Quantum computing in power systems

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ABSTRACT

Electric power systems provide the backbone of modern industrial societies. Enabling scalable grid analytics is the keystone to successfully operating large transmission and distribution systems. However, today's power systems are suffering from ever-increasing computational burdens in sustaining the expanding communities and deep integration of renewable energy resources, as well as managing huge volumes of data accordingly. These unprecedented challenges call for transformative analytics to support the resilient operations of power systems. Recently, the explosive growth of quantum computing techniques has ignited new hopes of revolutionizing power system computations. Quantum computing harnesses quantum mechanisms to solve traditionally intractable computational problems, which may lead to ultra-scalable and efficient power grid analytics. This paper reviews the newly emerging application of quantum computing techniques in power systems. We present a comprehensive overview of existing quantum-engineered power analytics from different operation perspectives, including static analysis, transient analysis, stochastic analysis, optimization, stability, and control. We thoroughly discuss the related quantum algorithms, their benefits and limitations, hardware implementations, and recommended practices. We also review the quantum networking techniques to ensure secure communication of power systems in the quantum era. Finally, we discuss challenges and future research directions. This paper will hopefully stimulate increasing attention to the development of quantum-engineered smart grids.

Governments across the world are reaching a consensus to increase the use of renewable resources so as to fulfill their countries' ever-increasing energy demands. For instance, the U.S. federal government has recently been committed to reducing greenhouse gas emissions 50–52 percent below 2005 levels in 2030, reaching a 100% carbon pollution-free power sector by 2035, and achieving a net-zero economy by no later than 2050[1]. As an example of climate laws at the state level, New York State's Climate Leadership and Community Protection Act (Climate Act)[2] has set a series of nation-leading climate targets, including the grid integration of 9 gigawatts of offshore wind power carbon neutral economy by 2035, 6 gigawatts of distributed solar by 2025, and 3 gigawatts of energy storage by 2030. Despite the tremendous benefits of decarbonization and emission reduction, interconnecting hundreds of gigawatts of renewables causes severe impacts on the power grids, such as congested transmission and distribution corridors, and weakened power grids due to reducing system inertia, widespread intermittency and uncertainty, compromised situational awareness, and destabilized electricity markets. Two major challenges have contributed to this worsening situation: (1) The state-of-the-practice computing capabilities of power grids are unable to handle the gigantic volumes of data generated from, and commands needed by the real-time operation of the large interconnected grids[3,4]; and (2) The unprecedentedly ultra-scale computational requirements make existing analysis algorithms, from probabilistic power flow to electromagnetic transients pro-

gram (EMTP), unscalable and unable to offer real-time, high-fidelity results needed for managing massive distributed energy resources (DERs) and ensuring resilient operations[5,6]. Those challenges are further escalating as today's power grids are subject to more frequent weather events and targeted by malicious, well-equipped and motivated adversaries.

Recently, the successes in exploiting the potential of quantum supremacy[7,8] shed light on a 'quantum leap' of the computational capabilities, which could empower an unprecedentedly resilient power system. In general, the representation of complex power systems' states on a classical computer scale exponentially with the size of the problem, while on a quantum computer they scale polynomially in theory. Furthermore, highly entangled states, which are prohibitively difficult for classical computers to model, can be readily represented on a quantum computer[9,10]. This implies that those intractable power system problems, which remain formidable problems even solved on powerful and expensive real-time simulators or high-performance computers, if formulated properly, can be executed much more efficiently on quantum computers. Quantum computing, however, is a double-edged sword. An immediate concern, for instance, is that the advent of quantum computers will invalidate computational-hardness assumptions that underpin the data security schemes being used in today's power systems[11].

Inspired by the aforementioned challenges and opportunities, since 2018, the Power Systems Laboratory at Stony Brook Uni-

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versity has pioneered the research in Quantum-Engineered Smart Grids (Quantum Grids, or QGrids, see Figure 1; the term Quantum Grids originally came from our proposal to the National Science Foundation —P. Zhang, et al., ASCENT: Quantum grid: Empowering a resilient and secure power grid through quantum engineering, Proposal# 2023915, February 2020). The quantum grids group at Stony Brook, including power engineers, computer scientists, and quantum physicists, has been integrating quantum computing and quantum networking into a quantum-engineered grid infrastructure to form scalable, self-protecting, autonomic and sustainable power grids capable of coordinating gigantic distributed energy systems and fostering future resilient communities and smart cities.
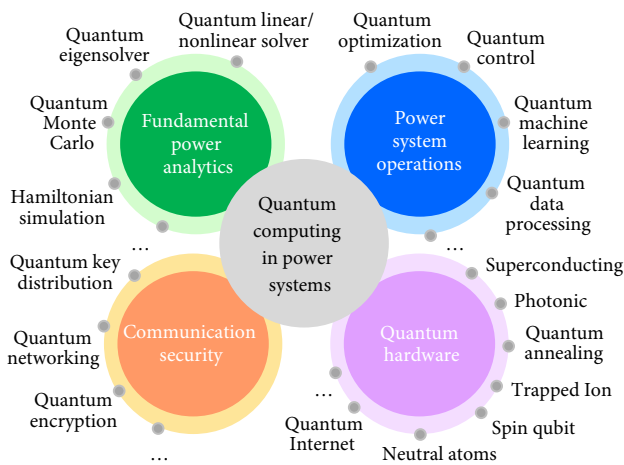

Fig. 1   A schematic of quantum computing for power systems.

The main purpose of this paper is to review Stony Brook's new contributions to Quantum-Engineered Smart Grids[12–29], partially supported by the U.S. Department of Energy's Office of Electricity (P. Zhang, Practical Quantum Analytics for Ultra-Efficient and Resilient Bulk Power Systems Operations. U.S. DOE Office of Electricity Agreement No. 37533. Proposal submitted in June 2020) and Stony Brook University's Quantum Information Science and Technology seed grant. We will introduce a series of quantum analytics for power systems that are feasible to pursue on noisy-intermediate-scale quantum (NISQ) computers. Meanwhile, we will also describe a few quantum grid analytics designed for noise-free quantum computers that may emerge in the next decade. We further discuss quantum networking, which provides a level of security for key distribution that is unattainable through classical cyber systems.

1   A brief introduction of quantum computing

1.1   Quantum computing: From bits to qubits

This subsection introduces the basic knowledge of quantum computing. We refer readers to the textbook by Nielsen and Chuang for a pedagogical introduction[9]. To understand what quantum computing is and how quantum computing can be implemented, we first discuss classical computing in terms of gates. The basic classical information carrier is a collection of bits, each of which can be in two binary states: 0 or 1. A '0' can be implemented by a voltage of 0 volt and a '1' can be implemented by a voltage of 5 volts in electronics. To flip a bit one has a NOT gate, i.e. NOT : $0 \leftrightarrow 1$. There are other gates that act on two bits at once,

such as the AND gate, which is the binary addition, e.g., $0 \text{ AND } 1 = 1 \text{ AND } 0 = 0$, $0 \text{ AND } 0 = 0$, and $1 \text{ AND } 1 = 0$. The OR gate has the actions: $0 \text{ OR } 1 = 1 \text{ OR } 0 = 1 \text{ OR } 1 = 1$, but $0 \text{ OR } 0 = 0$. Any logical expression can be constructed by a circuit using these three gates, and hence, they form a set of universal gates. In fact, only one kind of gate is needed, i.e. the so-called NAND gate, which takes two inputs as AND and OR gates and acts as an AND gate followed by a NOT gate on the output of the AND gate.

Quantum computing, in some sense, is a generalization of (1) classical bits to quantum bits (qubits), and (2) classical logical gates to reversible and general unitary gates.

Additionally, states can be 'added' or 'superposed'. For example, a quantum bit has two basis states corresponding to the logical 0 and 1, but written inside brackets: $|0\rangle$ and $|1\rangle$. Unlike classical bits, a quantum bit can be in any superposition: $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$, where α and β are two generally complex coefficients such that $|\alpha|^2 + |\beta|^2 = 1$, which is the normalization of a quantum state, i.e. $\||\psi\rangle\|^2 = \langle\psi|\psi\rangle = 1$. Here, one can regard a qubit as a two-component normalized complex vector. Given that the overall phase factor of a qubit does not have a physical meaning, we can choose to parameterize $\alpha = \cos(\theta/2)$ (i.e. real) and $\beta = \sin(\theta/2)e^{i\varphi}$, then the so-called density matrix $|\psi\rangle\langle\psi|$ (or the outer product of a column and a row vector of the complex vector ψ) is written as $(I + r_x X + r_y Y + r_z Z)/2$, where I is the 2 × 2 identity matrix X, Y, and Z are the so-called Pauli matrices, and moreover $r_x = \sin\theta\cos\varphi$, $r_y = \sin\theta\sin\varphi$, and $r_z = \cos\theta$ give the spherical coordinate of a unit sphere. This is the so-called Bloch sphere, where any qubit can point to any direction on or inside the sphere.

A quantum gate, as explained, generalizes classical gates and acts on one or possibly multiple qubits. We have seen the classical NOT gate, and its quantum version is the Pauli X matrix/operator, which flips between the basis states, $X : |0\rangle \leftrightarrow |1\rangle$. By using this rule, it is easy to see that $X|\psi\rangle = \alpha|1\rangle + \beta|0\rangle$. The quantum version of the NOT gate can act on any superposition of logical 0 and 1 states. Other one-qubit gates can be regarded as the rotation of the Bloch vector and are generally written as $\exp(-i\theta\vec{\sigma} \cdot \hat{n}/2)$, where $\vec{\sigma} = (X, Y, Z)$ is a vector whose three components are simply the Pauli matrices and $\hat{n}$ is a unit vector representing a direction. The meaning of this gate is to rotate a Bloch vector by an angle θ with respect to the axis defined by $\hat{n}$.

Beyond one qubit and one-qubit gates, for two qubits, there are four basis states, which are simply a juxtaposition of two single-qubit basis states: $|0\rangle \otimes |0\rangle = |00\rangle, |01\rangle, |10\rangle$, and $|11\rangle$, where $\otimes$ is the so-called tensor product notation but is usually ignored if there is no ambiguity. One can easily generalize to n qubits, where there are 2^n such basis states, and thus there are 2^n complex coefficients for a general n-qubit state. A general n-qubit gate is a $2^n \times 2^n$ unitary matrix that takes an n-qubit state to another n-qubit state, which is in general quite complicated. However, according to the matrix theorem that any such n-qubit unitary can always be decomposed into a sequence of one-qubit gates and two-qubit gates acting on appropriate qubits. In such a decomposition, one-qubit gates are the general Bloch vector rotations, and we only need one kind of two-qubit gates, such as the Controlled-NOT gate (CNOT or CX).

