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Gaussian Process Based Modeling and Control of Affine Systems with Control Saturation Constraints

Shulong Zhao, Qipeng Wang, Jiayi Zheng, and Xiangke Wang*

Abstract: Model-based methods require an accurate dynamic model to design the controller. However, the hydraulic parameters of nonlinear systems, complex friction, or actuator dynamics make it challenging to obtain accurate models. In this case, using the input-output data of the system to learn a dynamic model is an alternative approach. Therefore, we propose a dynamic model based on the Gaussian process (GP) to construct systems with control constraints. Since GP provides a measure of model confidence, it can deal with uncertainty. Unfortunately, most GP-based literature considers model uncertainty but does not consider the effect of constraints on inputs in closed-loop systems. An auxiliary system is developed to deal with the influence of the saturation constraints of input. Meanwhile, we relax the nonsingular assumption of the control coefficients to construct the controller. Some numerical results verify the rationality of the proposed approach and compare it with similar methods.

Key words: Gaussian process (GP); auxiliary system; credibility; constraints input

1 Introduction

In recent years, how to better construct the model of the controlled system has been a hot issue. Relatively generic and simplified models can only be helpful at specific operating points, and modeling errors can lead to control performance degradation. However, relatively complex and accurate models may contain more varying forms and stricter conditions of applications, which could be more detrimental to the application of controllers.

The presence of uncertainty greatly increases the difficulty of modeling. Uncertainty in a closed-loop system can come from many sources, such as noise from the measurement sensor, random characteristics inside the system, interference from the external environment, perturbation of model parameters, etc. During controller designing, every uncertainty must be addressed in a targeted manner, which is very complex and unrealistic.

In the field of uncertain compensation or estimation, popular choices are neural networks (NN)[1–3], fuzzy systems[4, 5], or other computational intelligence methods, which need to predetermine the system structure and optimize the parameters. However, selecting a suitable procedure in advance and avoiding the risk of overfitting is the biggest challenge those methods face.

Gaussian process (GP) is a non-parametric modeling technique based on Bayesian inference[6]. GP regression provides an incremental joint probability distribution based on input-output data. To overcome the problem of overfitting, Ref. [7] proposed a novel method of recognition of nonlinear systems based on GP. In addition, a Matlab toolbox for GP-based system identification was presented in Ref. [8]. However, this work did not give a detailed solution to uncertainty. To this end, Gijsberts and Metta[9] introduced a method based on GP and established a high-fidelity flight dynamics model. To better compensate for the impact of uncertainty, the latest research is divided into the

• Shulong Zhao, Qipeng Wang, Jiayi Zheng, and Xiangke Wang are with the College of Intelligence Science and Technology, National University of Defense Technology, Changsha 410073, China. E-mail: jaymaths@nudt.edu.cn; xkwang@nudt.edu.cn.
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following four types.

(1) Traditional modeling method. The model structure is fixed, and there is a risk of overfitting. Through the combination of fuzzy logic and observer[10], a more effective feedback tracking control method is obtained[11].

(2) Based on the assumption of bounded modeling error[12], a robust control method ensures the stability of the closed-loop system. However, the inappropriate parameters introduced by these assumptions will cause the controller to be relatively conservative.

(3) Optimal control adapts the dynamic programming control output through online optimization by incorporating uncertainty into the constraints[13, 14]. However, the stability and convergence analysis of these methods are relatively lacking.

(4) Combined modeling and control[15–18]. The Bayesian process can be used to build a non-parametric model offline or online to approximate the real physical model.

It should be noted that the above studies only partially consider controller design under input constraints. In the research of robot control, the control Lyapunov function (CLF) is widely used to deal with control constraints[19]. The work of combining GP and CLF was first proposed in Ref. [20]. Then, in Ref. [21], a composite kernel was proposed to incorporate CLF into minimum norm optimization. Reference [22] proposed a CLF based on uncertainty and used the knowledge of model fidelity to avoid uncertain regions.

Inspired by Refs. [20–23], we use the GP to learn the uncertainty of the nonlinear system. Combining the characteristics of the variance function, we cleverly propose an auxiliary system to deal with input saturation constraints. Based on the above results and their limitations, contributions include:

(1) To our knowledge, it is the first time that a GP based adaptive controller for multiple-input multiple-output (MIMO) nonlinear systems with input saturation constraint is considered. A novel control law with an auxiliary system is introduced to handle the influence of input saturation effectively.

(2) The condition that control matrix $g_i(x)$ is invertible is relaxed by using the spectral radius of the control coefficient matrix. In contrast to Refs. [23, 24], it is no longer necessary to assume that the control matrix is strictly positive definite. Unlike Ref. [21], the direction (sign) of the control matrix does not need to be obtained in advance.

Notations: I_n represents the $n\times n$ identity matrix, $E(\cdot)$ stands for the expectation of the variable, and $\mathrm{cov}(\cdot)$ expresses the variance of the variable. The Gaussian density $\mathcal{N}(m(\cdot),k(x,x'))$ is represented by the mean $m(\cdot)$ and the covariance $k(x,x')$. Given a matrix $M\in\mathbb{R}^{m\times m}$ and a vector $v\in\mathbb{R}^m$, $\|M\|^2=\mathrm{tr}(M^{\mathrm{T}}M)$, $\|v\|^2=v^{\mathrm{T}}v$, and $v^c\triangleq\mathrm{diag}\{|v_1|^c,|v_2|^c,\ldots,|v_m|^c\}\mathrm{sign}(v)$, where $\mathrm{sign}(\cdot)$ represents the sign function. $M>0$ indicates M is a positive definite matrix and $\lambda_{\min}(M)$ denotes the minimum eigenvalue of M.

