A Costas-Based Waveform for Local Range-Doppler Sidelobe Level Reduction

Nadav Neuberger and Risto Vehmas

Abstract—**Target masking due to high sidelobes in the range-Doppler domain is a common problem in radar signal processing. Masking occurs when there is large variation in targets' signal-to-noise-ratio levels, caused by differences in either range or radar cross section. In this letter, we consider the design of a waveform to reduce the masking of far range targets caused by near range targets. Our method is based on a time-frequency concatenation of a Costas sequence with shifted parts of itself. It allows the user to design a low sidelobe region in the range-Doppler domain. Moreover, it approximately preserves the desirable uniform sidelobe level and thumbtack behavior of the Costas ambiguity function. We demonstrate a 20 dB sidelobe level reduction within the desired region compared to a normal Costas code with the same bandwidth and pulse length.**

Index Terms—**Ambiguity function, radar detection, radar signal processing, sidelobe level reduction, waveform design.**

I. Introduction

THE ability of a radar system to estimate the range and Doppler of multiple targets heavily depends on the waveform of the transmitted signal. The range-Doppler (RD) response of the waveform is quantified by the ambiguity function (AF), which represents the theoretical matched filter (MF) output as a function of time delay (range) and Doppler frequency (radial velocity). It encapsulates the detection performance, range and Doppler estimation accuracies and target resolution capability of the waveform. Thus, one of the radar engineer's tasks is to design a waveform with an AF that satisfies the required performance criteria for their specific use-case.

To provide fine range resolution and a high signal-to-noise ratio (SNR), either phase modulated (PM), frequency modulated (FM) or frequency-coded long pulses are commonly used [1]. To maximize the SNR, the received signal is commonly pulse compressed using the MF. When multiple targets with different SNR levels are present, the MF sidelobes of a strong target may mask the response of a weaker target – preventing its detection. Depending on the operational scenario, target masking may be undesirable only for a limited region in the RD domain. Thus, the AF sidelobe level (SLL) can be decreased in a certain region at the expense of increasing it at other unimportant areas.

Most of the existing methods for SLL reduction aim to optimize the sidelobes of the zero-Doppler cut of the AF of PM waveforms (e. g. [2]–[6]). To extend these approaches for a number of Doppler cuts, optimization-based PM waveforms have been considered in [7]–[11]. However, these methods have some drawbacks: they rely on non-ideal numerical optimization and the sidelobe structure outside the minimization region may be highly non-uniform.

Costas frequency coding is a well-known method to achieve a nearly constant SLL over the RD domain, i. e. an almost ideal "thumbtack" AF [12]. The length of the Costas sequence – the number of distinct frequency sub-pulses called chips – controls the SLL. References [13]–[15] present methods for choosing a Costas sequence to obtain a low SLL close to the AF mainlobe. Another frequency-coded waveform, the so-called "pushing sequence," has also been proposed for the same purpose [16]. These methods are well-suited for reducing masking of closely spaced targets, thus enhancing detection performance in dense target environments.

Target masking may also present itself in a sparse target scenario when the sidelobes of a near target mask a far away target. This occurs when the pulse length is comparable to the target time delays and the relative range differences between the targets are large – implying large SNR differences. Thus, it is important to consider SLL reduction far away from the AF mainlobe as well.

In this letter, we propose a novel Costas-based frequency-coded waveform to achieve a low SLL for a rectangular area in the RD domain. The area consists of large time delays far away from the AF mainlobe and a limited set of Doppler frequencies around zero. Importantly, our waveform approximately achieves the desirable sidelobe properties of the Costas sequence. Using a numerical example, we demonstrate a significant SLL reduction within the region of interest compared to a normal Costas code with the same bandwidth and pulse length.

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II. Theoretical Background

A. Signal Model

We consider a monostatic radar system transmitting a signal x as a function of fast time t. Throughout this letter, we will describe frequency-coded waveforms. A baseband waveform can be expressed as

$$x(t) = \sum_{m=0}^{M-1} x_m(t), \tag{1}$$

where the mth chip is defined as

$$x_m(t) = a(t - mT_c) \exp(-i2\pi f_m t). \tag{2}$$

In (2), a is the amplitude envelope of the chip, T_c is the chip length and f_m is the frequency of the mth chip. Furthermore, $f_m = p_m \Delta f$, where Δf is the frequency spacing between the chips and p_m is the mth element of a permutation sequence $\boldsymbol{p} \in \mathbb{Z}^{M\times 1}$ of the integer set $\{\lfloor \frac{-(M-1)}{2} \rfloor, \ldots, 0, \ldots, \lfloor \frac{M-1}{2} \rfloor\}$, where $\lfloor \cdot \rfloor$ denotes the flooring function.