Given the CNOT gate acts on two qubits, it can be defined by the action on the four basis states; specifically, under CNOT: $|00\rangle \to |00\rangle$, $|01\rangle \to |01\rangle$, $|10\rangle \to |11\rangle$, and $|11\rangle \to |10\rangle$, where we assume the first qubit is the controlled bit and the second qubit is the target bit. The CNOT gate can generate the so-called quantum entanglement from a product state, for example, $(|0\rangle + |1\rangle)/\sqrt{2} \otimes |0\rangle \to (|00\rangle + |11\rangle)/\sqrt{2}$, which is an entangled

state that enables quantum teleportation, a scheme to transfer quantum states without physically sending them. For a review of quantum circuits and implementations of recent quantum devices, please see the review paper ref. [30].

A quantum computer usually begins with the initialization that all qubits are in the $|0\rangle$ state. Then a sequence of single- and two-qubit gates act on these qubits to achieve a certain n-qubit unitary operation. A famous such action is the quantum Fourier transform (QFT), which is used in Shor's factoring quantum algorithm. Another example which is quite popular for NISQ devices[31] is the variational quantum circuit (VQC). In the VQC, one has some pre-determined circuit structure, e.g., composed of fixed CNOTs and some single-qubit rotation gates, whose rotation angles are variational parameters. The VQCs are used in the variational quantum eigensolver (VQE) algorithm, in which the goal is to optimize some cost function or the expectation of a certain energy operator by using VQCs and measurement to yield some classical values, which in turn are used to infer how to change the variational parameters. This hybrid quantum-classical process is iterated until the cost is converged. For a recent review of the VQE algorithms and their applications, we refer the readers to a recent article published in *Nature Reviews Physics*[32]. Variational quantum circuits are also used in many quantum machine learning designs; for the latter, see a recent review[33].

So the one final piece that we have not explicitly explained is the measurement. Given the final readout of a quantum computer is to measure all or some of the qubits in the $|0/1\rangle$ basis, we explain the effect of such measurement applied to a single qubit: $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$. Given $|0\rangle$ and $|1\rangle$ are eigenstates of the Pauli Z operator (with eigenvalues +1 and −1, respectively), the measurement is also called Z measurement. The outcome is probabilistic, i.e. with a probability $|\alpha|^2$ one obtains $|0\rangle$ outcome, and with a probability $|\beta|^2$ one obtains $|1\rangle$ outcome. This also explains why we have chosen to normalize the coefficients of $|\psi\rangle$ such that $|\alpha|^2 + |\beta|^2 = 1$. This completes the circle of the brief introduction to quantum computation[9].

So what are quantum computers good for? It is worth mentioning that a quantum speedup occurs if the corresponding quantum task requires the depth of the quantum circuit to scale much less than the number of steps in any classical approach. If quantum computers can perform tasks that classical computers cannot efficiently simulate, then this is called quantum supremacy or quantum advantage[34,35]. One of the earliest suggestions made by Feynman is to use them to simulate other quantum systems, which would be more efficient than classical computers[36]. Given the need for an exponential number of parameters on classical computers to describe a quantum system, simulating a quantum system is generally hard for classical computers. Lloyd showed that Feynman's conjecture that 'quantum computers can be programmed to simulate any local quantum systems' is correct[37]. Hence, quantum computers may assist to solve fundamental science problems, such as the mass gap problem in the Yang–Mills theory (one of the Clay Mathematics Institution Millennium problems) and high-temperature superconductivity problem. Perhaps the most well-known application of quantum computers is to factor a large integer number by a quantum algorithm invented by Shor[38], which is superpolynomially faster than the current best classical approach. Other potential superpolynomial speedups by quantum computers include computing topological invariants of a topological field theory, such as the Jones polynomial[39], solving a system of linear equations with a large matrix[40], and computing the permanent of a matrix using boson sampling[41]. Recent experimental

progress towards quantum advantage includes random quantum circuits[7] and the Boson sampling problems[42].

1.2 Current status of quantum computers

Working on real quantum computers is not far from real life. As reported by IBM, one of the world's pioneering companies in providing quantum computing services, they already have over 400, 000 userbases running 1 trillion circuits so far[43].

Two mainstream paths for developing quantum computer hardware are gate-based and annealing-based approaches. Table 1 lists some of the major industry players in providing real quantum computers and cloud-based services:

- IBM is the first company to provide cloud access to quantum computers in 2016, i.e. through the IBM Q graphical user interface (GUI)[44] and Qiskit software development kit (SDK)[45]. So far, IBM offers commercial access to quantum devices up to 127 qubits and public access to quantum devices up to 32 qubits. Their roadmap is to launch the 1121-qubit Condor processor by 2023, which is capable of solving a range of complex scientific problems[46], and to achieve hundreds of thousands of qubits from 2026 and forward[43].

- Google is another major quantum computing company, especially in the quantum artificial intelligence (AI) area[47]. It provides several open-source packages, such as Cirq[48], OpenFermion[49], and TensorFlow Quantum[50], for customers to develop near-term applications compatible with noisy quantum machines. An impressive milestone is that Google claimed in 2019 that they had achieved quantum supremacy[7], which was a world-first experiment to demonstrate the quantum speedup. On a 54-qubit quantum processor "Sycamore", Google showed that the quantum computation for their benchmark testing could be accomplished in 200 s, while the world's fastest supercomputer may take 10,000 years to obtain a comparable result[7]. However, since then new classical algorithms were developed that improved the classical simulations for sampling random circuit outcomes[51–53].

- Xanadu is a Canadian company offering the first photonics-based quantum computing platform[54]. Rather than using superconductors like IBM or Google, Xanadu's system is based on light and can be operated at normal temperature, with a non-negligible advantage. Xanadu also provides cloud-based service through Xanadu Quantum Cloud and application libraries such as Strawberry Fields[55] and Pennylane[56].

- While most quantum devices are gate-based (e.g., IBM, Google, Xanadu), D-Wave pursues another path using specialized quantum annealing techniques[57]. A quantum annealer does not rely on quantum circuits for computing. Instead, it reformulates the problem into ground state searching problems, an excellent match to various optimizing issues. Annealing-based quantum computers appear to be more scalable than gate-based ones in terms of the number of qubits manufactured in a single processor. While most gate-based quantum computers possess no more than 200 qubits, D-Wave already achieves the level of thousands of qubits. With more than 5, 000 qubits and over 15 couplers per qubit, D-wave systems are capable of calculating problems with more than 10, 000 variables[58].

- Quantum computing in China is also under swift development. In 2017, Alibaba and Chinese Academy of Sciences jointly debuted an 11-qubit quantum computer[59], which was the first public-accessible quantum computing service in China. Origin Quantum, another superconducting-based quantum computing startup in China, has raised an ambitious

Table 1 Major providers for commercially accessible quantum computers and platforms

Provider	Type	Realization	Maximum qubits	Country/region
IBM	Gate-based	Superconducting	127	US
Google	Gate-based	Superconducting	72	US
Rigetti	Gate-based	Superconducting	32	US
Honeywell	Gate-based	Trapped ion	10	US
IonQ	Gate-based	Trapped ion	32	US
QuEra	Gate-based	Neutral atoms	256	US
Xanadu	Gate-based	Photonic	24	Canada
D-Wave	Analog-based	Annealing	5000+	Canada
Alibaba	Gate-based	Superconducting	11	China
Origin Quantum	Gate-based	Superconducting	64	China
OpenSuperQ	Gate-based	Superconducting	20	Europe
QuTech	Gate-based	Spin qubit	29	Europe
AQT	Gate-based	Trapped ion	20	Europe

Other large companies include Microsoft, Intel, Amazon, Hitachi, Hewlett-Packard (HP), etc. There is an online article describing quantum hardware outlook in ref. [61].

roadmap to achieve 144 qubits by 2022 and 1024 by 2025. Besides, University of Science and Technology of China has developed Jiuzhang, which is the first photonic quantum computer to have announced quantum supremacy[60].

Today we are still in the NISQ era, meaning that state-of-the-art quantum computers are sensitive to noisy environments, and there are not enough qubits and the gate error rates are still too high for error correction. Fault-tolerant quantum computers towards millions of qubits may still be decades away. Therefore, the executable scale of quantum circuits on today's quantum computers is significantly restricted by the quantum gate errors, insufficient number of qubits, low connectivity between qubits, etc[32].

2 Quantum computing for fundamental power analytics

Massive integration of renewable energies has significantly reshaped modern power systems by introducing highly uncertain and low-inertia inverters. Under such circumstances, developing ultra-efficient analytics for accurate static and transient simulation of power systems becomes prohibitively critical, especially in uncertain scenarios. The power of quantum computing is derived from the possibility of preparing and maintaining complex superpositions of quantum states across many quantum degrees of freedom as well as providing entanglement between the states of the system. Thus, most theoretical quantum computing models achieve exponential speedups over classical models. This section reviews quantum algorithms for fundamental power analytics, including both static and transient analyses, as well as their probabilistic versions, such as Monte Carlo-based power system tools. Such fundamental quantum-power analytics opens the door to opportunities to solve many traditionally complex problems for power systems.

2.1 Quantum-enabled static analysis

Power system static analysis, represented by power flow and state estimation, is the keystone of various power system analytics. Under the unprecedented integration of renewables, a tremendous amount of repetitive static analysis is required to analyze the

impact of uncertainties. However, if solved by the conventional iterative algorithms, the computation complexities of power flow and state estimation scale polynomially with the problem scale. Such circumstances significantly restrict their applications for tractable real-time operation demands. This subsection reviews the quantum-inspired power flow and state estimation methods, which offer a potential path toward more scalable power grid static analytics.

