Unsupervised Learning Strategy for Direction-of-Arrival Estimation Network

Ye Yuan, Shuang Wu, Minjie Wu, and Naichang Yuan

***Abstract*—In this letter, we proposed a novel unsupervised learning strategy for direction-of-arrival (DOA) estimation network. Inspired by the sparse power spectrum and ℓ_1-norm optimization, we develop a novel loss function to cooperate with the estimation network. Unlike the prior DL-based methods, the proposed method does not need any manual annotations for training and validation datasets. Compared with state-of-art methods, the proposed method can automatically increase the degree of freedom of the array without further pre-processing on the covariance matrix of array observation data. Moreover, the proposed method can obtain clear spectrum and precise DOAs under harsh estimation environments.**

***Index Terms*—Artificial intelligent, deep neural network, direction-of-arrival estimation, unsupervised learning.**

I. Introduction

DIRECTION-OF-ARRIVAL (DOA) estimation is widely studied by researchers in the past decades [1]–[6]. Many excellent estimation methods (such as multiple signal classification (MUSIC) algorithm [7] and estimating signal parameter via rotational invariance techniques (ESPRIT) algorithm [8]) have been proposed and used in morden rada, sonar, and wireless communication systems.

Most classical estimation methods are subspace-based methods, which rely on the prior information about the estimation environment [9], [10]. To develop efficient methods with better adaptation, researchers have already introduced the sparsity-inducing and ℓ_p-norm-based techniques into the DOA estimation field [11]–[13]. Although these methods have made some significant improvements, the optimization of ℓ_p-norm function can easily converge into local minimum [14]. Moreover, the sparsity-inducing and ℓ_p-norm-based estimation methods require a long and iterative process, which makes these methods lose their real-time capability [15].

With the rapid development of artificial intelligence (AI), researchers have designed many deep-learning-based (DL-based) networks to achieve data-driven estimation approaches [16]–[18]. There are generally two kinds of models to realize DL-based estimation: the classification model and the regression model [19], [20]. The classification models (such as deep neural network (DNN) group [14] used to against array imperfections and convolutional neural network (CNN) designed for multi-speaker DOA estimation [21]) divides the angle space into many discrete subregions, then use the classifier networks to estimate DOAs in each region. The regression models (such as the cascaded neural network (cascaded NN) [20] and the CNN used for broadband DOA estimation [22]) regard the DOA estimation problem as a regression problem, which can be solved by finding the optimal solution of DOAs. In [23], researchers propose a sparse-prior-based deep convolution network (DCN) estimator. This method combines the sparsity-inducing technology and regression neural network. It can recover the clear power spectrum of signals.

However, the existing DL-based estimation methods are all based on supervised learning [24], [25]. During training the networks, these methods require us to attach the correct labels for training and validation datasets [26]. In reality, the actual labels (such as angle and power spectrum) are hard to collect. This defect limits the effectiveness of the prior DL-based estimation methods. Inspired by [23], [15], and [27], we proposed a novel unsupervised learning strategy for DL-based estimation network. The proposed unsupervised-learning-based network can directly operate on the covariance matrix of array observation data without any further pre-processing or manual annotations for datasets. Moreover, by using the Khatri-Rao (KR) product [27], the proposed method can automatically increase the degree of freedom (DOF) of the array, and it can achieve multi-DOA estimation without prior information about the signal number.

The rest of this letter is organized as follows. First, we derive the sparse form of the vectorized covariance matrix and explain the effectiveness of the KR product in Section II. We then propose a ℓ_1-norm-based unsupervised estimation network in Section III. We test the estimation performance of the proposed method and make comparisons with some prior estimation methods in Section IV. At last, we summarise the contributions of this letter in Section V.

