Market Equilibria in Cross-Border Balancing Platforms

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***Abstract*—The next phase of electricity market integration in Europe will see the introduction of pan-European balancing platforms, MARI and PICASSO, for the trading of manual and automatic frequency restoration reserve. This article provides an analytical framework for the study of pricing asymmetries between European member states in this context. The pricing asymmetries are due to balancing incentive components and consist of the unilateral introduction by a member state of either (i) an adder on the imbalance price and balancing price, (ii) an adder on the imbalance price solely, or (iii) the introduction of a real-time price for the trading of real-time balancing capacity. Our analytical framework allows us to characterize the optimal bidding strategy of flexible assets under the different designs and to derive the resulting equilibria. Our analysis demonstrates that adders without the trading of balancing capacity create inefficiencies by distorting the merit order and tend to be detrimental to the member state that introduces it.**

***Index Terms*—Balancing market, cross-border balancing, frequency restoration reserve, real-time market for reserve.**

I. Introduction

A. The European Balancing Context

Following in reverse of the appropriate order [1], the next phase of European electricity market integration concerns balancing markets. These markets ensure the reliable operation of the grid by balancing, at all times, electricity generation with electricity consumption. European balancing markets coordinate interactions between three types of agents: (i) the *transmission system operator* or TSO, (ii) *balancing responsible parties* or BRPs, and (iii) *balancing service providers* or BSPs.

Balancing markets consist of energy auctions where TSOs restore system balance in response to real-time conditions. They cover the aggregated imbalance caused by BRPs' deviation from their forward position (*system imbalance*) by activating *balancing energy* from BSPs. BSPs are dispatched based on their *balancing energy bid*, a price-quantity pair that represents the limit price at which they are willing to be activated and the maximum quantity that they can deliver. BSPs' balancing energy is remunerated at the *balancing price*. Instead, BRPs' imbalance is charged an *imbalance price* which is based, among other factors, on the activation cost of reserve [2]. In order to ensure sufficient balancing energy bids in real time, TSOs contract *balancing capacity* in the day ahead. In this 2-step process, the procurement cost of reserve capacity is passed on to consumers through the grid tariff, and the activation cost for balancing energy is borne by the BRPs that cause the imbalance.

National balancing markets in Europe have been characterised by specific features in terms of (i) traded balancing products, (ii) balancing capacity procurement (market-based or mandatory contribution), settlement methods (regulated price, pay-as-bid, or pay-as-cleared) and cost recovery schemes (passed on to the customer through the grid tariff or covered by the BRPs causing the imbalance), (iii) balancing energy activation (pro-rata or based on a merit-order), settlement methods and cost recovery schemes, (iv) the *imbalance settlement period* duration (from 15 minutes to 1 h) and (v) imbalance price formation.[1] These rules reflect the approach of diverse TSOs for covering imbalances and are a legacy of a time with less coordination between European member states. There is, however, a push to harmonize balancing market operation in order to fully harness the benefit of the integration of the European electricity market. This push takes two forms: (i) the introduction of *balancing platforms* and their accompanying *pricing methodology* and (ii) the *imbalance settlement harmonization methodology* (ISHM) [3].

Balancing platforms are developed with the goal of centralizing balancing energy bids across Europe. Specifically, TSOs submit a demand for activation, and the platforms clear the offers that maximize social welfare. Rules for computing the platform prices are framed by the pricing methodology set by decision 2020/01 of the European agency of regulators (ACER). The connection to the platforms, coupled with the pricing methodology, create an impetus for standardizing the balancing products and for transitioning to "pay-as-cleared" settlement (also known as marginal pricing), for whichever Member States this is not the case yet, in accordance with Article 30.1.a of the Electricity Balancing Guideline (EBGL).

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[1] A complete comparison of the current European balancing market state of affairs can be found in the electronic supplement.

The ISHM provides a framework for the calculation of the imbalance price. It prompts TSOs to adopt an imbalance pricing scheme based on the balancing prices generated by the platforms. The ISHM also offers to the TSOs the possibility of unilaterally introducing a *scarcity component* or an *incentivizing component* in their imbalance price. Similarly, the possibility of introducing a scarcity component to the balancing price is foreseen by the pricing methodology. Such adders on the imbalance price are already present in Belgium and the Netherlands, with the stated goal of inducing BRPs to contribute towards balancing the system. They take the form of adders and aim at incentivizing BRPs to participate in the balancing process by penalizing (resp. rewarding) positions that hurt (resp. help) the system. Imbalances in the same (resp. opposite) direction as the SI are punished (resp. rewarded) with an increased imbalance price.

The intention of introducing adders to the imbalance price is to keep the system imbalance stable and to prevent long-lasting imbalances by BRPs [4]. This imbalance pricing scheme holds BRPs accountable for their consumption of balancing capacity and one possible interpretation is that this acknowledges the real-time value of balancing capacity. Unfortunately, this design choice fails to efficiently back-propagate the value of balancing capacity to the day-ahead market [5]. The most adequate measure for ensuring back-propagation would be to co-optimize balancing energy and balancing capacity in real time, as is already the case in various US markets, such as [6]. Otherwise, its second best approximation, the introduction of a real-time market for reserve and the use of an adder on the energy price, which has been implemented in ERCOT [7], would be more realistic to implement in the EU market in the immediate future.

The argument for an increased participation of flexible demand, which is a necessary component of electric systems dominated by renewable generation, can also be invoked for supporting the introduction of adders [8]. Most importantly, energy adders can assist in compensating for the inability of electricity markets to price and remunerate reliability. Completely offsetting this market failure will require customers to specify their preferred reliability. Until then, administrative measures, such as *shortage pricing functions* through adders on the energy price which can approximate the co-optimization of reserve and energy, can be helpful in ensuring an adequate level of reliability, since energy-only markets with dormant demand-side resources alone may face challenges in price discovery. This measure can improve the revenue profile of flexible assets and help overcome the reluctance of risk-averse investors to finance such assets due to their position at the end of the merit order, the high year-to-year volatility of energy prices, and the heavy reliance of these investments on infrequent high prices caused by shortages [9], [10].

Even though the volumes involved in balancing markets are much lower than the ones traded in day-ahead and intraday markets, the design that determines price formation in balancing and imbalance settlement should not be overlooked as the expectation of the real-time prices generated by the balancing markets drives the wholesale electricity price in the day-ahead and other forward markets [11]. As Hogan has stated repeatedly: "The last (price generated) should be (designed) first" [1]. Additionally, the potential synergies brought by the integration of balancing are estimated to be considerably more significant than the ones from coupling day-ahead and intraday markets [12]. The potential savings from *imbalance netting* and the use of cheaper balancing resources have been simulated to 400 million € for the Nordics alone [13].

B. Literature Review

This analysis is part of the balancing market design literature. Seminal work on coupled capacity and energy auctions includes [14], where the authors identify with an analytical model the optimal bidding strategy and the necessary conditions for an equilibrium in early bid scoring systems with discriminatory settlement rules. Chao and Wilson establish, by arguing through backward induction, that uniform pricing for both energy and reserve can create incentives for truthful bidding [15]. Similar analytical methods have been employed in order to investigate the interplay between the wholesale market and the balancing market [16] and the switch from "pay-as-bid" to "pay-as-cleared" auctions [17]. Agent-based [5], [18], [19], [20] and simulation-based methods [21] have also been used for analysing the impact of the imbalance pricing scheme on system cost [18], strategic bidding in joint or split balancing capacity and balancing energy markets [19], [20], the back-propagation of real-time prices to day-ahead prices [5], and the timing of the gate closure time [21].

