Learning with Missing Data

1 st Carlos A. Escobar *Global Research & Development General Motors* Warren, MI, USA carlos.1.escobar@gm.com

4 th Ruben Morales-Menendez

Escuela de Ingenieria y Ciencias Tecnologico de Monterrey Monterrey, NL, Mexico rmm@tec.mx

Abstract—Many real-world data sets contain missing values, therefore, learning with incomplete data sets is a common challenge faced by data scientists. Handling them in an intelligent way is important to develop robust data models, since there is no perfect approach to compensate for the missing values. Deleting the rows with empty cells is a commonly used approach, this *naive method* may lead to estimates with larger standard errors due to reduced sample size. On the other hand, imputing the missing records is a better approach, but it should be used with great caution, as it relies on often unrealistic specific assumptions which can potentially bias results. In this paper, a new greedy-like algorithm is proposed to maximize the number of records. The algorithm can be used to generate various maximized sub-sets by varying the number of columns (features) that can be used for learning. It salvages more records than the *naive method*, and it avoids the bias induced by imputation. The learning algorithms would be able to learn from real sub-sets without the bias induced by artificial data. Finally, the proposed algorithm is applied to a case study, the COVID-19 Open Research data set (CORD-19) that was prepared and posted by The White House and a coalition of leading research groups as a call to action to the world's artificial intelligence experts to answer high priority scientific questions. This data set contains missing records, therefore, resulting maximized sub-sets from this analysis can be further investigated by the research community.

Index Terms—machine learning, incomplete data, preprocessing, manufacturing

I. INTRODUCTION

Many real-world data sets contain missing values for various reasons, such as, corrupt data, failure to load the information, or incomplete recording of data entries. For example, in manufacturing, the unexpected faulty operating-conditions of sensors and cameras are the main causes of incomplete data sets, asi seen in Fig. 1. Handling the missing values, which are often encoded as NaN (Not a Number) or blanks, is one of the first relevant challenges faced by data scientists. Missing data results in loss of learning power, therefore, selecting the right approach on how to handle it, is very important to develop robust models.

Deleting rows or columns with missing data is a widely used approach. However, as it will be illustrated, if this *naive*

2nd Jorge Arinez *Global Research & Development General Motors* Warren, MI, USA jorge.arinez@gm.com

3 rd Daniela Macias *Escuela de Ingenieria y Ciencias Tecnologico de Monterrey* Monterrey, NL, Mexico a01195037@itesm.mx

Fig. 1. Incomplete data set derived from a real manufacturing process.

method is applied, oftentimes the sample size may be significantly reduced, leaving little information for the algorithm to learn the patterns. This method is not advised unless the proportion of eliminated records is very small $(<5\%)$ [1].

Imputing the missing records is a better approach, this statistical technique refers to the process of replacing missing data with guessed/estimated values. Before selecting the imputation method, the data set should be thoroughly assessed, to determine the most appropriate statistical method to handle missing data. Although imputation can be applied to preserve all samples, it relies on often unrealistic specific assumptions, which can potentially bias results. Therefore, this approach should be used with great caution. A review of imputation methods can be found in [1]–[3].

Learning with incomplete data sets requires careful planning

and attention, since there is no perfect way to compensate for the missing values. Whereas removing the data will lead to estimates with larger standard errors due to reduced sample size, imputation can potentially bias results.

In this paper, a new greedy-like algorithm is proposed, it maximizes the number of total records that can be used to learn. As it will be demonstrated, the algorithm salvages more records than the *naive method*, and may help to reduce the bias induced by imputation, as the algorithms would be able to effectively learn from real data without the biased induced by artificially created data.

The rest of this paper is organized as follows. The proposed algorithm is presented in Section III. Followed by two real case studies in Section IV, one of them contributes in the pandemic, by formally analyzing the COVID-19 Open Research data set (CORD-19) that was posted by The White House as a call to action to the world's artificial intelligence experts to answers high priority scientific questions. Finally, conclusions and future research are presented in Section V.