II. Problem Statement

Assuming that K narrowband far-field signals impinge on an uniform linear array (ULA). The ULA is assumed to have M sensors with inter-sensor space of d. We use $\boldsymbol{\theta} = [\theta_1, \theta_2, \ldots, \theta_K] \in \mathbb{R}^K$ to denote the direction-of-arrival (DOA) of signals $\boldsymbol{s}(n) \in \mathbb{C}^K$. The array observation data $\boldsymbol{x}(n) \in \mathbb{C}^M$ in the n-th snapshot can be modeled as (1):

$$\boldsymbol{x}(n) = \boldsymbol{A}\boldsymbol{s}(n) + \boldsymbol{v}(n), \tag{1}$$

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where $\boldsymbol{v}(n) \in \mathbb{C}^M$ denotes the noise vector received by ULA. $\boldsymbol{A} = [\boldsymbol{a}(\theta_1), \boldsymbol{a}(\theta_2), \ldots, \boldsymbol{a}(\theta_K)] \in \mathbb{C}^{M\times K}$ is the so-called array manifold matrix with the k-th steering vector $\boldsymbol{a}(\theta_k) \in \mathbb{C}^M$ of ULA expressed as (2):

$$\boldsymbol{a}(\theta_k) = [1, e^{-\mathrm{j}2\pi\frac{d}{\lambda}\sin\theta_k}, \ldots, e^{-\mathrm{j}2\pi\frac{d}{\lambda}(M-1)\sin\theta_k}]^{\mathrm{T}}, \quad (2)$$

where λ denotes the wavelength of signals.

To simplify the DOA estimation problem, we introduce some common assumptions as follows:

A1) The sensors of ULA are identical and isotropic. There are no coupling or position errors among sensors.

A2) All signals are independent and mutually uncorrelated. The amplitude of signals follows the Gaussian distribution with zero-mean.

A3) The noise $\boldsymbol{v}(n)$ in receive channels is Gaussian white noise with covariance matrix $E\{\boldsymbol{v}(n)\boldsymbol{v}^{\mathrm{H}}(n)\} = {\sigma_v}^2\boldsymbol{I}$, where ${\sigma_v}^2$ denotes the power of noise.

Under these assumptions, the covariance matrix $\boldsymbol{R}$ of $\boldsymbol{x}$ can be expressed as (3):

$$\begin{aligned}\boldsymbol{R} &\triangleq E\{\boldsymbol{x}\boldsymbol{x}^{\mathrm{H}}\} = \boldsymbol{A}\boldsymbol{R}_{ss}\boldsymbol{A} + {\sigma_v}^2\boldsymbol{I} \\ &= \boldsymbol{A}\begin{bmatrix}\sigma_1^2 & & & \\ & {\sigma_2}^2 & & \\ & & \ddots & \\ & & & {\sigma_K}^2\end{bmatrix}\boldsymbol{A} + {\sigma_v}^2\boldsymbol{I}, \end{aligned} \quad (3)$$

where $\boldsymbol{R}_{ss}$ is the power matrix of signals with the k-th diagonal element ${\sigma_k}^2$ denotes the power of the k-th signal.

We divide the potential space of θ into a discrete direction set $\boldsymbol{\Phi} = [\phi_1, \phi_2, \ldots, \phi_L]^{\mathrm{T}} \in \mathbb{R}^L$, where the true $\boldsymbol{\theta}$ is also contained in this set. Then (1) can be reformulated as (4):

$$\boldsymbol{x}(n) = \sum_{l=1}^{L} \boldsymbol{a}(\phi_l)\bar{s}_l(n) + \boldsymbol{v}(n), \quad (4)$$

here we define the sparse signal vector $\bar{\boldsymbol{s}}(n) \in \mathbb{C}^L$, where the l-th element $\bar{s}_l$ only has non-zero value when $\phi_l \in \boldsymbol{\theta}$. According to Assumption A2), the covariance matrix $\boldsymbol{R}$ also can be reformulated as follows:

