Evolutionary Analysis of Vaccination Strategies for Infectious Diseases Considering Neutral Strategy

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Abstract - **In this paper, we propose an evolutionary game model of epidemic vaccination strategies considering neutral strategy on the homogeneous network. By establishing a state layer and a strategy layer for each individual in the network, we conduct an evolutionary game analysis of epidemic vaccination strategies. Firstly, we take into account various factors such as vaccination effectiveness, government subsidy rate, treatment discount rate, vaccination cost and treatment cost based on the traditional SIR model. We fully analyze various risk factors affecting vaccination. In the strategy layer, we introduce a new neutral strategy. Then, we analyze the proportion of individuals and game benefit of each strategy and use the mean field theory to establish a dynamic equation based on the proposed model. Simulation results show that in order to increase the number of individuals vaccinated when the network evolution is stable, the vaccination effectiveness should be increased and vaccination cost should be reduced. For government decision making, choosing the appropriate vaccination cost determines whether the network evolves towards vaccination strategy.**

Keywords - **Evolutionary game, mean field theory, neutral strategy, vaccination of infectious disease**

I. INTRODUCTION

In recent years, infectious disease models based on complex networks have attracted the attention of many researchers. Many researchers have studied the spreading of epidemic on human contact networks. On human contact network, nodes correspond to individuals in human society and edges represent the mutual relationship between individuals in the contact network. Infectious diseases will spread along the edge of the network.

For the analysis of infectious disease vaccination strategy, many researchers have conducted evolutionary game analysis on different infectious disease models based on the traditional warehouse model. Liu et al. [1] study the dynamics of a stochastic delayed susceptible-infected-recovered (SIR) epidemic model with vaccination and double diseases which make the research more complex. Zaman et al. [2] propose a SIR epidemic model which describes the interaction between susceptible and infected individuals in a community and analyze the epidemic model through the optimal control theory and mathematical analysis. Xu et al. [3] propose a model of delayed stochastic SIRS type with temporary immunity and vaccination is investigated. Meanwhile, some researchers analyze the psychological status of strategy selection. Li [4] et al. proposed an evolutionary vaccination game model by integrating the prospect theory (PT), which accounts for individual subjective perception under uncertainty. Feng et al. [5] provide a model combining epidemic dynamics with evolutionary game theory which captures the voluntary vaccination dilemma. However, the different social benefits of each strategy also have an impact on individual strategy choices. For instance, Ichinose et al. [6] study the effect of social impact on the vaccination behavior resulting in preventing infectious disease in networks.

Recently, many researchers have applied evolutionary vaccination strategies to multi-layer networks to study the transmission mechanism of infectious diseases and the evolution of vaccination strategies on multi-layer networks [7,8]. There is also some work that applies cascading failures to infectious disease research. For instance, Zhan et al. [9] propose a nonlinear model to further interpret the coupling effect based on the susceptible-infected-susceptible (SIS) model. Hota et al. [10] study decentralized protection strategies against susceptible-infected-susceptible epidemics on networks. Taking into account the interactions and conflicts of interests among egoistic individuals (nodes) in a network, Li et al. [11] introduce the zero-determinant (ZD) strategy into the proposed non-cooperative networking vaccination game with the economic incentive mechanism to optimize the social cost against a SIS epidemic process. Si et al. [12] summarize the reliability optimization problems and methods of complex systems.

We propose a new model of vaccination strategy evolution considering neutral strategy by establishing the state layer and strategy layer of individuals in the network. We conduct an evolutionary game analysis of vaccination strategies for infectious diseases with neutral strategy. Firstly, different from the traditional SIR model, we have considered factors such as vaccination effectiveness, government subsidy rate, epidemic discount rate, vaccination cost and treatment cost. And we analyze these various risk factors affecting vaccination. Meanwhile, in the strategy layer, we introduce a new neutral strategy. Neutral strategy holders adopt a bystander attitude towards vaccination and adjust their strategies based on factors such as the impact of infectious diseases and their own interests.

