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Estimation of Line Parameters, Tap Changer Ratios, and Systematic Measurement Errors Based on Synchronized Measurements and a General Model of Tap-Changing Transformers

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ABSTRACT A primary requirement for the transmission system operator is an accurate knowledge of grid parameters. Moreover, the availability of effective and accurate monitoring tools allows the proper operation of power transmission grids. However, in spite of the now widespread possibility of having monitoring systems based on synchronized measurements, the monitoring applications can be affected not only by the inevitable uncertainty sources but also by the simplified or incomplete modeling of the network components. For this reason, the impact on power system monitoring and control applications of tap-changing transformer models is a key point. In this scenario, this article presents a method to estimate simultaneously line parameters, tap changer ratios, and systematic measurement errors associated with the instrument transformers. The method exploits a flexible model of the tap-changing transformer based on a parameter representing the ratio between the two winding impedances of the transformer. The proposal is based also on the suitable modeling of the measurement chain and on the constraints introduced by the equations of involved transmissions lines and transformers. The validation has been carried out by means of tests performed on the IEEE 14 Bus Test Case.

INDEX TERMS Current measurement, instrument transformers, measurement errors, parameter estimation, phasor measurement units (PMUs), power transmission lines, tap changers, voltage measurement.

I. INTRODUCTION

ACCURATE modeling of network components is increasingly in demand for modern power systems. In this context, the models of power transformers and particularly of tap-changing transformers can be critical.

Two are the most used tap-changing transformer models for power system studies (see, [1], [2], [3]), considered not only in the literature but also in software packages implementation. These two models represent the tap-changing transformer as a π-branch assuming that the transformer short circuit impedance is provided either by nominal winding or by the tapped winding. The values for the model parameters can be obtained from the nameplate data provided by manufacturers, reporting, for example, the short circuit impedance. Nevertheless, it is important to highlight that, commonly, the nameplate data do not provide detailed data on the nominal and tapped windings [4], because manufacturers are not compelled by regulations to make this information available to customers.

These models have been largely applied in studies concerning power systems: in [5], a single-phase estimator of both power system state and transformer tap positions is

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addressed; in [6], an augmented estimation method, based on phasor measurement units (PMUs), is proposed; in [7], a linear estimator is presented, to evaluate transmission-line and transformer parameters from unsynchronized supervisory control and data acquisition (SCADA) measurements; in [8], a steady-state power flow analysis algorithm is presented.

It is worth noting that tap-changing transformers are a crucial and critical component of the power system [9], [10]. In particular, inaccurate modeling of these transformers may affect several applications, such as voltage regulations, fault location identification, voltage stability analysis [11], power flow algorithms, and state estimation [12]. In the literature, several papers discussed this topic. In [13], for example, it is shown how the use of the traditional models of tap-changing transformers can lead to inconsistencies. In [11], an improved model for tap-changing transformers taking into account the impedances in each transformer winding and particularly suited to voltage stability studies is presented. In [14], a general tap-changing transformer model is proposed. This model includes a parameter k, which represents the nominal and tapped winding impedance ratio. Cano *et al.* [14] showed that the use of traditional models can be critical in case the operating point of the transformer is at extreme tap positions.

In this context, this article presents a novel estimation method for line parameters, tap ratios, and systematic errors that involves a general model of the tap-changing transformers and is based on measurements from PMUs. In particular, the new method includes the parameter k introduced in [14] to use the information available on the windings. The proposed methodology is then intended to improve the estimation, overcoming the criticality due to a model that does not include all the available information on the tap-changing transformers. In fact, even if the widespread availability of PMUs could allow to follow the evolution of voltages and currents and to manage the grid under some circumstances, it is important to highlight that individual measurements and network status monitoring can be significantly enhanced if parameters and systematic errors in the measurement chain are accurately estimated.

The proposed methodology is inspired by the outcomes of [15] and [16], where a traditional model of the tap-changing transformer was applied and, therefore, the simultaneous estimation of systematic measurement errors along with the parameters associated with π-models of transmission lines and tap-changing transformers is conducted: both the systematic and random errors of the measurement chain affecting synchronized measurements in modern power systems are appropriately modeled and taken into account. With respect to [15] and [16], the more general model of tap-changing transformer is now introduced in the estimation framework, which is therefore completely redesigned relying on new constraints among the measurements and the quantities to estimate.

The impact on the estimation performance of such a more complete model of tap-changing transformers within the estimator is analyzed. In addition, the impact of a mismatch between the actual impedances of the transformer windings and those modeled in the estimator is discussed, also in comparison with the approach based on a traditional model. The generalized approach leads to two main advantages with respect to previous methods: 1) flexibility and 2) robustness. It is indeed possible to include into the model all information available from the manufacturer. Moreover, in the absence of *a priori* knowledge on the transformer impedances, it is possible to choose a model that is more robust (thus leading to more accurate estimates) in case of a mismatch between actual and available information on the impedance configuration between the two sides of the transformer.

Performance evaluation tests are carried out on the IEEE 14 Bus Test Case. Both single-tap changer and full grid parameters estimations are applied. Different operating conditions and uncertainty sources are considered. Appropriate comparisons are then made, which highlight the effectiveness of the proposal.

This article is structured as follows: in Section II, the line and tap-changing transformer parameter estimation method that embeds the generalized model is presented; in Section III, performance evaluation tests for both single and multiple-branch approaches are considered and the results are analyzed and discussed. Section IV concludes this article, providing the final remarks.

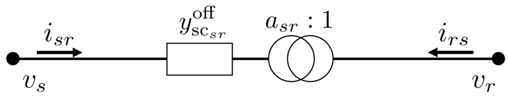

FIGURE 1. Tap-changing transformer model with short circuit impedance at the off-nominal turns side.

II. PROPOSED METHOD

A power network can be modeled as a set of branches, which are either transmission lines or branches corresponding to tap-changing transformers. In this article, both transformers and transmission lines are treated with π-equivalent models and, for the sake of an effective and simpler introduction, a single-phase model is considered. A detailed description of the models considered for each network element and for the measurement chain is reported in the following, along with the definition of the estimation problem.

A. TAP-CHANGING TRANSFORMER AND MEASUREMENT UNCERTAINTY MODELS

Tap-changing transformers can be represented with an impedance in series with off-nominal turn ratio as shown in Fig. 1 [14], where the off-nominal short circuit admittance $y^{\text{off}}_{\text{sc}_{sr}}$ can be defined depending on the assumed model. In particular, the ratio between the impedances of the nominal winding and of the tapped winding can be taken into account and described by a parameter k [14]. In this case,

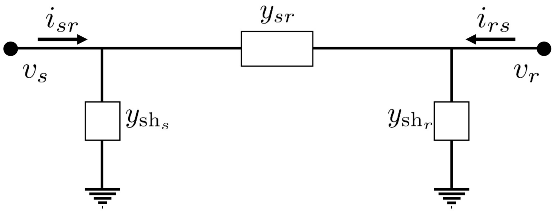

FIGURE 2. π-model of the tap-changing transformer.

