# A Brief Comparative Study of Uncertainty Modeling Techniques in Power System

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*Abstract*—**Modern power systems tend to have large penetrations of renewable energy resources (RERs). Since power generation from these resources is intermittent due to uncertain climate conditions, therefore their rising integration poses several challenges for power system engineers to ensure stable grid operation. To overcome these challenges, the impact of such variable sources on the system needs to be judiciously analyzed. This paper presents different approaches to handle uncertain parameters related to RERs, particularly solar and wind energies, with a critical assessment. Many models and approaches are critically assessed, particularly focusing on past and recent techniques applied to determine the available renewable energy at a specific interval of time. Moreover, the relative features of various models are compared while highlighting their potential applications. The discussion presented in this work will support researchers in choosing appropriate models for simulation studies in power systems with high penetrations of distributed RERs. It will also help outline the needs and requirements of novel models for uncertainty modeling in energy systems.**

*Keywords— uncertainty modeling, information gap decision theory, Monte Carlo simulation, probabilistic modeling, renewable energy resources, robust optimization*

## I. INTRODUCTION

In recent days, the demand for renewable energy is rising with the rise in the global population and restrictions imposed on greenhouse gas emissions. This scenario needs reshaping the traditional ways of energy generation, transmission and distribution to meet the energy requirements. The legacy power system is being reconstructed by the introduction of renewable energy resources (RERs) such as solar and wind. As these sources are uncertain, intermittent and weather dependent, therefore their higher penetrations cause difficulty in stable and economic operation of the power system. Besides the intermittency of RERs, there are some other uncertainty sources in the power system such as unplanned outages/interruptions, transmission capacity variations, changes in fuel price and energy price, varying market rules, time-varying load requirements and other so on [1]. During power system planning and operation, all these uncertainties are taken into account to ensure the reliable and efficient operation of the power grid.

Mathematical uncertainty arises due to calculation and observation errors and it causes differences in true/measured and estimated value [1]. A lot of traditional and deterministic approaches exist to deal with different types of uncertainties.

As sources of these uncertainties are different, therefore different types of uncertainty handling techniques have been developed for power systems so for. These approaches can be classified as the possibilistic approach, probabilistic approach, information gap decision theory (IGDT), hybrid probabilistic-possibilistic approach, interval optimization, robust optimization and some recent techniques (fuzzy logic, scenarios creation, 2-stage scheduling schemes, worst-case scenario method, etc.) as shown in Fig. 1 [2]. These techniques have been used for modeling the uncertainty of distributed generation resources and load.
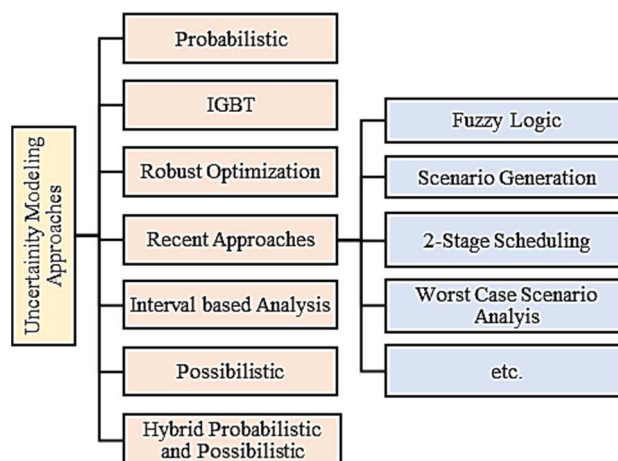


