Modelling of the Infectious Disease Process with Taking into Account of Small-Scale Spatially Distributed Influences

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Abstract— **The approach of modification the infectious disease models that allow to investigate small-scale spatially distributed influences on the process development by diffusion perturbation of the corresponding degenerate problems of the model is proposed. The singularly perturbed model problem with time-delay is reduced to a sequence of problems without time-delay and the corresponding representations of asymptotic expansions the solutions are constructed. The results of numerical experiments that characterize the influence of spatially distributed diffusion "redistributions" of infectious disease factors on the development of the process are given. The decrease in the maximum level of pathogenic antigens concentration in the infection locus due to their diffusion "redistribution" is illustrated.**

Keywords— infectious disease model, dynamic systems, asymptotic methods, singularly perturbed problems

I. Introduction

A various spectrum of mathematical models constructed according to different principles is developed to study the processes of immune defense of the organism against pathogenic bacteria and viruses [1-5]. In particular, the simplest (basic) infectious disease model by G.I. Marchuk describes the most general patterns of the pathogens reproduction process and organism immune system response to them. This model and the more detailed one that deals with the immune response [1] are based on the fundamental immune defense mechanisms formulated in the clonal-breeding theory by F. Burnet [6].

The infectious disease model is based on the principle of each active factor balance and is formulated as a nonlinear differential equations system with time-delay [1]. The main acting factors in the base model are $V(t)$ (the pathogenic antigens concentration), $C(t)$ (the concentration of plasma cells that are carriers and producers of antibodies), $F(t)$ (the concentration of antibodies that neutralize the antigens), $m(t)$ (the relative characteristic (contagion measure) of the infected organ, it is equal to zero for uninfected organ and is one for the fully infected). The model also assumes that the organ damage level influences the antibodies production [1].

Note that basic model of infectious disease by G.I. Marchuk (along with his more detailed immune response model [1]) is constructed on the assumption that the "organism" is some homogeneous closed volume where all process components are mixed uniformly. This approach allows to take into account large numbers of infectious disease factors on the one hand and does not determine effect of the spatial distributed spreading in the organism antigen, antibodies and other active factors on the development of the process of infectious disease on the other hand. According to contemporary immune protection ideas there are lymphocytes for every antigen in the organism that are able to recognize it [7]. However, the quantity of lymphocytes with corresponding, according to antigen, specific receptors is small, moreover, they are less moving than antigens are. So, it is considered that if the antigens enter the organism before recognition they are able to spread in the organism, infect cells and create some infection focus. In some time, the generated antigens will redistribute in the organism from the infection focus to uninfected places expanding the lesion and decreasing the antigens concentration in the original location.

The purpose of this study is taking into account the small spatially distributed diffusion influences on the development of the infectious disease process when it's research bases on the basic model [1].

II. Problem Statement. The Infectious Disease Process Modelling That Takes Into Account Small-Scale Diffusion Perturbations

We assume that small-scale spatially distributed influences on the development of the infectious disease process have diffusive character. The corresponding space-time dynamics of infectious disease model factors in the domain $G_Z = \{(x,y,t): -\infty < x < +\infty;\ -\infty < y < +\infty; 0 < t < +\infty\}$ is described as a generalized differential equations system with time-delay in respect of the infectious disease basic model

$$V_t'(x,y,t) = (\beta - \gamma F(x,y,t))V(x,y,t) + \varepsilon D_V (V_{xx}''(x,y,t) + V_{yy}''(x,y,t)),$$

$$C_t'(x,y,t) = \xi(m(x,y,t))\alpha V(x,y,t-\tau)F(x,y,t-\tau) - \mu_C \times (C(x,y,t) - C^*) + \varepsilon D_C (C_{xx}''(x,y,t) + C_{yy}''(x,y,t)),$$

$$F_t'(x,y,t) = \rho C(x,y,t) - (\mu_f + \eta\gamma V(x,y,t))F(x,y,t) + \varepsilon^2 D_F (F_{xx}''(x,y,t) + F_{yy}''(x,y,t)),$$

$$m_t'(x,y,t)=\sigma V(x,y,t)-\mu_m m(x,y,t)+\varepsilon^2 D_m\left(m_{xx}''(x,y,t)+\right.$$
$$\left.+m_{yy}''(x,y,t)\right) \quad (1)$$

