What if vaccinated individuals could be infected?

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***Abstract*—The COVID–19 pandemic has turned the spotlight on sanitary systems, on the weakness of surveillance network and on the need of models able to describe the different issues of the pandemic and to propose, by analysing possible scenarios, containment measures suitably tailored for the situation. In this paper, starting from the specific current emergency, an improvement of the well known epidemic $SEIR$ model is proposed, adding the categories of subjects that have received the vaccination, P, and V, corresponding to those that have received one or both doses respectively. Based on the current knowledge, the vaccination avoids the severe effects of the virus but not the contagion; moreover it is estimated that after about 8 months the immune memory can decrease, both for vaccinated subjects and healed ones. These two aspects are considered in this work along with the possible scheduling of the containment measures.**

***Index Terms*—epidemic modeling, COVID-19, control actions.**

I. Introduction

Since January 2020 the attention of large part of scientists has been devoted to analyse and face the emergency due to COVID–19 under different points of view.

In particular, starting from the first papers regarding mainly data analysis, for example [1] and [2], a lot of effort is put to describe the pandemic from a dynamical point of view by adapting well known epidemic models, or by introducing ad hoc ones to face the peculiarities of the virus Sars–Cov-2 responsible of the disease, [3], [4], [5], [6], [7], [8], [9], [10], [11], [12].

Up to December 2020, the unique way to mitigate the impact of such a disease and interrupt the virus diffusion was the social distancing, applied by the various countries with different severity. Since January 2021 the vaccination campaign has started, allowing to decrease the number of infected patients, and therefore the death due to COVID–19. In Europe, at the beginning of January, the weekly increase of the number of infected patients was of 18.07%, whereas in May 17 there was a decrease of -17%. The same trend is evident also regarding the weekly change of the number of dead subjects, passed from 0.21% in January to -1.86% at the end of May, [13].

Two issues are of interest in this phase of the pandemic. The first one regards the protection of the vaccine; it is well known that vaccination protects from the severe consequences of the Sars-Cov-2 but the subject can still be infected, especially before completing the cycle of vaccination. Depending on the kind of vaccine, the protection after the first dose is estimated about of 50% after a couple of weeks from the first dose, with significant higher values (up to 90%) after one week from the second dose. The second issue regards how long immunity last after vaccination or after having healed from COVID–19; a recent study assesses that the immune memory after COVID–19 could protect for a period of 8 months, [14].

In this paper, the impact of these two aspects on the virus diffusion is investigated; aiming to focus on the infection and re-infection issues, the COVID–19 is efficiently described with the $SEIR$ model, where S stand for healthy subjects that can be infected, E is the class of exposed individuals, infected but not yet infectious; the class I contains all the people infected and that can infect whereas R is the class of healed subjects. To this basic model two more classes are added, one, P, containing the people that have received one dose of vaccine; the other, V, containing the subjects that have received both doses and have an high level of immunity. Re–infection is introduced after healing or after vaccination. Basically, after having fixed the vaccination strategy (daily number of vaccine administration and kind of vaccine) the unique control parameter regards the *contact rate*, related to the characteristics of the virus and on the social distancing. Some scenarios are analysed to study how the infection could spread, clearly depending on the vaccination campaign, but also on the precautions needed to avoid undesirable new epidemic waves. It is stressed the importance of a continuous monitoring of the number of infected patients to increase, if needed, the social distancing.

The paper is organized as follows. In Section II the proposed model is described and analysed, determining the reproduction number and stressing its dependency from the model parameters. In Section III, after a deep description of the choices of the numerical values for the model parameters, some interesting scenarios are analysed. Conclusions and future work are outlined in Section IV.

II. Materials and methods

In this section a new mathematical model is proposed describing a pandemic situation, with the availability of a vaccination with two doses. Both the subjects that have received one doses (indicated with P, Pre–vaccinated subjects) and those that have completed the vaccination cycle (V, Vaccinated subjects) can be infected and therefore infect, see Fig. 1.

