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Multidirection Update-Based Multiobjective Particle Swarm Optimization for Mixed No-Idle Flow-Shop Scheduling Problem

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Abstract: The Mixed No-Idle Flow-shop Scheduling Problem (MNIFSP) is an extension of flow-shop scheduling, which has practical significance and application prospects in production scheduling. To improve the efficacy of solving the complicated multiobjective MNIFSP, a MultiDirection Update (MDU) based Multiobjective Particle Swarm Optimization (MDU-MoPSO) is proposed in this study. For the biobjective optimization problem of the MNIFSP with minimization of makespan and total processing time, the MDU strategy divides particles into three subgroups according to a hybrid selection mechanism. Each subgroup prefers one convergence direction. Two subgroups are individually close to the two edge areas of the Pareto Front (PF) and serve two objectives, whereas the other one approaches the central area of the PF, preferring the two objectives at the same time. The MDU-MoPSO adopts a job sequence representation method and an exchange sequence-based particle update operation, which can better reflect the characteristics of sequence differences among particles. The MDU-MoPSO updates the particle in multiple directions and interacts in each direction, which speeds up the convergence while maintaining a good distribution performance. The experimental results and comparison of six classical evolutionary algorithms for various benchmark problems demonstrate the effectiveness of the proposed algorithm.

Key words: multiobjective optimization; Particle Swarm Optimization (PSO); Mixed No-Idle Flow-shop Scheduling Problem (MNLFSP); multidirection update

1 Introduction

The shop scheduling problem is the most important issue in manufacturing systems that restricts production efficiency. Therefore, optimizing the shop scheduling problem has great significance and far-reaching impact. Johnson published the first paper on the Flow-shop Scheduling Problem (FSP). Since then, extensive research has been conducted on the FSP[1]. In the FSP, there are n jobs to be processed on m machines in a manufacturing shop. All jobs must have the same processing sequence on each machine. Thus, there are $n!$ possible solutions for the FSP. The No-Idle FSP (NIFSP) is more complicated than the FSP. In the NIFSP, each machine should run continuously from the time when it starts the first job to the time it finishes processing the last job. In other words, idle time is not allowed between processing adjacent jobs on one machine. The NIFSP often appears in a production system where the operating cost of the machine is very high, so it is necessary to guarantee that the machine does not run empty during the production operation. When some machines are idle while others are not, the Mixed NIFSP (MNIFSP) appears as a general issue of

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NIFSP. The MNIFSP is often used in production systems, such as integrated circuits, ceramic frits, and the steel industry[2].

The FSP is identified as a Nondeterministic Polynomial (NP)-hard problem when there are more than three machines[3]. As an extension topic of the FSP, the MNIFSP is also an NP-hard problem[4]. The exact algorithm (mathematical method) needs time to search each permutation and combination to find the best solution. For middle- and large-scale problems, the exact algorithm could take an unacceptably long time to seek the best solution, which greatly affects industrial efficiency.

A heuristic algorithm has significant advantages in solving NP problems, and it can find the approximate optimal solution in an acceptable time. Metaheuristic algorithms are different from traditional heuristic algorithms. Metaheuristic algorithms have specific steps to solve the problem and do not need to consider the problem type. After setting algorithm parameters, the approximate optimal solution can be found by the algorithm. Moreover, metaheuristic algorithms do not require several prior experiences as compared to traditional heuristic algorithms. Metaheuristic algorithms are also more intelligent and more suitable for solving the MNIFSP.

As a metaheuristic algorithm, Particle Swarm Optimization (PSO) is proposed based on the foraging characteristics of birds[5]. PSO drives a set of particles to search the solution space. In the evolution process of PSO, the velocity of each particle is randomly adjusted according to its historical best position and all particles' global best position, which is named as *pbest* and *gbest*, respectively. The change in particle velocity will cause the particle position to be perturbed accordingly. Accordingly, PSO has the capability of finding the approximate optimal solution. Compared with other metaheuristic algorithms, PSO has the following advantages: PSO has fewer parameters and is easier to adjust. PSO has a simpler structure and is not sensitive to the scale size and nonlinearity of the problem. Each particle records *pbset* and *gbest* in PSO. Therefore, PSO has a more efficient storage capacity than a Genetic Algorithm (GA)[6].

In 2002, Coello and Lechuga[7] proposed a Multiobjective PSO (MoPSO) to deal with the multiobjective problem. MoPSO mainly examines the following parts: the selection of *pbest* and *gbest*, the improvement of the velocity formula, and the improvement of the searched solution. Coello and Lechuga[7] improved the exploration ability of MoPSO by introducing a mutation operator whose scope changes over time[8]. Zhan et al.[9] then proposed a coevolution technique and applied it to MoPSO. In the algorithm, each objective has a population used to find the optimal solution. This algorithm speeds up the search for every single objective, and the distribution of the algorithm may not be good. Al Moubayed et al.[10] designed an MoPSO by considering the decomposition and dominance with archiving (D2MoPSO for short), that uses the decomposition strategy and crowding distance method for archiving. The D2MoPSO incorporates dominance with decomposition to improve the quality of the particle. However, the number of subgroups is high, so the number of particles in each subgroup is small, which could slow down the update ability of PSO in each direction.

In this paper, a MultiDirection Update (MDU) based Multiobjective PSO (MDU-MoPSO) is proposed to deal with the biobjective MNIFSP problem by simultaneously minimizing the makespan and total processing time. The MDU-MoPSO is inspired by the D2MoPSO algorithm to decompose the particle swarm into multiple subgroups, and each subgroup only serves for a specific direction. Unlike the commonly used MultiObjective Evolutionary Algorithms (MOEAs) that decompose the problem into many sub-problems to be optimized separately, the MDU strategy divides particles into three subgroups according to a hybrid selection mechanism. Each subgroup only serves for a specific direction, so particles could be driven to evolve in three directions. This strategy could avoid the MoPSO based on multidirectional decomposition from weakening the overall convergence ability. The hybrid selection mechanism combines a single-objective-based selection mechanism from the Vector Evaluated Genetic Algorithm (VEGA)[11] and a Pareto-based selection strategy according to a Pareto Dominance and Dominance Relationship-based Fitness Function (PDDR-FF)[12]. These three subgroups attempt to quickly converge to three areas of the Pareto Front (PF), where each subgroup prefers one convergence direction: the two subgroups are individually close to the two edge areas of the PF and separately serve two objectives, and the other one is close to the central area of PF, benefiting the two objectives at the same time. When updating a particle, *gbest* and *pbest* are selected

according to the subgroup where the particle is located, so the direction of updating the particle also serves the edge area and central area of PF. This method will improve the strong convergence performance in the three directions of the entire PF and could maintain a certain uniform distribution performance.

Moreover, different from real number-based particle representation and particle update operations, the MDU-MoPSO adopts a job-based sequence encoding method and an exchange sequence-based particle update strategy. Because the MNIFSP is a variant of the FSP, we still use the job-based sequence encoding method, but in the particle update, the sequence difference is used to reflect the difference between the particles. Updating the particles according to the length of the parameters and sequence differences can better avoid generating illegal solutions and could ensure the diversity of the population.

The main contributions are summarized as follows.

● The hybrid selection strategy can easily decompose particles into three subgroups, which serve three convergence directions of PF.

● The update strategy of particles in three directions can easily improve the overall convergence performance and also bring a certain improvement for an even distribution ability. Each subgroup is updated in the direction of the nearest PF.

● The particle update operation based on the exchange sequence is well adapted to the problem characteristics and PSO operation.

The rest of this paper is organized as follows. Section 2 introduces the related work of the MNIFSP. Section 3 presents the description of the MNIFSP. Section 4 describes the detailed implementation of the MDU-MoPSO. Section 5 demonstrates the experimental results and analysis. Lastly, Section 6 describes the conclusions.

2 Related Work

The MNIFSP is a shop scheduling problem raised in recent years. To the best of our knowledge, only a few research works have investigated the MNIFSP. The following section summarizes some algorithms used to solve the MNIFSP and similar NIFSP problems.

Although many exact algorithms are used to solve the FSP, only a few exact algorithms have been proposed for the MNIFSP or NIFSP. Bekteş et al.[13] proposed an exact algorithm to solve the MNIFSP, which is combined with Benders decomposition and enhancement strategy using a referenced local search to speed up convergence. The proposed exact algorithm uses a single additional optimality cut at each iteration and combines it with combinatorial cuts, which could solve instances with up to 500 jobs and 15 machines.

In addition to exact algorithms, heuristic and metaheuristic algorithms have also been proposed to solve the MNIFSP or NIFSP. Rossi and Nagano[14] proposed a heuristic method to evaluate the total time of permutation sequences and an acceleration method for calculating the total flow time for the MNIFSP. Rossi and Nagano proposed a constructive heuristic for the MNIFSP with the minimization of makespan[15] and total tardiness[16] under sequence-dependent setup times. Nagano et al.[17] proposed an efficient constructive heuristic method by combining an iterated greedy algorithm for the NIFSP with minimization of the total flow time.

A greedy method is also used to solve the MNIFSP or NIFSP. Pan and Ruiz[2] proposed an improved Iterative Greedy Algorithm (IGA) for the MNIFSP by improving the initialization, destruction, and reconstruction operators, and designed a set of formulas to speed up the insert neighborhood search. For solving the distributed NIFSP with the objective of minimizing the makespan, an iterated reference greedy algorithm was proposed by Ying et al. in 2017[18]. To solve the distributed NIFSP, Cheng et al.[4] proposed an IGA based on cloud theory, which combines a temperature cooling scheme based on cloud theory and a neighborhood-based local search. Riahi et al.[19] developed an IGA for the NIFSP with the objective of minimizing the total tardiness.

Hybrid methods have been treated as an effective method that combines different heuristics or metaheuristics for the complicated MNIFSP or NIFSP. Deng and Gu[20] proposed a hybrid discrete Differential Evolution (DE) algorithm for NIFSP, which combines a speedup method and an insert neighborhood-based local search. For minimizing the maximum completion time of the NIFSP, Shao et al.[21] developed a memetic algorithm, which combines a population-based global search strategy and a local refinement mechanism. Shao et al.[22] proposed a metaheuristic based on a hybrid discrete teaching-learning algorithm to deal with the NIFSP with the minimization of the total tardiness. Chen et al.[23] developed a collaborative optimization algorithm for the energy-efficient distributed NIFSP using several properties and four collaborative mechanisms in different strategies and ways to

simultaneously optimize the minimization of the makespan and total energy consumption. For optimizing the NIFSP with the minimization of total tardiness, a hybrid discrete water wave optimization algorithm was proposed by Zhao et al.[24], which combines the priority rule, self-adaption selection neighborhood search, and variable neighborhood search strategies.

An Evolutionary Algorithm (EA) based on biological mechanisms and natural selection theories has received widespread attention because of its potential to solve complex problems. As one of the EAs, the traditional GA easily falls into the local optimum but cannot find the best individual. Therefore, the GA needs to be combined with other nature-inspired metaheuristics to further improve the search quality.

The PSO algorithm is much faster than the GA in terms of running speed and has a powerful search ability. Therefore, the PSO has great advantages in solving the NIFSP. A novel hybrid discrete PSO was proposed by Pan and Wang[25] for the NIFSP by combining a permutation representation-based discrete PSO and an inserted neighborhood-based local search. Bewoor et al.[26] proposed a hybrid PSO based on evolution for the NIFSP by combining a random key conversion mechanism with PSO to convert the continuous position information of particles into discrete information.

As for the single-objective FSP, Nouha and Talel[27] proposed a PSO for the blocking FSP, in which the velocity and position are encoded as the job replacement list, Nawaz-Enscore-Ham heuristic technique is used to generate particles, and a neighborhood search method is designed to update it. In addition, Eddaly et al.[28] developed a combinatorial PSO for the blocking FSP, where an iterative local search algorithm based on probability perturbation is designed to improve the quality. Jiao and Yan[29] proposed a coevolutionary PSO based on the niche sharing strategy, which enhances the diversity of particles for solving the FSP. Mokhtari and Noroozi[30] developed a chaotic-based PSO for the batch-processing FSP with earliness and tardiness criteria, which improves the particle velocity formula and studies the influence of chaotic characteristics on search performance. Although the proposed PSOs are mostly aimed at the FSP, their research results are still helpful for PSO design on the MNIFSP, which is a variant of the FSP.

The MoPSO algorithm mainly solves multiobjective problems and is more practical than the PSO algorithm. Su and Chi[31] proposed an MoPSO, which combines PSO with a DE, and uses the simulated annealing strategy to generate a scale factor to dynamically adjust the particle swarm and DE. Cui et al.[32] proposed the hybrid MoPSO algorithm, which uses different operators to generate a new offspring and then uses the environmental selection mechanism to screen out the next generation of individuals.

In general, all of the above algorithms can be used to solve the MNIFSP or NIFSP, but there is a lack of research on the multiobjective MNIFSP using MoPSO. Therefore, improving the MoPSO algorithm to solve the MNIFSP still faces challenges.

3 Problem Definition

The difference between the MNIFSP and shop scheduling problem is that different processing operations for all jobs must be processed on all machines within a specific operating order. The specific description of the MNIFSP is as follows: All jobs will be operated on machines, and the order of machines is from the first to the last, which is fixed. The machine must only process one job at a time, and one job cannot be simultaneously operated on two machines. Some machines have no idle time and cannot be interrupted once the processing starts. The objective is to find a suitable order of jobs to satisfy the minimization of the makespan and total processing time.

The MNIFSP contains n jobs (j_1, j_2, …, j_n) to be processed on m machines (M_1, M_2, …, M_m). The processing time of job j_i on machine M_k is set to $d(j_i, M_k)$, and the completion time of job j_i operated on machine M_k is set to $C(j_i, M_k)$. The set of $X = (j_1, j_2, \ldots, j_n)$ is a particular execution sequence. Assume that the preparation time of each job is zero, and the delivery time is not considered after the job is operated. The production status of the job and the status of the machines in the factory can be calculated by the following formulas. For the first machine, jobs are always processed individually, so the completion time of the jobs operated on the first machine can be calculated with the following formulas:

$$C(j_1, M_1) = d(j_1, M_1) \tag{1}$$

$$C(j_{i+1}, M_1) = C(j_i, M_1) + d(j_{i+1}, M_1), \quad i = 1, 2, \ldots, n-1 \tag{2}$$

Except for the first machine, it is necessary to determine whether the machine has a no-idle machine before the completion time on the computing machine. If it is a normal machine, then the calculation method is as follows:

$$C(j_1, M_{k+1}) = C(j_1, M_k) + d(j_1, M_{k+1}), \quad k = 1, 2, \ldots, m-1 \tag{3}$$

$$C(j_i, M_k) = \max\{C(j_{i-1}, M_k), C(j_i, M_{k-1})\} + d(j_i, M_k), \quad i = 2, 3, \ldots, n;\ k = 2, 3, \ldots, m \tag{4}$$

If there is a no-idle machine, then the calculation method of the completion time is as follows: First, determine the completion time of the last job according to a normal machine by Eqs. (3) and (4). Then, calculate the completion time of each job backward as follows:

$$C(j_{i-1}, M_k) = C(j_i, M_k) - d(j_i, M_k), \quad i = 2, 3, \ldots, n;\ k = 2, 3, \ldots, m \tag{5}$$

Because the first job must be operated on the first machine, $C(j_1,M_1) = d(j_1, M_1)$ as presented in Eq. (1). Because the first step of each job is performed on machine M_1, the completion time of machine M_1 can be represented by Eq. (2). Because Job j_1 is the first job to be operated, it is the first operation on each machine, so the completion time of Job 1 can be represented by Eq. (3). Equation (4) calculates the completion time of each job. Equation (5) is used on the basis of Eq. (4). When the current machine is no-idle, Eq. (5) infers the completion time of each job processed on the current machine. In this study, two common objectives are used:

$$\min\ f_1 = C(j_n, M_m) \tag{6}$$

$$\min\ f_2 = \sum_{i=1}^{n} C(j_i, M_m) \tag{7}$$

Equation (6) is the minimization of the makespan and Eq. (7) is the minimization of the total processing time.

4 MDU-MoPSO

Because the MNIFSP is an NP-hard problem, its solution space is too large, and the multiobjective makes the problem more complicated. Therefore, MDU-MoPSO is proposed to solve the MNIFSP. In this section, the overview of MDU-MoPSO, the details of MDU, encoding and decoding, and exchange sequence will be described in detail.

4.1 Overview of MDU-MoPSO

In MDU-MoPSO, first, particles are divided into three subgroups using the hybrid selection strategy where each subgroup serves one convergence direction. The subgroup *sub*1 located in the upper-edge area of the PF generated based on a single-objective selection strategy serves one objective, the other subgroup *sub*2 located in the lower-edge area serves another objective, and the central-area subgroup *sub*3 generated based on the PDDR-FF selection mechanism simultaneously serves the two objectives.

After decomposing into three subgroups, the update strategy of the particles is executed. Each subgroup is updated in the direction of the nearest PF. The *pbest* and *gbest* selected by the particles of the subgroup *sub*1 are both biased toward the upper-edge region of the PF. Therefore, the particles in *sub*1 will be updated toward the upper-edge area of the PF. Similarly, the particles of *sub*2 will be updated toward the lower-edge area of the PF, and the particles of *sub*3 will be adjusted toward the center area of the PF. The update strategy could quickly tune the search directions to the center and edge area of the PF to make the algorithm converge faster and distribute well.

In particular, the particle update can better integrate the problem characteristics and particle update operation and make full use of the job exchange sequence difference to update the particles, thereby improving the convergence performance of the particles and maintaining the diversity of the particles.

The framework of MDU-MoPSO is shown in Fig. 1, and its details can be described in the following steps:

Step 1: The particle swarm is divided into three subgroups through the hybrid selection strategy (VEGA-based selection and PDDR-FF-based selection). The VEGA-based selection strategy is used to select two single-objective directions, whereas the PDDR-FF-based selection strategy is used to select particles that are good on both objectives. *sub*1 tends to perform well on Objective 1(Eq. (6)), *sub*2 on Objective 2 (Eq. (7)), and *sub*3 on both objectives under the PDDR-FF.

Step 2: For each subgroup, the particle position is updated with the velocity formula and position formula. In the velocity formula, *sub*1 uses good *pbest* and *gbest* on Objective 1, *sub*2 prefers good *pbest* and *gbest* on Objective 2, and *sub*3 uses *pbest* and *gbest* with good PDDR-FF values. In this way, each subgroup will be updated in different directions. Because this is a multiobjective problem, the binary tournament method is used to select *pbest* and *gbest*. *pbest* is selected from the set of the particle's historical

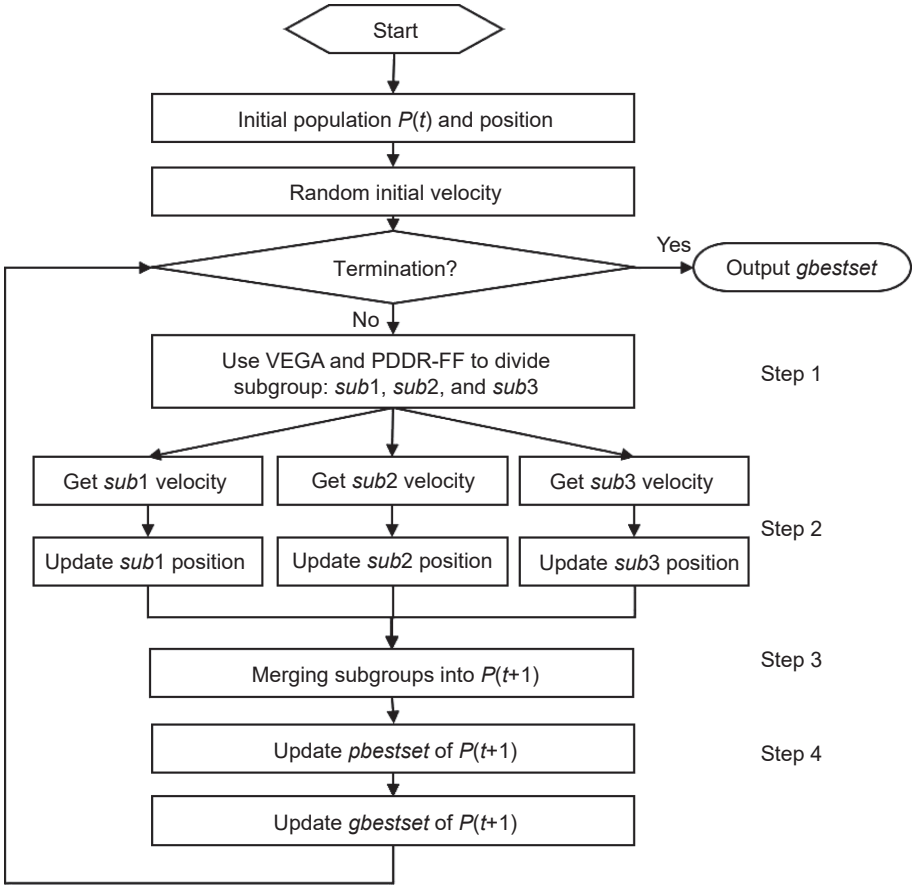

Fig. 1 Framework of MDU-MoPSO.

best position (*pbestset*), and *gbest* is selected from the set of the particle's global best position (*gbestset*).

Step 3: The three subgroups are recombined into a new particle swarm, which is the next-generation particle swarm. There are many ways to combine particles. The method used in this study is the mixing of particles into new particle swarms in order. In addition, the size of the particles remains unchanged.

Step 4: *pbestset* and *gbestset* are updated. Each particle has a *pbestset*, which contains all the excellent positions experienced by the particle from the beginning to the current generation. *pbestset* is updated according to the dominance relationship between particles; the main operation is to delete the dominated particles. The particles in *gbestset* are filtered from the particles in *pbestset* according to the dominance relationship.

The algorithm can be divided into the above steps, and the specific details of MDU-MoPSO will be described later.

4.2 Encoding and decoding

This study adopts the fixed-length sorting construction method. Each particle is constructed from a sequence of fixed-length real numbers. The particle consists of all the jobs of the MNIFSP. The value range of a bit is 1-*Jobnum* in a particle sequence, and the real numbers at any two bits are not repeated. For example, when solving an MNIFSP with six jobs and three machines, a particle can be constructed as $\boldsymbol{x} = \{2,3,1,5,4,6\}$.

The decoding operation is described as follows. After the particle sequence is determined, the entire processing flow is determined, and the processing schedule of the normal machine can be calculated by Eqs. (1)–(4). The processing schedule of no-idle machines can be calculated by Eq. (6). Each fitness function can be calculated according to the processing schedule. Figure 2 a shows a schedule of normal machines, which can be calculated from the Eqs. (1)–(4). Figure 2 b shows a schedule of mixed no-idle machines, where the previous machine must be fully determined before calculating the processing schedule of the current machine, and the type of machine must be evaluated before calculation.

The completion time of jobs on normal machines can be calculated by Eqs. (1)–(4). As for the no-idle

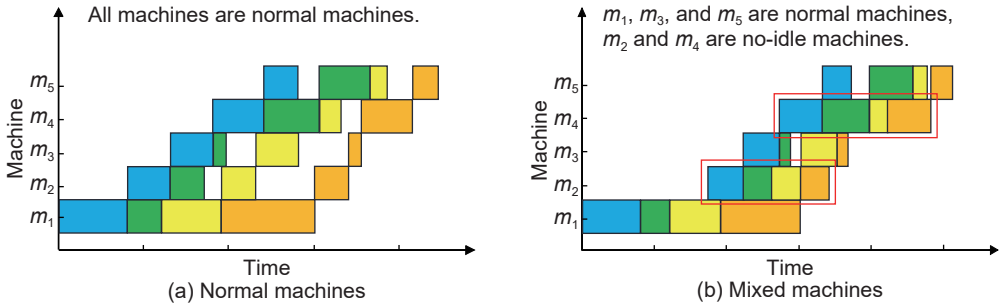

Fig. 2 Decoding method of MNIFSPs and the difference between decoding with normal machines.

machine, the completion time of the last job on a normal machine is first determined, and then the completion time of all jobs is calculated according to Eq. (6). The operation time results of the jobs in Figs. 2a and 2b are the same. After replacing m_2 and m_4 in Fig. 2a with no-idle machines, Fig. 2b is obtained.

