An Adaptive Formulation of the Sliding Innovation Filter

Andrew S. Lee, S. Andrew Gadsden, *Senior Member, IEEE*, and Mohammad Al-Shabi, *Senior Member, IEEE*

***Abstract*—In this paper, an adaptive formulation of the sliding innovation filter (SIF) is presented. The SIF is a recently proposed estimation strategy that has demonstrated robustness to modeling errors and uncertainties. It utilizes a switching gain that is a function of the innovation (measurement error) and sliding boundary layer term. In this paper, a time-varying sliding boundary layer is derived based on minimizing the state error covariance. The resulting solution creates an adaptive formulation of the SIF. The adaptive SIF is applied on a linear aerospace system, and is compared with the well-known Kalman filter (KF) and the standard SIF. The results demonstrate the robustness of the new estimation strategy in the presence of modeling uncertainties and system faults.**

***Index Terms*—Estimation theory, extended Kalman filter, fault detection and diagnosis strategies, magnetorheological damper, sliding innovation filter.**

I. Introduction

THE objective of estimation theory is to extract useful information on system states in the presence of unknown system and measurement noise. The Kalman filter (KF) is the most well-studied estimation method as it provides an optimal estimate for linear systems in the presence of known systems and white noise [1]–[3] The KF has a wide-range of applications such as target tracking, signal processing, and fault detection [4]–[6] For nonlinear systems, the KF has been modified to approximate the nonlinearities. The extended Kalman Filter (EKF) can be used to estimate the states of a nonlinear dynamic system. The filter uses local linearization of the system model at the operating point in order to calculate the corrective gain [1], [4] However, if the system is highly nonlinear, the EKF solution may diverge from the true state trajectory leading to numerical instabilities and poor estimation results [7]–[9]. Another well-known nonlinear version is the unscented Kalman filter (UKF) which utilizes sigma-points to formulate a weighted statistical linear regression which approximates the nonlinearities [10]. The UKF works well for a number of signal processing applications, however it can be resource intensive and can be sensitive to modeling uncertainties and disturbances.

Similar to control theory, a trade-off exists between estimation accuracy and robustness to disturbances [7], [11]. A number of robust strategies have been presented in the literature, and include estimating bounds on the uncertainties or minimizing the maximum estimation error [1], [12], [13]. Other strategies can be classified as entropy-based, distribution-based, or sliding mode type. Entropy-based strategies utilize a different criterion than the KF, which minimizes the well-known mean square error (MSE) [14], [15]. Both methods attempt to improve robustness to heavy-tailed non-Gaussian noises in order to provide a stable estimate. Distribution-based strategies typically utilize variational Bayesian methods that provide an approximation to the probability density function of estimated states, and may also be used to improve robustness to non-Gaussian noise [16]. Assumed density filters, which are part of distribution-based methods, utilize a tractable parametric distribution in Bayesian networks [17]. Note that these types of filters are also known as moment matching, online Bayesian learning, and weak marginalization [17].

Sliding mode observers were introduced based on sliding mode and variable structure theory [18]. The observer gain is calculated based on the innovation and is implemented in an attempt to force the error surface to zero [19]. Sliding mode observers define a hyperplane (i.e., a sliding surface) and apply a discontinuous switching force on the estimate to keep the estimate bounded within an area of the hyperplane [20]. This strategy provides estimates that are robust to modeling uncertainties and external disturbances.

Based on sliding mode observers, the smooth variable structure filter (SVSF) was developed [4], [21]. This method has demonstrated robustness to modeling uncertainties while providing a sub-optimal solution in terms of estimation accuracy [4], [22], [23]. More recently, the sliding innovation filter (SIF) was proposed, and it utilizes a simpler gain and yields a more accurate solution when compared with the SVSF [24]. A nonlinear version, similar to the EKF, was also presented called the extended SIF (ESIF). The SIF and ESIF applied in [24]. utilized a fixed-width sliding boundary layer which assumes a constant upper limit to modeling uncertainty and noise.