The contact rate of the subjects in P and V classes is of course different with respect to the contact rate regarding the susceptible individuals S. The key role is just represented by

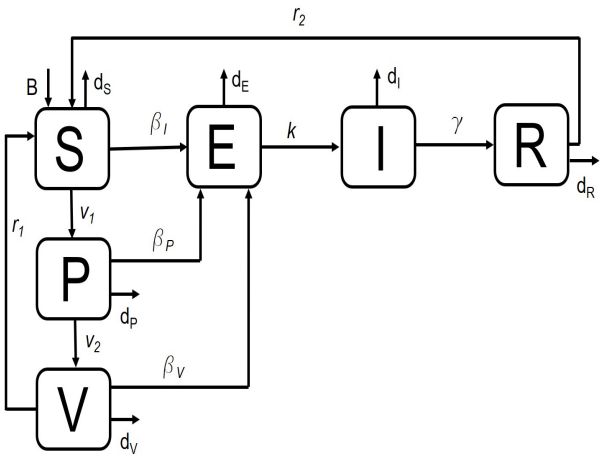

Fig. 1. Block diagram of the proposed model.

the contact rates of individuals in S, P, V; their different values account for the probability of being infected, depending also if the vaccination have occurred and on its efficacy.

A. The mathematical model

The new mathematical model proposed in this paper considers the following classes:

- S: compartment of susceptible subjects, composed by the healthy part of the population which is not vaccinated yet;
- P: class of healthy subjects that received the first vaccination dose;
- V: class of healthy subjects that received both the vaccination doses.
- E: compartment of exposed individuals, i.e. the subjects in the incubation period; they are infected but can not infect;
- I: class of infected patients, that are infected and can infect the susceptible individuals, but also the subjects in P and V compartments;
- R: class of removed subjects, including individuals immunized because healed from the virus.

The six dimensional system proposed may be described as follows:

$$\dot{S} = A - \beta_I SI - d_S S - v_1 S + r_1 V + r_2 R \tag{1}$$

$$\dot{P} = v_1 S - \beta_P PI - d_P P - v_2 P \tag{2}$$

$$\dot{V} = v_2 P - \beta_V VI - d_V V - r_1 V \tag{3}$$

$$\dot{E} = \beta_I SI + \beta_P PI + \beta_V VI - kE - d_E E \tag{4}$$

$$\dot{I} = kE - d_I I - \gamma I \tag{5}$$

$$\dot{R} = \gamma I - d_R R - r_2 R \tag{6}$$

where: d_S, d_P, d_V, d_E, d_I and d_R are the death rates in the class indicated in the subscripts; A is the rate of new incoming individuals; k accounts for the incubation time and γ is related to the healing rate; r_1 and r_2 allow the re-infections after vaccination and healing respectively: they account for the temporary immunity. The parameters v_1 and v_2 regards the rate of vaccination and the interval between the two doses respectively. The contact rates β_I, β_P and β_V take into account the different probabilities of infection, depending on whether the subject is vaccinated or not, and the number of doses received.

The initial condition for the system (1)–(6) is denoted by $\begin{pmatrix} S_0 & P_0 & V_0 & E_0 & I_0 & R_0 \end{pmatrix}^T$.

B. Model analysis

The proposed model is now analysed, aiming at determining the *reproduction number* $\mathcal{R}_0$; it is an important parameter that yields information about the spread of the virus: it represents the number of secondary cases produced by one infected individual in a population of susceptible subjects, in the period in which she/he can infect. By using the approach proposed in [15], the first step is the determination of the *disease free equilibrium* P^e_{DF}. From the system (1)–(6), by equating all the equations to 0, it is obtained:

$$P^e_{DF} = \begin{pmatrix} S_{DFE} & P_{DFE} & V_{DFE} & E_{DFE} & I_{DFE} & R_{DFE} \end{pmatrix}^T$$

with $E_{DFE} = I_{DFE} = R_{DFE} = 0$ and

$$S_{DFE} = \frac{A}{v_1 + d_S - r_1 \bar{v}_1 \bar{v}_2} \tag{7}$$

$$P_{DFE} = \bar{v}_1 S_{DFE} \qquad V_{DFE} = \bar{v}_1 \bar{v}_2 S_{DFE} \tag{8}$$

with

$$\bar{v}_1 = \frac{v_1}{d_P + v_2} \qquad \bar{v}_2 = \frac{v_2}{d_V + r_1} \tag{9}$$