There is also a broad literature on optimizing the strategy of different agents in balancing markets. Such literature includes analyses of optimal trading strategies [22], [23], the minimization of portfolio imbalance [24], the optimal activation of balancing energy by system operators [25], and the optimal scheduling of batteries that perform reactive balancing [26]. Nevertheless, this line of work is tangent to our analysis, which is focused on the *design* of balancing markets.

The introduction of a real-time market for reserve links this discussion to the *scarcity pricing* literature based on *Operating Reserve Demand Curves* (ORDCs). The concept of scarcity pricing with ORDCs was formalized by Hogan in [8] and has since gained traction in the US, with ERCOT, PJM, ISO-NE, MISO, SPP and CAISO having implemented it [27]. The authors in [28] and [29] investigate the adaptation of the mechanism to European balancing markets. These works advocate for the introduction of a real-time market for reserve as a non-disruptive no-regret measure.

A similar analysis on the effect of uncoordinated regulation in spatially integrated electricity markets has been conducted by Bushnell in the context of carbon reduction policies [30]. The authors conclude that regulatory interventions can lead to a distortion of the merit order, which may in turn lead to inefficiencies.

C. Objective and Contributions

This work uses an analytical model to investigate how adders foreseen by the ISHM and the pricing methodology can be applied in European balancing markets connected with cross-border balancing platforms. Three designs are examined: (i) The *adder on BRPs* design, which is currently used in Belgium by ELIA, the Belgian TSO, where the adder is applied on the imbalance price [31], (ii) the *adder on BRPs and BSPs* design, suggested by the Dutch TSO TenneT, where an adder is applied on both the balancing and imbalance price [32], and (iii) the *Real-Time (RT) market for reserve* suggested by [5], [28], [29]. This design proposes to remunerate available but non-activated balancing capacity in addition to balancing energy. The coupling of the balancing capacity and balancing energy market uplifts the balancing and imbalance price by an adder equal to the balancing capacity price. The value of balancing capacity, the *reserve price*, is based on an ORDC which represents the probability of losing load given the current state of the system [33].

The paper completes and extends the model proposed in [5] in order to assess the back-propagation induced by different imbalance and balancing pricing schemes. Our modelling and theoretical contributions can be stated as follows: (i) we characterize an equilibrium for the "adder on BRPs" design, which allows us to abandon the agent-based modeling method used in [5], (ii) we provide a novel analysis of a newly proposed pricing scheme, the "adder on BRPs and BSPs" design, and (iii) we apply our analysis to a cross-border setting. From a policy standpoint, our analysis shows the inability of both the "adder on BRPs" and the "adder on BRPs and BSPs" designs to support an optimal dispatch in a cross-border setting, in contrast to the "RT market for reserve" design which can achieve this objective.

The remainder of the paper is structured as follows. Section II goes on to describe the functioning of European balancing markets and balancing platforms, and introduces our notation and model. Sections III and IV describe the optimal strategies of agents under the different designs analyzed in this work, as well as the resulting market equilibrium. Section V illustrates and compares the market equilibria in a two-zone setting. Section VI concludes.

II. European Balancing Market

This section presents a single-zone balancing market and then introduces balancing platforms for representing cross-zonal integration in balancing operations.

A. Single-Zone Balancing Market

The functioning of European balancing markets is outlined in the *Electricity Balancing Guideline* (EBGL) and described in a stylised manner hereunder.

TSOs are responsible for the operational security of the grid. They hold reservation auctions for ensuring an adequate level of available reserve capacity of different types in real time. Balancing capacities can be differentiated according to their activation time. They include the following types of reserve. (i) *Frequency containment reserve* (FCR) is based on automatic control. (ii) *Automatic frequency restoration reserve* (aFRR) is also driven by automatic controllers, and strives to control the grid frequency. It has a full activation time of 5 to 7.5 minutes. (iii) *Manual frequency restoration reserve* (mFRR) is activated manually by the controller in order to relieve aFRR. It has a full activation time of 15 minutes. (iv) *Replacement reserve* (RR) has a full activation time that ranges between 15 minutes and hours. The discussion in the paper is targeted at manual frequency restoration reserve that is dispatched through balancing energy auctions. Replacement reserve and automatic frequency restoration reserve are also dispatched through balancing energy auctions, but they are ignored in order to highlight the effect of introducing adders on the interaction between two cash flows with comparable settlement timeframes: mFRR balancing energy and imbalance. Any subsequent reference to balancing capacity and energy will respectively refer to the mFRR capacity available for activation by the TSOs and to the mFRR capacity dispatched by the TSOs.

BRPs are owners of portfolios that consist of residential, commercial and industrial load, as well as generation assets. According to the EBGL, they shall strive to be balanced or to help the system be balanced (article 17.1 of EBGL) and they are financialy responsible for their imbalance (article 17.2 of EBGL). Their imbalance relative to their ex-ante position is charged at the *imbalance price*. For the sake of this analysis, BRPs can be considered as price-inelastic energy bids.

BSPs are flexibility providers that participate in the balancing energy auctions. They belong to a BRP portfolio, and they include a wide range of assets, such as classical thermal units (CCGT, OCGT,...), battery aggregations, and industrial and/or commercial demand response. BSPs can offer various reserves, depending on their characteristics and on the qualification criteria set by the TSO. BSPs can be considered as elastic suppliers of balancing energy in the context of our models.

The term "balancing the market" refers to the process whereby a TSO activates balancing energy from BSPs, in order to cover the aggregation of the BRPs' inelastic imbalance. We proceed now with a description of the balancing process. We have voluntarily left aside the reserve procurement auctions, as their representation is not required in order to highlight the issues that emerge from pricing asymmetries resulting from adders.

Firstly, BSPs submit their balancing energy bids to a balancing energy auction which is organised by the TSO. The balancing energy auction is assumed to clear at a uniform price, following the pricing scheme of the European balancing platforms [34].

Secondly, the aggregation of the BRPs' inelastic imbalance is revealed. The TSO clears the balancing energy auction in order to balance the market and a *platform price*, λ_P, is generated as the marginal cost of balancing energy activation. Given the available information, one can also compute a *scarcity component price*, λ_R. This represents the value of balancing capacity at the time of clearing and can be set by an ORDC. ORDCs are downward-sloping functions of the reserve in the system. A version based on the *loss of load probability* (LOLP) and the *value of lost load* (VOLL) has been introduced in the market clearing process of numerous ISOs in order to approximate the dispatch and expected payoff that would be generated by a stochastic *economic dispatch*. These ORDCs represent the marginal value of an additional MW of reserve in terms of preventing load curtailment. As indicated in (1), they are a

TABLE I
BALANCING AND IMBALANCE PRICES UNDER THE VARIOUS DESIGNS THAT ARE DEBATED IN EUROPEAN BALANCING MARKET DESIGN

	λ_{bal}	λ_{imb}	Res. price
No adder	λ_P	λ_P	0
Adder on BRPs	λ_P	$\lambda_P + \lambda_R$	0
Adder on BSPs and BRPs	$\lambda_P + \lambda_R$	$\lambda_P + \lambda_R$	0
RT Market for reserve	$\lambda_P + \lambda_R$	$\lambda_P + \lambda_R$	λ_R

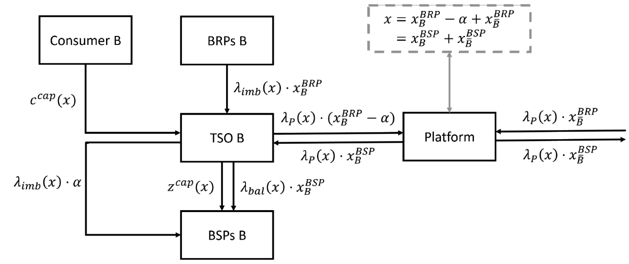

Fig. 1. Cash flow over multiple zones for a general European cross-border balancing market.

function of the level of reserve in the system (r), the value of lost load ($VOLL$), the loss of load probability ($LOLP(\cdot)$) and an approximation of the marginal cost of the marginal resource ($\widehat{MC}$).

$$ORDC(r) = (VOLL - \widehat{MC}) \cdot LOLP(r) \tag{1}$$

Other types of ORDCs include fixed reserve requirements and stepped ORDCs.