4.3 Exchange sequence

The exchange sequence is a one-step operation to exchange two serial numbers in the sequence. For example, if a particle $\boldsymbol{x}=\{2,5,4,6,3,1\}$ has been changed to a new particle $\boldsymbol{x}_1=\{2,3,4,6,5,1\}$, then the 2nd and 5th position in $\boldsymbol{x}$ are swapped, and then the $(2,5)$ position is a pair of exchange sequence. The exchange sequence is very different from the vector, as it cannot add or subtract. However, the exchange sequence can be combined or deleted. A single exchange sequence only exchanges two serial numbers, and a set of exchange sequences can be used to exchange multiple serial numbers.

In this study, the difference between particles is represented by the exchange sequence, which can be useful for PSO operators. The exchange sequence is only suitable for the temporary storage of changes. Once the exchange sequence between two particles is obtained, this exchange sequence must be used as soon as possible, and the particle to be updated cannot participate in other updates. As shown in Fig. 3, $\boldsymbol{x}$ is changed to $\boldsymbol{x}_1$ through (1, 3), (4, 6). Particle $\boldsymbol{x}_1$ passes (1, 2), (4, 6), and (2, 4) to generate $\boldsymbol{x}_2$, and $\boldsymbol{x}_1$ passes (2, 4) to generate $\boldsymbol{x}_3$. Clearly, $\boldsymbol{x}_2$ and $\boldsymbol{x}_3$ are two different particles. The findings prove that the exchange sequence can only be used as a tool to temporarily store the difference of particles and does not have the additivity of vectors[33].

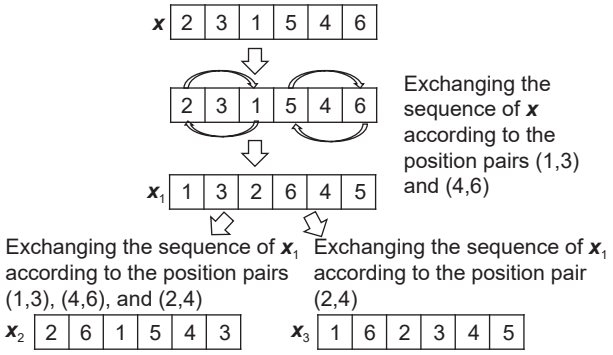

Fig. 3 Influence of the exchange sequence.

4.4 Details of MDU

4.4.1 Subgroup division

Unlike the MOEA/D that divides a population into more directions to evolve, MDU-MoPSO divides particles into three directions, where the decomposition time and calculation time can be reduced. However, the number of particles in the three subgroups is still large, which can greatly improve the convergence performance without reducing the distribution performance.

The MDU strategy consists of two parts: VEGA-based selection and PDDR-FF-based selection. The VEGA can separate a multiobjective problem into multiple single-objective problems through selection, which could store the particles of the multiobjective problem into multiple subgroups according to the objective, and each subgroup serves one objective. The differentiation of particles in the edge area is beneficial to drive PSO to improve the particles in the edge area of PF.

PDDR-FF is a fitness function in calculating the dominance relationships among particles. It evaluates particles and assigns them with real numbers, which could identify the dominated and nondominated relationship among particles and the position where the particle is located in the PF[12]. The formula of the PDDR-FF will be shown using an example that calculates the fitness value of an individual $\boldsymbol{x}$,

$$eval(\boldsymbol{x})=q(\boldsymbol{x})+\frac{1}{p(\boldsymbol{x})+1},i=1,2,\ldots,popsize \tag{8}$$

where $q(\boldsymbol{x})$ denotes the number of particles, which dominate the particle $\boldsymbol{x}$; $p(\boldsymbol{x})$ represents the number of particles dominated by particle $\boldsymbol{x}$; and *popsize* is the size of particle swarm. Therefore, particles with a smaller fitness value are better than those with a larger fitness value.

The MDU divides particles into three subgroups; each of the subgroups is different. The VEGA-based selection strategy prefers to select particles in the edge area of the PF, and the PDDR-FF-based selection strategy prefers particles in the central area of the PF. The VEGA selects better particles for Objective 1 into *sub*1 and better particles for Objective 2 into *sub*2. Then, the PDDR-FF selects better particles for both objectives into *sub*3. The directions of each subgroup update particle are different, as shown in Fig. 4.

4.4.2 Particle update

The difference in the job sequences between two particles is represented by the exchange sequence in this study. The velocity in the particle update formula consists of an exchange sequence. Because the exchange sequence can only be used for temporary difference storage, the initial velocity in the particle velocity formula has no meaning. To simulate the effect of the initial velocity of the particles, a pair of random exchange sequences are used to replace the initial velocity. The velocity and position update formulas of particle $\boldsymbol{x}(t)$ in generation t are defined as follows:

$$\boldsymbol{v}(t+1) = \boldsymbol{rs}(t) + c_1 r_1 (pbest - \boldsymbol{x}(t)) + c_2 r_2 (gbest - \boldsymbol{x}(t)) \quad (9)$$

$$\boldsymbol{x}(t+1) = \boldsymbol{x}(t) + \boldsymbol{v}(t+1) \quad (10)$$

where $\boldsymbol{rs}(t)$ is a random exchange sequence. It is a one-step neighborhood search in the case of the current sequence. c_1 is a cognitive learning factor, and c_2 is a social learning factor. The values of both are equal to 1. The range of the random numbers r_1 and r_2 of the original PSO is 0–1. In this study, the range 0–0.2 for r_1 and r_2 is the best for the MNIFSP. The experimental part of the parameter setting is presented in Section 5.1.

Figure 5 shows the PSO update process with the exchange sequence for the particle $\boldsymbol{x}(t) = \{2,3,1,5,4,6\}$ to generate $\boldsymbol{x}(t+1)$ according to Eqs. (9) and (10). In general, it contains three parts, where the first part uses a one-step neighborhood search to fine-tune the particles, the second part lets the particles evolve in the direction to the *pbest*, and the third part makes the particles evolve in the direction to the *gbest*.

MDU divides the particles into three subgroups. *sub*1 is biased toward the upper edge of the PF, which is better on Objective 1; *sub*2 is biased toward the lower edge of the PF, which is better on Objective 2; and *sub*3 is biased toward the central area of the PF, which is better on the two objectives. The update direction of each subgroup is different. The update direction of *sub*1 is the area of Objective 1, that of *sub*2 is the area of Objective 2, and that of *sub*3 is the center area of the PF.

The particles in each subgroup are updated according to the PSO update formula, but the difference between *gbest* and *pbest* selected in each subgroup makes the direction of evolution different. As shown in Fig. 6a, *gbest* and *pbest* are good particles on Objective 1 and can be selected using the fitness value of Objective 1. As shown in Fig. 6b, *gbest* and *pbest* are good particles on Objective 2, which can be selected using the fitness value of Objective 2. As shown in Fig. 6c, *gbest* and *pbest* are particles close to the central area of the PF, which can be selected by the *eval* value.

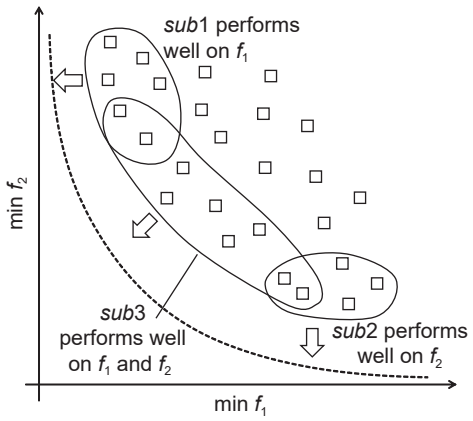

Fig. 4 Division of the particle swarm by direction.

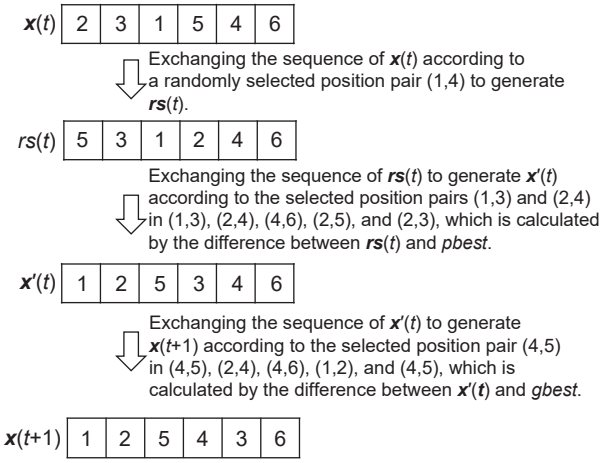

Fig. 5 Update process of PSO with the exchange sequence.

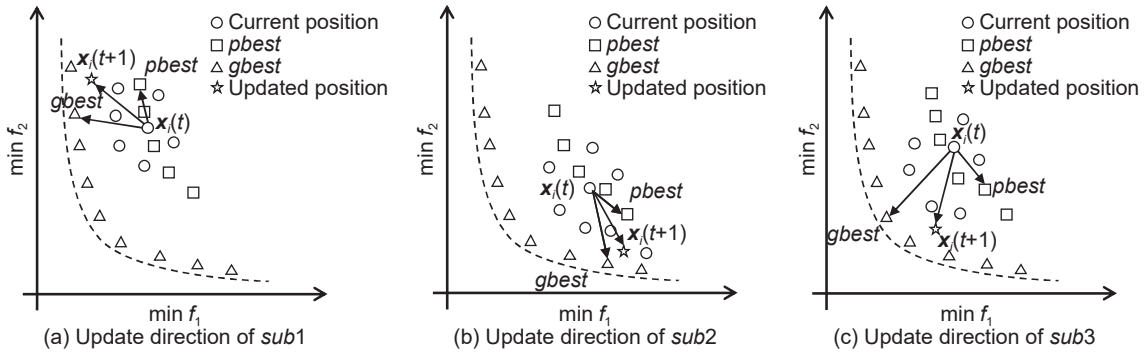

Fig. 6 Update directions of *sub*1, *sub*2, and *sub*3.

With the update method, the particles in *sub*1 have superior performance on Objective 1, and the particles in *sub*2 have superior performance on Objective 2. Particles in *sub*3 will search for better solutions on the central area of the PF. The particles are updated in three directions, so the solution searched can cover the entire PF, which naturally makes a better distribution of the solutions. The subgroup searches the solution space in a single direction, and the number of particles in each subgroup is large, which easily improves the convergence performance of the algorithm.

5 Experimental Study

In this research, we use the benchmark problem of the MNIFSP by Rossi and Nagano in 2019[14]. The scale of the benchmark problem is set from small to large, the number of jobs is set from 50 to 500, and the number of machines is set from 10 to 50. The benchmark problems can be divided into seven problem sets, and the characteristics of each problem set are presented as follows:

- **Problem set 1:** The first half of the machines are no-idle machines.
- **Problem set 2:** The second half of the machines are no-idle machines.
- **Problem set 3:** A normal machine and no-idle machine are arranged alternately. The first machine is a no-idle machine, and the second machine is a normal machine.
- **Problem sets 4, 5, and 6:** 25%, 50%, and 75% of the machines are randomly set with no-idle machines, respectively.
- **Problem set 7:** All machines are no-idle machines.

The problems in each problem set include all combinations of jobs and machines, totaling 350 ($7\times10\times5$) types of problems. Each type of problem contains five similar problems. This study only uses the first problem in each type of problem. The name of each problem consists of a problem set, number of jobs, number of machines, and problem number in each type of problem. For example, 1_50_10_1 is the first problem from Problem set 1 with 50 jobs and 10 machines.

As for the evaluation of the algorithms, the HyperVolume (HV) and Inverted Generational Distance (IGD) are selected as the performance indices, which are two comprehensive performance indicators to evaluate the convergence and distribution performance of the MOEA. Because of the different scales of objectives, the obtained objective values are normalized.

5.1 Parameter settings

In this section, the main purpose of finding the optimal parameter settings for the velocity and position update formula in MDU-MoPSO is presented. Because the exchange sequence is used to represent the difference between particles, the values of c_1 and c_2 in the velocity Eq. (9) are both 1. The important part is to compare the ranges of r_1 and r_2 to find the best r_1 and r_2 for the MNIFSP. Therefore, we consider two parameters: the population size (*popsize*) and the ranges of r_1 and r_2. The levels of two parameters are set as follows: *popsize* at four levels (50,100,150, and 200) and ranges of r_1 and r_2 at five levels (0.2, 0.4, 0.6, 0.8, and 1.0). Each level has the ranges of r_1 and r_2, which is set from 0 to level. Two parameters have 20 (4×5) different combinations. The design of experiments is performed on five instances with 100_30 (100 jobs and 30 machines), 200_30,300_30,400_30, and 500_30, with each instance running 30 independent replicates.

Therefore, a full factorial experimental design with 3000 $(20\times30\times5)$ treatments is verified to decide parameter settings. Particularly, the size of the three sub-particle swarms is set to one-third of the entire particle swarm size.

Figure 7 shows the main effect of each parameter, where the best parameter settings are *popsize* = 200 and ranges of r_1 and r_2 are both 0–0.2.

5.2 Results and discussions

MDU-MoPSO is verified by comparing it with NSGA-II[34], SPEA2[35], MoPSO, MOEA/D[36], MultiObjective Hybrid EA (MOHEA)[12], and MOHEA with DE (MOHEA-DE)[37]. For the stability of the experiment, each algorithm was run 30 times. We set the same population size for all the comparison algorithms. The maximum generation is set as 500. In the MDU-MoPSO algorithm, the sizes of *sub*1, *sub*2, and *sub*3 are 66, 66, and 68, respectively. Table 1 shows the parameters of algorithms.

The parameters of NSGA-II, SPEA2, MOEA/D, and MOHEA have been tuned in previous paper[33], where the crossover and mutation rates are set as 0.8 and 0.1, respectively. Because there is no special local search in the proposed algorithm, we set mutation rate as 0.3 for all comparison algorithms to improve the exploitation performance. Moreover, the crossover and mutation rates of MOHEA-DE in the original paper[37] are set as 0.9 and 1/(length of variables), respectively, and the DE parameters K and F are both set as 0.5. Considering the different scales of MNIFSP problem and FSP[37], we set the parameters of the MOHEA-DE algorithm to be the same as those of other MOEAs.

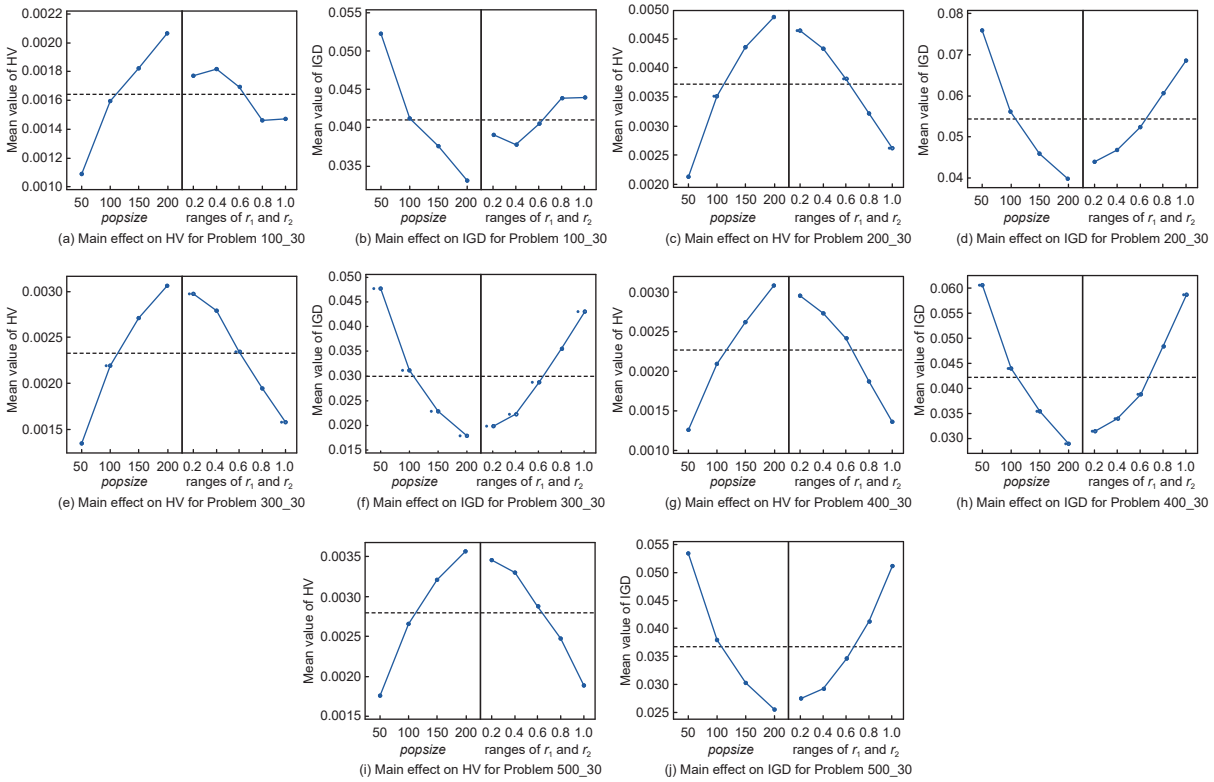

Fig. 7 Main effect of each parameter for different instances. The horizontal coordinates of the left and right are *popsize* and rangs of r_1 and r_2, respectively, and the vertical coordinate is the mean value of HV or IGD.

Table 1 Parameter settings.

Algorithm	Population size	Elite population size	Crossover rate	Mutation rate	Ranges of r_1 and r_2	DE paremeter	Maximum generation
NSGA-II	200	200	0.8	0.3	–	–	500
SPEA2	200	200	0.8	0.3	–	–	500
MoPSO	200	–	–	–	0−0.2	–	500
MOEA/D	200	–	0.8	0.3	–	–	500
MOHEA	200	100	0.8	0.3	–	–	500
MOHEA-DE	200	100	0.8	0.3	–	K, F = 0.5	500
MDU-MoPSO	200	–	–	–	0−0.2	–	500

The parameters of MoPSO and the proposed MDU-MoPSO are also the same.

The HV and IGD indicators evaluate the convergence and distribution performances. The significance analysis of different algorithms for different evaluation indicators is also presented in the results, where "+", "_", and "=" describe that the index of the compared method is significantly better than, worse than, and similar to the MDU-MoPSO on the Wilcoxon's rank-sum test at a 0.05 significant level, respectively. Moreover, the best mean values of index are highlighted in bold font.

The HV and IGD index results of Problem set 1 are shown in Table 2. Problem set 1 contains 50 different problems, where the first half of the machines are no-idle machines. Based on the HV index, the mean results of MDU-MoPSO on problems 1_50_10_1, 1_50_20_1, and 1_100_10_1 are not as good as those of MoPSO, but it performs best on the remaining 47 problems. Hence, the MDU-MoPSO solution has a larger dominance area than the solutions of the compared algorithms. Therefore, MDU-MoPSO has good convergence and distribution performances on Problem set 1. Moreover, the same results can be observed from the IGD indicator, where MDU-MoPSO on problems 1_50_10_1, 1_50_20_1, and 1_50_40_1 are not as good as MoPSO and MOHEA, but it performs best on the remaining 47 problems. In general, the MDU-MoPSO algorithm has good convergence and distribution performances on Problem set 1.

In Problem set 2, the latter half of the machines are a no-idle machines. Table 3 shows the HV and IGD index results of Problem set 2, which contains 50 different problems. Based on the HV index, the mean results of MDU-MoPSO on 2_100_10_1 are not as good as those of MoPSO, but it performs best on the remaining 49 problems. This finding indicates that MDU-MoPSO has good convergence and distribution performances on Problem set 2. As for the IGD indicator, MDU-MoPSO on problems 2_50_30_1 and 2_100_10_1 are not as good as MOHEA and MoPSO, but it performs best on the remaining 48 problems. Therefore, the MDU-MoPSO algorithm has good convergence and distribution performances on Problem set 2.

In Problem set 3, a normal machine and no-idle machine are arranged alternately. Table 4 shows the HV and IGD index results of Problem set 3, which contains 50 different problems. Based on the HV index, the mean results of MDU-MoPSO on 3_50_10_1 are not as good as those of MoPSO, but it performs best on the remaining 49 problems. This finding indicates that MDU-MoPSO has good convergence and distribution performances on most of the problems in Problem set 3. Moreover, for the IGD indicator, the results of MDU-MoPSO on 3_50_10_1, and 3_100_10_1 are not as good as those of MoPSO, but it performs best on the remaining 48 problems. In general, the MDU-MoPSO algorithm has good convergence and distribution performances on most of the problems in Problem set 3.

In Problem set 4, 25% of machines are no-idle machines. Table 5 shows the HV and IGD index results of Problem set 4, which contains 50 different problems. Based on the HV index, the mean results of MDU-MoPSO on 4_100_100_1 is not as good as those of MoPSO, but it performs best on the remaining 49 problems. This finding means that MDU-MoPSO has good convergence and distribution performances on Problem set 4. Furthermore, for the IGD indicator, the results of MDU-MoPSO on 4_50_20_1 and 4_100_10_1 are not as good as those of MoPSO, but it performs best on the remaining 48 problems. For the two indicators, the MDU-MoPSO algorithm has good convergence and distribution performances on Problem set 4.

In Problem set 5, 50% of machines are no-idle machines. Table 6 shows the HV and IGD index results of Problem set 5, which contains 50 different problems. Based on the HV index, MDU-MoPSO performs best on all the problems. The IGD indicator also performs best on all the problems. Therefore, the MDU-MoPSO algorithm has good convergence and distribution performances on Problem set 5.

In Problem set 6, 75% of machines are no-idle machines. Table 7 shows the HV and IGD index results of Problem set 6, which contains 50 different problems. Based on the HV index, the mean results of MDU-MoPSO on 6_100_10_1 is not as good as those of MoPSO, but it performs best on the remaining 49 problems. Furthermore, for the IGD indicator, the results of MDU-MoPSO on 6_50_10_1 and 6_100_10_1 are not as good as those of MoPSO, but it performs best on the remaining 48 problems. Therefore, the MDU-MoPSO algorithm has good convergence and distributions on most of problems in Problem set 6.

In Problem set 7, all machines are no-idle machines. Table 8 shows the HV and IGD index results of

Table 2 Results of HV and IGD for Problem set 1.