We analyze the proportion of individuals and game benefit of each strategy. Then we use the mean field theory to establish dynamic equations based on the proposed model. We conduct a numerical analysis of factors in the form of a heat map. The simulation results show that in

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order to increase the number of people vaccinated when the network evolution is stable, the vaccination effectiveness should be increased and the cost of vaccination should be reduced. And when the cost of vaccination increases, the network gradually evolves towards neutral strategy.

II. METHODOLOGY

A. Assumptions

Based on the traditional SIR model, we have considered factors such as vaccination effectiveness, government subsidy rate, treatment discount rate, vaccination cost and treatment cost. We fully analyze various risk factors affecting the evolution of vaccination strategy. Meanwhile, in the strategy layer, we add a new neutral strategy. Neutral strategy holders adopt a bystander attitude towards vaccination and adjust their strategies based on factors such as the impact of infectious diseases and their own interests. Before presenting our model, we make the following assumptions about the model:

1) We don't consider the natural birth rate and natural mortality rate of the population in the network. And the number of people in the population is large enough.

2) We assume that once the individual is vaccinated, the effective period of the vaccination is long-term, which is greater than the individual's life span.

3) We assume that the recovered individuals get permanent immunity and will not become susceptible.

4) We assume that there are no side effects after vaccination.

5) We assume that the virus has no incubation period. And infected individuals show symptoms of the disease within a short time after infection.

B. Infectious Disease Vaccination Model

We define two attributes for each individual in the network. One attribute is the state of each individual and the other attribute is the strategy of each individual. In the state layer, there are three states, including susceptible, infected and remover (or recovered) state. The susceptible individuals will be infected with the infectious disease with a certain probability β. Once an individual becomes removed or recovered, the individual leaves the system. In the strategy layer, each individual has three strategies for whether to be vaccinated, including the vaccination strategy, the non-vaccination strategy and the neutral strategy. Individuals will consider various risk factors when deciding on vaccination strategies, including treatment cost, vaccination cost, government subsidy rate, treatment discount rate and vaccination effectiveness. Neutral strategy means that the individual takes neither vaccination strategy nor non-vaccination strategy, but a wait-and-see attitude. Once any party is found to be profitable, it will change its strategy.

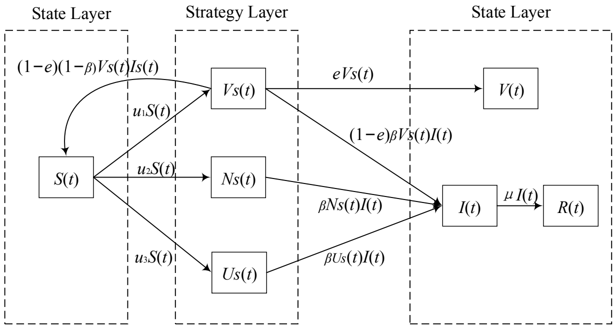

Fig. 1. Infectious disease vaccination model

In Fig. 1, $S(t), I(t), R(t)$ and $V(t)$ represent the proportion of susceptible, infected, removed and vaccinated effectively individuals, respectively in the network at time t. $V_S(t)$ represents the proportion of individuals vaccinated in the network at time t. $N_S(t)$ represents the proportion of individuals who hold a neutral strategy in the network. And $U_S(t)$ represents the proportion of individuals who do not receive the vaccine. At the initial moment, each individual will measure whether to be vaccinated based on various risk factors such as vaccination cost, treatment cost and so on. We assume that in the susceptible group, u_1, u_2 and u_3 represent the proportion of individuals who adopt vaccination strategy, neutral strategy and non-vaccination strategy, respectively. We use e to denote the effectiveness of the vaccine. Then if the vaccine is ineffective, the vaccinated individuals will be infected by the neighbor nodes in the proportion of $(1-e)\beta V(t)I(t)$ and become the infected individuals. If the vaccine is ineffective and vaccinated individuals are not infected by the neighbor nodes, they will maintain a healthy state and become susceptible in the proportion of $(1-e)(1-\beta)V(t)I(t)$. Taking the homogeneous network as the research object, we obtain the following dynamic equations.