II. STATE OF THE ART

In the current state of the art, handling missing values has been tested thoroughly. There is an existing concept, called missing data mechanism, where there is a probability for each data point to be missing [1]. That probability is governed by three categories, which are:

- Missing completely at random (MCAR) Where the probability of being missing is the same for all the data points in the set. This is due to unforeseen circumstances that are not related to a problem from the data.
- Missing at random (MAR) When the data is missing at random, the probability is caused by a group defined by the observed data. A circumstance that may cause missing data could vary from one condition to another. When those conditions are known, and the data is collected knowing the difference between those conditions, then the data will be MAR. This is a more realistic approach than MCAR, and may be the starting point for analysis.
- Not missing at random (NMAR) When there is a known cause to be missing data, which could be a situation or condition, then the data will be NMAR. The obtained data will need more in-depth analysis to handle and fix the origin of those missing values.

There is no distinction between MCAR, MAR or MNAR approaches. Practically, MCAR is unrealistic and the least plausible of the three, leaving MAR or MNAR mechanisms. For practical issues, modern missing data methods use the MAR assumption as the standard for analyzing the data.

Imputation is one of the most widely used methods for dealing with missing values; assuming the NMAR approach, but not knowing the data, could lead to use it in the wrong way. As imputation mechanisms, obtaining mean and mode of the existing data, among other old techniques, and replacing it in the NaN values is proven to be ineffective, because imputation has to be adequate and follow the pattern of the NaN values [4]. Hot deck imputation has also shown inadequate results due to a high error probability caused by multiple missing values in the same row.

To avoid any possible bias induced by imputation methods, authors recommend -when possible- to apply the proposed algorithm to maximize the number of records used for learning. Thus, the learning algorithm will learn only real patterns. However, imputation methods should be evaluated and selected as part of the modeling solution to ensure estimates are readily available when needed, to keep the predictive systems running.

III. INFORMATION OPTIMIZATION ALGORITHM

The proposed algorithm is based on a greedy-like selection, algorithms in this category make whatever choice seems the best at each iteration. The pool of options depends on the choices made so far, but not on the future ones. It iteratively makes one greedy choice after another, reducing the problem into smaller subproblems. A greedy algorithm is not subject to combinatorial explosion, as it never reconsiders its choices, therefore, there is no guarantee of finding an optimal solution. The basic idea of the proposed algorithm, is to maximize the number of total records that can be used to learn. This problem is solved iteratively by a two-step approach that considers the number of empty (NaN) cells. First, the column(s) with the minimum number of NaN is selected, if there are two or more columns with the same number of NaNs, the one with the maximum number of associated NaNs is selected. Since the rows of the NaNs in the selected column will be deleted, in this second step the algorithm is selecting the column that will minimize the deletion of useful information. The algorithm has three components, inputs, outputs and initialization. The pseudo-code is presented in Fig. 2:

• Inputs

-*X*, matrix with m rows (samples) and n columns (features).

• Outputs

-*Order*, vector with the final order of the columns in the solution.

-*RowsSalvagedbyCol (RSC)*, vector with the number of rows included in the solution by column. Either constant or decreasing.

-*PercentofRowsSalvagedbyCol(PRSC)*, vector with the percentage of *RowsSalvagedbyCol* relative to m.

-*CumulativeRecordsSalvagedbyCol(CRSC)*, vector with the number of records (cells) by column.

-*PercentofCumulativeRecordsSalvagedbyCol(PCRSC)*, vector with the percentage of *CumulativeRecordsSalvagedbyCol* relative to the matrix size $(m \times n)$.

-*TotalRecordsSalvaged(TRS)*, number that describes the

total number of records salvaged by the solution (*sum of RowsSalvagedbyCol*).

-*PercentofTotalRecordsSalvaged(PTRS)*, *Total-RecordsSalvaged* expressed in percentage relative to the matrix size $(m \times n)$. -*PermutedMatrix*, matrix sorted based on *Order*.

- Initialization -Define nc as 1 to n vector.
	- -Define Remaining as 1 to n vector.
	- -Define Tracker as 1 to n vector.
	- -Define SelectedF as an empty vector of size n.
	- -Define RSC as an empty vector of size n .
	- -Define PRSC as an empty vector of size n .
	- -Define CRSC as an empty vector of size n.
	- -Define PCRSC as an empty vector of size n .
	- -Define TRS as an empty vector of size n .
	- -Define PTRS as an empty vector of size n .