Problem	HV														IGD													
	MDU-MoPSO		MoPSO		MOHEA		NSGA-II		SPEA2		MOEA/D		MOHEA-DE		MDU-MoPSO		MoPSO		MOHEA		NSGA-II		SPEA2		MOEA/D		MOHEA-DE	
	Mean	Variance	Mean	Variance	Mean	Variance	Mean	Variance	Mean	Variance	Mean	Variance	Mean	Variance	Mean	Variance	Mean	Variance	Mean	Variance	Mean	Variance	Mean	Variance	Mean	Variance	Mean	Variance
	(×10⁻²)	(×10⁻⁶)	(×10⁻²)	(×10⁻⁶)	(×10⁻²)	(×10⁻⁶)	(×10⁻²)	(×10⁻⁶)	(×10⁻²)	(×10⁻⁶)	(×10⁻²)	(×10⁻⁶)	(×10⁻²)	(×10⁻⁶)	(×10⁻¹)	(×10⁻⁴)	(×10⁻¹)	(×10⁻⁴)	(×10⁻¹)	(×10⁻⁴)	(×10⁻¹)	(×10⁻⁴)	(×10⁻¹)	(×10⁻⁴)	(×10⁻¹)	(×10⁻⁴)	(×10⁻¹)	(×10⁻⁴)
1_50_10_1	0.53	1.05	**0.57**	1.76 =	0.51	1.62 =	0.35	1.35 +	0.46	1.76 +	0.24	1.76 +	0.27	1.48 +	0.26	0.79	**0.23**	0.93 =	0.25	1.17 +	0.43	1.90 +	0.31	1.83 =	0.52	2.85 +	0.54	2.87 +
1_50_20_1	0.33	1.85	**0.38**	2.23 =	0.33	0.98 =	0.20	0.61 +	0.26	0.76 +	0.14	0.91 +	0.13	0.54 +	0.37	2.03	**0.31**	2.76 =	0.34	1.08 +	0.52	1.44 +	0.44	1.23 +	0.63	2.29 +	0.64	2.51 +
1_50_30_1	**0.39**	1.02	0.34	1.27 =	0.38	1.15 =	0.22	1.10 +	0.27	0.92 +	0.14	0.44 +	0.21	0.86 +	**0.21**	1.02	0.27	1.66 =	0.22	1.55 +	0.42	2.07 +	0.35	1.74 +	0.58	1.84 +	0.54	2.63 +
1_50_40_1	**0.34**	1.20	0.33	1.15 =	0.34	1.34 =	0.18	0.70 +	0.25	0.70 +	0.15	0.82 +	0.15	0.52 +	0.36	1.70	0.37	1.91 =	**0.36**	1.82 +	0.58	1.95 +	0.47	1.60 +	0.62	2.70 +	0.62	2.01 +
1_50_50_1	**0.21**	0.55	0.14	0.52 +	0.17	0.60 =	0.08	0.32 +	0.14	0.34 +	0.07	0.30 +	0.09	0.42 +	**0.24**	0.99	0.37	1.78 +	0.30	1.66 +	0.47	1.73 +	0.35	1.46 +	0.52	2.02 +	0.53	2.59 +
1_100_10_1	0.48	0.46	**0.48**	0.74 =	0.25	0.48 +	0.13	0.20 +	0.19	0.39 +	0.10	0.23 +	0.10	0.26 +	**0.17**	0.16	0.19	0.21 +	0.37	0.69 +	0.53	0.68 +	0.45	0.90 +	0.61	0.98 +	0.58	1.04 +
1_100_20_1	**0.71**	1.41	0.69	1.93 =	0.38	0.66 +	0.19	0.88 +	0.28	0.84 +	0.14	0.63 +	0.17	0.61 +	**0.23**	0.88	0.23	1.33 =	0.54	0.94 +	0.80	2.73 +	0.66	1.56 +	0.86	2.09 +	0.81	1.98 +
1_100_30_1	**0.41**	0.23	0.41	0.46 =	0.24	0.48 +	0.12	0.26 +	0.19	0.41 +	0.09	0.17 +	0.13	0.54 +	**0.13**	0.22	0.14	0.34 =	0.33	1.02 +	0.52	1.44 +	0.41	1.19 +	0.58	0.82 +	0.51	1.96 +
1_100_40_1	**0.54**	0.81	0.52	1.47 =	0.26	0.83 +	0.08	0.43 +	0.16	0.52 +	0.07	0.20 +	0.07	0.34 +	**0.26**	0.72	0.27	1.31 =	0.57	1.70 +	0.89	2.66 +	0.73	1.86 +	0.90	1.58 +	0.94	2.46 +
1_100_50_1	**0.70**	1.39	0.56	2.46 +	0.24	0.84 +	0.09	0.42 +	0.16	0.60 +	0.07	0.27 +	0.09	0.35 +	**0.28**	1.08	0.40	2.03 +	0.77	1.70 +	1.06	2.72 +	0.90	2.07 +	1.07	2.00 +	1.03	2.02 +
1_150_10_1	**0.30**	0.25	0.24	0.18 +	0.11	0.15 +	0.05	0.06 +	0.10	0.17 +	0.05	0.11 +	0.06	0.11 +	**0.12**	0.25	0.20	0.31 +	0.48	0.76 +	0.63	0.49 +	0.51	0.76 +	0.62	1.18 +	0.66	0.92 +
1_150_20_1	**0.78**	1.08	0.64	1.31 +	0.30	1.04 +	0.10	0.38 +	0.23	0.58 +	0.09	0.28 +	0.12	0.61 +	**0.19**	0.69	0.27	1.05 +	0.65	1.77 +	0.98	2.01 +	0.75	1.43 +	0.96	1.70 +	0.95	2.43 +
1_150_30_1	**0.70**	1.42	0.54	1.31 +	0.25	0.64 +	0.10	0.43 +	0.17	0.46 +	0.08	0.29 +	0.09	0.26 +	**0.22**	1.13	0.36	1.12 +	0.69	1.30 +	0.97	2.16 +	0.81	1.19 +	0.99	1.49 +	0.98	1.51 +
1_150_40_1	**0.61**	0.64	0.48	1.17 +	0.20	0.61 +	0.09	0.30 +	0.12	0.21 +	0.08	0.19 +	0.08	0.23 +	**0.12**	0.36	0.24	0.99 +	0.58	1.53 +	0.80	1.54 +	0.72	0.89 +	0.81	1.51 +	0.82	1.56 +
1_150_50_1	**0.80**	1.26	0.62	1.66 +	0.26	0.77 +	0.08	0.35 +	0.17	0.60 +	0.08	0.31 +	0.08	0.24 +	**0.16**	0.68	0.30	1.39 +	0.70	1.58 +	1.03	2.02 +	0.83	1.66 +	1.03	2.32 +	1.00	1.30 +
1_200_10_1	**0.46**	0.24	0.37	0.22 +	0.19	0.28 +	0.09	0.17 +	0.15	0.16 +	0.07	0.15 +	0.08	0.09 +	**0.10**	0.30	0.19	0.40 +	0.49	0.80 +	0.68	0.80 +	0.56	0.59 +	0.66	0.82 +	0.69	0.68 +
1_200_20_1	**0.76**	0.80	0.55	0.99 +	0.27	0.56 +	0.14	0.71 +	0.23	0.51 +	0.10	0.29 +	0.15	0.73 +	**0.17**	0.53	0.34	0.89 +	0.67	1.02 +	0.90	2.89 +	0.74	1.20 +	0.93	1.69 +	0.86	2.09 +
1_200_30_1	**0.88**	0.64	0.70	0.72 +	0.28	0.83 +	0.13	0.64 +	0.22	0.52 +	0.09	0.31 +	0.15	0.50 +	**0.13**	0.28	0.27	0.57 +	0.71	1.43 +	0.96	2.56 +	0.79	1.32 +	1.03	2.02 +	0.91	2.02 +
1_200_40_1	**0.60**	0.90	0.50	0.66 +	0.18	0.40 +	0.04	0.08 +	0.12	0.37 +	0.05	0.13 +	0.07	0.25 +	**0.14**	0.57	0.23	0.57 +	0.62	1.12 +	0.95	1.02 +	0.73	1.49 +	0.93	1.81 +	0.90	2.26 +
1_200_50_1	**0.71**	0.91	0.53	0.99 +	0.22	0.34 +	0.04	0.10 +	0.18	0.69 +	0.07	0.17 +	0.09	0.21 +	**0.17**	0.45	0.32	0.93 +	0.70	0.76 +	1.08	1.25 +	0.76	2.26 +	0.99	1.51 +	0.93	1.22 +
1_250_10_1	**0.36**	0.15	0.27	0.13 +	0.13	0.14 +	0.08	0.11 +	0.12	0.07 +	0.05	0.06 +	0.06	0.04 +	**0.13**	0.25	0.26	0.31 +	0.53	0.52 +	0.66	0.93 +	0.56	0.35 +	0.63	0.40 +	0.69	0.36 +
1_250_20_1	**0.50**	0.50	0.42	0.57 +	0.19	0.46 +	0.07	0.14 +	0.13	0.19 +	0.05	0.07 +	0.07	0.21 +	**0.12**	0.49	0.20	0.74 +	0.53	1.32 +	0.77	1.24 +	0.63	0.72 +	0.80	0.89 +	0.77	1.56 +
1_250_30_1	**0.59**	0.62	0.45	0.44 +	0.16	0.27 +	0.06	0.13 +	0.12	0.26 +	0.04	0.07 +	0.07	0.13 +	**0.15**	0.49	0.29	0.48 +	0.68	0.91 +	0.90	1.10 +	0.74	1.20 +	0.97	0.94 +	0.87	1.11 +
1_250_40_1	**0.98**	1.52	0.78	1.73 +	0.31	1.08 +	0.08	0.20 +	0.23	1.00 +	0.09	0.35 +	0.14	0.73 +	**0.15**	0.73	0.30	1.13 +	0.78	1.76 +	1.18	1.49 +	0.89	2.23 +	1.13	2.12 +	1.09	3.28 +
1_250_50_1	**1.02**	1.24	0.81	1.24 +	0.35	1.36 +	0.14	0.56 +	0.28	0.72 +	0.14	0.59 +	0.17	0.51 +	**0.13**	0.57	0.28	0.78 +	0.73	1.90 +	1.05	2.67 +	0.81	1.46 +	1.02	2.15 +	0.96	1.52 +
1_300_10_1	**0.40**	0.25	0.31	0.16 +	0.15	0.12 +	0.07	0.14 +	0.12	0.22 +	0.06	0.09 +	0.06	0.09 +	**0.11**	0.37	0.23	0.31 +	0.50	0.41 +	0.68	0.88 +	0.56	1.12 +	0.67	0.84 +	0.71	0.85 +
1_300_20_1	**0.62**	0.25	0.49	0.46 +	0.20	0.27 +	0.09	0.18 +	0.14	0.24 +	0.05	0.14 +	0.08	0.16 +	**0.09**	0.20	0.21	0.47 +	0.58	0.67 +	0.80	1.29 +	0.68	0.80 +	0.87	1.20 +	0.81	1.19 +
1_300_30_1	**1.25**	1.34	0.98	1.77 +	0.37	1.34 +	0.14	0.50 +	0.31	1.00 +	0.11	0.40 +	0.19	1.17 +	**0.13**	0.40	0.29	0.89 +	0.84	1.90 +	1.18	2.16 +	0.92	2.06 +	1.20	1.34 +	1.10	3.60 +
1_300_40_1	**0.88**	1.03	0.69	1.25 +	0.28	0.43 +	0.08	0.37 +	0.24	0.68 +	0.08	0.37 +	0.15	0.32 +	**0.19**	0.55	0.34	0.98 +	0.78	0.84 +	1.15	3.00 +	0.84	1.43 +	1.10	1.86 +	1.00	1.70 +
1_300_50_1	**0.77**	0.60	0.59	0.67 +	0.22	0.30 +	0.05	0.20 +	0.17	0.46 +	0.06	0.18 +	0.09	0.29 +	**0.12**	0.34	0.27	0.56 +	0.70	0.79 +	1.07	2.21 +	0.78	1.39 +	1.03	1.70 +	0.96	1.77 +
1_350_10_1	**0.38**	0.16	0.30	0.08 +	0.14	0.09 +	0.06	0.08 +	0.11	0.13 +	0.04	0.04 +	0.05	0.10 +	**0.09**	0.21	0.19	0.18 +	0.49	0.40 +	0.67	0.75 +	0.55	0.67 +	0.69	0.42 +	0.69	0.95 +
1_350_20_1	**0.74**	0.44	0.59	0.47 +	0.26	0.49 +	0.10	0.54 +	0.24	0.35 +	0.09	0.20 +	0.14	0.41 +	**0.09**	0.22	0.23	0.44 +	0.63	1.21 +	0.92	3.00 +	0.65	0.79 +	0.91	1.68 +	0.81	1.56 +
1_350_30_1	**0.84**	0.73	0.65	0.56 +	0.31	0.29 +	0.11	0.36 +	0.23	0.54 +	0.10	0.23 +	0.15	0.50 +	**0.11**	0.33	0.25	0.40 +	0.61	0.48 +	0.94	2.09 +	0.73	1.19 +	0.96	1.52 +	0.85	1.73 +
1_350_40_1	**1.18**	1.32	0.98	1.48 +	0.37	1.03 +	0.12	0.61 +	0.31	0.79 +	0.11	0.53 +	0.16	0.71 +	**0.14**	0.31	0.26	0.74 +	0.80	1.50 +	1.21	2.72 +	0.87	1.58 +	1.19	1.88 +	1.10	2.14 +
1_350_50_1	**1.49**	2.13	1.21	2.54 +	0.46	2.11 +	0.11	0.49 +	0.30	1.36 +	0.11	0.41 +	0.21	0.62 +	**0.16**	0.62	0.32	1.10 +	0.93	2.24 +	1.42	2.00 +	1.11	2.55 +	1.42	2.69 +	1.24	1.54 +
1_400_10_1	**0.31**	0.03	0.24	0.07 +	0.11	0.10 +	0.06	0.07 +	0.11	0.06 +	0.04	0.08 +	0.06	0.07 +	**0.09**	0.07	0.24	0.33 +	0.54	0.77 +	0.68	0.61 +	0.55	0.38 +	0.66	1.01 +	0.70	0.76 +
1_400_20_1	**0.61**	0.24	0.50	0.19 +	0.18	0.32 +	0.06	0.13 +	0.15	0.21 +	0.05	0.12 +	0.09	0.21 +	**0.12**	0.25	0.22	0.21 +	0.65	0.99 +	0.90	1.25 +	0.69	0.66 +	0.93	1.47 +	0.83	1.21 +
1_400_30_1	**0.85**	0.41	0.66	0.69 +	0.30	0.57 +	0.10	0.47 +	0.25	0.49 +	0.09	0.28 +	0.15	0.33 +	**0.15**	0.24	0.31	0.53 +	0.70	0.99 +	1.05	2.68 +	0.76	1.08 +	1.03	1.40 +	0.91	1.17 +
1_400_40_1	**0.96**	0.81	0.76	0.90 +	0.30	0.70 +	0.11	0.47 +	0.26	0.68 +	0.08	0.33 +	0.14	0.52 +	**0.12**	0.32	0.27	0.55 +	0.73	1.39 +	1.07	3.13 +	0.79	1.43 +	1.09	1.74 +	0.99	2.19 +
1_400_50_1	**1.37**	1.20	1.14	1.87 +	0.47	2.20 +	0.13	0.68 +	0.41	1.55 +	0.15	0.62 +	0.26	1.56 +	**0.18**	0.43	0.32	0.79 +	0.87	2.30 +	1.36	3.13 +	0.93	2.31 +	1.29	2.64 +	1.12	3.16 +
1_450_10_1	**0.32**	0.10	0.25	0.10 +	0.10	0.11 +	0.05	0.04 +	0.09	0.08 +	0.03	0.03 +	0.06	0.08 +	**0.13**	0.25	0.22	0.31 +	0.52	0.54 +	0.66	0.38 +	0.54	0.41 +	0.68	0.53 +	0.66	0.78 +
1_450_20_1	**0.41**	0.12	0.32	0.17 +	0.13	0.17 +	0.05	0.10 +	0.12	0.18 +	0.04	0.07 +	0.06	0.11 +	**0.08**	0.09	0.17	0.26 +	0.46	0.67 +	0.66	0.99 +	0.50	0.75 +	0.67	0.75 +	0.64	1.06 +
1_450_30_1	**0.66**	0.37	0.53	0.31 +	0.23	0.37 +	0.06	0.29 +	0.17	0.30 +	0.05	0.10 +	0.09	0.23 +	**0.10**	0.29	0.21	0.27 +	0.57	0.86 +	0.94	2.68 +	0.67	0.94 +	0.93	1.19 +	0.82	1.55 +
1_450_40_1	**0.56**	0.30	0.46	0.26 +	0.17	0.27 +	0.04	0.19 +	0.14	0.26 +	0.04	0.06 +	0.07	0.10 +	**0.10**	0.27	0.19	0.30 +	0.58	0.86 +	0.88	2.01 +	0.63	1.23 +	0.87	0.81 +	0.77	0.76 +
1_450_50_1	**1.19**	1.41	0.97	1.04 +	0.37	1.34 +	0.10	1.11 +	0.31	1.03 +	0.10	0.23 +	0.19	0.52 +	**0.18**	0.61	0.34	0.55 +	0.88	1.86 +	1.33	4.72 +	0.95	1.72 +	1.26	1.05 +	1.12	1.37 +
1_500_10_1	**0.32**	0.07	0.26	0.08 +	0.13	0.08 +	0.07	0.09 +	0.11	0.07 +	0.05	0.07 +	0.07	0.05 +	**0.13**	0.01	0.18	0.11 +	0.41	0.37 +	0.56	0.68 +	0.44	0.37 +	0.56	0.75 +	0.53	0.41 +
1_500_20_1	**0.56**	0.25	0.46	0.36 +	0.19	0.20 +	0.10	0.28 +	0.17	0.28 +	0.06	0.09 +	0.10	0.22 +	**0.11**	0.13	0.20	0.38 +	0.56	0.55 +	0.74	1.54 +	0.59	1.09 +	0.82	0.96 +	0.71	1.11 +
1_500_30_1	**0.66**	0.25	0.51	0.40 +	0.21	0.16 +	0.05	0.18 +	0.19	0.32 +	0.06	0.14 +	0.10	0.26 +	**0.08**	0.16	0.22	0.39 +	0.58	0.41 +	0.92	1.88 +	0.62	0.95 +	0.89	1.46 +	0.78	1.24 +
1_500_40_1	**0.84**	0.74	0.64	0.70 +	0.21	0.57 +	0.08	0.30 +	0.20	0.39 +	0.06	0.21 +	0.11	0.20 +	**0.17**	0.42	0.33	0.57 +	0.83	1.39 +	1.09	2.30 +	0.84	1.18 +	1.14	2.01 +	1.00	1.05 +
1_500_50_1	**1.37**	1.02	1.03	1.72 +	0.45	1.65 +	0.12	1.26 +	0.34	1.40 +	0.12	0.44 +	0.23	0.95 +	**0.15**	0.31	0.37	0.83 +	0.87	2.11 +	1.38	6.04 +	0.99	2.19 +	1.33	2.35 +	1.15	2.56 +
+/−/=	–		42/0/8		45/0/5		50/0/0		50/0/0		50/0/0		50/0/0		–		43/0/7		50/0/0		50/0/0		49/0/1		50/0/0		50/0/0	

Table 3 Results of HV and IGD for Problem set 2.