Fig. 1. The uncertainty modeling approaches

The objective of different types of approaches is to see the influence of intermittent input parameters on various output parameters of the system. Different functions are used for modeling the uncertainty of variable input parameters in different methods. In the case of probabilistic methods, probability density function (PDF) is used while possibilistic methods employ membership function. This paper gives brief details for various uncertainty handling techniques in power systems. The organization of the paper is as follows. Probabilistic methods are presented in Section II. Modeling of possibilistic uncertainty is discussed in Section III and Section IV presents the hybrid probabilistic-possibilistic approach. Interval optimization is briefly overviewed in Section V. Concept of uncertainty handling through information gap theory is explored in Section VI while Section VII presents robust optimization. Some recent algorithms are discussed in Section VIII and Section IX concludes the paper.
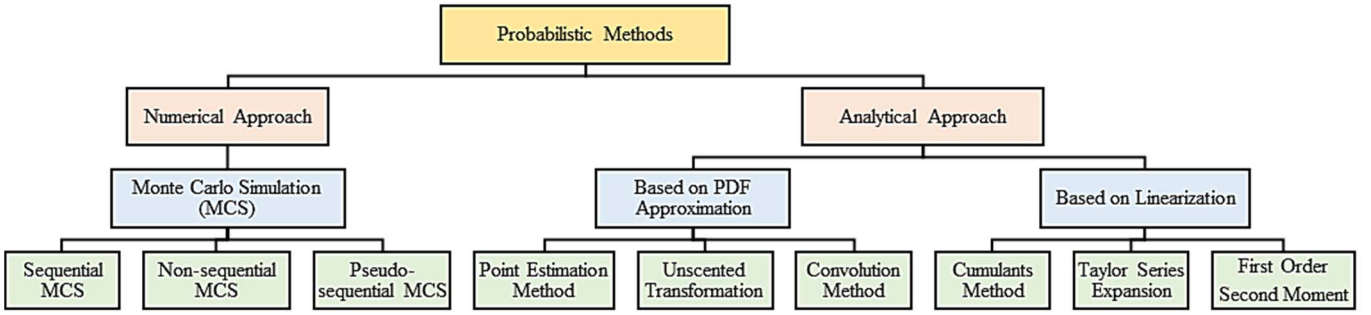
Fig. 2. Probabilistic approaches of uncertainty modeling

## II. PROBABILISTIC METHODS

In this approach, a multivariant function $y = f(x)$ is considered in which $x = \{x_1, x_2, x_3, \ldots, x_n\}$ is a vector containing variable inputs having known PDF. Now the question arises that how we can determine the PDF of $y$. The Weibull PDF usually used for generating wind speed patterns is defined by (1). Similarly, normal PDF which is generally employed for load variations is given by (2).

$$f(x) = \frac{\beta}{\alpha^\beta} x^{\beta-1} \exp\left[-\left(\frac{x}{\beta}\right)^\alpha\right] \tag{1}$$

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}} \tag{2}$$

Here $\mu$ and $\sigma$ represent the mean and standard deviation respectively while $\alpha$ and $\beta$ are scale and shape parameters of Weibull PDF respectively.

### A. Monte Carlo Simulation

Several methods of probabilistic uncertainty analysis can be found in the literature. These methods mainly use Monte Carlo Simulation (MCS) to solve the problem of variable power generation due to distributed generators (DGs). A combination of MCS and Taylor series expansion is employed in Ref. [3] to assess the reliability of the bulk power system. MCS has further three types: sequential MCS, non-sequential MCS and pseudo sequential MCS as shown in Fig. 2.

#### 1) Sequential MCS

For the determination of posterior distribution, Sequential Monte Carlo (SMC) schemes are assessed as the most effective approaches. The SMC methods are flexible and easy to implement. The influence of wind on the power distribution system is precisely examined by SMC which is build up by a time-based wind storm sampling scheme [4]. In [5], the authors deployed the SMC technique for simulating the adaptability of the power systems with wind farms. For the reliability estimation of a complex power system, MCS is used in [6].

#### 2) Non-sequential MCS

Non-sequential MCS, also known as the state sampling method, is an MCS based approach. In this technique, the probability of each component state is evaluated by sampling its probability of component lying on that state for any state of the system in a group of all component states. It is mainly used for the estimation of risk in the power system. A non-sequential MCS based optimization tool was developed in [7] for optimal dispatch of thermal power. The objective of this study was to reduce environmental pollution in the presence of variable wind power.

#### 3) Pseudo-sequential MCS

Pseudo-sequential MCS is faster than sequential and non-sequential MCS schemes because it is a hybrid of these two techniques. A pseudo sequential MCS based method is used in [8] to observe the customers' nodal reliability and reserve deployment with high penetration of PV. Generally, a large number of random numbers are generated for MC but this MCS technique generates results quickly.

