Discrete Anamorphic Transform for Image Compression

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***Abstract*—To deal with the exponential increase of digital data, new compression technologies are needed for more efficient representation of information. We introduce a physics-based transform that enables image compression by increasing the spatial coherency. We also present the Stretched Modulation Distribution, a new density function that provides the recipe for the proposed image compression. Experimental results show pre-compression using our method can improve the performance of JPEG 2000 format.**

***Index Terms*—Anamorphic transform, diffractive data compression, dispersive data compression, image compression, physics based data compression, space-bandwidth engineering, warped stretch transform.**

I. Introduction

IMAGE compression leading to efficient representation of information is critical for dealing with the storage and transmission of high resolution images and videos that dominate the internet traffic. JPEG [1] and JPEG 2000 [2] are the most commonly used methods for image compression. To reduce the data size, JPEG and JPEG 2000 use frequency decomposition via the discrete cosine transform (DCT) [1] or wavelet transform [2] as well as the frequency dependence of the human psychovisual perception.

In this letter, we introduce the Discrete Anamorphic Stretch Transform (DAST) and its application to image compression. DAST is a physics-inspired transformation that emulates diffraction of the image through a physical medium with specific nonlinear dispersive property. By performing space-bandwidth compression, it reduces the data size required to represent the image for a given image quality. This diffraction-based compression is achieved through a mathematical restructuring of the image and not through modification of the sampling process as in compressive sensing (CS) [3]–[7]. Our technique does not need feature detection and is non-iterative.

Compared to frequency dependent frequency decomposition transforms such as warped DCT [8], [9] or chirped z- or wavelet transforms [10], [11], DAST is a nonlinear transform, both in terms of amplitude and in terms of the phase operation, as shown below.

DAST is related to the recently introduced method for analog time-bandwidth compression of one-dimensional temporal signals [12]–[14]. Such a need arises in scientific research and medicine where large numbers of real-time measurements must be made in order to find statistically rare but important information. To give an example, for rare cancer cell detection in blood, screening of millions of cells in a high speed flow stream is required. Such problems has fueled development of record throughput real-time instruments such as the time-stretch camera that allowed the detection of cancer cells in blood with sensitivity of one cell in a million [15] and the time-stretch spectrum analyzer enabling the discovery of Optical Rogue Waves [16]. These instruments produce a fire hose of temporal data approaching 1 Tbit/s. The challenges of managing such data loads led to the development of the Anamorphic Stretch Transform, an analog time-domain transform for capturing and compressing high speed temporal signals in optical domain [12]–[14].

Here we introduce the discrete-time representation of the Anamorphic Stretch Transform and its generalization to N-th order and to two dimensional data [17]. We also present the Stretched Modulation Distribution, a new discrete density function that provides the recipe for the proposed image compression. It explains graphically, how the transform reshapes the image and how image compression is achieved. Experimental demonstrations study the effect of DAST on image coherence and bandwidth and show its application to enhance JPEG 2000 format when used as pre-compression.

II. Technical Description

Different steps for implementation of DAST for application to image compression are shown in Fig. 1. The original image is represented by $B[n, m]$ where n and m represent the two dimensional discrete spatial variables. To compress the image using our method, it is first passed through the DAST and then is uniformly re-sampled (down-sampled) at a rate below the Nyquist rate of original image. To recover the original image form the compressed one, the compressed image is first up-sampled and then inverse DAST is applied to recover the original image.