Problem	HV														IGD													
	MDU-MoPSO		MoPSO		MOHEA		NSGA-II		SPEA2		MOEA/D		MOHEA-DE		MDU-MoPSO		MoPSO		MOHEA		NSGA-II		SPEA2		MOEA/D		MOHEA-DE	
	Mean ($\times10^{-2}$)	Variance ($\times10^{-6}$)	Mean ($\times10^{-2}$)	Variance ($\times10^{-6}$)	Mean ($\times10^{-2}$)	Variance ($\times10^{-6}$)	Mean ($\times10^{-2}$)	Variance ($\times10^{-6}$)	Mean ($\times10^{-2}$)	Variance ($\times10^{-6}$)	Mean ($\times10^{-2}$)	Variance ($\times10^{-6}$)	Mean ($\times10^{-2}$)	Variance ($\times10^{-6}$)	Mean ($\times10^{-1}$)	Variance ($\times10^{-4}$)	Mean ($\times10^{-1}$)	Variance ($\times10^{-4}$)	Mean ($\times10^{-1}$)	Variance ($\times10^{-4}$)	Mean ($\times10^{-1}$)	Variance ($\times10^{-4}$)	Mean ($\times10^{-1}$)	Variance ($\times10^{-4}$)	Mean ($\times10^{-1}$)	Variance ($\times10^{-4}$)	Mean ($\times10^{-1}$)	Variance ($\times10^{-4}$)
2_50_10_1	**0.45**	0.85	0.39	1.94 =	0.37	1.37 +	0.21	0.88 +	0.34	1.48 +	0.15	1.09 +	0.19	1.32 +	**0.19**	0.79	0.25	2.27 =	0.27	2.20 +	0.48	2.08 +	0.32	2.43 +	0.61	4.61 +	0.52	3.69 +
2_50_20_1	**0.25**	0.95	0.19	0.79 +	0.14	0.60 +	0.08	0.38 +	0.13	0.49 +	0.06	0.13 +	0.08	0.38 +	**0.33**	2.09	0.40	2.27 =	0.50	1.84 +	0.65	2.41 +	0.51	2.13 +	0.67	1.27 +	0.72	2.99 +
2_50_30_1	**0.27**	0.65	0.19	0.75 +	0.27	0.75 =	0.09	0.42 +	0.18	0.54 +	0.11	0.29 +	0.10	0.47 +	0.26	1.08	0.38	2.20 +	**0.25**	1.40 =	0.58	2.36 +	0.40	1.43 +	0.51	1.47 +	0.55	2.44 +
2_50_40_1	**0.30**	0.88	0.15	0.51 +	0.23	0.63 +	0.11	0.50 +	0.16	0.76 +	0.09	0.51 +	0.12	0.48 +	**0.27**	1.65	0.51	2.25 +	0.37	1.53 +	0.61	3.10 +	0.49	2.93 +	0.64	3.45 +	0.64	2.67 +
2_50_50_1	**0.30**	1.21	0.17	0.54 +	0.23	0.52 +	0.13	0.68 +	0.14	0.45 +	0.11	0.52 +	0.12	0.24 +	**0.25**	1.47	0.42	1.54 +	0.33	1.26 +	0.53	3.26 +	0.48	1.57 +	0.55	2.52 +	0.51	0.98 +
2_100_10_1	0.35	0.54	**0.36**	0.61 =	0.16	0.35 +	0.06	0.14 +	0.15	0.29 +	0.07	0.22 +	0.07	0.14 +	0.23	1.01	**0.18**	0.74 ≈	0.59	1.33 +	0.85	1.59 +	0.63	1.03 +	0.78	2.07 +	0.81	1.18 +
2_100_20_1	**0.60**	1.90	0.57	2.09 =	0.22	0.64 +	0.06	0.18 +	0.13	0.64 +	0.09	0.54 +	0.12	0.62 +	**0.25**	1.27	0.29	1.52 =	0.67	1.58 +	1.00	1.28 +	0.84	2.78 +	0.93	3.37 +	0.96	2.57 +
2_100_30_1	**0.74**	2.24	0.66	2.12 =	0.32	1.11 +	0.13	0.47 +	0.25	0.97 +	0.16	0.85 +	0.17	0.56 +	**0.29**	1.59	0.35	1.72 =	0.71	2.10 +	1.02	2.09 +	0.81	2.11 +	0.95	3.24 +	0.95	1.91 +
2_100_40_1	**0.89**	1.93	0.80	3.33 =	0.37	1.34 +	0.12	0.54 +	0.21	1.00 +	0.14	0.89 +	0.18	1.09 +	**0.24**	1.03	0.31	2.12 =	0.73	1.91 +	1.11	2.59 +	0.94	1.97 +	1.06	3.17 +	1.01	3.38 +
2_100_50_1	**0.61**	2.34	0.50	1.65 +	0.25	0.91 +	0.07	0.19 +	0.15	0.57 +	0.07	0.22 +	0.12	0.42 +	**0.28**	1.92	0.38	1.92 +	0.69	1.99 +	1.03	1.77 +	0.84	2.22 +	1.02	1.69 +	0.91	1.95 +
2_150_10_1	**0.24**	0.07	0.21	0.20 +	0.11	0.12 +	0.06	0.10 +	0.11	0.09 +	0.05	0.13 +	0.07	0.10 +	**0.08**	0.19	0.13	0.62 +	0.47	0.92 +	0.63	1.29 +	0.50	0.54 +	0.65	1.70 +	0.62	1.28 +
2_150_20_1	**0.70**	1.13	0.53	1.29 +	0.23	0.74 +	0.07	0.25 +	0.17	0.50 +	0.07	0.36 +	0.11	0.47 +	**0.18**	0.91	0.34	1.22 +	0.72	1.56 +	1.04	1.92 +	0.83	1.38 +	1.04	2.98 +	0.97	2.87 +
2_150_30_1	**0.74**	2.63	0.54	1.29 +	0.20	0.73 +	0.05	0.12 +	0.17	0.60 +	0.07	0.35 +	0.11	0.64 +	**0.28**	1.83	0.44	1.31 +	0.88	2.03 +	1.24	2.04 +	0.94	2.42 +	1.12	2.22 +	1.06	2.59 +
2_150_40_1	**0.93**	2.35	0.75	1.75 +	0.35	1.23 +	0.13	0.32 +	0.27	1.24 +	0.12	0.63 +	0.21	0.73 +	**0.19**	1.32	0.32	1.21 +	0.73	2.20 +	1.07	1.67 +	0.83	2.47 +	1.10	3.15 +	0.93	2.25 +
2_150_50_1	**0.66**	1.09	0.49	1.84 +	0.23	0.60 +	0.06	0.30 +	0.20	0.82 +	0.08	0.29 +	0.15	0.49 +	**0.21**	0.95	0.37	1.97 +	0.70	1.41 +	1.05	2.56 +	0.75	2.37 +	0.99	1.94 +	0.84	1.92 +
2_200_10_1	**0.36**	0.12	0.29	0.19 +	0.12	0.16 +	0.06	0.19 +	0.11	0.14 +	0.04	0.07 +	0.06	0.15 +	**0.07**	0.22	0.21	0.57 +	0.64	1.05 +	0.87	1.91 +	0.70	1.11 +	0.87	1.25 +	0.86	1.72 +
2_200_20_1	**0.63**	1.30	0.54	1.19 +	0.25	0.62 +	0.09	0.29 +	0.18	0.43 +	0.06	0.18 +	0.12	0.56 +	**0.21**	1.10	0.30	1.11 +	0.70	1.61 +	1.00	1.92 +	0.83	1.34 +	1.06	1.89 +	0.93	1.99 +
2_200_30_1	**1.16**	1.87	0.86	1.49 +	0.40	1.85 +	0.15	0.79 +	0.37	1.98 +	0.19	1.13 +	0.24	1.22 +	**0.24**	0.86	0.46	0.96 +	0.91	2.64 +	1.28	3.52 +	0.95	3.22 +	1.20	3.55 +	1.12	2.73 +
2_200_40_1	**0.79**	1.58	0.59	1.09 +	0.23	1.00 +	0.07	0.37 +	0.18	0.51 +	0.08	0.26 +	0.12	0.40 +	**0.16**	0.95	0.32	0.96 +	0.76	2.39 +	1.09	2.97 +	0.82	1.54 +	1.02	1.79 +	0.93	1.66 +
2_200_50_1	**1.00**	1.84	0.77	1.75 +	0.35	1.79 +	0.12	0.73 +	0.23	1.17 +	0.12	0.49 +	0.17	0.67 +	**0.19**	0.79	0.36	1.24 +	0.77	2.92 +	1.15	3.82 +	0.94	2.70 +	1.12	1.92 +	1.05	2.43 +
2_250_10_1	**0.38**	0.14	0.30	0.19 +	0.14	0.20 +	0.05	0.12 +	0.11	0.16 +	0.05	0.08 +	0.07	0.11 +	**0.09**	0.16	0.21	0.35 +	0.58	0.82 +	0.82	1.47 +	0.67	1.09 +	0.76	1.21 +	0.76	0.96 +
2_250_20_1	**0.82**	1.45	0.67	0.80 +	0.25	0.79 +	0.06	0.20 +	0.18	0.72 +	0.07	0.28 +	0.10	0.46 +	**0.17**	0.73	0.28	0.64 +	0.79	1.57 +	1.18	1.99 +	0.91	2.54 +	1.13	2.22 +	1.08	2.46 +
2_250_30_1	**0.93**	1.24	0.72	0.98 +	0.31	0.77 +	0.09	0.39 +	0.24	0.85 +	0.10	0.32 +	0.17	0.94 +	**0.20**	0.75	0.37	0.78 +	0.83	1.66 +	1.23	3.21 +	0.92	2.04 +	1.17	1.83 +	1.05	3.05 +
2_250_40_1	**1.21**	2.10	0.91	1.71 +	0.35	1.40 +	0.10	0.46 +	0.25	1.23 +	0.14	0.60 +	0.20	0.58 +	**0.21**	0.93	0.42	1.03 +	0.96	2.10 +	1.38	3.25 +	1.09	2.60 +	1.28	2.43 +	1.17	1.48 +
2_250_50_1	**1.14**	2.42	0.87	2.37 +	0.35	1.42 +	0.09	0.80 +	0.23	0.82 +	0.12	0.46 +	0.17	0.65 +	**0.19**	1.07	0.38	1.59 +	0.88	2.31 +	1.33	4.78 +	1.05	2.05 +	1.24	2.07 +	1.15	2.13 +
2_300_10_1	**0.17**	0.03	0.14	0.07 +	0.05	0.04 +	0.02	0.02 +	0.05	0.02 +	0.02	0.02 +	0.03	0.02 +	**0.08**	0.11	0.18	0.34 +	0.65	0.80 +	0.89	1.48 +	0.66	0.46 +	0.84	0.72 +	0.82	1.34 +
2_300_20_1	**0.78**	0.80	0.63	0.84 +	0.31	0.56 +	0.13	0.40 +	0.24	0.45 +	0.14	0.40 +	0.17	0.35 +	**0.15**	0.55	0.29	0.76 +	0.71	1.02 +	1.02	2.34 +	0.82	1.16 +	0.96	1.92 +	0.94	1.14 +
2_300_30_1	**0.91**	1.56	0.64	1.04 +	0.27	0.83 +	0.08	0.39 +	0.19	0.67 +	0.08	0.35 +	0.14	0.34 +	**0.23**	0.88	0.46	0.85 +	0.89	1.69 +	1.28	3.76 +	1.03	2.16 +	1.25	2.17 +	1.13	1.67 +
2_300_40_1	**1.17**	1.68	0.91	2.03 +	0.38	1.14 +	0.07	0.25 +	0.26	1.10 +	0.12	0.39 +	0.16	0.71 +	**0.19**	0.67	0.37	1.13 +	0.88	1.56 +	1.40	2.09 +	1.04	2.93 +	1.27	1.80 +	1.20	2.75 +
2_300_50_1	**1.36**	2.31	1.04	1.91 +	0.42	1.55 +	0.13	0.97 +	0.35	1.76 +	0.15	0.82 +	0.20	0.95 +	**0.21**	0.90	0.42	0.98 +	0.96	1.95 +	1.42	4.74 +	1.05	2.72 +	1.36	3.66 +	1.25	2.45 +
2_350_10_1	**0.24**	0.06	0.20	0.09 +	0.08	0.03 +	0.04	0.04 +	0.09	0.05 +	0.03	0.02 +	0.06	0.04 +	**0.07**	0.12	0.17	0.42 +	0.54	0.22 +	0.70	0.90 +	0.53	0.56 +	0.70	0.64 +	0.63	0.59 +
2_350_20_1	**0.86**	0.87	0.71	0.96 +	0.28	0.88 +	0.09	0.30 +	0.23	0.48 +	0.07	0.31 +	0.15	0.55 +	**0.17**	0.50	0.29	0.71 +	0.80	1.79 +	1.16	2.10 +	0.87	1.30 +	1.18	2.25 +	1.01	2.22 +
2_350_30_1	**0.82**	0.64	0.66	0.81 +	0.28	0.65 +	0.09	0.65 +	0.20	0.56 +	0.11	0.30 +	0.14	0.20 +	**0.14**	0.51	0.29	0.76 +	0.73	1.22 +	1.09	3.71 +	0.86	1.73 +	1.02	1.41 +	0.96	0.99 +
2_350_40_1	**1.14**	1.92	0.93	1.35 +	0.36	1.10 +	0.08	0.54 +	0.25	0.98 +	0.09	0.46 +	0.17	0.92 +	**0.24**	0.89	0.38	0.72 +	0.93	1.62 +	1.43	3.39 +	1.08	2.06 +	1.38	3.23 +	1.21	2.84 +
2_350_50_1	**1.24**	1.91	0.97	1.72 +	0.44	1.42 +	0.10	0.81 +	0.30	1.06 +	0.14	0.50 +	0.20	1.16 +	**0.16**	0.85	0.35	0.96 +	0.82	1.94 +	1.35	4.36 +	0.99	1.87 +	1.25	2.13 +	1.15	3.29 +
2_400_10_1	**0.20**	0.06	0.17	0.06 +	0.07	0.04 +	0.03	0.03 +	0.06	0.04 +	0.02	0.02 +	0.04	0.02 +	**0.10**	0.29	0.17	0.31 +	0.54	0.41 +	0.69	0.84 +	0.58	0.52 +	0.66	0.48 +	0.65	0.43 +
2_400_20_1	**0.82**	0.62	0.71	0.75 +	0.26	0.61 +	0.07	0.25 +	0.19	0.61 +	0.08	0.26 +	0.15	0.45 +	**0.26**	0.48	0.35	0.68 +	0.89	1.43 +	1.30	2.71 +	1.01	2.09 +	1.23	1.94 +	1.08	1.74 +
2_400_30_1	**1.14**	1.60	0.91	1.20 +	0.46	1.17 +	0.12	0.50 +	0.32	1.21 +	0.14	0.56 +	0.26	0.87 +	**0.18**	0.80	0.34	0.73 +	0.76	1.50 +	1.27	3.14 +	0.93	2.09 +	1.19	1.96 +	1.02	1.80 +
2_400_40_1	**1.27**	1.39	0.97	1.75 +	0.40	1.80 +	0.12	0.80 +	0.29	1.11 +	0.13	0.51 +	0.24	1.21 +	**0.13**	0.44	0.32	0.98 +	0.84	2.46 +	1.29	4.10 +	0.98	1.99 +	1.24	2.61 +	1.07	2.61 +
2_400_50_1	**1.08**	2.17	0.88	2.08 +	0.39	1.45 +	0.07	0.39 +	0.27	1.12 +	0.09	0.34 +	0.15	0.25 +	**0.18**	1.04	0.33	1.21 +	0.80	2.10 +	1.35	3.22 +	0.95	2.37 +	1.25	1.66 +	1.14	1.02 +
2_450_10_1	**0.24**	0.05	0.18	0.06 +	0.07	0.06 +	0.03	0.04 +	0.07	0.05 +	0.03	0.05 +	0.04	0.05 +	**0.06**	0.16	0.17	0.26 +	0.55	0.61 +	0.77	0.97 +	0.59	0.55 +	0.71	1.34 +	0.71	0.89 +
2_450_20_1	**0.63**	0.38	0.51	0.53 +	0.21	0.48 +	0.07	0.33 +	0.16	0.42 +	0.07	0.12 +	0.12	0.30 +	**0.09**	0.29	0.21	0.68 +	0.65	1.44 +	0.96	2.98 +	0.75	1.74 +	0.91	1.09 +	0.84	1.80 +
2_450_30_1	**1.41**	1.56	1.09	1.33 +	0.47	1.53 +	0.11	0.60 +	0.35	1.16 +	0.18	0.69 +	0.28	1.12 +	**0.16**	0.57	0.37	0.73 +	0.92	1.86 +	1.46	3.24 +	1.05	1.78 +	1.30	2.44 +	1.16	2.23 +
2_450_40_1	**0.92**	1.11	0.74	0.91 +	0.28	0.80 +	0.04	0.22 +	0.23	0.63 +	0.10	0.31 +	0.16	0.41 +	**0.11**	0.68	0.26	0.65 +	0.76	1.59 +	1.27	2.74 +	0.82	1.62 +	1.07	1.61 +	0.96	1.36 +
2_450_50_1	**1.75**	3.12	1.47	3.13 +	0.77	2.92 +	0.29	2.39 +	0.60	2.28 +	0.32	1.30 +	0.50	2.77 +	**0.16**	0.93	0.32	1.16 +	0.80	1.97 +	1.33	6.22 +	0.95	2.65 +	1.25	2.38 +	1.06	3.08 +
2_500_10_1	**0.29**	0.09	0.24	0.11 +	0.11	0.09 +	0.04	0.07 +	0.10	0.08 +	0.04	0.07 +	0.06	0.04 +	**0.09**	0.20	0.19	0.40 +	0.54	0.55 +	0.78	1.35 +	0.58	0.60 +	0.71	1.04 +	0.69	0.34 +
2_500_20_1	**0.67**	0.57	0.54	0.52 +	0.25	0.60 +	0.07	0.23 +	0.18	0.52 +	0.08	0.21 +	0.13	0.24 +	**0.13**	0.52	0.26	0.57 +	0.68	1.41 +	1.06	2.35 +	0.79	1.80 +	0.97	1.18 +	0.87	1.33 +
2_500_30_1	**1.19**	1.68	0.94	0.80 +	0.43	0.58 +	0.11	0.64 +	0.34	1.15 +	0.14	0.43 +	0.27	0.57 +	**0.18**	0.73	0.35	0.57 +	0.84	0.84 +	1.35	3.55 +	0.95	1.72 +	1.26	2.06 +	1.04	1.12 +
2_500_40_1	**1.45**	1.72	1.08	1.52 +	0.48	1.82 +	0.16	1.75 +	0.42	1.49 +	0.18	0.79 +	0.29	0.89 +	**0.15**	0.67	0.39	0.80 +	0.90	1.93 +	1.39	7.28 +	0.97	1.88 +	1.30	2.26 +	1.13	1.69 +
2_500_50_1	**0.97**	0.99	0.72	0.90 +	0.33	0.92 +	0.08	0.77 +	0.19	0.51 +	0.08	0.25 +	0.17	0.62 +	**0.13**	0.48	0.32	0.64 +	0.73	1.34 +	1.20	4.81 +	0.93	1.43 +	1.15	1.63 +	0.96	1.76 +
+/−/=	−		45/0/5		49/0/1		50/0/0		50/0/0		50/0/0		50/0/0		−		44/1/5		49/0/1		50/0/0		50/0/0		50/0/0		50/0/0	

Table 4 Results of HV and IGD for Problem set 3.

Problem	HV														IGD													
	MDU-MoPSO		MoPSO		MOHEA		NSGA-II		SPEA2		MOEA/D		MOHEA-DE		MDU-MoPSO		MoPSO		MOHEA		NSGA-II		SPEA2		MOEA/D		MOHEA-DE	
	Mean ($\times10^{-2}$)	Variance ($\times10^{-6}$)	Mean ($\times10^{-2}$)	Variance ($\times10^{-6}$)	Mean ($\times10^{-2}$)	Variance ($\times10^{-6}$)	Mean ($\times10^{-2}$)	Variance ($\times10^{-6}$)	Mean ($\times10^{-2}$)	Variance ($\times10^{-6}$)	Mean ($\times10^{-2}$)	Variance ($\times10^{-6}$)	Mean ($\times10^{-2}$)	Variance ($\times10^{-6}$)	Mean ($\times10^{-1}$)	Variance ($\times10^{-4}$)	Mean ($\times10^{-1}$)	Variance ($\times10^{-4}$)	Mean ($\times10^{-1}$)	Variance ($\times10^{-4}$)	Mean ($\times10^{-1}$)	Variance ($\times10^{-4}$)	Mean ($\times10^{-1}$)	Variance ($\times10^{-4}$)	Mean ($\times10^{-1}$)	Variance ($\times10^{-4}$)	Mean ($\times10^{-1}$)	Variance ($\times10^{-4}$)
3_50_10_1	0.30	1.37	**0.31**	1.47 =	0.29	0.68 =	0.14	0.92 +	0.24	0.86 +	0.13	0.87 +	0.15	0.87 +	0.32	2.43	**0.27**	2.42 =	0.32	1.02 =	0.58	2.44 +	0.41	1.83 +	0.56	2.83 +	0.56	3.24 +
3_50_20_1	**0.45**	1.52	0.37	3.43 =	0.41	1.22 =	0.20	1.16 +	0.29	1.41 +	0.19	1.37 +	0.18	1.38 +	**0.29**	1.69	0.38	6.59 =	0.32	1.30 =	0.61	2.51 +	0.47	2.61 +	0.59	3.55 +	0.64	4.01 +
3_50_30_1	**0.33**	1.82	0.20	1.00 +	0.22	0.62 +	0.09	0.45 +	0.18	0.62 +	0.09	0.55 +	0.12	0.51 +	**0.31**	2.29	0.49	2.46 +	0.46	1.45 +	0.71	2.41 +	0.53	1.30 +	0.72	3.16 +	0.67	2.69 +
3_50_40_1	**0.37**	1.16	0.25	1.40 +	0.35	0.97 =	0.11	0.87 +	0.23	1.57 +	0.14	1.39 +	0.13	0.86 +	**0.30**	1.68	0.46	2.99 +	0.32	1.42 =	0.73	4.54 +	0.50	3.35 +	0.65	5.27 +	0.67	3.12 +
3_50_50_1	**0.47**	2.20	0.23	1.65 +	0.36	1.40 +	0.19	0.90 +	0.27	1.59 +	0.23	0.82 +	0.22	0.63 +	**0.24**	2.62	0.55	5.31 +	0.35	2.07 +	0.60	2.71 +	0.48	3.04 +	0.53	2.42 +	0.53	1.47 +
3_100_10_1	**0.39**	0.16	0.36	0.45 =	0.24	0.28 +	0.13	0.41 +	0.23	0.34 +	0.10	0.36 +	0.11	0.30 +	0.16	0.27	**0.15**	0.39 =	0.38	0.93 +	0.59	2.14 +	0.42	0.95 +	0.56	1.41 +	0.62	1.72 +
3_100_20_1	**0.36**	0.30	0.31	0.75 +	0.17	0.54 +	0.05	0.16 +	0.13	0.49 +	0.06	0.22 +	0.07	0.23 +	**0.14**	0.34	0.20	0.98 +	0.43	1.89 +	0.72	1.91 +	0.52	1.80 +	0.64	1.77 +	0.64	2.03 +
3_100_30_1	**0.46**	1.27	0.43	1.66 =	0.16	0.53 +	0.07	0.24 +	0.17	0.69 +	0.10	0.42 +	0.12	0.62 +	**0.26**	1.46	0.28	1.93 =	0.66	1.57 +	0.88	2.30 +	0.65	2.56 +	0.78	2.34 +	0.75	2.93 +
3_100_40_1	**0.89**	4.05	0.71	3.05 +	0.34	2.66 +	0.09	0.42 +	0.23	1.28 +	0.11	0.94 +	0.14	1.02 +	**0.33**	2.41	0.47	2.36 +	0.86	4.01 +	1.28	3.00 +	1.01	3.26 +	1.24	4.62 +	1.16	3.46 +
3_100_50_1	**1.06**	6.50	0.87	4.67 +	0.46	2.16 +	0.17	1.20 +	0.35	2.17 +	0.17	1.06 +	0.23	1.02 +	**0.32**	3.04	0.45	2.77 +	0.82	2.73 +	1.21	3.53 +	0.95	3.21 +	1.21	3.74 +	1.10	2.35 +
3_150_10_1	**0.68**	0.82	0.57	1.24 +	0.32	0.68 +	0.15	0.28 +	0.24	0.63 +	0.14	0.78 +	0.19	0.69 +	**0.16**	0.52	0.27	1.13 +	0.62	1.25 +	0.90	1.12 +	0.76	1.42 +	0.88	2.54 +	0.84	1.66 +
3_150_20_1	**0.84**	1.16	0.67	2.13 +	0.28	1.08 +	0.08	0.59 +	0.20	0.83 +	0.14	0.55 +	0.14	0.54 +	**0.25**	0.78	0.39	2.08 +	0.85	2.03 +	1.25	4.07 +	0.98	2.55 +	1.05	2.23 +	1.08	2.71 +
3_150_30_1	**1.36**	3.04	1.01	3.54 +	0.48	2.77 +	0.15	1.62 +	0.39	2.88 +	0.19	0.80 +	0.25	1.09 +	**0.25**	1.01	0.48	1.93 +	0.94	2.94 +	1.38	4.63 +	1.04	3.69 +	1.30	3.10 +	1.21	2.72 +
3_150_40_1	**1.18**	2.82	0.94	2.46 +	0.49	2.89 +	0.17	1.53 +	0.37	1.79 +	0.21	1.43 +	0.25	1.84 +	**0.25**	1.27	0.42	1.31 +	0.82	2.87 +	1.26	4.21 +	0.95	2.39 +	1.17	3.87 +	1.12	4.07 +
3_150_50_1	**1.04**	2.12	0.80	3.93 +	0.42	1.31 +	0.10	0.37 +	0.24	0.74 +	0.14	0.64 +	0.21	0.88 +	**0.19**	0.92	0.37	2.34 +	0.71	1.66 +	1.22	2.55 +	0.94	1.65 +	1.12	2.63 +	1.00	2.92 +
3_200_10_1	**0.62**	0.57	0.52	0.63 +	0.28	0.55 +	0.11	0.36 +	0.22	0.19 +	0.09	0.27 +	0.13	0.28 +	**0.14**	0.44	0.23	0.73 +	0.56	1.35 +	0.86	1.77 +	0.65	0.45 +	0.86	1.20 +	0.80	1.17 +
3_200_20_1	**0.56**	0.43	0.47	0.57 +	0.22	0.58 +	0.07	0.22 +	0.15	0.83 +	0.07	0.20 +	0.12	0.44 +	**0.15**	0.35	0.24	0.63 +	0.62	1.52 +	0.93	1.94 +	0.75	3.10 +	0.88	1.57 +	0.81	2.32 +
3_200_30_1	**1.18**	1.99	1.02	2.68 +	0.36	1.99 +	0.10	0.59 +	0.25	1.72 +	0.09	0.52 +	0.18	1.06 +	**0.16**	0.84	0.27	1.29 +	0.89	3.11 +	1.33	3.94 +	1.04	4.09 +	1.29	2.85 +	1.15	2.96 +
3_200_40_1	**1.67**	10.17	1.27	7.07 +	0.43	2.30 +	0.11	0.59 +	0.26	1.46 +	0.15	1.18 +	0.20	1.11 +	**0.25**	3.04	0.48	2.92 +	1.16	3.16 +	1.65	3.37 +	1.39	3.29 +	1.57	4.52 +	1.46	2.89 +
3_200_50_1	**1.04**	2.81	0.76	2.94 +	0.30	1.30 +	0.09	0.41 +	0.24	1.13 +	0.10	0.58 +	0.16	0.85 +	**0.22**	1.39	0.43	1.99 +	0.91	2.50 +	1.28	2.91 +	1.00	2.60 +	1.25	2.50 +	1.14	2.89 +
3_250_10_1	**0.42**	0.18	0.33	0.18 +	0.16	0.19 +	0.06	0.14 +	0.12	0.19 +	0.04	0.07 +	0.09	0.20 +	**0.11**	0.19	0.23	0.28 +	0.55	0.89 +	0.78	1.26 +	0.63	0.82 +	0.79	1.20 +	0.70	1.25 +
3_250_20_1	**1.27**	1.95	1.10	2.07 +	0.40	1.60 +	0.16	0.60 +	0.33	1.33 +	0.18	1.21 +	0.24	1.07 +	**0.18**	0.76	0.28	0.99 +	0.94	2.37 +	1.31	3.17 +	1.02	2.06 +	1.21	3.75 +	1.15	2.71 +
3_250_30_1	**1.68**	4.62	1.33	3.28 +	0.56	2.88 +	0.17	1.01 +	0.41	2.05 +	0.20	0.75 +	0.31	1.65 +	**0.20**	1.37	0.40	1.13 +	1.00	2.74 +	1.50	3.93 +	1.16	2.56 +	1.44	2.35 +	1.29	2.46 +
3_250_40_1	**1.40**	3.68	1.10	2.98 +	0.43	2.48 +	0.16	1.11 +	0.34	1.34 +	0.19	0.97 +	0.28	1.12 +	**0.25**	1.20	0.44	1.53 +	1.03	3.49 +	1.40	4.28 +	1.12	2.02 +	1.33	3.00 +	1.20	2.06 +
3_250_50_1	**1.76**	4.92	1.36	5.80 +	0.61	3.75 +	0.15	2.47 +	0.44	3.65 +	0.22	1.17 +	0.31	1.06 +	**0.24**	1.21	0.47	2.17 +	1.02	3.13 +	1.66	7.96 +	1.20	5.40 +	1.47	3.14 +	1.34	1.83 +
3_300_10_1	**0.32**	0.09	0.27	0.11 +	0.12	0.11 +	0.06	0.12 +	0.11	0.16 +	0.06	0.07 +	0.07	0.09 +	**0.07**	0.14	0.17	0.25 +	0.55	0.63 +	0.73	1.37 +	0.58	0.87 +	0.66	0.66 +	0.68	0.84 +
3_300_20_1	**0.54**	0.21	0.46	0.31 +	0.18	0.36 +	0.05	0.14 +	0.14	0.20 +	0.05	0.11 +	0.11	0.35 +	**0.10**	0.17	0.17	0.34 +	0.60	1.08 +	0.89	1.95 +	0.67	0.76 +	0.88	1.42 +	0.75	1.83 +
3_300_30_1	**1.48**	3.44	1.27	3.23 +	0.46	1.33 +	0.10	0.59 +	0.28	1.32 +	0.13	0.77 +	0.21	1.00 +	**0.27**	1.33	0.40	1.45 +	1.07	1.71 +	1.63	3.54 +	1.30	2.57 +	1.56	3.61 +	1.41	2.43 +
3_300_40_1	**1.92**	6.52	1.47	3.77 +	0.63	3.70 +	0.15	1.62 +	0.44	2.39 +	0.20	1.10 +	0.32	1.64 +	**0.22**	1.70	0.46	1.37 +	1.09	3.11 +	1.72	6.23 +	1.28	2.91 +	1.58	2.95 +	1.42	2.89 +
3_300_50_1	**1.50**	3.96	1.15	2.24 +	0.48	1.46 +	0.13	1.11 +	0.35	2.80 +	0.16	0.96 +	0.24	1.25 +	**0.23**	1.32	0.45	1.00 +	1.00	1.77 +	1.53	5.09 +	1.17	5.07 +	1.45	4.08 +	1.30	2.89 +
3_350_10_1	**0.39**	0.13	0.33	0.15 +	0.16	0.21 +	0.07	0.10 +	0.14	0.09 +	0.04	0.08 +	0.09	0.09 +	**0.10**	0.20	0.17	0.31 +	0.50	0.71 +	0.72	1.08 +	0.55	0.38 +	0.73	0.83 +	0.63	0.61 +
3_350_20_1	**0.76**	0.58	0.60	0.66 +	0.28	0.62 +	0.06	0.29 +	0.19	0.59 +	0.08	0.27 +	0.13	0.73 +	**0.16**	0.40	0.30	0.60 +	0.71	1.10 +	1.15	2.34 +	0.83	1.65 +	1.06	2.27 +	0.96	2.71 +
3_350_30_1	**1.23**	1.18	1.02	1.12 +	0.41	0.99 +	0.14	1.00 +	0.41	2.32 +	0.20	0.81 +	0.27	1.32 +	**0.14**	0.52	0.29	0.58 +	0.85	1.36 +	1.28	5.26 +	0.87	3.41 +	1.14	2.46 +	1.05	3.45 +
3_350_40_1	**1.41**	1.87	1.09	1.87 +	0.48	2.00 +	0.09	0.76 +	0.35	1.63 +	0.17	0.99 +	0.25	1.42 +	**0.14**	0.61	0.34	0.88 +	0.87	2.19 +	1.47	4.61 +	1.02	3.02 +	1.28	3.03 +	1.15	3.14 +
3_350_50_1	**1.64**	3.94	1.28	3.27 +	0.53	2.29 +	0.11	1.21 +	0.34	1.67 +	0.18	0.69 +	0.32	2.39 +	**0.16**	1.32	0.37	1.34 +	0.96	2.56 +	1.60	6.88 +	1.17	2.79 +	1.39	2.24 +	1.20	3.53 +
3_400_10_1	**0.36**	0.12	0.28	0.13 +	0.13	0.13 +	0.05	0.04 +	0.11	0.10 +	0.04	0.06 +	0.07	0.08 +	**0.11**	0.17	0.23	0.33 +	0.54	0.60 +	0.76	0.59 +	0.59	0.58 +	0.75	0.93 +	0.68	0.71 +
3_400_20_1	**0.95**	0.61	0.83	0.80 +	0.38	0.64 +	0.16	0.96 +	0.34	1.13 +	0.18	0.86 +	0.24	0.73 +	**0.10**	0.27	0.19	0.53 +	0.68	0.87 +	1.04	4.04 +	0.73	2.05 +	0.97	2.69 +	0.88	1.51 +
3_400_30_1	**1.13**	0.76	0.94	1.10 +	0.42	0.93 +	0.17	1.61 +	0.34	1.46 +	0.17	0.56 +	0.24	0.81 +	**0.09**	0.31	0.22	0.58 +	0.72	1.18 +	1.12	6.47 +	0.82	2.61 +	1.06	2.10 +	0.95	1.89 +
3_400_40_1	**1.42**	1.55	1.08	1.93 +	0.39	1.02 +	0.09	0.66 +	0.30	1.03 +	0.07	0.27 +	0.18	0.88 +	**0.11**	0.48	0.31	0.91 +	0.91	1.30 +	1.42	3.89 +	1.03	1.73 +	1.43	1.90 +	1.21	2.73 +
3_400_50_1	**2.37**	1.74	1.82	4.32 +	0.87	4.13 +	0.23	2.63 +	0.75	4.68 +	0.36	2.85 +	0.51	2.82 +	**0.12**	0.36	0.39	1.28 +	1.00	2.44 +	1.69	6.47 +	1.11	3.37 +	1.49	4.89 +	1.32	3.17 +
3_450_10_1	**0.29**	0.06	0.24	0.07 +	0.13	0.10 +	0.07	0.07 +	0.13	0.10 +	0.07	0.11 +	0.10	0.05 +	**0.07**	0.08	0.16	0.20 +	0.51	0.62 +	0.73	0.84 +	0.53	0.72 +	0.65	1.09 +	0.62	0.58 +
3_450_20_1	**0.71**	0.43	0.60	0.40 +	0.28	0.43 +	0.07	0.26 +	0.23	0.61 +	0.10	0.31 +	0.13	0.33 +	**0.10**	0.29	0.19	0.37 +	0.61	0.92 +	1.01	2.27 +	0.68	1.67 +	0.91	2.14 +	0.86	1.38 +
3_450_30_1	**1.46**	2.02	1.21	2.01 +	0.49	1.63 +	0.10	0.48 +	0.30	1.15 +	0.18	0.75 +	0.25	1.11 +	**0.26**	0.75	0.41	0.90 +	1.02	1.89 +	1.61	2.87 +	1.24	2.14 +	1.42	2.30 +	1.32	2.45 +
3_450_40_1	**1.53**	1.57	1.21	3.53 +	0.46	2.04 +	0.09	0.69 +	0.42	2.02 +	0.20	1.24 +	0.25	1.51 +	**0.12**	0.58	0.32	1.49 +	0.96	2.35 +	1.53	4.34 +	1.01	2.75 +	1.29	3.47 +	1.23	3.53 +
3_450_50_1	**2.33**	5.02	1.92	4.93 +	0.77	5.53 +	0.28	3.24 +	0.65	4.38 +	0.26	1.77 +	0.44	1.91 +	**0.28**	1.11	0.49	1.34 +	1.24	3.62 +	1.77	7.24 +	1.35	4.21 +	1.77	3.32 +	1.54	2.41 +
3_500_10_1	**0.51**	0.15	0.43	0.15 +	0.26	0.22 +	0.12	0.21 +	0.23	0.22 +	0.10	0.17 +	0.15	0.12 +	**0.10**	0.22	0.20	0.24 +	0.48	0.52 +	0.73	1.05 +	0.53	0.60 +	0.72	0.75 +	0.65	0.44 +
3_500_20_1	**0.91**	0.81	0.74	1.21 +	0.35	0.94 +	0.09	0.42 +	0.30	1.04 +	0.11	0.26 +	0.18	0.44 +	**0.12**	0.50	0.26	0.96 +	0.70	1.87 +	1.15	3.30 +	0.77	1.93 +	1.06	1.53 +	0.94	1.40 +
3_500_30_1	**1.42**	2.50	1.15	1.42 +	0.49	1.06 +	0.12	0.96 +	0.35	1.39 +	0.15	1.04 +	0.24	0.68 +	**0.15**	0.63	0.31	0.59 +	0.86	1.25 +	1.45	5.53 +	1.03	2.28 +	1.33	3.58 +	1.17	1.60 +
3_500_40_1	**1.70**	2.15	1.39	2.17 +	0.62	2.08 +	0.12	1.62 +	0.50	2.38 +	0.18	1.02 +	0.31	1.24 +	**0.19**	0.67	0.37	0.84 +	0.96	1.81 +	1.64	6.34 +	1.07	2.71 +	1.49	3.31 +	1.29	2.26 +
3_500_50_1	**2.07**	5.23	1.71	4.18 +	0.75	4.38 +	0.16	2.17 +	0.60	4.68 +	0.26	1.46 +	0.42	3.16 +	**0.17**	1.25	0.35	1.30 +	1.00	3.26 +	1.73	7.80 +	1.14	5.09 +	1.51	3.06 +	1.32	4.60 +
+/−/=	−		46/0/4		47/0/3		50/0/0		50/0/0		50/0/0		50/0/0		−		46/0/4		47/0/3		50/0/0		50/0/0		50/0/0		50/0/0	