Note that in (7) the quantity $v_1 + d_S - r_1 \bar{v}_1 \bar{v}_2$ is positive, thus guaranteeing the existence of the disease free equilibrium. The calculus of the expression of the reproduction number requires the study of the part of the system (1)–(6) involving the evolution of the subjects infected, that is the individuals belonging to the classes E and I. The reduced system (4)–(5) may be written enhancing the contributions due the infection, $\mathcal{F}$, and the ones due to changing the health condition, $\mathcal{V}$:

$$\begin{pmatrix} \dot{E} \\ \dot{I} \end{pmatrix} = \mathcal{F} - \mathcal{V} \tag{10}$$

where

$$\mathcal{F} = \begin{pmatrix} \beta_I SI + \beta_P PI + \beta_V VI \\ 0 \end{pmatrix} \tag{11}$$

$$\mathcal{V} = \begin{pmatrix} d_E E + kE \\ -kE + d_I I - \gamma I \end{pmatrix} \tag{12}$$

The variations of these matrices with respect to the variables E, I, evaluated in the disease free equilibrium point P^e_{DF}, yield the matrices F and V respectively:

$$F = \left.\frac{\partial \mathcal{F}}{\partial(E,I)}\right|_{P^e_{DF}} = \begin{pmatrix} 0 & \beta_I S_{DEF} + \beta_P P_{DEF} + \beta_V V_{DEF} \\ 0 & 0 \end{pmatrix} \tag{13}$$

and

$$V = \left.\frac{\partial \mathcal{V}}{\partial(E,I)}\right|_{P^e_{DF}} = \begin{pmatrix} d_E + k & 0 \\ -k & d_I + \gamma \end{pmatrix} \tag{14}$$

Under these positions, the reproduction number $\mathcal{R}_0$ is given by the dominant eigenvalue of the matrix FV^{-1}:

$$FV^{-1} = \frac{(\beta_I S_{DFE} + \beta_P P_{DFE} + \beta_V V_{DFE})}{(d_E + k)(d_I + \gamma)} \begin{pmatrix} k & (d_E + k) \\ 0 & 0 \end{pmatrix} \tag{15}$$

the computation easily yields:

$$\mathcal{R}_0 = \frac{k(\beta_I S_{DFE} + \beta_P P_{DFE} + \beta_V V_{DFE})}{(d_E + k)(d_I + \gamma)} \tag{16}$$

By recalling (7) and (8), it can be stressed the role of the contact rates: the $\mathcal{R}_0$ is proportional to the weighted sum of β_I, β_P and β_V. In some epidemic diseases, like the COVID-19, a specific medication is not available; the possibility of reducing the spread mainly depends on improving the social distancing and, when possible, in the availability of a vaccination, as formula (16) evidenced.

III. Numerical results

The proposed model describes common situations in which a vaccinated subject can be infected anyway, with lower probability than in absence of vaccination; the effects of such an infection on the vaccinated patients are in general not fatal, but imply, anyway, a social cost. Moreover, the possibility of having not–permanent immunity, both after having the vaccination or having healed from infection, implies a scenario in which the organization of a vaccination campaign is not an isolated event, but must be a normal prevention campaign. The proposed model describes a quite general epidemic situation, nevertheless, due to the particular historical period, in the numerical analysis that will be carried on we will refer to COVID-19 pandemic that seems to be adequately described by the proposed model in the current phase in which most of the world's population is participating to a vaccination campaign and the antibodies resist for about 8 months.

As far as the choice of the model parameters, some of them are characteristic of COVID-19, and are independent from the specific population; some fluctuations are possible, and the following average values are assumed:

$$k = \frac{1}{6} \qquad \gamma = \frac{1}{10} \tag{17}$$

corresponding to incubation period of 6 days and infection period of 10 days (average values). Note that for this model the patients in the I compartment are those that can infect; this is the reason for which the parameter γ is chosen referring only to the period in which a patient can infect and not to the period requested for healing. As far as the possibility of re-infection, it is supposed that the antibodies of Sars–CoV–2 disappear after about 8 months (about 240 days) from the infection or from the vaccination; thus

$$r_1 = r_2 = \frac{1}{240} \tag{18}$$

The parameter v_1 accounts for the number of daily doses administered, and v_2 is related with the interval between the two doses, considering also that the maximum of immunity is obtained after 2 weeks after the second dose. Referring to N_d doses administered in a day and T days between the doses, it is assumed that

$$v_1 = \frac{N_d}{N} \qquad v_2 = \frac{1}{T + 14} \tag{19}$$

being N the number of individuals in the considered population.