$$\begin{aligned}\boldsymbol{R} &= \bar{\boldsymbol{A}}\begin{bmatrix}\bar{\sigma}_1^2 & & & \\ & \bar{\sigma}_2^2 & & \\ & & \ddots & \\ & & & \bar{\sigma}_L^2\end{bmatrix}\bar{\boldsymbol{A}} + {\sigma_v}^2\boldsymbol{I} \\ &= \sum_{l=1}^{L}\bar{\sigma}_l^2\boldsymbol{a}(\phi_l)\boldsymbol{a}^{\mathrm{H}}(\phi_l) + {\sigma_v}^2\boldsymbol{I}, \end{aligned} \quad (5)$$

where $\bar{\boldsymbol{A}} = [\boldsymbol{a}(\phi_1), \boldsymbol{a}(\phi_2), \ldots, \boldsymbol{a}(\phi_L)] \in \mathbb{C}^{M\times L}$ denotes the over-complete manifold matrix. $\bar{\sigma}_l^2 = {\sigma_k}^2$ when $\phi_l = \theta_k$, otherwise, $\bar{\sigma}_l^2 = 0$.

Let vetor $\boldsymbol{y}$ denotes the vectorized form of $\boldsymbol{R}$:

$$\begin{aligned}\boldsymbol{y} &= \mathrm{vec}(R) = \mathrm{vec}\left[\sum_{l=1}^{L}\bar{\sigma}_l^2\boldsymbol{a}(\phi_l)\boldsymbol{a}^{\mathrm{H}}(\phi_l)\right] + {\sigma_v}^2\vec{\mathbf{1}} \\ &= (\bar{\boldsymbol{A}}^* \odot \bar{\boldsymbol{A}})\bar{\boldsymbol{\sigma}} + {\sigma_v}^2\vec{\mathbf{1}}, \end{aligned} \quad (6)$$

where $\vec{\mathbf{1}} = [{\boldsymbol{e}_1}^{\mathrm{T}}, {\boldsymbol{e}_2}^{\mathrm{T}}, \ldots, {\boldsymbol{e}_M}^{\mathrm{T}}]^{\mathrm{T}} \in \{0,1\}^{M^2}$ with $\boldsymbol{e}_m$ being a column vector of all zeros except a 1 at the m-th position. Vector $\bar{\boldsymbol{\sigma}} \in \mathbb{R}^L$ is the sparse power vector, which is also the expected output of our proposed estimation network. Symbol $\odot$ denotes the KR product [27]. The KR product can generate the so-called *difference co-array* (detailedly investigated in [27]–[29]), which can increase the degree of freedom (DOF) of ULA to $2(M-1)$. This advantage allows us to estimate more than M signals with only M sensors.

III. Unsupervised Learning for DL-Based Estimation Network

Assuming that the proposed network contains L output elements $\boldsymbol{z} = [z_1, z_2, \ldots, z_L]^{\mathrm{T}} \in \mathbb{R}^L$. The optimization goal of DL-based estimation methods can be expressed as a mean square error (MSE) based loss function shown in (7), where $\boldsymbol{W}$ and $\boldsymbol{b}$ denotes the weights and bias of network.

$$\{\boldsymbol{W}, \boldsymbol{b}\} = \underset{\{\boldsymbol{W},\boldsymbol{b}\}}{\arg\min}\,\frac{1}{L}\|\bar{\boldsymbol{\sigma}} - \boldsymbol{z}\|_2^2. \quad (7)$$

Many prior articles use $\bar{\boldsymbol{\sigma}}$ as the label of training and validation data samples [23], [30], however, it is difficult to obtain the actual $\bar{\boldsymbol{\sigma}}$ in reality.

It is noticeable that (7) is equivalent to finding the minimum solution of a ℓ_0-norm function [15]. Let us denote $(\bar{\boldsymbol{A}}^* \odot \bar{\boldsymbol{A}})$ as $\boldsymbol{A}_{\mathrm{KR}}$, then (7) can be formulated as (8):

$$\min\|\boldsymbol{z}\|_0, \text{ subject to } \|\boldsymbol{y} - \boldsymbol{A}_{\mathrm{KR}}\boldsymbol{z}\|_2 = {\sigma_v}^2 M. \quad (8)$$