Table 5 Results of HV and IGD for Problem set 4.

Problem	HV														IGD											
	MDU-MoPSO		MoPSO		MOHEA		NSGA-II		SPEA2		MOEA/D		MOHEA-DE		MDU-MoPSO		MoPSO		MOHEA		NSGA-II		SPEA2		MOEA/D		MOHEA-DE	
	Mean ($\times10^{-2}$)	Variance ($\times10^{-6}$)	Mean ($\times10^{-2}$)	Variance ($\times10^{-6}$)	Mean ($\times10^{-2}$)	Variance ($\times10^{-6}$)	Mean ($\times10^{-2}$)	Variance ($\times10^{-6}$)	Mean ($\times10^{-2}$)	Variance ($\times10^{-6}$)	Mean ($\times10^{-2}$)	Variance ($\times10^{-6}$)	Mean ($\times10^{-2}$)	Variance ($\times10^{-6}$)	Mean ($\times10^{-1}$)	Variance ($\times10^{-4}$)	Mean ($\times10^{-1}$)	Variance ($\times10^{-4}$)	Mean ($\times10^{-1}$)	Variance ($\times10^{-4}$)	Mean ($\times10^{-1}$)	Variance ($\times10^{-4}$)	Mean ($\times10^{-1}$)	Variance ($\times10^{-4}$)	Mean ($\times10^{-1}$)	Variance ($\times10^{-4}$)	Mean ($\times10^{-1}$)	Variance ($\times10^{-4}$)
4_50_10_1	**0.53**	1.85	0.46	2.67 =	0.42	2.05 +	0.29	0.83 +	0.37	0.86 +	0.17	1.16 +	0.19	1.20 +	**0.25**	1.78	0.30	2.44 =	0.34	2.73 +	0.48	1.75 +	0.39	1.20 +	0.66	3.74 +	0.63	3.28 +
4_50_20_1	**0.38**	0.98	0.38	1.44 =	0.36	0.93 =	0.21	0.99 +	0.32	0.67 +	0.19	1.33 +	0.14	0.76 +	0.26	0.67	**0.24**	1.35 =	0.26	0.70 =	0.45	2.53 +	0.30	0.99 =	0.47	3.67 +	0.55	2.67 +
4_50_30_1	**0.25**	0.48	0.16	0.93 +	0.19	0.63 +	0.10	0.32 +	0.14	0.39 +	0.08	0.33 +	0.09	0.18 +	**0.19**	0.96	0.35	2.79 +	0.28	1.30 +	0.44	1.53 +	0.36	1.47 +	0.50	2.12 +	0.47	1.31 +
4_50_40_1	**0.36**	1.64	0.17	1.01 +	0.26	0.92 +	0.10	0.42 +	0.15	0.57 +	0.07	0.29 +	0.10	0.62 +	**0.31**	2.48	0.59	2.91 +	0.44	2.07 +	0.74	2.69 +	0.61	2.50 +	0.80	2.60 +	0.78	4.36 +
4_50_50_1	**0.33**	1.63	0.17	1.00 +	0.23	1.45 +	0.08	0.24 +	0.15	0.79 +	0.09	0.47 +	0.11	0.56 +	**0.24**	2.41	0.48	3.20 +	0.38	3.14 +	0.65	1.72 +	0.52	2.43 +	0.64	3.04 +	0.61	3.00 +
4_100_10_1	0.61	0.93	**0.64**	1.13 =	0.33	1.07 +	0.15	0.44 +	0.25	0.54 +	0.12	0.36 +	0.19	0.67 +	0.23	0.63	**0.17**	0.51 -	0.49	1.95 +	0.75	1.48 +	0.61	0.95 +	0.80	2.20 +	0.69	1.81 +
4_100_20_1	**0.62**	1.67	0.52	0.98 +	0.24	0.66 +	0.10	0.43 +	0.20	0.75 +	0.08	0.43 +	0.12	0.80 +	**0.26**	1.38	0.34	0.91 +	0.71	1.48 +	0.98	2.30 +	0.77	1.83 +	0.99	2.22 +	0.93	3.33 +
4_100_30_1	**0.75**	2.16	0.72	2.16 =	0.36	1.95 +	0.12	0.42 +	0.27	1.37 +	0.11	0.52 +	0.13	0.61 +	**0.21**	1.34	0.23	1.25 =	0.58	2.79 +	0.93	1.84 +	0.70	2.61 +	0.96	2.51 +	0.91	2.63 +
4_100_40_1	**0.80**	2.59	0.66	3.24 +	0.37	1.17 +	0.10	0.43 +	0.19	0.99 +	0.10	0.57 +	0.12	0.73 +	**0.31**	1.69	0.42	2.72 +	0.71	1.60 +	1.16	3.03 +	0.97	2.63 +	1.13	2.94 +	1.10	2.94 +
4_100_50_1	**0.73**	1.95	0.55	4.35 +	0.29	1.76 +	0.10	0.64 +	0.20	0.58 +	0.06	0.30 +	0.14	0.81 +	**0.25**	1.41	0.43	4.19 +	0.71	3.10 +	1.04	2.87 +	0.84	1.76 +	1.14	2.79 +	0.99	2.92 +
4_150_10_1	**0.30**	0.16	0.26	0.21 +	0.13	0.19 +	0.08	0.14 +	0.13	0.17 +	0.06	0.13 +	0.06	0.13 +	**0.10**	0.33	0.17	0.54 +	0.52	0.82 +	0.69	1.50 +	0.53	0.97 +	0.64	1.25 +	0.73	1.19 +
4_150_20_1	**0.89**	1.02	0.72	1.04 +	0.35	1.08 +	0.14	0.31 +	0.22	0.37 +	0.11	0.56 +	0.15	0.35 +	**0.21**	0.60	0.34	0.78 +	0.74	1.92 +	1.06	1.16 +	0.91	0.98 +	1.11	3.11 +	1.02	1.30 +
4_150_30_1	**0.86**	1.88	0.64	1.52 +	0.31	0.92 +	0.11	0.28 +	0.27	0.58 +	0.12	0.50 +	0.14	0.53 +	**0.23**	1.12	0.41	1.21 +	0.76	1.49 +	1.09	1.67 +	0.82	1.06 +	1.06	2.74 +	1.03	2.28 +
4_150_40_1	**1.21**	3.15	0.94	4.19 +	0.45	2.27 +	0.14	0.86 +	0.30	1.63 +	0.14	0.57 +	0.18	0.63 +	**0.19**	1.24	0.37	2.48 +	0.81	2.94 +	1.25	3.41 +	0.99	2.97 +	1.24	2.86 +	1.17	2.07 +
4_150_50_1	**0.99**	4.38	0.66	2.12 +	0.31	1.52 +	0.07	0.27 +	0.20	0.66 +	0.08	0.27 +	0.11	0.50 +	**0.44**	2.10	0.70	1.68 +	1.08	2.56 +	1.52	2.13 +	1.24	2.05 +	1.46	1.79 +	1.44	3.10 +
4_200_10_1	**0.54**	0.41	0.43	0.21 +	0.20	0.31 +	0.12	0.21 +	0.18	0.25 +	0.09	0.27 +	0.11	0.19 +	**0.12**	0.51	0.22	0.35 +	0.54	0.78 +	0.68	0.88 +	0.57	0.81 +	0.72	1.38 +	0.68	0.87 +
4_200_20_1	**0.77**	1.36	0.61	1.38 +	0.25	0.85 +	0.07	0.27 +	0.18	0.40 +	0.08	0.25 +	0.16	0.48 +	**0.22**	0.70	0.35	1.20 +	0.77	1.62 +	1.12	1.54 +	0.88	1.20 +	1.08	1.74 +	1.02	2.10 +
4_200_30_1	**0.93**	2.33	0.68	1.23 +	0.26	0.74 +	0.08	0.30 +	0.17	0.42 +	0.06	0.23 +	0.08	0.36 +	**0.21**	1.03	0.39	1.00 +	0.85	1.60 +	1.17	1.86 +	0.99	1.39 +	1.20	1.78 +	1.20	2.54 +
4_200_40_1	**1.25**	2.41	0.85	3.43 +	0.33	1.12 +	0.06	0.19 +	0.25	1.07 +	0.07	0.33 +	0.12	0.51 +	**0.22**	1.03	0.50	2.18 +	1.01	2.06 +	1.50	1.90 +	1.12	2.90 +	1.43	2.49 +	1.34	2.79 +
4_200_50_1	**1.24**	3.26	0.90	4.92 +	0.39	1.62 +	0.08	0.57 +	0.22	1.07 +	0.10	0.75 +	0.14	0.80 +	**0.30**	1.33	0.54	2.93 +	1.01	2.07 +	1.54	3.84 +	1.24	2.96 +	1.48	4.48 +	1.38	3.20 +
4_250_10_1	**0.48**	0.19	0.39	0.27 +	0.23	0.31 +	0.14	0.11 +	0.22	0.23 +	0.11	0.20 +	0.15	0.16 +	**0.08**	0.30	0.19	0.50 +	0.46	0.80 +	0.61	0.43 +	0.48	0.61 +	0.62	1.30 +	0.60	0.62 +
4_250_20_1	**0.74**	0.68	0.58	0.86 +	0.28	0.59 +	0.11	0.21 +	0.21	0.40 +	0.09	0.23 +	0.12	0.38 +	**0.13**	0.46	0.27	0.83 +	0.63	1.01 +	0.91	1.00 +	0.73	1.04 +	0.94	1.55 +	0.88	1.73 +
4_250_30_1	**0.88**	1.62	0.70	1.15 +	0.26	0.89 +	0.10	0.28 +	0.24	0.65 +	0.07	0.24 +	0.12	0.51 +	**0.18**	0.84	0.32	0.99 +	0.80	1.74 +	1.10	1.75 +	0.84	1.35 +	1.17	2.44 +	1.05	2.48 +
4_250_40_1	**0.83**	1.32	0.59	0.99 +	0.26	0.93 +	0.07	0.22 +	0.19	0.72 +	0.09	0.28 +	0.12	0.26 +	**0.16**	0.67	0.36	0.94 +	0.75	1.71 +	1.13	2.23 +	0.86	2.41 +	1.04	1.72 +	0.99	1.40 +
4_250_50_1	**1.29**	2.50	0.89	4.15 +	0.42	2.09 +	0.12	0.66 +	0.26	1.26 +	0.10	0.42 +	0.19	1.02 +	**0.17**	0.90	0.45	2.63 +	0.89	2.51 +	1.33	2.97 +	1.08	2.64 +	1.36	2.30 +	1.21	3.55 +
4_300_10_1	**0.32**	0.22	0.25	0.12 +	0.10	0.11 +	0.04	0.09 +	0.09	0.10 +	0.04	0.06 +	0.05	0.05 +	**0.11**	0.31	0.24	0.37 +	0.66	0.60 +	0.87	1.39 +	0.70	0.95 +	0.84	0.86 +	0.82	0.79 +
4_300_20_1	**0.73**	0.73	0.59	0.50 +	0.23	0.92 +	0.08	0.27 +	0.19	0.69 +	0.07	0.24 +	0.12	0.30 +	**0.14**	0.52	0.25	0.53 +	0.72	2.26 +	1.02	2.07 +	0.80	2.05 +	1.03	2.04 +	0.93	1.51 +
4_300_30_1	**0.98**	1.06	0.78	1.97 +	0.39	1.39 +	0.11	0.64 +	0.31	0.90 +	0.11	0.59 +	0.19	0.85 +	**0.15**	0.45	0.31	1.28 +	0.71	2.31 +	1.16	3.65 +	0.82	1.60 +	1.14	3.26 +	1.00	2.96 +
4_300_40_1	**1.22**	1.65	0.93	1.87 +	0.36	1.55 +	0.07	0.26 +	0.29	1.31 +	0.13	0.87 +	0.20	1.34 +	**0.15**	0.69	0.35	1.06 +	0.89	2.23 +	1.40	2.28 +	1.00	2.60 +	1.25	3.85 +	1.14	4.78 +
4_300_50_1	**1.01**	1.21	0.75	1.99 +	0.35	1.70 +	0.07	0.58 +	0.21	1.41 +	0.10	0.60 +	0.17	0.92 +	**0.21**	0.57	0.41	1.34 +	0.82	3.14 +	1.32	3.69 +	1.02	3.35 +	1.22	3.19 +	1.09	3.21 +
4_350_10_1	**0.46**	0.10	0.34	0.20 +	0.18	0.19 +	0.07	0.11 +	0.14	0.10 +	0.05	0.09 +	0.08	0.10 +	**0.09**	0.07	0.20	0.37 +	0.47	0.64 +	0.70	0.89 +	0.55	0.43 +	0.73	1.05 +	0.68	0.63 +
4_350_20_1	**1.03**	0.88	0.83	1.47 +	0.33	0.57 +	0.14	0.52 +	0.26	0.67 +	0.08	0.29 +	0.14	0.51 +	**0.16**	0.49	0.31	1.26 +	0.81	0.92 +	1.12	2.28 +	0.91	1.48 +	1.22	2.40 +	1.09	1.73 +
4_350_30_1	**1.19**	1.17	0.96	1.22 +	0.43	1.23 +	0.11	0.74 +	0.29	0.58 +	0.12	0.45 +	0.20	0.75 +	**0.19**	0.53	0.35	0.69 +	0.84	1.49 +	1.35	4.19 +	1.01	1.26 +	1.27	2.20 +	1.15	2.04 +
4_350_40_1	**1.50**	1.93	1.19	1.76 +	0.59	2.89 +	0.20	1.06 +	0.51	2.37 +	0.20	1.00 +	0.31	1.33 +	**0.21**	0.69	0.41	0.76 +	0.90	2.72 +	1.38	4.22 +	0.97	2.51 +	1.34	2.74 +	1.21	3.06 +
4_350_50_1	**1.59**	4.67	1.32	3.93 +	0.59	3.74 +	0.16	1.43 +	0.47	2.89 +	0.25	1.01 +	0.31	1.69 +	**0.26**	1.56	0.42	1.56 +	0.98	2.97 +	1.54	5.36 +	1.10	3.05 +	1.36	2.22 +	1.29	2.96 +
4_400_10_1	**0.38**	0.11	0.31	0.11 +	0.17	0.10 +	0.07	0.13 +	0.15	0.08 +	0.05	0.13 +	0.10	0.09 +	**0.08**	0.18	0.19	0.23 +	0.51	0.50 +	0.74	1.21 +	0.54	0.33 +	0.74	1.00 +	0.65	0.53 +
4_400_20_1	**0.66**	0.36	0.52	0.76 +	0.21	0.18 +	0.06	0.25 +	0.17	0.26 +	0.06	0.11 +	0.11	0.20 +	**0.10**	0.25	0.22	0.68 +	0.63	0.44 +	0.98	2.80 +	0.70	0.95 +	0.94	1.07 +	0.84	1.07 +
4_400_30_1	**1.17**	1.40	0.95	1.19 +	0.34	0.72 +	0.09	0.67 +	0.23	0.96 +	0.08	0.39 +	0.18	0.66 +	**0.14**	0.61	0.28	0.52 +	0.85	1.22 +	1.31	3.88 +	1.01	1.90 +	1.28	2.31 +	1.10	2.39 +
4_400_40_1	**1.17**	1.13	0.88	1.43 +	0.36	1.03 +	0.09	0.55 +	0.29	1.06 +	0.08	0.29 +	0.17	0.66 +	**0.15**	0.47	0.34	0.82 +	0.84	1.70 +	1.29	3.71 +	0.93	1.80 +	1.27	2.02 +	1.10	2.16 +
4_400_50_1	**1.28**	2.60	0.94	2.75 +	0.50	1.51 +	0.10	0.72 +	0.32	1.36 +	0.14	0.72 +	0.20	0.93 +	**0.17**	0.93	0.41	1.58 +	0.80	1.88 +	1.41	4.84 +	1.00	2.20 +	1.28	3.17 +	1.18	2.59 +
4_450_10_1	**0.39**	0.08	0.31	0.08 +	0.16	0.11 +	0.10	0.09 +	0.15	0.12 +	0.05	0.05 +	0.10	0.06 +	**0.08**	0.13	0.17	0.20 +	0.43	0.42 +	0.55	0.48 +	0.45	0.48 +	0.59	0.35 +	0.53	0.32 +
4_450_20_1	**0.45**	0.14	0.36	0.19 +	0.19	0.43 +	0.06	0.23 +	0.16	0.16 +	0.04	0.08 +	0.10	0.16 +	**0.10**	0.20	0.21	0.34 +	0.52	1.35 +	0.82	1.97 +	0.56	0.62 +	0.79	1.12 +	0.69	0.84 +
4_450_30_1	**1.08**	1.25	0.87	0.78 +	0.34	1.16 +	0.08	0.49 +	0.27	0.65 +	0.10	0.16 +	0.19	0.35 +	**0.13**	0.48	0.26	0.46 +	0.80	1.94 +	1.27	3.52 +	0.89	1.25 +	1.16	0.80 +	1.01	1.07 +
4_450_40_1	**1.24**	1.61	0.95	1.94 +	0.39	1.37 +	0.11	0.57 +	0.32	0.96 +	0.08	0.26 +	0.19	0.97 +	**0.12**	0.63	0.32	1.18 +	0.83	1.75 +	1.30	3.69 +	0.92	1.82 +	1.30	1.42 +	1.12	2.68 +
4_450_50_1	**1.67**	2.61	1.29	2.60 +	0.53	3.09 +	0.12	1.39 +	0.44	2.32 +	0.15	0.54 +	0.30	1.49 +	**0.15**	0.71	0.37	1.03 +	0.97	3.01 +	1.57	6.15 +	1.07	2.76 +	1.46	2.31 +	1.24	2.77 +
4_500_10_1	**0.34**	0.04	0.26	0.08 +	0.15	0.09 +	0.08	0.08 +	0.13	0.09 +	0.04	0.07 +	0.08	0.08 +	**0.05**	0.06	0.17	0.16 +	0.42	0.47 +	0.59	0.66 +	0.48	0.40 +	0.61	0.68 +	0.59	0.73 +
4_500_20_1	**0.61**	0.24	0.50	0.36 +	0.21	0.39 +	0.06	0.23 +	0.15	0.31 +	0.05	0.09 +	0.11	0.22 +	**0.10**	0.14	0.20	0.36 +	0.59	1.13 +	0.90	1.81 +	0.68	1.05 +	0.89	0.94 +	0.78	1.31 +
4_500_30_1	**1.07**	0.77	0.88	0.61 +	0.42	1.67 +	0.15	1.36 +	0.38	1.01 +	0.14	0.38 +	0.24	0.80 +	**0.10**	0.33	0.23	0.32 +	0.68	2.06 +	1.11	5.61 +	0.72	1.53 +	1.06	1.43 +	0.91	1.86 +
4_500_40_1	**1.35**	1.73	1.07	1.08 +	0.51	1.00 +	0.14	0.93 +	0.41	1.77 +	0.16	0.76 +	0.28	1.62 +	**0.20**	0.75	0.40	0.53 +	0.87	1.08 +	1.40	4.27 +	0.99	2.61 +	1.33	2.49 +	1.17	3.77 +
4_500_50_1	**1.59**	3.27	1.23	2.72 +	0.49	2.41 +	0.14	1.08 +	0.42	1.36 +	0.11	0.61 +	0.26	2.52 +	**0.20**	1.00	0.42	1.22 +	1.02	2.57 +	1.53	4.87 +	1.09	1.62 +	1.53	2.24 +	1.31	4.83 +
+/−/=	−		46/0/4		49/0/1		50/0/0		50/0/0		50/0/0		50/0/0		−		46/1/3		49/0/1		50/0/0		49/0/1		50/0/0		50/0/0	