The other parameters are strictly dependent on the population, for example the death rates, and the parameter A. Referring to the population of Italy, from the $ISTAT$ website, [16], the following values have been taken:

$$d_S = d_P = d_V = d_E = d_R = 2.81 \cdot 10^{-5}$$
$$d_I = 1.2 \cdot d_S \qquad A = 1.69 \cdot 10^3 \qquad N = 59 \cdot 10^6 \tag{20}$$

Note that d_I is an average value; in fact, not all the patients have the same death rate, mainly depending on age, but also on whether they have been vaccinated or not. As far as the contact rate β_I, a value of the same order of degree as the one estimated in [5] is assumed:

$$\beta_I = 1.6 \cdot 10^{-9} \tag{21}$$

taking into account that it is influenced by the containment measures applied. The effect of vaccination reduces the probability of being infected, since the administration of the first dose, thus affecting the contact rates β_P and β_V of factors α_P and α_V respectively, depending on the efficacy of the vaccine. In the modelling of these contact rates, it has been decided to include also a factor regarding the possibility of granting more freedom to the vaccinated subjects: these parameters, indicated by F_P and F_V (depending on the number of doses received) obviously imply an increase in the contact rates. The following assumptions have been made:

$$\beta_P = \alpha_P \cdot F_P \cdot \beta_I \qquad \beta_V = \alpha_V \cdot F_V \cdot \beta_I \tag{22}$$

Obviously, also the parameter β_I could be modified, reduced or increased, with a factor F_I, in order to simulate more or less control in social distancing. By varying the parameters related to the restrictions of the containment measures, F_I, F_P, F_V, and the ones related to the efficacy of vaccination, α_P and α_V, interesting scenarios can be analysed. The same initial conditions $\begin{pmatrix} S_0 & P_0 & V_0 & E_0 & I_0 & R_0 \end{pmatrix}^T$ represent an important element of analysis, being different the impact of the vaccination campaign on the basis of the specific period of the pandemic in which it is extensively applied.

As said, the model parameters used have been tailored on the COVID-19 emergency, considering values of the same order of magnitude as the ones identified for the model in [5]; nevertheless, being the herein proposed model simplified with respected to the cited one, and having introduced the two novelties of vaccination and possible re–infection, as a first step it is useful to check whether the model (1)–(6) is able to get the main characteristics of the pandemic, such as the the number of vaccinated subjects (with one and two doses), as well as the number of infected patients. Since the vaccination campaign had strongly different trends in Italy from January to March and from March to May, these two periods are investigated separately, assuming for the period January–February $N_d = 90000$ and $T = 50$, as an average between the interval time requested by the different kind of vaccines available. For the parameters F_I, F_P and F_V the constant value 1.1 is chosen.

For the initial conditions the following realistic values are set:

$$\begin{aligned} S_0 &= 5.5 \cdot 10^7 \quad E_0 = 168954 \quad I_0 = 168954 \\ R_0 &= 1479988 \quad P_0 = 39902 \quad V_0 = 0 \end{aligned} \tag{23}$$

for the first period and

$$\begin{aligned} S_0 &= 5.1 \cdot 10^7 \quad E_0 = 170448 \quad I_0 = 198448 \\ R_0 &= 2.4 \cdot 10^6 \quad P_0 = 3 \cdot 10^6 \quad V_0 = 1.4 \cdot 10^6 \end{aligned} \tag{24}$$

for the second one.

For the period March–April, the average values $N_d = 380000$ and $T = 77$ are chosen; the restrictions of the Easter period are modelled reducing F_I, F_P and F_V to 1. At the end of each period, the modulus of the errors, e_I, e_P and e_V with respect to real data [17] of the number of infected patients, of the subjects vaccinated with one dose and of the individuals vaccinated with two doses are evaluated; these quantities are shown in Table I. The real values used to evaluate the error quantities in Table I for the infected patients are estimated by using the sum of the infected patients identified in the successive 10 days after the end of the period under analysis: for example for the period January-February the number of infected patients assigned to March 1 is given by the sum of the number of identified infected individuals in the first 10 days of March. This is of course an approximation coherent with the meaning of subjects in the class I: according to the proposed model they are the patients that can infect. The results of Table I suggests that the simplified model can be assumed as a satisfactory description of the current situation.