$y^{\mathrm{off}}_{\mathrm{sc}_{sr}}$ can be obtained as

$$y^{\mathrm{off}}_{\mathrm{sc}_{sr}} = \frac{1}{z_{\mathrm{tap}} + a_{sr}^2 z_{\mathrm{nom}}} = \frac{(1+k)}{\left(1 + a_{sr}^2 k\right)} y_{\mathrm{sc}_{sr}} \tag{1}$$

where $y_{\mathrm{sc}_{sr}}$ represents the short circuit admittance of the power transformer (parameter provided by the manufacturer), a_{sr} is the tap changer ratio, and k is the ratio between z_{nom} and z_{tap}. The π-equivalent model of a tap-changing transformer is shown in Fig. 2, for a generic transformer branch (s, r). It is important to recall that the shunt admittances y_{sh_s} and y_{sh_r} are not negligible in the case of off-nominal turns ratio. The model can be represented by means of its nodal admittance matrix $\mathbf{Y}_{\mathrm{tap}}$, which can be written as follows:

$$\mathbf{Y}_{\mathrm{tap}} = \begin{bmatrix} \mathbf{Y}_{\mathrm{tap}_{ss}} & \mathbf{Y}_{\mathrm{tap}_{sr}} \\ \mathbf{Y}_{\mathrm{tap}_{rs}} & \mathbf{Y}_{\mathrm{tap}_{rr}} \end{bmatrix} = \begin{bmatrix} y_{\mathrm{tap}_{ss}} & y_{\mathrm{tap}_{sr}} \\ y_{\mathrm{tap}_{rs}} & y_{\mathrm{tap}_{rr}} \end{bmatrix} \tag{2}$$

where $\mathbf{Y}_{\mathrm{tap}}$ can be divided into its submatrices associated with the corresponding sending node (node s, i.e., subscript ss), receiving node (node r, i.e., subscript rr) and with mutual interactions (subscripts sr and rs). In the last equality, according to the single-phase model, each submatrix is a complex value with analogous meaning.

The components of the corresponding π-model shown in Fig. 2 can be obtained as follows:

$$\begin{aligned} y_{sr} &= -y_{\mathrm{tap}_{sr}} = \frac{a_{sr}(1+k)}{\left(1 + a_{sr}^2 k\right)} y_{\mathrm{sc}_{sr}} \\ y_{\mathrm{sh}_s} &= y_{\mathrm{tap}_{ss}} + y_{\mathrm{tap}_{sr}} = \frac{(1-a_{sr})(1+k)}{\left(1 + a_{sr}^2 k\right)} y_{\mathrm{sc}_{sr}} \\ y_{\mathrm{sh}_r} &= y_{\mathrm{tap}_{rr}} + y_{\mathrm{tap}_{sr}} = \frac{a_{sr}(a_{sr}-1)(1+k)}{\left(1 + a_{sr}^2 k\right)} y_{\mathrm{sc}_{sr}}. \end{aligned} \tag{3}$$

The general model of the tap-changing transformer considered in this article is then function of three parameters a_{sr}, $y_{\mathrm{sc}_{sr}}$, and k (see [14] for details).

To monitor these network components, in this article, the presence of PMUs on both sides of every considered transmission line [generic branch (i, j)] and tap-changing transformer [corresponding to branch (s, r) as in Fig. 2] is assumed. Taking as an example the generic tap-changing transformer branch (analogous measurements are supposed for the generic line), the following measurement configuration is thus provided:

1) two voltage synchrophasor measurements v_s and v_r, at the start and end node, respectively;
2) two current synchrophasor measurements flowing from the start and end nodes, i.e., i_{sr} and i_{rs}, respectively.

These synchrophasor measurements are referred to the same time instant t, i.e., an coordinated universal time (UTC) timestamp. The π-models of transmission lines and transformers, like in [16], allow the definition of a measurement model that links synchrophasor measurements to the magnitude and phase-angle measurement errors and to line parameters deviations from their nominal (or available) values. Thanks to this model, it is possible to express each synchrophasor measurement as a function of its reference value (indicated in this article with superscript ^{R}) and all associated measurement errors as follows:

$$\begin{aligned} v_h &= V_h e^{j\varphi_h} = V_h^r + jV_h^x \\ &= \left(1 + \xi_h^{\mathrm{sys}} + \xi_h^{\mathrm{rnd}}\right) V_h^R e^{j\left(\varphi_h^R + \alpha_h^{\mathrm{sys}} + \alpha_h^{\mathrm{rnd}}\right)} \\ i_{sr} &= I_{sr} e^{j\theta_{sr}} = I_{sr}^r + jI_{sr}^x \\ &= \left(1 + \eta_{sr}^{\mathrm{sys}} + \eta_{sr}^{\mathrm{rnd}}\right) I_{sr}^R e^{j\left(\theta_{sr}^R + \psi_{sr}^{\mathrm{sys}} + \psi_{sr}^{\mathrm{rnd}}\right)} \end{aligned} \tag{4}$$

where V_h and φ_h are the amplitude and phase-angle voltage synchrophasor measurements of node h (with $h \in \{s, r\}$) while I_{sr} and θ_{sr} are the measured amplitude and phase-angle of the branch current flowing from node s to node r (similar definitions can be obtained for the measured current synchrophasor i_{rs}, flowing from node r to node s). The quantities ξ_h and η_{sr} indicate the aforementioned errors in voltage and current magnitude (ratio errors), while α_h and ψ_{sr} represent voltage and current phase errors. Finally, $^{\mathrm{sys}}$ and $^{\mathrm{rnd}}$ refer to the systematic and random errors, respectively. In this work, systematic measurement errors (thus considered the same across repeated measurements) are associated mainly with ITs and random errors (different for each observation) with PMUs. Considering IT accuracy classes [17], [18], and PMU specifications [19], it is realistic to assume that all these magnitude and phase-angle errors are small, i.e., their absolute values are $\ll 1$. Thanks to this assumption, it is possible to adopt a first-order approximation and rewrite (4) to express the reference values as a function of the measurements and the above-defined errors

$$\begin{aligned} v_h^R &\simeq \left(V_h^r + jV_h^x\right)\left(1 - \xi_h^{\mathrm{sys}} - \xi_h^{\mathrm{rnd}} - j\alpha_h^{\mathrm{sys}} - j\alpha_h^{\mathrm{rnd}}\right) \\ i_{sr}^R &\simeq \left(I_{sr}^r + jI_{sr}^x\right)\left(1 - \eta_{sr}^{\mathrm{sys}} - \eta_{sr}^{\mathrm{rnd}} - j\psi_{sr}^{\mathrm{sys}} - j\psi_{sr}^{\mathrm{rnd}}\right) \end{aligned} \tag{5}$$

where $v_h = V_h^r + jV_h^x$ and $i_{sr} = I_{sr}^r + jI_{sr}^x$, and an analogous expression can be also used for i_{rs}. The validity of the approximation used to obtain (5) has been also verified through Monte Carlo (MC) trials using the same two measurement scenarios of Section III. The approximation has been evaluated by means of the introduced relative phasor error (i.e., the total vector error, TVE). Considering v_h^R (similar results can be obtained also for current synchrophasor measurements), the additional TVE due to the approximation is, in the worst case, two order of magnitudes lower than the TVE introduced by the PMU measurement chain, thus confirming the validity of the approximation used in (5). For instance, in the best measurement scenario of Section III, the maximum additional TVE is $4\cdot 10^{-3}$ % while the corresponding TVE of the measurement chain is 0.91 %.

