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Information entropy and supervision of the transient of an epidemic model

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***Abstract*— A supervisory monitoring scheme is designed for the control of infectious diseases. The design is based on the tested accumulated information entropy of the absolute error of the model versus the observed data. Such a supervisory loss function is minimized at each supervision time-interval occurring in-between each two consecutive switching time instants. The design method allows to set through time the active model, within a prescribed parallel structure of potential models, each with its own coefficient transmission rate. Such an active model generates the vaccination and or treatment controls to be injected to the monitored population.**

Keywords—infectious disease; entropy; epidemic model; supervisory loss funtio; vaccination and treattment controls.

I. INTRODUCTION

The controls of epidemic models are typically of vaccination and antiviral or antibiotic treatment, [1-6]. The first one is applied to the susceptible population while the second one is applied to the infectious one. There are also other biological problems where the control actions are of interest. See, for instance, [7] where the environment carrying capacity can be manipulated via appropriately fixing variables(such as temperature) to improve the production efficiency, for instance, in aquaculture or fisheries exploitations. The epidemic models are described by a set of parameters which parameterize a differential system. Some of those models are related to the kind of illness, for instance, influenza while others are related to the population, for instance, humans, rabbits etc…. The most relevant illness parameter is the so-called coefficient transmission rate which depends on each particular kind on disease under study and which is typically available from medical data with a certain approximation. It can vary according to the environment conditions (temperature, humidity, etc) and, in some cases on the hygienic conditions of the habitat. It is well- known that the coefficient transmission rate can also be time-varying accordingly to seasonality concerns. To run with the drawbacks of such a seasonality influencing the illness transmission power, this paper proposes the design of a finite predefined set of running simpler models which are described by a system of coupled differential equations. The whole set of models is allocated in a parallel disposal so as to process the registered data from the disease. Such a whole discrete set covers a range of variation of such a coefficient transmission rate within known lower-bound and upper-bound limits which reflect the real variation of the transmission coefficient rate. In that way, each one of the models in the parallel disposal has its own transmission coefficient rate. A supervisory monitoring algorithm is proposed which chooses the, so-called, active model which minimizes a loss function built with the accumulated information entropy of the absolute error of the data of the model related to the measured registered data within each supervision interval. A switching rule chooses another active model as soon as it is detected that the current active model becomes more uncertain than others related to the observed data. The active model supplies the controls to be injected to the real epidemic process.

II. SUPERVISORY ALGORITHM

One considers an epidemic disease with seasonal illness coefficient transmission rate $\beta(t)\in\left[\beta^0\,,\beta^1\right]$, where β^0 and β^1 are known, which is given by the following differential system of n first-order differential equations:

$$\dot{x}(t)=\Psi\left(x(t),\,\beta(t),\,\boldsymbol{p}\right)x(t)+\Gamma u(t)\;\;;\;x(0)=x_0 \qquad (1)$$

where $x(t)\in\boldsymbol{R}^n$ is the state-vector and $\boldsymbol{p}$ is the vector of parameters containing all other parameters that the coefficient transmission rate, like recovery rate mortality rate, average survival rate, average expectation of life irrespective of the illness etc. In practice, the above description could be substituted by a non-parameterized description, based on the state measurements through time. where $x(t)$ is given by provided experimental on-line data on the subpopulation which in this case, should be

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discretized with a small sampling period. The state vector contains the subpopulations integrated in the model which depend on the type of model itself such as susceptible (E), infectious (I) and recovered (or immune) (R) in the so-called SIR models to which it is added, in the so-called SEIR models the exposed subpopulation (E) which are those in the first infection stages with no external symptoms. The models can also contain a vaccinated subpopulation (V) and they can have also several nodes or patches, describing, for instance, different environments, in general coupled, each having their own set of coupled subpopulations which interact with the remaining ones though population fluxes. The vector $u(t)\in \boldsymbol{R}^m$ is the control vector. There are typically either one control, namely, vaccination on the susceptible, or two controls, namely, vaccination on the susceptible and (either antiviral or antibiotic) treatment on the infectious in the case when there is only one node. Those controls might be applied to each subsystem associated to one patch if there are several patches integrated in the model. The matrix function of dynamics $\Psi(x(t),\beta(t),\boldsymbol{p})$ is a real $n\times n$-matrix for each time instant $t\in \boldsymbol{R}_{0+}$. The control matrix Γ has as many columns as controls are applied and it typically consists of entries being "o" (i.e., no control applied on the corresponding state component associated to one subpopulation) , "-1" if the control leads to a decrease of the rate of growing of a subpopulation, for instance, vaccination effort on the susceptible) and "+1" if it leads to a compensatory increase rate of a subpopulation due to a corresponding decrease of another one, for instance, the increase in the recovered in the vaccination case (when the susceptible are decreased via vaccination) or again the recovered in the treatment case (when the infectious are decreased via treatment). For simplicity, it is assumed that $\boldsymbol{p}$ is constant and there are no delays in the dynamics.