In this study, we propose a time-varying sliding boundary layer that is optimized by minimizing the trace of the updated state error covariance with respect to the sliding boundary layer at each time step. This method provides an advantage to manually-tuned sliding boundary layers by adapting to changes in the system dynamics. In addition, significant changes in the time-varying sliding boundary layer can be used to indicate the presence of faults within the system. The motivation of this work

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Andrew S. Lee and S. Andrew Gadsden are with the College of Engineering and Physical Sciences, University of Guelph, ON N1G 2W1, Canada (e-mail: alee32@uoguelph.ca; gadsden@uoguelph.ca).

Mohammad Al-Shabi is with the Department of Mechanical and Nuclear Engineering, University of Sharjah, Sharjah, UAE (e-mail: malshabi@sharjah.ac.ae).

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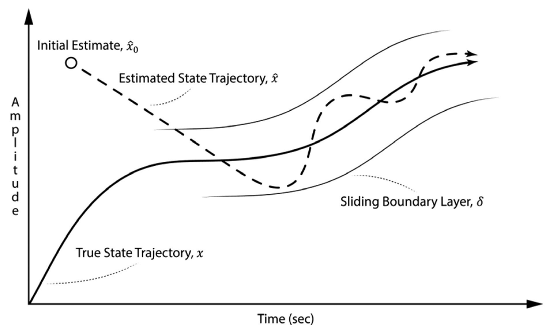

Fig. 1. The sliding innovation filter (SIF) concept illustrating the effects of the switching gain and sliding boundary layer [24].

is to improve the estimation accuracy of the SIF method, which is considered robust but sub-optimal.

The paper is organized as follows. The SIF estimation process is summarized in Section II, followed by the proposed time-varying sliding boundary layer derivation in Section III. The results of applying the KF, the standard SIF, and the proposed adaptive SIF are shown in Section IV, followed by concluding remarks.

II. The Sliding Innovation Filter

The sliding innovation filter (SIF) is a predictor-corrector estimator based on sliding mode concepts [24]. The difference between the KF and SIF is the structure of the corrective gain matrix. The SIF gain is calculated using the measurement matrix, innovation, and sliding boundary layer term. An initial estimate is pushed towards the sliding boundary layer which is based on the upper limit of uncertainties in the estimation process [24]. If the estimate is within the sliding boundary layer, the estimates are forced to switch about the true state trajectory by the SIF gain. Fig. 1 illustrates the SIF estimation concept.

This section describes the linear SIF estimation process. The prediction stage is given by the following equations:

$$\hat{x}_{k+1|k} = A\hat{x}_{k|k} + Bu_k \tag{1}$$

$$P_{k+1|k} = AP_{k|k}A^T + Q_{k+1} \tag{2}$$

$$\tilde{z}_{k+1|k} = z_{k+1} - C\hat{x}_{k+1|k} \tag{3}$$

where x refers to the state, $\hat{x}$ refers to the estimated state, u refers to the system input, z refers to the measurement, $\tilde{z}$ refers to the innovation (or measurement error), and k refers to the time step. In addition, A, B, C, P, Q, and R, are respectively defined as the system matrix, input gain matrix, measurement matrix, state error covariance matrix, system noise covariance, and measurement noise covariance. Note also that $k+1|k$ and $k+1|k+1$ refer to predicted and updated values, respectively.

The states are predicted in (1) before being updated in (5) using the innovation defined in (3) which is also used in the gain formulation in (4). The state error covariance matrix is predicted in (2) before being updated in (6). Note that the gain (4) is also used to update the state error covariance (6). The update state is summarized by the following equations:

$$K_{k+1} = C^+ \, \overline{sat}\left(\left|\tilde{z}_{k+1|k}\right| / \delta\right) \tag{4}$$

$$\hat{x}_{k+1|k+1} = \hat{x}_{k+1|k} + K_{k+1}\tilde{z}_{k+1|k} \tag{5}$$

$$P_{k+1|k+1} = \left(I - K_{k+1}C^+\right) P_{k+1|k}\left(I - K_{k+1}C^+\right)^T \ldots + K_{k+1}R_{k+1}K_{k+1}^T \tag{6}$$

where C^+ refers to the pseudoinverse of the measurement matrix, $|\tilde{z}_{k+1|k}|$ refers to the absolute innovation value, T refers to transpose of a vector or matrix, δ refers to the fixed sliding boundary layer width, and $\overline{sat}$ refers to the diagonal matrix of the saturated vector values. The sliding boundary layer term may be tuned based on designer knowledge of the system (e.g., level of noise) in an effort to minimize the state estimation error.