DAST warps the image such that the intensity bandwidth is reduced without proportional increase the image spatial size. This increases the spatial coherence and reduces the amount of

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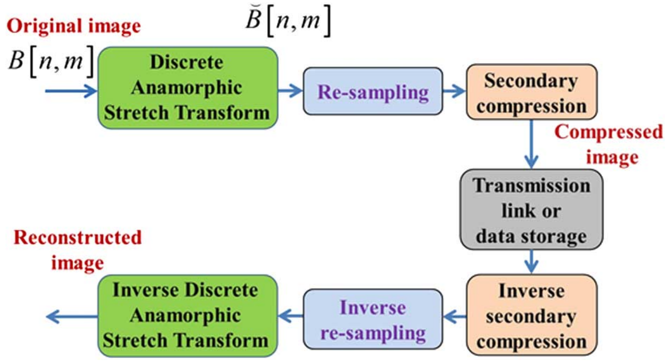

Fig. 1. In the proposed method for image comression, Discrete Anamorphic Stretch Transform (DAST) is operated on the original image followed by re-sampling (down-sampling) and secondary compression such as spatial or entropy encoding as well as other standard image compression algorithms. To recover the original image, the inverse operation is performed on the compressed image.

data needed to represent the image. Mathematically, DAST is defined as follows:

$$\breve{B}[n,m] = \left| \sum_{k_1,k_2=-\infty}^{\infty} K[n-k_1, m-k_2] \cdot B[k_1,k_2] \right|^N \quad (1)$$

where | | is the absolute operator. For DAST operation, the original image is convolved with DAST Kernel $K[n,m]$, and then the N-th power magnitude of the result is computed. In this paper we have used the case of $N = 1$. The Kernel $K[n,m]$ is described by a nonlinear phase operation,

$$K[n,m] = e^{j \cdot \Phi[n,m]}. \quad (2)$$

To compress the image, the nonlinear phase profile $\Phi[n,m]$ should be chosen such that DAST applies a spatial warp to the image with a particular profile described below. To describe the applied warp, we define the DAST Local Frequency (LF) profile as the 2D spatial gradient (derivative) of the DAST Kernel phase function. LF is the equivalent of time domain instantaneous frequency but in 2D spatial domain.

For better understanding on selection of $\Phi[n,m]$ or the DAST Kernel, we introduce a mathematical tool to describe the image bandwidth and the resulting image spatial size after it is subjected to the transformation. As we will show in Figs. 2 and 3, this tool visualizes the DAST operation on the image and the resulting data compression. To this end, we define discrete-domain Stretched Modulation (S_M) Distribution as follows:

$$S_M[n,m,p,q] = \sum_{k_1,k_2} \tilde{B}[k_1,k_2] \cdot \tilde{B}^*[p+k_1, q+k_2] \cdot \tilde{K}[k_1,k_2] . \tilde{K}^*[p+k_1, q+k_2] \cdot e^{j \cdot (n \cdot k_1 + m k_2)}, \quad (3)$$

where the symbol $*$ represents complex conjugation. S_M or Anamorphic Distribution provides a tool for engineering the image brightness space-bandwidth product through proper choice of $\Phi[n,m]$. The distribution can be described as the cross-correlation of the complex output image spectrum with its spatially shifted version. It shows spatial and spectral distributions of image intensity after diffractive propagation through a dispersive medium that imparts a nonlinear phase operation described by the Kernel $K[n,m]$. To show use of the Distribution to design the DAST Kernel, we have plotted it for the Lena image. For simplicity, we consider one randomly chosen linescan of the image. The S_M Distribution for the linescan is shown in Fig. 2(b). At $n = 0$ (i.e. spatial shift of zero) the Distribution becomes the autocorrelation of the brightness spectrum and its width gives the output intensity bandwidth. Also the maximum absolute amount of spatial shift that complex cross-correlation has non-zero values is image spatial size. This is given by the half-height of the plot (see Fig. 2(b)).