It is proposed to run a set of $Q+1$ approximated models of the same dimension, in a parallel disposal for data acquisition, each one being described via a constant coefficient transmission rate β_i ; $\forall i\in\overline{Q+1}$ being chosen as:

$$\dot{x}_i(t)=\Psi_i(x_i(t),\beta_i,\boldsymbol{p}_i)x(t)+\Gamma u_i(t);\ x_i(0)=x_0,\ \forall i\in\overline{Q+1} \tag{2}$$

with $x_i(t)\in \boldsymbol{R}^n$,where

$$\beta_{i+1}=\beta_i+\frac{\beta^1-\beta^0}{Q}\ ;\ \forall i(>1)\in\overline{Q}\ ,\ \beta_1=\beta^0 \tag{3}$$

in such way that $\beta_1=\beta^0$ and $\beta_{Q+1}=\beta^1$. The set of models are initialized to the initial conditions of the true data. We consider a set of $(Q+1)$ event errors $\boldsymbol{E}=\{\boldsymbol{E}_i : i\in\overline{Q+1}\}$ of the states of the models (27)-(28) with respect to the real data, that is, $e_i(t)=x_i(t)-x(t)$; $\forall i\in\overline{Q+1}$. Each event $\boldsymbol{E}_i$ is integrated by a set of events E_{ij} which are the errors of each of its integrating subpopulations with respect to the real system, that is, $\boldsymbol{E}_i=\{E_{ij} : j\in\overline{n}\};\ \forall i\in\overline{Q+1}$.

Define the instantaneous error entropies of each error event by summing up all the component-wise contributions, that is, $H(\boldsymbol{E}_i,t)=-\sum_{j=1}^n p_{ij}(t)ln\ p_{ij}(t)$; $\forall i\in\overline{Q+1}$; p_{ij} being the probabilities of the respective subpopulations per model ,while the corresponding accumulated continuous-time and discrete-time entropies on the time interval $[t,t+T)$ are defined in a natural way from the instantaneous ones, respectively, as follows:

$$H_{ca}(\boldsymbol{E}_i,[t,t+T]) = -\int_t^{t+T} H(\boldsymbol{E}_i,t+j\tau)d\tau = -\int_t^{t+T}\sum_{j=1}^n p_{ij}(\sigma)ln\ p_{ij}(\sigma)d\sigma \tag{4}$$

and

$$H_{da}(\boldsymbol{E}_i,[t,t+T]) = -\sum_{j=0}^{\alpha} H(\boldsymbol{E}_i,t+j\tau) = -\sum_{k=0}^{\alpha}\sum_{j=1}^n p_{ij}(t+k\tau)ln\ p_{ij}(t+k\tau) \tag{5}$$

provided that $T=\alpha\tau$ so that $\alpha=T/\tau$ is the set of sampling intervals on T of period τ which is a sub-multiple of T with T and τ being design parameters satisfying these constraints. The control efforts are calculated by applying on a time interval $[t,t_{i+1})$ the control which has made the accumulated entropy of the error on an error event to be smallest one among all the error events on a tested previous time interval $[t-t_i,t)$ which defines the so-called active model on $[t,t_{i+1})$. It can be pointed out that the use of adaptive sampling or non-periodic sampling, in general, can improve in certain control problems either the transients by reducing, for instance, the overshoots or the numerical errors in computational computations as, for instance, the initial conditions in observability problems. It can be also useful to accommodate different signals integrated in the same problems whose natural running sampling periods are distinct because of the specific nature of the various interacting signals. See, for instance, [8-13] and some of the references therein. To simplify the coming exposition, and with no loss in generality, the accumulated discrete-time entropy is the particular one used for testing in the sequel. Then, the following switching algorithm is proposed to select online the active model along the next evaluation time period:

Algorithm

Step 0- *Auxiliary design parameters*:
Define the prefixed minimum inter-sample period threshold $T_{min}>0$ and σ being an auxiliary time interval, $0<\sigma<<T_{min}$, to measure the possible degradation of the current active model in operation what foresees a new coming switching.