$$
\begin{aligned}
\frac{dS(t)}{dt} &= (1-e)(1-\beta)\langle k\rangle V_S(t)I(t) - u_1S(t) - u_2S(t) - u_3S(t) \\
\frac{dI(t)}{dt} &= (1-e)\beta\langle k\rangle V_S(t)I(t) + \beta\langle k\rangle I(t)(N_S(t)+U_S(t)) - \mu I(t) \\
\frac{dR(t)}{dt} &= \mu I(t) \\
\frac{dV(t)}{dt} &= eV_S(t)
\end{aligned} \tag{1}
$$

where $\langle k\rangle$ represents the average degree of the homogeneous network.

C. Vaccination Model Considering Individual Benefit

In each round of the game, we consider the influencing factors affecting evolutionary vaccination strategy game, including vaccination effectiveness e, government subsidy rate s, treatment discount rate d, vaccination cost C_V and treatment cost C_I. Because vaccination has the risk of being ineffective. And considering vaccination cost, some individuals will choose not to be vaccinated.

If node i is vaccinated and the vaccine is effective, then node i only needs to pay the cost of vaccination C_V. If the vaccination of node i is ineffective and not infected, then the government will compensate for the cost at a certain subsidy rate. At this time, the cost that node i needs to pay is $(1-s)C_V$. If the vaccination of node i is ineffective andnode i is infected by the adjacent node, then both the cost of vaccination and the cost of treatment need to be paid. In the end, the cost that node$i$ needs to pay is $(1-s)C_V - dC_I$.

If node i adopts a non-vaccination strategy and is infected, the treatment cost C_I needs to be paid. If node i adopts a non-vaccination strategy but is not infected, then only the cost βC_I is needed. Without vaccination, the node has a tendency to avoid the risk of infection, which will affect its social and economic activities. We assume that the cost of such individuals is proportional to the treatment cost and the scale factor is β.

There is another type of node that adopts a neutral strategy. Individuals who adopt neutral strategy have a bystander attitude towards vaccination. We assume that at time t, the proportion of the individuals who choose the vaccination strategy is x. The proportion of individuals who choose the neutral strategy is y. And the proportion of individuals who choose not to be vaccinated is z. The vaccination effectiveness is$e$, the government subsidy rate is s and the treatment discount rate is d. In real life, if node i is an uninfected individual, the probability of being infected should be related to the number of effectively vaccinated individuals in its neighbors. At this time, the probability of node i being infected is $\lambda(x)$. So the benefit of individuals under different strategies is shown in Table □ and the proportion of individuals under different strategies is shown in Table □.

Table □ INDIVIDUAL BENEFIT UNDER DIFFERENT STRATEGIES

Strategy/State	Healthy	Infected
Effective vaccination	$-C_V$(EVH)	
Ineffective vaccination	$-(1-s)C_V$ (IVH)	$-(1-s)C_V - dC_I$ (IVI)
Neutral	$-\sigma\beta C_I$(NH)	$-\sigma C_I$(NI)
Unvaccinated	$-\beta C_I$(UH)	$-C_I$(UI)

Table □ PROPORTION OF INDIVIDUALS UNDER DIFFERENT STRATEGIES

Strategy/State	Healthy	Infected
Effective vaccination	xe	
Ineffective vaccination	$x(1-e)(1-\lambda(x))$	$x(1-e)\lambda(x)$
Neutral	$y(1-\lambda(x))$	$y\lambda(x)$
Unvaccinated	$z(1-\lambda(x))$	$z\lambda(x)$

We use π_V to represent the average benefit of all individuals who choose the vaccination strategy in the network. π_N represents the average benefit of individuals who choose the neutral strategy in the network. π_U represents the average benefit of individuals who choose not to be vaccinated in the network. π_A represents the average benefit of all individuals in the network.

$$\pi_V = xe(-C_V) + x(1-e)\lambda(x)(-(1-s)C_V - dC_I) + x(1-e)(1-\lambda(x))(-(1-s)C_V)$$
$$\pi_N = y(1-\lambda(x))(-\sigma\beta C_I) + y\lambda(x)(-\sigma C_I)$$
$$\pi_U = z(1-\lambda(x))(-\beta C_I) + z\lambda(x)(-C_I)$$
$$\pi_A = x\pi_V + y\pi_N + z\pi_U \tag{2}$$

III. RESULTS

We use the Fermi Update Rule as the policy update rule for individuals in the network, and then establish the dynamic equations.