To ensure that the chips have no overlap (the cross-correlation between different chips is approximately zero), the frequency spacing $\Delta f = 1/T_c$ [12]. Given the bandwidth $B = M\Delta f$ and the pulse length $T = MT_c$ of the waveform, the number of chips can be obtained as $M = \lfloor \sqrt{BT} \rceil$, where $\lfloor \cdot \rceil$ denotes rounding to the nearest integer.

The formulation of the AF function is given by [1]

$$AF(\tau, f_d) = \int_{-\infty}^{\infty} x(t) x^*(t - \tau) e^{-i2\pi f_d t} dt, \tag{3}$$

where τ is the time delay and f_d is the Doppler frequency. As the total energy of the AF is fixed, reducing the SLL in a certain region will increase it elsewhere.

B. Target Masking

Target masking may occur when the SLL of a high SNR target is greater than the mainlobe response of a target with smaller SNR. Since the MF output is used in the target detection process, the weak target will be undetected and the dynamic range of the target detection will be degraded. Hence, it becomes important to design a waveform with such sidelobes that target masking is reduced.

The extent of time delays in the AF is limited by the length T of the transmitted pulse. By translating the time delays τ into radial distances R according to $R = c\tau/2$, where c is the speed of light, we see that target masking occurs only if the range separation $\Delta R \leq cT/2$. Since the target SNR is inversely proportional to the fourth power of the range, long pulses may introduce major SNR differences between near and far range targets with similar radar cross section (RCS), leading to unwanted masking. For short pulses, masking may be caused due to differences in target RCS.

The masking phenomena may occur not only for the zero-Doppler cut (i. e. $AF(\tau, 0)$), but for any Doppler value, depending on the AF shape caused by the waveform modulation. For many applications, it is sufficient to define a bounded area free of potential masking in the RD domain (a practical scenario is given in Section V).

III. Problem Formulation

Our objective is to design the waveform x such that we overcome the masking problem, i. e. achieve the lowest possible SLL in a rectangular area

$$A = \{(\tau, f_d) \mid T_0 < \tau \leq T \wedge |f_d| < f_{th}\}. \tag{4}$$

This helps to prevent near range targets from masking far range targets, whose SNR is likely to be much smaller. We assume that there is a threshold Doppler shift $\pm f_{th}$, which is determined by the maximum possible difference between target radial velocities in the given scenario. We choose the delay limit T_0 as an integer fraction of T. We define two more areas

$$A^c = \{(\tau, f_d) \mid 0 \leq \tau \leq T_0 \wedge |f_d| < f_{th}\}$$
$$A^0 = \{(\tau, f_d) \mid 0 \leq \tau \leq T \wedge |f_d| \geq f_{th}\} \tag{5}$$

and summarize the practical requirements for the waveform design method as follows:

a) Minimal (ideally zero) SLL in A
b) As uniform SLL as possible in A^c
c) Optimal range resolution
d) No RD coupling
e) No restriction of any kind in A^0

In general, we note that it is possible to reduce the SLL by using mismatched filters [17]. However, because they degrade the SNR, we do not consider them in this letter. While the optimization problem to satisfy requirements (a) and (e) is easily formulated for PM waveforms [7]–[11], the problem is non-convex and highly non-linear, leading to computationally intensive numerical global optimization. Thus, the results are affected by the initialization stage and other heuristic parameters. Moreover, it is difficult to fulfill condition (b), i. e. to control the SLL outside the minimization region. Also, the available bandwidth is not fully exploited in PM waveforms, leading to a violation of requirement (c).

Due to these reasons, we consider FM and frequency-coded waveforms. The most common coding scheme is the Linear Frequency Modulation (LFM) [1]. Despite the decreasing SLL with increasing time delay, the AF of the LFM exhibits a diagonal "ridge" – a coupling between range and Doppler. This coupling hinders unambiguous range and Doppler detection and estimation. Therefore, LFM is ruled out by failing to meet requirements (a), (b) and (d).

In the Costas waveform [12], each frequency index appears once in the code, in such a way that only one chip may have overlap for any given time and frequency shift. There is no RD coupling, but the SLL is not monotonically decreasing as a function of time delay, as in the LFM case. The SLL is nearly constant – inversely proportional to the code length M – over the entire RD domain (excluding the vicinity of the mainlobe). Since the range resolution is determined by the bandwidth, it remains the same for both LFM and Costas waveforms. Next, we propose a waveform meeting all of the above requirements.