2.1.1 Quantum power flow

Power flow analysis aims at solving the nodal power balance equations formulated by power generation, load, and grid topology[62–64]. Prominent AC power flow algorithms include the Newton–Raphson algorithm[65], the Gauss–Seidel algorithm[66] and fast-decoupled methods[67]. An indispensable step of the aforementioned algorithms (i.e. the iterative nonlinear algorithms) is to solve a set of linear algebraic equations. Therefore, the critical bottleneck of power flow analysis lies in the inefficiency of the linear solvers.

In the quantum computing area, the Harrow–Hassidim–Lloyd (HHL) algorithm is a significant landmark for solving linear equations in the quantum space[40]. The HHL employs a quantum circuit to realize a unitary transformation for the quantum superposition of the linear solution. A salient advantage of the HHL (or any of its variants) is that it enables an exponential speedup over classical methods for analyzing sparse systems, which exactly matches the characteristics of power systems. One requirement of the HHL algorithm is that the input matrix should be Hermitian (otherwise, the matrix A should be reformulated as $H = [0, A; A^{\dagger}, 0]$ so it becomes Hermitian). In ref. [12], a quantum power flow (QPF) method is proposed (see Figure 2), which is the first quantum-inspired algorithm to underpin the AC power flow issue. QPF innovatively integrates the fast-decoupled power flow philosophy with the HHL algorithm, which makes full use of the Hermitian and sparse jacobian matrix of power grids to enable a realizable implementation of the HHL. As shown in Figure 2, the quantum circuit of the HHL-based fast-decoupled QPF consists of four components, i.e. a quantum phase estimation (QPE) for determining the eigenvalues of Jacobian matrices, a controlled rotation for generating the reciprocal of the eigenvalues, an inverse
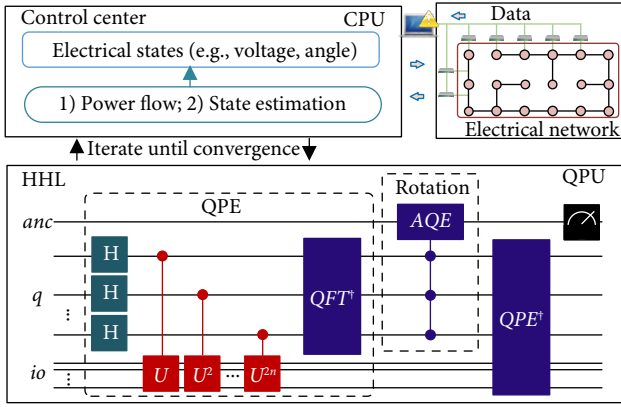
Fig. 2 Quantum circuit architecture for quantum-inspired power grid static analytics[12,13].

QPE for disentangling the qubits, and a measurement for the final states. The proof-of-concept of QPF on a small test system is provided in ref. [12].

2.1.2 Quantum state estimation

Power system state estimation (SE) produces the best possible estimation of the true system states based on the data from supervisory control and data acquisition (SCADA) systems and phasor measurement units (PMUs), which are extremely valuable for online system operations[68,69]. The most widely-used algorithm for SE is the weighted least square (WLS) algorithm, which minimizes the sum of weighted squared errors between measurements and estimations.

The major computation burden of WLS lies in an iterative calculation of a series of linear equation systems characterized by the SE gain matrix. To tackle the challenge, ref. [13] establishes HHL-enabled quantum state estimation (QSE) algorithms. The complexity of HHL, i.e. $O(\log(N)\eta^2\kappa^2)$[70], is closely related to three factors, i.e. the system dimension N, the matrix sparsity η and the condition number κ. While the sparsity of the SE problem can always be guaranteed because of the naturally sparse feature of power grids, the condition number may vary from case to case. For the well-conditioned scenario, the HHL can be straightforwardly implemented. However, for the ill-conditioned scenario, the HHL may lose the quantum speedup or even fail to provide reasonable results. Therefore, ref. [13] proposes a preconditioned-HHL for the QSE implementation. The overall idea is to use a preconditioned iterative optimization to obtain the power system states instead of directly calculating them through the HHL algorithm. The performance of QSE under both well-conditioned and ill-conditioned scenarios has been validated on a microgrid system.

In sum, the HHL-based quantum linear solver provides a promising tool for power system static analysis, which allows for exponential improvement of the computational complexity for linear equation solving. However, the HHL suffers from an excessively large depth of quantum circuits that are sensitive to the noises and short coherence time, and its implementation requires fault-tolerant quantum computers. Therefore, a noteworthy future direction could be exploiting variational quantum linear solvers (VQLSs)[71] to accelerate power system static analysis in the NISQ era.

2.2 Quantum-enabled transient analysis

Transient analysis is another cornerstone for power system ana-

lytics. Today's power systems are facing a risk of diminishing inertia due to the deep integration of inverter-based resources and the retirement of synchronous generators powered by fissile fuel or nuclear reactors[72–75]. To capture the wide-band electromagnetic transients of power electronic devices, EMTP becomes indispensable[76,77]. Although EMTP is capable of precisely tracing the electromagnetic waveforms, its daunting computational complexity, which scales polynomially with the system size, formidably hinders its application in very large power systems. This subsection reviews quantum-enabled EMTP (QEMTP) algorithms, which tackle the EMT computation problem through quantum computing. Such analytics lays the foundations for power grid transient analysis on both current NISQ computers and noise-free quantum computers of a distant future.

2.2.1 Quantum-encoded EMTP formulation

Classically, EMTP applies the trapezoidal discretization at each time step to transform the dynamic equations of a power network into numerical equations of an equivalent resistance network, which can be formulated as

$$\boldsymbol{G}_0\boldsymbol{v}(t) = \boldsymbol{i}(t), \tag{1}$$

where $\boldsymbol{v}$ and $\boldsymbol{i}$, respectively, denote the vectors of nodal voltages and equivalent nodal current injections; $\boldsymbol{G}_0$ is the equivalent conductance matrix. The mathematical essence of Eq. (1) is a linear system problem (LSP). Therefore, for a power system with N dimension, classical EMTP performs the computation in an N-dimensional Euclidean space. The inverse operation of the matrix $\boldsymbol{G}_0$ is at the computational complexity of $O(N)$.

Quantum computing holds the promise for a logarithmically-growing computational complexity for LSP[78], which sheds light on unprecedentedly scalable EMTP tools for power systems. The very first step towards developing a QEMTP algorithm is to encode the EMTP formulation in Eq. (1) into a quantized version. To this end, quantum EMTP models have been developed in refs. [14, 15]. Denote the normalised quantum representations of $\boldsymbol{v}(t)$ and $\boldsymbol{i}(t)$ as $|v\rangle = \sum_k \frac{v_k}{\sqrt{\sum_k v_k^2}} |k\rangle$ and $|i\rangle = \sum_k \frac{i_k}{\sqrt{\sum_k i_k^2}} |k\rangle$. An attractive fact is that such a quantum formulation only requires $\lceil \log_2 N \rceil$ qubits, which can be ultra-scalable compared with the classical EMT formulation.

Correspondingly, Eq. (1) can be embedded into the Hilbert space as

$$\boldsymbol{G}|v\rangle = |i\rangle, \tag{2}$$

where $\boldsymbol{G}$ is the padded and normalized counterpart of $\boldsymbol{G}_0$. Eq. (2) therefore establishes the quantum counterpart of the classical EMTP. A salient feature of the QEMTP formulation is that any operators on Eq. (2) will be performed in the Hilbert space with exponential scalability.

Mathematically, Eq. (2) is a quantum linear system problem (QLSP). There exist two main types of approaches to solving QLSP: while noise-free methods usually rely on ideal quantum machines which may not be available in the near future, noisy intermediate-scale methods provide a practical solution for quantum computing on near-term quantum computers. In the following, we explain how QEMTP can be achieved by both a noise-free approach and a noisy intermediate-scale approach.

2.2.2 HHl-enabled QEMTP: A noise-free approach

Ref. [14] is the first attempt to resolve the QEMTP issue through

the noise-free HHL algorithm. As introduced in Section 2.1, the HHL algorithm is well-known for its capability to estimate the solution of an LSP with a computational complexity of $O(\log(N))$, which realizes an exponential speedup compared with its classical counterpart.

The mathematical basis of the HHL-enabled QEMTP is to decompose Eq. (2) into the eigenbasis of the power system's equivalent conductance matrix $\boldsymbol{G}$:

$$|\boldsymbol{v}\rangle = \boldsymbol{G}^{-1}|\boldsymbol{i}\rangle = \sum_j \lambda_j^{-1} b_j |\boldsymbol{u}_j\rangle, \tag{3}$$

where $(\lambda_j, \boldsymbol{u}_j)$ are the jth eigenpair of $\boldsymbol{G}$ and b_j is the corresponding decomposition coefficient of $|\boldsymbol{i}\rangle$.

To achieve the eigendecomposition logic, the HHL QEMTP adopts three quantum registers for calculation. As illustrated in Figure 3, register io stores the input (e.g., nodal current injections) and output (e.g., nodal voltages) of QEMTP; register w performs the computation of QEMTP; and register a stores ancilla qubits for the HHL algorithm. Consequently, at each timestep, the HHL-enabled QEMTP updates the equivalent current injections based on the quantities at the previous timestep and performs the QPE calculation of $\boldsymbol{G}$ as well as other necessary quantum computations (e.g., controlled rotation, inverse QPE). Finally, the nodal voltages $\boldsymbol{v}$ are output on the io register. More details are explained in ref. [14].

Although the HHL algorithm theoretically provides an ultra-scalable path towards QEMTP, its practicability is still hindered. The major challenge is that the HHL usually adopts extremely complicated quantum circuits[40]. For example, ref. [14] demonstrates that even for a simple RLC circuit, the HHL quantum circuit reaches 102-depth and involves 54 CNOT gates. Such quantum circuits, unfortunately, may to be executed correctly by today's NISQ computers because of the non-negligible quantum errors, insufficient qubits for EMT correction, limited connectivity between qubits, etc[71].

2.2.3 VQLS-enabled QEMTP: A noisy intermediate-scale approach

Motivated by the aforementioned challenges of the HHL-enabled QEMTP, ref. [15] further develops a VQLS-enabled QEMTP algorithm to unlock a practical and noise-resilient approach for EMTP analysis on today's NISQ devices.