Equations (1) through (6) represent the SIF estimation process for linear systems and measurements. The SIF proof of stability was discussed in detail in [24]. A Lyapunov function was defined based on the updated innovation, and was used to prove stability. Note that the nonlinear version of the SIF, the extended SIF (ESIF), is similar to the SIF with the main difference being the formulation of the gain [24]. Similar to the EKF, the ESIF uses Jacobian matrices to linearize the nonlinear system $f(\hat{x}_{k|k}, u_k)$ and nonlinear measurement $h(\hat{x}_{k+1|k})$ functions, respectively as follows:

$$F_k = \frac{\partial f}{\partial x}\Big|_{\left(\hat{x}_{k|k}, u_k\right)} \tag{7}$$

$$H_{k+1} = \frac{\partial h}{\partial x} Big|_{(\hat{x}_{k+1|k})} \tag{8}$$

In its current formulation, the state error covariance matrix P defined in the SIF estimation process is not used to update the state estimates. However, as will be shown in Section III, it is used to derive a time-varying sliding boundary layer.

III. Derivation of the Sliding Boundary Layer

This section presents the derivation of the time-varying sliding boundary layer, which forms the basis of the adaptive SIF strategy. For simplicity, in the presented derivation, the measurement matrix C is assumed to be constant. For nonlinear measurements, the derivation would remain the same, but the linearized measurement matrix H_{k+1} would be used instead as per (8). The adaptive SIF is derived based on minimizing the trace of the updated state error covariance $P_{k+1|k+1}$. The trace is taken as it represents the total amount of state error present in the estimation process.

The updated state error covariance matrix given by (6) can be expanded to the following:

$$P_{k+1|k+1} = P_{k+1|k} - K_{k+1}CP_{k+1|k} - P_{k+1|k}C^TK_{k+1}^T \ldots\ldots + K_{k+1}S_{k+1}K_{k+1}^T \tag{9}$$

where S_{k+1} is the innovation covariance defined by:

$$S_{k+1} = CP_{k+1|k}C^T + R_{k+1} \tag{10}$$

The SIF gain defined by (4) is rewritten to include a full δ matrix (a sliding boundary layer term for each innovation term and cross-innovation term). We also ignoring the saturation term in order to calculate the optimal solution. In this case, the modified SIF gain is defined as follows:

$$K_{k+1} = C^+ \left|\overline{\tilde{z}_{k+1|k}}\right| \delta^{-1} \tag{11}$$

By substituting (11) into (9), the updated state error covariance can be written in terms of the boundary layer δ:

$$P_{k+1|k+1} = P_{k+1|k} - P_{k+1|k}C^T\delta^{-T}\left|\overline{\tilde{z}_{k+1|k}}\right|C^{+T}\ldots\ldots$$
$$- C^{+}\left|\overline{\tilde{z}_{k+1|k}}\right|\delta^{-1}CP_{k+1|k}\ldots$$
$$\ldots + C^{+}\left|\overline{\tilde{z}_{k+1|k}}\right|\delta^{-1}S_{k+1}\delta^{-T}\left|\overline{\tilde{z}_{k+1|k}}\right|C^{+T} \quad (12)$$

The partial derivative of the trace of the updated state error covariance matrix with respect to the sliding boundary layer δ is the basis for obtaining a time-varying δ, as shown in (13).

$$\frac{\partial\left(trace\left(P_{k+1|k+1}\right)\right)}{\partial\delta} \quad (13)$$