$\mathbf{1}$.	$i = 1$
2.	$[m,n]=size(X)$
3.	$for i = i:n$
4.	Define SumEmptyinRowbyCol as an empty vector of size $n - i$
5.	NumofEmptyinCol as an empty vector of size $n - i$
6.	SumDeletesbyCol as an empty vector of size $n - i$
7.	for $il = l:n$
8.	$SumEmptyByRow = 0;$
9.	$EmptyinCol = find(isnan(X(:,ii)))$;
10.	for iii = 1:numel(EmptyinCol);
11.	$EmptyRow = sum(isnan(X(EmptyInCol(iii),:)))$
12.	$SumEmptyRow = SumEmptybyRow + EmptybyRow;$
13.	end
14.	$SumEmpty inRowbyCol(ii - i + 1) = SumEmptybyRow;$
15.	$Num of EmptvinCol(ii - i + 1) = numel(EmptvinCol);$
16.	$SumDelete=sbyCol(ii-i+1) = NumofEmptyinCol(ii-i+1)*n = SumEmptyinRowbyCol(ii-i+1);$
	17. end
18.	$PreSelCol = find(min(NumofEmptyinCol) == NumofEmptyinCol);$
19.	if $numberFe1Co1 > 1$
20.	$Selected = PreSelCol(min(find(max(SumEmptylinRowbyCol(PreSelCol))) == SumEmptylinRowbyCol(PreSelCol)))$;
21.	else
22.	$Selected = PreSelCol;$
23.	end
24.	$SelectedF(i) = Remaining(Selected);$
25.	$Remaining = setdiff(nc, SelectedF);$
26.	$UpdateOrder = [SelectedF(1:i), Remaining]$;
27.	$[member, index] = is member (UpdateOrder, Tracker);$
28.	$X = X(:,index);$
29.	$Tracker = UpdatedOrder;$
30.	$Row2Del = find(isnan(X(:, i)))$;
31.	$X(Row2Del,:) = []$
32.	$[mxwip, nxwip] = size(X);$
33.	$Fills = NaN(m - maxwip, 1);$
34.	$Xwip(:, i) = [X(:, i); Fills];$
35.	$RowsSalvagedbyCol(i) = m x wip;$
36.	$Percent of RowsSalvaged byCol(i) = RowsSalvagedbyCol(i)/m;$
37.	$Cumulative Records SalvadorCol(i) = i * RowsSalvagedbyCol(i);$
38.	$Percent of Cumulative Records Salvador (i) = Cumulative RecordsS always (i)/(m*n);$
	39. end
40.	$Total Records Salvador = sum(RowsSalvagedbyCol);$
41.	$Percent of Total Records Salvador = Total Records Salvador 1(m*n):$
42.	$Order = UpdatedOrder;$
	43. $PermutedMatrix = Xwip;$

Fig. 2. Records optimizer pseudo-code.

Lines 1-2 define initial values used by the algorithm. In lines 3-39, n number of iterations are performed to develop the solution, where n is the number of columns. In lines 4-6, the vectors used in this process are defined, since the number of remaining columns decreases by one after each iteration, their size also decreases by one. In lines 7-17, the information required to select the next column is generated. In lines 8-9, the variable that stores the sum of empties by row is reset and the positions of the NaNs by column are identified. In lines 10- 13, these positions, are then used to determine the associated NaNs by row (line 11). The sum of these values is stored in line 12. The vectors defined in lines 4-6, are populated in lines 14-16. In line 18, the column with the minimum number of NaNs is selected. If there are two or more columns with the same number of NaNs, the column with the maximum number of associated NaNs by row is selected, this can be observed in lines 18-23. Line 24 populates the vector with the solution. Line 25 keeps track of the remaining columns. Since the selected columns are moved to the left, their order needs to be updated, stated in line 26. In lines 27-28, the original matrix X is rearranged, based on the updated order. Line 29 helps to keep track of the original column number in each iteration. In lines 30-31, the rows of the empty cells in the selected column are deleted. In line 32, the size of the matrix is recorded. In lines 33-34, the $PermutedMatrix$ is generated; the empty records of each preprocessed column are filled with NaN. In lines 35-43 statistics are reported and, at the end of the iterations, they store the information of the solution. Where *Order* contains the order of the matrix and RowsSalvagedbyCol contains the number of associated rows, this number exhibits a constant or a decreasing pattern. Finally, $PermutedMatrix$ shows the overall solution.