The VQLS-enabled QEMTP employs a hybrid quantum-classical framework. A VQC is constructed for solving Eq. (2), which does not involve the complicated eigendecomposition quantum circuits required by QPE:

$$|v\rangle = U_{\text{EMTP}}(\boldsymbol{p})|0\rangle. \tag{4}$$

Here, U_{EMTP} denotes a VQC whose parameters $\boldsymbol{p}$ are undetermined. The key concept of VQLS is to optimize $\boldsymbol{p}$ so that the output of $U_{\text{EMTP}}(\boldsymbol{p})$ conforms with the desired solution of Eq. (2). To this end, a cost function can be constructed to indicate the similarity between the current injection state $|i\rangle$ and the quantum state $|\varphi\rangle = \boldsymbol{G}|v\rangle$. Here, the word "variational" or "hybrid" means that the quantum circuit is executed on quantum devices to obtain the output quantum state, and the circuit optimization procedure is performed on classical computers (see Figure 3). The two routines interact until Eq. (2) is achieved, which provides a qualified VQC for EMTP analysis. Various algorithms can be employed for optimizing a VQC, such as quantum gradient descent[79,80] and its variants.

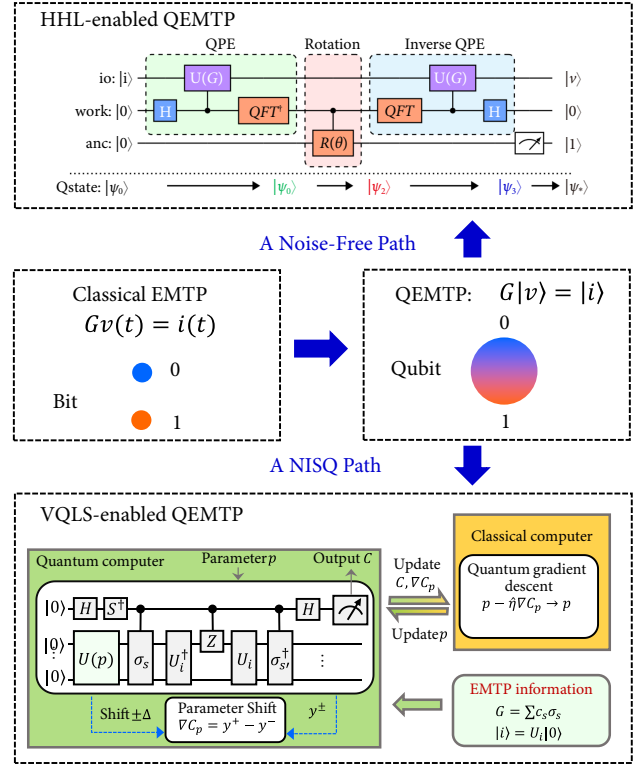Besides the parameters to be optimized, another configurable

Fig. 3   Schematic of quantum-enabled power system electromagnetic transient analysis[14,15].

setting of the VQC $U_{\text{EMTP}}(\boldsymbol{p})$ is its structure, which can significantly impact the performance of the algorithm. Ref. [15] designs a layered quantum circuit with the RZ-SX-RZ sequence, which has been demonstrated as an effective structure for the QEMTP calculation. However, because of the limitations of the state-of-the-art quantum devices, it is non-trivial to design a quantum circuit that not only realizes the EMTP functionality but also is executable on the NISQ devices. In Section 3.2, we will provide more discussions on the design of VQCs.

Although VQLS is much more NISQ-friendly compared with HHL, it still faces several challenges for real-world implementation, such as quantum circuit depth, quantum state measurement, and small discretization steps. To this end, ref. [15] has also established practical variants of the VQLS-enabled QEMTP by making full use of the characteristics of both power grids and quantum computing, such as analyzing the diagonally dominant feature of power grid conductance matrix to decompose the QEMTP formulation to enable measurable quantum states, and exploring the superposition of quantum computing to simultaneously solve different basis nodal voltages of EMTP. In addition, we have also employed the philosophy of shifted frequency analysis (SFA)[81–83] in QEMTP to develop a quantum shifted frequency analysis (QSFA) to enable QEMTP computation with larger timestep.

The most attractive superiority of the VQLS-enabled QEMTP is that, it not only achieves exponential scalability of EMTP computation, but can also be executed on today's noisy quantum machines. Ref. [15] has demonstrated the implementation of the VQLS-enabled QEMTP on real IBM quantum computers (i.e. a 27-qubit quantum computer ibmq_sydney). The real-hardware experiments show that the VQLS-enabled QEMTP can achieve satisfactory precision under noisy quantum environments with shallow quantum circuits. Meanwhile, even for the large-scale 906-bus feeder, QEMTP only requires 10 qubits, which is promisingly scalable. More importantly, the noise impact analysis shows that

QEMTP remains high performance under the general noise level of today's quantum computers, which ensures the universal practicality of QEMTP on arbitrary near-term hardware.

However, it should be noted that quantum linear solvers (either noise-free or noisy approaches) can only approximately estimate the solution of LSP, which is different from the classical solvers which can return the full solution. Therefore, error correction is still indispensable for the QEMTP algorithms[14, 15].

2.3 Quantum-enabled stochastic analysis

With the increasing deployment of renewables, static, dynamic, or transient analysis under a single scenario becomes insufficient for ensuring the high reliability of power system operations. Therefore, it is critical to verify the system performance under numerous stochastic scenarios initiated by heterogeneous uncertainties and unforeseen faults.

In large-scale power systems integrating transmission grids, distribution grids, and even microgrids, the number of system states grows swiftly, which potentially leads to complex computational problems and NP-hardness in stochastic analysis[84, 85]. Monte Carlo simulation (MCS) is a representative simulation-based stochastic method. The sampling size of MCS is a deciding factor in generating the desired probability distribution function, which results in heavy computational burdens and slow convergence[86].

The quantum estimation algorithm aims at mitigating the number of usages of a randomized algorithm in classical MCS technique[87]. A correct design of a quantum algorithm containing various gates and unitary operators can achieve an acceptable approximation of the distribution function with quantum speedup. It is proven that the convergence rate of the classical MCS method with S sampling size is $O(\frac{1}{\sqrt{S}})$[88]. On the other hand, quantum computing can achieve a quadratic speedup with convergence $O(\frac{1}{S})$[89–91]. The Quantum amplitude estimation (QAE) algorithm[90], which takes advantages of the Grover's search algorithm[92], has already demonstrated the quadratic speedup and convergence over the classical MCS technique in estimating an uncertain variable.

Ref. [16] tackle the power system reliability assessment using the QAE algorithm. With a small number of qubits and sampling size, ref. [16] realize the quantum speed-up and better convergence over the classical MCS-based power system reliability analysis. Classical MCS mainly includes three steps: first, the uncertain parameters should be modeled as random variables $X = \{X_1, X_2, \cdots, X_R\}$, where R is the total number of random variables. Second, S samples should be generated for each random variable using the probability distribution function of each variable as $\{x_1, x_2, \cdots, x_S\}$, where x is a random sample of a variable. Finally, the expected value of a real-valued function $f(x)$ for each random variable is calculated as follows:

$$E[f(X_j)] = \sum_{i=1}^{S} \frac{1}{S} f(X_{ij}), \ \forall j \in \{1, 2, \cdots, R\}. \tag{5}$$

To solve such a problem using the quantum-amenable MCS method, the aforementioned steps of the classical method should be translated into the quantum blocks. In Figure 4, a schematic circuit of the quantum estimation method is depicted on running the MCS steps.

In the quantum circuit of QAE, the first quantum block P aims at generating the n-bit string result x with probability $p(x)$. This quantum block outputs:
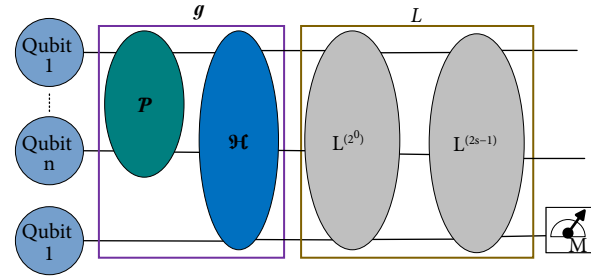

Fig. 4 Schematic of quantum circuits of the quantum estimation algorithm.

$$P|0\rangle_n = \sum_{i=0}^{2^n-1} \sqrt{p(x_i)} \cdot |x_i\rangle_n, \tag{6}$$

where, n is the number of input qubits.

In the second block of the quantum circuit, the unitary operator H is applied to $(n+1)$ qubits. In this step, a rotation is executed onto an ancilla qubit. The operator H outputs:

$$H|x\rangle_n |0\rangle = |x\rangle_n \left(\sqrt{f(x)} \cdot |1\rangle + \sqrt{1-f(x)} \cdot |0\rangle \right), \tag{7}$$

where, $f(x)$ is a function $f(x) : \{0,1\}^n \to \mathbb{R}$ which is mapped to real numbers from n-bit strings.

The output state ψ is resulted after applying quantum blocks $(P \otimes I)$ and H to the states $|0\rangle_n |0\rangle$:

$$
\begin{aligned}
|\psi\rangle &= H(P \otimes I) \cdot |0\rangle_n |0\rangle \\
&= \sum_{i=0}^{2^n-1} \left[\sqrt{p(x_i)} \cdot |x_i\rangle_n \left(\sqrt{f(x_i)} \cdot |1\rangle + \sqrt{1-f(x_i)} \cdot |0\rangle \right) \right],
\end{aligned} \tag{8}
$$

where, I is the identity operator.

According to Figure 4, there exists a block L with s sampling qubits and $S = 2^s$ application of operator L. To achieve an efficient estimation of a function, operator L is employed. The purpose of QAE is to estimate the probability of measurement $|1\rangle$ in Eq. (8). At the end of the quantum circuit, the measuring units are used to measure the amplitude of $|1\rangle$.

3 Quantum computing for power system operations

As one of the earth's most extensive and complicated dynamical systems, the power system requires efficacious modeling, monitoring, planning, and controlling methodologies to support its efficient, reliable, and resilient operations. In addition to promoting fundamental power system analytics, quantum computing can also benefit various aspects of power system operations. This section reviews the recent progress in leveraging cutting-edge quantum computing techniques, including quantum optimization, quantum machine learning and quantum control, to tackle vital power system operation issues.