Step 1-*Initial control*:
$u(t)=u_i(t)$ for $t=t_0$, with $t_0=0$, for some arbitrary model $i\in\overline{Q+1}$ and make $k(\in \boldsymbol{Z}_{0+})=0$, the initial active control being $a(0)=i\in\overline{Q+1}$ an the initial running integer for switching time instants is $k=0$.

Step 2- *Eventually switched control*:

$$u(t)=\left\{u_i(t): H_{da}(\boldsymbol{E}_i,[t_k,t))=\min_{1\le j\le Q+1} H_{da}(\boldsymbol{E}_j,[t_k,t))\right\} \quad ;$$

$$\forall t\in[t_k,t_k+T_k)$$

$$\wedge\left(H_{da}(\boldsymbol{E}_i,[t_k,t_k+\tau))>\min_{1\le j(\ne i)\le Q+1} H_{da}(\boldsymbol{E}_j,[t_k,t_k+\tau))\right)\Bigg\} \tag{6}$$

such that

- *current active model*: $a(t)=i\in\overline{Q+1}$; $\forall t\in[t_k,t_{k+1})$ is the active model in the set of $(Q+1)$ models which generates the control on $[t_k,t_{k+1})$,

- *next active model*: $a(t)=j(\ne i)\in\overline{Q+1}$,

such that

$$H_{da}(\boldsymbol{E}_j,[t_{k+1},t_{k+1}-\sigma])<\min_{1\le\ell\le Q+1} H_{da}(\boldsymbol{E}_\ell,[t_{k+1},t_{k+1}-\sigma])$$

is the next active model to be in operation at the next switching time instant $t=t_{k+1}$.

- The switching time instants $t_{k+1}=t_{k+1}(t_k)=t_k+T_k$ and the inter-switching time periods $T_k=T_k(t_k)\ge T_{min}$ for $j\in\boldsymbol{Z}_{0+}$ depend on the set of preceding sampling instants $\{t_0=0,t_1(t_0),t_2(t_1),\cdots,t_k(t_{k-1}),\ldots\}$.

- The control law (6) is calculated by computing the accumulated discrete- time entropies with the following probabilistic rule:

$$p_{ij}(t+k\tau)=1-\frac{k_{ij}(t)\left|e_{ij}(t+k\tau)\right|}{\sum_{j=1}^{n}\left|e_{ij}(t+k\tau)\right|} \quad ; \quad \forall i\in\overline{Q+1} \ , \ \forall j\in\overline{n} \ ,$$

$$\forall k\in\overline{T/\tau}\cup\{0\},\ \forall t\in\boldsymbol{R}_{0+} \tag{7}$$

with $k_{ij}(t)\in(0,\bar{k}_{ij}(t))$; $\forall i\in\overline{Q+1}$, $\forall j\in\overline{n}$, and

$$e_{ij}(t)=x_{ij}(t)-x_j(t);\ \forall i\in\overline{Q+1},\ \forall j\in\overline{n},\ \forall t\in\boldsymbol{R}_{0+}$$

Step 3-*Updating the activation of the next active control and inter-switching time interval*:

Do $k\leftarrow k+1$ with $t_{k+1}\leftarrow t_k+T_k$ being the next controller switching time instant to the next active model $a(t_{k+1})\in\overline{Q+1}$, re-initialize all the models to the measured data, that is, $x_i(t_{k+1})=x(t_{k+1})$; $\forall i\in\overline{Q+1}$ and Go to Step 2.

Remarks. **1.** Note that the initial control runs on a time interval lasting at least the designed time interval length T_{min} . In the case of availability of some "a priori" knowledge about the adequacy of the various models to the epidemic process in the initial stage of the disease, this knowledge can be used to overcome the arbitrariness in the selection of the initial controller.

2. Note also that if $\bar{k}_{ij}\le\dfrac{\sum_{j=1}^{n}\left|e_{ij}(t+j\tau)\right|}{\left|e_{ij}(t+j\tau)\right|}$; $i\in\overline{Q+1}$, $\forall j\in\overline{n}$, $\forall t\in\boldsymbol{R}_{0+}$ then the global probability of the summed out partial probabilities of the error event $\boldsymbol{E}_i$ cannot be inflated (that is, exceeding unity) but it can be deflated (that is, being less than unity). This is due to the fact that they are estimated from measured errors related to the true available data. If $\bar{k}_{ij}(t)=n-1$; $i\in\overline{Q+1}$, $\forall j\in\overline{n}$, $\forall t\in\boldsymbol{R}_{0+}$ then such a global probability is neither inflated nor deflated for all time.

$$\Sigma_{j=1}^{n}p_{ij}(t+j\tau)=\Sigma_{j=1}^{n}\left(1-(n-1)\frac{\left|e_{ij}(t+j\tau)\right|}{\left(\sum_{j=1}^{n}\left|e_{ij}(t+j\tau)\right|\right)}\right) \quad ;$$

$\forall i\in\overline{Q+1}$, $\forall t\in\boldsymbol{R}_{0+}$. Note also that the fact that $k_{ij}(t)$ can be time-varying so that the probabilities have a margin for experimental design adjustment relies with the problem statement of probabilities subject to possible errors in the theoretical statements of the above sections.