2 Problem Formulation

2.1 Gaussian process

Gaussian process (GP) is a prevalent method in machine learning. It is a stochastic process composed of the mean function $m(x):\mathbb{R}^n\mapsto\mathbb{R}$ and the covariance function $k(x,x'):\mathbb{R}^n\times\mathbb{R}^n\mapsto\mathbb{R}_0^+$[25]. A known function $f(x)$ described as a GP can be written as

$$f(x)\sim\mathrm{GP}(m(x),k(x,x')) \tag{1}$$

To show the process of GP regression, we assume a training dataset $\mathcal{D}=\{X,Y\}$ consisting of N inputs $X=\{x^{(1)},x^{(2)},\ldots,x^{(N)}\}$ and N outputs $Y=\{y^{(1)},y^{(2)},\ldots,y^{(N)}\}$, which consists of noisy observations $y^{(i)}=f(x^{(i)})+\varsigma$. To estimate the y^* at a given test input x^*, we can obtain that

$$\begin{pmatrix} Y \\ y^* \end{pmatrix}\sim\mathrm{GP}\left(\begin{pmatrix} m(X) \\ m(x^*) \end{pmatrix},\begin{pmatrix} K(X,X)+\sigma_N^2I_N & K(X,x^*) \\ K(x^*,X) & k(x^*,x^*) \end{pmatrix}\right) \tag{2}$$

where σ_N^2 denotes measurement noise, and K is the covariance matrix. The Bayesian formula is employed to get a posterior conditional probability distribution. Moreover, the posterior probability obeys the Gaussian distribution. The mean function and covariance function are defined as

$$\begin{aligned} m(y^*)&=m(x^*)+K(x^*,X)(K(X,X)+\sigma_N^2I_N)^{-1}(Y-m(X)),\\ \mathrm{cov}(y^*)&=k(x^*,x^*)-K(x^*,X)(K(X,X)+\sigma_N^2I_N)^{-1}K(X,x^*) \end{aligned} \tag{3}$$

Usually, $k(x,x')$ adopts square exponential function:

$$k(x,x'|\theta)=\nu\exp(-\tfrac{1}{2}(x-x')^{\mathrm{T}}\Omega^{-1}(x-x')) \tag{4}$$

where $\Omega^{-1}=\mathrm{diag}\{\omega_1,\omega_2,\ldots,\omega_N\}$, $\omega_i=\dfrac{1}{\lambda_i^2}$, and $\theta=\{\nu,\Omega,\sigma_N^2\}$ represents hyperparameters that need to be optimized by maximizing the log-likelihood function:

$$\log(p(Y|X,\theta))=-\frac{1}{2}Y^{\mathrm{T}}K^{-1}Y-\frac{1}{2}\log|K|-\frac{N}{2}\log(2\pi) \tag{5}$$

2.2 System model

In this section, we consider the nonlinear system:

$$\begin{aligned} &\dot{x}_i = f_i(\underline{x}_i) + (g_i(\underline{x}_i) + \Delta g_i(\underline{x}_i))x_{i+1} + d_i,\\ &\dots\\ &\dot{x}_n = f_n(\underline{x}_n) + (g_n(\underline{x}_n) + \Delta g_n(\underline{x}_n))\mathrm{sat}(u) + d_n,\\ &y = x_1 \end{aligned} \tag{6}$$

where $x_i \in \mathbb{R}^m, i = 1,2,\dots,n$ represents the state vectors and $\underline{x}_i = [x_1, x_2, \dots, x_i] \in \mathbb{R}^{m\times i}$; $f_i \in \mathbb{R}^m$ expresses unknown dynamics functions; $\mathrm{sat}(u) \in \mathbb{R}^m$ means the input vectors; $g_i \in \mathbb{R}^{m\times m}$ indicates known control matrices and $\Delta g_i \in \mathbb{R}^{m\times m}$ indicates unknown perturbations of control matrices; $d_i \in \mathbb{R}^m$ means unknown disturbances; and $y \in \mathbb{R}^m$ means output vector.

In system (6), we employ GPs to estimate the unknown dynamics f_i and external disturbances d_i. Further, the control input of the system satisfies the input saturation constraint, and the saturation $\mathrm{sat}(u_i)$ can be described as

$$\mathrm{sat}(u_i) = \begin{cases} \mathrm{sign}(u_i)u_{i\max}, & |u_i| \geqslant u_{i\max};\\ u_i, & |u_i| < u_{i\max} \end{cases} \tag{7}$$

where $u_{i\max}$ is a known constant. In the actual controller design process, it is possible that the ideal input u_i is larger than the practical input $\mathrm{sat}(u_i)(i = 1,2,\dots,m)$, and $\|\mathrm{sat}(u)\| \leqslant u_{\max}$. Then, there will be a certain deviation between the ideal input and the practical input. In fact, saturated nonlinearity can be approximated by some smoothing functions. As cited in Ref. [26], $\varphi(u) = [\varphi(u_1), \varphi(u_2), \dots, \varphi(u_m)]^{\mathrm{T}}$ and

$$\varphi(u_i) = u_{i\max}\tanh(\frac{u_i}{u_{i\max}}) = u_{i\max}\frac{\mathrm{e}^{\frac{u_i}{u_{i\max}}} - \mathrm{e}^{-\frac{u_i}{u_{i\max}}}}{\mathrm{e}^{\frac{u_i}{u_{i\max}}} + \mathrm{e}^{-\frac{u_i}{u_{i\max}}}} \tag{8}$$

Here, we define $\Delta u = \mathrm{sat}(u) - \varphi(u)$ and $\|\Delta u\| \leqslant \epsilon_m$.

Some necessary assumptions and lemmas are given first to facilitate the control law design.

Assumption 1 The unmodeled parts of the nonlinear system, f_i, have a bounded reproducing kernel Hilbert space (RKHS) norm B with a certain kernel (such as squared exponential (SE) kernel), $\|f_i\|_k^2 < B$.

Lemma 1 (Ref. [27], Theorem 6) A standard Gaussian process trains and learns the data of a certain system $\tilde{\Phi}_x$. The error between the prediction output $m(x)$ and the true value at point x is bounded by $\mathcal{P}\{\|m(x) - f(x)\| \leqslant \beta\Sigma(x|\mathcal{D}_N)\} \geqslant (1-\delta)$, for $x \in \mathcal{D}_N$ and a probability $\delta \in (0,1)$, where $\Sigma(x|\mathcal{D}_N)$ is the standard deviation at point x and β is defined as

$$\beta = \sqrt{2B + 300\gamma\log^3(\frac{N+1}{\delta})} \tag{9}$$

where γ represents the maximum information gain and

$$\gamma = \max_{x^1, x^2, \dots, x^{N+1} \in X} \frac{1}{2}\log|I_N + \sigma_N^{-2}K(x, x')| \tag{10}$$

There is a quasilinear relationship between the maximum information gain γ_j and the amount of training data. As cited in Ref. [27], γ_j can be approximated to a constant value with the training data increasing. Here, we keep the number of training sets constant.