Between the first and second step, BSPs, as part of a BRP portfolio, can decide to perform reactive balancing and self-activate their assets. In this case, the activated energy is considered part of the BRP's imbalance and is charged at the imbalance price.

The ISHM and the pricing methodology leave the door open for the introduction of scarcity components in the balancing price, λ_{bal}, and the imbalance price, λ_{imb}. These prices are used respectively for remunerating BSPs' balancing energy and for settling BRPs' imbalance. The application of such adders can result in different designs, as shown in Table I. The default design is the "no adder" policy, where the balancing and imbalance price are equal to the platform price. The "adder on BRPs" and "adder on BRPs and BSPs" designs introduce an adder on the imbalance price and on the imbalance and balancing price, respectively. Finally, the "RT market for reserve" design has an adder on the imbalance and balancing price, and additionally trades balancing capacity in real time. In this last design, the balancing capacity that has not been activated is entitled to the real-time reserve price, which is equal to an adder computed from an ORDC. More specifically, the "RT market for reserve" design proposes to introduce a market for *balancing capacity imbalance* which is equivalent to a market for balancing capacity that is conducted in real time. Stylized models that represents balancing markets with a real-time market for reserve, as well as the existing EU balancing market, are presented in the electronic supplement.

B. Cross-Border Balancing Platforms

The transition from one zone to multiple zones requires cross-border coordination, which is the goal of the European balancing platforms. These platforms aim at coordinating the dispatch of balancing energy from different zones, and are called PICASSO (for aFRR) and MARI (for mFRR). Their objective is to cover the TSOs' demand, at least cost, by activating balancing energy from the BSPs of multiple zones. They have gone live in 2022 and are operating over Germany, Austria and the Czech Republic for MARI, with the addition of Italy for PICASSO. The other European TSOs are expected to join the platforms in 2023 or 2024. MARI clears every 15 minutes and PICASSO clears every 4 seconds.[2]

TSOs that are connected to the platforms first receive the balancing bids from the BSPs. They filter these bids in order to suppress the ones that could create congestion and transmit the others to the platform. Afterwards, they send their demand for the activation of balancing energy to the platform.[3] The platform clears a balancing energy auction and informs TSOs on which bids of their control area have been accepted. Finally, TSOs inform BSPs the bids of which are accepted. TSOs are also responsible for the settlement between the BRPs and BSPs and the platform. This balancing process, and the cash flow between the different agents, is represented graphically in Fig. 1 for the case of two zones: zone B and zone $\bar{B}$, where zone $\bar{B}$ corresponds to the rest of the system.

The demand for activation of balancing energy in zone i is denoted as x_i^{BRP} if there is no reactive balancing and as $x_i^{BRP} + \alpha$ in the case with reactive balancing. Note that the TSO cannot distinguish between the inelastic imbalance from BRPs and reactive balancing. The activated balancing energy in zone i is denoted as x_i^{BSP}. Settlements between the platform and the TSOs are charged at the platform price cleared by the balancing energy auction, but TSOs are free to unilaterally introduce adders, corresponding to the designs proposed in Table I, for the settlements with the BRPs and BSPs.

These potential pricing asymmetries between the platform price and the balancing and imbalance price might result in a capacity cost component c^{cap} borne by all the consumers in the zone introducing the adder, which is socialized through the grid tariff. Outside of balancing settlement, BSPs are also entitled to a capacity settlement z^{cap} for their unused balancing capacity if their zone operates a "RT market for reserve". Both of these components are described for the different designs in Table II.

III. OPTIMAL DECISIONS OF AGENTS

The objective of the analytical model is to identify the optimal strategy of risk-neutral BSPs[4] that maximize profit. BSPs are assumed to participate in a single-product balancing market with

[2]MARI can clear more than one time per 15 minutes at the TSOs' request.
[3]In practice, MARI accepts elastic demand bids from TSOs, but we restrict our investigation to inelastic demand.
[4]Market agents may exhibit risk-averse behaviour in practice.

TABLE II
COST OF CAPACITY AND CAPACITY SETTLEMENT

	$c^{cap}(x)$	$z^{cap}(x)$
No adder	0	0
Adder on BRPs	$-\lambda_R(x) \cdot (x_B^{BRP} - \alpha)$	0
Adder on BSPs and BRPs	$\lambda_R(x) \cdot (x_B^{BSP} - x_B^{BRP})$	0
RT Market for reserve	$\lambda_R(x) \cdot (P^{max} - x_B^{BRP})$	$\lambda_R(x) \cdot (P^{max} - x_B^{BSP})$

a uniform balancing price and an imbalance price which is based on the balancing price. The agents can either participate in the energy balancing auction or perform reactive balancing. BRPs are assumed to be inelastic. We will first describe the profit function of a fringe BSP with P^+ MW of upward balancing capacity and a marginal cost of C € /MWh. We will then derive its optimal strategy and discuss the impact of BSPs' level of information on the demand for balancing energy on their bidding strategy. The notation and analysis presented here are an extension of the model presented in [5].

The model has been kept simple on purpose to highlight the effects brought forth by the introduction of adders. Inefficiencies encountered in a simple stylized model are symptoms of underlying design flaws. In this spirit, well-documented sources of distortion or inefficiencies in electricity markets such as market power [35], [36], collusive bidding by suppliers [17], arbitrage opportunities between different products [37], gaming opportunities created by congestion management actions [38], and frictions resulting from ramping or commitment constraints [21] are ignored. Nevertheless, the modelling framework is quite flexible, and additional components can be introduced in the analysis. For example, the introduction of reservation cost and a day-ahead balancing capacity market is discussed in the electronic supplement.

The focus on upward balancing capacity is justified by the fact that scarcity adders are expected to be activated when the system is under stress due to insufficient generation, i.e. when it is short. BSPs with downward balancing capacity will not be affected by the introduction of scarcity adders as such adders will be zero when these BSPs are dispatched.[5]

A. Balancing and Imbalance Payoffs of Agents

The decision process under the "no adder", "adder on BRPs" and "adder on BRPs and BSPs" designs is represented as a two-stage process. In the first stage, the agent submits a price-quantity bid (p, q) to the energy balancing auction and decides on its level of reactive balancing ai. Note that the sum of the level of reactive balancing and of the bid quantity must be lower than the total capacity of the agent. In the second stage, the TSO dispatches balancing energy through the balancing energy auction in order to cover the system imbalance x, which is the demand for balancing energy. This demand is assumed to be drawn from a known distribution with probability measure $\mu(x)$. The resulting price from the balancing energy auction, $\lambda_P(x)$, is a function of the random demand for balancing energy. It can be defined as the price offer of the most expensive accepted energy bid. Alternatively, the platform price is equal to the dual variable of the market clearing constraint of the economic dispatch problem that is solved by the system operator in order to balance the system.