### B. Analytical Approaches

The analytical approaches for uncertainty modeling are divided into two groups. One group of these approaches relies on linearization while the 2nd group of analytical methods is based on PDF approximation as shown in Fig. 2. Gram-Charlier series, Convolution method, Cornish-Fisher expansion, cumulant method, Taylor series, Edgeworth expansion and first-order second-moment methods are based on linearization while unscented transformation and point estimation method are examples of PDF approximation-based methods. The technical merits and demerits of some techniques are given in Table I.

## III. POSSIBILISTIC METHOD

The idea of uncertainty modeling through the possibilistic approach was given by Zadeh [9]. In this method, uncertain parameters having fuzzy boundaries were demonstrated by linguistic categories. This technique is applied by finding out the membership function of the output variable with known input membership function. $\alpha$-cut and defuzzification methods are used for it. In the presence of uncertainties, the impact of DGs' operation on active losses was analyzed and load supply ability of distribution network was determined in [10]. In [11], the authors performed the harmonic analysis by introducing a harmonic load flow in presence of power generation through wind farms.

## IV. JOINT PROBABILISTIC-POSSIBILISTIC METHODS

The combined probabilistic-possibilistic method is useful to model certain input data probabilistically and remaining data possibilistically. The coding of such type of problem is done by two loops. The outer loop is based on the probabilistic method while the inner loop is based on the possibilistic method. In Possibilistic-Monte Carlo, the inner loop is of possibilistic approach ($\alpha$-cut method) and the outer is Monte-Carlo. For Possibilistic-Scenario based scheme, scenarios are generated in the outer loop and the Possibilistic loop runs completely for each scenario generation.

TABLE I. SUMMARY OF UNCERTAINTY MODELING ATTRIBUTES

| Group | Examples | Main Idea | Advantages | Disadvantages |
|---|---|---|---|---|
| Probabilistic (based on simulation)[12] | MCS | Real state simulation | Accurate for complex problems having large amounts of data. | Very time consuming |
| Probabilistic (analytical) | Cumulants | Based on linearization | Fast | High order instants cannot be attained correctly. |
| Probabilistic (based on different scenarios) | Scenario-based methods | Scenarios creation | It is faster than MCS and the selection of scenarios is important for accuracy in it. | Only the mean value is achieved at the output. |
| | Point estimation method | Probability distribution function approximation | Simple and computationally efficient. | Only valid for PDF-based problems. |
| | Two-point estimation method | Probability distribution function approximation | Accurate and fast | The uncertain variable determines the execution time. |
| Possibilistic | α-cut Method | Fuzzy membership function (FMF) used | Member function of variable output can be attained. | Simulation takes a long time. |
| Joint probabilistic-possibilistic | Fuzzy-Monte Carlo, Fuzzy-scenario based | Uncertainty modeling of Joint probabilistic-possibilistic approaches | Both probabilistic and possibilistic uncertainties can be modeled. | It is time-consuming. |
| Interval Analysis | Method of interval analysis | Using intervals | More effective when intervals exist. | Intervals correlation cannot be modeled. |
| Information gap theory (IGT) [13] | IGDT | Predicted values of parameters are used | Decision making becomes easy in severe intermittency. | It is more complex. |
| Roust optimization [14] | Robust optimization method | Using intervals | Effective when a specific interval exists. | Non-linear problems are difficult to solve. |

Various applications of this combined approach can be found in the literature. In [15], an assessment tool is developed by this hybrid method to analyze the active losses due to uncertainty caused by DG integration.

## V. INTERVAL OPTIMIZATION

In this uncertainty handling method, when the system has intermittent input parameters then upper and lower bounds are each variable are taken that can have representation through an interval. Assuming a multivariable function of the form $f(x_1, x_2, x_3, \ldots x_n)$ and $lo_i \leq x_i \leq up_i$ with upper and lower bounds $up_i$ and $lo_i$ respectively.