A. Policy Update Rule

Considering that game players with limited rationality also make certain mistakes when making strategic adjustments, the Fermi update rule allows irrational probabilistic imitation, which introduce noise parameters into the policy adjustment to characterize the irrational choices of game individuals and their motivations The learning mechanism is that the game individual obtains game benefit by playing games with all its neighbors. When a game individual i wants to update its own game strategy, it randomly chooses its own neighbor j to compare the benefit. The probability that the individual i will adopt the strategy of neighbor j in the next game as $P_{(i\leftarrow j)} = \frac{1}{1+e^{(\pi_i-\pi_j)/\kappa}}$, where π_i and π_j represent the benefits obtained by individuals i and j in this game, respectively. $\kappa(\kappa \geq 0)$ characterizes the noise effect, which means that individuals are allowed to make irrational choices. That is, those strategies with lower benefits still have a small probability of being adopted by individuals with higher returns. So we can get

$$P^{+}_{(i\leftarrow j)} = 1/(1+e^{(\pi_i-\pi_j)/\kappa}) \tag{3}$$

$$P^{-}_{(i\leftarrow j)} = 1/(1+e^{(\pi_i-\pi_j)/\kappa}) \tag{4}$$

$$P^{+}_{(i\leftarrow j)} - P^{-}_{(i\leftarrow j)} = 1/(1+e^{(\pi_i-\pi_j)/\kappa}) - 1/(1+e^{(\pi_i-\pi_j)/\kappa}) \tag{5}$$

We make Taylor expansion of the above formula to get

$$P^{+}_{(i\leftarrow j)} - P^{-}_{(i\leftarrow j)} = -\frac{1}{2}\left(\frac{\pi_i-\pi_j}{\kappa}\right) + \frac{1}{24}\left(\frac{\pi_i-\pi_j}{\kappa}\right)^3 - \frac{1}{240}\left(\frac{\pi_i-\pi_j}{\kappa}\right)^5 + \cdots \triangleq -\frac{1}{2}\left(\frac{\pi_i-\pi_j}{\kappa}\right) \tag{6}$$

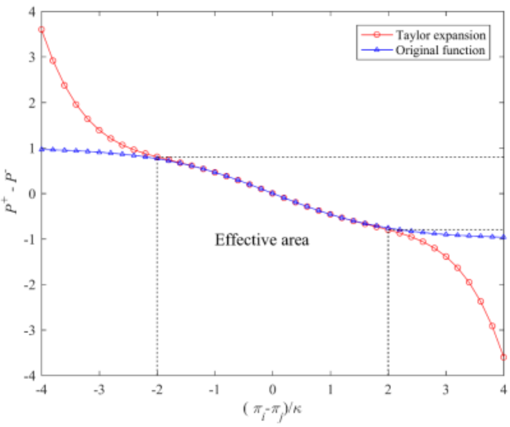

Fig. 2. The original function curve and its Taylor expansion curve of Fermi process

From Table □, we can see that the value range of the benefit of individual i is $0 \le \pi_i \le 2$. So the value range of the difference between the benefit of individual i and $j$is $-2 \le \pi_i - \pi_i \le 2$. From the Fig. 2above, we can see that the original function curve and its Taylor expansion curve of the Fermi process almost overlap in the interval $[-2,2]$. And they are almost linearly distributed. So here we use Taylor expansion form instead of the original function for calculation.

