Prevention and Control Strategy for Multi-group Epidemics Based on Delay and Isolation Control

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***Abstract*—Aiming at the prevention and control of multi-group epidemics, the dynamic model of disease transmission is established, and an optimal delay and isolation strategy is proposed. The control variables in the model represent the strength of various measures taken to block and isolate contact between groups. The optimal control problem is solved by Pontryagin maximum principle and the corresponding numerical iterative algorithm is given. The results show that the delay and isolation control strategy can effectively control the epidemic of infectious diseases while minimizing the cost of infectious diseases. The outbreak of the COVID-19 in 2020 is a serious threat to people's lives and health. It is because of the timely adoption of delay and isolation control strategy of "early detection and early isolation" and "joint-prevention and joint-control", China took the lead in the fight against the epidemic situation and made an example for the whole world.**

Keywords*—*delay and isolation control, epidemics control, multi-group epidemics model, COVID-19

I. Introduction

Infectious disease is one of the most serious problems in the world. In 1927, Kermack and McKendrick [1] first established the mathematical model of infectious diseases by means of nonlinear dynamics, that is, the so-called K-M model. Since then, it is of great practical significance to study, predict and control infectious diseases by using mathematical models, which can provide useful information and effective measures for people to prevent and treat infectious diseases. It has become a consensus that mathematical models can help to discover the transmission mechanism of infectious diseases and predict the epidemic trend of infectious diseases.

Multi-group usually refers to dividing people into several subgroups according to their individual habits. Each of the two groups has the same habit[2-3]. Multi group model is mainly used to study the spread of viral diseases, such as SARS, HIV/AIDS, COVID-19, etc. in this kind of diseases, individual habits are an extremely important factor in the probability of infection. If each subgroup is regarded as a population, the multi group model can also be used to study zoonotic diseases, such as avian influenza. In addition, groups can be divided according to regions, such as SARS and COVID-19.

In most of the literatures on epidemic models, most of them only use mathematical models to simulate and predict the spread of diseases, or discuss the epidemic trend near the equilibrium point by solving the basic reproductive number, without considering the time-varying disease control strategy. In 1973, Gupta and rink [4] regarded active and passive immunity as control variables and applied dynamic optimal control to infectious disease control. Since then, based on Pontryagin maximum principle [5], time-varying optimal control has been used to study the control of HIV [6], insect borne diseases [7], dengue fever [8], tuberculosis [9], SARS [10] and COVID-19 [11].

In the process of transmission of infectious diseases, the contact infection rate of susceptible and infected persons is one of the important factors to determine the outbreak or elimination of the disease. In the control of infectious diseases, especially COVID-19, which is prevalent in 2020, quarantine and isolation strategies have been proved to be very effective means to reduce the contact rate. Although the optimal control theory has been widely used in the spread and treatment of various infectious diseases, most of the literatures on infectious diseases only use data fitting, simulation, prediction of epidemic trend or analysis of basic regeneration number to study the effect of quarantine and isolation and other prevention and control measures, but do not consider the time-varying disease control strategy. In view of this, this paper attempts to use the method of optimal control theory to study the dynamic quarantine and isolation strategy of infectious diseases.

Based on the *m*-group SIRS model of Hethcote [2-3], the application of optimal control in multi group infectious disease control is discussed in this paper. In order to block and isolate the contact and infection of susceptible and infected persons among groups, m^2 control variables were introduced to represent the implementation intensity of various measures (such as strengthening public prevention education, blocking personnel contact, compulsory quarantine and isolation, etc.). Because the contact infection rate of susceptible and infected people directly determines the transmission characteristics of infectious diseases, the optimal control of blocking and isolating the contact between susceptible and infected people among groups is called optimal blocking and isolation control, referred to as barrier control.

II. Multi -group Epidemic Model

A. K-M model

Dynamics of infectious diseases is an important method for theoretical and quantitative study of infectious diseases. It is based on the characteristics of population growth, the occurrence of diseases, the spread and development of diseases in the population, and the related social factors, to establish a mathematical model that can reflect the dynamic characteristics of infectious diseases. Through qualitative and quantitative analysis and numerical simulation of the dynamic morphology of the model, the development process of the disease, the epidemic law and the change and development trend of the disease can be predicted In order to

This research was supported by the project of school-enterprise cooperation practice education base in Anhui (2018sjjd079), the emergent engineering research and practice project in Anhui (2020xgkxm32), national innovation and entrepreneurship training program for college students (201910375042, 201910375009).

provide theoretical basis and quantitative basis for people's decision-making of prevention and control, this paper analyzes the causes and key factors of epidemic disease, and seeks the optimal strategy for its prevention and control.