Assumption 2 It is assumed that system (6) has a nominal model:

$$\dot{x}_i = \hat{f}_i(\underline{x}_i) + (g_i(\underline{x}_i) + \Delta g_i(\underline{x}_i))x_{i+1} + d_i \tag{11}$$

which can be used to generate suitable training data.

Assumption 3 For a known control matrix g_i, we do not require absolute invertibility of g_i. Instead, it is assumed that there is a constant $\varsigma_i > 0$ such that $\|g_i(\underline{x}_i)\| \leqslant \varsigma_i, \forall \underline{x}_i \in \Omega_i$ with compact subset Ω_i containing the origin.

Assumption 4 The desired state of the system is x_{1d}. We assume $\|x_{1d}^{(i)}\| \leqslant o_i$ for $o_i > 0$.

Assumption 5 For $i = 1,2,\dots,n$, there exist known constants $\iota_i \geqslant 0$ such that $\|\Delta g_i(\underline{x}_i)\| \leqslant \iota_i$.

Assumption 6 The deviation between the ideal control input and the practical control input is bounded, such that

$$\|\mathrm{sat}(u) - u\| \leqslant \mu \tag{12}$$

where $\mu > 0$ is an unknown constant.

Remark 1 Assumption 1 and Lemma 1 mean that the unknown nonlinear part of the system can be estimated by a GP, and the estimation error is bounded. Assumption 3 is reasonable for most nonlinear systems, and the boundedness of the known control matrix can be well guaranteed. Assumption 4 means that all derivatives of the desired state of the system exist and are bounded. Assumption 5 means that the perturbed part of the control matrix is also bound. For many practical systems, unbounded disturbances or unbounded control matrices are difficult to control. Assumption 6 means the expected control input is bound since the state errors are bounded in our controller design process (see Theorem 1). In addition, Assumption 6 is common in many works, such as Refs. [26, 28].

Remark 2 (Singularity problem) In Refs. [21−23, 29], the control matrix g_i is assumed to be invertible. However, this is very difficult for many practical nonlinear systems. Especially when there is a time-varying control matrix perturbation Δg_i, the invertibility of g_i is more difficult to ensure in advance. In this paper, we no longer require the control matrix g_i to be invertible, but only ensure that $\|g_i\|$ is bounded, which is very useful for most nonlinear systems.

Lemma 2 (Ref. [28]) For a square matrix $g \in \mathbb{R}^{m\times m}$ with spectral radius $\varrho(g)$, $\zeta > 0$ is a constant, such that $g + (\varrho(g) + \zeta)I_m$ is non-singular. $\varrho(g_i)$ satisfies $\varrho(g_i) \leqslant \rho_i$. Therefore, it can be directly obtained that $g_i(\underline{x}_i) + (\rho_i + \varpi_i)I_m$ are nonsingular for $\varpi_i > 0$.

3 Main Work

In this section, we employ an auxiliary system to handle input saturation. Combined with the excellent estimation performance of the GP, the adaptive control law of the MIMO system is finally obtained.

Step 1 For system (6), we select the error $e_1 = x_1 - x_{1d}$ and $e_2 = x_2 - \nu_1$, where $\nu_1 \in \mathbb{R}^m$ is the virtual control input, which will be defined later. Differentiating e_1 to obtain

$$\dot{e}_1 = f_1(x_1) + g_1(x_1)(e_2 + \nu_1) + \Delta g_1(x_1)x_2 + d_1 - \dot{x}_{1d} \tag{13}$$

Choose the candidate Lyapunov function

$$V_1^* = \frac{1}{2}e_1^{\mathrm{T}}e_1 \tag{14}$$

Differentiating it can obtain

$$\begin{aligned}\dot{V}_1^* =& e_1^{\mathrm{T}} f_1(x_1) + e_1^{\mathrm{T}} g_1(x_1)(e_2 + \nu_1) + \\ & e_1^{\mathrm{T}}\Delta g_1(x_1)x_2 + e_1^{\mathrm{T}} d_1 - e_1^{\mathrm{T}}\dot{x}_{1d} \leqslant \\ & e_1^{\mathrm{T}}\bar{f}_1(x_1) + e_1^{\mathrm{T}} g_1(x_1)(e_2 + \nu_1) + \\ & \iota_1\|e_1\|\|x_2\| - e_1^{\mathrm{T}}\dot{x}_{1d}\end{aligned} \tag{15}$$

where $\bar{f}_1(x_1) = f_1(x_1) + d_1$.

Design the virtual control input as

$$\nu_1 = [g_1(x_1) + (\rho_1 + \varpi_1)I_m]^{-1}(-K_1e_1 - \hat{\bar{f}}_1(x_1) + \dot{x}_{1d}) \tag{16}$$

where $K_1 \in \mathbb{R}^{m\times m}$ and $K_1 - \frac{1}{2}I_m > 0$, $\hat{\bar{f}}_1(x_1)$ is estimated by a Gaussian process.

Substituting Eq. (16) into Eq. (15) yields

$$\begin{aligned}\dot{V}_1^* \leqslant& e_1^{\mathrm{T}}\tilde{f}_1(x_1) + e_1^{\mathrm{T}}[g_1(x_1) + (\rho_1 + \varpi_1)I_m - \\ & (\rho_1 + \varpi_1)I_m](e_2 + \nu_1) + \iota_1\|e_1\|\|x_2\| - e_1^{\mathrm{T}}\dot{x}_{1d} \leqslant \\ & e_1^{\mathrm{T}}\tilde{f}_1(x_1) + e_1^{\mathrm{T}} g_1(x_1)e_2 - e_1^{\mathrm{T}}K_1e_1 + \\ & \iota_1\|e_1\|\|x_2\| - (\rho_1 + \varpi_1)e_1^{\mathrm{T}}\nu_1\end{aligned} \tag{17}$$

where $\tilde{f}_1(x_1) = \bar{f}_1(x_1) - \hat{\bar{f}}_1(x_1)$. The first term on the right side of Eq. (17) is the Gaussian estimation error term, the second will be eliminated in the iterative process, and the third term $-e_1^{\mathrm{T}}K_1e_1$ is negative definite. The remaining terms $\iota_1\|e_1\|\|x_2\| - (\rho_1 + \varpi_1)e_1^{\mathrm{T}}\nu_1$ will be subtly handled in the stability analysis.