B. Evolutionary Dynamic Equations

Evolutionary stabilization strategies and replicating dynamic equations are the core concepts of evolutionary game theory, reflecting stable states and dynamic processes converging to such stable states, respectively. The criterion of evolutionary stability emphasizes the role of variation in evolution and the dynamic equations emphasize the role of choice.

$$\begin{aligned}\frac{dx}{dt} &= xye\big(1-\lambda(x)\big)\big(P(\mathrm{NH}\leftarrow \mathrm{EVH}) - P(\mathrm{EVH}\leftarrow \mathrm{NH})\big)\\
&+xye\lambda(x)\big(P(\mathrm{NI}\leftarrow \mathrm{EVH}) - P(\mathrm{EVH}\leftarrow \mathrm{NI})\big)\\
&+xze\big(1-\lambda(x)\big)\big(P(\mathrm{UH}\leftarrow \mathrm{EVH}) - P(\mathrm{EVH}\leftarrow \mathrm{UH})\big)\\
&+xze\lambda(x)\big(P(\mathrm{UI}\leftarrow \mathrm{EVH}) - P(\mathrm{EVH}\leftarrow \mathrm{UI})\big)\\
&+xy(1-e)\big(1-\lambda(x)\big)^2\big(P(\mathrm{NH}\leftarrow \mathrm{IVH}) - P(\mathrm{IVH}\leftarrow \mathrm{NH})\big)\\
&+xy(1-e)\big(1-\lambda(x)\big)\lambda(x)\big(P(\mathrm{NI}\leftarrow \mathrm{IVH}) - P(\mathrm{IVH}\leftarrow \mathrm{NI})\big)\\
&+xz(1-e)\big(1-\lambda(x)\big)^2\big(P(\mathrm{UH}\leftarrow \mathrm{IVH}) - P(\mathrm{IVH}\leftarrow \mathrm{UH})\big)\\
&+xz(1-e)\big(1-\lambda(x)\big)\lambda(x)\big(P(\mathrm{UI}\leftarrow \mathrm{IVH}) - P(\mathrm{IVH}\leftarrow \mathrm{UI})\big)\\
&+xy(1-e)\big(1-\lambda(x)\big)\lambda(x)\big(P(\mathrm{UH}\leftarrow \mathrm{IVI}) - P(\mathrm{IVI}\leftarrow \mathrm{UH})\big)\\
&+xy(1-e)\lambda(x)^2\big(P(\mathrm{NI}\leftarrow \mathrm{IVI}) - P(\mathrm{IVI}\leftarrow \mathrm{NI})\big)\\
&+xz(1-e)\big(1-\lambda(x)\big)\lambda(x)\big(P(\mathrm{UH}\leftarrow \mathrm{IVI}) - P(\mathrm{IVI}\leftarrow \mathrm{UH})\big)\\
&+xz(1-e)\lambda(x)^2(P(\mathrm{UI}\leftarrow \mathrm{IVI}) - P(\mathrm{IVI}\leftarrow \mathrm{UI}))\\
&= \frac{\kappa}{2}x(yC_V + zC_V - \beta zC_I - \lambda zC_I - syC_V - szC_V + \beta\lambda zC_I\\
&+d\lambda yC_I + d\lambda zC_I - \beta\sigma yC_I + esyC_V + eszC_V - y\lambda\sigma C_I\\
&-yde\lambda C_I - zde\lambda C_I + y\beta\lambda\sigma C_I) \qquad (7)\end{aligned}$$

Similarly, we can get

$$\frac{dy}{dt} = -\frac{\kappa}{2}y(xC_V + z\beta C_I + z\lambda C_I - xsC_V + xd\lambda C_I - z\beta\lambda C_I - z\beta\sigma C_I + xesC_V - x\lambda\sigma C_I - z\lambda\sigma C_I - xde\lambda C_I + x\beta\lambda\sigma C_I + z\beta\lambda\sigma C_I) \qquad (8)$$

$$\frac{dz}{dt} = -\frac{\kappa}{2}z(xC_V - x\beta C_I - y\beta C_I - x\lambda C_I - y\lambda C_I - xsC_V + x\beta\lambda C_I + y\beta\lambda C_I + xd\lambda C_I + y\beta\sigma C_I + xesC_V + y\lambda\sigma C_I - xde\lambda C_I - y\beta\lambda\sigma C_I) \qquad (9)$$