for conditions (Cauchy problem)

$$C(x,y,t_0)=C^0(x,y),\ m(x,y,t_0)=m^0(x,y),$$
$$V(x,y,\tilde{t})=V^0(x,y,\tilde{t}),\ F(x,y,\tilde{t})=F^0(x,y,\tilde{t}),\ t_0-\tau\le\tilde{t}\le t_0, \quad (2)$$

where εD_V , εD_C , $\varepsilon^2 D_F$, $\varepsilon^2 D_m$ are the coefficients of space-diffusion "redistribution" of antigens, antibodies, plasma and affected cells respectively. The function

$$\xi(m)=\begin{cases}1, & 0\le m<m^*,\\ (m-1)/(m^*-1), & m^*\le m<1,\end{cases}$$

is intended to taking into account the reducing of the antibodies production efficiency when significant damage of important target organs takes place [1]. The function $\xi(m)$ equals one on the interval $0\le m<m^*$, it means that the immunological organs functionality is complete and does not depend on the disease severity. Further, for $m^*\le m<1$, the organ functionality efficiency decreases rapidly.

Taking into account the values of the functions with time-delay argument that are defined in the previous step, the solution of the problem(1)-(2) with time-delay are reduced to a sequence of solutions of problems without delay [8] (for $\xi(m)=1$):

$$\begin{cases}V_{0\,t}'=(\beta-\gamma F_0)V_0+\varepsilon D_V\left(V_{0\,xx}''+V_{0\,yy}''\right),\\ C_{0\,t}'=\alpha V^0(x,y,t-\tau)F^0(x,y,t-\tau)-\mu_C\left(C_0-C^*\right)+\\ \quad+\varepsilon D_C\left(C_{0\,xx}''+C_{0\,yy}''\right),\\ F_{0\,t}'=\rho C_0-\left(\mu_f+\eta\gamma V_0\right)F_0+\varepsilon^2 D_F\left(F_{0\,xx}''+F_{0\,yy}''\right),\\ m_{0\,t}'=\sigma V_0-\mu_m m_0+\varepsilon^2 D_m\left(m_{0\,xx}''+m_{0\,yy}''\right),\\ C_0(x,y,t_0)=C^0(x,y),\ m_0(x,y,t_0)=m^0(x,y),\\ V_0(x,y,t_0)=V^0(x,y,t_0),\ F_0(x,y,t_0)=F^0(x,y,t_0),\\ t_0\le t\le t_0+\tau,\end{cases} \quad (3)$$

$$\begin{cases}V_{1\,t}'=(\beta-\gamma F_1)V_1+\varepsilon D_V\left(V_{1\,xx}''+V_{1\,yy}''\right),\\ C_{1\,t}'=\alpha V_0(x,y,t-\tau)F_0(x,y,t-\tau)-\mu_C\left(C_1-C^*\right)\\ \quad+\varepsilon D_C\left(C_{1\,xx}''+C_{1\,yy}''\right),\\ F_{1\,t}'=\rho C_1-\left(\mu_f+\eta\gamma V_1\right)F_1+\varepsilon^2 D_F\left(F_{1\,xx}''+F_{1\,yy}''\right),\\ m_{1\,t}'=\sigma V_1-\mu_m m_1+\varepsilon^2 D_m\left(m_{1\,xx}''+m_{1\,yy}''\right),\\ C_1(x,y,t_0+\tau)=C_0(x,y,t_0+\tau),\\ m_1(x,y,t_0+\tau)=m_0(x,y,t_0+\tau),\\ V_1(x,y,t_0+\tau)=V_0(x,y,t_0+\tau),\\ F_1(x,y,t_0+\tau)=F_0(x,y,t_0+\tau),\ t_0+\tau\le t\le t_0+2\tau,\end{cases} \quad (4)$$

