A Brief History of Computational Electromagnetics

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***Abstract*—Computational Physics, i.e. computational methods applied to physics, is much older than computers, dating back to Bernoulli, Newton and Gauss. Yet true application of stochastic algorithms or applications of finite differences to partial differential equations is feasible only by electronic calculators. We will briefly review the development of Computational Physics, with a focus on the solution of partial differential equations and boundary value problems and a particular attention to the field of Computational Electromagnetics. If all areas of physics had benefited from computer algorithms, in the area of Electromagnetics Computer Aided Design and Computer Aided Engineering allowed the opening of thousands of engineering applications, especially in the area of telecommunications, with a striking impact on society and our way of life.**

Keywords—Electromagnetics, Computational methods, History

I. Introduction

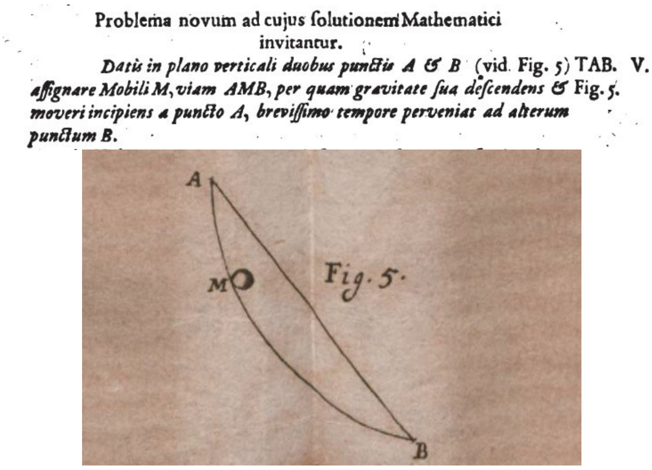
Computational Physics is almost as old as modern science itself. Modern science beginning can be set around 1611, when Galileo Galilei [Pisa, Italy, 1564 – Arcetri, Florence, Italy, 1642] first wrote about the moon not being a perfect sphere but having mountains *"per sensata esperienza et per necessaria dimostrazione* [for manifest experiences and necessary demonstration]" [1].

The point on *demonstration* was a critical one, needing mathematics and calculus. And indeed only two years later we have the first recorded occurrence of the word "computer" as referred to a person doing computations [2]. And calculus was fundamental to another contemporary of Galileo, Johannes Kepler [Weil der Stadt, Germany, 1571 – Regensburg, Germany, 1630] to devise his laws and shortly later to Ole Rømer [Å arhus, Denmark, 1644 – Copenhagen, Denmark, 1710] who manage to make the first extimation of the speed of light [4].

We might recognize a fundamental step forward in this path towards computational physics in the "Brachistochrone challenge" issued by Johann Bernoulli [Basel, Switzerland, 1667 – Basel, Switzerland, 1748] on the June 1696 issue of the Acta Eruditorum [3] (Fig. 1):

> *Given in a vertical plane two points A and B, assign to the moving body M, the path AMB, by means of which, descending by its own weight from point A, it would arrive at the other point B in the shortest time.*

Seldom in the history of science a challenge lead to so fruitful results. Five mathematicians responded with solutions: Isaac Newton, Jakob Bernoulli (Johann's brother), Gottfried Wilhelm Leibniz, Ehrenfried Walther von Tschirnhaus and Guillaume de l'Hôpital.


Problema novum ad cujus ſolutionem Mathematici invitantur.

Datis in plano verticali duobus punctis A & B (vid. Fig. 5) TAB. V. *aſſignare Mobili M, viam AMB, per quam gravitate ſua deſcendens &* Fig. 5. *moveri incipiens a puncto A, breviſſimo tempore perveniat ad alterum punctum B.*

Fig. 1 The "Brachistochrone challenge" as published in [3].

What really matters is not the problem itself and indeed nor the relevant, particular, solution proposed, but how the problem was solved. The new method was to be elaborated by Leonard Euler [Basel, Switzerland, 1707 – Saint Petersburg, Russia, 1783] who worked on the geodesic problem in 1732, and significantly improved an intuition by Joseph-Louis Lagrange [Turin, Italy, 1736 – Paris, France, 1813] which the latter communicated to the former in a letter dated August 12, 1755. Finally, in 1756, Euler himself gave this technique it its current name: Calculus of Variations [5], [6].