Step *i* $(1 < i \leqslant n-1)$ We define the error as $e_{i+1} = x_{i+1} - \nu_i$, where $\nu_i \in \mathbb{R}^m$ is the virtual control input, which will be defined later. Considering system (6) and the derivation of e_i, we obtain

$$\dot{e}_i = f_i(\underline{x}_i) + g_i(\underline{x}_i)(e_{i+1} + \nu_i) + \Delta g_i(\underline{x}_i)x_{i+1} + d_i - \dot{\nu}_{i-1} \tag{18}$$

Choose the Lyapunov function candidate:

$$V_i^* = \frac{1}{2}e_i^{\mathrm{T}}e_i \tag{19}$$

The differential of V_i^* can be obtained:

$$\begin{aligned}\dot{V}_i^* =& e_i^{\mathrm{T}} f_i(\underline{x}_i) + e_i^{\mathrm{T}} g_i(\underline{x}_i)(e_{i+1} + \nu_i) + \\ & e_i^{\mathrm{T}}\Delta g_i(\underline{x}_i)x_{i+1} + e_i^{\mathrm{T}} d_i - e_i^{\mathrm{T}}\dot{\nu}_{i-1} \leqslant \\ & e_i^{\mathrm{T}}\bar{f}_i(\underline{x}_i) + e_i^{\mathrm{T}} g_i(\underline{x}_i)(e_{i+1} + \nu_i) + \\ & \iota_i\|e_i\|\|x_{i+1}\| - e_i^{\mathrm{T}}\dot{\nu}_{i-1}\end{aligned} \tag{20}$$

where $\bar{f}_i(\underline{x}_i) = f_i(\underline{x}_i) + d_i$.

The virtual control input is selected as

$$\begin{aligned}\nu_i =& [g_i(\underline{x}_i) + (\rho_i + \varpi_i)I_m]^{-1}\cdot \\ & (-g_{i-1}^{\mathrm{T}}(x_{i-1})e_{i-1} - K_ie_i - \hat{\bar{f}}_i(\underline{x}_i) + \dot{\nu}_{i-1})\end{aligned}$$

where $K_i \in \mathbb{R}^{m\times m}$ and $K_i - \frac{1}{2}I_m > 0$, and $\hat{\bar{f}}_i(\underline{x}_i)$ is estimated by a Gaussian process.

Similar with Eq. (16) and Formula (17), we obtain

$$\begin{aligned}\dot{V}_i^* \leqslant& e_i^{\mathrm{T}}\tilde{f}_i(\underline{x}_i) + e_i^{\mathrm{T}} g_i(\underline{x}_i)e_{i+1} - \\ & e_i^{\mathrm{T}} g_{i-1}^{\mathrm{T}}(x_{i-1})e_{i-1} - e_i^{\mathrm{T}}K_ie_i + \\ & \iota_i\|e_i\|\|x_{i+1}\| - (\rho_i + \varpi_i)e_i^{\mathrm{T}}\nu_i\end{aligned} \tag{21}$$

The augmented Lyapunov function candidate is

$$V_i = V_{i-1} + V_i^* \tag{22}$$

and the time derivative of V_i is given by

$$\begin{aligned}\dot{V}_i \leqslant& \sum_{j=1}^{i} e_j^{\mathrm{T}}\tilde{f}_j(\underline{x}_j) + e_i^{\mathrm{T}} g_i(\underline{x}_i)e_{i+1} - \\ & \sum_{j=1}^{i} e_j^{\mathrm{T}}K_je_j + \sum_{j=1}^{i} \iota_j\|e_j\|\|x_{j+1}\| - \\ & \sum_{j=1}^{i}(\rho_j + \varpi_j)e_j^{\mathrm{T}}\nu_j\end{aligned} \tag{23}$$

Step *n* Inspired by the design ideas of the auxiliary system in Ref. [24], we define $e_n = x_n - \nu_{n-1} - \eta$, η is an auxiliary signal, and we can obtain the time derivative of e_n as

$$\dot{e}_n = f_n(\underline{x}_n) + g_n(\underline{x}_n)\mathrm{sat}(u) + \Delta g_n(\underline{x}_n)\mathrm{sat}(u) + d_n - \dot{v}_{n-1} - \dot{\eta} \tag{24}$$

According to the previous form of the smooth function approximating the saturation function, it can be known that

$$\mathrm{sat}(u) = \varphi(u) + \Delta u \tag{25}$$

the auxiliary system is designed as

$$\dot{\eta} = [g_n(\underline{x}_n) + (\rho_n + \varpi_n)I_m](\varphi(u) - u) - k_a\eta \tag{26}$$

where $k_a > 0$.

From Eqs. (6), (24), and (26), we obtain

$$\dot{e}_n = \bar{f}_n(\underline{x}_n) + g_n(\underline{x}_n)\mathrm{sat}(u) + k_a\eta - [g_n(\underline{x}_n) + (\rho_n + \varpi_n)I_m](\varphi(u) - u) + \Delta g_n(\underline{x}_n)\mathrm{sat}(u) - \dot{v}_{n-1} \tag{27}$$

where $\bar{f}_n(\underline{x}_n) = f_n(\underline{x}_n) + d_n$.