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$$\begin{cases}V_{n\,t}'=(\beta-\gamma F_n)V_n+\varepsilon D_V\left(V_{n\,xx}''+V_{n\,yy}''\right),\\ C_{n\,t}'=\alpha V_{n-1}(x,y,t-\tau)F_{n-1}(x,y,t-\tau)-\mu_C\left(C_n-C^*\right)+\\ \quad+\varepsilon D_C\left(C_{n\,xx}''+C_{n\,yy}''\right),\\ F_{n\,t}'=\rho C_n-\left(\mu_f+\eta\gamma V_n\right)F_n+\varepsilon^2 D_F\left(F_{n\,xx}''+F_{n\,yy}''\right),\\ m_{n\,t}'=\sigma V_n-\mu_m m_n+\varepsilon^2 D_m\left(m_{n\,xx}''+m_{n\,yy}''\right),\\ C_n(x,y,t_0+n\tau)=C_{n-1}(x,y,t_0+n\tau),\\ m_n(x,y,t_0+n\tau)=m_{n-1}(x,y,t_0+n\tau),\\ V_n(x,y,t_0+n\tau)=V_{n-1}(x,y,t_0+n\tau),\\ F_n(x,y,t_0+n\tau)=F_{n-1}(x,y,t_0+n\tau),\ t_0+n\tau\le t\le t_0+(n+1)\tau,\end{cases} \quad (5)$$

Note that in order to ensure the proper smoothness of the corresponding solutions for $t=\tau$, $t=2\tau$, $t=3\tau$, … it is necessary besides the traditional conditions of smoothness with respect to functions (2), impose also their consistency conditions for $t=-\tau$, $t=0$ [10]. In particular, the condition

$$C_t'(x,y,t_0)=\alpha V^0(x,y,t_0-\tau)F^0(x,y,t_0-\tau)-\mu_C\times$$
$$\times\left(C(x,y,t_0)-C^*\right)+\varepsilon D_C\left(C_{xx}''(x,y,t_0)+C_{yy}''(x,y,t_0)\right)$$

must be satisfied.

In cases when the spatially distributed diffusive effects on the infectious disease dynamic are small compared to other process components (parameter ε is small) the usage of asymptotic methods for solving the corresponding singularly perturbed model problems [9,10] is effective. In particular, the solutions of problems (3) - (5) can be formally represented as asymptotic series

$$V_j(x,y,t)=\sum_{i=o}^{N}\varepsilon^i V_{ij}(x,y,t)+R_{Nj}^V(x,y,t,\varepsilon),$$

$$C_j(x,y,t)=\sum_{i=o}^{N}\varepsilon^i C_{ij}(x,y,t)+R_{Nj}^C(x,y,t,\varepsilon),\qquad F_j(x,y,t)=$$

$$=\sum_{i=o}^{N}\varepsilon^i F_{ij}(x,y,t)+R_{Nj}^F(x,y,t,\varepsilon)\ ,\quad m_j(x,y,t)=\sum_{i=o}^{N}\varepsilon^i m_{ij}(x,y,t)+$$
$$+R_{Nj}^m(x,y,t,\varepsilon)$$

as perturbing the solutions to the corresponding degenerate tasks [11], where $j=0,1,...,n,...$, $V_{ij}(x,y,t)$, $C_{ij}(x,y,t)$, $F_{ij}(x,y,t)$, $m_{ij}(x,y,t)$ are the members of regular parts of asymptotics, $R_{Nj}^V(x,y,t,\varepsilon)$, $R_{Nj}^C(x,y,t,\varepsilon)$, $R_{Nj}^F(x,y,t,\varepsilon)$, $R_{Nj}^m(x,y,t,\varepsilon)$ are the corresponding remaining members. After the asymptotic series substituting and equating the coefficients of identical powers of ε using the standard procedure, we obtain such problems for determination of the functions V_{ij} , C_{ij} , F_{ij} , m_{ij} $(i=0,1,...,N$, $j=0,1,...,n,...)$:

$$\begin{cases}V_{0,0\,t}'=(\beta-\gamma F_{0,0})V_{0,0},\\ C_{0,0\,t}'=\alpha V^0(x,y,t-\tau)F^0(x,y,t-\tau)-\mu_C\left(C_{0,0}-C^*\right),\\ F_{0,0\,t}'=\rho C_{0,0}-\left(\mu_f+\eta\gamma V_{0,0}\right)F_{0,0},\\ m_{0,0\,t}'=\sigma V_{0,0}-\mu_m m_{0,0},\\ C_{0,0}(x,y,t_0)=C^0(x,y),\ m_{0,0}(x,y,t_0)=m^0(x,y),\\ V_{0,0}(x,y,t_0)=V^0(x,y,t_0),\ F_{0,0}(x,y,t_0)=F^0(x,y,t_0),\\ t_0\le t\le t_0+\tau,\end{cases} \quad (6)$$