The transformer short circuit impedance can be defined as $z_{\mathrm{sc}_{sr}} = (1/y_{\mathrm{sc}_{sr}})$ (indicated in the following as z_{sr}) and, thus, the tap-changing transformer parameters can be expressed as

$$\begin{aligned} z_{sr} &= jX^0_{sr}(1+\beta_{sr}) \\ a_{sr} &= a^0_{sr}(1+\tau_{sr}) \end{aligned} \tag{6}$$

where β_{sr} and τ_{sr} are the unknown relative deviations from the nominal transformer reactance and tap ratio, while X^0_{sr} and a^0_{sr} are the associated nominal (or available) values. Like in [16], tap changer ratios are here considered uncertain because they cannot be known exactly.

All things considered, focusing on the generic branch (s, r), the following elements are unknown and can be estimated from the measurements: β_{sr}, τ_{sr}, ξ^{sys}_s, ξ^{sys}_r, η^{sys}_{sr}, η^{sys}_{rs}, α^{sys}_s, α^{sys}_r, ψ^{sys}_{sr}, and ψ^{sys}_{rs}.

In order to estimate these quantities, it is important to define the constraints that link measured values and measurement errors to parameters. Using the nodal admittance matrix (2) that connects the branch current and nodal voltage phasors and the expressions in (3), it is possible to define two complex equations for each transformer π-model. In particular, the following equations hold true:

$$(1+k)(v^R_s - a_{sr}v^R_r) = (1+ka^2_{sr})z_{sr}i^R_{sr} \tag{7}$$

$$i^R_{rs} = -a_{sr}i^R_{sr}. \tag{8}$$

Then, according to the first-order approximation for the voltage and current phasors in (5) and transformer parameters definitions (6), it is possible to obtain two complex-valued linear equations in the unknowns or, using rectangular coordinates, to write four real-valued equations for the considered tap-changing transformer. From (7), the two following linear equations (corresponding to real and imaginary parts) can indeed be obtained neglecting all second-order terms

$$\begin{aligned} &(1+k)V^r_s - a^0_{sr}(1+k)V^r_r + \left(1+ka^{0\,2}_{sr}\right)X^0_{sr}I^x_{sr} \\ &\simeq \left(\xi^{\mathrm{sys}}_s + \xi^{\mathrm{rnd}}_s\right)(1+k)V^r_s - \left(\alpha^{\mathrm{sys}}_s + \alpha^{\mathrm{rnd}}_s\right)(1+k)V^x_s \\ &\quad - \left(\xi^{\mathrm{sys}}_r + \xi^{\mathrm{rnd}}_r\right)a^0_{sr}(1+k)V^r_r \\ &\quad + \left(\alpha^{\mathrm{sys}}_r + \alpha^{\mathrm{rnd}}_r\right)a^0_{sr}(1+k)V^x_r \\ &\quad + \left(\eta^{\mathrm{sys}}_{sr} + \eta^{\mathrm{rnd}}_{sr}\right)\left(1+ka^{0\,2}_{sr}\right)X^0_{sr}I^x_{sr} \\ &\quad + \left(\psi^{\mathrm{sys}}_{sr} + \psi^{\mathrm{rnd}}_{sr}\right)\left(1+ka^{0\,2}_{sr}\right)X^0_{sr}I^r_{sr} \\ &\quad - \beta_{sr}\left(1+ka^{0\,2}_{sr}\right)X^0_{sr}I^x_{sr} \\ &\quad + \tau_{sr}\left[a^0_{sr}(1+k)V^r_r - 2ka^{0\,2}_{sr}I^x_{sr}X^0_{sr}\right] \end{aligned} \tag{9}$$

$$\begin{aligned} &(1+k)V^x_s - a^0_{sr}(1+k)V^x_r - \left(1+ka^{0\,2}_{sr}\right)X^0_{sr}I^r_{sr} \\ &\simeq \left(\xi^{\mathrm{sys}}_s + \xi^{\mathrm{rnd}}_s\right)(1+k)V^x_s + \left(\alpha^{\mathrm{sys}}_s + \alpha^{\mathrm{rnd}}_s\right)(1+k)V^r_s \\ &\quad - \left(\xi^{\mathrm{sys}}_r + \xi^{\mathrm{rnd}}_r\right)a^0_{sr}(1+k)V^x_r \\ &\quad - \left(\alpha^{\mathrm{sys}}_r + \alpha^{\mathrm{rnd}}_r\right)a^0_{sr}(1+k)V^r_r \\ &\quad - \left(\eta^{\mathrm{sys}}_{sr} + \eta^{\mathrm{rnd}}_{sr}\right)\left(1+ka^{0\,2}_{sr}\right)X^0_{ij}I^r_{sr} \\ &\quad + \left(\psi^{\mathrm{sys}}_{sr} + \psi^{\mathrm{rnd}}_{sr}\right)\left(1+ka^{0\,2}_{sr}\right)X^0_{sr}I^x_{sr} \\ &\quad + \left(1+ka^{0\,2}_{sr}\right)\beta_{sr}X^0_{sr}I^r_{sr} \\ &\quad + \tau_{sr}\left[a^0_{sr}(1+k)V^x_r + 2ka^{0\,2}_{sr}I^r_{sr}X^0_{sr}\right]. \end{aligned} \tag{10}$$

Using the current balance (8), other two real-valued equations can be obtained

$$\begin{aligned} &I^r_{rs} + a^0_{sr}I^r_{sr} \\ &\simeq \left(\eta^{\mathrm{sys}}_{rs} + \eta^{\mathrm{rnd}}_{rs}\right)I^r_{rs} - \left(\psi^{\mathrm{sys}}_{rs} + \psi^{\mathrm{rnd}}_{rs}\right)I^x_{rs} \\ &\quad + \left(\eta^{\mathrm{sys}}_{sr} + \eta^{\mathrm{rnd}}_{sr}\right)a^0_{sr}I^r_{sr} - \left(\psi^{\mathrm{sys}}_{sr} + \psi^{\mathrm{rnd}}_{sr}\right)a^0_{sr}I^x_{sr} \\ &\quad - \tau_{sr}a^0_{sr}I^r_{sr} \end{aligned} \tag{11}$$

$$\begin{aligned} &I^x_{rs} + a^0_{sr}I^x_{sr} \\ &\simeq \left(\eta^{\mathrm{sys}}_{rs} + \eta^{\mathrm{rnd}}_{rs}\right)I^x_{rs} + \left(\psi^{\mathrm{sys}}_{rs} + \psi^{\mathrm{rnd}}_{rs}\right)I^r_{rs} \\ &\quad + \left(\eta^{\mathrm{sys}}_{sr} + \eta^{\mathrm{rnd}}_{sr}\right)a^0_{sr}I^x_{sr} + \left(\psi^{\mathrm{sys}}_{sr} + \psi^{\mathrm{rnd}}_{sr}\right)a^0_{sr}I^r_{sr} \\ &\quad - \tau_{sr}a^0_{sr}I^x_{sr}. \end{aligned} \tag{12}$$