However, solving (8) requires us to screen all the non-zero elements in $\boldsymbol{z}$, which has been proven to be an NP-hard problem [15]. Considering that ℓ_1-norm convex optimization is the closest approximation to the ℓ_0-norm optimization problem, we then use the ℓ_1-norm relaxation objective function to obtain the optimal solution of $\boldsymbol{z}$:

$$\min\|\boldsymbol{z}\|_1, \text{ subject to } \|\boldsymbol{y} - \boldsymbol{A}_{\mathrm{KR}}\boldsymbol{z}\|_2 \le \beta. \quad (9)$$

In (9), the equality constraint is also relaxed by an inequality constraint, and the constant $\beta > 0$ performs as a regularization that controls the proportion of the ℓ_2-norm term in (9). A large β can lead to a more sparse spectrum while a small β makes the spectrum less sparse but more accurate.

We then construct the unconstrained optimization function $\mathcal{L}(\bullet)$ [31] as:

$$\mathcal{L}(\boldsymbol{z}, \mu) = \sum_{l=1}^{L}|z_l| + \mu\left(\sqrt{(\boldsymbol{y} - \boldsymbol{A}_{\mathrm{KR}}\boldsymbol{z})^{\mathrm{H}}(\boldsymbol{y} - \boldsymbol{A}_{\mathrm{KR}}\boldsymbol{z})} - \beta\right), \quad (10)$$

where μ denotes the Lagrangian multiplier [31]. In our proposed estimation network, we use (10) as the loss function to find the optimal $\boldsymbol{z}$. Compared with the MSE-based loss function $\frac{1}{L}\|\bar{\boldsymbol{\sigma}} - \boldsymbol{z}\|_2^2$ shown in (7), the proposed loss function (10) does not rely on the actual power or angle spectrum,which helps the network to realize unsupervised learning.

According to the optimization theory, we also need to update μ during each epoch of training:

$$\begin{aligned}\mu_{\mathrm{new}} &= \mu_{\mathrm{old}} + \alpha\frac{\partial}{\partial\mu}\mathcal{L}(\boldsymbol{z}, \mu) \\ &= \mu_{\mathrm{old}} + \alpha\left(\sqrt{(\boldsymbol{y} - \boldsymbol{A}_{\mathrm{KR}}\boldsymbol{z})^{\mathrm{H}}(\boldsymbol{y} - \boldsymbol{A}_{\mathrm{KR}}\boldsymbol{z})} - \beta\right), \end{aligned} \quad (11)$$

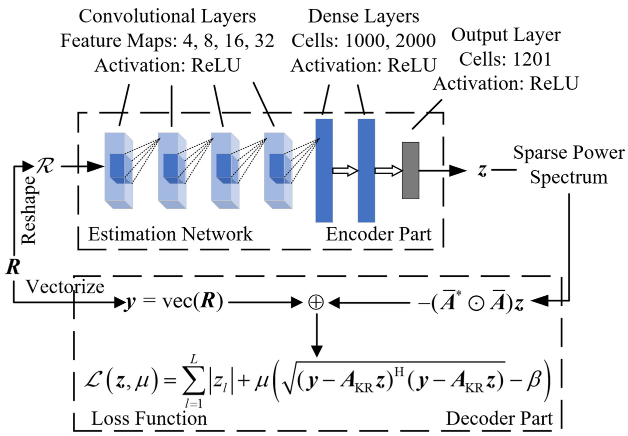


Fig. 1. Unsupervised learning strategy and framework of the proposed estimation network.

where $\alpha > 0$ is the learning rate. Since μ is also optimized, it should be considered to be part of the network, but the value of μ is determined by the received signals in the training dataset. It is noticeable that (9) and (10) are similar to the LASSO-type function [32], which use ℓ_2-norm reconstruction loss and ℓ_1-norm constraint. However, the LASSO-type loss function is unsuitable for our method. It is because the network will continuously reduce the ℓ_1-norm term with the increase of μ, and this can lead to an all-zero output of $\boldsymbol{z}$.