$$\begin{cases} V_{1,0\,t}' = \beta V_{1,0} - \gamma\left(a_{0,0}F_{1,0} + b_{0,0}V_{1,0}\right) + \Phi_{V1,0}, \\ C_{1,0\,t}' = \alpha\left(a_{0,0}(x,y,t-\tau)F_{1,0}(x,y,t-\tau) + b_{0,0}(x,y,t-\tau)\times\right. \\ \qquad \left.\times V_{1,0}(x,y,t-\tau)\right) - \mu_C C_{1,0} + \Phi_{C1,0}, \\ F_{1,0\,t}' = \rho C_{1,0} - \mu_f F_{1,0} - \eta\gamma\left(a_{0,0}F_{1,0} + b_{0,0}V_{1,0}\right), \\ m_{1,0\,t}' = \sigma V_{1,0} - \mu_m m_{1,0}, \\ C_{1,0}(x,y,t_0) = 0,\ m_{1,0}(x,y,t_0) = 0, \\ V_{1,0}(x,y,t_0) = 0,\ F_{1.0}(x,y,t_0) = 0,\ t_0 \le t \le t_0 + \tau; \end{cases} \tag{7}$$

. . . . . .

$$\begin{cases} V_{i,0\,t}' = \beta V_{i,0} - \gamma\left(a_{0,0}F_{i,0} + b_{0,0}V_{i,0}\right) + \Phi_{Vi,0}, \\ C_{i,0\,t}' = \alpha\left(a_{0,0}(x,y,t-\tau)F_{i,0}(x,y,t-\tau) + b_{0,0}(x,y,t-\tau)\times\right. \\ \left.\times V_{i,0}(x,y,t-\tau)\right) - \mu_C C_{i,0} + \Phi_{Ci,0}, \\ F_{i,0\,t}' = \rho C_{i,0} - \mu_f F_{i,0} - \eta\gamma\left(a_{0,0}F_{i,0} + b_{0,0}V_{i,0}\right) + \Phi_{Fi,0}, \\ m_{i,0\,t}' = \sigma V_{i,0} - \mu_m m_{i,0} + \Phi_{m\,i,0}, \\ C_{i,0}(x,y,t_0) = 0,\ m_{i,0}(x,y,t_0) = 0, \\ V_{i,0}(x,y,t_0) = 0,\ F_{i.0}(x,y,t_0) = 0,\ t_0 \le t \le t_0 + \tau; \end{cases} \tag{8}$$

$$\begin{cases} V_{0,1\,t}' = \left(\beta - \gamma F_{0,1}\right)V_{0,1}, \\ C_{0,1\,t}' = \alpha V_{0,0}(x,y,t-\tau)F_{0,0}(x,y,t-\tau) - \mu_C\left(C_{0,1} - C^*\right), \\ F_{0,1\,t}' = \rho C_{0,1} - \left(\mu_f + \eta\gamma V_{0,1}\right)F_{0,1}, \\ m_{0,1\,t}' = \sigma V_{0,1} - \mu_m m_{0,1}, \\ C_{0,1}(x,y,t_0+\tau) = C_{0,0}(x,y,t_0+\tau), \\ m_{0,1}(x,y,t_0+\tau) = m_{0,0}(x,y,t_0+\tau), \\ V_{0,1}(x,y,t_0+\tau) = V_{0,0}(x,y,t_0+\tau), \\ F_{0,1}(x,y,t_0+\tau) = F_{0,0}(x,y,t_0+\tau),\ t_0+\tau \le t \le t_0+2\tau, \end{cases} \tag{9}$$