An interval arithmetic approach is used to see the variation in load demand that is based on probabilistic distribution.

$$Prob = \int A1 \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}} \qquad (3)$$

$$Prob = \frac{1}{\sigma\sqrt{2\pi}}\left[\int_a^b \frac{x-a}{b-a} e^{\frac{-(x-\mu)^2}{2\sigma^2}} + \int_b^c e^{\frac{-(x-\mu)^2}{2\sigma^2}} + \int_c^d \frac{x-d}{c-d} e^{\frac{-(x-\mu)^2}{2\sigma^2}} \right] \qquad (4)$$

where a, b, c and d are edge points of the slope in the interval of range a to d. Coordination of directional overcurrent relay is among the major problems in the power system. In [16], the authors proposed an interval analysis technique to cope with this protection problem.

## VI. INFORMATION GAP DECISION THEORY

The IGDT is adopted for those problems that cannot be solved by PDF due to less historical data. In [13], the authors applied IGDT to overcome the variation in wind power generation. Ref. [17] presents a model of uncertainties using the non-probabilistic information gap theory for short term scheduling of generation companies. In [18], the authors introduced an IGDT model to meet the customer load demand by helping network operators in the selection of specific supplying resources. Similarly, a non-probabilistic IGDT was applied in [19] to model uncertainty in the day-ahead price of electricity in the electricity market.

## VII. Robust Optimization

Robust optimization is considered as an effective and new approach for solving uncertainty problems in case of lack of information. In [20], the authors developed a strategy of scheduling energy generation in microgrid using robust optimization for system stability and cost minimization. Various applications of this scheme can be found in the literature. A home energy management scheme is presented in [14] considering the uncertainty of PV storage. A flexible robust optimization method is proposed in [21] for economic power dispatch in multiple intervals of time. Uncertainty of battery charging and discharging is modeled using a distinct robust optimization [22]. In [23], the authors did microgrid planning by considering the uncertainty of load, renewable generation and market price.

TABLE II. SOME RECENT UNCERTAINTY MODELS

| Sr. no. | Method | Model Applications | Features | Ref. |
|---|---|---|---|---|
| 1 | 2-stage scheduling schemes | Power produced from RESs. | • Day-ahead scheduling is done in the first stage<br>• Hours ahead scheduling is done 2$^{nd}$ stage to adjust possible errors | [24]-[25] |
| 2 | Determining worst-case scenario | Day-ahead price forecasting, wind speed | • Upper and lower bounds of uncertain inputs are determined<br>• High system stability is achieved | [26] |
| 3 | By estimating Kernel density | Heat and power demand in the renewable energy system | • Distance between reference and net distribution is determined through Kullback-Liebler divergence<br>• Historical data is required | [27] |
| 4 | Using Hyper-Heuristics (HHs) | Generation of renewable power | • Used for heuristic selection | [28] |
| 5 | Fuzzy logic | Load demand, load forecasting | • On each uncertain parameter, a specific fuzzy membership function is applied | [29]-[30] |
| 6 | Scenarios creation | Load power, wind speed, solar insolation | • For load – Normal distribution<br>• For wind -- Weibull distribution<br>• For solar insolation –Beta Distribution | [31]-[32] |
| 7 | Using discrete states combination | Solar irradiance, wind speed | • Generation of different PDFs for each source<br>• Splitting up each one-hour continuous distribution into N number of states | [33] |
| 8 | Hong's point estimation method (PEM) method | Solar irradiance, wind speed | • The location and weighing factors are evaluated for each random variable.<br>• The variance, mean and skewness coefficient are determined for central moments | [34] |

## VIII. RECENT APPROACHES

Power system stability and operation is affected by large renewable energy penetration. Some new approaches to overcome the uncertainty problem mainly arise due to RERs have been discussed in Table II.

## IX. CONCLUSION

Recent smart developments, higher penetration of the intermittent RERs and other uncertain parameters have risen the uncertainty in the energy system. Researchers are looking for better modeling approaches to tackle the uncertainty while solving various problems related to the power system. A brief comparative analysis of different uncertainty modeling approaches was presented in this paper while taking into account their various features and applications. Effectiveness of different uncertainty modeling methods such as possibilistic, probabilistic, joint possibilistic-probabilistic, interval analysis, IGDT, robust optimization and some recent techniques was presented to resolve emerging uncertainty problems. Additionally, the merits and drawbacks of these methods were also presented. This discussion of this comparative study can be helpful for power system researchers to choose and develop better uncertainty modeling methods for the power system simulation studies and for the advancement of scientific research in this domain.

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