3.1 Power system operation via quantum optimization

Optimization plays an essential role in power system operations. Various power system tasks, such as unit commitment[93], energy management[94], energy trading[95], and emergence control[96], can be formulated as optimization problems, among which large-scale combinatorial optimization is one of the most intractable optimization problems. While the traditional combinatorial optimization is an NP-hard problem, quantum optimization leveraging quantum mechanisms is expected to achieve a super-polynomial advantage for complicated combinatorial optimization problems.

Quantum approximation optimization algorithm (QAOA) is one of the most prominent quantum optimization algorithms[97]. As established in ref. [98], the solution to a quadratic unconstrained binary optimization (QUBO) problem is equivalent to the ground state of a corresponding Ising Hamiltonian. Several methods have been developed to establish the Hamiltonian of the Ising model[99]. QAOA aims to find feasible solutions to the QUBO problems by minimizing the expected value of the Hamiltonian. The expectation is taken with respect to quantum states, which, in turn, are obtained by rotating the initial state that entangles all possible states with equal probabilities. The minimum expected value of the Hamiltonian can therefore be obtained by obtaining feasible rotation angles by using a classical optimizer. Due to the limited number of qubits, the scalability of quantum optimization in the centralized mode might be restricted. To resolve this issue, the quantum distributed optimization idea is proposed, where QAOA can serve as an essential sub-routine for calculating sub-problems[97].

Refs. [17, 18] explore the efficacy of quantum optimization and its distributed variants in power system unit commitment (UC). Originally, the UC problem consists of continuous variables for the active power output of generators and binary variables for the commitment status of generators. To fit the requirement of QAOA, refs. [17, 18] translate the UC model into sub-problems, where the binary variables are formulated by QUBO sub-problems. Then, a multi-block decomposition of the alternating direction method of multipliers (ADMM) is used for coordinating different sub-problems. The overall procedure of the quantum ADMM (Q-ADMM)-enabled UC is as follows. First, initialize the iteration index s, decision variables, penalty factor, and stopping criteria. Second, solve the QUBO sub-problem to update the binary decision variables. Then, solve the continuous sub-problem to update the continuous decision variables. Following, update the dual variables of Q-ADMM based on the obtained decision variables. The above sub-routines interact until convergence, which returns the optimized solution. Figure 5 depicts the schematic diagram of the quantum distributed UC for power systems. More details are presented in refs. [17, 18].
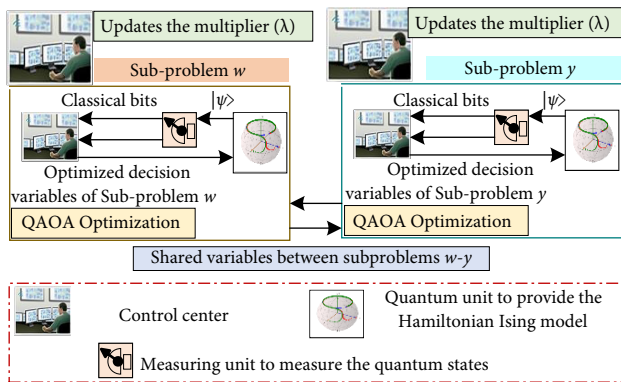

Fig. 5 Schematic of distributed optimization of power systems.

Ref. [19] proposes another path towards the quantum optimization-enabled UC. It incorporates the QAOA approach into the surrogate Lagrangian relaxation (SLR) method[19]. Specifically, the UC problem is decomposed into the time-unit-split binary sub-problems and continuous sub-problems, which are respectively solved through quantum and classical computing. Meanwhile, sub-problems are effectively coordinated by updating the Lagrangian multipliers. Because of the adoption of the SLR philosophy, QSLR

achieves improved convergence performance with the integration of the contraction-mapping stepsize[19].

3.2 Power system stability assessment via quantum machine learning

Transient stability assessment (TSA) is a long-standing obstacle for power system operations. It evaluates whether a power system can reach a steady-state operating point after large disturbances[100]. In modern power systems, the high penetration of renewable energies and electronic devices may potentially induce unprecedented stability risks, bringing ever-strict requirements for scalable and efficient TSA. Classical TSA methods mainly rely on numerical integration to perform time-domain simulations of power systems, which can be extremely time-consuming for complicated power systems. Data-driven methods provide an alternative path. By employing offline-trained neural networks to establish stability rules, data-driven TSA can be potentially scalable to realize online stability verification. A plethora of classical data-driven algorithms have been applied to power systems stability analysis, such as kernel machines[101], deep neural networks[102], and reinforcement learning[103].

Quantum machine learning (QML) is a confluence of quantum computing and machine learning[104–106]. In recent years, there has been an explosive growth of QML algorithms in supervised learning, unsupervised learning and even reinforcement learning. Targeting different learning purposes, quantum versions of various machine learning techniques have been proposed, such as quantum principal component analysis[107,108], quantum kernel estimation[109,110], quantum classifier[111], quantum clustering[112], quantum generative adversarial network[113,114], quantum Boltzmann machine[115], quantum Q-learning[116], etc. Classically, machine learning involves significant computational burden for inner production and its performance heavily depends on the choice of learning models. Since quantum states can be efficiently operated in the Hilbert space and are capable of representing entangled correlations, QML is promisingly powerful for data processing and model training in ultra-high dimensional space that are intractable for classical algorithms[107,110,117,118].

Tackling the power system stability issues, ref. [20] establishes a QML-enabled TSA approach. For an arbitrary power system, its transients model can be functionally formulated as a set of nonlinear differential algebraic equations (DAE) $F(\dot{Z}, Z) = 0$, where Z denotes the system states including both the differential variables and algebraic variables. For most DAE systems, the orbit is uniquely determined by the initial point. According to the stability region theory, if the post-disturbance state is within the stability region of a stable equilibrium point (SEP), the system will finally reach a steady-state, i.e. the SEP. Therefore, it inspires various data-driven TSA methods[119,120] to establish a classification-based mapping between the post-disturbance states and the stability judgment. QTSA inherits such a learning-based idea and novelly exploits the expressibility and scalability of QML techniques for power system TSA.

The following briefly introduces the basic idea of QTSA[20]. A unique feature of QTSA is that it embeds the transient stability features (i.e. the post-disturbance states such as frequency, active and reactive power, voltages) into quantum states through a VQC (see Figure 6):

$$|\varphi\rangle = U_{\text{TSA}}(\boldsymbol{p}, \boldsymbol{Z})|0\rangle. \tag{9}$$

Here, U_{TSA} denotes the VQC; $\boldsymbol{p}$ denotes the parameters to be optimized; $\boldsymbol{Z}$ denotes the power system stability features, which
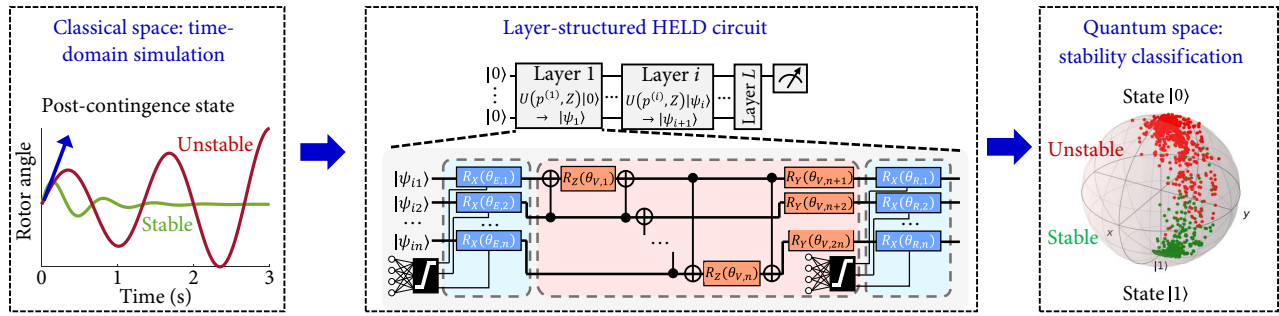
Fig. 6 Schematic of quantum machine learning-enabled power system stability assessment.

are inputs of the QML algorithm. Therefore, the transient stability assessment is performed based on the measurements in the Hilbert space. Consequently, a cost function is constructed as the conformance between the true stability judgment and the prediction from QTSA:

$$\min C(\boldsymbol{p}) = \sum_{i=1}^{n} c(y_i(\boldsymbol{p}, \boldsymbol{Z}_i)). \tag{10}$$

Here, $\boldsymbol{Z}_i$ and y_i respectively denote the stability features and quantum stability judgment of the i-th sample. As a variational quantum algorithm, QTSA also employs a hybrid quantum-classical framework for the quantum circuit training[20].

While the parameters of QTSA-VQC can be optimized by Eq. (10), the structure of the VQC should be pre-determined. Designing an effective yet noise-resilient VQC is crucial for the success of QTSA. On the one hand, QTSA requires high-expressibility quantum circuits because of the strong nonlinearity of power system stability issues. On the other hand, only low-depth quantum circuits can be executed considering the restrictions of current NISQ devices. However, even in the quantum area, it is still an open problem to design an expressive VQC that can well represent the solution space of a specific problem. Regarding VQC design, ref. [121] quantitatively analyzes the expressibility of different types of ansatzs by assessing their capability to explore the Hilbert space. Ref. [122] demonstrates that certain single-qubit rotations can be reduced without significantly deteriorating the performance of quantum circuits. Ref. [123] shows that while increasing the circuit depth may enhance the expressibility, it unavoidably hurts the noise-resilience of the algorithm. Specifically for the QTSA issue, ref. [20] designs a high expressibility, low-depth circuit (HELD) by integrating both the quantum operators and the classical activation functions (see Figure 6). The authors also demonstrate that generic expressibility indices, such as the Kullback–Leibler (KL) divergence-based ones, may not provide a reasonable assessment of quantum circuits under specific objectives because they mainly focus on the uniform exploration of the solution space without considering the probability distribution.