The scarcity component, $\lambda_R(x)$, is obtained through an operating reserve demand curve. Scarcity pricing based on ORDC adders takes the leftover capacity in the system as an input, but leftover capacity can be equivalently expressed as a function of the demand for balancing energy by assuming that the leftover capacity in the system is the total capacity in the system minus the inelastic energy demand. The scarcity component is a non-decreasing function of demand. The balancing payoff for a BSP, given a random demand for balancing energy, is the uniform balancing price multiplied by the quantity bid if the price bid is lower than the platform price, or 0 if the bid is not accepted.

$$z_B(p, q, x) = \begin{cases} \lambda_{bal}(x) \cdot q & \text{if } p \leq \lambda_P(x), \\ 0 & \text{else} \end{cases} \tag{2}$$

Note that bids are assumed to be either fully selected, if they are at-the-money or in-the-money, or not selected at all. The expectation of this payoff can be reformulated as follows, with the operator $\mathbb{E}_\mu$ being the expectation over the probability measure μ:

$$z_B(p, q) = \mathbb{E}_\mu[z_B(p, q, \cdot)] = \int_{\lambda_P(x) \geq p} (\lambda_{bal}(x) - C) d\mu(x) \cdot q \tag{3}$$

The payoff of reactive balancing is found by first deriving the level of reactive balancing performed by an agent. This is found by solving the following optimization problem:

$$\max_{ai} \quad (\mathbb{E}_\mu[\lambda_{imb}] - C) \cdot ai \tag{4}$$

$$s.t. \quad ai + q \leq P^+ \tag{5}$$

$$ai \geq 0 \tag{6}$$

If $C \geq \mathbb{E}_\mu[\lambda_{imb}]$, the optimal level of reactive balancing $ai^\star$ is 0, else $ai^\star$ is equal to the leftover capacity from the balancing energy auction. The reactive balancing payoff is then described as follows:

$$z_I(q) = \begin{cases} (\mathbb{E}_\mu[\lambda_{imb}] - C) \cdot (P^+ - q) & \text{if } C \leq \mathbb{E}_\mu[\lambda_{imb}], \\ 0 & \text{else.} \end{cases} \tag{7}$$

The optimal strategies for the "no adder", "adder on BRPs" and "adder on BRPs and BSPs" designs are then found by maximizing the sum of the balancing and of the reactive balancing payoff.

A real-time market for balancing capacity requires the introduction of a new component in the payoffs. The unused balancing capacity of BSPs is now remunerated by the reserve price. As the balancing and imbalance prices are equal to the platform price plus the reserve component in the "RT market for reserve" design (see Table I), the real-time payoff can be

[5]The paper focuses on upward reserve but the rise in solar generation and the resulting steep increasing ramp in the morning, the so-called "duck curve", has increased the need for downward reserve.

reformulated as follows for a random demand x, with $z_I(ai, x)$ being the reactive balancing payoff for self-activating ai MWh:

$$z_B(p,q,x) + z_I(ai,x)$$
$$= \begin{cases} (\lambda_P(x) + \lambda_R(x) - C) \cdot (q + ai) + \lambda_R(x) \\ \cdot (P^+ - q - ai) \quad \text{if } \lambda_P(x) \leq p, \\ (\lambda_P(x) + \lambda_R(x) - C) \cdot ai + \lambda_R(x) \cdot (P^+ - ai) \\ \quad \text{else,} \end{cases} \tag{8}$$

$$= \begin{cases} (\lambda_P(x) - C) \cdot (q + ai) + \lambda_R(x) \cdot P^+ & \text{if } \lambda_P(x) \leq p, \\ (\lambda_P(x) - C) \cdot ai + \lambda_R(x) \cdot P^+ & \text{else.} \end{cases} \tag{9}$$

This allows us to rewrite both the balancing and imbalance payoff as a function of the platform price:

$$z_I(q) = \begin{cases} (\mathbb{E}_\mu[\lambda_P] - C) \cdot (P^+ - q) & \text{if } C \leq \mathbb{E}_\mu[\lambda_P] \\ 0 & \text{else,} \end{cases} \tag{10}$$

$$z_B(p,q) = \int_{\lambda_P(x) \geq p} ((\lambda_P(x) - C) \cdot q)\, d\mu(x) + \mathbb{E}_\mu[\lambda_R] \cdot P^+. \tag{11}$$

The objective of this reformulation is to isolate the scarcity component remuneration $\mathbb{E}_\mu[\lambda_R] \cdot P^+$ from the standard imbalance and balancing payoff, in order to highlight the correspondence between the payoffs of the "RT market for reserve" and "no adder" designs up to a constant.

B. Optimal Balancing Market Bid

The strategy for characterizing the optimal behaviour under the different designs is an extension of [5] where we additionally analyze the "adder on BRPs and BSPs design". Only the statement and a brief intuition of the results are provided here. The complete proofs are available in the electronic supplement. These proofs consider fringe agents reacting to an exogenous platform price. The platform price is also assumed to be strictly monotonic increasing. The inverse of the platform price, i.e. the supply function, is assumed to be differentiable almost everywhere.[6] Section IV shows that these assumptions hold at equilibrium.

Proposition 1 (Bidding Strategy – No Adder): The optimal strategy for a fringe agent under a "no adder" design is to bid truthfully in the balancing auction.

Bidding more or less than the marginal cost in the balancing energy auction will result in a lower balancing payoff for an agent, as the agent can respectively lose some potential payoff (in the case of overbidding) or be unprofitable (in the case of underbidding). One can then conclude that it is optimal for every agent to bid its full capacity in the balancing auction, since the payoff of the balancing auction will be (i) equal to the payoff from self-activation whenever the agent is in the money in the balancing auction, and (ii) is higher than the payoff of self-activating whenever the agent is out of the money.

Proposition 2 (Bidding Strategy – Adder on BRPs): The optimal strategy for a fringe agent under an "adder on BRPs" design is to bid truthfully in the balancing market if

$$C \geq \mathbb{E}_\mu[\lambda_P + \lambda_R] - \int_{\lambda_P(x) \geq C} (\lambda_P(x) - C) d\mu(x),$$

else to perform reactive balancing with its full capacity.

The pricing asymmetry of this design can incentivize BSPs to self-activate when they have a low marginal cost. If a BSP is very likely to be activated, the little it would lose when the imbalance price is not sufficient to cover its cost can be compensated by the additional payoff of the imbalance settlement, compared to the balancing energy auction, due to the scarcity component in the former.

In order to analyze the next design, we define $(\lambda_P + \lambda_R)^{-1}$ as the inverse function of the sum of the platform price and the scarcity component under the adder on BRPs and BSPs design. This sum is equal to both the balancing and imbalance price (see Table I). The expression $(\lambda_P + \lambda_R)^{-1}(p)$ is then the level of demand for balancing energy such that the balancing price p is attained. Note that $\lambda_P + \lambda_R$ has a well-defined inverse everywhere because the platform price is strictly monotonic increasing and the reserve component is non-decreasing.