Here we use a simple deep neural network (shown in Fig. 1) to verify the availability of the proposed unsupervised method. The proposed estimation network contains seven layers, where the number and type of layers are chosen to reach the trade-off between the nonlinear expressivity and overfitting risk of the network. The first four layers are convolutional layers with 4, 8, 16, and 32 feature maps. The middle two layers are dense layers with 1000 and 2000 cells. For the output layer, the number of output cells L is related to the scale of discrete direction set $\boldsymbol{\Phi}$. Here we sample the potential angle space (from -60° to 60°) with interval $\Delta\phi = 0.1^\circ$, thus $L = 1201$. Because $\bar{\boldsymbol{\sigma}} \geq \boldsymbol{0}$, we use ReLU function [33] as the activations of all layers to keep the output vector $\boldsymbol{z} \geq \boldsymbol{0}$.

For the input tensor of the proposed estimation network, We reshape the matrix $\boldsymbol{R}$ into a $2@M \times M$ tensor $\mathcal{R}$, where the real part $\Re\{\boldsymbol{R}\}$ and the imaginary part$\Im\{\boldsymbol{R}\}$ are two independent channels of $\mathcal{R}$.

Unlike the so-called *deep unrolling* strategy [16], the proposed network acts as an optimizer but not an unrolling model. The iteration of original optimization is replaced by the training procedure but not the layers. In fact, the proposed estimation netowrk and loss function (10) together construct an auto-encoder network [34]. The estimation network encode $\boldsymbol{R}$ into its power spectrum, while $(\bar{\boldsymbol{A}}^* \odot \bar{\boldsymbol{A}})\boldsymbol{z}$ in the proposed loss function performs as a decoder to reconstructe $\boldsymbol{R}$. Then the loss function evaluate the distance between $\boldsymbol{y}$ and $(\bar{\boldsymbol{A}}^* \odot \bar{\boldsymbol{A}})\boldsymbol{z}$ and find the optimal $\boldsymbol{z}$ with minimum $\|\boldsymbol{z}\|_1$, which is also the main goal of the network.

IV. Numerical Results

In this section, we present the performance of the proposed unsupervised estimation method. We consider ULA with

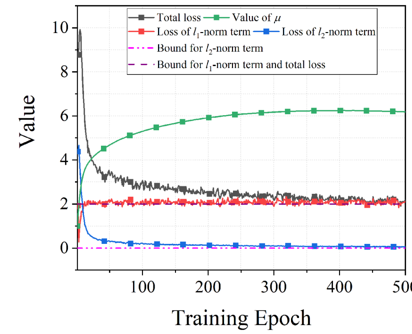


Fig. 2. Trajectories of training loss versus training epoches.

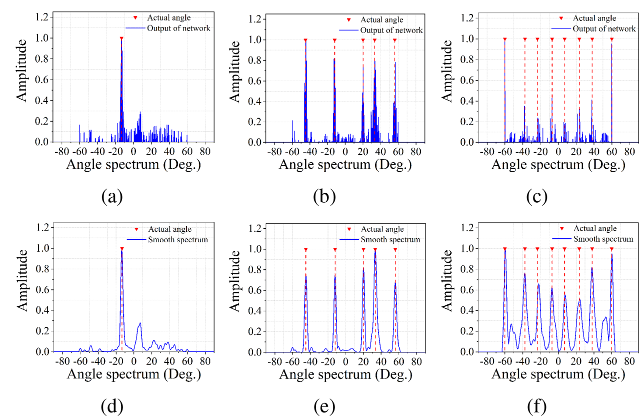


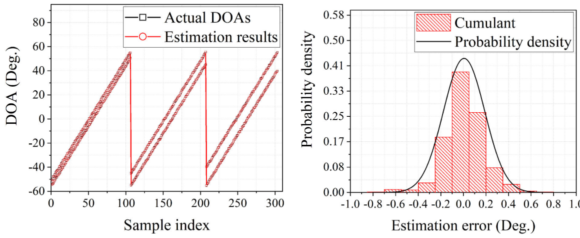
Fig. 3. Output sparse power spectrum of the proposed method. (a)-(c) Original output spectrum (d)-(f) Smoothed spectrum after processed by (12).