A. Solution Process Overview

A virtual data set is created to show the solution step by step. The data set contains 11 columns and 10 rows with empty cells in the diagonal of columns 1-10, Table I. In this case, if rows with empty cells are eliminated, all the data set is deleted, if columns with empty cells are deleted only one column is salvaged (11), leaving very little information for the algorithm to learn from. Besides the data set, the table has two more rows at the bottom: (1) *Empty by column (EBC)*, which describes the number of empty cells in each column, and (2) *Associated empties (AE)*, which describes the sum of the row's associated empty cells to each of the empty cells in a column. Moreover, another column is added at the end which describes the number of empty cells by row, this information is used to determine the values of *AE*. A color convention is also included to keep track of the decisions and actions of the algorithm. The feasible options (i.e., minimum number of *EBC*) at each step are highlighted in gray, if there is a tie, the *AE* values (maximum) are used as tiebreaker to maximize the information extraction. If still, there are two or more columns with the same numbers, the smallest index is arbitrarily selected. The green color keeps track of the selected columns. Finally, based on the empty cells of the selected column, the yellow color is used to highlight which rows will be deleted. The full solution process is graphically described in Table II, the solutions are summarized in Table III and Table IV shows the $PermutedMatrix$. The solutions are interpreted as follows: in iteration one, column 11 is selected, which includes include 10 rows, the updated order column keeps track of selected columns (in bold) in each iteration, as well as remaining columns.

In iteration 1 (Table I), column 11 is selected since the *EBC* value is the smallest one (0), which means all the records in that column are available. In iteration 2 (upper left, Table II), there is a tie between columns eight and five, as both have a $EBC = 1$, however, column eight has three associated empty cells $(AE = 3)$ vs two $(AE = 2)$ of column five. In this situation, it is better to remove row eight (in yellow) to salvage more records. The remaining steps follow the same logic. Figure 3 shows how the number of records decreases (rows

TABLE I VIRTUAL DATA SET.

3 9 Index 2 8 10 11 Empty by row (EBR) 1 4 5 6	
3 NaN NaN NaN 1 1 1 1 1	
2 NaN $\overline{2}$ \overline{a} 2 2 $\overline{2}$ $\overline{2}$ 2 2 2 1	
3 3 3 3 2 3 3 3 3 3 NaN NaN	
3 4 4 NaN 4 NaN 4 4 NaN 4 4 4	
5 5 5 5 5 3 NaN NaN 5 NaN 5 5	
Samples 6 3 6 6 6 6 6 NaN 6 NaN 6 NaN	
7 7 7 NaN NaN NaN 7 7 7 NaN 7 4	
8 NaN 8 NaN NaN 8 8 8 8 4 8 NaN	
9 $\overline{2}$ 9 9 ٩ 9 ٩ 9 NaN ٩ 9 NaN	
10 10 10 10 10 10 10 10 NaN 10 10 1	
3 2 5 26 Empty by column (EBC) 2 3 3 $\mathbf 0$ 4 2 1	
7 $\overline{2}$ 9 2 9 8 2 5 5 3 0 Associated empties (AE)	

TABLE II SOLUTION PROCESS (ITERATIONS FROM LEFT TO RIGHT).

salvaged) by including more columns, whereas the number of cumulative records increases and eventually decline. In this case, if all columns are included, no rows are salvaged.

This algorithm addresses the tradeoff between the number of columns (features) and rows (samples). In real life problems, this is a common situation, as an example, medical files of patients tend to have a lot of missing records, or in manufacturing, as described earlier, sensors may randomly break down or may be systematically turned off. Whereas this algorithm helps to maximize the information extraction, the main question remains: which sub-data set should be used to

learn? In this context, this algorithm cannot straight forward answer this question, instead, it is a guide on how to cleverly define sub-data sets to explore. For example, in this data set, two useful insights can be derived, per Table III, the subdata set generated in iteration six is a better solution than the sub-data set generated in iteration five, as both have the same number of rows but the former has more features (6 vs 5) or cumulative records (30 vs 25). Finally, if all features are included, there are no samples left to learn from. In real life problems, this situation refers to the columns on the extreme right, that oftentimes have very little information, making the

TABLE III SOLUTIONS.