Proposition 3 (Bidding Strategy – Adder on BRPs and BSPs): The optimal strategy for a fringe agent under an "adder on BRPs and BSPs" design is to bid its full capacity in the balancing energy market at price

$$\lambda_P((\lambda_P + \lambda_R)^{-1}(C)).$$

BSPs should internalize the value of the adder in their balancing energy bid so as to ensure that they are always activated when the balancing price is higher than or equal to their marginal cost. This corresponds to bidding the platform price $\lambda_P(x')$ for x' such that

$$(\lambda_P + \lambda_R)(x') = C.$$

Proposition 4 (Bidding Strategy – RT Market for Reserve): The optimal strategy for a fringe agent under a RT market for reserve design is to bid truthfully in the balancing energy market.

The profit function of the "RT market for reserve" design is equal to the one of the "no adder" design up to a constant $\mathbb{E}_\mu[\lambda_P] \cdot P^+$, independent of the agent's strategy.

The optimal bidding strategies for a BSP under the different designs are not modified by the transition to multiple zones. As long as (i) the platform price and the reserve component price as a function of the zonal demands for balancing energy and (ii) the probability measures of these demands are known, the optimal strategies described above remain valid.

C. Level of Information Regarding the Demand for Balancing Energy

In practice, one could argue that BSPs have more freedom concerning the self-activation of their plant than what has been described earlier. Current Belgian bidding rules require BSPs to

[6] At equilibrium, the supply function may exhibit breaking points due to BSPs resorting to reactive balancing. This can cause the supply function to not be differentiable everywhere.

bid in the balancing energy auction and allow them to retract their bid up to 25 minutes before the clearing of the balancing auction [39]. Under this regulation, BSPs can decide to self-activate their plant only when the state of the system at time t minus 25 guarantees them a favorable outcome by self-activating. If the state of the system is unfavorable, they can opt to bid in the balancing market instead, and thus avoid being exposed to any risk.

From a modelling standpoint, we can introduce information on the demand for balancing energy by using a random variable y, with probability measure ν and support $\mathcal{Y}$. The random variable y is assumed to be observable at the moment when the BSP is called to reach a decision about reactive balancing, e.g. it can include the realized imbalance of the previous interval, wind forecasts, load forecasts, etc. This random variable thus provides information that is revealed to the BSPs when submitting their energy bid. BSPs can then consider whether to self-activate their assets or participate in the balancing auction, depending on the distribution of the demand given y, $X|y$, its probability measure $\mu(x|y)$, and the platform price, balancing price, and imbalance price for a given demand given y, $\lambda_P(x|y)$, $\lambda_{bal}(x|y)$ and $\lambda_{imb}(x|y)$.

The balancing payoff of an agent can then be expressed as follows:

$$z_B(p,q,y) = \int_{\lambda_{bal}(x|y)\geq p} (\lambda_{bal}(x|y)\cdot q)d\mu(x|y) \tag{12}$$

The payoff from reactive balancing as a function of the information is found by solving the following optimization problem:

$$\max_{ai(y)} \quad \int_{\mathcal{Y}}\int_{\mathcal{X}|y} (\lambda_{imb}(x|y) - C)\cdot ai(y))d\mu(x|y)d\nu(y) \tag{13}$$

$$s.t. \quad ai(y) + q(y) \leq P^+ \tag{14}$$

$$ai \geq 0 \tag{15}$$

Notice that both the balancing and imbalance profit functions are separable for y, meaning that they are parametrized by y but do not couple different values of y with each other. This allows us to model the level of information by combining the optimal strategy of agents for different distributions of demand for balancing energy without information and to fall back to the basic setting.

IV. Market Equilibrium

This section commences by characterizing the Nash equilibria resulting from the optimal strategy outlined earlier in a single zone. These equilibria correspond to the ones that would emerge in a balancing market that is not connected to a cross-border platform. We then extend the analysis to multiple zones and we discuss the ensuing inefficiencies.

The result presented here assumes a truthful merit order curve $MC(x)$, which is strictly monotonic increasing and differentiable, as well as a system capacity $P^{\max}$ which is greater than the upper bound of the support of the distribution of the random demand for balancing energy.

A. Single-Zone

Proposition 5 (Equilibrium – "No Adder" and "RT market for reserve"): The Nash equilibrium generated by fringe agents under the "no adder" and "RT market for reserve" designs is characterized by all agents participating truthfully in the balancing energy auction and the following platform price:

$$\lambda_P(x) = MC(x).$$

Proof: The agents' optimal strategies consisting of bidding truthfully are independent from the other agents' bidding behavior. This behavior, coupled with the balancing energy auction selecting bids in increasing price order, results in the platform price following the merit order, and being strictly monotone increasing and differentiable. This confirms the validity of propositions 1 and 4. ∎

We now define $\lambda_P(x,\alpha)$ as the platform price for energy demand x, a total of α MWh of reactive balancing from the cheapest BSPs with upward balancing capacity, and we consider what happens when other BSPs bidding truthfully. The platform price in this situation is as follows:

$$\lambda_P(x,\alpha) = \begin{cases} MC(x-\alpha) \text{ if } x<\alpha, \\ MC(x) \text{ if } x>\alpha, \\ \text{price indeterminacy between } MC(0) \text{ and } MC(\alpha) \text{ else.} \end{cases} \tag{16}$$

If $x<\alpha$, $x-\alpha$ MWh of downward balancing capacity has to be activated in order to balance the excessive self-activation by the agents, resulting in $\lambda_P(x,\alpha) = MC(x-\alpha)$. If $x>\alpha$, there is no price distortion and $\lambda_P(x,\alpha) = MC(x)$. If $x=\alpha$, no balancing energy is activated through the balancing energy auction and there is a price indeterminacy.

In a single-zone setting, the reserve component is not impacted by the level of reactive balancing, due to the presence of downward balancing capacity. To see this, note that if the level of reactive balancing is greater than the demand for balancing energy, then the potential curtailment of downward balancing capacity that was dispatched to cover the excessive reactive balancing can be assimilated as upward balancing energy. In case of an increased demand for balancing energy, reducing the level of the dispatched downward balancing capacity will contribute towards reducing the imbalance and will therefore not impact the level of upward balancing capacity.

The opportunity cost of participating in the balancing auction given a level α of self-balancing from the cheapest agents with upward balancing capacity for an agent with marginal cost C is:

$$z(\alpha,C) = (\mathbb{E}_\mu[\lambda_P(\cdot,\alpha)+\lambda_R(\cdot)] - C) - \int_{\lambda_P(x,\alpha)\geq C} (\lambda_P(x,\alpha)-C)d\mu(x). \tag{17}$$

If $z(\alpha,C)<0$, an agent with marginal cost C should bid truthfully in the balancing auction for a level α of reactive balancing. If $z(\alpha,C)>0$, the agent should self-activate its capacity.

Proposition 6 (Equilibrium – Adder on BRPs): If $z(\alpha, MC(\alpha))$ is continuous, there exists a unique Nash equilibrium generated by fringe agents under the "adder on BRPs" design characterized by an equilibrium level of reactive balancing, α^*, such that $0 \leq \alpha^* \leq P^{\max}$, and with other BSPs bidding truthfully. This optimal level of reactive balancing is equal to (i) 0 if $z(0, MC(0)) < 0$, (ii) $P^{\max}$ if $z(P^{\max}, MC(P^{\max})) > 0$ or (iii) α^* characterized by the identity

$$z(\alpha^*, MC(\alpha^*)) = 0. \tag{18}$$

This equilibrium level of reactive balancing generates platform prices equal to $\lambda_P(x, \alpha^*)$.