IV. Waveform Design Method

Our waveform design is based on a particular concatenation of a pure Costas code. The purpose is to deflect sidelobe energy from A to A^0. The code structure, SLL and design procedure are presented next.

A. Code Structure and Design

We describe how to construct the frequency sequence $\boldsymbol{f} = [f_0 \ \ldots \ f_{M-1}]^T$ to satisfy our requirements (a)–(e). We start with a given bandwidth B and transmission length T.

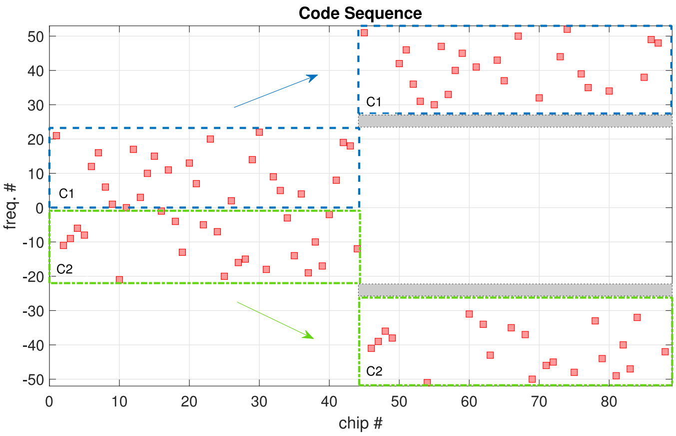

Fig. 1. The time-frequency coding concept of our waveform. After a pure Costas code (first half of the chips), we have an additional concatenated part containing the shifted positive (C1) and negative (C2) frequency parts. The maximum expected target Doppler frequency dictates the width f_{th} of the empty gray areas.

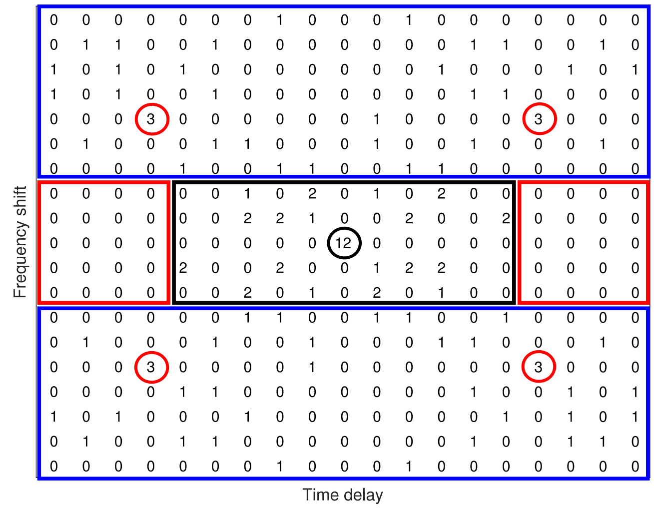

Fig. 2. The DAF of our waveform for code length $M = 12$. The area A^c is enclosed by the black rectangle, while A is enclosed by the red rectangles. The mainlobe response (black circle) is accompanied by four ambiguous peaks (red circles), which are located inside the non-feasible area A^0 (blue rectangles).

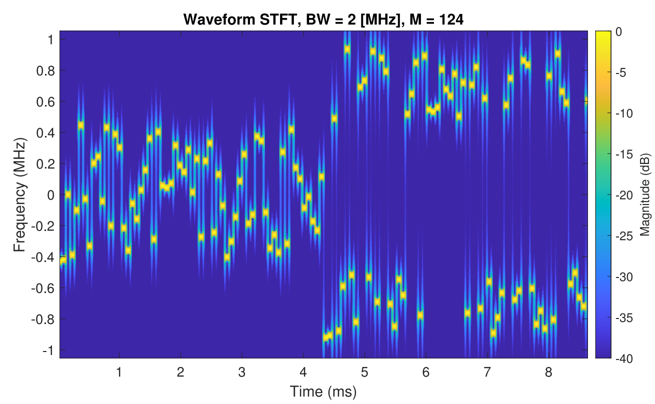

Fig. 3. The STFT of our waveform. The frequency gap in the second part produces a region with a low SLL for long time delays and Doppler frequencies close to zero.

Then, two input parameters are needed to define the low SLL region A in (4): the delay limit $T_0 = T/L$, where $L = 2, 3, 4, \ldots$ and the maximal expected target Doppler shift $f_{th} \ll B/2$. For simplicity and clarity of the concept, we will focus on $L = 2$ from here on.