$$\begin{cases} V_{1,1\,t}' = \beta V_{1,1} - \gamma\left(a_{0,1}F_{1,1} + b_{0,1}V_{1,1}\right) + \Phi_{V1,1}, \\ C_{1,1\,t}' = \alpha\left(a_{0,1}(x,y,t-\tau)F_{1,1}(x,y,t-\tau) + b_{0,1}(x,y,t-\tau)\times\right. \\ \qquad \left.\times V_{1,1}(x,y,t-\tau)\right) - \mu_C C_{1,1} + \Phi_{C1,1}, \\ F_{1,1\,t}' = \rho C_{1,1} - \mu_f F_{1,1} - \eta\gamma\left(a_{0,1}F_{1,1} + b_{0,1}V_{1,1}\right), \\ m_{1,1\,t}' = \sigma V_{1,1} - \mu_m m_{1,1}, \\ C_{1,1}(x,y,t_0+\tau) = C_{1,0}(x,y,t_0+\tau), \\ m_{1,1}(x,y,t_0+\tau) = m_{1,0}(x,y,t_0+\tau), \\ V_{1,1}(x,y,t_0+\tau) = V_{1,0}(x,y,t_0+\tau), \\ F_{1.1}(x,y,t_0+\tau) = F_{1,0}(x,y,t_0+\tau),\ t_0+\tau \le t \le t_0+2\tau; \end{cases} \tag{10}$$

$$\begin{cases} V_{i,1\,t}' = \beta V_{i,1} - \gamma\left(a_{0,1}F_{i,1} + b_{0,1}V_{i,1}\right) + \Phi_{Vi,1}, \\ C_{i,1\,t}' = \alpha\left(a_{0,1}(x,y,t-\tau)F_{i,1}(x,y,t-\tau) + b_{0,1}(x,y,t-\tau)\times\right. \\ \qquad \left.\times V_{i,1}(x,y,t-\tau)\right) - \mu_C C_{i,1} + \Phi_{Ci,1}, \\ F_{i,1\,t}' = \rho C_{i,1} - \mu_f F_{i,1} - \eta\gamma\left(a_{0,1}F_{i,1} + b_{0,1}V_{i,1}\right) + \Phi_{Fi,1}, \\ m_{i,1\,t}' = \sigma V_{i,1} - \mu_m m_{i,1} + \Phi_{m\,i,1}, \\ C_{i,1}(x,y,t_0+\tau) = C_{i,0}(x,y,t_0+\tau), \\ m_{i,1}(x,y,t_0+\tau) = m_{i,0}(x,y,t_0+\tau), \\ V_{i,1}(x,y,t_0+\tau) = V_{i,0}(x,y,t_0+\tau), \\ F_{i,1}(x,y,t_0+\tau) = F_{i,0}(x,y,t_0+\tau),\ t_0+\tau \le t \le t_0+2\tau; \end{cases} \tag{11}$$

$$\begin{cases} V_{0,n\,t}' = \left(\beta - \gamma F_{0,n}\right)V_{0,n}, \\ C_{0,n\,t}' = \alpha V_{0,n-1}(x,y,t-\tau)F_{0,n-1}(x,y,t-\tau) - \mu_C\left(C_{0,n} - C^*\right), \\ F_{0,n\,t}' = \rho C_{0,n} - \left(\mu_f + \eta\gamma V_{0,n}\right)F_{0,n}, \\ m_{0,n\,t}' = \sigma V_{0,n} - \mu_m m_{0,n}, \\ C_{0,n}(x,y,t_0+n\tau) = C_{0,n-1}(x,y,t_0+n\tau), \\ m_{0,n}(x,y,t_0+n\tau) = m_{0,n-1}(x,y,t_0+n\tau), \\ V_{0,n}(x,y,t_0+n\tau) = V_{0,n-1}(x,y,t_0+n\tau), \\ F_{0,n}(x,y,t_0+n\tau) = F_{0,n-1}(x,y,t_0+n\tau), \\ t_0+n\tau \le t \le t_0+(n+1)\tau, \end{cases} \tag{12}$$