The existence of an equilibrium relies on the continuity of z. The stability of α^* is derived analytically. Stability in this context refers to a level of reactive balancing for which no agent has an incentive to deviate from its decision. BSPs after α^* on the merit order prefer to participate in the balancing auction and agents before α^* prefer to resort to reactive balancing. The uniqueness of the equilibrium results from the monotonicity of z. The complete proof can be found in the electronic supplement.

Note that assuming a positive distribution for the demand for balancing energy results in z being continuous. Under this assumption, the probability of a particular demand occurring is infinitesimal. Discrete random demand can generate a price indeterminacy if the level of reactive balancing is equal to the imbalance. This breaks the continuity of z and an example of a system without a pure-strategy equilibrium can be found in the appendix.

Proposition 7 (Equilibrium – Adder on BRPs and BSPs): If $MC(x) - \lambda_R(x)$ is strictly monotonic increasing, there exists a Nash equilibrium generated by fringe agents under an "adder on BRPs and BSPs" design. It is characterized by all agents participating in the balancing energy auction and internalizing the value of the adder in their balancing energy bid. The produced platform price is described as follows:

$$\lambda_P(x) = MC(x) - \lambda_R(x).$$

Proof: The agents' optimal strategy is to bid at their marginal cost minus the scarcity component. This bidding behavior, combined with the balancing energy auction selecting bids in increasing price order, results in the platform price following the merit order minus the scarcity component and being strictly monotonic increasing and differentiable. This confirms the validity of proposition 3. ■

If $MC(x) - \lambda_R(x)$ is not strictly monotonic increasing, the optimal strategy derived in proposition 3 could modify the order of activation.

In terms of efficiency, the "no adder", "adder on BRPs and BSPs" and "RT market for reserve" designs support the optimal dispatch for a single zone, as they do not modify the order of activation specified by the truthful merit order. The "adder on BRPs" design increases the cost by inducing the dispatch of assets out of the merit order.

B. Multiple Zones

The characterization of an equilibrium in a setting with multiple zones requires introducing an aggregation operator $\cup$ for the aggregation of offer curves from different zones. Given $B_i(q)$, the offer curve in zone i, the aggregated offer curve, $B(q)$, can be obtained through the aggregation operator, as follows:

$$B(q) = \cup_i B_i(q) = \{\pi : B_i(q_i) = \pi \text{ for all } i \text{ and } \sum_i q_i = q\}. \tag{19}$$

TABLE III
OFFER CURVES UNDER DIFFERENT DESIGNS

Design in zone i	$B_i(x)$
No adder	$MC_i(x)$
Adder on BRPs and BSPs	$MC_i(x) - \lambda_{R,i}(x)$
Adder on BRPs	$\begin{cases} MC_i(x-\alpha_i) & \text{if } x \leq \alpha_i \\ MC_i(x) & \text{else} \end{cases}$
RT market for reserve	$MC_i(x)$

The optimal strategies derived in Section III remain valid in a multi-zone setting, and are used in order to derive offer curves under different designs, as shown in Table III. $\lambda_{R,i}$ is the reserve demand curve in zone i and α_i is the optimal level of self-activation in zone i.

For the "adder on BRPs" design, the opportunity cost function has to be modified in order to account for multiple zones. The assumption regarding the scarcity component not being impacted by the level of self-activation needs to be revisited. Excessive self-activation in a multi-zone setting is covered by activating downward balancing capacity from all zones. This reduces the total level of available upward balancing capacity in the zone with self-dispatched assets. This means that we need to define the scarcity component as a function of both the level of aggregated demand for balancing energy over all zones, as well as the level of self-activation in the zone with the "adder on BRPs", $\lambda_R(x, \alpha)$, and to update the opportunity cost of self-activation, as follows:

$$z(\alpha, C) = (\mathbb{E}_\mu[\lambda_P(\cdot, \alpha) + \lambda_R(\cdot, \alpha)] - C) - \int_{\lambda_P(x,\alpha) \geq C} (\lambda_P(x, \alpha) - C) d\mu(x), \tag{20}$$

This modifies the condition for an equilibrium level of self-activation and might lead to multiple equilibria if $z(\alpha, MC(\alpha))$ is not strictly monotonic decreasing in α.

Two conclusions can be drawn from the aggregation of the offer curves presented in Table III. First, only the introduction of a "RT market for reserve" does not affect the optimal dispatch. Both the "adder on RBPs and BSPs" and the "adder on BSPs" modify the bidding incentives in the zone implementing an adder, and result in a suboptimal aggregated offer curve. Second, the suboptimal aggregated offer curves generate lower platform prices than the one generated by the aggregation of the truthful merit order curves.

V. Illustration on a Stylized Example

The examples presented in this section assume a maximum level of upward balancing capacity $P^{\max}$, and a BSP merit order curve $MC(x)$ which is a function of the level of demand for balancing energy, x. The demand is drawn from a known distribution with probability measure μ. The scarcity component

TABLE IV
EXAMPLE 1 – EXPECTED PLATFORM AND SCARCITY PRICES, LEVEL OF REACTIVE BALANCING AND ACTIVATION COST FOR THE DIFFERENT DESIGNS

	No adder	Adder on BRPs and BSPs	Adder on BRPs	RT market for reserve
$\mathbb{E}_\mu[\lambda_P]$ (€/MWh)	60.00	55.83	60.00	60.00
$\mathbb{E}_\mu[\lambda_R]$ (€/MWh)	0.00	4.17	4.17	4.17
α (MWh)	0.00	0.00	0.00	0.00
Activation cost (€)	833.38	833.38	833.38	833.38

λ_R is obtained from an operating reserve demand curve defined as a function of the level of demand for balancing energy in the system.

This section presents four examples: (i) a single-zone example without information on the level of demand for balancing energy in the system, (ii) a single-zone example with information on the level of demand for balancing energy, (iii) a two-zone example without cross-border congestion and with information on the level of demand for balancing energy, and (iv) a two-zone example with cross-border congestion and with information on the level of demand for balancing energy.

A. Example 1: Single Zone Without Information on the Demand for Balancing Energy

In this example, we assume that the demand is uniformly distributed between -100 MWh and 100 MWh and that the merit order curve is described as follows:

$$MC(x) = x/2 + 60 \ \text{€}/MWh. \tag{21}$$

The scarcity price component is defined as

$$\lambda_R(x) = \begin{cases} 0 \ \text{€}/MWh & \text{if } x \le 0, \\ x/6 \ \text{€}/MWh & \text{else,} \end{cases} \tag{22}$$

which can be equivalently formulated as a function of the leftover capacity in the system, $\lambda_R^r(r)$, assuming a maximum level of balancing capacity in the system $P^{\max}$, equal to 200 MW in our case.

$$\lambda_R^r(r) = \lambda_R(P^{\max} - r) = \begin{cases} (P^{\max} - r)/6 & \text{if } r \le P^{\max}, \\ 0 & \text{else.} \end{cases} \tag{23}$$

All BSPs (i) bid truthfully under the no-adder and RT market for reserve design, thus $\lambda_P(x) = MC(x)$ (see proposition 5); (ii) bid at their marginal cost minus the level of the adder at their position on the merit order under the adder on BRPs and BSPs design, thus $\lambda_P(x) = MC(x) - \lambda_R(x)$ (Proposition 7); (iii) bid in the balancing energy auction at their marginal cost under the adder on BRPs design, thus $\lambda_P(x) = MC(x)$ (Proposition 6). No BSP does reactive balancing, as the opportunity cost of the cheapest generator when no asset is self-activating is negative. If participating in the balancing auction is more profitable for the cheapest generator, then this is also the case for every generator.