TABLE I
FITTING ERROR

Period	$\|e_I\|$	$\|e_P\|$	$\|e_V\|$
January–February	2.2%	2.0%	6.8%
March– April	10.9%	7.5%	6.3%

The availability of a realistic model allows to investigate interesting scenarios, depending on the daily number of doses administered, N_d, on their period required for immunization, $T + 14$, and on the application of more or less severe containment measures, both for susceptible and vaccinated subjects. The following scenarios, all starting in May 1 2021, are investigated:

- S_1 Scenario 1: $T = 30$ with $F_I = F_P = F_V = 1$; this case corresponds to the application of severe lock down for 365 days, during the vaccination campaign with a vaccine like *Pfizer*, with about one months of interval between the two doses;
- S_2 Scenario 2: $T = 30$ with $F_I = F_P = F_V = 1$ up to day 30 and then $F_I = 1.5; F_P = F_V = 2$; this case corresponds to the strategy in which the severe lock down is applied for one month and successively the containment measures are relaxed of about 30%
- S_3 Scenario 3: $T = 30$ with $F_I = 1$, $F_P = F_V = 1.5$ up to day 30 and then $F_I = F_P = F_V = 2$; this case corresponds to the strategy in which the severe lock down is applied for one month for the subjects in S, while milder restrictions are applied to the subjects in P and V classes; successively the containment measures are relaxed for all the subjects of about 50%;
- S_4 Scenario 4: like Scenario 3 with $T = 77$, corresponding to a vaccine like *Astrazeneca* with an interval between the two doses of more than two months;
- S_5 Scenario 5: again with $T = 77$, with $F_I = F_P = F_V = 1.5$ up to day 30 and then strongly reduced control actions with $F_I = F_P = F_V = 3$.

In all the cases considered, an average value of 400000 daily vaccination is assumed. To compare the consequences of the choices of each scenarios, there are evaluated also the normalized difference of the number of infected patients I, subjects vaccinated with one dose P and subjects vaccinated with two doses V with respect to the same quantities of the reference condition of Scenario 1, indicated with e_I, e_P, and e_V respectively; moreover also the peak of infected patients M_I and the period of its occurrence is found. The results are summarised in Table II. Of course, the most conservative scenario is the S_1, with only 3 infected patients after one year; nevertheless, it is also the less realistic, for economic and social reasons. The significant differences between scenarios S_2 and S_3 are due to the more or less prolonged strong containment measures: it appears important to consolidate the decrease of the infection before relaxing the control effort. The use of a vaccine with long interval time between two doses (parameter T) seems to affect the peak of infected patients when compared with analogous conditions (scenarios S_3 and S_4), but the general trend appears similar. The introduction of reduced control actions, as in scenario 5, represents a strong hazard, being in that case possible a significant increase in the number of infected patients. It can be noted that in the cases S_3–S_5 the peaks of infected patients occur after about 4 months from the beginning of the analysis; this suggests the importance of monitoring the trend of the epidemic after relaxing containment measures. To show the impact of th different

control strategies in the realistic scenarios S_i, $i = 1, \ldots, 4$ the trends of the number of infected patients and of the vaccinated individuals are shown in Fig.2 and Fig. 3.

By analysing the mentioned scenarios, it is possible to propose a strategy, acceptable from social and economical points of view, to reduce the impact of pandemic also allowing to trace the contacts in order to reduce the diffusion. The possibility of tracing the contacts of an infected subjects depends on the percentage of infected individuals per 100000 habitants. For example, aiming at reducing the infected patients under the daily threshold of 250, by using a two doses vaccine with $T = 77$ and increasing the daily administered doses up to 500000, the tracing possibility may be obtained at day 43 by prolonging the severe measures for non vaccinated subjects up to 2 months, while the subjects in P and V conditions could mildly relax their condition ($F_P = F_V = 1.5$); then it could be chosen $F_I = 1.5$ and $F_P = F_V = 2$, thus allowing a sensible decrease in restrictions. In Fig. 4 this situation is shown (continuous line) along with other two possible strategies. In the less severe one (dotted line) after two months, for the vaccinated individuals (both the ones in P and in V conditions) restrictions are almost totally relaxed ($F_P = F_V = 3$), leaving them unchanged for the non vaccinated individuals. It can be seen that the tracing is possible only for a limited period (about from day 50 to 100); the interesting situation is the bold one, in which after day 60 for the subjects if P and V less restrictions are allowed ($F_P = F_V = 2.5$) without exceeding the threshold (horizontal dotted line positioned at value 147500 corresponding to 250 people over 100000 in a population of $59 \cdot 10^5$ habitants, as in Italy).