$$\begin{cases} V_{1,n\,t}' = \beta V_{1,n} - \gamma\left(a_{0,n}F_{1,n} + b_{0,n}V_{1,n}\right) + \Phi_{V1,n}, \\ C_{1,n\,t}' = \alpha\left(a_{0,n}(x,y,t-\tau)F_{1,n}(x,y,t-\tau) + b_{0,n}(x,y,t-\tau)\times\right. \\ \qquad \left.\times V_{1,n}(x,y,t-\tau)\right) - \mu_C C_{1,n} + \Phi_{C1,n}, \\ F_{1,n\,t}' = \rho C_{1,n} - \mu_f F_{1,n} - \eta\gamma\left(a_{0,n}F_{1,n} + b_{0,n}V_{1,n}\right), \\ m_{1,n\,t}' = \sigma V_{1,n} - \mu_m m_{1,n}, \\ C_{1,n}(x,y,t_0+n\tau) = C_{1,n-1}(x,y,t_0+n\tau), \\ m_{1,n}(x,y,t_0+n\tau) = m_{1,n-1}(x,y,t_0+n\tau), \\ V_{1,n}(x,y,t_0+n\tau) = V_{1,n-1}(x,y,t_0+n\tau), \\ F_{1.n}(x,y,t_0+n\tau) = F_{1,n-1}(x,y,t_0+n\tau), \\ t_0+n\tau \le t \le t_0+(n+1)\tau, \end{cases} \tag{13}$$

. . . . . ;

$$\begin{cases} V_{i,n\,t}' = \beta V_{i,n} - \gamma\left(a_{0,n}F_{i,n} + b_{0,n}V_{i,n}\right) + \Phi_{Vi,n}, \\ C_{i,n\,t}' = \alpha\left(a_{0,n}(x,y,t-\tau)F_{i,n}(x,y,t-\tau) + b_{0,n}(x,y,t-\tau)\times\right. \\ \qquad \left.\times V_{i,n}(x,y,t-\tau)\right) - \mu_C C_{i,n} + \Phi_{Ci,n}, \\ F_{i,n\,t}' = \rho C_{i,n} - \mu_f F_{i,n} - \eta\gamma\left(a_{0,n}F_{i,n} + b_{0,n}V_{i,n}\right) + \Phi_{Fi,n}, \\ m_{i,n\,t}' = \sigma V_{i,n} - \mu_m m_{i,n} + \Phi_{m\,i,n}, \\ C_{i,n}(x,y,t_0+n\tau) = C_{i,n-1}(x,y,t_0+n\tau), \\ m_{i,n}(x,y,t_0+n\tau) = m_{i,n-1}(x,y,t_0+n\tau), \\ V_{i,n}(x,y,t_0+n\tau) = V_{i,n-1}(x,y,t_0+n\tau), \\ F_{i,n}(x,y,t_0+n\tau) = F_{i,n-1}(x,y,t_0+n\tau), \\ t_0+n\tau \le t \le t_0+(n+1)\tau; \end{cases} \tag{14}$$

where $a_{0,j}(x,y,t) = V_{0,j}(x,y,t)$, $b_{0,j}(x,y,t) = F_{0,j}(x,y,t)$;

$$\Phi_{V1,j}(x,y,t) = D_V\left(V''_{0,j\,xx}(x,y,t) + V''_{0,j\,yy}(x,y,t)\right),$$

$$\Phi_{C1,j}(x,y,t) = D_C\left(C''_{0,j\,xx}(x,y,t) + C''_{0,j\,yy}(x,y,t)\right);$$

$$\Phi_{Vi,j}(x,y,t) = -\gamma\sum_{k=1}^{i-1} V_{k,j}(x,y,t)F_{i-k,j}(x,y,t) + D_V\left(V''_{i-1,j\,xx}(x,y,t) + V''_{i-1,j\,yy}(x,y,t)\right),$$

$$\Phi_{Ci,j}(x,y,t) = \alpha\sum_{k=1}^{i-1} V_{k,j}(x,y,t-\tau)F_{i-k,j}(x,y,t-\tau) + D_C\left(C''_{i-1,j\,xx}(x,y,t) + C''_{i-1,j\,yy}(x,y,t)\right),$$

$$\Phi_{Fi,j}(x,y,t) = -\eta\gamma\sum_{k=1}^{i-1} V_{k,j}(x,y,t)F_{i-k,j}(x,y,t) + D_F\left(F''_{i-2,j\,xx}(x,y,t) + F''_{i-2,j\,yy}(x,y,t)\right),$$

$$\Phi_{m\,i,j}(x,y,t) = D_m\left(m''_{i-2,j\,xx}(x,y,t) + m''_{i-2,j\,yy}(x,y,t)\right),$$

$i = 2,3,\ldots,N$, $j = 0,1,\ldots,n,\ldots$.