III. EXAMPLE

This section contains a simulation example illustrating the application of the proposed Entropy paradigm to the parallel structure of the multi-model epidemic system. Thus, the behavior of the supervisory algorithm will be shown in this section through numerical examples in open and closed-loop operation. The accurate model considered as the one generating the actual or true data is the SEIR (susceptible-exposed-infectious- recovered - or immune-) one described in with vaccination:

$$\frac{dS}{dt}=\mu-\beta(t)S(t)I(t)-\mu S(t)+\delta R(t)-V(t)$$

$$\frac{dE}{dt}=\beta(t)S(t)I(t)-(\mu+\varepsilon)E(t)$$

$$\frac{dI}{dt}=\varepsilon E(t)-(\mu+\gamma)I(t)$$

$$\frac{dR}{dt}=\gamma I(t)-(\mu+\delta)R(t)+V(t)$$

where $\mu=2.0$ years^{-1} is the growth and death rate of the population, $\varepsilon=1.0$ years^{-1}, $\delta=0.1$ days^{-1}, $\gamma=0.02$ days^{-1} are the instantaneous per capita rates of leaving the exposed, infected and recovered stages, respectively, and $V(t)$ denotes the vaccination. This model fits in the parallel structure given by where $u(t)=V(t)$ acts as the control command. The initial conditions are given by $S(0)=E(0)=I(0)=R(0)=0.1$. All the parameters are assumed to be constant except $\beta(t)$, the disease transmission coefficient, which describes the seasonality in the infection rate and is given by the widely accepted Dietz's model, $\beta(t)=\beta_0(1+b\cos(2\pi t))$ with $\beta_0=6.2$ and $b=0.6$. The function $\beta(t)$ describes annual seasonality in this example. Figure 1 shows the behavior of this system in the absence of any external action (i.e. in open loop). As it can be observed in Figure 1, the disease is persistent since the infectious do not converge to a zero steady-state value asymptotically. This situation will be tackled more efficiently by means of the vaccination function in order to generate a closed-loop system whose infectious tend to zero.

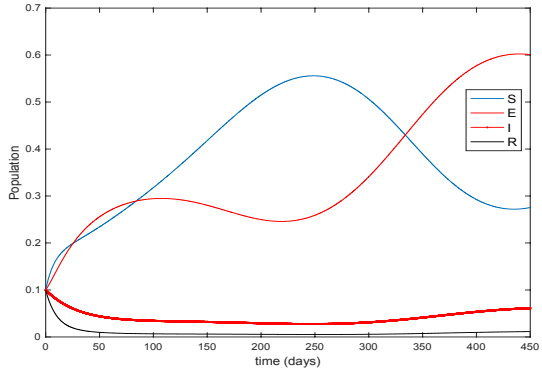

Figure 1. Dynamics of the seasonal epidemics.

This accurate time-varying model is described by a number of simplified time-invariant models running in parallel with the same constant parameter values and fixed values of β. In this example. There are nine models in parallel, that is $Q=9$, so that 10 models will be running in parallel. Now, a vaccination control is employed to avoid the persistency of the disease though time. To this end, the following state-feedback type vaccination law is used:

$$V(t)=K_S S_{active}(t)+K_I I_{active}(t)$$

with $K_S=0.1$ and $K_I=0.01$ being the state-feedback control gains and $S_{active}(t)$, $I_{active}(t)$ the state components of the corresponding active model according to the supervisory algorithm. Along this section, $T=5$ days and $k_{ij}=15.8$. It can be seen that both control commands are very similar with only some peaks associated with the switching process making the difference between one and another. It can be concluded that the proposed approach has been shown to be a powerful tool to model the complex time-varying system.