In real life, if node i is a susceptible individual, the probability of being infected should be related to the number of effectively vaccinated individuals in its neighbors. So the expression of $\lambda(x)$is as follows.

$$\lambda(x) = 1-(1-\beta)^{<k>ex} \overset{TE}{\Rightarrow} -ex{<}k{>}\mathrm{In}(1-\beta) -\frac{1}{2}e^2x^2{<}k{>}^2\mathrm{In}((1-\beta)^2) \qquad (10)$$

where TE means Taylor expansion.

IV. DISCUSSION

For the partial differential equation above, we plot the proportion of individuals with a neutral strategy and the proportion of individuals vaccinated. We set $C_I = 1.0, e = 0.99, \beta = 0.5, \sigma = 0.5, {<}k{>} = 6, \kappa = 0.1, d = s = 0.1$.

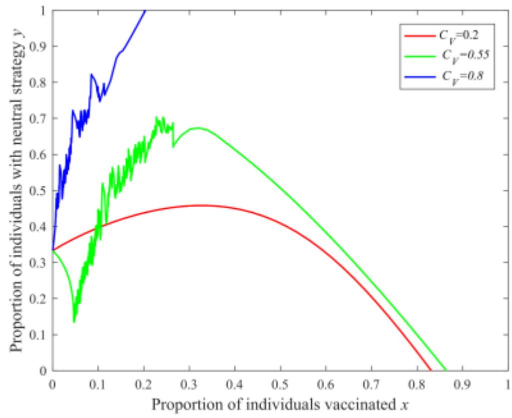

Fig. 3. Evolution map of vaccination strategy and neutral strategy

From the Fig. 3, we can find that when $C_V = 0.2$ or 0.55, individual strategies in the network evolve towards vaccination strategies. When $C_V = 0.8$, individual strategies in the network evolve towards the neutral strategy. Overall, when the cost of vaccination increases, the network gradually evolves towards neutral strategy. Because with the increase of the vaccination cost, the relative benefits of individuals vaccinated compared with non-vaccinated individuals are getting less and less, which make the majority of individuals hold a free-rider mentality.

As can be seen from the Fig. 4, in order to increase the proportion of people vaccinated when the network evolution is stable, vaccination effectiveness can be increased and vaccination cost can be reduced.

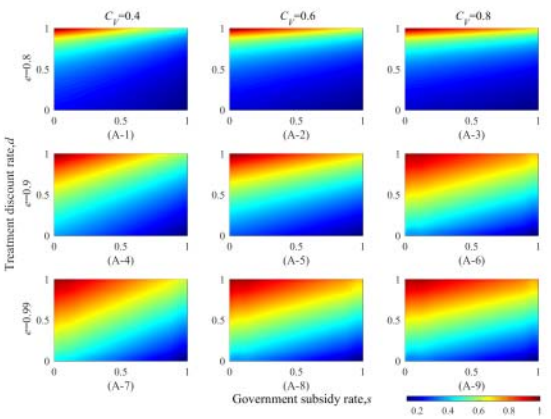

Fig. 4. Heat map of the proportion of individuals vaccinated when evolution is stable

Meanwhile, by increasing the government subsidy rate, individual strategies are more conducive to transformation to neutral strategy when evolution is stable.