Real-hardware testing is an indispensable step in ensuring the efficacy of quantum computing techniques in noisy quantum environments. To this end, ref. [20] has established a systematical scheme for evaluating QTSA's performance on real quantum machines. Four different angles are recommended, including accuracy, expressibility, fidelity, and noise-resilience, to comprehensively study whether the quantum circuit can accurately and effectively express the transient stability region and whether the QTSA judgments are trustworthy, especially in noisy quantum environments. Such indices can also be expanded to evaluate other quantum-inspired power system analytics. Experiments show that QTSA achieves an accuracy over 98% even for large-scale systems such as a 300-bus power grid and remains satisfactory noise-resilience, which indicates its potential for the NISQ appli-

cations. Some research has also demonstrated that QML can be potentially more expressible for complicated data relationships, e. g., achieving a comparable accuracy against classical machine learning while saving more parameters for the neural network[124].

3.3 Resilient control for power systems via quantum distributed control

The increasing integration of DERs is challenging the control and synchronization of modern power systems[125, 126], not only because they are highly uncertain and inverter-interfaced, but also because of their distributed natures and the privacy requirements. Distributed control has become the most promising solution for resilient operations of power systems with high penetration of DERs as it offers flexible plug-and-play architecture[127–129]. Distributed control of power systems can be functionally described as a network of differential equations over a simple, connected graph $G = (V, E)$, where the node-set V represents n DERs, and the edge-set E depicts the allowed communication among DERs.

Although distributed control strategies can significantly improve power system resilience, the vulnerability of communication networks to cyberattacks induces potential risks for third-party agents to drive the system toward inconsistent performance and instability. Addressing cybersecurity challenges in distributed control is an active area[130]. However, the existing solutions may become obsolete due to the development of supercomputers and the emergence of quantum computers[131].

Inspired by quantum mechanisms, the next generation of communication technologies is aiming to leverage the quantum nature of light, which gives rise to novel capabilities unattainable with classical transmission media[132–141]. In these schemes, information is encoded in the particle's quantum state, which cannot be copied, and any attempt to do so can be detected. Therefore, the critical aspect is unconditional information security which is impossible with classical information processing[142]. The most exciting benefit of using quantum-secured information is that the lifetime of the security is "infinite", i.e. it will be secure against any future advance in computation capability[142].

Refs. [22, 23] are a confluence of power system distributed control and quantum communication. Leveraging the potential to establish synchronization throughout a network of quantum nodes via exchanging qubits, a novel quantum distributed controller (QDC) has been proposed. The overarching goal of QDC is to construct a network of differential equations to control a network of DERs. In contrast to classical synchronization, QDC encodes the information into quantum states and exchanges them among the nodes over quantum channels (see Figure 7). In this way, the control process, such as power/current sharing and frequency/voltage regulation, is provably guaranteed and secured.

In the QDC design, each DER is equipped with a quantum computing device, which prepares a quantum state and seeks a
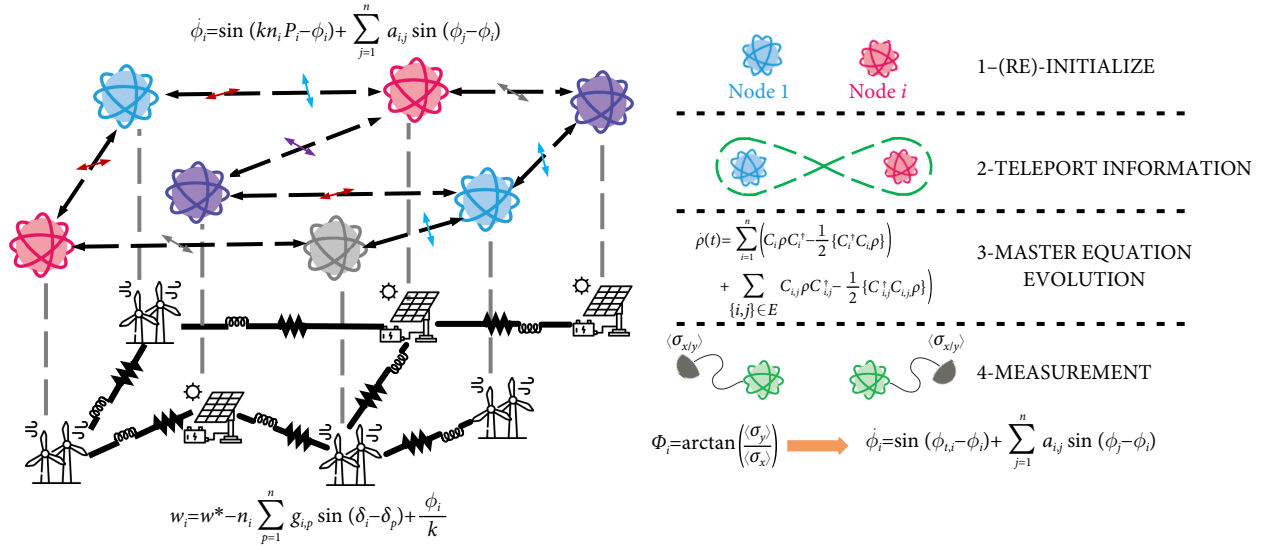
$$\dot{\phi}_i = \sin(kn_iP_i - \phi_i) + \sum_{j=1}^{n} a_{ij}\sin(\phi_j - \phi_i)$$

$$\dot{\rho}(t) = \sum_{i=1}^{n}\left(C_i\rho C_i^\dagger - \frac{1}{2}\{C_i^\dagger C_i, \rho\}\right)$$

$$+ \sum_{\{i,j\}\in E} C_{i,j}\rho C_{i,j}^\dagger - \frac{1}{2}\{C_{i,j}^\dagger C_{i,j}, \rho\}$$

$$\Phi_i = \arctan\left(\frac{\langle\sigma_y\rangle}{\langle\sigma_x\rangle}\right) \quad\longrightarrow\quad \dot{\phi}_i = \sin(\phi_{t,i} - \phi_i) + \sum_{j=1}^{n} a_{i,j}\sin(\phi_j - \phi_i)$$

1–(RE)-INITIALIZE

2-TELEPORT INFORMATION

3-MASTER EQUATION EVOLUTION

4-MEASUREMENT

Node 1   Node i

$$w_i = w^* - n_i\sum_{p=1}^{n} g_{i,p}\sin(\delta_i - \delta_p) + \frac{\phi_i}{k}$$

Fig. 7 Coupling of power grids to the network of quantum controllers (left), and schematic depiction of the QDC algorithm (right)[22].

consensus among all the quantum devices in a distributed manner. The following briefly introduces the mathematical formulation of QDC. The state of each quantum device can be described by a positive Hermitian density matrix ρ. Since synchronization requires interaction among all quantum devices, it is assumed that each device has access to its neighbors' (quantum) information. Let $|\psi\rangle = |q_1 q_2 \cdots q_n\rangle$ be the state of the whole quantum network and $\rho = |\psi\rangle\langle\psi|$. The following quantum master equation has been introduced to construct the network of differential equations:

$$\dot{\rho}(t) = \sum_{i=1}^{n}\left(C_i\rho C_i^\dagger - \frac{1}{2}\{C_i^\dagger C_i, \rho\}\right)$$
$$+ \sum_{\{ij\}\in E}\left(C_{ij}\rho C_{ij}^\dagger - \frac{1}{2}\{C_{ij}^\dagger C_{ij}, \rho\}\right), \tag{11}$$

where C_i and C_{ij} are unitary jump operators described by rotation-Z, $R_z(\varphi)$, and swapping operators, respectively. $R_z(\varphi)$ is the single-qubit rotation-Z operator by an angle φ radians around the Z-axis. The swapping operator specifies the external interaction between quantum computing devices i and $,j$ forming a connected communication graph.

In an abstract sense, the goal is to encode the classical information into the φ angle of the qubits, teleport information throughout the network, evolve the state of each node's qubit for one time step utilizing its received information (i.e. according to the synchronization protocol Eq. (11)), and finally retrieve the classical information from the φ angle of the qubit. Such classical information will be used as the control signal later.

Since the dynamic of the φ angle is the dynamic of interest, at each time step, qubits are (re)-initialized on the equator of the Bloch sphere. Hence, as the first step, qubits are initialized as points on the first quarter of the equator of the Bloch sphere. Next, quantum information is transmitted throughout the network such that each quantum node receives the quantum information from its adjacent nodes. After each node i receives the quantum information of the adjacent nodes, the target state for node i ($\varphi_{t,i}$) is integrated into the synchronization protocol Eq. (11) through the rotation-Z operator such that it acts as a pinning term for the synchronization protocol. Then, the quantum state of node i is evolved according to the master Eq. (11) for one time step δt using the swapping and rotation-Z operators.

After the master equation evolution, the processed information

needs to be retrieved from the qubit by measuring the φ angle. Generally, the master equation results in states becoming more mixed; however, the system is allowed to evolve in a short time and is re-initialized in a product of pure qubit states. Thus, the projection of the state vector (qubit i) on X and Y axes of the Bloch sphere, given by the expectations of Pauli-X and Pauli-Y, can be obtained as $\text{tr}(\rho\sigma_x) = \cos\varphi_i$ and $\text{tr}(\rho\sigma_y) = \sin\varphi_i$, respectively.

If the procedure of the master equation evolution is repeated in a short duration, the approximated equations for φ_i's in the limit $dt \to 0$ can be obtained. Note that for an observable A, $\frac{d}{dt}\langle A\rangle = \frac{d}{dt}\text{tr}(\rho A) = \text{tr}(\dot{\rho}A)$. Hence, utilizing $\text{tr}(\rho\sigma_x)$, $\text{tr}(\rho\sigma_y)$ and Eq. (11), the dynamic of the phase angles φ_i can be obtained as follows:

$$\dot{\varphi}_i = \sin(\varphi_{t,i} - \varphi_i) + \sum_{j=1}^{n} a_{ij}\sin(\varphi_j - \varphi_i), \tag{12}$$

where a_{ij} are the entries of the $n \times n$ adjacency matrix of graph G, denoted as A. $a_{ij} = 1$ if $C_{ij} \neq 0$ and $a_{ij} = 0$ otherwise. In Eq. (12), the pinning term, $\sin(\varphi_{t,i} - \varphi_i)$, forces the phase φ_i to stick at the value $\varphi_{t,i}$ and the coupling mechanism, $\sum_{j=1}^{n} a_{ij}\sin(\varphi_j - \varphi_i)$, synchronizes the entire system.

