 \in [h_{\min}, h_{\max}]} [GDOP(h_{\min}, 0), GDOP(h_{\max}, 0)] \tag{36}$$

Therefore, the layout optimization location for pseudolites can be expressed as

$$\boldsymbol{Pos} = \begin{bmatrix} 0 & 0 & \zeta_0 \\ 0 & -R' & h_{\max} \\ 0 & R' & h_{\max} \\ -R' & 0 & h_{\max} \\ R' & 0 & h_{\max} \end{bmatrix} \tag{37}$$

4. Simulation

The number of pseudolites in this subsection is 5, and other conditions are the same as those in the above subsection. The positions of pseudolites are calculated through the method in this paper, the empirical layout method, and the layout algorithm based on the NSGA-II algorithm.

Figs.11–14 show the average value, mean square error, the maximum and minimum value of GDOP under these four methods respectively. It can be seen from the figure that with the increase of the users' elevation cutoff angle, in order to meet the five-fold signal coverage in the 100 km area, the GDOP value of users has deteriorated greatly. It can be seen from the statistical results that, the robustness of the empirical methods is very poor. The results obtained through empirical methods make the users' GDOP value deteriorate more obviously, and when the users' elevation cutoff angle is greater than 8°, the empirical methods can no longer meet the requirements on five-fold signal coverage. The NSGA-II method has better results than the empirical method, but its deterioration effect is more obvious than the proposed method. It can be clearly seen from

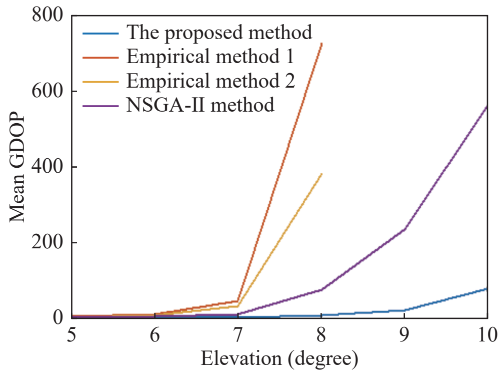

Fig. 11. Mean GDOP.

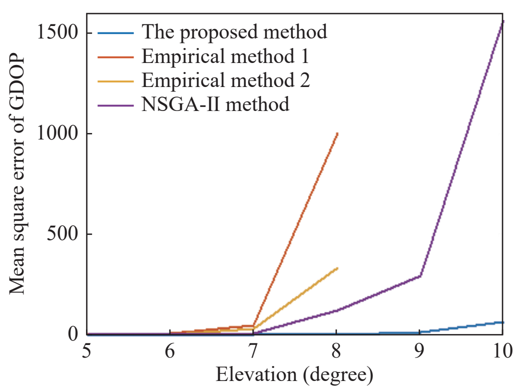

Fig. 12. Mean square error of GDOP.

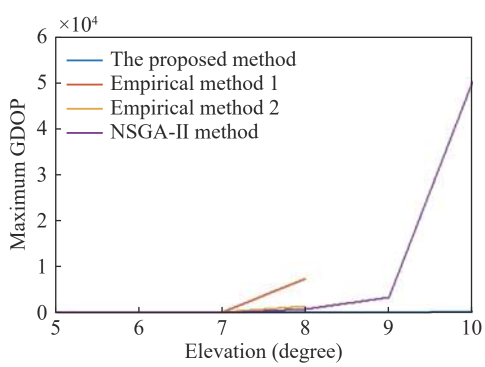

Fig. 13. Maximum GDOP.

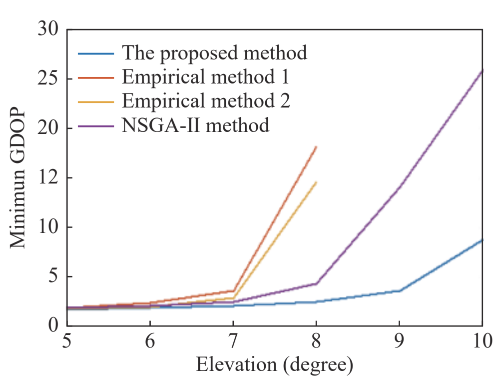

Fig. 14. Minimum GDOP.

the figure that when the users' elevation cutoff angle increases, the method proposed in this paper is more robust and can provide better positioning service with limited resources.

IV. Conclusions

For an independent-networked pseudolite system, the premise of providing a regional spatiotemporal positioning service is to form signal coverage for target areas, and to ensure that users there can receive at least four or more pseudolite signals. To meet the requirements on positioning accuracy, the system also has to provide good GDOP to users. For the problems that the robustness of the empirical layout method is not strong, and the layout method based on NSGA-II is time-consuming, a layout method is proposed in this paper based on the smallest GDOP value in the central area. The pseudolite deployment proposal obtained through the proposed method can provide better positioning performance for users in target area, and compared with other methods, a better average GDOP value can be achieved for users in the entire area, and the GDOP value can be better balanced for users. The method proposed in this paper is more robust and has its advantages when the users' elevation cutoff angle is large.

Our future work is to operate the UAVs according to the proposed method to conduct real-world experiments and analyze the effect of the proposed method in the actual system. Moreover, we will continue studying how to better serve users in a wider area when there are more than five UAVs.

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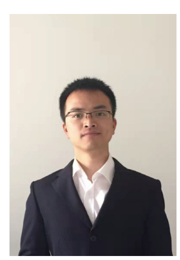
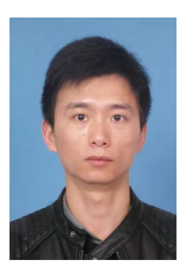
YANG Xin was born in 1995. received the B.S. degree in electronic science and technology from the National University of Defense Technology in 2017, and M.S. degree in 2019. Currently she is working towards a Ph.D. degree in the College of Electronic Science and Engineering, National University of Defense Technology, Changsha, China. His current research interests involve array antennas and precise positioning. (Email: YANGXIN_NUDT@163.com)

WANG Feixue (corresponding author) was born in 1971. received the Ph.D. degree in electronic science and technology from the National University of Defense Technology in Changsha, China, in 1998. He is currently a Professor with the College of Electronic Science and Engineering, National University of Defense Technology, Changsha, China. His research interests include Beidou positioning.
(Email: wangfeixue_nnc@163.com)

LIU Wenxiang received the M.S. and Ph.D. degree in electronic science and technology from the National University of Defense Technology in 2008 and 2011, respectively. Currently, he is a teacher in the College of Electronic Science and Engineering, National University of Defense Technology, Changsha, China. His research interests include precise positioning and system simulation.
(Email: liuwenxiang8888@163.com)



XIAO Wei was born in 1992 and received the Ph.D. degree in information and communication from the National University of Defense Technology in 2019. He is currently a Lecturer at the National University of Defense Technology. His main research field is navigation signal analysis.
(Email: eqiq2008@yeah.net)



YE Xiaozhou was born in 1991 and received the Ph.D. degree in information and communication from the National University of Defense Technology in 2020. He is currently a Lecturer at the National University of Defense Technology. His main research field is navigation positioning.
(Email: beiqiuzhiye@163.com)