In order to find out the transmission mechanism of infectious diseases, Kermack and McKendrick constructed the famous K-M model [1]. The model divides the population into three categories: susceptible class (s), infectious class (I) and removed class (R). Suppose that a susceptible person (s) is infected by any infected person (I) with constant β, and the infected person (I) randomly recovers in a constant proportion of γ and becomes a lifelong immune person (R). Then, the model of infectious disease transmission is as follows:

$$\begin{cases} \dot{S} = -\beta SI \\ \dot{I} = \beta SI - \gamma I \\ \dot{R} = \gamma I \end{cases} \tag{1}$$

Define the number of susceptible persons $S(0)=S_0$, $\rho=\gamma/\beta$, $R_0=S_0\beta/\gamma= S_0/\rho$ at the initial moment, and get $dI(t)/dt<0$ from equation (1) and equation (4) if and only if $S_0<\rho$. Therefore, when the number of susceptible individuals $S_0>\rho$ at the initial time, the number of infected people $I(t)$ will increase to the maximum value $I(\rho)$, and then gradually decrease to zero, which indicates that the worm will be popular; when $S_0< \rho$, the virus will not be popular, and the number of infected people $I(t)$ will monotonically decrease and tend to zero. In other words, $R_0=1$ is the threshold to distinguish the prevalence of the disease. This conclusion is also called the threshold theorem of K-M model.

B. SIRS model

On the basis of K-M model, Hethcote established a multi group SIRS model [2-3], which consists of three ordinary differential equations:

$$\begin{cases} \dot{S}_i = (1-P_i)b_i - (d_i+\theta_i)S_i + \sigma_i R_i - \sum_{j=1}^{m}\left[1-u_{ij}(t)\right]\beta_{ij}S_i I_j \\ \dot{I}_i = \sum_{j=1}^{m}\left[1-u_{ij}(t)\right]\beta_{ij}S_i I_j - (d_i+\gamma_i\varepsilon_i)I_i \\ \dot{R}_i = P_i b_i + \gamma_i I_i + \theta_i S_i - (d_i+\sigma_i)R_i \end{cases} \tag{2}$$

Where i, j=1, …, m, S_i, I_j and R_i represent the number of susceptible, infected and displaced persons in group i respectively, and the initial values $S_i(0)$, $I_i(0)$ and $R_i(0)$ are given. d_i and b_i were the mortality rate and neonatal input rate of group i, respectively, and $b_i= d_i(S_i+ I_i+ R_i)$. ε_i is the mortality caused by the disease. γ_i is the cure rate (removal rate). The cured patients have temporary immunity, and those with $\sigma_i R_i$ will lose immunity and may be infected again. Suppose that all the new-born population are susceptible, and P_i is the rate of vaccination at birth, and thereafter, the vaccination rate of susceptible persons is θ_i. In the absence of control, the infection rate β_{ij} is determined by the habits of each group, indicating the degree of mixing among the populations.

The control function $u_{ij}(t)$ is bounded and Lebesgue integrable. The control rate $u_{ij}(t)$ represents the intensity of the measures taken to reduce the contact infection rate β_{ij} between the susceptible group i and the infected group J (e.g. strengthening public preventive education, blocking personnel contact, compulsory quarantine isolation, etc.). 1-$u_{ij}(t)$ indicates the degree of influence on the contact infection rate after taking measures. For example, $u_{ij}(t)$= 0 means that no measures are taken, and the infection rate β_{ij} is completely determined by the habits of each group. $u_{ij}(t)$=1 indicates that the contact between the susceptible group i and the infected group j is completely isolated.

III. Delay and Isolation Control Principle

The delay and isolation control strategy adopted in this paper is based on SIRS model.