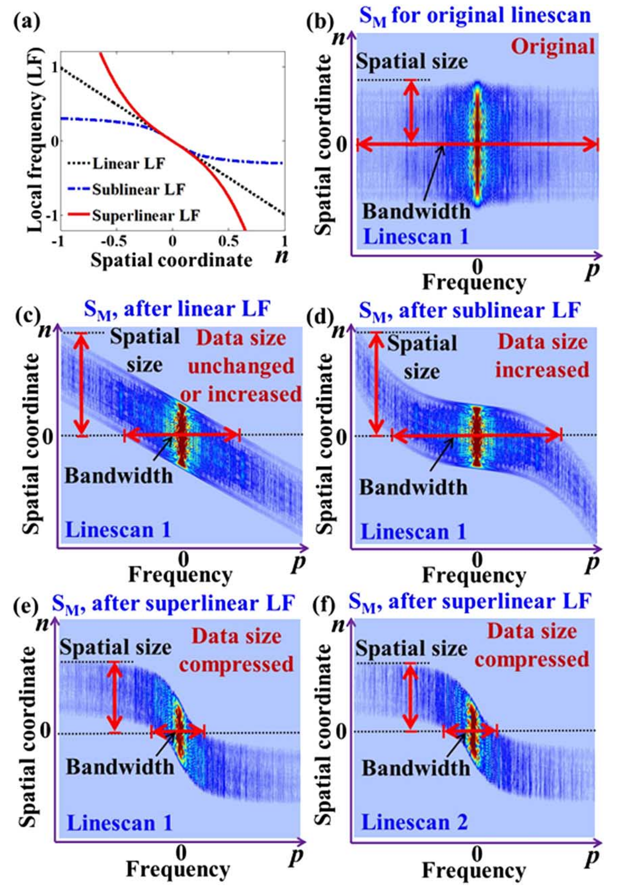

Fig. 2. Stretched Modulation (S_M) Distribution is a new density function used to design the Kernel of Discrete Anamorphic Stretch Transform (DAST). It shows the dependence of the image brightness on spatial and frequency variables. (a) Three different DAST Kernel Local Frequency (LF) profiles, (b) S_M Distribution of a randomly selected linescan in Lena image without the DAST, (c)-(e) S_M Distribution of the linescan after it is subjected to DAST with LF profiles in (a). Figure shows that only the case with superlinear LF profile results in image compression. (f) S_M Distribution corresponding to another randomly selected linescan in Lena image, for the case of superlinear LF profile confirming the generality of operation on randomly chosen linescan (see also Fig. 3).

The S_M Distribution provides a powerful and intuitive tool for identifying the general shape of the Kernel that results in image compression. As shown in Fig. 2(c), when the Kernel has a linear local frequency, the Distribution is linearly tilted resulting in a reduced bandwidth. However, the image size is proportionally stretched. This offers no compression. Similarly, if the Distribution is warped in the manner shown in Fig. 2(d), corresponding to sublinear LF profile shown in (a),

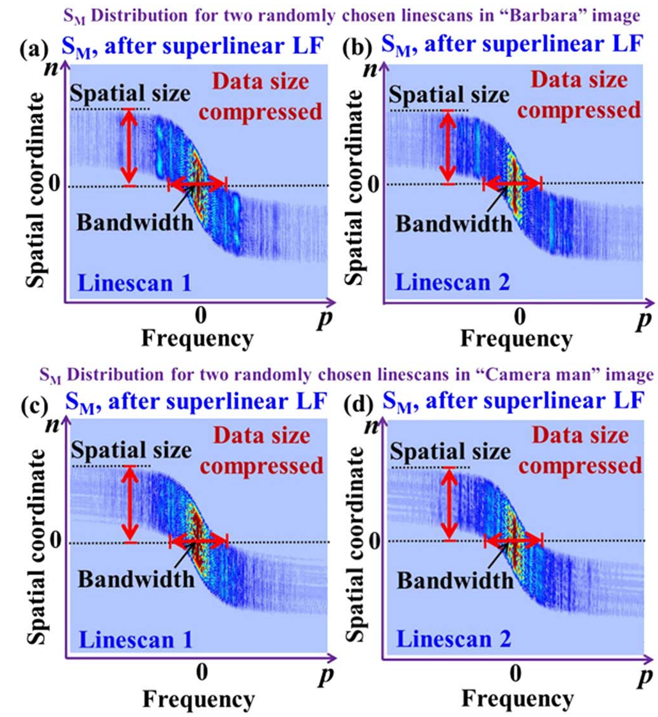

Fig. 3. Stretched Modulation (S_M) Distribution of two randomly selected linescans in "Barbara" and "Camera man" images after they are subjected to Discrete Anamorphic Stretch Transform (DAST). (a) and (b) show the S_M Distribution for two linescans in Barbara image and (c) and (d) show the Distribution for two linescans in Camera man image. For these plots we have used the superlinear Local Frequency (LF) profile shown in Fig. 2(a) normalized to the image spatial size.