Table IV presents the expected platform price, the expected scarcity component, the level of self-activation, and the cost of reserve activation under the four designs. The four designs result in the same activation cost as the merit order is not distorted.

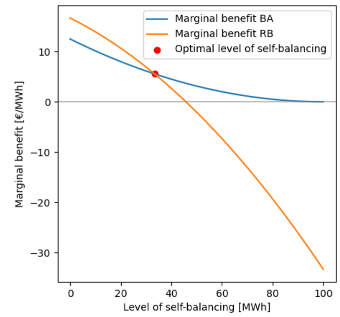

Fig. 2. Comparison of marginal benefit of reactive balancing and balancing auction for the frontier agent in the case of example 2.

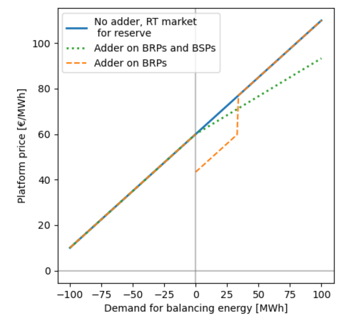

Fig. 3. Offer curve in example 2.

B. Example 2: Single Zone With Information on the Demand for Balancing Energy

In this example, BSPs have some information on the level of demand that the system will be exposed to, and are specifically aware of the sign of the required balancing activation. Specifically, we assume a draw with a probability 0.5 of having a negative demand that is distributed uniformly between -100 MWh and 0 MWh and a probability 0.5 of having a positive demand that is distributed uniformly between 0 MWh and 100 MWh.

All the parameters are identical to the previous example. The BSPs' strategy under the "no-adder," "adders on BRPs and BSPs" and "RT market for reserve" designs are not modified by the introduction of information on the imbalance, but there is an impact on the "adder on BRPs" design. The optimal level of reactive balancing in the "adder on BRPs" design is (i) 0 MWh if the system imbalance is in the interval $[-100, 0]$ MWh, and (ii) 33.33 MWh if the system imbalance is in the interval [0,100] MWh. The optimal level of reactive balancing is found by resolving the identity of (18) for α, the level of self-activation. In the first interval, no generator self-balances, as $z(\alpha, MC(\alpha)) \le 0$ for all α. For the second interval, $\alpha = 33.33$ MWh does satisfy the identity. This process is illustrated graphically in Fig. 2 by splitting the opportunity cost between the balancing auction and reactive balancing component. This figure represents the difference in profits for the frontier agent (i.e. the last agent to self-activate) for different levels of reactive balancing.

The platform prices for the four designs are presented in Fig. 3 as a function of the level of demand for balancing energy. Agents bid truthfully under the "no adder" and "RT market for

TABLE V
EXAMPLE 2 – IMPACT OF ADDITIONAL INFORMATION ON THE DEMAND FOR BALANCING ENERGY IN THE MARKET EQUILIBRIUM

Add. information	With			Without
Imbalance interval	[-100; 0]	[0; 100]	[-100; 100]	[-100; 100]
$\mathbb{E}_\mu[\lambda_P]$ (€/MWh)	35.00	79.42	57.21	60.00
$\mathbb{E}_\mu[\lambda_R]$ (€/MWh)	0.00	8.33	4.17	4.17
α (MWh)	0.00	33.33	16.67	0.00
Activation cost (€)	-2166.63	3925.99	879.68	833.38

reserve" designs, and they internalize the reserve adder under the "adder on BRPs and BSPs" design. Some of the BSPs decide to resort to reactive balancing if they know that the demand will be between 0 MWh and 100 MWh under the "adder on BRPs". This self-activation results in a translation of the merit order curve for negative balancing activation up to the level of reactive balancing.

The metrics concerning both intervals are presented in the first two columns of Table V. Columns 3 and 4 compare the result for the "adder on BRPs" design with and without additional information on the demand for balancing energy. The reactive balancing results in an inefficient dispatch that increases the total activation cost of the system. It also decreases the platform price, which is beneficial to the BRPs.

C. Example 3 – Two Zones Without Cross-Border Congestion and With Information on the Demand for Balancing Energy

We now refer to the system mentioned in examples 1 and 2 as zone B, and connect it to a new zone with an unlimited interconnector capacity between the two zones. The system in this new zone, called zone D, is four times larger than the one in zone B, resulting in a less steep merit order curve (see (24)).

$$MC_D(x) = x/8 + 60 \;\; €/MWh \tag{24}$$

The example is intended to mimic, in a highly stylized setting, the interaction between Belgium and Germany, hence the initials of the zones. We limit the exposition to the 2-zone case with zone D implementing a "no adder" policy in order to analyze the effect of the pricing asymmetry between the zones, since this has also dominated the policy discussion thus far [40].

The demand in zone D is distributed as in example 2, except for the distributions being uniform between 0 MWh and 400 MWh, and -400 MWh and 0 MWh. The combination of the probability distributions in zone B and zone D results in an equiprobable four-branch probability tree with distributions $\mathcal{U}[0,100]+\mathcal{U}[0,400]$ MWh, $\mathcal{U}[0,100]+\mathcal{U}[-400,0]$ MWh, $\mathcal{U}[-100,0]+\mathcal{U}[0,400]$ MWh, and $\mathcal{U}[-100,0]+\mathcal{U}[-400,0]$ MWh.

The equilibrium prices are presented in Table VI and Fig. 4 compares the surplus distribution with respect to the "no adder" benchmark. The results are based on the aggregated offer curves that are generated from the optimal BSP bids in zone B and D. The aggregated curve is constructed with (19) and is described in Section IV of the electronic supplement. Consumer surplus refers to the cost of serving the inelastic BRP imbalance plus the capacity cost borne by all the consumers of zone B or D.

TABLE VI
EXAMPLE 3 – EXPECTED PRICES (€ /MWH)

	No adder	Adder on BRPs and BSPs	Adder on BRPs	RT market for reserve
Platform price	60.00	59.07	59.74	60.00
Scarcity component	0.00	4.64	3.71	3.40
Balancing price (zone B)	60.00	63.71	63.45	63.40
Imbalance price (zone B)	60.00	63.71	67.08	63.40

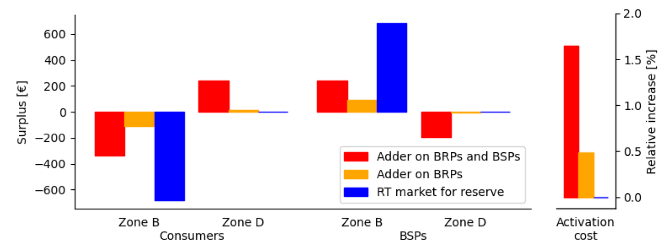

Fig. 4. Differences in surplus and activation cost relative increase compared to the "no-adder" benchmark in example 3.