The first part of our code consists of a pure Costas code of bandwidth $B_0 = \frac{B}{2} - f_{th}$ and length $T_0 = T/2$. Thus, the number of chips in the first part is $M_0 = \lfloor \sqrt{B_0 T_0} \rceil$. We denote this Costas permutation sequence by $\boldsymbol{p}_0 \in \mathbb{Z}^{M_0 \times 1}$. It can be chosen e. g. by the methods described in [15]. Thus, the frequency sequence of the first part is $\boldsymbol{f}_0 = \Delta f \boldsymbol{p_0}$.

For the next part of the code we choose frequencies f_m satisfying

$$\frac{B}{2} \geq |f_m| \geq \frac{B_0}{2} + f_{th}. \tag{6}$$

This creates a gap in the time-frequency representation of the waveform, which controls the Doppler extent of the desired low SLL area in the AF. To maintain a nearly uniform SLL in A^c, we exploit the Costas sequence $\boldsymbol{c_0}$ in the following manner. First, we calculate the required shift in the frequency indices p_m for $m > M_0$ to satisfy (6) as

$$M_s = \left\lfloor M_0 \left(\frac{1}{2} + \frac{f_{th}}{B_0} \right) \right\rceil. \tag{7}$$

Then, we obtain the shifted frequency indices of the second part as

$$\boldsymbol{p}_1 = \boldsymbol{p}_0 + \operatorname{sgn}(\boldsymbol{p}_0) \times M_s \tag{8}$$

and the corresponding frequency sequence $\boldsymbol{f}_1 = \Delta f \boldsymbol{p_1}$ satisfying (6). Finally, we obtain the frequency sequence of the waveform by concatenating the first and second parts as

$$\boldsymbol{f} = \left[\boldsymbol{f}_0^T \ \boldsymbol{f}_1^T \right]^T. \tag{9}$$

Thus, we have a waveform consisting of $M = 2M_0$ chips with a bandwidth B containing two small gaps of width f_{th} in its spectrum (see Fig. 1). We note that it is possible to implement the frequency code $\boldsymbol{f}$ as FM (with a continuous phase between the chips) e. g. by using the polyphase-coded FM method [18].

The above-described procedure can readily be extended for a different delay drop point $L > 2$. However, it should be noted that as L increases, the length of the concatenated parts decreases, raising the SLL.

In Fig. 1, we illustrate the time-frequency coding concept of our waveform.

B. Correlation Properties

To demonstrate the correlation properties of our waveform, we have plotted the discrete ambiguity function (DAF) [15] in Fig. 2. The DAF is the 2D cross-correlation of the permutation matrix of ones and zeros representing the waveform's time-frequency coding scheme. For illustration purposes, we consider a code of length $M = 2M_0 = 12$ and a frequency gap f_{th} corresponding to two chips. The rows correspond to shifts in Doppler, while the columns represent different delays (integer multiples of chips).

As seen from the DAF in Fig. 2, for delays longer than the length M_0 and Doppler shifts below f_{th}, there are no overlapping chips (zero correlation). This results in the desired rectangular area A in the AF, which is illustrated with red rectangles in Fig. 2

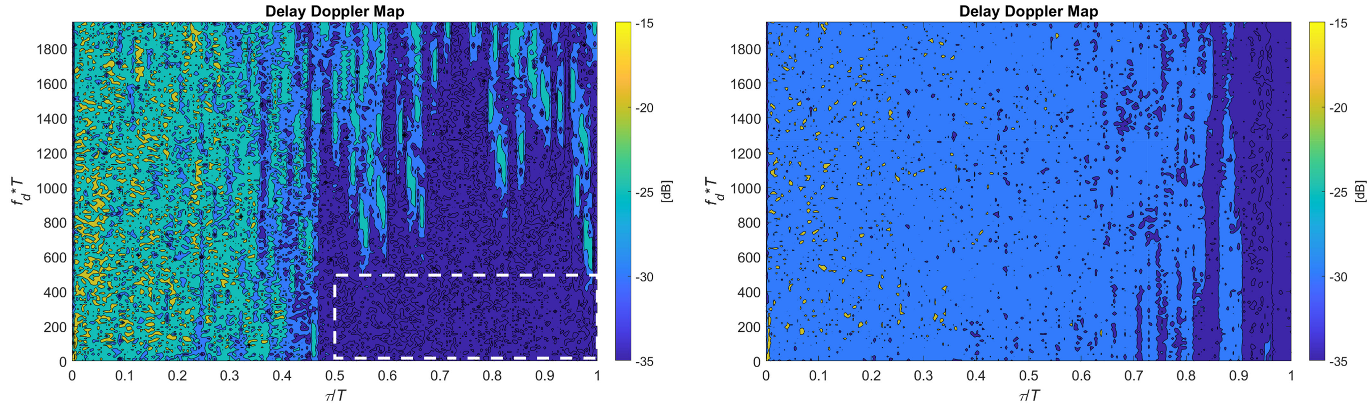

Fig. 4. The power level of the AF for our waveform (left). The low SLL area A is marked in white. The energy within A is 20 dB lower than for a pure Costas code (right), while for A^c it is 0.9 dB higher.