Table 6 Results of HV and IGD for Problem set 5.

Problem	HV														IGD													
	MDU-MoPSO		MoPSO		MOHEA		NSGA-II		SPEA2		MOEA/D		MOHEA-DE		MDU-MoPSO		MoPSO		MOHEA		NSGA-II		SPEA2		MOEA/D		MOHEA-DE	
	Mean ($\times10^{-2}$)	Variance ($\times10^{-6}$)	Mean ($\times10^{-2}$)	Variance ($\times10^{-6}$)	Mean ($\times10^{-2}$)	Variance ($\times10^{-6}$)	Mean ($\times10^{-2}$)	Variance ($\times10^{-6}$)	Mean ($\times10^{-2}$)	Variance ($\times10^{-6}$)	Mean ($\times10^{-2}$)	Variance ($\times10^{-6}$)	Mean ($\times10^{-2}$)	Variance ($\times10^{-6}$)	Mean ($\times10^{-1}$)	Variance ($\times10^{-4}$)	Mean ($\times10^{-1}$)	Variance ($\times10^{-4}$)	Mean ($\times10^{-1}$)	Variance ($\times10^{-4}$)	Mean ($\times10^{-1}$)	Variance ($\times10^{-4}$)	Mean ($\times10^{-1}$)	Variance ($\times10^{-4}$)	Mean ($\times10^{-1}$)	Variance ($\times10^{-4}$)	Mean ($\times10^{-1}$)	Variance ($\times10^{-4}$)
5_50_10_1	**0.47**	1.47	0.38	1.20 +	0.35	1.23 +	0.23	0.49 +	0.30	1.00 +	0.16	0.84 +	0.22	0.68 +	**0.22**	0.61	0.27	0.76 +	0.29	1.05 +	0.45	1.11 +	0.35	1.01 +	0.50	1.69 +	0.44	1.49 +
5_50_20_1	**0.39**	2.17	0.34	1.70 =	0.33	1.14 =	0.17	0.81 +	0.26	0.89 +	0.15	0.76 +	0.15	0.80 +	**0.30**	1.97	0.38	2.29 =	0.36	1.28 =	0.58	1.97 +	0.45	1.45 +	0.65	3.30 +	0.61	2.68 +
5_50_30_1	**0.74**	5.77	0.48	4.03 +	0.66	1.93 =	0.32	2.24 +	0.44	1.73 +	0.26	2.32 +	0.34	2.19 +	**0.41**	3.98	0.65	4.22 +	0.47	1.44 =	0.82	3.69 +	0.68	2.37 +	0.93	6.31 +	0.79	3.74 +
5_50_40_1	**0.47**	2.85	0.24	1.35 +	0.37	1.14 +	0.18	1.09 +	0.26	1.01 +	0.14	0.47 +	0.19	1.33 +	**0.39**	3.11	0.67	3.20 +	0.48	1.80 +	0.77	4.24 +	0.63	2.15 +	0.81	1.67 +	0.76	4.51 +
5_50_50_1	**0.43**	1.11	0.20	1.09 +	0.30	1.16 +	0.13	0.89 +	0.22	0.85 +	0.09	0.76 +	0.14	0.87 +	**0.29**	1.23	0.59	2.60 +	0.45	2.20 +	0.74	3.66 +	0.56	2.33 +	0.82	3.74 +	0.76	4.43 +
5_100_10_1	**0.50**	0.68	0.42	0.90 +	0.22	0.55 +	0.10	0.34 +	0.18	0.53 +	0.08	0.28 +	0.12	0.58 +	**0.16**	0.23	0.25	0.47 +	0.46	1.24 +	0.65	1.42 +	0.52	1.33 +	0.72	1.76 +	0.62	2.42 +
5_100_20_1	**0.79**	3.25	0.70	3.48 =	0.39	0.81 +	0.18	0.80 +	0.32	0.88 +	0.16	0.73 +	0.19	0.80 +	**0.29**	2.05	0.37	2.63 =	0.67	1.28 +	0.98	2.34 +	0.75	1.47 +	0.97	3.01 +	0.95	2.11 +
5_100_30_1	**0.63**	1.84	0.50	1.85 +	0.31	1.52 +	0.12	0.82 +	0.20	0.97 +	0.13	0.62 +	0.17	0.79 +	**0.33**	1.46	0.45	1.94 +	0.69	2.91 +	1.02	3.59 +	0.85	2.51 +	0.97	2.61 +	0.91	2.60 +
5_100_40_1	**0.88**	2.59	0.80	4.03 +	0.43	1.60 +	0.19	1.01 +	0.32	1.75 +	0.19	0.94 +	0.25	1.10 +	**0.34**	1.51	0.40	2.59 =	0.74	2.03 +	1.07	4.18 +	0.87	2.85 +	1.06	2.47 +	0.96	2.49 +
5_100_50_1	**0.43**	0.82	0.35	1.19 +	0.20	0.59 +	0.08	0.35 +	0.12	0.49 +	0.07	0.36 +	0.08	0.26 +	**0.14**	0.79	0.23	1.91 +	0.43	1.54 +	0.69	2.21 +	0.59	2.14 +	0.73	2.64 +	0.67	1.74 +
5_150_10_1	**0.51**	0.45	0.40	0.56 +	0.19	0.37 +	0.08	0.18 +	0.14	0.22 +	0.07	0.25 +	0.13	0.44 +	**0.14**	0.52	0.25	0.81 +	0.66	1.29 +	0.90	1.40 +	0.76	1.52 +	0.88	1.71 +	0.91	2.28 +
5_150_20_1	**0.73**	0.82	0.60	1.33 +	0.27	1.21 +	0.09	0.35 +	0.22	0.75 +	0.12	0.78 +	0.13	0.57 +	**0.15**	0.27	0.25	0.75 +	0.64	2.37 +	0.96	1.98 +	0.72	1.90 +	0.87	3.64 +	0.87	2.51 +
5_150_30_1	**1.07**	3.38	0.83	2.12 +	0.27	1.43 +	0.07	0.18 +	0.20	1.44 +	0.08	0.30 +	0.13	0.52 +	**0.27**	1.70	0.44	1.37 +	1.02	3.12 +	1.37	1.39 +	1.14	3.49 +	1.35	2.30 +	1.24	2.08 +
5_150_40_1	**1.38**	4.24	1.04	7.74 +	0.43	2.16 +	0.12	0.88 +	0.33	1.62 +	0.15	1.02 +	0.25	2.21 +	**0.28**	1.62	0.50	4.05 +	1.03	2.76 +	1.49	4.60 +	1.15	2.79 +	1.44	4.02 +	1.28	5.23 +
5_150_50_1	**0.79**	2.00	0.62	2.71 +	0.31	1.36 +	0.09	0.50 +	0.26	1.25 +	0.15	0.50 +	0.17	0.55 +	**0.22**	1.27	0.36	1.96 +	0.70	2.65 +	1.08	3.16 +	0.78	2.35 +	0.94	2.38 +	0.91	1.77 +
5_200_10_1	**0.49**	0.21	0.38	0.26 +	0.20	0.15 +	0.10	0.28 +	0.17	0.14 +	0.11	0.35 +	0.10	0.19 +	**0.09**	0.21	0.25	0.51 +	0.67	0.67 +	0.92	2.05 +	0.74	0.61 +	0.85	2.25 +	0.91	1.41 +
5_200_20_1	**1.17**	1.86	0.97	1.72 +	0.35	1.59 +	0.15	0.84 +	0.27	1.01 +	0.09	0.43 +	0.19	1.10 +	**0.18**	0.66	0.31	0.93 +	0.90	2.38 +	1.20	2.88 +	1.00	1.97 +	1.31	2.73 +	1.13	3.30 +
5_200_30_1	**0.96**	2.23	0.78	1.79 +	0.30	1.48 +	0.11	0.61 +	0.23	1.04 +	0.14	1.03 +	0.19	0.65 +	**0.22**	1.32	0.36	1.27 +	0.90	3.24 +	1.22	3.08 +	0.99	2.54 +	1.14	4.09 +	1.05	2.08 +
5_200_40_1	**1.48**	4.15	1.19	5.50 +	0.48	2.38 +	0.15	0.68 +	0.34	1.15 +	0.18	0.98 +	0.28	2.02 +	**0.21**	1.27	0.39	2.44 +	0.97	2.75 +	1.42	2.97 +	1.13	1.73 +	1.36	3.27 +	1.22	3.95 +
5_200_50_1	**0.98**	2.41	0.75	2.13 +	0.29	1.08 +	0.07	0.48 +	0.23	0.77 +	0.10	0.34 +	0.14	0.63 +	**0.27**	1.17	0.44	1.43 +	0.92	1.89 +	1.33	3.41 +	0.99	1.83 +	1.22	1.71 +	1.15	2.06 +
5_250_10_1	**0.43**	0.21	0.39	0.31 +	0.15	0.25 +	0.06	0.14 +	0.12	0.09 +	0.06	0.16 +	0.08	0.07 +	**0.09**	0.11	0.15	0.44 +	0.58	1.01 +	0.79	1.50 +	0.64	0.57 +	0.74	1.43 +	0.72	0.80 +
5_250_20_1	**0.92**	1.54	0.73	0.81 +	0.33	0.89 +	0.08	0.47 +	0.24	0.87 +	0.13	0.60 +	0.16	0.68 +	**0.20**	0.75	0.35	0.68 +	0.82	1.73 +	1.27	3.36 +	0.94	2.11 +	1.12	2.71 +	1.09	2.70 +
5_250_30_1	**1.38**	2.58	1.17	2.41 +	0.40	2.23 +	0.13	0.87 +	0.34	2.04 +	0.12	0.53 +	0.18	0.66 +	**0.20**	0.92	0.32	1.00 +	0.99	2.77 +	1.42	4.01 +	1.08	3.69 +	1.37	2.20 +	1.28	1.93 +
5_250_40_1	**1.49**	3.73	1.07	3.70 +	0.49	2.43 +	0.13	1.46 +	0.35	2.41 +	0.12	0.43 +	0.16	0.63 +	**0.27**	1.24	0.53	1.77 +	1.03	2.90 +	1.55	5.21 +	1.19	4.35 +	1.53	2.40 +	1.45	2.39 +
5_250_50_1	**0.74**	1.08	0.54	0.63 +	0.24	1.12 +	0.06	0.34 +	0.19	0.71 +	0.05	0.15 +	0.07	0.25 +	**0.17**	0.77	0.34	0.61 +	0.72	2.41 +	1.07	2.61 +	0.78	1.99 +	1.07	1.41 +	1.03	1.83 +
5_300_10_1	**0.29**	0.08	0.24	0.12 +	0.09	0.04 +	0.04	0.06 +	0.08	0.07 +	0.03	0.05 +	0.05	0.08 +	**0.07**	0.05	0.13	0.27 +	0.50	0.31 +	0.68	0.79 +	0.54	0.57 +	0.66	0.72 +	0.67	0.98 +
5_300_20_1	**1.00**	1.38	0.82	1.21 +	0.26	0.99 +	0.09	0.28 +	0.22	0.88 +	0.09	0.28 +	0.16	0.75 +	**0.12**	0.54	0.25	0.79 +	0.89	2.65 +	1.22	2.46 +	0.96	2.67 +	1.16	1.88 +	1.06	2.98 +
5_300_30_1	**1.34**	3.62	1.08	2.71 +	0.39	1.55 +	0.10	0.37 +	0.25	1.06 +	0.14	0.93 +	0.20	1.08 +	**0.17**	1.40	0.33	1.50 +	0.95	2.38 +	1.43	2.42 +	1.14	2.23 +	1.33	3.82 +	1.23	2.80 +
5_300_40_1	**1.22**	2.61	0.97	1.42 +	0.48	2.18 +	0.15	1.11 +	0.33	0.94 +	0.17	0.75 +	0.28	1.08 +	**0.17**	1.08	0.34	0.82 +	0.79	2.43 +	1.25	4.33 +	0.95	1.43 +	1.20	2.89 +	1.02	2.30 +
5_300_50_1	**1.54**	4.59	1.24	5.85 +	0.46	3.22 +	0.14	2.13 +	0.36	2.24 +	0.14	0.63 +	0.25	1.36 +	**0.24**	1.58	0.43	2.48 +	1.06	3.92 +	1.53	5.99 +	1.17	3.40 +	1.50	2.82 +	1.32	2.82 +
5_350_10_1	**0.49**	0.17	0.39	0.23 +	0.18	0.14 +	0.05	0.14 +	0.17	0.23 +	0.07	0.12 +	0.10	0.14 +	**0.09**	0.17	0.21	0.47 +	0.60	0.54 +	0.93	1.48 +	0.62	0.85 +	0.78	1.00 +	0.78	0.87 +
5_350_20_1	**1.10**	0.96	0.91	1.09 +	0.42	0.58 +	0.12	0.48 +	0.32	1.10 +	0.14	0.57 +	0.25	0.63 +	**0.14**	0.56	0.28	0.73 +	0.79	0.86 +	1.25	2.87 +	0.91	2.22 +	1.18	2.78 +	1.01	1.83 +
5_350_30_1	**1.03**	0.55	0.90	1.37 +	0.35	1.16 +	0.08	0.53 +	0.32	0.68 +	0.12	0.54 +	0.22	1.43 +	**0.12**	0.28	0.22	0.81 +	0.76	1.91 +	1.25	3.66 +	0.82	1.31 +	1.11	2.38 +	0.96	3.56 +
5_350_40_1	**0.98**	0.99	0.80	1.19 +	0.33	0.92 +	0.07	0.29 +	0.27	0.83 +	0.13	0.73 +	0.21	0.57 +	**0.16**	0.55	0.30	0.81 +	0.79	1.64 +	1.26	2.85 +	0.87	1.70 +	1.12	2.79 +	0.96	1.93 +
5_350_50_1	**1.65**	3.87	1.34	3.68 +	0.56	3.97 +	0.10	0.81 +	0.40	3.05 +	0.17	0.61 +	0.26	1.53 +	**0.19**	0.93	0.37	1.39 +	0.97	4.04 +	1.64	5.08 +	1.15	5.17 +	1.43	1.83 +	1.31	2.70 +
5_400_10_1	**0.27**	0.04	0.21	0.06 +	0.11	0.09 +	0.03	0.05 +	0.09	0.11 +	0.04	0.06 +	0.05	0.07 +	**0.07**	0.13	0.19	0.20 +	0.51	0.51 +	0.74	0.82 +	0.56	0.97 +	0.68	1.11 +	0.66	0.92 +
5_400_20_1	**0.73**	0.59	0.60	0.44 +	0.30	0.57 +	0.10	0.39 +	0.22	0.87 +	0.08	0.36 +	0.15	0.47 +	**0.11**	0.36	0.23	0.43 +	0.60	1.08 +	0.94	2.52 +	0.72	2.15 +	0.97	2.72 +	0.83	1.95 +
5_400_30_1	**1.46**	1.58	1.15	1.47 +	0.48	2.00 +	0.10	0.81 +	0.36	1.84 +	0.13	0.65 +	0.29	1.16 +	**0.13**	0.57	0.32	0.66 +	0.91	2.26 +	1.49	4.67 +	1.05	3.40 +	1.38	2.57 +	1.13	2.18 +
5_400_40_1	**1.58**	2.23	1.27	3.07 +	0.54	2.55 +	0.11	1.18 +	0.39	1.88 +	0.16	0.73 +	0.33	1.36 +	**0.21**	0.73	0.40	1.28 +	0.99	2.73 +	1.61	6.13 +	1.15	3.23 +	1.47	2.85 +	1.21	2.32 +
5_400_50_1	**1.87**	3.03	1.47	2.85 +	0.64	4.21 +	0.15	1.40 +	0.45	2.38 +	0.21	1.50 +	0.30	1.47 +	**0.21**	0.80	0.43	1.02 +	1.04	3.40 +	1.65	4.40 +	1.22	2.79 +	1.51	3.48 +	1.39	2.61 +
5_450_10_1	**0.44**	0.14	0.36	0.21 +	0.19	0.20 +	0.08	0.09 +	0.16	0.28 +	0.08	0.16 +	0.12	0.13 +	**0.08**	0.08	0.15	0.35 +	0.45	0.57 +	0.70	0.83 +	0.53	1.18 +	0.65	1.06 +	0.59	0.61 +
5_450_20_1	**0.89**	0.72	0.72	0.77 +	0.30	0.60 +	0.05	0.19 +	0.21	0.84 +	0.07	0.16 +	0.16	0.38 +	**0.10**	0.45	0.23	0.60 +	0.70	1.08 +	1.19	2.17 +	0.85	2.37 +	1.10	1.69 +	0.93	2.12 +
5_450_30_1	**0.86**	0.62	0.75	0.96 +	0.29	0.74 +	0.09	0.45 +	0.28	1.01 +	0.12	0.62 +	0.18	0.60 +	**0.11**	0.39	0.20	0.62 +	0.70	1.45 +	1.07	3.03 +	0.72	1.95 +	0.98	2.62 +	0.85	1.87 +
5_450_40_1	**1.86**	1.71	1.52	2.23 +	0.73	3.22 +	0.20	1.53 +	0.57	1.81 +	0.28	1.17 +	0.44	1.78 +	**0.14**	0.35	0.32	0.80 +	0.88	2.47 +	1.50	5.61 +	1.02	1.66 +	1.35	2.44 +	1.15	2.05 +
5_450_50_1	**1.78**	4.92	1.39	2.86 +	0.52	2.20 +	0.14	2.29 +	0.45	2.33 +	0.16	0.84 +	0.33	1.99 +	**0.25**	1.34	0.48	1.08 +	1.15	2.58 +	1.73	7.83 +	1.23	2.59 +	1.61	3.21 +	1.37	3.41 +
5_500_10_1	**0.28**	0.05	0.24	0.06 +	0.12	0.04 +	0.07	0.09 +	0.11	0.02 +	0.05	0.05 +	0.09	0.06 +	**0.11**	0.20	0.20	0.30 +	0.52	0.23 +	0.67	0.79 +	0.56	0.17 +	0.65	0.53 +	0.62	0.49 +
5_500_20_1	**0.52**	0.28	0.44	0.21 +	0.17	0.36 +	0.04	0.14 +	0.12	0.31 +	0.03	0.05 +	0.08	0.21 +	**0.13**	0.24	0.22	0.30 +	0.62	1.10 +	0.95	1.79 +	0.73	1.22 +	0.93	0.91 +	0.81	1.38 +
5_500_30_1	**1.35**	1.45	1.07	1.37 +	0.46	1.53 +	0.16	1.66 +	0.38	1.61 +	0.13	0.68 +	0.25	0.97 +	**0.12**	0.46	0.30	0.65 +	0.84	2.06 +	1.27	5.42 +	0.92	2.59 +	1.29	3.08 +	1.09	2.51 +
5_500_40_1	**1.78**	1.85	1.41	2.90 +	0.58	2.48 +	0.14	2.15 +	0.46	2.73 +	0.21	1.22 +	0.36	1.75 +	**0.18**	0.68	0.40	1.33 +	1.02	2.17 +	1.68	9.34 +	1.16	3.79 +	1.47	3.23 +	1.28	2.87 +
5_500_50_1	**1.87**	2.94	1.52	3.20 +	0.62	2.86 +	0.16	1.55 +	0.48	3.37 +	0.19	1.52 +	0.31	1.95 +	**0.16**	0.85	0.35	1.12 +	1.00	2.58 +	1.61	6.28 +	1.15	4.35 +	1.53	4.63 +	1.35	3.47 +
+/−/=	−		48/0/2		48/0/2		50/0/0		50/0/0		50/0/0		50/0/0		−		47/0/3		48/0/2		50/0/0		50/0/0		50/0/0		50/0/0	

Table 7 Results of HV and IGD for Problem set 6.