B. TRANSMISSION LINE MODEL

In this article, the simultaneous estimation of the transformer parameters discussed in Section II-A and of transmission line parameters is proposed. For this reason, the generic transmission line (i, j) is also considered, assuming the presence of PMUs on both sides of the branch. Also in this case, two complex equations that represent the line voltage drop and current balance equations can be defined as follows:

$$\begin{aligned} \left(v^R_i - v^R_j\right) &= z_{ij}\left(i^R_{ij} - j\frac{B_{\mathrm{sh},ij}}{2}v^R_i\right) \\ \left(i^R_{ij} + i^R_{ji}\right) &= j\frac{B_{\mathrm{sh},ij}}{2}\left(v^R_i + v^R_j\right) \end{aligned} \tag{13}$$

where z_{ij} is the line impedance of the branch and $B_{\mathrm{sh},ij}$ its shunt susceptance divided in two in the π-model. All the other symbols have analogous meaning as in (4) and (5). It is possible to use (13) similarly to (7) and (8) with the same assumptions as in Section II-A, thus writing the reference phasors v^R_i, v^R_j, i^R_{ij}, and i^R_{ji} as the function of measured values, measurement errors, and line parameter deviations. The counterparts of (9)–(12) can be defined for every transmission line (i, j) (see [20] for details). In particular, transmission line parameters are expressed in terms of the parameter values currently available to the transmission system operator and their relative deviations as follows:

$$\begin{aligned} z_{ij} &= R^0_{ij}\left(1+\gamma_{ij}\right) + jX^0_{ij}\left(1+\beta_{ij}\right) \\ B_{\mathrm{sh},ij} &= B^0_{\mathrm{sh},ij}\left(1+\rho_{ij}\right) \end{aligned} \tag{14}$$

where γ_{ij}, β_{ij}, and ρ_{ij} are the unknowns relative deviations for the resistance, reactance, and transversal susceptance, while R^0_{ij}, X^0_{ij}, and $B^0_{\mathrm{sh},ij}$ are the corresponding nominal values.

C. TRANSFORMER AND LINE PARAMETERS ESTIMATION METHOD

The assumptions about the measurement chain model (see Section II-A) and the relationships including deviations of line and transformer parameters defined in (6) and (14) can be used to obtain a system of linear approximated equations for each tap-changing transformer expressed by the general model (3) [(9)–(12)] and for each transmission line.

Considering, for the sake of brevity, only the case of the generic tap-changing transformer branch (s, r), a system of four linear equations can be written for each PMU measurement timestamp t as follows:

$$\mathbf{b}_{sr,t} = \mathbf{H}_{sr,t} \begin{bmatrix} \xi_s^{\text{sys}} \\ \alpha_s^{\text{sys}} \\ \xi_r^{\text{sys}} \\ \alpha_r^{\text{sys}} \\ \eta_{sr}^{\text{sys}} \\ \psi_{sr}^{\text{sys}} \\ \eta_{rs}^{\text{sys}} \\ \psi_{rs}^{\text{sys}} \\ \beta_{sr} \\ \tau_{sr} \end{bmatrix} + \mathbf{E}_{sr,t} \begin{bmatrix} \xi_{s,t}^{\text{rnd}} \\ \alpha_{s,t}^{\text{rnd}} \\ \xi_{r,t}^{\text{rnd}} \\ \alpha_{r,t}^{\text{rnd}} \\ \eta_{sr,t}^{\text{rnd}} \\ \psi_{sr,t}^{\text{rnd}} \\ \eta_{rs,t}^{\text{rnd}} \\ \psi_{rs,t}^{\text{rnd}} \end{bmatrix}$$
$$= \mathbf{H}_{sr,t}\mathbf{x}_{sr} + \mathbf{E}_{sr,t}\mathbf{e}_{sr,t} = \mathbf{H}_{sr,t}\mathbf{x}_{sr} + \boldsymbol{\epsilon}_{sr,t} \quad (15)$$

where $\mathbf{b}_{sr,t}$ is the vector of known terms of the associated measurements equations, which are the first terms in (9)–(12), and $\mathbf{H}_{sr,t}$ is the measurement matrix linking all the components of the state vector of unknowns $\mathbf{x}_{sr}$ to $\mathbf{b}_{sr,t}$. $\mathbf{E}_{sr,t}$ is the Jacobian matrix transforming the random errors $\mathbf{e}_{sr,t}$ associated with t into the vector of equivalent measurement random errors $\boldsymbol{\epsilon}_{sr,t}$. A similar system of equations can be defined for the generic transmission line (i, j) as discussed in Section II-B.

When considering one timestamp t, there are four approximated linear equations and ten unknowns (the number of unknowns for a transmission line is typically 11). Multiple PMU measurements (indeed, multiple time instants $t_1, \ldots, t_{M_t}$) can be used to estimate $\mathbf{x}_{sr}$ (see [21], [22], [23]), thus permitting to define an augmented multiple-timestamp system of equations as follows:

$$\mathbf{b}_{sr} = \begin{bmatrix} \mathbf{H}_{sr,t_1} \\ \vdots \\ \mathbf{H}_{sr,t_{M_t}} \end{bmatrix} \mathbf{x}_{sr} + \begin{bmatrix} \mathbf{E}_{sr,t_1} & \mathbf{0} & \\ \mathbf{0} & \ddots & \\ & & \mathbf{E}_{sr,t_{M_t}} \end{bmatrix} \begin{bmatrix} \mathbf{e}_{sr,t_1} \\ \vdots \\ \mathbf{e}_{sr,t_{M_t}} \end{bmatrix}$$
$$= \mathbf{H}_{sr}\mathbf{x}_{sr} + \mathbf{E}_{sr}\mathbf{e}_{sr} = \mathbf{H}_{sr}\mathbf{x}_{sr} + \boldsymbol{\epsilon}_{sr} \quad (16)$$

where $\mathbf{b}_{sr}$ is the vector containing all known terms for timestamps $t_1, \ldots, t_{M_t}$ and $\mathbf{H}_{sr,t_i}$, $\mathbf{E}_{sr,t_i}$, and $\mathbf{e}_{sr,t_i}$ have a similar meaning to that in (15) for $i \in \{1, 2, \ldots, M_t\}$. $\mathbf{H}_{sr}$, $\mathbf{E}_{sr}$, and $\boldsymbol{\epsilon}_{sr}$ are thus the overall measurement matrix, random error transformation matrix, and equivalent random error vector, respectively, for the new problem.