V. CONCLUSION

We propose a new model of vaccination strategy evolution. By establishing the state layer and strategy layer of individuals in the network, we conduct an evolutionary game analysis of vaccination strategies for infectious diseases with neutral strategies. Firstly, based on the traditional SIR model, we fully consider various risk factors such as vaccination effectiveness, government subsidy rate, epidemic discount rate, vaccination cost and treatment cost. Meanwhile, in the strategy level, we introduce a new neutral strategy. Neutral strategy means that individuals take a bystander attitude toward vaccination and adjust their strategies based on factors such as the external response to infectious diseases and their own interests. Then based on the proposed model, we analyze the proportion and game benefit of each strategy. Next we use the mean field theory to establish dynamic equations. We use the heat map to numerically analyze the factors. Simulation results show that in order to increase the number of individuals vaccinated when the network evolution is stable, the vaccination effectiveness can be increased and vaccination cost can be reduced. Meanwhile, by increasing the government subsidy rate, individual strategies are more conducive to transformation to neutral strategies when evolution is stable. In actual social networks, the intimacy between people is different. The intimacy between people in social networks corresponds to the weight of the edges in complex networks. And a network is not static in reality. Its structure will change with the movement of nodes. Therefore, it is necessary to analyze the evolution mechanism of epidemic in weighted networks and in dynamic networks.

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REFERENCES

[1] Q. Liu, D. Q. Jiang, N. Z. Shi, T. Hayat, "Dynamics of a stochastic delayed SIR epidemic model with vaccination and double diseases driven by Lévy jumps," *Physica A Statistical Mechanics & Its Applications*, vol. 492, no. 2018, pp. 2010–2018, Feb. 2018.

[2] G. Zaman, Y. H. Kang, G. Cho, I1 H. Jung, "Optimal strategy of vaccination & treatment in an SIR epidemic model," *Mathematics and computers in simulation*, vol. 136, no. 2017, pp. 63–77, Jun. 2017.

[3] C. Y. Xu, X. Y. Li, "The threshold of a stochastic delayed SIRS epidemic model with temporary immunity and vaccination," *Chaos, Solitons & Fractals*, vol. 111, no. 2018, pp. 227–234, Jun. 2018.

[4] X. J. Li, X. Li, "Perception Effect in Evolutionary Vaccination Game Under Prospect-Theoretic Approach," *IEEE Transactions on Computational Social Systems*, vol. 7, no. 2, pp. 329–338, Jan. 2020.

[5] X. Feng, B. Wu, L. Wang, "Voluntary vaccination dilemma with evolving psychological perceptions," *Journal of Theoretical Biology*, vol. 439, no. 2018, pp. 65–75, Feb. 2018.

[6] G. Ichinose, T. Kurisaku, "Positive and negative effects of social impact on evolutionary vaccination game in networks," *Physica, A. Statistical mechanics and its applications*, vol. 468, no. 2017, pp. 84–90, Feb. 2017.

[7] L. Alvarez-Zuzek, M. A. Di Muro, S. Havlin, L. A. Braunstein, "Dynamic vaccination in partially overlapped multiplex network," *Physical Review E*, vol. 99, no. 1, pp. 1–25, Jan. 2019.

[8] K. M. A. Kabir, J. Tanimoto, "Analysis of epidemic outbreaks in two-layer networks with different structures for information spreading and disease diffusion," *Communications in Nonlinear Science and Numerical Simulation*, vol. 72, no. 2019, pp. 565–574, Jun. 2019.

[9] X. X. Zhan, C. Liu, G Zhou, et al, "Coupling dynamics of epidemic spreading and information diffusion on complex networks," *Applied Mathematics & Computation*, vol. 332, no. 2018, pp. 437–448, Sep. 2018.

[10] A. R. Hota, S. Sundaram, "Game-Theoretic Vaccination Against Networked SIS Epidemics and Impacts of Human Decision-Making," *IEEE Transactions on Control of Network Systems*, vol. 6, no. 4, pp. 1461–1472, Dec. 2019.

[11] X. J. Li, C. Li, X. Li, "Minimizing Social Cost of Vaccinating Network SIS Epidemics," *IEEE Transactions on Network Science & Engineering*, vol. 5, no. 4, pp. 326–335, Dec. 2018.

[12] S. B. Si, J. B. Zhao, Z. Q. Cai, H. Y. Dui, "Recent advances in system reliability optimization driven by importance measures," *Frontiers of Engineering Management*, vol. 7, no. 3, pp. 335–358, Jan. 2020.