(requirement (a)). For delays shorter than M_0 and Doppler shifts below f_{th}, there are two possible overlapping chips (see Figs. 1 and 2): one from the overlap of the first block (pure Costas code) with itself and the second one from the overlap of the concatenated part (shifted C1 and C2) with itself. This results in the maximum SLL of two in the DAF inside A^c, which is illustrated using a black rectangle in Fig. 2.

Thus, we suffer a penalty of twice the SLL of a Costas code of length M, but retain a nearly uniform (either 0, 1 or 2) SLL inside A^c (requirement (b)). Because we use a Costas code as a basis of our method, requirements (c)-(d) are also fulfilled (provided that $f_{th} \ll B/2$, the spectrum remains close to uniform). Moreover, our approach is analytical without any numerical optimization.

We note that the SLL in A^0 can be higher than two due to additional overlapping chips. Moreover, the code structure produces four ambiguous peaks, whose magnitude is one fourth of the AF mainlobe (the red circles in Fig. 2). This intentional effect happens for a certain delay-Doppler combination, where a perfect overlap occurs between the first code block C1 and the second part of the code (a shifted C1). However, these peaks are located in A^0, which we consider to be a non-feasible area. By non-feasible we mean that observed targets are not expected to have these differences in τ and f_d.

As in any waveform, sidelobe behavior for small τ is influenced by the frequency spectrum of the signal. The separation between the frequency blocks C1 and C2, controlled by f_{th}, impacts the uniformity of the spectrum. This will alter the sidelobe behavior near the AF mainlobe, and must be considered as a trade-off in the waveform design.

V. Numerical Results

For a practical demonstration, we consider using our waveform in a space surveillance scenario, where the span of target ranges and pulse length are very large. Here, the objective is to detect and track space debris with various sizes and ranges (corresponding to different orbital heights). It is straightforward to determine the maximal radial velocity a debris target can have based on a Kepler orbit model [19].

We simulate the GESTRA system [20], [21] with the following parameters: carrier frequency $f_c = 1.33$ GHz, $B =$ 2 MHz, and $T = 8.5$ ms. For simplicity, we simulate the MF response of a single pulse. The maximum target range is 3000 km and the maximal target radial velocity is $v_r = 7$ km/s. The associated Doppler shift is therefore $f_{th} = 2v_r f_c/c =$ 62 kHz. Choosing $L = 2$, we can now derive the desired low SLL AF area as $A = [4.25, 8.5] \times [0, 62]$ (ms $\times$ kHz) $= [0.5, 1] \times [-527, 527]$ $(T \times 1/T)$.

Following the design procedure of section IV, we construct the waveform using the above parameters. The short time Fourier transform (STFT) of our waveform is seen in Fig. 3. We then proceed to calculate the AFs according to (3), which are shown in Fig. 4. We calculate the mean SLL in A (enclosed by the white rectangle) to be approximately 20 dB lower than for a pure Costas code of equal length. This has the potential to drastically improve the unwanted masking. As a penalty, the mean SLL in A^c is 0.9 dB higher for our waveform. By further increasing f_{th} and minimizing the spectral overlap between the chips, the SLL within A could decrease beyond 20 dB to allow a higher dynamic range of target detection (on the account of increased range sidelobes near the mainlobe).

VI. Conclusion

We presented an analytical method to design the AF of a frequency-coded waveform based on Costas sequences to reduce target masking far away from the AF mainlobe. Using two user-defined parameters, a rectangular area in the AF domain with reduced SLL can be designed. To achieve this, we assumed a threshold value limiting possible differences in target Doppler frequencies. The proposed waveform design method can be adjusted for a wide variety of applications where the multiple target masking problem presents itself.

Future work includes a careful examination of code sequences: for a given number of chips, several choices of the corresponding equal length Costas code are possible. Through optimization, it is possible to choose a sequence to decrease the needed frequency gap between the code blocks, producing a more uniform waveform spectrum.

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