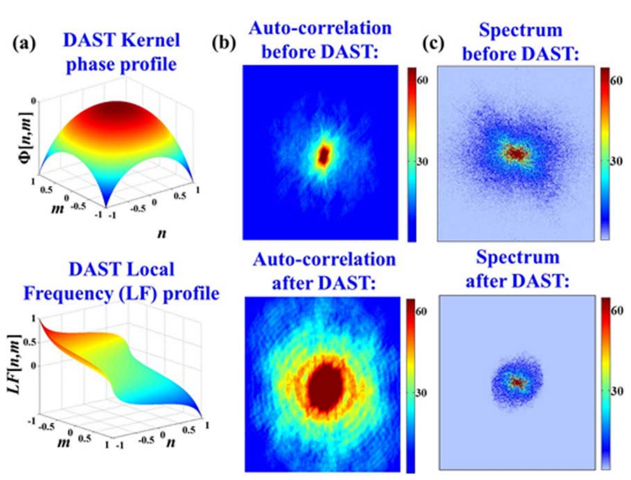

Fig. 4. Analysis of Discrete Anamorphic Stretch Transform (DAST) effect on Lena image, (a) Phase and Local Frequency (LF) profiles of the DAST Kernel used for the examples studied in this letter, these plots are normalized to the spatial dimensions of the image in each case. (b) Auto-correlation of original image ($B[n, m]$) compared to auto-correlation of transformed image using DAST. The broader autocorrelation indicates increased coherency. This is done without an image size increase, i.e. the original and transformed images have 512×512 pixels with 8 bits/pixel accuracy. (c) Image intensity (brightness) spectrum before and after DAST operation which shows that the intensity bandwidth is compressed after DAST. The compression is a result of the reshaping of the image where sharp features are stretched more than coarse ones.

the space-bandwidth product is expanded and the data size is increased. However, if we cause a nonlinear tilt (i.e. a warp) having the shape shown in Fig. 2(e), corresponding to superlinear LF profile shown in (a), the bandwidth is reduced but the size is not stretched proportionally. In this case, one achieves image compression. In Fig. 2(f), we have plotted the S_M Distribution corresponding to another randomly chosen linescan in the Lena image confirming the generality of operation on randomly chosen linescan. To show that DAST works on arbitrary images, in Fig. 3 we have plotted the S_M Distribution for two randomly selected linescans in "Barbara" and "Camera man" images. For these plots we have used the superlinear LF profile shown in Fig. 2(a) normalized to the image spatial size. As shown in Figs. 2(e) and (f) as well as Fig. 3, DAST operation with superlinear LF profile, results in image compression by reducing the spatial intensity bandwidth without proportional increase in the image spatial size.

As suggested by S_M Distribution plots, superlinear LF profile in DAST Kernel results in image compression. One of the simplest (e.g. least number of parameters) yet effective such profiles is the tangent function:

$$LF[n, m] = a_1 \cdot tan\left(b_1 \cdot n\right) + a_2 \cdot tan\left(b_2 \cdot m\right), \quad (4)$$

where a_1, b_1, a_2 and b_2 are real-valued numbers. This LF profile results in the following DAST Kernel phase profile:

$$\Phi[n, m] = \frac{a_1}{b_1} \cdot ln(cos(b_1 \cdot n)) + \frac{a_2}{b_2} \cdot ln(cos(b_2 \cdot m)), \quad (5)$$

where *ln* is the natural logarithm and *cos* is the Cosine function. The b_1, b_2 parameters are always normalized to the image size to make sure $b_1.n$ & $b_2.m < \pi/2$. An example of this phase function is shown in Fig. 4(a). The slope of the LF profile at the origin (related to a_1 and a_2) determines the amount of intensity bandwidth compression. After the proper choice of a_1 and a_2, the resulting spatial image size is related to the warping strength (related to b_1 and b_2).