The objective cost function of optimal control is

$$J(u_{ij}, i, j=1,\cdots,m) = \int_0^{t_f}\left[\sum_{i=1}^{m}B_i I_i + \sum_{i=1}^{m}\sum_{j=1}^{m}\frac{C_{ij}}{2}u_{ij}^2(t)\right]dt \tag{3}$$

Where t_f is the final moment. J consists of the number of infected persons in each group and the cost of implementing the control strategy, and it is assumed that the cost of implementing the control strategy has a quadratic form with respect to the control law. B_i and C_{ij} (i, j= 1, …, m) is the corresponding weight. The purpose of this paper is to find the optimal control law u_{ij}^* so that

$$J(u_{ij}^*) = \min_{\Omega} J(u_{ji}) \quad i, j=1,\cdots,m \tag{4}$$

Where $\Omega = \left\{u_{ij} \in L^1(0,t_f) \middle| a_{ij} \le ui_j \le b_{ij}, i, j=1,\cdots,m\right\}$ is the admissible control set, a_{ij},b_{ij} are nonnegative constants.

Pontryagin maximum principle [5] gives the necessary conditions for optimal control. Its Hamiltonian function is

$$H = \sum_{i=1}^{m}B_i I_i + \sum_{i=1}^{m}\sum_{j=1}^{m}\frac{C_{ij}}{2}u_{ij}^2(t) + \sum_{i=1}^{m}\lambda_i^T f_i \tag{5}$$

Where f_i is the column vector formed on the right side of equation (1), $\lambda_i=(\lambda_{i1},\lambda_{i2},\lambda_{i3})^T$ is the undetermined Lagrangian multiplier vector. Let $X_i=(S_i, I_i, R_i)^T$. For the above optimal control problems, we have the following theorem.

Theorem 1 The system (2) has an optimal control law u_{ij}^* $\in \Psi$ and corresponding optimal trajectory X_{ij}^*, so that J reaches the minimum value. The co-state equation is

$$\lambda_i(t) = \begin{bmatrix}\lambda_{i1}(t)\\ \lambda_{i2}(t)\\ \lambda_{i3}(t)\end{bmatrix} = \begin{bmatrix} \lambda_{i1}(d_i+\theta_i) + (\lambda_{i1}-\lambda_{i2})\sum_{j=1}^{m}\left[1-u_{ij}^*\right]\beta_{ij}I_j - \lambda_{i3}\theta_i \\ -B_i + \sum_{k=1}^{m}(\lambda_{k1}-\lambda_{k2})\left[1-u_{ki}^*(t)\right]\beta_{ki}S_k + \lambda_{ki}(d_i+\gamma_i+\varepsilon_i) - \lambda_{i3}\gamma_i \\ -\lambda_{i1}\sigma_i + \lambda_{i3}(d_i+\sigma_i) \end{bmatrix} \tag{6}$$

The cross-section condition is

$$X_i(t_f)=0 \tag{7}$$

and

$$u_{ij}(t)=\min\left\{\max\left\{a_{ij},\frac{\beta_{ij}S_i^*I_j^*(\lambda_{i2}-\lambda_{i1})}{C_{ij}}\right\},b_{ij}\right\} \tag{8}$$

Proof Because the integrand of objective function J is about (u_{ij}, *i*, *j*=1, …, *m*), the equation of state is Lipschitz property with respect to the state variable and bounded for a given initial value. According to Pontryagin's maximum principle, there are

$$\lambda_i(t)=[\lambda_{i1}(t),\lambda_{i2}(t),\lambda_{i3}(t)]^T=-\frac{\partial H}{\partial X_i}=-\left[\frac{\partial H}{\partial S_i^*},\frac{\partial H}{\partial I_i},\frac{\partial H}{\partial R_i}\right]^T$$

$$X_i(t_f)=0$$

Substituting the equation of state (1) into the equation of state (5). Using the extremum condition, there are

$$\frac{\partial H}{\partial u_{ij}}=C_{ij}u_{ij}+\lambda_{i1}\beta_{ij}S_iI_j-\lambda_{i2}\beta_{ij}S_iI_j=0$$

Namely

$$u_{ij}(t)=\frac{\beta_{ij}S_iI_j(\lambda_{i2}-\lambda_{i1})}{C_{ij}}$$

Considering $\{a_{ij}\le u_{ij}^*\le b_{ij}\}$, the optimal control law formula (7) is obtained.