Eq. (12) offers a universal form for quantum distributed control of dynamical systems. It can be applied to various power system control problems. Ref. [22] shows how Eq. (12) can be utilized as a secondary control for distributed frequency control of AC microgrids, which allows microgrids to be profited from quantum communication advantages. Classically, the distributed frequency control in AC microgrids regulates the system frequency to a rated value and guarantees active power sharing among the DERs, whose dynamics can be described by:

$$\omega_i = \omega^* - n_iP_i + \Phi_i,$$
$$\dot{\Phi}_i = f(\Phi_i, P_i, \Phi_j, j \in N_i), \tag{13}$$

where ω_i and P_i respectively represent the frequency and active power injection of DER; ω^* is a nominal frequency; and n_i is the droop gain. Φ_i is the secondary control variable, whose dynamics is a function of its current value, P_i, and its neighbors' Φ_j. Therefore, Φ_i will eventually evolve toward an (weighted) average of its neighbours' Φ_j such that all control variables Φ_i converge to the

common value $\Phi_i = \Phi_j = n_i P_i$. As can be seen, Eq. (12) is a synchronization rule consisting pinning terms and coupling mechanism. Therefore, in order to apply the QDC, we need to define the target which is done through scaling $n_i P_i$. Hence, the developed QDC for AC microgrids is formulated as follows :

$$
\begin{aligned}
\omega_i &= \omega^* - n_i P_i + \frac{\varphi_i}{k}, \\
\dot{\varphi}_i &= \sin\left(k n_i P_i - \varphi_i\right) + \sum_{j=1}^{n} a_{ij} \sin\left(\varphi_j - \varphi_i\right),
\end{aligned}
\tag{14}
$$

where φ_i / k is the secondary control variable. Ref. [23] further discusses more applications of QDC. It demonstrates the cyber-security for QDC, where the distributed control of AC and DC microgrids can be provably secured by encoding the control signals into quantum qubits and exchanging information via quantum channels among participating DERs.

In summary, due to the superposition feature of qubits, QDC provides a foundation for allowing more enhanced quantum-secure distributed control for power systems through randomizing the θ angle of qubits in the initialization step. Such a control scheme is unprecedentedly secured such that even if a third-party agent can measure the exchanged qubits, the measurement outcomes would be some random values that do not reveal information to the eavesdropper.

3.4 Power system scenario generation via quantum generative learning

With the increasingly high penetration of renewable energy in power grids nowadays, renewable scenario generation that captures renewable uncertainties has been indispensable in power system planning, scheduling, and operations[143,144]. Traditional methods are model-based. That is, a specific statistical model is utilized to find the probabilistic distribution of renewable scenarios, and samples are extracted from the distribution to generate new scenarios. Model-based methods are easy to comprehend and operate. However, they are difficult to adapt to the time-varying weather and are inflexible in scaling due to the complexity and non-linearity of renewable scenarios[145,146].

Machine learning methods provide a model-free path[147,148]. They use historical data to generate new scenarios without specifying a model or a distribution. An example is the generative adversarial network (GAN)[149]. Two neural networks are involved in a GAN, i.e. a generator and a discriminator. The generator generates fake scenarios to fool the discriminator, and the discriminator tries to distinguish between actual samples and fake ones. The two neural networks contest with each other in a game until a Nash equilibrium is reached. However, while GAN provides a flexible and scalable solution, training GAN models may require an unexpectedly long time, especially when a large dataset is used.

A quantum version of GAN is the quantum generative adversarial network (QGAN). It uses two quantum components to represent the generator and the discriminator, respectively. Through the quantum-mechanical phenomenon, it promises to reduce computational complexity. However, many existing works on QGAN only focus on simple cases where limited input data points are involved. They use amplitude or angle encoding to represent classical data in a quantum circuit[150]. Amplitude encoding encodes data into amplitudes of a quantum state. Therefore, n qubits can store 2^n data points. However, the disadvantage is that the circuit depth will be significant with a giant n, and the circuit will be challenging to construct. Angle encoding encodes each data point into a rotation angle. However, when n is large, the circuit

also becomes complicated. Using a single QGAN to cover all the input features for renewable scenario generation with a large dataset is difficult and impractical.

To bridge the gaps, ref. [21] presents a Multi-QGAN framework. Instead of relying on a single QGAN, it uses multiple QGANs to generate scenarios, thus avoiding using a complicated QGAN. Specifically, n QGANs are constructed for n classical data points in the dataset. The value of each classical data point is first normalized and is then represented by a rotation angle. In Multi-QGAN, QGANs are trained one by one following the sequence of the data point. Compared with the single QGAN, Multi-QGAN has a simpler circuit topology for each QGAN, is easier to construct, and is more scalable and flexible.

While Multi-QGAN provides a promising way to generate scenarios, it neglects the correlation between neighboring data points, which is considerable in many cases, e.g., in a time-series dataset. To address this issue, ref. [21] further presents a correlation-based Multi-QGAN (CMulti-QGAN) method. Unlike Multi-QGAN, which associates each QGAN with one input data point, CMulti-QGAN generates each scenario using the corresponding true value and its neighbors. Specifically, when the ith quantum discriminator is trained, its cost function is not only determined by the ith real quantum data source's output and the ith quantum generator's output, but also is associated with the outputs of the ith generator's neighbors. Data from real photovoltaic systems in the State of Connecticut are collected for studies. Results demonstrate the effectiveness and robustness of Multi-QGAN and CMulti-QGAN and validate the superiority of CMulti-QGAN over Multi-QGAN.

It is worth noting that ref. [21] only presents a preliminary study of using QGAN for renewable scenario generation. While it provides a quantum way to generate renewable scenarios effectively, it is still under investigation that the computational speed can be improved. More research needs to be conducted to further enhance the performances of Multi-QGAN and CMulti-QGAN, especially with a larger dataset.

4 Quantum communication for provably-secured power systems

While quantum computing promises to address power system problems, it poses security threats to power system communications. The security of public-key cryptographic systems such as the Diffie–Hellman key exchange (DH)[151] and Rivest–Shamir–Adleman (RSA)[152] heavily relies on the computational difficulty of specific mathematical problems such as discrete logarithm and factoring problems[38]. These problems are at high risk of being addressed by quantum computers. For instance, it has been shown that Shor's quantum factoring algorithm can effectively break RSA cryptosystems with the help of enough qubits[153]. Developing provably-secured power systems in the quantum era has become essential and urgent. We refer the readers to the book by Nielsen and Chuang[9] for a pedagogical introduction to this subject.

A powerful solution to securely transfer information between two remote parties in the quantum era is quantum communication. It uses the fundamental laws of quantum physics, which provide a more solid foundation in the quantum era than mathematical assumptions. According to the quantum-mechanical property, measuring an unknown quantum state will, in general, change that state. Therefore, the two parties can detect when an adversary is trying to gain knowledge of the qubits. After proper post-pro-

cessing procedures, including error correction and privacy amplification, the final information shared between two parties will be information-theoretically secure[154]. This section reviews quantum communication-related research in power systems, including quantum key distribution (QKD) and quantum networks.

4.1   Quantum key distribution

QKD is a key-growing approach that generates and distributes keys for two communicating parties[155]. A QKD system typically consists of a quantum channel and a classical one. The quantum channel transmits qubits between two communicating parties for generating raw keys. The classical channel is used to conduct post-processing procedures for distilling final secret keys. The final keys are utilized to encrypt and decrypt data messages transmitted through the classical channel. Note that QKD only generates private keys; encryption methods are still classical, and the encrypted data messages are sent over the classical channel. In reality, QKD can be combined with symmetric-key encryption methods like the One-Time Pad[156] or the Advanced Encryption Standard[157]. A few research efforts have recently been conducted to integrate QKD systems into power grids.

Ref. [24] is the first work that develops a QKD-enabled quantum-secured microgrid. It devises a QKD simulator in Python to simulate QKD protocols with great flexibility to update QKD parameters. The formulations of the decoy-state BB84 QKD protocol, i.e. a practical, mature, and widely used scheme to implement QKD, are presented in detail. This simulator simulates the probabilities of various events occurring and outputs the number of final keys generated in real-time. Then, the simulator is integrated with the real-time digital simulator (RTDS), i.e. a real-time power system simulator. Specifically, a medium-voltage microgrid system is used as the test system. It comprises a 5.5 MVA diesel generator, a 1.74 MW PV system, a 2 MW doubly-fed induction generator wind turbine system, a lithium-ion battery storage, seven loads, and a control center. The control center receives load information from loads and sends control signals (i.e. the real and reactive power references of the P-Q control) to the local controller of the battery storage, which employs the P-Q control to regulate its output power. The QKD simulator is implemented in the control center running on a remote server. It continuously generates key bits and stores them in a key pool. When a control signal needs to be sent out, a specific number of key bits (e.g., 64 bits) are consumed from the key pool to simulate the encryption process. In this way, the quantum key generation and key consumption are integrated successfully. The QKD simulator determines the key generation speed, and the key consumption speed is affected by the actual data transmission between the control center and the battery storage.

The QKD-enabled quantum-secured microgrid is extended to quantum-secured networked microgrids in ref. [25]. Three microgrids are established. Each microgrid has a control center, which communicates with one of the local controllers in the same microgrid, and communicates with the neighboring two control centers. The user datagram protocol is utilized for data transmission. The three control centers run on the same server with a specific IP address. Three port numbers are assigned to each control center. One is used to receive data (i.e. the load information) from the RTDS hardware, and the other two are used to obtain data (i.e. control signals) from the neighboring control centers. Six QKD systems are installed, three of which are for the communications between control centers and their local controllers, and three are for the communications between neighboring microgrids. This design successfully combines multiple QKD systems, making the

system more complicated. Therefore, more research work can be conducted with this system. For instance, researchers can investigate the impact of attacks on multiple QKD systems and the corresponding defending strategies.

While QKD offers a promising solution to securely distribute keys between two distant parties, itself is vulnerable to denial of service (DoS) attacks. To address this issue in QKD-enabled networked microgrids, ref. [26] presents a programmable quantum networked microgrids architecture, where software-defined networking (SDN) is utilized to manage the communication network. SDN is a practical and promising technique to achieve a fast and flexible communication environment[158–160]. The decoupling of control and data planes and the centralization of the control logic in the SDN controller make each SDN switch a simple forwarding device. The SDN controller obtains a global knowledge of network states, and enables a rapid development of sophisticated applications. In ref. [26], a software-defined adaptive post processing approach, a two-level key pool sharing stategy, and an SDN-enabled event-triggered communication scheme are developed to mitigate the impact of DoS attacks through programmable post processing and secure key sharing among QKD systems.