The Lyapunov function is considered as

$$V_n^* = \frac{1}{2}e_n^{\mathrm{T}}e_n \tag{28}$$

The differentiation of V_n^* can be expanded as

$$\begin{aligned}\dot{V}_n^* = {} & e_n^{\mathrm{T}}\bar{f}_n(\underline{x}_n) - e_n^{\mathrm{T}}\dot{v}_{n-1} + k_a e_n^{\mathrm{T}}\eta + \\ & e_n^{\mathrm{T}}[g_n(\underline{x}_n) + (\rho_n + \varpi_n)I_m](\mathrm{sat}(u) - \varphi(u)) - \\ & (\rho_n + \varpi_n)e_n^{\mathrm{T}}I_m\mathrm{sat}(u) + e_n^{\mathrm{T}}\Delta g_n(\underline{x}_n)\mathrm{sat}(u) + \\ & e_n^{\mathrm{T}}[g_n(\underline{x}_n) + (\rho_n + \varpi_n)I_m]u\end{aligned} \tag{29}$$

Choose the virtual control law as

$$\begin{aligned}u = {} & [g_n(\underline{x}_n) + (\rho_n + \varpi_n)I_m]^{-1} \cdot \\ & (-g_{n-1}^{\mathrm{T}}(\underline{x}_{n-1})e_{n-1} - K_n e_n - \hat{\bar{f}}_n(\underline{x}_n) + \\ & \dot{v}_{n-1} - k_a\eta - \frac{e_n H(e_i, x_i, v_i)}{\vartheta^{\mathrm{T}}\vartheta + e_n^{\mathrm{T}}e_n})\end{aligned} \tag{30}$$

where $K_n \in \mathbb{R}^{m\times m}$ and $K_1 - \frac{1}{2}I_m > 0$, and

$$\begin{aligned}H(e_i, x_i, v_i) = {} & -(\rho_n + \varpi_n)e_n^{\mathrm{T}}I_m\mathrm{sat}(u) + \\ & \sum_{j=1}^{n-1}\iota_j\|e_j\|\|x_{j+1}\| + \iota_n\|e_n\|u_{\max} - \sum_{j=1}^{n-1}(\rho_j + \varpi_j)e_j^{\mathrm{T}}v_j + \\ & e_n^{\mathrm{T}}[g_n(\underline{x}_n) + (\rho_n + \varpi_n)I_m](\mathrm{sat}(u) - \varphi(u))\end{aligned} \tag{31}$$

and ϑ is an auxiliary system. The adaptive laws for ϑ are designed as

$$\dot{\vartheta} = \begin{cases} -\dfrac{\vartheta H(e_i, x_i, v_i)}{\vartheta^{\mathrm{T}}\vartheta + e_n^{\mathrm{T}}e_n} - k_b\vartheta, & \|e_n\| \geqslant \mu_n; \\ 0, \quad \|e_n\| < \mu_n \end{cases} \tag{32}$$

where $k_b > 0$ and $\mu_n > 0$ are constants.

Remark 3 The parameter μ_n needs to be cleverly designed. The actual control output is associated with the initial state of the system, error magnitude, etc. The role of μ_n here is: (1) The adaptive update law of ϑ will not have singular values with a denominator of 0. (2) To ensure that the control input is less than u_m when the tracking error e_n is less than μ_n. That is, there is no input saturation.

The following Theorem 1 will prove that if $\|e_n\| < \mu_n$, it will never escape from μ_n, thus the adaptive rate of θ will not cause buffeting.

To draw the controller design process and conclusions, we give the main results for the adaptive controller of system (6) in Theorem 1. The proof of Theorem 1 is given in the Appendix.

Theorem 1 Considering the nonlinear system (6) with input saturation constraint (7), unknown functions f_i and disturbances d_i are estimated by Gaussian processes. Under Assumptions 1−6, the control law (30), and parameter updated law (32), the closed-loop system is semi-globally stable with probability at least $(1-\delta)^n$ for all x_i. The following conclusions are obtained: (1) All of the states e_i, η, and ϑ are bounded; (2) The tracking error e_1 converges to a small region near the origin with probability at least $(1-\delta)^n$.

Remark 4 The main idea of the algorithm can be divided as three parts. First, GP models are studied to estimate the unknown dynamic functions f_i. Then, the backstepping technique is used in the design of control law while an invertible control matrix $g + (\varrho(g) + \zeta)I_m$ is put forward to address the singular problem. Finally, an auxiliary system (adaptive law) is designed to compensate for input saturation. The design process of the controller is shown in Fig. 1.

4 Validation

A numerical example of an MIMO nonlinear system with input saturation is implemented in this section. We demonstrate that GP can estimate the uncertainty of the system well. Meanwhile, an unmodeled dynamic compensation is gradually introduced, and the tracking error will gradually decrease. The tracking performance of the proposed method is compared with that of Ref. [24] as well.

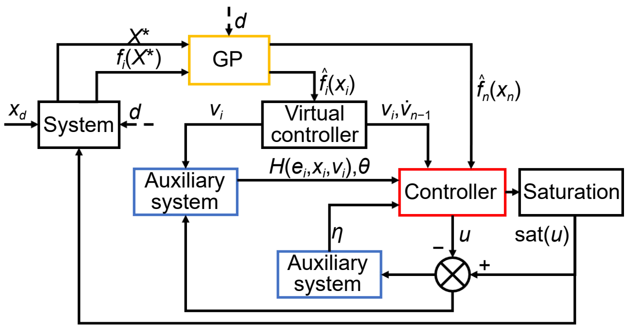

Fig. 1 Design process of the controller.

An MIMO nonlinear system with input saturation is considered:

$$\begin{aligned}\dot{x}_1 &= f_1(x_1)+(g_1(x_1)+\Delta g_1(x_1))x_2+d_1,\\ \dot{x}_2 &= f_2(\underline{x}_2)+(g_2(\underline{x}_2)+\Delta g_2(\underline{x}_2))\mathrm{sat}(u)+d_2,\\ y &= x_1\end{aligned} \tag{33}$$

where

$$x_1=[x_{11},x_{12}]^{\mathrm{T}},\qquad x_2=[x_{21},x_{22}]^{\mathrm{T}},$$

$$f_1(x_1)=\begin{bmatrix}0.2\sin(x_{11})\cos(x_{12})\\ 0.2x_{11}x_{12}\end{bmatrix},$$