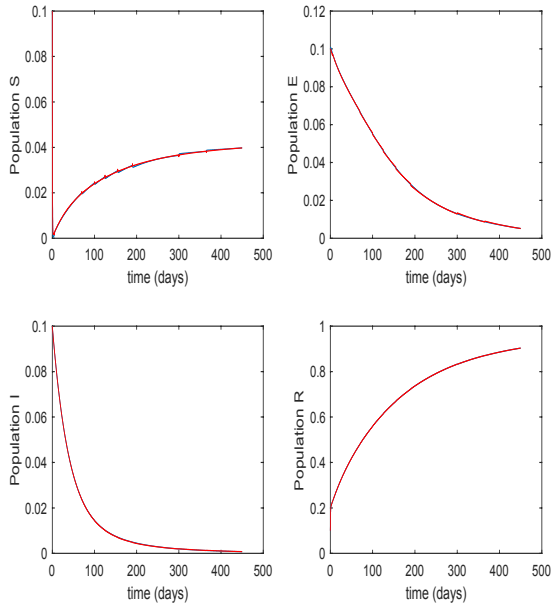

Figure 2. Evolution of the closed-loop system when vaccination is applied and the algorithm is used.

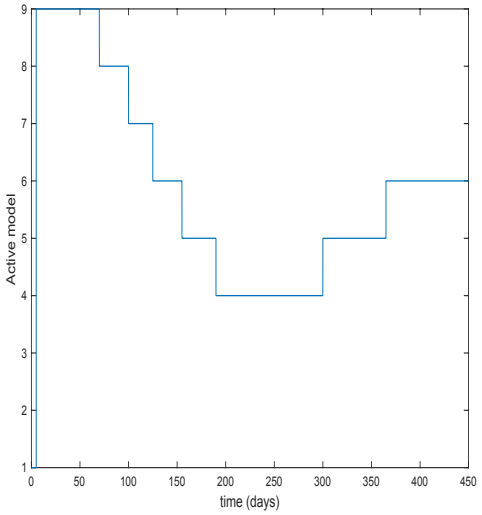

Figure 3. Active model when Algorithm 1 is employed.

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REFERENCES

[1] E.Y. Erten, J.T. Lizier, M. Piraveenan and M. Prokopenko, "Criticality and information dynamics in epidemiological models", *Entropy*, Vol. 2017, No. 19, article ID 194, pp. 1-11, 2017.

[2] M. De la Sen, "On the approximated reachability of a class of time-varying systems based on their linearized behaviour about the equilibria: Applications to epidemic models", *Entropy*, Vol. 2019, No. 21, Article ID 1045, pp. 1-31, 2019.

[3] R. Nistal, M. De la Sen, S. Alonso-Quesada and A. Ibeas, *International Journal of Innovative Computing, Information and Control*, Vol. 15, No. 6, pp. 2053-2067, 2019.

[4] Y. Nakata and T. Kuniya, "Global dynamics of a class of SEIRS epidemic models in a periodic environment", *Journal of Mathematical Analysis and Applications,* vol. 363, pp. 230-237, 2010.

[5] L. Meyers, "Contact network epidemiology: bond percolation applied to infectious disease prediction and control", *Bull. Am. Math. Society*, Vol. 44, No. 1, pp. 63-86, 2007.

[6] Q. Cui, Z. Qiu, W. Liu and Z. Hu. "Complex dynamics of an SIR epidemic model with nonlinear saturated incidence and recovery rate", *Entropy*, Vol. 2017, No. 19, Article ID 305, pp. 1-16, 2017.

[7] M. De la Sen and S. Alonso-Quesada, "Model-matching based control for the Beverton-Holt equation in ecology", *Discrete Dynamics in Nature and Society*, Vol. 2008, Article number 793512, 2008.

[8] M. Delasen, "Multirate hybrid adaptive-control", *IEEE Transactions on Automatic Control,* Vol. 31, No. 6, pp.582-586, 1986.

[9] M. Delasen, "A method for improving the adaptation transient using adaptive sampling", *International Journal of Control*, Vol. 40, No. 4, pp.639-665, 1984.

[10] M. Delasen, "Application of the non-periodic sampling to the identifiability and model-matching problems in dynamic systems", *International Journal of Systems Science*, Vol. 14, No. 4, pp.367-383, 1983.

[11] T.C. Hsia, "Comparison of adaptive sampling control laws*", IEEE Transactions on Automatic Control*, Vol. AC17, No. 6, pp . 830-831, 1972.

[12] T.C. Hsia, "Analytic design of adaptive sampling control laws in sampled-data systems*", IEEE Transactions on Automatic Control*, Vol. AC19, No. 1, pp. 39-42, 1974.

[13] E. Nagy, "Variable sampling interval linear stochastic control", *Proceedings of the 2000 American Control Conference*, Vol. 1-6, pp. 2780-2781, 2000.