The partial derivative of each term in (13) yields the following:

$$\frac{\partial\left(trace\left(P_{k+1|k}\right)\right)}{\partial\delta} = 0 \quad (14)$$

$$\frac{\partial\left(trace\left(-P_{k+1|k}C^T\delta^{-T}\left|\overline{\tilde{z}_{k+1|k}}\right|C^{+T}\right)\right)}{\partial\delta}$$
$$= \delta^{-T}\left|\overline{\tilde{z}_{k+1|k}}\right|C^{+T}P_{k+1|k}C^T\delta^{-T} \quad (15)$$

$$\frac{\partial\left(trace\left(-C^{+}\left|\overline{\tilde{z}_{k+1|k}}\right|\delta^{-1}CP_{k+1|k}\right)\right)}{\partial\delta}$$
$$= \delta^{-T}\left|\overline{\tilde{z}_{k+1|k}}\right|C^{+T}P_{k+1|k}C^T\delta^{-T} \quad (16)$$

$$\frac{\partial\left(trace\left(C^{+}\left|\overline{\tilde{z}_{k+1|k}}\right|\delta^{-1}S_{k+1}\delta^{-T}\left|\overline{\tilde{z}_{k+1|k}}\right|C^{+T}\right)\right)}{\partial\delta}$$
$$= -2\delta^{-T}\left|\overline{\tilde{z}_{k+1|k}}\right|C^{+T}C^{+}\left|\overline{\tilde{z}_{k+1|k}}\right|\delta^{-1}S_{k+1}\delta^{-T} \quad (17)$$

Combining (13) with (14) through (17) yields the following expression:

$$2\delta^{-T}\left|\overline{\tilde{z}_{k+1|k}}\right|C^{+T}P_{k+1|k}C^T\delta^{-T}\ldots$$
$$\ldots - 2\delta^{-T}\left|\overline{\tilde{z}_{k+1|k}}\right|C^{+T}C^{+}\left|\overline{\tilde{z}_{k+1|k}}\right|\delta^{-1}S_{k+1}\,\delta^{-T} = 0 \quad (18)$$

Simplifying the terms yields:

$$CP_{k+1|k}C^TS_{k+1}^{-1} - \left|\overline{\tilde{z}_{k+1|k}}\right|\,\delta^{-1} = 0 \quad (19)$$

Finally, simplifying (19) yields the following time-varying sliding boundary layer:

$$\delta_{k+1} = S_{k+1}\left(CP_{k+1|k}C^T\right)^{-1}\left|\overline{\tilde{z}_{k+1|k}}\right| \quad (20)$$

$$\delta_{k+1} = S_{k+1}\left(S_{k+1} - R_{k+1}\right)^{-1}\left|\overline{\tilde{z}_{k+1|k}}\right| \quad (21)$$

Equation (20) represents the time-varying sliding boundary layer δ_{k+1} that is used by the proposed adaptive SIF. The width of δ_{k+1} is found to be a function of the innovation covariance matrix S_{k+1}, the measurement matrix C, the state error covariance matrix $P_{k+1|k}$, and the absolute magnitude of the innovation $\tilde{z}_{k+1|k}$. Note that (20) may be simplified even further using (10) as follows:

The adaptive SIF strategy remains the same as the standard SIF strategy presented in Section II, except that δ in (4) is no longer fixed and is calculated at each time step as per (21).

IV. Simulation Setup and Results

To demonstrate the effectiveness of the proposed adaptive SIF strategy, a linear aerospace system with noise is simulated and discussed in this section. An electrohydrostatic actuator (EHA) is a type of flight surface actuator used in aerospace, and has been studied extensively in [6]. The linear form of the EHA system and measurements are described as per the following equations, respectively:

$$x_{k+1} = \begin{bmatrix} 1 & T & 0 \\ 0 & 1 & T \\ -557 & -28.6 & 0.94 \end{bmatrix} x_k + \begin{bmatrix} 0 \\ 0 \\ 557 \end{bmatrix} u_k + w_k \quad (22)$$

$$z_{k+1} = Cx_{k+1} + v_{k+1} \quad (23)$$

where the sample rate T is defined as $1\ ms$, C is an identity matrix of dimension 3×3, and u is the controller input for the system (a square wave with amplitude $0.5\ rad/s$ and frequency 2π). The values in the system and input gain matrices of (22) were found experimentally as per the linear model in [6]. The system and measurement noises (w and v) are normally distributed with zero mean and covariance's Q and R defined by (24) and (25), respectively.