The platform prices from the "adder on BRPs and BSPs" and the "adder on BRPs" design are lower than the ones in the designs that induce truthful bidding, due to the lower offer curve in zone B. These altered offer curves result in an over-dispatch of the assets in zone B, and in an increased level of adders compared to the "RT market for reserve" design. The self-activation of assets for the "adder on BRPs" design generates particularly high imbalance prices in zone B.

Three adverse effects resulting from the "adder on BRPs and BSPs" and the "adder on BRPs" designs can be observed. First, the induced out-of-merit activations lead to an increased activation cost and an inefficient dispatch. The compliance with the objective outlined in article 3(m) of the Clean Energy Package [41] might be questioned.

Second, these designs give rise to cross-zonal distributive effects between consumers. The cost of decreasing the platform price is borne by the consumers in zone B, either through an increased imbalance price or through the capacity cost. This suggests that the consumers in the zone with the adder subsidize the consumption of the consumers in the zone without the adder.

Third, these designs result in discrimination between BSPs from different zones. At similar marginal costs, BSPs in zone B are more likely to be activated than BSPs in zone D due to the increased balancing price or the possibility of resorting to reactive balancing. This leads to an increased surplus for BSPs in zone B compared to the "no-adder" benchmark and an opposite effect for zone D.

Only the "RT market for reserve" manages to introduce adders without inducing inefficiencies. In addition, this design only influences the surplus distribution between BRPs and BSPs in zone B and does not generate cross-zonal distributional effects.

TABLE VII
EXAMPLE 4 – EXPECTED PRICES (€ /MWH) WITH AN INTERCONNECTOR CAPACITY OF 50 MW IN EXAMPLE 4

	No adder	Adder on BRPs and BSPs	Adder on BRPs	RT market for reserve
Platform price (Zone B)	60.00	58.24	59.8	60.00
Platform price (Zone D)	60.00	59.54	59.9	60.00
Scarcity component	0.00	3.62	3.18	3.00

TABLE VIII
EXAMPLE 4 – RELATIVE INCREASE IN ACTIVATION COST (%) RELATIVE TO LEAST-COST ACTIVATION

	Relative cost activation increase		Reference cost
Interconnector capacity	Adder on BRPs and BSPs	Adder on BRPs	No adder
$F = \infty$ (MW)	1.65	0.48	100.0
$F = 50$ (MW)	0.73	0.23	110.5
$F = 0$ (MW)	0.00	1.11 (5.56 for zone B)	147.1

The complete characterization of the equilibrium prices and surplus for each branch of the probability tree and for both zones can be found in the electronic supplement.

D. Example 4 – Two Zones With Cross-Border Congestion and With Information on the Demand for Balancing Energy

We revisit example 3 by limiting the interconnector capacity between zone B and D at F MW. The analytical model with congestion requires the disaggregation of the demand for balancing energy between zone B and D. In a two-zone setting, congestion can be foreseen by BSPs in zone B when the difference between the demand for balancing energy in their zone and the activated balancing energy in their zone if there wese no congestion, $x_B^{BSP,uncon}(x_B^{BRP}, x_D^{BRP})$, exceeds the interconnector capacity in either direction:

$$x_B^{BRP} - x_B^{BSP,uncon}(x_B^{BRP}, x_D^{BRP}) \geq F, \quad (25)$$

or

$$x_B^{BRP} - x_B^{BSP,uncon}(x_B^{BRP}, x_D^{BRP}) \leq -F. \quad (26)$$

The activated balancing energy in zone B as a function of the demand for balancing energy in zones B and D follows from these bounds and is presented in (27):

$$x_B^{BSP}(x_B^{BRP}, x_D^{BRP}) = \begin{cases} x_B^{BRP} + F \text{ if } x_B^{BRP} - x_B^{BSP,uncon}(x_B^{BRP}, x_D^{BRP}) \geq F, \\ x_B^{BRP} - F \text{ if } x_B^{BRP} - x_B^{BSP,uncon}(x_B^{BRP}, x_D^{BRP}) \leq -F, \\ x_B^{BSP,uncon}(x_B^{BRP}, x_D^{BRP}) \text{ else.} \end{cases} \quad (27)$$

The platform price in zone B, $\lambda_{P,B}$, can be derived by evaluating the offer curve in zone B at the activated balancing energy,

$$\lambda_{P,B}(x_B^{BRP}, x_D^{BRP}) = B_B(x_B^{BSP}(x_B^{BRP}, x_D^{BRP})). \quad (28)$$

The same process can be reproduced to obtain the platform price and activated balancing energy in zone D.

Table VII displays the expected zonal platform prices for an interconnector with a capacity of 50 MW. The expected prices are equal in both zones for the "no adder" and the "RT market for reserve" designs, due to the symmetrical bidding in upward and downward balancing energy.

Table VIII presents the relative increase in activation cost resulting from the suboptimal bidding for the "adder on BRPs" and the "adder on BRPs and BSPs" designs and for three levels of interconnector capacity: (i) the uncongested case with unlimited capacity (example 3), (ii) the case with some level of congestion (example 4), and (iii) the case with isolated systems (example 2). The last column reports the cost of the "no adder" design standardized to the case with unlimited capacity. Activation cost increases by up to 47%, depending on the availability of the interconnector.

The adverse effects encountered in example 3 are present for the case with some congestion, as displayed by the inefficient dispatch, but the efficiency losses due to limited cross-border capacity interfere with the analysis. There is also a question of how the congestion rent is allocated. Note that the relative increase in activation cost is not representative of the intensity of the distributional effects, as exemplified in Fig. 4 of the previous example. We have observed empirically that even minor changes in relative cost can be associated with significant changes in the distribution of welfare.

There is no distributional effect for the case with isolated systems. As already shown in example 1, the "adder on BRPs and BSPs" design does not generate inefficiencies in an isolated balancing market. The inefficiencies encountered in the "adder on BRPs" design is concentrated in zone B, the zone with the adder.

The complete characterization of the equilibrium prices and surplus for each branch of the probability tree and for both zones for an interconnector limit of 50 MW can be found in the electronic supplement.

VI. CONCLUSION

This article investigates the unilateral application of an adder to balancing prices in a cross-border setting. An analytical model is used in order to identify the optimal strategies of flexibility providers under three different designs: the "adder on BRPs" design where the adder is applied to the imbalance price, the "adder on BRPs and BSPs" design, where the adder is applied to the balancing and imbalance price, and the "RT market for reserve" design that additionally introduces a real-time balancing capacity market. Market equilibria are derived based on these optimal strategies, extended to a cross-border setting, and illustrated on a two-zone example.

Adders, either on the imbalance price or on the balancing and imbalance price, in the absence of a real-time market for reserve, induce out-of-merit dispatch and increase the activation cost that is required for balancing the system. In a cross-border setting, this increased cost is borne by the consumers in the zone with the adder, as they face higher balancing and imbalance prices, whereas consumers in other zones enjoy lower prices. The introduction of a real-time market for reserve restores truthful bidding incentives and ensures that the increased cost to consumers in a zone, due to the adder, is fully distributed back to the flexibility suppliers in that zone.

In future work, we are interested in extending the analysis to equilibria for multiple products (aFRR and mFRR) on cross-border balancing platforms. An alternative direction of future work would apply the present methodology for investigating cross-zonal distributional effects between the Iberic peninsula and mainland Europe resulting from market interventions in Spain and Portugal in the context of the 2022 European gas crisis.

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