All the scenarios studied up to now consider daily administration of 400000 doses; it is quite intuitive that if the interval between the doses is augmented (and therefore the almost total immunity is reached with some delay) the peak of infected patients M_I is increased. Nevertheless, with higher daily doses administration the peak could be reduced. For example, the Scenario S_4, that represents a worrying situation, could halve the peak of infected patients by applying 700000 daily administrations; in that case the peak of infected patients should occur at day 98 with $3.2 \cdot 10^5$ individuals.

It is useful to study the possible effects of reduction of antibodies, and therefore coming back to the susceptibility condition after an average period of 8 months, both for healed subjects and vaccinated ones; this analysis is performed on a time interval of 5 years, simulating a period of 3 months in which $F_I = 1.5$, $F_P = 2$ and $F_V = 3$, thus implying a distinction also among people in the P and V conditions, and then $F_I = 2$, $F_P = 3$ and $F_V = 3$, corresponding to relaxing the restriction, but preserving still some attentions to the subjects in S that have lost the immunity. In Fig. 5 the trends of the quantities in the S, E, I, R, P, V classes are shown; it can be noted an oscillating trend up to year 3 for all the quantities. In particular, the evolution of the number of infected patients reaches its minimum before the end of the second year, but then it increases again, remaining anyway limited under 330000 units. This suggests again a careful monitoring of the epidemic situation, eventually increasing the rate of vaccination or, at least for limited period, increase the containment measures.

TABLE II
SCENARIOS

	S_1	S_2	S_3	S_4	S_5
$I(365)$	3	7633	111810	107190	407260
$\lvert e_I \rvert$ (%)		$2.5 \cdot 10^5$	$3.7 \cdot 10^6$	$3.5 \cdot 10^6$	$1.3 \cdot 10^7$
$P(365)$	$6.2 \cdot 10^6$	$6.1 \cdot 10^6$	$5.6 \cdot 10^6$	$1 \cdot 10^7$	$8.2 \cdot 10^6$
$\lvert e_P \rvert$ (%)		0.67	9.1	69.8	32
$V(365)$	$3.1 \cdot 0^7$	$3 \cdot 10^7$	$2.8 \cdot 10^7$	$2.4 \cdot 10^7$	$1.4 \cdot 10^7$
$\lvert e_V \rvert$ (%)		1.15	11.9	23.3	55.7
M_I					
Day	7	7	139	142	106
Patients	$2.1 \cdot 10^5$	$2.1 \cdot 10^5$	$6.8 \cdot 10^5$	$7.6 \cdot 10^5$	$4.8 \cdot 10^6$

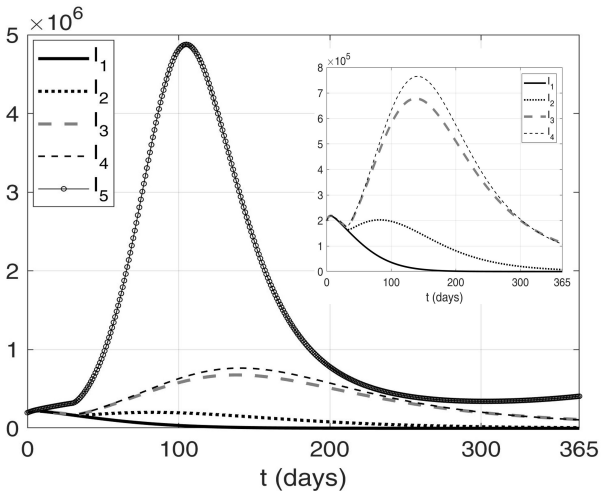

Fig. 2. Infected patients in the scenarios S_1, S_2, S_3, S_4 and S_5.