$$Q = diag\left(\left[10^{-5}\ 10^{-3}\ 0.1\right]\right) \quad (24)$$

$$R = diag\left(\left[10^{-4}\ 10^{-2}\ 1\right]\right) \quad (25)$$

The states in (22) represent the kinematic states of the EHA (e.g., position, velocity, and acceleration). The initial states, measurements and estimated states were set to zero. The initial state error covariance values were set to $P_{0|0} = 10Q$. The sliding boundary layer width for the SIF was manually tuned to yield the smallest estimation error, and was found for this simulation to be $\delta = [0.05\ 0.5\ 3]^T$. The simulation was coded in MATLAB. A total of 100 Monte Carlo simulations were run, and the results were averaged.

The KF, SIF, and proposed adaptive SIF (ASIF in the legend of the figures) were applied on the EHA. The estimated positions for one run are shown in Fig. 2. Note that all three estimation strategies were able to yield relatively good estimates, as the lines appear overlapping in the figure.

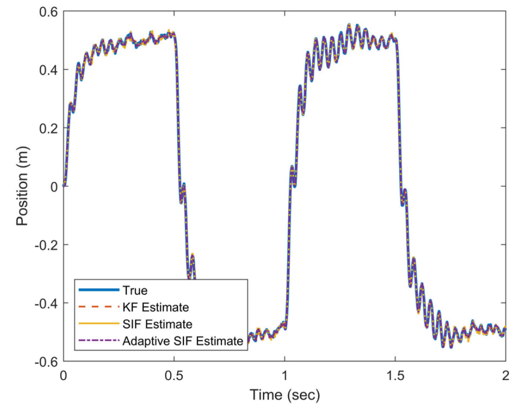

Fig. 2. The position estimation results for the linear EHA case under normal operating conditions. Note that the lines appear overlapping as the results are quite similar.

The root mean square error (RMSE) metric was used to measure and compare the estimation performance of the KF, SIF, and adaptive SIF, and is defined as:

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}\left(x_i - \hat{x}_i\right)^2}{n}} \quad (26)$$

TABLE I
RMSE RESULTS: NORMAL EHA OPERATION

State	KF	SIF	ASIF
Position (m)	3.698×10^{-3}	5.723×10^{-3}	3.698×10^{-3}
Velocity (m/s)	4.821×10^{-2}	5.298×10^{-2}	4.821×10^{-2}
Acceleration (m/s^2)	8.510×10^{-1}	9.386×10^{-1}	8.510×10^{-1}

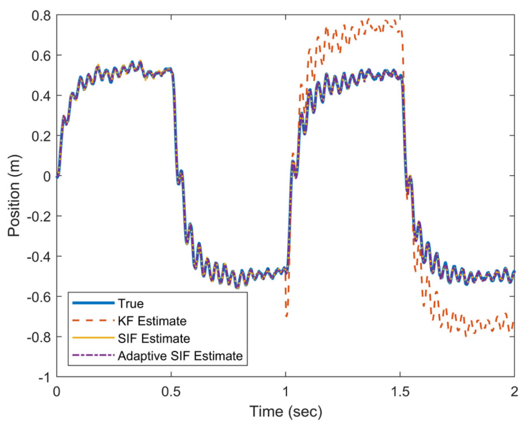

Fig. 3. The position estimation results for the linear EHA case under the presence of a fault introduced at 1 second. Note that the KF fails to provide a reliable estimate, whereas both the SIF and proposed adaptive SIF yield relatively good results.

where n in (26) is the number of time steps (total samples).

The RMSE results for these simulations are shown in Table I. As expected, the KF yields the best estimation result in terms of accuracy as it is the optimal solution for linear, known systems under the presence of white noise. The SIF yielded acceptable results, but was not as accurate as the KF. This was expected as the SIF is a sub-optimal yet robust filtering strategy. The proposed adaptive SIF yielded exactly the same result as the KF. This was due to the optimal sliding boundary layer derived in Section III. The sliding boundary layer defined in (21) changes with time based on the innovation covariance, measurement noise covariance, and innovation; essentially collapsing the SIF estimation process to an optimal gain (KF).