Further fundamental steps were due to Johann Friedrich Carl Gauss [Brunswick, Germany, 1777 – Göttingen, Germany, 1855] – let's only remember Gaussian quadrature and Gaussian distribution among his numberless contributions – and to Carl Gustav Jacob Jacobi [Potsdam, Germany, 1804 – Berlin, Germany, 1851] – we shall mention only his linear algebra contributions and its application to the solution of partial differential equations.

> *Indeed, the essential point in computational physics is not the use of machines, but the systematic application of numerical techniques in place of, and in addition to, analytical methods, in order to render accessible to computation as large a part of physical reality as possible [7].*

Such a computation was made by hand, or at most with mechanical aids for long. Indeed these mechanical aids could be very sophisticated, from Charles Babbage's [London, England, 1791 – London, England, 1871] engines (1823) [8] to Enrico Fermi's [Rome, Italy, – Chicago, Illinois, 1954] FERMIAC or trolley (1946) [9] used to compute statistics of neutron behavior in nuclear fission while waiting for ENIAC computer to be fully operational. It is indeed well known that JPL and NASA exploited pools of human computers (Fig. 2) for their rocket design up to 1950 and beyond, not fully trusting the new electronic computers.

Yet it was the introduction of electronic computers, and more precisely of Turing complete ones to make Computational Physics eventually blossom.

Among the various branches of Computational Physics we will focus on Computational Electromagnetics. Maxwell's equations have closed-form solutions only in simple cases and under simplified assumptions. An early fundamental contribution for an electromagnetic diffraction problem was due to Arnold Sommerfeld [Königsberg, Russia, 1868 – Munich, Germany, 1951] who gave the first exact solution of an electromagnetic diffraction problem [10], suitable for approximate computations via saddle point method. Sommerfeld's approach, extended by Joseph Keller [Paterson, New Jersey, 1923 – Palo Alto, California, 2016] [11] is at the basis of the geometrical theory of diffraction, and its evolutions, an approach to Computational Electromagnetics pertaining to the class of High Frequency methods, to which also physical optics belong. These will be analyzed in the following section and rely on approximation of the solutions.

A completely different approach, that of Full Wave Methods, has ancient roots, indeed it somewhat dates back to the brachistochrone problem sketched in the introduction, but of course it flourished after the introduction of computers. These will be matter of the third section and rely on the approximation of the equations.

These two main branches, and their further subdivision, can be seen in Fig. 3.

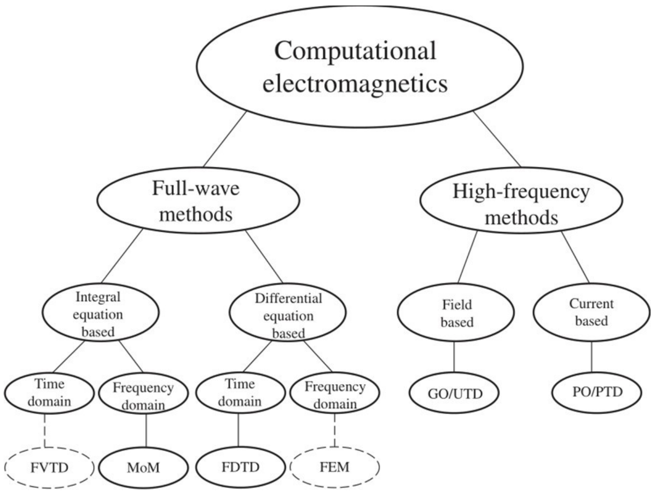


Fig. 2 The classification of Computational Electromagnetics techniques.

II. High-Frequency Methods

In high-frequency methods the aim is to find an approximation of the solution of Maxwell equations via simplifying hypothesis which can hold if the wavelength is much smaller than the geometrical characteristics of the object, that is lengths, curvature radii etc.

It develops in two lines, as Fig. 3 shows, field-based and current-based.