6 sensors, and the inter-sensor space $d = \lambda/2$. The proposed estimation network and loss function is implemented by Tensorflow 2.0 [35]. We use Adam optimizer [36] to train the network with the learning rate equals to 0.0001. We collect 60 000 samples of $\mathbb{R}$ as the training dataset and another 2000 samples as the testing and validation dataset. All samples are randomly generated by (3) with $K = 2$ and $\theta_k \in [-60^\circ, 60^\circ]$. The SNR of samples are randomly set between -10dB and 10dB. For the hyper-parameters shown in (10), we set $\beta = 0.5$, $\alpha = 0.1$, and the initial value of $\mu = 1$. The learning rate for μ is much larger than the rate of network learning because it can increase the optimizing priority of the second term in (10). After 500 epoch of training, both ℓ_1-norm term and ℓ_2-norm term shown in (10) closely reach their lower bound ($K = 2$ for ℓ_1-norm term and 0 for ℓ_2-norm term). The values of μ converges to its optimal value and $\mathcal{L}(\boldsymbol{z}, \mu)$ becomes stable. These results are shown in Fig. 2.

To obtain spectrum with clear peaks, we implement Gaussian weighted smoothing (GWS) on the output vector $\boldsymbol{z}^{(\mathrm{opt})}$:

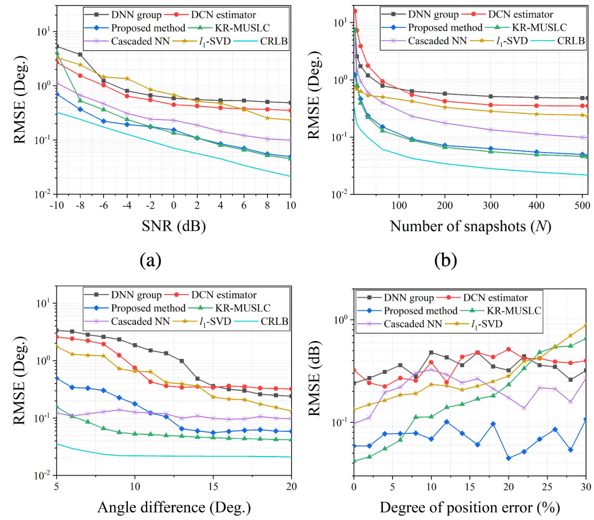
$$\hat{z}_l = \frac{1}{2P} \sum_{p=l-P}^{l+P} \frac{1}{\sqrt{2\pi}} e^{-(z_p^{(\mathrm{opt})} - z_l^{(\mathrm{opt})})^2} \cdot z_p^{(\mathrm{opt})}, \tag{12}$$

where P denotes the number of averaged points, and larger P can give better smoothing performance but less resolution. Comprehensively consider these two points, here we set $P = 20$. Fig. 3(a) shows the spectrum of 1 signal (actual $\theta = 13.62^\circ$), which contains 1 obvious peak around the actual position of



(a) (b)

Fig. 4. DOA estimation and probability density of corresponding errors. (a) Estimation results (b) Probability density of errors.



(a) (b) (c) (d)

Fig. 5. Statistic results of estimation performance. (a) RMSE results versus SNR (b) RMSE versus N (c) RMSE versus $\Delta\theta$ (d) RMSE versus ρ.

DOAs. However, $\boldsymbol{z}^{(\text{opt})}$ is a discontinuous spectrum formed by plenty of separate pulses. This is caused by the choice of β, here we use a relatively small β to sacrifice part of sparsity and obtain more accuracy. After smoothing by (12), we can obtain the relatively clear spectrum as Fig. 3(d). Figure 3(b) and (e) display the spectrums of 5 signals. This illustrates that the proposed network can estimate more than 2 signals using the network trained by only 2 signals. Fig. 3(c) and (f) show the estimation results of 8 signals with only 6 sensors, and it verifies that the proposed method can estimate DOAs when $K \geq M$. Moreover, unlike the KR-MUSIC algorithm, our proposed method can be directly used for Gaussian signals without KR-based spatial smoothing [27].