A brief review of the research work on QKD-enabled microgrids is presented in ref. [27]. The current status of developing quantum-secured microgrids and some future perspectives are discussed. The work of developing quantum-secured power grids has just started. More work needs to be conducted from both quantum cryptography and power grid sides. For instance, it is crucial to enhance QKD's resilience against disturbances and attacks, e.g., side-channel and DoS attacks. The experimental demonstration of QKD in power grids is essential but has not yet been carried out. Proper strategies are desired to make QKD more practical in power grids. The field is still in its infancy with substantial growth potential.

4.2   Quantum networks

There are two significant technical challenges impeding the broad adoption of QKD in power grids. One is the distance limitation issue, meaning that the QKD's key generation rate is reduced largely when the communication distance increases. This is particularly unacceptable in a power grid with a considerable distance between two communicating parties and a high data transmission rate. The other critical issue is that a QKD system is typically point-to-point. Therefore, using QKD systems becomes impractical in a power grid with intensive communicating devices.

A promising way to overcome these limitations is establishing a quantum network. When a number of quantum devices are interconnected, the quantum network greatly extends the communication distance, and maximally utilizes quantum resources to offer a more flexible environment for plug-and-play communicating devices. The realization of quantum networks requires quantum communication equipped with quantum cryptographic protocols, where two techniques stand out above all others: QKD and quantum direct communication[135].

A QKD-based quantum network architecture designed for power grids is presented in ref. [28]. It is also shown in Figure 8. Quantum repeaters are used to extend the distance between two communicating parties. A quantum repeater comprises two quantum sources and a Bell state measurement (BSM) device. Each quantum source independently generates two entangled qubits; one qubit is sent to one communicating party, and the other is sent to the BSM device. The BSM device publicly announces the measurement result of the two received qubits. If
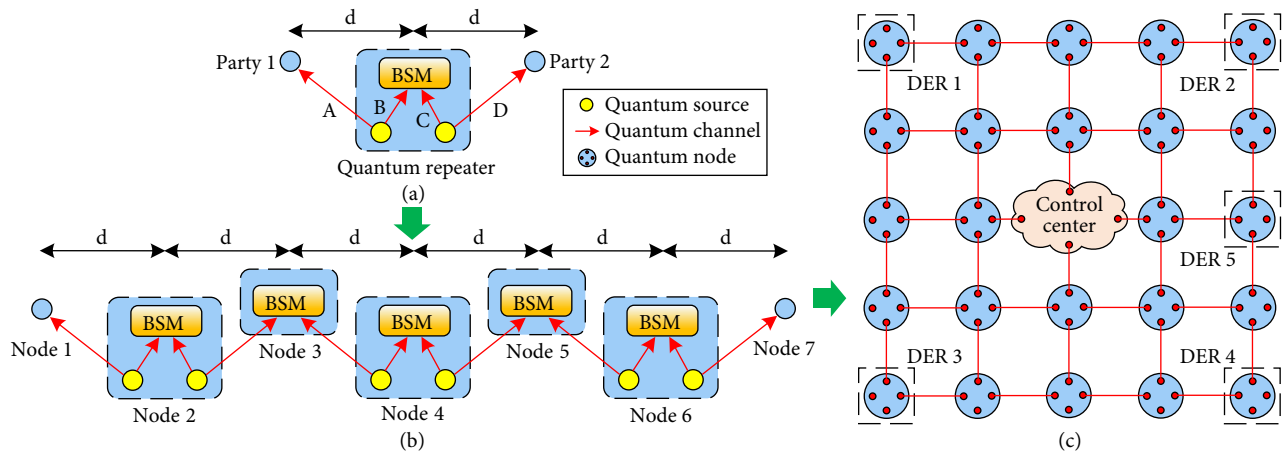
Fig. 8   A QKD-based quantum network using quantum repeaters [28]. **(a)** Illustration of a quantum repeater. **(b)** A chain of quantum repeaters. **(c)** The quantum network. BSM: Bell State Measurement. Quantum node: A quantum repeater (including a node with only the measurement capability) or a communicating party. d is the distance between two quantum nodes.

the measurement succeeds, the two qubits sent from the two quantum sources to the BSM device will be entangled. As a result, the two qubits forwarded to the two communicating parties will also be entangled. If multiple repeaters are connected, forming a chain of repeaters, the communication distance will be further extended.

After the two communicating parties share a series of pairs of entangled qubits, they can use a specific QKD protocol (such as the E91 QKD protocol[161]) to generate secret keys. With this quantum network architecture, ref. [28] develops a quantum network simulator, namely, QNSim, to simulate the performance of the network. Unlike other quantum network simulators, QNSim is easy to implement, allows for real-time simulation, and is flexible in altering network parameters and topologies. A routing strategy is also integrated into QNSim to simulate the quantum routing process. A quantum network-based power grid (QNetGrid) testbed is further designed to contain quantum communication, quantum routing, real SDN switches, and real-time networked microgrids operations.

In addition to QKD, quantum direct communication is another form of quantum communication. Unlike QKD, which generates secret keys and uses keys to encrypt data messages, quantum direct communication directly transmits confidential information over the quantum channel. Ref. [29] is the first work that develops a quantum direct communication-based quantum network for electric grids. The devised architecture has four layers: a quantum direct communication network layer for conducting quantum communication, a physical layer of the electric power system, a classical network layer operating in parallel with the quantum direct communication network for exchanging information, and an application layer for various power grid and quantum applications. The designed network, which supports teleportation and superdense coding protocols, comprises the following components: (a) quantum nodes to conduct communication, (b) entanglement generators to create EPR pairs, (c) quantum memories to store qubits, (d) quantum channels to distribute EPR pairs, and (e) quantum measurement devices to measure entangled states.

A quantum direct communication-enabled power grid testbed is established in ref. [29]. The network simulator is developed based on SQUANCH[162] and runs on a server, which functions as the grid control center. The control center receives data from and sends control signals to electric grids. The network simulator simulates the quantum data transmission process between the physical grid and the control center. A typical microgrid system is developed in MATLAB/Simulink running on a virtual machine. The microgrid system communicates with the control center using the User Datagram Protocol. With this testbed, the network performance under different power grid scenarios can be evaluated.

5   Conclusions and outlook

Quantum computing has been recognized by the National Science Foundation as part of the ten strategic research areas in the U.S.[163]. The pioneering work in quantum computing in the past decades, especially our breakthroughs in quantum-engineered smart grids, has helped ignite strong interests from the electricity sector and federal agencies such as the U.S. Department of Energy. This paper summarizes the existing research in developing quantum computing algorithms which will open the door for developing various power system solutions. By exploiting the inherent characteristics in power grid problems such as power flow, eigenanalysis, and real-time electromagnetic transient analysis, new and customized linear and non-linear quantum circuits and quantum solvers can be devised for both NISQ and fault-free quantum computer environments. Regarding the stringent cybersecurity and stability requirements from micro- and macro- grids and their controls, novel hybrid quantum and classical cyber-physical architectures will be established, and the quantum networking testbed, which has been successfully synchronized with the real-time digital power system simulators, can be used to optimize quantum device performances against various cyber attacks. Further, through the deep understanding of the grid operation and control mechanisms, novel quantum protocols requiring minimum quantum hardware (leading to low capital expenditure) will be developed, which are expected to be robust and self-adaptive to the frequency changes and reconfigurations in today's cyber-physical power grids. Enabled by quantum networking and quantum memory-assisted quantum nodes, various quantum controls will be devised to tackle the long-standing challenges in distributed algorithms such as cyber-vulnerability and slow convergence, leading to quantum (and hybrid) information fusion schemes and fast distributed approaches resilient against unstable communication, asynchronous clocks, and adversary attacks.

On the other hand, despite the experimental breakthroughs toward quantum technologies[7, 42], the jury is still deciding whether quantum supremacy has been achieved in real-world applications.

To demonstrate quantum supremacy, progress has been made continuously from different aspects. For instance, for NISQ devices, hybrid quantum-classical variational approaches provide promising solutions in specific applications[32], including engineering and optimization problems. The quality of qubits and quantum gates has been continuously improved. In addition, quantum error correction, a promising technique that leads to fault-tolerant quantum computing, will likely bring fundamental breakthroughs to demonstrate quantum supremacy. Further, our previous experience has shown that leveraging power system characteristics in quantum algorithm design can, in turn, enhance the algorithm's performance[15].

The recent progress in quantum computing has also attracted forward-thinking power utilities[164] to explore potential quantum applications in situational awareness, secured quantum networks, emergency preparedness, volatile renewable forecast, security-constrained unit commitment, service restoration, etc. Meanwhile, there exist several challenges in demonstrating practical quantum computing use cases in the field: (1) there is a lack of talents that have cross-domain knowledge needed for connecting quantum computing to power systems; (2) no standardized quantum programming environment has been established, which increases the opportunity cost for implementing a viable quantum computing application; (3) power and energy industry still needs demonstrations of quantum supremacy with a few practical applications. Therefore, the collaborations between universities, national labs, vendors and utilities are key to finding practical use cases for quantum computing algorithms and to address the aforementioned challenges. The ecosystem of quantum-engineered smart grids initiated by Stony Brook University is swiftly growing, and this will strongly support the multi-sector efforts in increasing participation of K-12 and university students as well as the general public in quantum computing workforce training.

In general, creating practical and replicable quantum algorithms to resolve the traditionally intractable computational problems and to support fast and resilient power system operations will be the central theme for quantum computing in power systems. Quantum grids analytics toolboxes allowing the power industry to exploit quantum supremacy in large power systems operations are in demand. With the swift growth of quantum computer capabilities, we are confident that the theoretical potentials of the NISQ algorithms will be unlocked in this decade, which will be able to ensure ultra-fast real-time decisions of large power systems, minimize customer outages and drastically increase the flexibility, responsiveness, and resilience of critical power infrastructures under small and large disturbances, as well as extreme events.

Acknowledgements

Additional information

Declaration of competing interest

The authors have no competing interests to declare that are relevant to the content of this article.

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