$$g_1(x_1)=\begin{bmatrix}1.2+g_{11} & -2\\ 5 & 1.3+g_{12}\end{bmatrix},$$

$$g_{11}=\cos(x_{11})\sin(x_{12}),$$

$$g_{12}=-\cos(x_{12})\sin(x_{11}),$$

$$\Delta g_1(x_1)=\begin{bmatrix}0.2\sin(x_{11}) & 0\\ 0 & 0.1\cos(x_{12})\end{bmatrix},$$

$$f_2(\underline{x}_2)=\begin{bmatrix}-x_{12}x_{21}\\ 2x_{11}x_{22}\end{bmatrix},$$

$$g_2(\underline{x}_2)=\begin{bmatrix}0.2+g_{21} & 0.3-\sin(x_{22})\\ \sin(x_{22}) & -0.2+\cos(x_{21})\end{bmatrix},$$

$$g_{21}=\cos(x_{21})\sin(x_{22}),$$

$$\Delta g_2(\underline{x}_2)=\begin{bmatrix}0.12\cos(x_{11}x_{21}) & 0.11\cos(x_{11}x_{21})\\ 0.15\sin(x_{11}x_{21}) & 0.13\cos(x_{21}x_{22})\end{bmatrix},$$

$$d_1=\begin{bmatrix}0.21\cos^2(x_{12})+0.04\sin(0.3x_{12}t)\\ 0.12\sin^2(x_{11})+0.03\sin(0.2x_{11}t)\end{bmatrix},$$

$$d_2=\begin{bmatrix}0.13\sin^2(x_{21})+0.05\sin(0.2x_{21}t)\\ 0.11\cos^2(x_{22})+0.21\sin(0.3x_{22}t)\end{bmatrix}.$$

It is worth noting that g_2 is bounded rather than invertible. It will be a challenge for the design of control law in Ref. [24], where g_2 needs to be invertible.

To estimate the values of $\bar{f}_{11}$, $\bar{f}_{12}$, $\bar{f}_{21}$, and $\bar{f}_{22}$, 3600 training inputs are equally distributed on the set $\Omega_{x_{11}}\times\Omega_{x_{12}}=[-1,1]\times[-1.5,1.5]$, $\Omega_{x_{11}}\times\Omega_{x_{12}}=[-1,1]\times[-1.5,1.5]$, $\Omega_{x_{12}}\times\Omega_{x_{21}}=[-1.5,1.5]\times[-0.3,0.3]$, and $\Omega_{x_{11}}\times\Omega_{x_{22}}=[-1,1]\times[-0.5,0.5]$, respectively, since the desired trajectory is bound.

Figure 2 demonstrates the estimated errors of the GP model with respect to $\bar{f}_{11}$, $\bar{f}_{12}$, $\bar{f}_{21}$, and $\bar{f}_{22}$. It can be noted that all errors are bounded in the simulation duration. It guarantees that the tracking error will exponentially converge to the region calculated in Formula (A4), which will be shown in the Appendix.

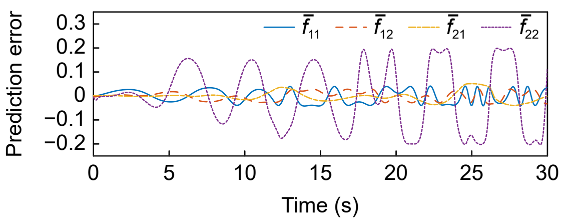

Fig. 2 Prediction errors of GP model.

In the simulation, the reference trajectory is described as

$$x_{11d}=0.5[\sin(1.5t)+\sin(0.5t)],$$

$$x_{12d}=0.8\sin(t)+0.5\sin(0.5t).$$

According to the previous control design (16), (26), and (30)−(32), the proposed controllers are designed as

$$v_1=[g_1+(\rho_1+\varpi_1)I_2]^{-1}(-K_1e_1-\hat{\bar{f}}_1+\dot{x}_{1d}),$$

$$\dot{\eta}=-k_a\eta+[g_2+(\rho_2+\varpi_2)I_2](\varphi(u)-u),$$

$$\begin{aligned}H=&-(\rho_2+\varpi_2)e_2^{\mathrm{T}}I_2\mathrm{sat}(u)+\iota_1\|e_1\|\|x_2\|+\\ &\iota_2\|e_2\|u_{\max}-(\rho_1+\varpi_1)e_1^{\mathrm{T}}v_1+\\ &e_2^{\mathrm{T}}[g_2+(\rho_2+\varpi_2)I_2](\mathrm{sat}(u)-\varphi(u)),\end{aligned}$$

$$\dot{\vartheta}=\begin{cases}-\dfrac{\vartheta H}{\vartheta^{\mathrm{T}}\vartheta+e_2^{\mathrm{T}}e_2}-k_b\vartheta, \|e_2\|\geqslant 0.1;\\ 0, \|e_2\|<0.1;\end{cases}$$

$$\begin{aligned}u=&[g_2+(\rho_2+\varpi_2)I_2]^{-1}(-g_1^{\mathrm{T}}e_1-K_2e_2-\hat{\bar{f}}_2+\\ &\dot{v}_1-k_a\eta-\frac{e_2H}{\vartheta^{\mathrm{T}}\vartheta+e_2^{\mathrm{T}}e_2}).\end{aligned}$$

The parameter values are set to $u_{1\max}=1.5, u_{2\max}=2$, $k_a=1$, $k_b=0.1$, $K_1=\mathrm{diag}\{5,10\}$, $K_2=\mathrm{diag}\{10,5\}$, $\rho_1=0$, $\rho_2=0.1$, $\varpi_1=\varpi_2=0$, $\iota_1=\iota_2=0.2$, $x_{11}(0)=1$, and $x_{12}(0)=1$.

Though the parameters of the proposed method are well designed in advance, there is some guidance on selecting the parameters. First, choosing two adaptive gains, k_a and k_b, is crucial. Too large a parameter will cause the control law to oscillate, and too small a parameter will increase the convergence time. Second, the smaller the ϖ_i and μ_n, the smaller the system error.