Problem	HV														IGD													
	MDU-MoPSO		MoPSO		MOHEA		NSGA-II		SPEA2		MOEA/D		MOHEA-DE		MDU-MoPSO		MoPSO		MOHEA		NSGA-II		SPEA2		MOEA/D		MOHEA-DE	
	Mean ($\times10^{-2}$)	Variance ($\times10^{-6}$)	Mean ($\times10^{-2}$)	Variance ($\times10^{-6}$)	Mean ($\times10^{-2}$)	Variance ($\times10^{-6}$)	Mean ($\times10^{-2}$)	Variance ($\times10^{-6}$)	Mean ($\times10^{-2}$)	Variance ($\times10^{-6}$)	Mean ($\times10^{-2}$)	Variance ($\times10^{-6}$)	Mean ($\times10^{-2}$)	Variance ($\times10^{-6}$)	Mean ($\times10^{-1}$)	Variance ($\times10^{-4}$)	Mean ($\times10^{-1}$)	Variance ($\times10^{-4}$)	Mean ($\times10^{-1}$)	Variance ($\times10^{-4}$)	Mean ($\times10^{-1}$)	Variance ($\times10^{-4}$)	Mean ($\times10^{-1}$)	Variance ($\times10^{-4}$)	Mean ($\times10^{-1}$)	Variance ($\times10^{-4}$)	Mean ($\times10^{-1}$)	Variance ($\times10^{-4}$)
6_50_10_1	**0.31**	1.65	0.31	1.69 =	0.27	1.71 =	0.13	0.57 +	0.22	1.32 +	0.12	0.55 +	0.13	0.56 +	0.44	2.80	**0.43**	2.99 =	0.50	2.72 =	0.72	2.75 +	0.57	3.55 +	0.71	2.37 +	0.76	2.67 +
6_50_20_1	**0.30**	1.41	0.22	0.85 +	0.22	0.81 +	0.07	0.40 +	0.16	0.85 +	0.11	0.91 +	0.10	0.77 +	**0.28**	2.15	0.38	2.17 +	0.38	1.99 +	0.67	2.74 +	0.48	2.68 +	0.59	4.17 +	0.66	3.82 +
6_50_30_1	**0.66**	2.49	0.36	2.69 +	0.44	2.12 +	0.22	1.58 +	0.30	2.12 +	0.17	1.49 +	0.24	1.26 +	**0.27**	1.92	0.59	4.60 +	0.49	2.47 +	0.78	3.86 +	0.66	3.31 +	0.86	4.50 +	0.83	3.95 +
6_50_40_1	**0.38**	1.41	0.18	2.20 +	0.29	1.56 +	0.10	0.80 +	0.21	0.99 +	0.13	0.53 +	0.14	0.53 +	**0.29**	1.86	0.59	5.82 +	0.41	3.40 +	0.74	3.97 +	0.51	2.43 +	0.67	2.94 +	0.63	2.33 +
6_50_50_1	**0.47**	1.91	0.23	1.58 +	0.37	1.67 +	0.16	0.92 +	0.24	1.73 +	0.14	1.20 +	0.18	0.82 +	**0.25**	2.12	0.57	4.16 +	0.36	2.05 +	0.69	3.83 +	0.54	3.38 +	0.72	5.08 +	0.64	2.80 +
6_100_10_1	0.53	0.72	**0.53**	1.15 =	0.24	0.57 +	0.14	0.51 +	0.21	0.49 +	0.12	0.38 +	0.15	0.54 +	0.23	0.72	**0.18**	0.88 =	0.64	1.69 +	0.83	2.56 +	0.70	1.16 +	0.77	1.35 +	0.80	2.08 +
6_100_20_1	**0.38**	0.85	0.35	1.33 =	0.15	0.37 +	0.06	0.21 +	0.09	0.25 +	0.06	0.50 +	0.08	0.20 +	**0.19**	0.96	0.22	1.62 =	0.51	1.39 +	0.74	1.98 +	0.65	1.63 +	0.74	3.62 +	0.67	1.40 +
6_100_30_1	**0.43**	0.92	0.41	2.01 =	0.21	0.77 +	0.05	0.17 +	0.15	0.43 +	0.06	0.28 +	0.11	0.55 +	**0.29**	0.98	0.32	2.26 =	0.58	2.03 +	0.92	1.59 +	0.69	1.62 +	0.89	2.64 +	0.82	2.89 +
6_100_40_1	**0.61**	1.26	0.55	2.66 =	0.27	1.26 +	0.07	0.23 +	0.16	0.65 +	0.11	0.48 +	0.13	1.00 +	**0.23**	1.12	0.28	2.63 =	0.60	2.56 +	0.97	2.23 +	0.77	2.48 +	0.88	2.43 +	0.87	4.97 +
6_100_50_1	**0.77**	2.07	0.60	2.33 +	0.30	1.12 +	0.07	0.29 +	0.19	1.66 +	0.10	0.42 +	0.11	0.34 +	**0.26**	1.34	0.41	2.10 +	0.74	1.85 +	1.18	2.82 +	0.94	5.90 +	1.09	2.73 +	1.12	2.20 +
6_150_10_1	**0.25**	0.05	0.21	0.13 +	0.13	0.12 +	0.07	0.08 +	0.12	0.07 +	0.07	0.12 +	0.08	0.16 +	**0.07**	0.14	0.14	0.23 +	0.45	0.77 +	0.66	0.54 +	0.47	0.28 +	0.58	0.66 +	0.61	1.06 +
6_150_20_1	**0.54**	0.32	0.47	0.43 +	0.22	0.61 +	0.08	0.25 +	0.14	0.31 +	0.06	0.19 +	0.09	0.30 +	**0.10**	0.17	0.17	0.36 +	0.47	1.33 +	0.73	1.45 +	0.60	1.22 +	0.78	1.44 +	0.70	1.70 +
6_150_30_1	**1.48**	5.76	1.18	6.63 +	0.43	1.73 +	0.11	0.65 +	0.28	1.33 +	0.15	0.90 +	0.16	1.24 +	**0.27**	1.80	0.46	2.86 +	1.08	2.12 +	1.58	3.90 +	1.27	2.83 +	1.48	3.09 +	1.47	3.87 +
6_150_40_1	**1.38**	4.49	1.04	4.28 +	0.41	1.71 +	0.13	1.21 +	0.29	1.67 +	0.18	0.55 +	0.23	2.11 +	**0.20**	1.72	0.42	2.01 +	0.97	2.03 +	1.41	5.30 +	1.12	3.04 +	1.28	1.63 +	1.22	4.44 +
6_150_50_1	**1.43**	4.86	0.99	3.97 +	0.43	5.10 +	0.08	0.70 +	0.28	1.50 +	0.14	0.88 +	0.18	1.18 +	**0.31**	1.66	0.60	2.21 +	1.12	6.72 +	1.65	4.22 +	1.28	3.04 +	1.50	3.29 +	1.43	3.64 +
6_200_10_1	**0.52**	0.58	0.44	0.38 +	0.23	0.43 +	0.09	0.34 +	0.20	0.36 +	0.08	0.36 +	0.13	0.42 +	**0.13**	0.42	0.21	0.48 +	0.56	1.06 +	0.83	2.02 +	0.60	1.11 +	0.79	2.35 +	0.73	1.86 +
6_200_20_1	**0.53**	0.26	0.47	0.57 +	0.17	0.31 +	0.07	0.30 +	0.17	0.58 +	0.08	0.25 +	0.11	0.26 +	**0.10**	0.22	0.17	0.77 +	0.67	1.33 +	0.93	2.73 +	0.68	2.01 +	0.84	1.39 +	0.81	1.37 +
6_200_30_1	**1.30**	3.14	1.02	2.79 +	0.41	1.97 +	0.12	0.50 +	0.29	1.46 +	0.15	0.57 +	0.23	1.61 +	**0.17**	1.07	0.35	1.47 +	0.91	2.76 +	1.35	3.46 +	1.05	2.63 +	1.24	2.26 +	1.13	3.39 +
6_200_40_1	**0.78**	1.00	0.61	1.54 +	0.26	0.60 +	0.08	0.36 +	0.23	0.89 +	0.09	0.46 +	0.13	0.96 +	**0.16**	0.68	0.31	1.34 +	0.72	1.27 +	1.07	2.61 +	0.77	2.15 +	1.01	2.55 +	0.94	3.42 +
6_200_50_1	**1.36**	3.21	1.00	5.96 +	0.40	1.54 +	0.10	0.76 +	0.32	1.16 +	0.14	0.92 +	0.20	1.06 +	**0.18**	1.21	0.42	2.91 +	0.95	2.01 +	1.45	5.53 +	1.05	1.96 +	1.33	3.96 +	1.22	2.95 +
6_250_10_1	**0.20**	0.08	0.16	0.14 +	0.06	0.07 +	0.02	0.04 +	0.05	0.06 +	0.02	0.02 +	0.03	0.04 +	**0.09**	0.15	0.18	0.55 +	0.63	0.97 +	0.81	0.92 +	0.67	0.84 +	0.78	0.90 +	0.78	0.98 +
6_250_20_1	**0.83**	0.99	0.68	0.99 +	0.27	0.91 +	0.07	0.15 +	0.19	0.82 +	0.07	0.26 +	0.13	0.69 +	**0.19**	0.69	0.31	0.77 +	0.82	1.81 +	1.20	1.72 +	0.94	2.45 +	1.16	1.93 +	1.06	3.38 +
6_250_30_1	**1.24**	2.36	1.03	3.92 +	0.38	1.63 +	0.12	1.42 +	0.29	1.40 +	0.16	0.55 +	0.22	1.31 +	**0.21**	0.91	0.36	2.08 +	0.96	2.49 +	1.39	6.04 +	1.07	2.51 +	1.27	2.52 +	1.18	3.18 +
6_250_40_1	**1.43**	3.35	1.18	3.60 +	0.46	1.98 +	0.15	0.94 +	0.37	1.92 +	0.18	0.79 +	0.26	1.75 +	**0.18**	1.18	0.34	1.52 +	0.93	2.21 +	1.37	3.72 +	1.04	2.76 +	1.29	2.49 +	1.18	3.96 +
6_250_50_1	**1.43**	4.81	1.04	4.48 +	0.36	1.76 +	0.07	0.34 +	0.27	1.95 +	0.10	0.47 +	0.22	1.61 +	**0.24**	1.79	0.49	2.48 +	1.10	2.72 +	1.62	2.98 +	1.23	4.13 +	1.51	2.62 +	1.30	3.50 +
6_300_10_1	**0.20**	0.04	0.17	0.04 +	0.07	0.05 +	0.03	0.04 +	0.07	0.03 +	0.03	0.03 +	0.05	0.05 +	**0.11**	0.20	0.22	0.12 +	0.57	0.57 +	0.76	0.79 +	0.61	0.41 +	0.71	0.58 +	0.68	0.74 +
6_300_20_1	**0.93**	1.18	0.74	0.74 +	0.26	0.68 +	0.07	0.60 +	0.22	0.68 +	0.09	0.36 +	0.16	0.64 +	**0.14**	0.54	0.28	0.51 +	0.85	1.47 +	1.27	4.27 +	0.92	1.68 +	1.14	2.63 +	1.02	2.54 +
6_300_30_1	**1.01**	1.32	0.84	1.86 +	0.33	0.99 +	0.10	0.48 +	0.22	0.96 +	0.13	0.61 +	0.17	0.98 +	**0.13**	0.62	0.25	1.07 +	0.79	1.95 +	1.18	2.93 +	0.94	2.43 +	1.10	3.25 +	1.03	3.19 +
6_300_40_1	**1.62**	2.62	1.29	3.61 +	0.52	2.37 +	0.13	0.91 +	0.38	1.90 +	0.20	1.03 +	0.26	0.91 +	**0.19**	0.72	0.38	1.43 +	0.99	2.48 +	1.54	4.91 +	1.15	3.03 +	1.38	2.98 +	1.31	1.93 +
6_300_50_1	**1.83**	4.21	1.42	3.56 +	0.48	1.99 +	0.11	0.93 +	0.32	2.95 +	0.14	1.30 +	0.24	2.47 +	**0.19**	1.08	0.41	1.20 +	1.13	2.32 +	1.70	4.68 +	1.33	4.53 +	1.62	4.03 +	1.46	5.15 +
6_350_10_1	**0.21**	0.05	0.17	0.05 +	0.07	0.05 +	0.04	0.04 +	0.07	0.05 +	0.03	0.03 +	0.05	0.05 +	**0.09**	0.18	0.18	0.24 +	0.50	0.44 +	0.66	0.88 +	0.53	0.59 +	0.61	0.42 +	0.61	0.77 +
6_350_20_1	**0.57**	0.27	0.46	0.62 +	0.17	0.59 +	0.05	0.26 +	0.14	0.27 +	0.06	0.21 +	0.14	0.37 +	**0.12**	0.28	0.24	0.70 +	0.70	1.70 +	1.00	2.17 +	0.76	1.08 +	0.95	2.16 +	0.82	2.33 +
6_350_30_1	**1.47**	1.42	1.09	2.05 +	0.44	2.52 +	0.11	1.37 +	0.30	1.21 +	0.13	0.62 +	0.23	1.07 +	**0.14**	0.59	0.39	1.08 +	0.96	3.31 +	1.49	6.28 +	1.13	2.47 +	1.41	3.33 +	1.23	2.56 +
6_350_40_1	**1.43**	2.04	1.14	2.35 +	0.49	2.67 +	0.21	2.54 +	0.41	1.83 +	0.21	1.43 +	0.29	1.13 +	**0.24**	0.76	0.43	1.16 +	0.99	2.56 +	1.40	8.40 +	1.07	2.06 +	1.34	3.48 +	1.22	2.16 +
6_350_50_1	**2.77**	9.79	2.15	10.02 +	1.01	7.02 +	0.28	3.56 +	0.69	8.40 +	0.32	1.90 +	0.50	2.16 +	**0.21**	1.80	0.49	2.50 +	1.17	3.82 +	1.89	8.21 +	1.42	6.00 +	1.80	3.92 +	1.59	2.71 +
6_400_10_1	**0.29**	0.09	0.22	0.15 +	0.09	0.10 +	0.04	0.05 +	0.09	0.07 +	0.03	0.04 +	0.07	0.08 +	**0.10**	0.23	0.24	0.58 +	0.54	0.70 +	0.71	0.67 +	0.57	0.42 +	0.71	0.85 +	0.62	0.65 +
6_400_20_1	**0.87**	0.42	0.68	0.70 +	0.29	0.61 +	0.07	0.18 +	0.20	0.85 +	0.10	0.34 +	0.14	0.52 +	**0.09**	0.23	0.24	0.56 +	0.71	1.24 +	1.13	1.79 +	0.84	2.03 +	1.02	1.72 +	0.94	2.52 +
6_400_30_1	**1.12**	1.27	0.89	0.88 +	0.36	1.17 +	0.09	0.69 +	0.30	0.93 +	0.13	0.46 +	0.20	0.67 +	**0.13**	0.55	0.29	0.48 +	0.82	1.98 +	1.28	4.68 +	0.90	1.80 +	1.16	1.86 +	1.05	1.85 +
6_400_40_1	**1.63**	2.54	1.31	3.01 +	0.58	3.81 +	0.14	1.21 +	0.38	2.04 +	0.18	0.98 +	0.37	2.08 +	**0.14**	0.74	0.33	1.23 +	0.90	3.23 +	1.49	4.79 +	1.11	3.06 +	1.40	3.34 +	1.13	3.67 +
6_400_50_1	**0.87**	1.15	0.68	0.89 +	0.29	0.86 +	0.08	0.57 +	0.23	0.80 +	0.08	0.25 +	0.14	0.47 +	**0.13**	0.63	0.29	0.68 +	0.72	1.49 +	1.14	3.93 +	0.80	1.94 +	1.10	2.26 +	0.95	1.75 +
6_450_10_1	**0.24**	0.09	0.20	0.08 +	0.09	0.05 +	0.03	0.04 +	0.08	0.06 +	0.04	0.05 +	0.05	0.03 +	**0.07**	0.05	0.13	0.25 +	0.57	0.69 +	0.84	1.80 +	0.62	0.92 +	0.74	0.76 +	0.71	0.59 +
6_450_20_1	**0.81**	0.43	0.70	0.51 +	0.32	0.70 +	0.10	0.34 +	0.27	0.57 +	0.14	0.31 +	0.20	0.73 +	**0.08**	0.25	0.19	0.45 +	0.68	1.58 +	1.08	2.51 +	0.75	1.41 +	0.94	1.75 +	0.85	2.39 +
6_450_30_1	**1.36**	1.15	1.08	1.76 +	0.44	1.77 +	0.15	1.24 +	0.41	1.59 +	0.21	1.25 +	0.29	1.49 +	**0.15**	0.47	0.34	0.96 +	0.92	2.30 +	1.35	4.67 +	0.96	2.23 +	1.23	3.78 +	1.10	2.85 +
6_450_40_1	**2.13**	2.93	1.66	3.00 +	0.80	4.01 +	0.24	3.20 +	0.66	3.70 +	0.34	1.86 +	0.51	2.35 +	**0.15**	0.71	0.39	1.03 +	0.97	2.78 +	1.60	7.73 +	1.10	3.14 +	1.41	2.72 +	1.24	2.66 +
6_450_50_1	**1.78**	1.83	1.50	4.06 +	0.71	2.19 +	0.19	2.37 +	0.54	2.38 +	0.26	1.18 +	0.40	2.25 +	**0.14**	0.53	0.31	1.32 +	0.86	1.51 +	1.49	7.32 +	1.02	2.24 +	1.34	2.42 +	1.17	2.93 +
6_500_10_1	**0.19**	0.03	0.14	0.03 +	0.05	0.04 +	0.02	0.02 +	0.05	0.02 +	0.02	0.02 +	0.03	0.02 +	**0.06**	0.04	0.15	0.15 +	0.50	0.57 +	0.67	0.52 +	0.51	0.29 +	0.63	0.49 +	0.60	0.58 +
6_500_20_1	**0.84**	0.63	0.73	0.65 +	0.30	0.66 +	0.08	0.33 +	0.26	0.55 +	0.10	0.19 +	0.18	0.57 +	**0.15**	0.39	0.25	0.57 +	0.75	1.19 +	1.18	2.96 +	0.81	1.36 +	1.08	1.22 +	0.93	2.16 +
6_500_30_1	**1.10**	0.82	0.88	0.97 +	0.38	1.20 +	0.11	0.66 +	0.33	1.21 +	0.13	0.78 +	0.24	1.27 +	**0.09**	0.28	0.25	0.59 +	0.75	1.60 +	1.18	3.41 +	0.81	1.99 +	1.14	3.45 +	0.93	2.91 +
6_500_40_1	**2.19**	3.78	1.78	4.73 +	0.74	2.27 +	0.17	1.57 +	0.61	3.46 +	0.26	1.09 +	0.48	2.75 +	**0.29**	0.92	0.50	1.42 +	1.20	1.63 +	1.89	5.65 +	1.32	3.07 +	1.71	2.65 +	1.45	3.19 +
6_500_50_1	**1.92**	2.82	1.48	4.69 +	0.58	2.71 +	0.11	1.46 +	0.55	2.67 +	0.18	1.53 +	0.32	2.01 +	**0.17**	0.69	0.41	1.58 +	1.07	2.62 +	1.76	7.51 +	1.10	2.81 +	1.58	4.14 +	1.36	3.18 +
+/−/=	−		45/0/5		49/0/1		50/0/0		50/0/0		50/0/0		50/0/0		−		45/0/5		49/0/1		50/0/0		50/0/0		50/0/0		50/0/0	

Table 8 Results of HV and IGD for Problem set 7.