The problem defined by (16) can be considered as composed of equations corresponding to different operating conditions of the grid (from here on called "cases") or to repeated measurements of the same condition obtained thanks to PMU high reporting rate. Furthermore, new equations can be added to (16) exploiting all the prior information about network parameters and systematic measurement errors to be estimated. The following problem can be written:

$$\mathbf{b}_{sr_*} = \begin{bmatrix} \mathbf{b}_{sr} \\ \mathbf{0}_{10\times 1} \end{bmatrix} = \begin{bmatrix} \mathbf{H}_{sr} \\ \mathbf{I}_{10} \end{bmatrix} \mathbf{x}_{sr} + \begin{bmatrix} \boldsymbol{\epsilon}_{sr} \\ \mathbf{e}_{\text{prior}} \end{bmatrix}$$
$$= \mathbf{H}_{sr_*}\mathbf{x}_{sr} + \boldsymbol{\epsilon}_{sr_*} \quad (17)$$

where $\mathbf{b}_{sr_*}$ is the concatenation of the known terms of the constraint equations $\mathbf{b}_{sr}$ and of a 10-size vector of zeros $\mathbf{0}_{10\times 1}$ (corresponding to the prior expected value of all the unknown quantities). $\mathbf{I}_{10}$ is the 10-size identity matrix corresponding to the prior measurement matrix and, finally, $\mathbf{e}_{\text{prior}}$ is the vector of random variables associated with prior lack of knowledge (prior errors). The single-branch tap-changing transformer estimation problem defined by (17) can be solved in a weighted least squares (WLSs) sense considering the following covariance measurement matrix:

$$\Sigma_{\boldsymbol{\epsilon}_{sr_*}} = \begin{bmatrix} \Sigma_{\boldsymbol{\epsilon}_{sr}} & \mathbf{0} \\ \mathbf{0} & \Sigma_{\mathbf{e}_{\text{prior}}} \end{bmatrix} \quad (18)$$

where $\Sigma_{\boldsymbol{\epsilon}_{sr}}$ is the covariance matrix of $\boldsymbol{\epsilon}_{sr}$ and $\Sigma_{\mathbf{e}_{\text{prior}}}$ is the diagonal covariance matrix including all the prior variances associated with each unknown quantity.

Therefore, the weight matrix $\mathbf{W}_{sr_*}$ obtained through the inversion of the covariance measurement matrix $\Sigma_{\boldsymbol{\epsilon}_{sr_*}}$ can be used to define the system of normal equations associated with the solution of the WLS estimation problem

$$\left(\mathbf{H}_{sr_*}^{\text{T}}\mathbf{W}_{sr_*}\mathbf{H}_{sr_*}\right)\hat{\mathbf{x}}_{sr} = \left(\mathbf{H}_{sr_*}^{\text{T}}\mathbf{W}_{sr_*}\right)\mathbf{b}_{sr_*} \quad (19)$$

where $\hat{}$ indicates the estimated quantity and $^{\text{T}}$ is the transpose operator.

Once the problem (17) is written for all the transformer branches (s, r) and for all the lines (i, j) of interest, a multiple-branch problem can be defined, resulting from merging all the equations. Clearly, if the branches do not have shared unknowns, the resulting merged problem is a juxtaposition of different single-branch problems, but since adjoined branches have nodes in common, the resulting WLS-based system becomes

$$\left(\mathbf{H}_*^{\text{T}}\mathbf{W}_*\mathbf{H}_*\right)\hat{\mathbf{x}} = \left(\mathbf{H}_*^{\text{T}}\mathbf{W}_*\right)\mathbf{b}_* \quad (20)$$

where $\mathbf{b}_*$ and $\mathbf{H}_*$ are obtained by merging, respectively, $\mathbf{b}_{sr_*}$ (and, with the corresponding meaning, $\mathbf{b}_{ij_*}$ for line branch) and $\mathbf{H}_{sr_*}$ (or $\mathbf{H}_{ij_*}$ for line branch) for all the branches involved. $\mathbf{W}_*$ is the overall weight matrix obtained as the inversion of the covariance matrix of all the vectors $\boldsymbol{\epsilon}_{sr_*}$ and, analogously, of vectors $\boldsymbol{\epsilon}_{ij_*}$ for line branches. $\mathbf{x}$ is the vector of all the unknowns (parameter deviations and measurement systematic errors) to be estimated in the multiple-branch set. Subscript $_*$ in (20) highlights that prior information is used for all the considered branches.

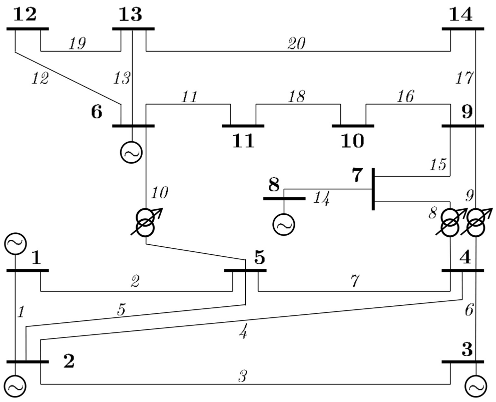

FIGURE 3. Single line diagram of IEEE 14 bus test case.

TABLE 1. Tap ratios of the IEEE 14 bus test case.

Branch	a^0_{sr}
8	0.978
9	0.969
10	0.932

III. TESTS AND RESULTS

In this section, the validation of the proposal is addressed by means of different performance evaluation tests carried out on the IEEE 14 Bus Test Case [24] shown in Fig. 3 (with node and branch indices). Table 1 shows the tap ratio values for all tap-changing transformers branches of the grid.

The general and detailed model described in Section II-A, which depends on k, is used for the three tap-changing transformers.

Different operating conditions have been considered: $C \in \{10, 200\}$ cases and, for each case, $M_C = 10$ repeated measurements have been taken into account. In particular, the high number of operating conditions (i.e., $C = 200$) has been dealt with following the findings of [20], where the WLS problem was framed as a Tikhonov regularization problem. Finally, $N_{\text{MC}} = 5000$ MC trials have been used to statistically validate the estimation results.

The reference values for every MC trial of the C cases are obtained by means of a power flow calculation. The powerflow is repeated for each value of k considered in the tests. In each MC trial, the actual values of tap ratios (i.e., τ_{sr} values) and line parameter values (i.e., β_{sr}, γ_{ij}, β_{ij}, and ρ_{ij} for all the branches) are extracted as discussed in the following. The reference values for voltage and current measurements are thus derived.

Following the modeling assumptions of the measurement chain discussed in Section II-A, in each MC trial, a different set of systematic errors is considered and kept constant across the cases and repeated measurements. Amplitude and phase-angle measurement errors affect the reference values of voltage and current and, in particular, systematic errors are attributed to the ITs (according to their accuracy class) and random errors are associated with PMUs.