A. Field-based High-Frequency Methods

In a nutshell, given the incident electromagnetic field and a complex, large, object, the evaluation of the scattered field is reduced to the computation of a limited number of contributions. These are computed, on the basis of Fermat's minimum path principle, on the basis of incident, reflected and diffracted rays.

Reflected rays form the *Geometrical Optics* (GO) solution, which is very approximate and unphysically discontinuous. Better approximations are achieved via the introduction of diffracted rays, in a *Geometrical Theory of Diffraction* (GTD) framework [11] or in a more refined *Uniform Geometrical Theory of Diffraction* (UTD) framework [12].

Of all diffracted ray contributions, those arising at abrupt discontinuities of the structure are the most relevant. These can be modeled via the wedge canonical problem, which is a generalization of the original Sommerfeld half plane problem, which has attracted much attention and has been solved with various techniques. The interested reader might refer to [13] for a survey.

B. Current-based High-Frequency Methods

A different approach is that of approximating the currents induced on the object by the impinging field. A first approximation, taking into account just the incident and reflected (GO) field leads to a discontinuous uniform current distribution which, if let to radiate, produces a field which is said to be a *Physical Optics* (PO) solution. The original idea behind this is due to Hector Munro McDonald [Edinburgh, Scotland, 1865 – Aberdeen, Scotland, 1935] [14].

More physical currents are obtained in the more refined *Physical Theory of Diffraction* (PTD) where a corrective term, extracted form the solution of the wedge problem, is applied [15]. he interested reader might refer to [16] for a survey.

C. High-frequency codes

Historically, researchers produced their own codes implementing their own solution. While these were important at a scientific level, true impact on engineering came with codes actually capable of handling generic and complex objects.

The first company to explore this field was TICRA, founded in 1971 as a company focused on satellite antennas. In 1976 TICRA launched GRASP, the world's first commercial reflector antenna code implementing GTD/UTD and PO-PTD. GRASP is still in use and, as many commercial electro- magnetic analysis software, now implements hybrid methods which includes also some of those in the next section.

We might also remember some early codes: NEC-BSC (Ohio State University, 1979) MISCAT (Northrop Grumman, 1981), McPTD (DEMACO, 1992) and Xpatch (DEMACO 1992), all based on GO-GTD/UTD or PO-PTD and mainly aimed at the evaluation of the radar cross section (RCS).

III. Full-Wave Methods

When the solution is not available, then the equations must be approximated. This is done by the so-called Full-Wave methods This indeed is something tracing back to the brachistochrone method and the calculus of variation stemmedfrom there, which is at the basis of indeed all these methods, since both the variational Rayleigh-Ritz and the projectiveFaedo-Galerkin are equivalent [17].

Full wave techniques can be divided into two broad families, depending of the version of Maxwell's equation which is approximated, either integral or differential. Indeed even ifthe naming is different, this subdivision matches the one in the previous section, since integral equations based techniques are indeed current-based, while differential equations based are,on the other hand, field-based.

A. Integral Equations-based Full-Wave Methods

Given a generic complex, perfectly conducting, object, the total field at its surface must satisfy the well known boundary condition of null tangential electric field. Such a total field is the sum of the incident field plus the scattered field radiated by the currents induced on the object by the incident field.

Unknowns are hence the induced currents, which generates, via an electric (or magnetic) field integral equation such a scattered field. Such a continuous, distributed, unknown is approximated with its expansion on a finite set of bases and a linear system of equation eventually obtained.

This system of linear equation is obtained in a projective framework: the residual error between the approximated and exact solution is forced to have a null projection over the subspace generated by the basis chosen for the unknown expansion. The development of this technique, known as Method of Moments (MoM) dates to the sixties [18], [19]. This method, typical of electromagnetism, belongs to the general class of methods called Boundary Element Methods (BEM).

From original simple wires, treated via the Thin Wire Approximation (TWA), the method evolved to treat surfaces subdivided into planar or curved patches (Surface patch Model, SPM), as well as non-perfectly conducting metals and dielectric materials. An early history of the method can be found in [17], [20]. The Method was meant initially for open, radiation and scattering problems, since the integral formulation naturally includes the radiation boundary condition.