After the anamorphic transform with proper phase profile the brightness bandwidth is compressed (the coherence increased). The transformed image can now be re-sampled at a lower rate without losing information given by the amount of brightness bandwidth compression after DAST. The compressed image including the re-sampled transformed image and its filtered one using the discriminator kernel (described below) along with the metadata a_1, a_2, b_1, b_2 and re-sampling factor is sent to the transmission channel or storage device. We note that only five parameters (real numbers) are required for reconstruction, resulting in negligible data overhead. The algorithm can also be combined with vector quantization [19] and entropy encoding to further reduce the image data size. Also, the re-sampled image can be compressed further by a secondary compression, e.g. JPEG or JPEG 2000. For application to color images the DAST image compression is applied to each of the constituent color components.

The decoding algorithm consists of up-sampling followed by inverse propagation through the DAST which incorporates 2D local frequency measurement. A number of techniques such as phase discrimination or iterative methods can be used for this purpose [12]–[14], [17], [18]. These phase discrimination techniques are based on measuring two instances of the signal's scalar amplitude (one filtered and one unfiltered). The phase is then recovered from these quantities. Although different discriminator kernels can be used, in this paper we have used a

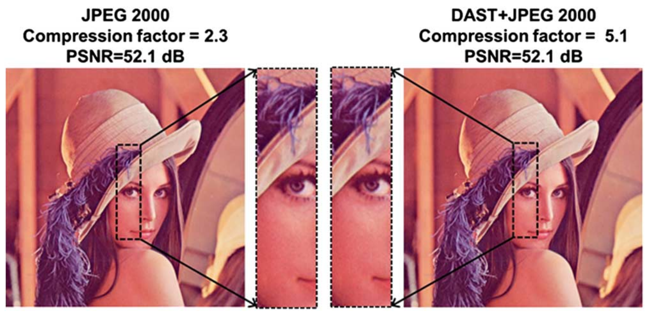

Fig. 5. Performance of Discrete Anamorphic Stretch Transform (DAST). Left column: image compressed by JPEG 2000. Right column: image pre-compressed using DAST followed by JPEG 2000. In both cases the decoded image PSNR is 52.1 dB but the case with DAST pre-compression has more than twice compression factor.

simple filter with linear frequency response (Ramp filter) for discrimination.

III. Experimental Results

In this section, we study an example to show the effect of DAST on images. We also examine the proposed image compression method and compare it to JPEG 2000 image compression format. The phase and Local Frequency (LF) profiles of the DAST Kernel for the examples presented in this letter are shown in Fig. 4(a). The parameters in these plots are normalized to the dimensions of the image in each case.

We first study the effect of DAST on gray-scale Lena image with 512×512 pixels with 8 bits/pixel accuracy in TIF format. To understand how the image data is compressed, in Fig. 4(b) we compare the auto-correlation of the original image ($B[n, m]$) with the transformed image. As it can be seen, the autocorrelation is broadened leading to increased spatial coherence and reduced spatial intensity bandwidth (Fig. 4(c)). This is done without an image spatial size increase, i.e. the original and transformed images have the same 512×512 pixels with 8 bits/pixel accuracy. The reduced spatial bandwidth (increased coherence) allows one to re-sample the transformed image at the lower rate, to achieve compression. However, it should be noted that compression is not merely obtained from the re-sampling, but rather from the increase in correlation caused by the reshaping. We prove this experimentally in the following example.