IV. Simulation Study on Emergency Control of Multi-groups of Infectious Diseases

The above optimal control problem is a two-point boundary value problem (TPBVP), which can be calculated by solving the iterative algorithm of state equation and costate equation. The steps are as follows:

(1) Given initial control;

(2) Taking $X_i(0)$(i= 1, …, m) as the initial value, the forward fourth-order Runge Kutta method is used to solve the equation of state (1) on [0, t_f];

(3) Using the current calculated state values, the backward fourth-order Runge Kutta method is used to solve the co state equation (6) on [0, t_f];

(4) The objective function J is calculated and the control law is updated by combining the gradient method [8] and the value of equation (8);

(5) Repeating (3) to (5) until the objective function J is close enough to its minimum value.

A model consisting of two groups (m = 2) is considered. It is assumed that the control cost of contact infection between susceptible and infected persons in block group is higher than that of susceptible and infected persons in block group. In the simulation, the weights are b_1=b_2= 1, c_{11}= 800, c_{12}= 500, c_{21}= 660, c_{22}= 900. Other infectious disease model parameters and numerical simulation parameters are shown in Tab.1 and Tab. 2 respectively.

Tab.1 model parameter values

Model parameters	Value
β_{11}	4.1×10^{-5}
β_{11}	1.0×10^{-5}
β_{21}	2.0×10^{-5}
β_{22}	5.0×10^{-5}
d_1	0.0005
d_2	0.0006
γ_1	0.06
γ_2	0.08
P_1, P_2	0
θ_1, θ_2	0
σ_1, σ_2	0
ε_1, ε_2	0

Tab. 2 numerical simulation parameter values

Parameters	Value
$S_1(0)$	5400
$I_1(0)$	565
$R_1(0)$	35
$S_1(0)$	8875
$I_1(0)$	980
$R_1(0)$	145
a_{11}, a_{22}	0.1
b_{11}, b_{22}	0.6
a_{12}, a_{21}	0.1
b_{12}, b_{21}	0.8
t_f	180 d
dt	1 d

A time-varying optimal blocking control law is shown in Fig. 1. From the beginning to most simulation time, the control law is in the upper bound of control, and then gradually decreases with time. The comparison of the number of infected persons under the optimal control and control law constant values (u_{11}= u_{12}= u_{21}= u_{22}= 0.3) is shown in Fig.2. Under constant control, the maximum number of infected people in the two groups will reach 2658 (44.3%) and 4929 (49.3%) respectively. As expected, the optimal blocking control is in the upper bound of the control during most of the initial simulation period, which effectively prevents the increase of infection population.