Tests have been carried out for different uncertainty values, operating conditions, and actual values of k. The following assumptions are applied.

1) IT accuracy class 0.5, with a maximum value of 0.5 % for voltage and current ratio errors (ξ^{sys} and η^{sys}, respectively),[1] and of 0.9 and 0.6 crad for the phase displacement of CTs and VTs (i.e., ψ^{sys} and α^{sys}), respectively. Systematic errors have been extracted according to uniform distributions, establishing for each MC trial the actual values of these errors.
2) PMU maximum errors equal to 0.1 % for amplitude and 0.1 crad for phase-angle.
3) As for transmission line parameters, maximum deviations of ±10 % from the nominal values (R^0_{ij}, X^0_{ij}, and $B^0_{\text{sh},ij}$).
4) As for transformer parameters, maximum deviations of ±10 % from nominal values of impedances (X^0_{sr}), and of ±1 % for a_{sr}.
5) In each of the C operating conditions, a maximum variability of ±10 % for active and reactive load/generator power.
6) In the tests, possible values of $k \in \{0, 0.1, 0.25, 0.50, 0.75, 1, 1.25, 2, 5, 100, 10000\}$.

The estimation performance concerning transformer and line parameters and systematic errors are evaluated by means of the following root mean square error (RMSE):

$$\text{RMSE}_\chi = \sqrt{\frac{1}{N_{\text{MC}}} \sum_{i=1}^{N_{\text{MC}}} \left(\hat{\chi}_i - \chi_i\right)^2} \tag{21}$$

where $\hat{\chi}_i$ represents the estimate of the unknown quantity χ_i included in the state vector $\mathbf{x}$ for the ith trial, which can thus be τ, β, γ, ρ, ξ^{sys}, α^{sys}, η^{sys}, and ψ^{sys} depending on the considered node or branch. In the following, for the sake of clarity, each tap-changing transformer branch or transmission line will be labeled with an index p (see Fig. 3) and, thus, the corresponding parameters will be indicated also as γ_p, β_p, ρ_p, or τ_p.

In the tests, the value of k assumed in the estimation method (in the measurement process) is indicated with k_m. Therefore, the proposed method depends on the considered k_m. In the following, two different approaches are used: "$k_m = k$" identifies the best option, i.e., when k is known (e.g., from the manufacturer or from specific analyses), while "$k_m = 1$" is a compromise adopted when the value of k is unknown and is thus assumed equal to 1, as suggested in [12] and [14].

The estimation performance of the proposed method has been then compared with the results obtained by the approach presented in [15], which is based on one of the most used tap changer models. Such a model corresponds to assuming $k_m = \infty$ and will be labeled thus as "$k_m = \infty$."

1. Subscripts are often dropped for the unknown quantities in the following when indicating a generic branch or node.

TABLE 2. Summary of Prior.

Parameter	prior standard deviation
$\gamma_p, \beta_p, \rho_p$	5.77 %
τ_p	0.58 %
ξ_h, η_p	0.29 %
α_h	0.35 crad
ψ_p	0.52 crad

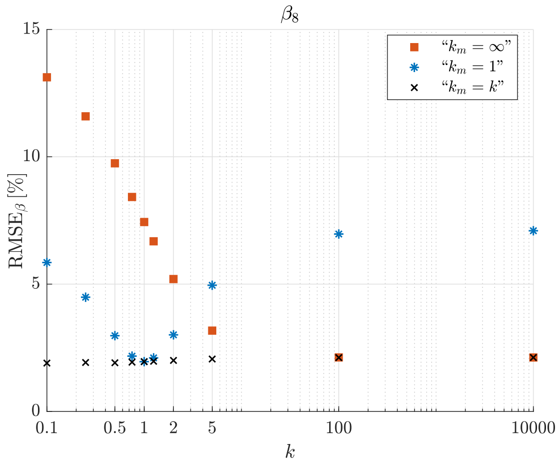

FIGURE 4. Single-branch approach, branch 10: RMSE of estimated transformer reactance deviations for different values of k.

Moreover, each RMSE can be compared with the prior error, i.e., the standard deviation of the errors extracted in the different configurations, hereinafter referred to as "Prior." Table 2 summarizes the Prior for all the parameters of interest that must be estimated. Each Prior is calculated as the maximum deviation of the associated parameter divided by $\sqrt{3}$.

A. SINGLE-BRANCH APPROACH - TAP-CHANGING TRANSFORMER ESTIMATION RESULTS

The first series of tests has been conducted using the single-branch approach on the branches of the network equipped with a tap-changing transformer (in Fig. 3 branches 8–10).

In this case, the estimation results of the methods "$k_m = k$" can be used as a benchmark since the method receives the value of k as an input and then assumes the "correct" value in the estimation process. This method is compared with both "$k_m = 1$" and "$k_m = \infty$" as mentioned above.

Fig. 4 displays the results concerning the estimates of the transformer reactance of branch 10 obtained for different values of k. The RMSE results for "$k_m = k$" represent clearly the best performance achievable in this test. It is possible to notice that the proposal "$k_m = 1$", in the presence of a strong mismatch with respect to the value of k, shows a greater robustness than "$k_m = \infty$", which has errors far beyond the Prior in the presence of low k values. The results for branch 10 were selected because they are the worst among those of the three tap branches (branches 8 and 9 have results with similar trends but lower errors). This confirms that the impact of the mismatch (the difference between k_m and k) due to the tap-changing transformer modeling is greater for operating points with tap ratio values farther from the ratio $a_{sr} = 1$, as discussed in [14].

TABLE 3. Single-branch approach: estimation results for different values of k.

Method	k	RMSE [%] β_8 [%]	τ_8 [%]	β_9 [%]	τ_9 [%]	β_{10} [%]	τ_{10} [%]
"$k_m = \infty$"	0.5	3.52	0.26	4.44	0.26	9.74	0.27
	1	3.07	0.26	3.53	0.26	7.44	0.27
	2	2.73	0.26	2.70	0.26	5.20	0.26
"$k_m = 1$"	0.5	2.45	0.26	1.98	0.26	2.98	0.26
	1	2.37	0.26	1.74	0.26	1.95	0.26
	2	2.49	0.26	2.04	0.26	3.00	0.26
"$k_m = k$"	0.5	2.38	0.26	1.72	0.26	1.91	0.26
	1	2.37	0.26	1.74	0.26	1.95	0.26
	2	2.40	0.26	1.77	0.26	2.00	0.26

As a further confirmation of this behavior, Table 3 summarizes the estimation results of reactance and tap ratio for branches 8–10 obtained varying $k \in \{0.5, 1, 2\}$. It is possible to observe that when the "$k_m = \infty$" method is applied the RMSE goes beyond the Prior (5.77 %) for the reactance of branch 10, for two out of three values of k (i.e., 0.5 and 1). Instead, as an example, the reactance estimation results of "$k_m = 1$" for branch 8 are close to the benchmark, with differences of about 3 % and 4 % for $k = 0.5$ and $k = 2$, respectively, (having obviously the same result for $k = 1$). Finally, in Table 3, it is possible to notice that the tap ratio estimations are similar for all three estimation methods and for all the branches of interest. Thus, the tap ratio estimation is robust with respect to a possible mismatch in the values of k and k_m.