To study the performance of DAST for image compression, in the first example we show that DAST pre-compression can improve the performance of JPEG 2000 for a given image quality. The original image is Lena color image with 512×512 pixels in TIF format. The left column in Fig. 5 shows the image compressed using JPEG 2000 with target Peak Signal to Noise Ratio (PSNR) of 52.1 dB. The compression factor (original over compressed image file sizes) in this case is 2.3 times. The right column shows the image pre-compressed by DAST followed by JPEG 2000 with the same recovered image PSNR of 52.1 dB. The normalized phase profile of the DAST Kernel is shown in Fig. 4(a). We note that the compressed file size in the case with DAST pre-compression includes re-sampled transformed image and its filtered one using the discriminator kernel (3 colors for each) followed by JPEG 2000 plus the metadata. In the case of using DAST pre-compression we achieved a compression factor

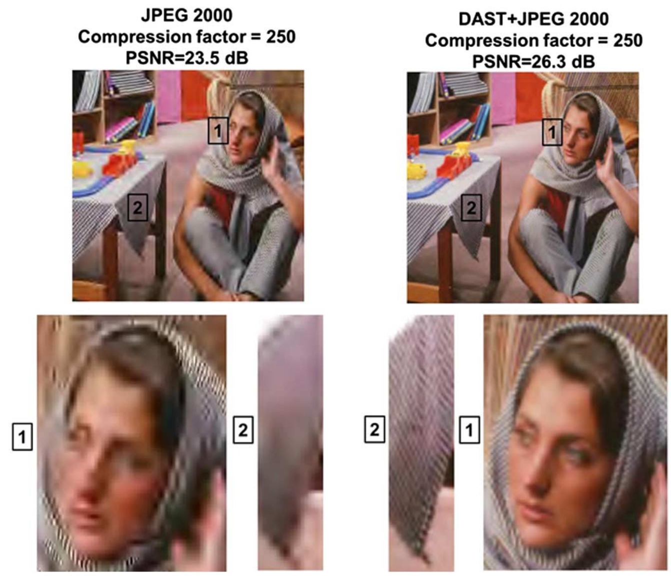

Fig. 6. Performance of Discrete Anamorphic Stretch Transform (DAST). Left column: image compressed by JPEG 2000. Right column: image encoded using DAST followed by JPEG 2000. In both cases the data size is reduced for 250 times but the case with DAST pre-compression shows superior compression performance in terms of PSNR.

of 5.1. Thus, both cases provide the same PSNR but the case with DAST pre-compression has more than twice compression factor. For this example, we have not used any down sampling, however since the spatial coherence of the image is increased, we achieved more than twice better compression factor.

In the next example, we show how DAST pre-compression can improve the performance of JPEG 2000 for a same high compression factor. Results are shown in Fig. 6. The original image for this example is Barbara color image with 576×576 pixels in TIF format. The left column in Fig. 6 shows the image compressed using JPEG 2000 with compression factor of 250. The right column shows the image pre-compressed by DAST followed by JPEG 2000 with same total compression factor of 250. This means that the total compressed data file size in the case with DAST (re-sampled transformed image and its filtered one using the discriminator kernel (3 colors for each) followed by JPEG 2000 plus the metadata) is the same as the case with JPEG 2000 alone. The normalized phase profile of the DAST Kernel is shown in Fig. 4(a). As seen, image pre-compressed with DAST has higher resolution even though the compressed file sizes are the same. To numerically compare the image compression performance, we have calculated the PSNR and Structural Similarity (SSIM) [20]. In particular, PSNR in the case of JPEG 2000 alone was 23.5 dB versus 26.3 dB for the case of using DAST pre-compression. Also SSIM for the case of JPEG 2000 without and with DAST pre-compression were 87.2% and 95.3% respectively. This shows that pre-compression using DAST has improved both PSNR and structural similarity of the compressed image to the original image.

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References

[1] W. B. Pennebaker and J. L. Mitchell, *JPEG still image data compression standard*, 3rd ed. Berlin, Germany: Springer, 1993.