For the adaptive SIF, the improved accuracy does not come at a loss of robustness to modeling uncertainty and disturbances. Consider the case where a fault is injected half-way through the simulation (at 1 second). At this point, the system equation used by the filters is changed to (27).

$$x_{k+1} = \begin{bmatrix} 1 & T & 0 \\ 0 & 1 & T \\ -240 & -28 & 0.94 \end{bmatrix} x_k + \begin{bmatrix} 0 \\ 0 \\ 557 \end{bmatrix} u_k + w_k \quad (27)$$

The results of injecting a fault into the system half-way through the simulation are shown in Fig. 3. At 1 second, the KF fails to provide a good estimate of the position (as well as the other states). However, both the SIF and proposed adaptive SIF are able to keep a relatively good track of the true position.

Table II summarizes the RMSE results for the second case, where a fault was injected half-way through the simulation. In this case, the KF fails to provide a reliable estimate of any state which could have yielded catastrophic control performance for the aerospace system. However, the SIF and proposed adaptive SIF still provide relatively good estimates, with the adaptive formulation yielding better results (approximately 17% more accurate position estimates than the standard SIF).

TABLE II
RMSE RESULTS: FAULTY EHA OPERATION

State	KF	SIF	ASIF
Position (m)	1.513×10^{-2}	6.173×10^{-3}	4.912×10^{-3}
Velocity (m/s)	1.721	5.802×10^{-2}	5.378×10^{-3}
Acceleration (m/s^2)	8.849	1.034	0.986

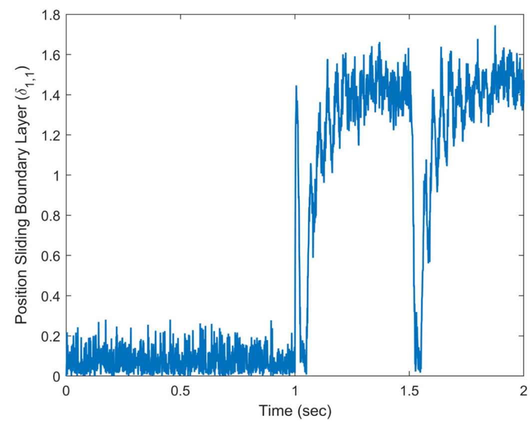

Fig. 4. The sliding boundary layer for the position state is shown in this figure. It represents the first row and first column of (21), which is a 3-by-3 matrix that changes with time. Note that the magnitude of δ_{11} increases significantly when the fault is injected at 1 seconds. The sliding boundary layer can be used as another indicator of performance or to detect a change in the system.

Fig. 4 shows the time-varying sliding boundary layer (21) for the position state for the faulty case. For the first half of the simulation, the value was largely based on the amount of noise present in the system and measurements and was relatively small. Once a fault was injected into the system as per (27), the magnitude of the sliding boundary layer grew significantly to account for the uncertainties in the estimation process. The time-varying sliding boundary layer was used as a secondary indicator for performance. For example, in this case, the term was able to detect a change in the system. To maintain robustness, once the change was detected, the adaptive SIF utilized the fixed sliding boundary layer terms in order to maintain robustness of the estimation process.

V. CONCLUSION

In this paper, an adaptive formulation of the sliding innovation filter (SIF) was presented. A time-varying sliding boundary layer was derived based on minimizing the state error covariance, which resulted in an adaptive formulation of the SIF estimation process. The proposed strategy was applied on a simulated linear aerospace system, and the results were compared with the well-known KF and standard SIF. Under normal operating conditions, the adaptive SIF yielded the same results as the KF. However, during the presence of a system fault, the KF failed while the adaptive SIF maintained a good estimate of the system states. Future work will look at implementing the adaptive SIF on a nonlinear experimental setup and perform a comprehensive comparison and study with other well-known robust estimation strategies.

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