B. MULTIPLE-BRANCH APPROACH–ESTIMATION RESULTS

Further performance evaluation tests have been carried out with a multiple-branch approach considering the entire network: all transmission lines and tap-changing transformers are involved. In the following, the results of the proposed method are reported only for "$k_m = 1$", compared with "$k_m = \infty$". This allows, depending on the test, to investigate the limits of the model and highlight the differences in the behavior.

Figs. 5 and 6 show the RMSEs of the estimates of β_p in all the branches for $k = 1$ and $k = 2$, respectively. It is possible to notice that in both figures the proposal has the best performance, in particular in the branches equipped with tap changers. Focusing on the RMSEs obtained with the proposed method for branches 8–10, and comparing Figs. 6 with 5, it is possible to see that moving away from the match condition (Fig. 5) the results worsen, by at least 72 % for β_8 up to a maximum of 335 % for β_{10} with an average RMSE

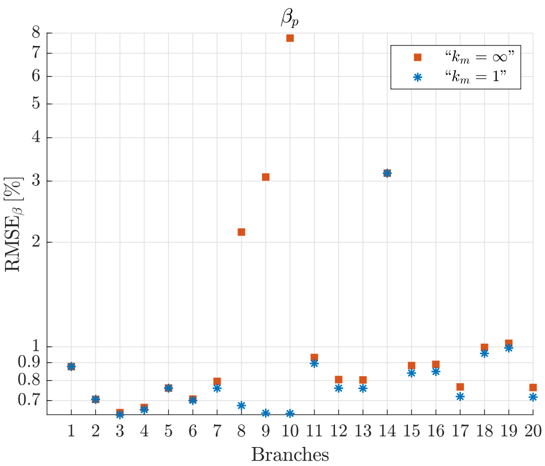

FIGURE 5. Multiple-branch approach, full grid configuration, $k = 1$: RMSE of reactance deviations (transformers and transmission lines).

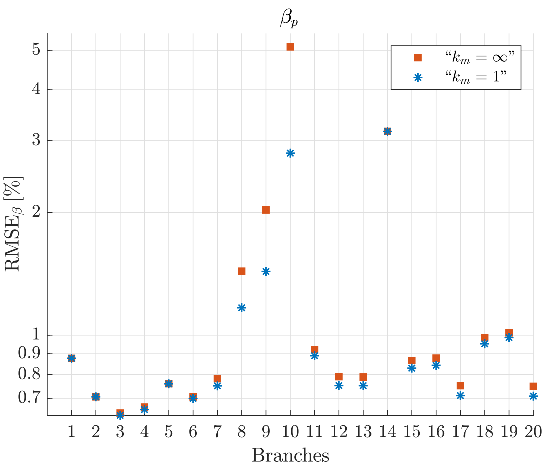

FIGURE 6. Multiple-branch approach, full grid configuration, $k = 2$: RMSE of reactance deviations (transformers and transmission lines).

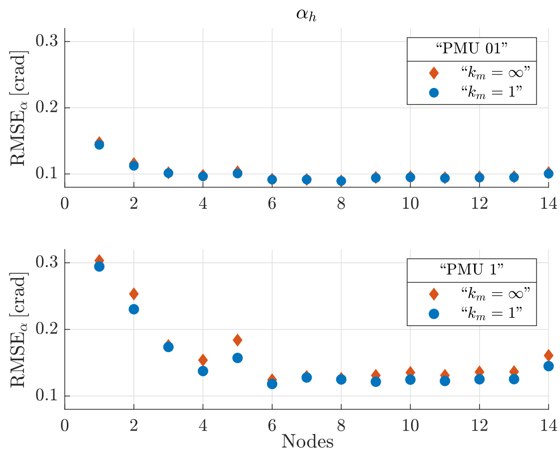

FIGURE 7. Multiple-branch approach, full grid configuration, $k = 1$, "PMU 01" (above) and "PMU 1" (below): RMSE of voltage phase-angle error.

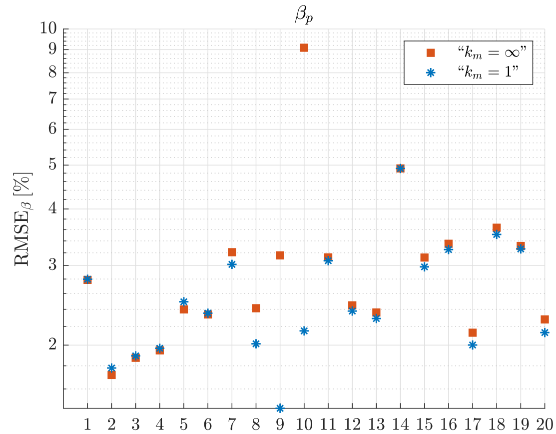

FIGURE 8. Multiple-branch approach, full grid configuration, $k = 0.5$, "PMU 1": RMSE of reactance deviations (transformers and transmission lines).

increase of about 177 %, but still remaining far below the Prior. On the other hand, despite, in Fig. 6, the estimation RMSEs for the three transformer branches obtained with "$k_m = \infty$" improve of about 34 % with respect to Fig. 5 (where the maximum RMSE value is 7.74 %), they are still significantly higher than those achieved with the proposed method.

As a stress test for the considered methods, another series of tests has been carried out by degrading the accuracy of the PMUs.

1) The first test is conducted like the previous ones, i.e., considering 0.1 % as maximum amplitude error and 0.1 crad as maximum phase-angle error. This PMU accuracy configuration is referred to as "PMU 01" in the following.
2) The second test is carried out assuming 0.707 % as maximum amplitude error and 0.707 crad as maximum phase-angle error, thus representing a maximum TVE of 1 %. This PMU is then referred to as "PMU 1."

Fig. 7 displays the RMSEs in the estimation of the phase-angle error α_h for all the nodes of the network, the two PMU configurations and $k = 1$. It is possible to notice that, using a "PMU 01", the results of the proposal do not show an appreciable improvement with respect to method "$k_m = \infty$". Instead, with a "PMU 1", the proposal results are always better than the others, showing a maximum RMSE reduction of about 17 % for α_5 and an average estimation improvement for α_h on all nodes of the network of about 7 %.

For the sake of completeness, Fig. 8 shows the β_p estimation results obtained with "PMU 1" when $k = 0.5$. As expected, the RMSE results worsen with respect to those obtained using the "PMU 01" (Figs. 5 and 6). Also in this case, the transformer reactance estimation accuracy of the method "$k_m = \infty$" is much worse than that obtained with the proposed estimation method. In particular, the RMSE of β_{10} estimated with "$k_m = \infty$" goes far beyond the associated Prior.

As a further test (with "PMU 01"), a comparison of the estimation performance between the single-branch and multiple-branch approaches has been conducted, considering

TABLE 4. Comparison of estimation performance for $k = 1$: different approaches and test configurations.