Iteration	Selected column	Rows salvaged	Columns included	Cumulative cells	Updated order
		10		10	11, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10
				18	11,8,1,2,3,4,5,6,7,9,10
				24	11,8,5,1,2,3,4,6,7,9,10
				28	11,8,5,2,1,3,4,6,7,9,10
				25	11,8,5,2,4,1,3,6,7,9,10
h			6	30	11,8,5,2,4,7,1,3,6,9,10
				28	11,8,5,2,4,7,1,3,6,9,10
				16	11,8,5,2,4,7,1,3,6,9,10
	6			9	11,8,5,2,4,7,1,3,6,9,10
10	9		10	10	11,8,5,2,4,7,1,3,6,9,10
11	10		11	0	11,8,5,2,4,7,1,3,6,9,10

TABLE IV PermutedMatrix, OVERALL SOLUTION.

data set ineffective, as illustrated in the following section.

Fig. 3. Salvaged rows and cumulative cells by columns.

IV. CASE STUDY

In this section, two public data sets are analyzed, they are particular relevant because they were not preprocessed before being posted, therefore they have a lot of missing records: (1) COVID-19 Open Research Data Set [5], this data set calls to action to the world's artificial intelligence experts to develop text and data mining tools that can help the medical community develop answers to high priority scientific questions, in this context, by analyzing this data set, we support the ongoing COVID-19 response efforts worldwide.

A. COVID-19 Open Research Data Set

This data set contains 111 features and 5644 samples, most of its cells are empty, as shown in Fig. 4. If rows with empty cells are eliminated, no rows would be salvaged, whereas if columns with empty cell are eliminated, only six columns would be salvaged, leaving the potential contribution of some of the 105 remaining features out of the learning part. The proposed algorithm is applied, Table V displays the set of solutions, in which column one describes the number of features, column two the feature index, and column three describes the number of salvaged rows, refer to Table VI in Appendix A to match the indexes with the actual names. Based on the set of solutions, a couple of relevant sub-data sets are exhibited, the first sub-data set would contain six features with 5644 records, the second data set would contain 23 features with 1352 records, then the third data set would contain 37 features with 362 records, if necessary, a few more sub-data sets can be created, but they would contain very little information, this information is graphically displayed in Fig. 5.

V. CONCLUSIONS AND FUTURE WORK

Learning with incomplete data sets is a common challenge faced by data scientists. Handling them in an intelligent way is important to develop robust data models. While deleting rows/columns or imputing missing values are common approaches, they may have serious implications such as loosing most of the training data (as demonstrated in the real case studies) or biasing the estimates. In this context, the authors

Fig. 4. Screen-shot of COVID-19 data set, empty cells in black.

Fig. 5. Salvaged rows by number of columns, the green color describes good data sets.

advocate –when possible– to learn from real data and impute missing values on deployment.

In this paper, a new greedy-like algorithm is presented, the algorithm is aimed at maximizing the information (cells) of incomplete data sets. It develops different sub-sets that solve the trade-off between number of features and number of samples differently. The user can choose a sub-set with either more features and less number of samples or the other way around. Since the best sub-set cannot be determined in advance, it is recommended to apply the learning algorithm to select the best, like wrapper-type feature selection algorithms [6].

Moreover, this paper reports a small contribution in addressing the pandemic challenge from an analytical perspective, by formally analyzing the COVID-19 Open Research Data Set (CORD-19) that was prepared and posted by The White House and a coalition of leading research groups as a call to action to the world's artificial intelligence experts to answer high priority scientific questions.

The proposed algorithm iteratively considers the number of empty cells in each remaining column to select the next one in a greedy fashion. This study can be enhanced by considering the predictive information associated to each columns too. This information can be obtained by using a filter-type feature selection algorithm [6]. Thus, columns are selected based on two criteria, the number of cells and their quality.

TABLE V SOLUTIONS, THE GREEN COLOR DESCRIBES GOOD DATA SETS.

APPENDIX A

This appendix includes the table with the names of the COVID-19 features.

TABLE VI NAMES OF COVID-19 FEATURES.

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