Note that the proposed approach is easily transferred to other, in particular, finite sets G_Z. More complex series should be used instead of those described above in this case.

The estimates of the remaining members R_{Nj}^V, R_{Nj}^C, R_{Nj}^F, R_{Nj}^m are similar to [9,10,11].

III. The Numerical Experiments Results

Fig. 1 shows a model dependence (under N=2) of the dynamics of the pathogenic antigens concentration on the diffusion influence intensity (ε parameter value) in the single infection locus epicenter in chronic disease. In particular, when the intensity of diffusion "redistribution" increases, the maximum value of the model concentration of pathogenic antigens in the epicenter of infection decreases. The dynamics of other model factors of infectious disease changes similarly. Thus, the predicted development of infectious disease at higher intensity values of the diffusion influence is characterized by lower exacerbation levels.

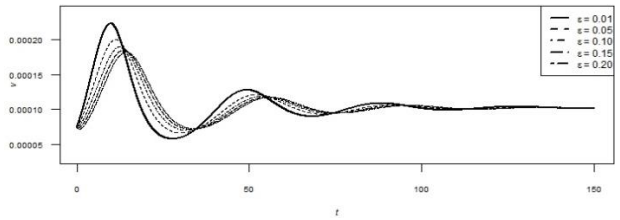

Fig. 1. Dynamics of the pathogenic antigens concentration at different levels of the diffusion influence intensity for the chronic disease

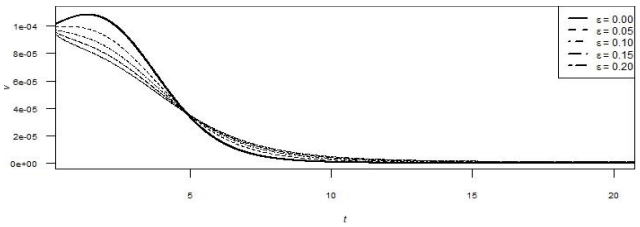

Fig. 2. Dynamics of the pathogenic antigens concentration at different levels of the diffusion influence intensity for the acute disease

Fig. 2 presents model dynamics of the pathogenic antigens concentration under conditions of acute disease at different diffusion "redistribution" intensities (in particular, when ε = 0) in the epicenter of the infection locus. In this model, if the absence of diffusion "redistribution" of antigens concentration in the organism increases to certain maximum level that causes an increase in the concentration of plasma cells and antigens. The latter concentration decreases and is established over time at some level due to the interaction of antibodies and antigens. If the diffusion redistribution model intensity increases, then the rate of change in antigens concentration decreases. The maximum of the model concentration of antigens in the infection locus does not increase beginning from a certain value of the diffusion "redistribution" intensity. The accepted force level of immune response in the model in the presence of a diffusion redistribution of the corresponding intensity is already sufficient that to prevent increasing of the model value of the maximum antigens concentration in the infection locus and without "exacerbations" to reduce their concentration to some stationary level starting from the moment of infected.

IV. Conclusions

The approach that takes into account small-scale spatially distributed diffusion influences on the development of infectious disease based on the model of infectious disease is presented [1]. The corresponding model problem with time-delay is reduced to a sequence of problems without time-delay. Also the representations of the desired functions in the form of asymptotic series as perturbation of the solutions of the corresponding degenerate problems were constructed**.**

The numerical experiments results illustrate the model dynamics of the maximum value of the antigens concentration decrease due to their diffusion "redistribution" from the infection locus. They demonstrate that even when the initial antigens dose V^0 in the part of the infection zone exceeds a certain critical value V^* then diffusion "redistribution" over a certain period of time may reduce the antigen concentration critical values to a level below the critical one, and it's further reduction may be provided by the available antibody level. That is, the nature of the infectious disease will change, for example, from acute to subclinical form in this model. In this case, the sequence of the corresponding singularly perturbed problems solutions (which determine the stepwise (time-delay τ) prediction of the distribution of antigens, antibodies, plasma cells, and measure of contagion in space and time) leads to some stable value and asymptotically stable stationary value, in particular.

It is perspective to take into account of this kind of spatially distributed diffusion influences in the research of the infectious disease process that is based on the more general models, in particular, the antiviral immune response model by Marchuk-Petrov [1].

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