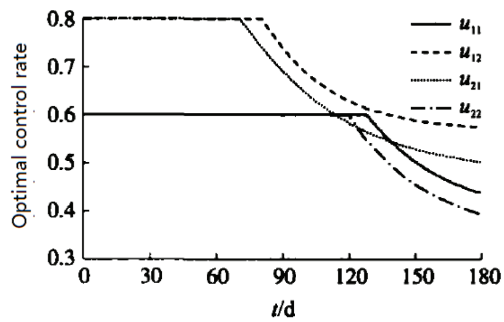

Fig. 1 optimal control law

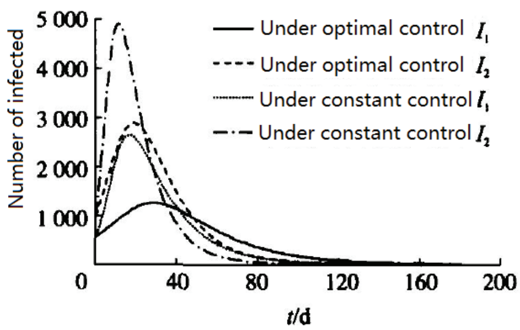

Fig. 2 number of infected persons under different control laws

V. Emergency Control Under COVID-19 Epidemic Situation

Since December 2019, novel coronavirus has been raging in China, forming a public health crisis that covers the whole country. In the face of the sudden outbreaks, general secretary Xi Jinping has repeatedly instructed and directed the work of anti-epidemic. In January 20, 2020, novel coronavirus novel coronavirus infection leading group was launched, and the State Council's joint-prevention and joint-control mechanism was launched. The national health and Health Committee has led the establishment of a joint-prevention and joint-control mechanism for dealing with the new coronavirus infection. Under the joint prevention and control working mechanism, there are working groups for epidemic prevention and control, medical treatment, scientific research, publicity, foreign affairs, logistics support, and front work. The leaders of relevant ministries and commissions are responsible for clear responsibilities and division of labor to form an effective joint force for epidemic prevention and control. Novel coronavirus infection was held by Premier Li Keqiang in January 26, 2020. The meeting stressed the importance of "paying great attention to the prevention and control of epidemic situation in rural areas, giving full play to the role of grassroots organizations and village doctors, and strengthening group prevention and control." In the following ten days, China gradually formed an emergency management mechanism of joint-prevention and joint-control from the central government to the grass-roots level, from the government to the people, and the grass-roots social mass-prevention and mass-control.

In essence, the COVID-19's emergency control is an emergency control strategy, which applies the principle of barrier control, actively adopts hierarchical blocking and isolation measures in the emergency state, weakens or completely cuts off the connection between subsystems, and avoids the collapse of the whole system due to local disasters. The barrier control in reality includes two parts: delay and isolation. Delay refers to reducing the connection between systems by means of control to achieve the purpose of protecting some important systems; isolation refers to completely cutting off the connection between some subsystems and dividing the large-scale system to form multiple completely isolated subsystem sets.

Joint-prevention and joint-control and mass-prevention and mass-control are common emergency management methods in China, which have been shown in SARS prevention and control, Wenchuan earthquake and air pollution and other crisis events [12]. Through joint prevention and control and mass prevention and control, Party members interact with the masses, the state and society, and realize the full coverage of Party and government institutions, enterprises and institutions, and urban and rural communities, which is an operational mechanism of emergency management with Chinese characteristics. In the prevention and control of this major epidemic situation, it is an important measure to control the spread of the epidemic by collecting all the infected patients, treating them as much as possible, and isolating the source of infection thoroughly [13-14]. After more than two months of hard work, the outbreak of covid-19 was successfully contained in China. On March 12, 2020, the National Health Commission announced that "the epidemic peak of this round has passed" [15], and on March 20, 2020, China realized zero reporting of new local confirmed cases and suspected cases [16].

Although we have achieved the initial victory of epidemic control, we must also be soberly aware that in the early stage of the outbreak, the whole country's public health emergency response system exposed the inadequacy of our country's infectious disease prevention and control and public health emergency response capacity in the aspects of research and judgment, prediction, control and diagnosis and treatment of the epidemic. If we can understand the dynamics model of disease transmission and find the targeted prevention and control strategy, then we can introduce effective control variables (such as strengthening public prevention education, blocking personnel contact, compulsory quarantine and isolation, etc.) to prevent and control the epidemic spread without specific drugs and treatment measures.

VI. Conclusion

For a long time, human beings have waged an indomitable struggle against various infectious diseases. However, the road for human beings to conquer infectious diseases is still tortuous and long. The world health report issued by the World Health Organization shows that infectious diseases are still the first killer of human beings and one of the greatest threats to human beings. No matter it is the black death, avian influenza, SARS, COVID-19 and other unknown new infectious viruses, quarantine and isolation of infected people is almost the only effective means to control the spread and outbreak of diseases, and has been proved by history and practice. At the same time, history and reality have warned people that human beings are facing a long-term and severe threat of infectious diseases. The importance of the research on the pathogenesis, infection law and control strategy of infectious diseases has become increasingly prominent, and has become a major problem to be solved urgently in today's world.

The optimal barrier control strategy can minimize the cost of infectious diseases and effectively control the epidemic of multi group infectious diseases. According to the transmission characteristics of different infectious diseases and adopting specific model parameters, the simultaneous interpreting method can be used to study the control of various infectious diseases. Although time-varying control is difficult to apply in reality, it can be used as the evaluation standard of similar optimization strategies in practice. It should be pointed out that barrier control is not a drug or treatment. Without effective treatment, it can only delay the peak outbreak time of patients. Therefore, "early detection and early isolation" is very important for epidemic prevention and control. The outbreak of the new epidemic in 2020 is a serious threat to people's lives and health. It is precisely because of the nationwide "joint-prevention and joint-control, mass-prevention and mass-control" that China took the lead in the initial victory in the fight against the epidemic situation.

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