We then test the estimation performance of two signals in the angle sector of $[-55°, 55°]$. The angle sparations are set to be $\Delta\theta = \{5° + \delta, 10° + \delta, 15° + \delta\}$, where $\delta \in (-1°, 1°)$ represents a fractional angle biase. We set SNR$=$ 10dB and $N = 512$, then the estimation results after smoothing are shown in Fig. 4. It is implied in Fig. 4(a) that the proposed unsupervised estimation network can obtain precise prediction of DOAs in the whole potential angle space. We also investigate the statistic features of estimation errors in Fig. 4(b). It may be obvious that estimation errors obey the normal distribution $\mathcal{N}(\epsilon, \gamma^2)$ with mean value $\epsilon \approx 0°$ and standard deviation $\gamma = 0.1827°$.

To further evaluate the performance of estimation methods, we use average root mean square error (RMSE) to measure the estimation error. We choose 5 prior methods to make comparison with the proposed method: DNN group [14], DCN estimator [23], Cascaded NN [20], KR-MUSIC algorithm with nested array [27], and l_1-SVD method [15]. The RMSE results of the proposed method are obtained after the smoothing because it performs better than the unsmoothed results.

First, we investigate the estimation performance under the different values of SNR. We set $K = 2$, $N = 512$, and $\boldsymbol{\theta} = [-5.23°, 19.76°]$. It is shown in Fig. 5(a) that the proposed unsupervised learning strategy highly increases the estimation precision of DL-based methods and can work with lower SNR. We then fix the SNR at 10dB and study the RMSE response versus N. Although Fig. 5(b) illustrates that all methods except the l_1-SVD method cannot obtain correct DOAs when $N \leq 4$, the proposed method and KR-MUSIC perform relatively better than other methods when $N \geq 16$.

Fig. 5(c) illustrates the RMSE versus angle difference $\Delta\theta$. The cascaded NN performs better than our proposed method when angle difference $\Delta\theta \leq 6$. However, the framework of cascaded NN only allows it to estimate two signals. Our proposed method can estimate more signals than sensor number, which has generalization to K. Fig. 5(d) shows the RMSE versus array position errors ρ. We give a random position disturbance for each sensor, then record the RMSE performance in Fig. 5(d). The results imply that DL-based methods have the robustness to position errors, and the proposed method can obtain accurate estimation results when the model $\bar{\boldsymbol{A}}$ may not be known perfectly.

We also record the time needs (averaged by 1000 independent experiments) of all methods. The experiments are implemented on a PC with one Intel i7-1065G7 CPU, and the results are shown in Tab. I. The l_1-SVD is time-consuming, which cannot achieve real-time estimation. The run time of the proposed method and DCN estimator are in the same order of magnitude, which is faster than other methods.

TABLE I
RUN TIME OF ESTIMATION METHODS

Method	DNN group	DCN estimator	Proposed Method	Cascaded NN	ℓ_1-SVD
Run Time	1.32×10^{-2} s	2.3×10^{-3} s	4.4×10^{-3} s	1.07×10^{-2} s	2.3 s

One final point worth noting is that the proposed unsupervised learning strategy can be used for all array structures. For example, if we want to implement the proposed method in an ad-hoc array such as the nested array, we only need to replace the training dataset generated by the nested array. The network and learning strategy do not need further modifications.

V. CONCLUSION

In this letter, we develop a novel unsupervised learning strategy based on sparse power spectrum and ℓ_1-norm optimization. According to the numerical results shown in Section IV, the proposed method performs much better than the prior DL-based estimation methods, and it can obtain precise estimate results even when $K \geq M$. Because the proposed method does not rely on further manual annotations or pre-processing, it is easier for us to implement in reality.

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