Problem	HV														IGD													
	MDU-MoPSO		MoPSO		MOHEA		NSGA-II		SPEA2		MOEA/D		MOHEA-DE		MDU-MoPSO		MoPSO		MOHEA		NSGA-II		SPEA2		MOEA/D		MOHEA-DE	
	Mean ($\times10^{-2}$)	Variance ($\times10^{-6}$)	Mean ($\times10^{-2}$)	Variance ($\times10^{-6}$)	Mean ($\times10^{-2}$)	Variance ($\times10^{-6}$)	Mean ($\times10^{-2}$)	Variance ($\times10^{-6}$)	Mean ($\times10^{-2}$)	Variance ($\times10^{-6}$)	Mean ($\times10^{-2}$)	Variance ($\times10^{-6}$)	Mean ($\times10^{-2}$)	Variance ($\times10^{-6}$)	Mean ($\times10^{-1}$)	Variance ($\times10^{-4}$)	Mean ($\times10^{-1}$)	Variance ($\times10^{-4}$)	Mean ($\times10^{-1}$)	Variance ($\times10^{-4}$)	Mean ($\times10^{-1}$)	Variance ($\times10^{-4}$)	Mean ($\times10^{-1}$)	Variance ($\times10^{-4}$)	Mean ($\times10^{-1}$)	Variance ($\times10^{-4}$)	Mean ($\times10^{-1}$)	Variance ($\times10^{-4}$)
7_50_10_1	0.13	0.09	**0.14**	0.10 =	0.11	0.08 +	0.05	0.06 +	0.08	0.08 +	0.07	0.14 +	0.06	0.08 +	0.16	0.55	**0.12**	0.55 -	0.22	0.61 +	0.36	0.78 +	0.28	0.73 +	0.27	1.49 +	0.36	0.71 +
7_50_20_1	**0.36**	1.65	0.25	1.64 +	0.30	0.83 =	0.15	0.52 +	0.28	0.84 +	0.17	0.56 +	0.17	0.93 +	**0.27**	2.20	0.40	3.32 +	0.32	1.37 =	0.56	2.54 +	0.36	1.61 +	0.51	2.26 +	0.53	2.93 +
7_50_30_1	**0.33**	1.56	0.21	0.98 +	0.30	1.13 =	0.11	0.50 +	0.23	1.00 +	0.13	0.62 +	0.14	0.71 +	**0.27**	2.30	0.44	2.42 +	0.31	2.44 =	0.63	2.81 +	0.42	2.25 +	0.58	2.83 +	0.56	2.04 +
7_50_40_1	**0.26**	1.05	0.16	0.66 +	0.21	0.64 =	0.06	0.41 +	0.16	0.80 +	0.10	0.81 +	0.09	0.36 +	**0.31**	1.81	0.47	2.45 +	0.37	1.63 =	0.71	2.95 +	0.48	2.47 +	0.62	4.10 +	0.64	2.72 +
7_50_50_1	**0.22**	0.84	0.11	0.46 +	0.16	0.36 +	0.05	0.15 +	0.12	0.57 +	0.08	0.46 +	0.08	0.30 +	**0.30**	1.99	0.50	2.50 +	0.39	1.33 +	0.65	1.39 +	0.48	2.61 +	0.60	3.08 +	0.57	1.95 +
7_100_10_1	0.41	0.47	**0.43**	1.16 =	0.17	0.26 +	0.09	0.31 +	0.18	0.55 +	0.08	0.33 +	0.09	0.22 +	0.30	0.68	**0.21**	1.32 -	0.64	0.92 +	0.84	2.26 +	0.64	1.68 +	0.80	2.25 +	0.80	1.63 +
7_100_20_1	**0.59**	0.87	0.57	1.61 =	0.28	1.37 +	0.10	0.97 +	0.23	0.65 +	0.14	0.33 +	0.15	0.71 +	**0.23**	0.69	0.24	1.02 =	0.61	2.40 +	0.96	4.44 +	0.67	1.76 +	0.81	1.40 +	0.81	2.63 +
7_100_30_1	**0.65**	2.24	0.61	1.41 =	0.25	0.94 +	0.08	0.16 +	0.19	0.80 +	0.09	0.33 +	0.11	0.81 +	**0.20**	1.78	0.23	1.12 =	0.63	1.93 +	0.96	1.43 +	0.72	2.12 +	0.94	2.63 +	0.89	3.38 +
7_100_40_1	**0.90**	4.65	0.78	4.98 +	0.34	2.18 +	0.09	0.36 +	0.26	1.03 +	0.11	0.56 +	0.16	0.64 +	**0.29**	2.91	0.38	3.35 =	0.82	3.83 +	1.22	2.56 +	0.92	2.29 +	1.17	3.45 +	1.08	2.71 +
7_100_50_1	**0.87**	3.39	0.70	3.19 +	0.42	1.32 +	0.13	0.95 +	0.32	1.67 +	0.19	0.71 +	0.20	1.43 +	**0.32**	2.08	0.47	2.45 +	0.73	1.73 +	1.17	4.27 +	0.85	2.90 +	1.03	1.95 +	1.04	3.77 +
7_150_10_1	**0.31**	0.08	0.25	0.29 +	0.14	0.21 +	0.08	0.09 +	0.13	0.12 +	0.08	0.19 +	0.09	0.17 +	**0.09**	0.15	0.18	0.75 +	0.52	0.94 +	0.68	0.73 +	0.54	0.64 +	0.63	1.06 +	0.65	0.97 +
7_150_20_1	**0.97**	2.24	0.82	2.53 +	0.28	1.13 +	0.08	0.47 +	0.20	0.72 +	0.10	0.25 +	0.18	0.93 +	**0.23**	1.22	0.33	1.48 +	0.92	2.32 +	1.28	3.16 +	1.03	2.27 +	1.20	1.50 +	1.06	3.30 +
7_150_30_1	**0.82**	2.11	0.68	1.69 +	0.29	1.37 +	0.09	0.46 +	0.22	0.58 +	0.12	0.46 +	0.14	0.46 +	**0.28**	1.39	0.40	1.20 +	0.84	2.59 +	1.22	3.41 +	0.94	1.42 +	1.12	2.86 +	1.08	1.59 +
7_150_40_1	**1.29**	5.38	0.93	3.09 +	0.34	1.88 +	0.08	0.45 +	0.25	1.24 +	0.13	0.52 +	0.18	1.47 +	**0.45**	1.96	0.69	1.77 +	1.25	2.71 +	1.69	2.80 +	1.37	2.51 +	1.58	2.62 +	1.54	4.38 +
7_150_50_1	**1.04**	2.60	0.75	1.70 +	0.32	1.74 +	0.08	0.45 +	0.18	0.70 +	0.10	0.47 +	0.16	0.74 +	**0.22**	1.06	0.43	1.16 +	0.87	2.73 +	1.30	3.13 +	1.07	1.97 +	1.23	2.42 +	1.10	1.95 +
7_200_10_1	**0.39**	0.28	0.32	0.22 +	0.16	0.21 +	0.08	0.18 +	0.14	0.29 +	0.08	0.25 +	0.09	0.13 +	**0.13**	0.48	0.23	0.43 +	0.62	0.82 +	0.83	1.68 +	0.67	1.45 +	0.76	1.59 +	0.79	1.20 +
7_200_20_1	**1.00**	1.30	0.86	1.84 +	0.38	1.37 +	0.12	0.65 +	0.32	1.05 +	0.19	0.53 +	0.21	0.84 +	**0.18**	0.78	0.29	1.36 +	0.78	2.07 +	1.21	3.30 +	0.86	1.97 +	1.03	1.69 +	1.02	2.10 +
7_200_30_1	**1.23**	2.32	0.99	2.39 +	0.31	1.32 +	0.10	0.55 +	0.27	1.58 +	0.15	0.68 +	0.20	1.06 +	**0.16**	0.85	0.32	1.31 +	0.96	2.14 +	1.32	3.10 +	1.02	2.95 +	1.22	2.98 +	1.15	4.04 +
7_200_40_1	**1.82**	4.74	1.47	4.75 +	0.64	2.82 +	0.22	1.56 +	0.52	2.12 +	0.31	2.05 +	0.40	1.62 +	**0.19**	1.31	0.39	1.72 +	0.99	2.33 +	1.48	4.48 +	1.10	2.09 +	1.35	3.83 +	1.24	2.27 +
7_200_50_1	**1.52**	5.68	1.13	5.65 +	0.39	2.42 +	0.07	0.30 +	0.29	2.81 +	0.14	0.91 +	0.22	1.37 +	**0.27**	1.86	0.52	2.56 +	1.15	3.07 +	1.68	2.22 +	1.29	5.30 +	1.54	4.19 +	1.38	3.47 +
7_250_10_1	**0.18**	0.07	0.16	0.07 +	0.06	0.05 +	0.02	0.01 +	0.05	0.05 +	0.02	0.02 +	0.03	0.04 +	**0.09**	0.18	0.17	0.23 +	0.61	0.67 +	0.84	0.66 +	0.66	0.95 +	0.78	0.74 +	0.75	0.79 +
7_250_20_1	**0.70**	0.79	0.58	1.16 +	0.22	0.87 +	0.06	0.20 +	0.19	0.53 +	0.07	0.33 +	0.14	0.26 +	**0.11**	0.49	0.21	1.06 +	0.67	2.03 +	1.01	1.68 +	0.71	1.42 +	0.98	2.30 +	0.80	0.84 +
7_250_30_1	**1.01**	0.79	0.79	1.15 +	0.36	1.20 +	0.13	0.89 +	0.29	0.92 +	0.12	0.48 +	0.23	1.69 +	**0.16**	0.41	0.34	0.73 +	0.79	1.70 +	1.17	4.70 +	0.88	1.61 +	1.14	1.90 +	0.98	3.95 +
7_250_40_1	**1.69**	4.83	1.37	4.27 +	0.50	2.58 +	0.11	0.84 +	0.38	2.16 +	0.20	0.77 +	0.29	1.64 +	**0.30**	1.44	0.48	1.68 +	1.16	2.93 +	1.75	5.32 +	1.29	3.08 +	1.54	1.95 +	1.41	3.12 +
7_250_50_1	**1.54**	2.26	1.24	2.59 +	0.57	2.76 +	0.13	1.36 +	0.39	2.03 +	0.21	0.74 +	0.31	1.54 +	**0.17**	0.64	0.35	1.06 +	0.87	2.43 +	1.49	5.69 +	1.06	2.73 +	1.30	2.02 +	1.16	2.59 +
7_300_10_1	**0.36**	0.10	0.30	0.16 +	0.13	0.12 +	0.04	0.09 +	0.12	0.23 +	0.05	0.06 +	0.08	0.09 +	**0.10**	0.19	0.19	0.40 +	0.62	0.75 +	0.89	1.65 +	0.65	1.22 +	0.80	0.91 +	0.76	0.70 +
7_300_20_1	**1.19**	2.32	0.97	2.42 +	0.35	1.60 +	0.09	0.49 +	0.29	1.39 +	0.11	0.52 +	0.17	0.59 +	**0.23**	0.97	0.40	1.30 +	1.00	2.56 +	1.44	3.19 +	1.09	2.63 +	1.39	2.95 +	1.27	1.98 +
7_300_30_1	**0.91**	1.61	0.79	1.20 +	0.34	0.96 +	0.09	0.49 +	0.27	0.96 +	0.13	0.66 +	0.20	1.00 +	**0.22**	1.12	0.33	0.93 +	0.81	1.55 +	1.25	3.47 +	0.90	1.89 +	1.12	2.63 +	1.02	2.88 +
7_300_40_1	**1.52**	4.10	1.22	2.62 +	0.41	2.37 +	0.12	1.61 +	0.35	1.98 +	0.14	1.02 +	0.24	1.19 +	**0.18**	1.36	0.37	1.19 +	1.05	3.08 +	1.54	6.67 +	1.12	2.89 +	1.45	3.97 +	1.27	2.31 +
7_300_50_1	**1.64**	3.04	1.24	3.16 +	0.49	3.12 +	0.15	2.28 +	0.34	1.48 +	0.14	0.82 +	0.23	1.29 +	**0.16**	0.89	0.40	1.25 +	1.01	3.70 +	1.53	8.51 +	1.17	2.06 +	1.48	3.05 +	1.33	2.95 +
7_350_10_1	**0.45**	0.21	0.40	0.29 +	0.17	0.33 +	0.08	0.19 +	0.15	0.22 +	0.06	0.15 +	0.10	0.24 +	**0.13**	0.29	0.20	0.39 +	0.59	1.13 +	0.78	1.30 +	0.63	0.85 +	0.83	1.53 +	0.73	1.43 +
7_350_20_1	**0.48**	0.17	0.41	0.21 +	0.16	0.40 +	0.04	0.16 +	0.14	0.29 +	0.06	0.13 +	0.09	0.31 +	**0.08**	0.17	0.17	0.31 +	0.60	1.41 +	0.91	1.79 +	0.66	1.23 +	0.83	1.40 +	0.77	1.51 +
7_350_30_1	**1.57**	1.01	1.17	4.56 +	0.46	2.56 +	0.11	1.30 +	0.38	1.42 +	0.16	1.35 +	0.28	1.53 +	**0.17**	0.32	0.41	2.00 +	1.02	3.08 +	1.58	5.77 +	1.11	2.17 +	1.43	4.28 +	1.24	3.02 +
7_350_40_1	**1.92**	5.04	1.58	3.85 +	0.52	3.39 +	0.11	1.46 +	0.38	3.12 +	0.17	0.95 +	0.24	1.91 +	**0.24**	1.37	0.43	1.28 +	1.21	2.86 +	1.83	6.20 +	1.38	4.87 +	1.66	3.13 +	1.56	3.37 +
7_350_50_1	**2.28**	6.13	1.80	4.01 +	0.65	2.25 +	0.18	2.19 +	0.51	5.95 +	0.22	1.26 +	0.41	2.92 +	**0.28**	1.36	0.52	1.18 +	1.30	1.89 +	1.90	8.06 +	1.45	6.33 +	1.80	3.47 +	1.55	4.20 +
7_400_10_1	**0.23**	0.04	0.19	0.03 +	0.08	0.05 +	0.04	0.04 +	0.07	0.06 +	0.04	0.03 +	0.05	0.05 +	**0.07**	0.12	0.18	0.11 +	0.70	0.70 +	0.92	1.31 +	0.73	0.95 +	0.86	0.68 +	0.84	1.24 +
7_400_20_1	**0.55**	0.47	0.46	0.49 +	0.16	0.34 +	0.05	0.15 +	0.16	0.39 +	0.04	0.12 +	0.10	0.26 +	**0.14**	0.48	0.24	0.54 +	0.70	0.98 +	0.99	1.70 +	0.73	1.43 +	1.00	1.92 +	0.85	1.74 +
7_400_30_1	**1.30**	1.72	1.14	1.71 +	0.47	2.20 +	0.13	1.01 +	0.38	1.83 +	0.16	0.65 +	0.28	1.30 +	**0.14**	0.65	0.24	0.67 +	0.82	2.68 +	1.33	4.76 +	0.91	2.70 +	1.23	2.24 +	1.06	3.23 +
7_400_40_1	**1.18**	1.45	1.04	1.42 +	0.48	1.39 +	0.14	1.62 +	0.40	1.93 +	0.22	1.05 +	0.27	1.12 +	**0.13**	0.63	0.23	0.71 +	0.71	1.47 +	1.23	7.00 +	0.81	2.68 +	1.04	2.50 +	0.96	1.85 +
7_400_50_1	**2.20**	3.14	1.82	1.92 +	0.89	5.51 +	0.21	2.59 +	0.71	3.77 +	0.27	1.19 +	0.54	2.49 +	**0.14**	0.64	0.33	0.55 +	0.93	3.28 +	1.66	7.10 +	1.07	3.00 +	1.54	2.46 +	1.22	2.49 +
7_450_10_1	**0.32**	0.08	0.25	0.11 +	0.10	0.09 +	0.03	0.06 +	0.07	0.12 +	0.03	0.07 +	0.05	0.06 +	**0.08**	0.11	0.17	0.33 +	0.55	0.57 +	0.78	1.09 +	0.64	1.13 +	0.73	1.50 +	0.68	0.89 +
7_450_20_1	**1.32**	1.24	1.11	1.00 +	0.53	1.33 +	0.14	0.94 +	0.40	1.37 +	0.23	0.69 +	0.30	1.04 +	**0.18**	0.55	0.32	0.46 +	0.84	1.38 +	1.39	4.91 +	0.98	1.93 +	1.19	2.26 +	1.10	1.63 +
7_450_30_1	**1.17**	1.01	0.96	1.11 +	0.44	1.22 +	0.08	0.54 +	0.32	1.33 +	0.15	0.70 +	0.23	1.35 +	**0.18**	0.46	0.33	0.68 +	0.82	1.60 +	1.40	3.37 +	0.97	2.37 +	1.22	2.62 +	1.10	3.20 +
7_450_40_1	**1.20**	1.15	1.00	1.49 +	0.38	1.75 +	0.09	0.64 +	0.34	0.78 +	0.12	0.80 +	0.22	1.08 +	**0.13**	0.51	0.27	0.77 +	0.85	2.61 +	1.34	4.52 +	0.88	1.22 +	1.26	3.50 +	1.07	2.21 +
7_450_50_1	**2.57**	9.27	2.01	6.40 +	0.66	5.75 +	0.15	1.80 +	0.52	3.06 +	0.25	1.55 +	0.36	1.71 +	**0.26**	1.89	0.52	1.66 +	1.41	4.85 +	2.08	8.00 +	1.54	3.26 +	1.86	3.46 +	1.71	2.56 +
7_500_10_1	**0.27**	0.03	0.21	0.07 +	0.09	0.07 +	0.04	0.06 +	0.08	0.07 +	0.04	0.04 +	0.06	0.05 +	**0.05**	0.10	0.18	0.34 +	0.59	0.72 +	0.79	1.43 +	0.62	0.67 +	0.76	0.85 +	0.71	0.71 +
7_500_20_1	**0.46**	0.13	0.39	0.14 +	0.17	0.23 +	0.06	0.16 +	0.16	0.21 +	0.07	0.20 +	0.11	0.16 +	**0.07**	0.14	0.16	0.25 +	0.60	0.84 +	0.90	2.12 +	0.64	0.90 +	0.82	1.70 +	0.74	1.18 +
7_500_30_1	**1.42**	0.96	1.19	1.72 +	0.63	1.76 +	0.19	2.27 +	0.55	2.53 +	0.25	1.66 +	0.36	1.74 +	**0.11**	0.32	0.26	0.78 +	0.72	1.56 +	1.29	6.60 +	0.80	2.50 +	1.14	3.87 +	1.00	2.72 +
7_500_40_1	**1.72**	1.41	1.43	2.48 +	0.65	3.73 +	0.18	2.73 +	0.46	2.11 +	0.22	1.68 +	0.34	2.54 +	**0.13**	0.38	0.29	0.91 +	0.87	3.07 +	1.48	8.12 +	1.05	2.33 +	1.37	4.30 +	1.21	4.54 +
7_500_50_1	**1.84**	2.26	1.44	2.43 +	0.59	2.02 +	0.13	1.11 +	0.55	2.22 +	0.18	0.91 +	0.34	1.12 +	**0.13**	0.57	0.35	0.88 +	0.98	1.74 +	1.61	5.32 +	1.02	2.30 +	1.49	3.14 +	1.25	2.11 +
+/−/=	−		46/0/4		47/0/3		50/0/0		50/0/0		50/0/0		50/0/0		−		45/2/3		47/0/3		50/0/0		50/0/0		50/0/0		50/0/0	

Problem set 7, which contains 50 different problems. Based on the HV index, the mean results of MDU-MoPSO on 7_50_10_1 and 7_100_10_1 are not as good as those of MoPSO, but it performs best on the remaining 48 problems. For the IGD indicator, the results of MDU-MoPSO on 7_50_10_1 and 7_100_10_1 are not as good as those of MoPSO, but it performs best on the remaining 48 problems. Therefore, the MDU-MoPSO algorithm has good convergence and distribution performances on Problem set 7.

There are a total of $350\,(7\times50)$ problems. Each problem is run 30 times, and the index values of each time are recorded to analyze their significance. The significance analysis results by the Wilcoxon rank sum test are listed in Table 9 , where "w" means significantly good, "t" means significantly bad, and "l" means not significant. When the index results of MDU-MoPSO are significantly better, R+ is increased by one. When the index result of MDU-MoPSO is significantly poor, R_ increased by one. The p-value is the test probability of all indicators, and h is the significance result under $\alpha = 0.05$, where $h = 1$ means significant and $h = 0$ means not significant. From the overall significant results (h value), MDU-MoPSO is significantly better than MoPSO, NSGA-II, SPEA2, MOEA/D, MOHEA, and MOHEA-DE on 350 problems.

In addition, due to the page limitation, we just list the calculation time comparisons of all algorithms for the selected problems (as shown in Table 10). In Table 10, the proposed algorithm has no evident advantages in calculation time except SPEA2 and MOHEA-DE. Bacause MDU-MoPSO and MoPSO should maintain *gbestset* and *pbestset* for each individual, the computation time is more than that of NSGA-II, MOEA/D, and MOHEA, and less than that of SPEA2 and MOHEA-DE.

In summary, based on the results of HV and IGD indices, MDU-MoPSO has good convergence and distribution performances, whereas MDU can enhance the convergence performance while maintaining a better distribution performance. The main reason is the decomposition of the particle swarm into three subgroups according to the three convergence directions of the PF and the update of particles within the three subgroups. Because the particle swarm is divided into three subgroups and each subgroup only serves for a specific direction, the particles could be seperately updated to evolve in three directions. The particles in each subgroup could be updated by selecting *gbest* and *pbest* according to the three directions, i.e., the three subgroups attempt to fast converge to three areas of the PF. Two subgroups are individually close to the two edge areas of the PF and serve two objectives individually, whereas the other one is close to the central area of the PF. This strategy could update the particle in multiple directions and interact in each direction to speed up the convergence while guaranteeing a good distribution performance.

6 Conclusion

In this paper, MDU-MoPSO is proposed to solve the MNIFSP with consideration of minimizing the makespan and total processing time. MDU-MoPSO could divide the particles into three subgroups according to the hybrid selection strategy, and each subgroup is updated in its own direction, which is a convergence direction to the PF. Therefore, the MDU could increase the attention of particles located at the edge and center area of the PF. In MDU-MoPSO, the update strategy of particles in the three directions could guarantee the ability to converge quickly in multiple directions with a good distribution performance. The particle update operation based on the exchange sequence could make full use of the job exchange sequence difference to update the particles, improve the convergence performance of the particles, and maintain the diversity of the particles. The experimental results demonstrate that the proposed MDU-MoPSO has advantages in terms of faster convergence and better distribution performances than those of the other algorithms for benchmark MNIFSPs.

In the future, how to reduce the computational time in MoPSO and MDU-MoPSO is worth investigating. Furthermore, the problem-dependent local search and hybrid local search for multiobjective MNIFSP will also be examined.

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Table 9 Significance analysis result obtained by the Wilcoxon rank sum test.

Indicator	Algorithm	w/t/l	R+	R_	p-value	h ($\alpha = 0.05$)
HV	MDU-MoPSO vs. MoPSO	318/32/0	8858	1642	0	1
	MDU-MoPSO vs. NSGA-II	350/0/0	10 428	72	0	1
	MDU-MoPSO vs. SPEA2	350/0/0	10 287	213	0	1
	MDU-MoPSO vs. MOEA/D	350/0/0	10 438	62	0	1
	MDU-MoPSO vs. MOHEA	334/16/0	10 138	362	0	1
	MDU-MoPSO vs. MOHEA-DE	350/0/0	10 123	377	0	1
IGD	MDU-MoPSO vs. MoPSO	316/30/4	8813	1687	0	1
	MDU-MoPSO vs. NSGA-II	350/0/0	10 426	74	0	1
	MDU-MoPSO vs. SPEA2	348/2/0	10 297	221	0	1
	MDU-MoPSO vs. MOEA/D	350/0/0	10 432	68	0	1
	MDU-MoPSO vs. MOHEA	334/16/0	10 124	376	0	1
	MDU-MoPSO vs. MOHEA-DE	350/0/0	10 467	33	0	1

Table 10 Mean of CPU time of different algorithms for the selected problem set. (ms)

Problem	MDU-MoPSO	MoPSO	MOHEA	NSGA-II	SPEA2	MOEA/D	MOHEA-DE
1_100_30_1	3296	2217	2594	2788	63 274	**2120**	5037
1_200_30_1	7131	5969	4951	4543	71 091	**4057**	9650
1_300_30_1	12 409	11 281	7066	6073	71 895	**5817**	14 577
1_400_30_1	17 116	17 291	9480	7783	64 057	**7697**	19 781
1_500_30_1	23 685	24 981	11 801	**9476**	69 651	10 400	25 212
2_100_30_1	3052	2096	2452	2680	61 079	**2081**	5153
2_200_30_1	6514	5405	4777	4392	64 399	**3977**	10 123
2_300_30_1	10 982	10 126	7101	6114	70 953	**5899**	15 071
2_400_30_1	15 974	15 684	9415	7840	76 510	**7801**	19 658
2_500_30_1	21 499	22 316	11 692	**9557**	78 814	9707	24 994
3_100_30_1	3218	2197	2530	2816	56 448	**2086**	5376
3_200_30_1	6476	5425	4739	4363	66 162	**3948**	12 116
3_300_30_1	11 014	10 103	7140	6159	74 782	**5946**	14 660
3_400_30_1	16 556	16 293	9999	8352	77 561	**7855**	19 933
3_500_30_1	21 322	21 920	11 538	**9408**	77 303	9578	25 206
4_100_30_1	2947	1970	2328	2535	63 809	**1919**	4931
4_200_30_1	7087	5913	4662	4205	102 245	**3608**	9439
4_300_30_1	10 616	9722	6647	5643	67 579	**5407**	14 313
4_400_30_1	16 070	16 229	9019	**7471**	79 553	7538	19 389
4_500_30_1	23 604	23 439	11 668	9525	88 420	**9091**	24 977
5_100_30_1	3114	2129	2509	2730	57 413	**2109**	5218
5_200_30_1	6523	5403	4797	4490	75 847	**4360**	10 130
5_300_30_1	12 003	10 255	7174	6192	73 681	**5965**	15 189
5_400_30_1	15 997	15 758	9482	7920	73 658	**7883**	20 617
5_500_30_1	22 660	23 791	12 103	**10 268**	96 889	10 730	26 205
6_100_30_1	3359	2350	2747	2988	57 953	**2339**	5485
6_200_30_1	7212	6000	5411	5020	70 267	**4501**	10 626
6_300_30_1	11 703	10 668	7823	6802	72 593	**6568**	16 073
6_400_30_1	16 821	16 379	10 195	8622	76 806	**8600**	21 759
6_500_30_1	21 978	22 029	12 248	**10 199**	78 159	11 255	27 775
7_100_30_1	3344	**2366**	2779	3049	58 887	2409	5773
7_200_30_1	7096	5879	5405	5080	65 553	**4651**	11 136
7_300_30_1	12 134	10 765	8164	7201	73 351	**6954**	16 755
7_400_30_1	17 249	16 416	10 731	9201	79 974	**9174**	22 613
7_500_30_1	22 111	22 407	12 844	**10 951**	76 083	11 100	28 794

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