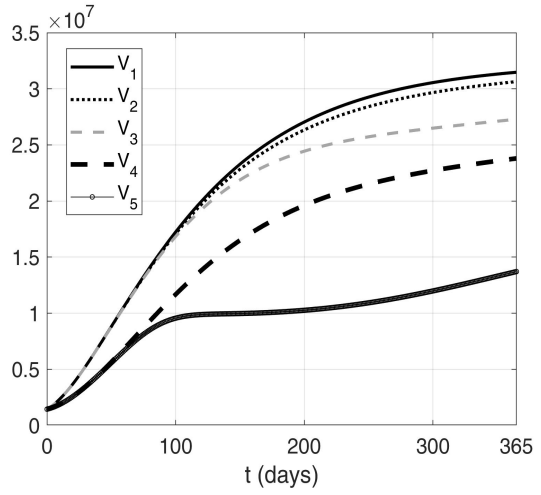

Fig. 3. Vaccinated individuals in the scenarios S_1, S_2, S_3, S_4 and S_5.

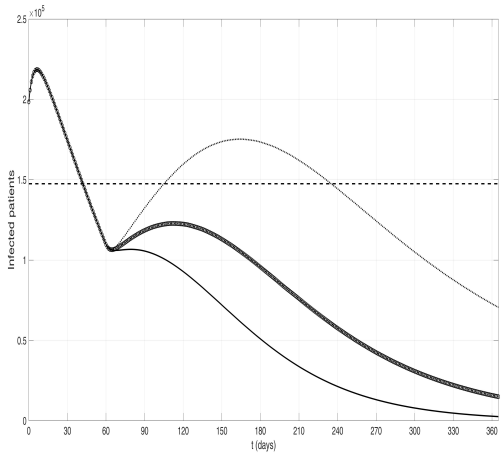

Fig. 4. Infected patients evolution; up to day 60: $FI = 1$, $F_P = F_V = 1.5$. After day 60 three cases. 1) Continuous line: $F_I = 1.5$, $F_P = F_V = 2$; 2) Dotted line: $F_I = 1.5$, $F_P = F_V = 3$; 3) Bold line: $F_I = 1.5$, $F_P = F_V = 2.5$;

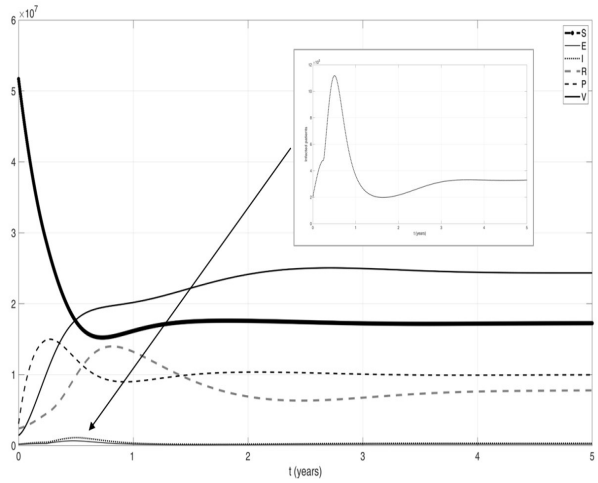

Fig. 5. Effects of lost of immunity over 5 years; evolution of the number of subjects in S, E, I, R, P, V classes; in the small box evolution of the number of infected patients in I.

IV. Conclusions

In this paper there are investigated possible scenarios when there is an epidemic for which a vaccination is available, assuming possible a re–infection after some months from vaccination or after healing from the infection. This framework well describes the current phase of COVID-19, that, after about 15 months from the official statement of pandemia by the World Health Organization, seems to be under control in Europe and North America. Nevertheless, the progressive decrease in the containment measures should be applied taking into account parameters related to the kind of vaccine used. It appears evident that the lowest number of infected patients is obtained in the discussed scenario 1 in which the severe containment measures are prolonged for more than one year. This scenario appears not realistic at this stage of pandemic, being not acceptable for economic and social reasons; nevertheless it is a reference one for other more sustainable possible choices. The results suggest not to cancel at once all the controls but to preserve, especially for non vaccinated subjects, a sufficient level of cautiousness. Despite the simplicity of the model, it is possible to improve it by considering different kind of vaccination as well as the introduction of a medication specific for COVID–19. Moreover the approach proposed could be applied to other infectious diseases for which a vaccination is available but with temporary immunity only.

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