Figure 3 indeed places MoM under the label "frequency domain" but MoM exist also in time domain [21], [22], even if its application is less widespread, due to stability issues. The same figure places the Finite Volume Time Domain (FVTD) technique in the integral-equation time-domain niche. This much more recent technique [23], [24] is indeed a derivation of finite differences time domain (FDTD, see below) and is field based, and not current based.

B. Differential Equations-based Full-Wave Methods

The last group, but not of least importance, is that of the differential equation based methods, the first of which has been Finite Differences (FD), firstly applied by Euler in one dimension (ca. 1768) and Carl Runge [Bremen, Germany, 1856 – Göttingen, Germany, 1927] in two dimensions (ca. 1908), by hand to non-electromagnetic problems. Early simple electromagnetic applications were due, for example, to Frederick C. Trutt in 1962 [25], but the first application to time varying fields in three dimensions is due to Kane S. Yee [Guangzhou, China, 1934] in 1966 [26].

FDTD has a very simple implementation, as compared to all other method, but, by discretizing a three dimensional domain, needs the domain to be bounded and is limited by the memory available and CPU speed. These latter issue gradually become less and less critical, as computer technology advanced; the first, after many different approaches, was excellently solved by Jean-Pierre Berenger (1994) [27] with the introduction of the Perfectly Matched Layer (PML) in two dimensions to simulate open, infinite, domains. A technique soon extended to three dimensions [28]. The interested reader may refer to [29] for an accurate history.

On the Frequency domain side we can find the Finite Element Method (FEM) which originated in a variational paradigm in structural engineering [30], [31], but which soon moved into a Galerkin framework. Its first application to electromagnetics is due to Peter P. Silvester [Tallin, Estonia, 1935 – Victoria, British Columbia, 1996] in 1969 [32]. FEM relies on an unstructured grid of elements, as opposed to the structured FDTD grid. On such a grid the field is approximated by piecewise polynomial functions of arbitrary order and, by field integration over the elements, a matrix system of equations is obtained. Even if born in frequency domain, FEM has been extended to time domain in the eighties [33]. The interested reader may refer to [34], [35] for a detailed history of FEM.

C. Full wave codes

Historically, many different approaches fall within the full wave techniques, but all of them can be classified in either one of the two previous classes.

For what concerns the Method of Moments Jack H. Richmond and Kenneth Kwai-Hsiang Mei [Shangai, China, 1932 – Oakland, California, 2017] independently developed point- matching solutions for Pocklington and Hallen equations, respectively. This lead to a first code, BRACT (Air Force Space and Missile System Organzation, 1967) which under- went several evolutions (and name changes) eventually to be released as NEC (Numerical Electromagnetic Code – Air Force Weapons Lab, 1977) [36]. NEC, even if limited to the thin wire approximation, become the basic for several evolutions and was the benchmark for commercial codes.

For FEM we must acknowledge that first softwares were for structural mechanics (like for example SAP-IV, 1974) and then extended to electromagnetics. In particular the author of SAP-IV then started ADINA which went commercial in 1986 featuring also an Electromagnetic module. Zoltan Cendes developed the High Frequency Structure Simulator (HFSS) in the eghties and commercialized it since 1989 through Hewlett-Packard, than via Ansoft, later to become part of ANSYS, a general-purpose code appeared in 1970.

We might also remember early codes such as MacFEM (Pierre et Marie Curie University, 1987) later developed in FreeFEM and still maintained, and JMAG, (JSol Co. 1983) for magnetostatic problems.

IV. Conclusions

While, historically, computational electromagnetics followed several different paths, as outlined in this paper, and indeed still does, the pressure to solve more and more complex problems called for the hybridization of techniques, so as to get the advantages of two or more numerical approaches at the same time.

Currently available codes hence tend to present more numerical solutions within an integrated framework, so that the user, once the modelization is done, can choose one of the many techniques here presented (and many more) as well as hybridization between two or more different technique. This makes the distinction between state-of-the art codes hazy and the user's preference often focuses on the interface rather than on the underlying mathematics soundness, which, in most case, is nowadays well assessed.

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