Method	Approch	C	RMSE γ_7 [%]	β_7 [%]	β_8 [%]	β_9 [%]	β_{10} [%]	ξ_4^+ [%]	α_4^+ [crad]
"$k_m = \infty$"	Single Branch	10	5.56	4.28	3.07	3.53	7.44	0.22	0.25
		200	4.93	2.27	2.33	3.30	7.70	0.22	0.24
	Multiple Branch	10	2.10	0.80	2.14	3.08	7.64	0.12	0.10
		200	1.30	0.48	2.07	3.06	7.74	0.13	0.09
"$k_m = 1$"	Single Branch	10	5.56	4.28	2.37	1.74	1.95	0.22	0.25
		200	4.93	2.27	0.80	0.57	0.73	0.22	0.24
	Multiple Branch	10	2.08	0.76	0.68	0.64	0.64	0.12	0.10
		200	1.29	0.44	0.42	0.41	0.43	0.13	0.09

$k = 1$ and different numbers of cases $C \in \{10, 200\}$. As an example, Table 4 shows results for branch 7 (which is close to the three tap-changing transformers), for the tap-changing transformer branches (8–10) and for node 4. In Table 4, superscript $^+$ specifies that, for the single-branch approach, the average of the RMSEs obtained for all branches converging on node 4 has been evaluated. Comparing the results obtained with the single-branch approach and $C = 10$ with those obtained with the multiple-branch approach and $C = 200$, it is possible to observe average reductions for β_p and γ_p up to about 41 % and 81 % for the methods "$k_m = \infty$" and "$k_m = 1$", respectively. This confirms the ability of the proposal to exploit the measurements of multiple operating conditions, if available, as discussed in [20].

Focusing on the voltage-related RMSEs in Table 4, it is possible to notice that moving from a single-branch approach to a multiple-branch approach, the α_4^+ errors are more than halved for all the methods and the ξ_4^+ errors are close to be halved.

It is also worth reporting that applying the multiple-branch approach, with any number of cases, the RMSE results for τ_p estimation (not reported here for space reason) are halved compared to those obtained in Table 3.

As a final comment, it is possible to highlight that the appropriate model of the tap-changing transformer, taking into account possible different ratios between nominal and tapped winding impedances through the specific parameter k and, therefore, being able to leverage the information on k if available, leads to significant improvements in the estimation results for both single-branch and multiple-branch approaches.

C. COMPARISON ANALYSIS

Finally, further tests have been carried out to assess the proposal performance with respect to other methods, with the same measurement setup used above. In the following, the multiple-branch approach with "$k_m = 1$" (Proposed Method) is compared with two estimation methods from the literature. In particular, the first considered method (Method A) is a direct estimation method for transmission line parameters based on the equations in [25] and [26]. For the sake of a more comprehensive comparison, an extension of such direct method to include tap-changing transformer branches (using traditional model, i.e., with "$k_m = \infty$") has been designed and applied. Method A thus allows also obtaining τ and β estimates for each transformer. The second method (Method B) is derived from the least squares method and can be applied only to transmission lines [27, Sec. IV]. Method B can estimate line parameters and compensation factors (systematic measurement errors) of the end node in a branch, assuming as accurate the voltage and current measurements of the start node.

TABLE 5. Comparison of estimation performance between the proposed method and other methods, $k = 1$.

Method	PMU Accuracy	C	RMSE γ_1 [%]	β_1 [%]	ξ_2 [%]	α_2 [crad]	β_9 [%]	τ_9 [%]	β_{10} [%]	τ_{10} [%]
Proposed Method	"PMU 01"	10	2.37	0.88	0.12	0.11	0.64	0.13	0.64	0.13
		200	1.38	0.41	0.13	0.09	0.41	0.13	0.43	0.12
	"PMU 1"	10	4.33	2.75	0.14	0.23	1.60	0.14	1.54	0.15
		200	3.77	1.98	0.16	0.28	1.75	0.13	1.69	0.15
Method A	"PMU 01"	10	13.69	5.35	–	–	6.32	0.41	9.43	0.41
		200	13.81	5.39	–	–	6.32	0.41	9.33	0.40
	"PMU 1"	10	13.83	5.37	–	–	6.35	0.42	9.43	0.42
		200	13.82	5.39	–	–	6.32	0.41	9.33	0.40
Method B	"PMU 01"	10	7.79	2.55	0.37	0.41	–	–	–	–
		200	2.63	1.04	0.30	0.36	–	–	–	–

Table 5 reports the comparison among the estimation results for one transmission line and two tap-changing transformer branches (corresponding to branch indices 1, 9, and10 in Fig. 3, respectively). The symbol "–" indicates that the associated parameter estimation is not available for the considered method. It has to be highlighted that the estimation results for Method B using the "PMU 1" scenario are not reported in the table, since they are far beyond (at least three times) the prior standard deviation results.

Focusing on Method A, it has to be noticed that its RMSEs, for all test configurations, are always significantly higher than those of the Proposed Method for both line parameters and tap ratios, and even higher than the prior standard deviations for γ_1, β_9, and β_{10}. It is also interesting to highlight that Method A results are stable varying the number of cases C and the PMU accuracy considered; thus, the method is not able to significantly leverage additional constraints and more accurate measurements as happens with the Proposed Method.

The performance of Method B, unlike Method A, takes the advantage of a higher C, specifically for γ_1 and β_1 estimation results, while it is strongly and negatively sensitive to the increase in PMU measurement errors (as mentioned above, its estimates for PMU 1 are critically affected). Method B allows estimating also systematic errors ξ_2 and α_2 (systematic errors in v_2) but it is possible to notice that the corresponding RMSE values are beyond prior standard deviations (reported in Table 2) for both $C = 10$ and $C = 200$ cases. Similar results (not reported here for the sake of brevity) can be observed for η_{21} and ψ_{21}. The results in Table 5 thus prove that the Proposed Method achieves the best performance for all considered line and transformer parameters. Furthermore, as proven above, it shows, with respect to others, also a significant robustness in the presence of higher measurement uncertainty.

IV. CONCLUSION

This article presented a novel method for the concurrent estimation of line parameters, tap ratios, and systematic errors, based on measurement chains composed of ITs and PMUs. The proposal involved a general model of the tap-changing transformers, which allows including all possible information available on the impedance windings. The performed analysis proved that the proposed methodology improves the estimation with respect to methods implementing traditional models for the tap-changing transformers (even if the measurement uncertainty model is the same). In particular, in the absence of detailed information on the transformer parameters, the choice of assuming the impedances equally divided between windings proves robust in the case of mismatches between assumptions on transformer parameters and actual values. All the tests carried out, in the presence of several operating conditions and different measurement uncertainties, proved also that the estimates of the tap ratios are always as accurate as those obtained with perfect match. This further confirms the effectiveness of the proposed procedure that estimates the parameters of interest and at the same time compensates for the systematic errors involved in the process.

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