At the same time, the controllers in Ref. [24] are designed as

$$\bar{\alpha}_1=\kappa_1e_1^{\frac{1}{2}}+c_1e_1^3+e_1+\frac{\hat{\sigma}e_1\Psi_1^{\mathrm{T}}\Psi_1}{2a_1^2},$$

$$\begin{aligned}\bar{\alpha}_2=&\kappa_2e_2^{\frac{1}{2}}+c_2e_2^3+(I_2+\frac{1}{2}g_2^2)e_2+g_1^{\mathrm{T}}e_1+\eta+\\ &\frac{\hat{\sigma}e_2\Psi_2^{\mathrm{T}}\Psi_2}{2a_2^2},\end{aligned}$$

$$\dot{\eta}=-\eta+g_2(\varphi(u)-u),$$

$$\dot{\hat{\sigma}}=\lambda\tau_2-r_1\hat{\sigma}-\frac{r_2}{\lambda}\hat{\sigma}^3,$$

$$\tau_2=\left(\|e_1\|^2\Psi_1^{\mathrm{T}}\Psi_1/2a_1^2\right)+\left(\|e_2\|^2\Psi_2^{\mathrm{T}}\Psi_2/2a_2^2\right),$$

$$\alpha_i=-g_i^{-1}\frac{e_i\|\bar{\alpha}_i\|^2}{\sqrt{\|e_i\|^2\|\bar{\alpha}_i\|^2+l_i^2}},\quad i=1,2,$$

$$u=\alpha_2,$$

where Ψ_1 and Ψ_2 are calculated via neural network.

The parameters are set to $u_{1\max} = 1.5, u_{2\max} = 2$, $c_1 = c_2 = 1, a_1 = a_2 = 40, \kappa_1 = \text{diag}\{15, 15\}$, $\kappa_2 = \text{diag}\{2, 2\}$, $l_1 = l_2 = 0.35, r_1 = r_2 = 0.1, \varpi_1 = \varpi_2 = 0, \lambda = 0.01, x_{11}(0) = 1$, and $x_{12}(0) = 1$.

Figure 3 shows the tracking performance of proposed method (solid line) compared with that in Ref. [24] (dashed line). It is obvious that the tracking errors in the proposed method are minor. It is noted that the parameters in Ref. [24] are selected carefully, otherwise g_2 will be singular and the controller in Ref. [24] will be out of work.

Figures 4 and 5 display the desired control signals u (dashed line) and saturation inputs sat(u) (solid line) in the proposed method and the method in Ref. [24], respectively. The saturation of control inputs is presented in the simulation duration and the performance of the proposed method is smoother.

5 Conclusion

An improved Gaussian process for modeling and control of an affine system with saturation input is proposed in this article as a novel concept. The presented model is nonparametric and can be dynamically optimized globally. The Gaussian process naturally has the advantage of dealing with noise and uncertainty. Through specific compensation for saturation inputs, the tracking error is reduced and asymptotically converges to a small neighborhood. With the help of an auxiliary design system, the adaptive control method is proposed to improve control performance. The proposed approach can also be useful when the system has unmodeled dynamics.

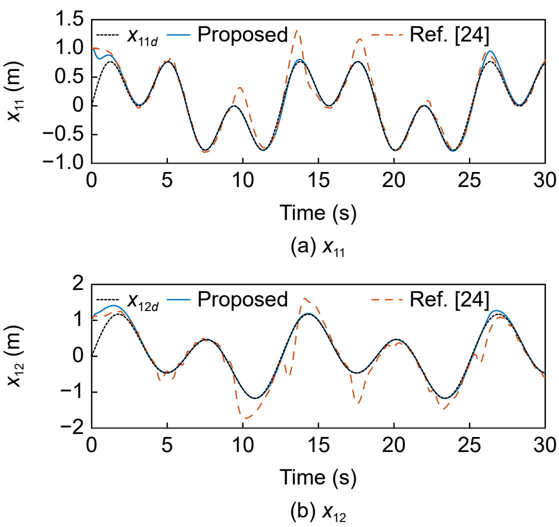

Fig. 3 Outputs x_{11} and x_{12} in proposed method and Ref. [24].

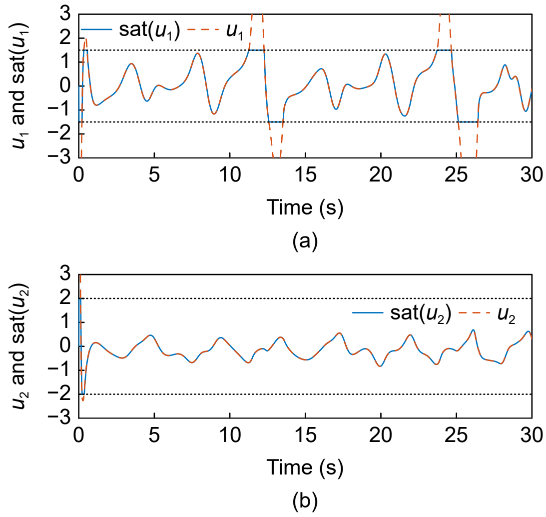

Fig. 4 Desired control signals *u* (dashed line) and saturation inputs sat(*u*) (solid line) in proposed method.

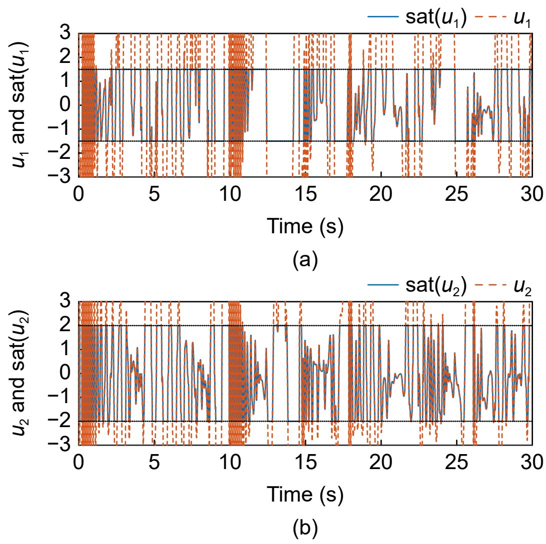

Fig. 5 Desired control signals *u* (dashed line) and saturation inputs sat(*u*) (solid line) in the method in Ref. [24].