[2] A. Skodras, C. Christopoulos, and T. Ebrahimi, "The JPEG 2000 still image compression standard"," *IEEE Signal Process. Mag.*, vol. 18, pp. 36–58, 2001.

[3] E. J. Candes and M. B. Wakin, "An introduction to compressive sampling," *IEEE Signal Process. Mag.*, vol. 25, pp. 21–30, 2008.

[4] R. Baraniuk, "Compressive sensing," *IEEE Signal Process. Mag.*, vol. 24, pp. 118–121, 2007.

[5] M. Lustig, D. Donoho, and J. M. Pauly, "Sparse MRI: The application of compressed sensing for rapid MR imaging," *Magn. Reson. Med.*, vol. 58, pp. 1182–1195, 2007.

[6] J. Mait, R. Athale, and J. Gracht, "Evolutionary paths in imaging and recent trends," *Optics Exp.*, vol. 11, pp. 2093–2101, 2003.

[7] A. Ashok, P. Baheti, and M. A. Neifeld, "Compressive imaging system design using task-specific information," *Appl. Opt.*, vol. 47, pp. 4457–471, 2008.

[8] A. Oppenheim and D. Johnson, "Computation of spectra with unequal resolution using the fast fourier transform," *Proc. IEEE*, vol. 59, pp. 299–301, 1971.

[9] N. I. Cho and S. K. Mitra, "Warped discrete cosine transform and its application in image compression," *IEEE Trans. Circuits Syst. Video Technol.*, vol. 10, pp. 1364–1373, 2000.

[10] L. R. Rabiner, R. W. Schafer, and C. M. Rader, "The chirp z-transform algorithm and its applications"," *Bell Syst. Tech. J.*, vol. 48, pp. 1249–1292, 1969.

[11] G. Evangelista and S. Cavaliere, "Discrete frequency warped wavelets: Theory and applications," *IEEE Trans. Signal Process.*, vol. 46, pp. 874–885, 1998.

[12] M. H. Asghari and B. Jalali, "Anamorphic transformation and its application to time-bandwidth compression," *Appl. Opt.*, vol. 52, pp. 6735–6743, 2013.

[13] B. Jalali and M. H. Asghari, "The anamorphic stretch transform: Putting the squeeze on big data," *Opt. Photon. News*, vol. 25, pp. 24–31, 2014.

[14] M. H. Asghari and B. Jalali, "Experimental demonstration of optical real-time data compression," *Appl. Phys. Lett.*, vol. 104, no. 111101, pp. 1–4, 2014.

[15] K. Goda, A. Ayazi, D. R. Gossett, J. Sadasivam, C. K. Lonappan, E. Sollier, A. M. Fard, S. C. Hur, J. Adam, C. Murray, C. Wang, N. Brackbill, D. Di Carlo, and B. Jalali, "High-throughput single-microparticle imaging flow analyzer," *Proc. Nat. Acad. Sci.*, vol. 109, pp. 11630–11635, 2012.

[16] D. R. Solli, C. Ropers, P. Koonath, and B. Jalali, "Optical rogue waves," *Nature*, vol. 450, pp. 1054–1057, 2007.

[17] M. H. Asghari and B. Jalali, "Image compression using the anamorphic stretch transform," in *13th IEEE Int. Symp. Signal Processing and Information Technology (ISSPIT 2013)*, Athens, Greece, Paper: S.B2.D.

[18] B. Boashash, "Estimating and interpreting the instantaneous frequency of a signal, I. fundamentals," *Proc. IEEE*, vol. 80, pp. 520–538, 1992.

[19] N. M. Nasrabadi and R. A. King, "Image coding using VQ: A review," *IEEE Trans. Commun.*, vol. 36, pp. 957–971, 1988.

[20] Z. Wang, A. C. Bovik, H. R. Sheikh, and E. P. Simoncelli, "Image quality assessment: From error measurement to structural similarity," *IEEE Trans. Image Process.*, vol. 13, pp. 600–612, 2004.