Appendix

Proof of Theorem 1

Proof For system (6), if there exists input saturation constraint, and $\|e_n\| \geqslant \mu_n$. The augmented Lyapunov function candidate

$$V_n = V_{n-1} + V_n^* + \frac{1}{2}\vartheta^{\mathrm{T}}\vartheta \tag{A1}$$

The time derivative of V_n is

$$\dot{V}_n = \dot{V}_{n-1} + \dot{V}_n^* + \vartheta^{\mathrm{T}}\dot{\vartheta} \tag{A2}$$

Substituting Eq. (30) into the time derivative of V_n, we obtain

$$\dot{V}_n \leqslant \sum_{j=1}^{n} e_j^{\mathrm{T}} \tilde{f}_j(\underline{x}_j) - (\rho_n + \varpi_n) e_n^{\mathrm{T}} I_m \mathrm{sat}(u) - \sum_{j=1}^{n} e_j^{\mathrm{T}} K_j e_j +$$

$$\sum_{j=1}^{n-1} \iota_j \|e_j\| \|x_{j+1}\| - \sum_{j=1}^{n-1} (\rho_j + \varpi_j) e_j^{\mathrm{T}} \nu_j + e_n^{\mathrm{T}} \cdot$$

$$[g_n(\underline{x}_n) + (\rho_n + \varpi_n) I_m](\mathrm{sat}(u) - \varphi(u)) + e_n^{\mathrm{T}} \Delta g_n(\underline{x}_n) \mathrm{sat}(u) -$$

$$e_n^{\mathrm{T}} \frac{e_n H(e_i, x_i, \nu_i)}{\vartheta^{\mathrm{T}} \vartheta + e_n^{\mathrm{T}} e_n} + \vartheta^{\mathrm{T}} \dot{\vartheta} \leqslant -\sum_{j=1}^{n} e_j^{\mathrm{T}} (K_i - \frac{1}{2} I_m) e_j +$$

$$\frac{1}{2} \sum_{j=1}^{n} \tilde{f}_j^{\mathrm{T}}(\underline{x}_j) \tilde{f}_j(\underline{x}_j) - \frac{e_n^{\mathrm{T}} e_n H(e_i, x_i, \nu_i)}{\vartheta^{\mathrm{T}} \vartheta + e_n^{\mathrm{T}} e_n} + \vartheta^{\mathrm{T}} \dot{\vartheta} +$$

$$\sum_{j=1}^{n-1} \iota_j \|e_j\| \|x_{j+1}\| + \iota_n^2 \|e_n\| u_{\max} - (\rho_n + \varpi_n) e_n^{\mathrm{T}} I_m \mathrm{sat}(u) -$$

$$\sum_{j=1}^{n-1} (\rho_j + \varpi_j) e_j^{\mathrm{T}} \nu_j + e_n^{\mathrm{T}} [g_n(\underline{x}_n) + (\rho_n + \varpi_n) I_m] \cdot$$

$$(\mathrm{sat}(u) - \varphi(u)) = -\sum_{j=1}^{n} e_j^{\mathrm{T}} (K_i - \frac{1}{2} I_m) e_j +$$

$$\frac{1}{2} \sum_{j=1}^{n} \tilde{f}_j^{\mathrm{T}}(\underline{x}_j) \tilde{f}_j(\underline{x}_j) + \frac{\vartheta^{\mathrm{T}} \vartheta H(e_i, x_i, \nu_i)}{\vartheta^{\mathrm{T}} \vartheta + e_n^{\mathrm{T}} e_n} + \vartheta^{\mathrm{T}} \dot{\vartheta} =$$

$$-\sum_{j=1}^{n} e_j^{\mathrm{T}} (K_i - \frac{1}{2} I_m) e_j - k_b \vartheta^{\mathrm{T}} \vartheta + \frac{1}{2} \sum_{j=1}^{n} \tilde{f}_j^{\mathrm{T}}(\underline{x}_j) \tilde{f}_j(\underline{x}_j) \leqslant$$

$$-2\chi_\vartheta V_n + C_\vartheta \tag{A3}$$

where $\chi_\vartheta = \min\{\lambda_{\min}(K_i - \frac{1}{2} I_m), k_b\}$ $(i = 1, 2, \ldots, n)$, $C_\vartheta = \frac{1}{2} \sum_{j=1}^{n} \tilde{f}_j^{\mathrm{T}}(\underline{x}_j) \tilde{f}_j(\underline{x}_j)$.

Invoking Lemma 1, we also have that C_ϑ is bounded with probability. The tracking error converges to the region that

$$\|e_1\| \leqslant \sqrt{\frac{C_\vartheta}{\chi_\vartheta} + (2V_n(0) - \frac{C_\vartheta}{\chi_\vartheta}) \mathrm{e}^{-2\chi_\vartheta t}} \tag{A4}$$

with probability at least $(1-\delta)^n$.

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Shulong Zhao received the BS degree from Beihang University (BUAA), China in 2011, and the MS and PhD degrees from National University of Defense Technology (NUDT), China in 2014 and 2017, respectively. Currently he is an associate professor at the College of Intelligence Science and Technology, National University of Defense Technology. His research interests include data driven control and curved path following of UAV.

Qipeng Wang received the BS degree from Chinese People’s Liberation Army Aviation Institute, China in 2019 and the MS degree from National University of Defense Technology, China in 2022. He is currently pursuing the PhD degree at the College of Intelligence Science and Technology, National University of Defense Technology. His research interests include nonlinear system control and cooperative control of distributed multi-agent systems.

Jiayi Zheng received the BS degree from Harbin Engineering University, China in 2022. She is currently pursuing the MS degree at the College of Intelligence Science and Technology, National University of Defense Technology. Her research interests include nonlinear system control and cooperative control of distributed multi-agent systems.

Xiangke Wang received the BS, MS, and PhD degrees from National University of Defense Technology, China, in 2004, 2006, and 2012, respectively. He was a visiting student with Australian National University, Canberra, ACT, Australia, from 2009 to 2011. Since 2012, he has been with the College of Intelligence Science and Technology, National University of Defense Technology, where he is currently a professor. His research interests include multi-agent systems and nonlinear control.