Linear Sintonization of Two Cascade Vectorial Controllers from the Numerical Prediction of a Mechanical Ventilator System

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***Abstract*- With the increasing number of COVID-19 cases, the demand of ventilators has increased to such an extent that there is not enough supply of equipment available. For this reason, this research is based on an open source project presented by the Massachusetts Institute of Technology which involves the conditioning of an Artificial Manual Breathing Unit (AMBU bag), that acts as an aid for patients to maintain a constant respiratory cycle. This document presents the calculation of the control matrix for the mechatronic and bio pneumatic systems of the ventilator by implementing a Space-State Model that allows to obtain the control gains that may reduce the error concerning the open loop operation of the ventilator; furthermore, this paper also presents the SIMULINK simulations that demonstrate the efficiency of the calculated control matrixes. Due to these graphic demonstrations, it was observed that the implemented controllers provided an increase in the response time and reduced the error, thus, theoretically, both controllers must provide stability and a higher performance to the ventilator prompting an appropriate assistance to the hospitalized victims of the COVID-19; however, currently, this project is not viable, as it has not been tested in real life and it does not meet the quality and medical requirements for its use in emergencies.**

Keywords- Control, system, state space, model, matrix, eigenvalues

I. INTRODUCTION

Today, on the date this document is being written, July 15, 2020, Mexico is still experimenting the global pandemic of the COVID-19 (SARS-CoV-2). This disease originated in the city of Wuhan, the capital of Hubei, China, where its first outbreak began in December 2019, but the first confirmed case in Mexico took place in the capital on February 27th of 2020, and the first death was announced later on March 28th of 2020, therefore the government of Mexico in coordination with the World Health Organization implemented a series of preventive measures to avoid more infections in the country [1].

To help the medical community around the world, the MIT created an open source mechanical ventilator design [2], while in Mexico, CONACYT and CIDESI developed two different models of mechanical ventilators named Ehécatl 4t and Gätsi which have already passed several quality tests and sanitary safety requirements as well as preclinical trials after being tested in artificial lungs and living systems [3].

The most important reason for this project is to contribute to the medical research for improving the performance of the mechanical ventilator as they are the main life support that allows COVID-19 patients to keep breathing due to the respiratory damages provoked by the disease.

Due to this previous situation, it is really important to properly design the controller of the mechanical ventilator to improve its performance for fulfilling the medical specifications required for ensuring the survival of the patient given its nonlinear dynamics with high level of uncertainties. Thus, a robust controller must be designed and tested considering a variety of scenarios to really understand its behavior in normal and in atypical situations where a miscalculation in the feedback gains can lead to an improper air operation and a potential risk of a cardiorespiratory arrest to the patient. In this context, the nonlinear dynamics require accurate control gains that respond to the particular dynamic model characteristics, having a priority in the behavior of the patients' lungs given the fact that the anatomy they present is diverse according to previous respiratory diseases, smoking habits or even age and sex; therefore, the robustness of the controller is a priority for ensuring the survival of the patients in need of a mechanical ventilator.

As a response to his need, mechanical engineers from the Eindhoven University of Technology proposed a variable-gain control method for a mechanical respiratory system in which a high gain pressure controller was applied in the system to achieve a fast pressure buildup and release during assisted ventilation whereas a low-gain pressure controller was utilized to decrease unwanted oscillations in order to avoid triggers that could compromise the patient's health [4].

From simulations, they realized that the low-gain integral feedback controller could target pressure accurately but at a slow pace with a favorable response to avoid triggers while the high-gain integral feedback controller managed to target pressure quickly but it had unwanted oscillations that could result on false triggers. Therefore, they decided to opt for a variable-gain control to balance the tradeoff between each controller.

Besides their simulations they performed experimental trials to compare the results obtained. From their results, they realized that even though they were similar, there were still some differences which they concluded where related to the lung's resistance value since they used a linear resistance in calculations and a quadratic resistance in the mechanical test that was performed at different air flows δ and with different gains α for the variable-gain linear controller to achieve the closest result as possible (Fig 1).

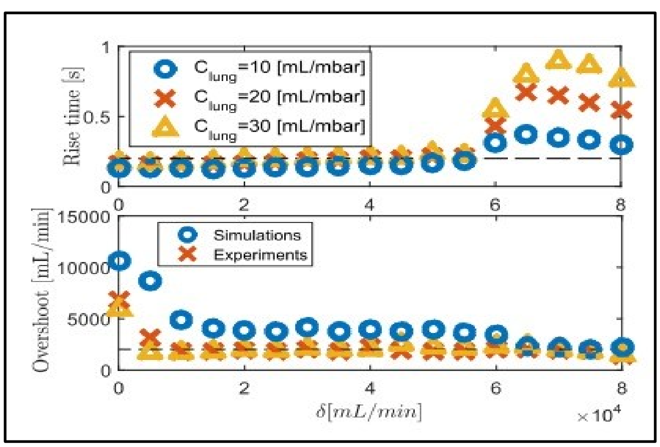


Fig. 1. Real results comparison against simulations from the variable-gain control experimental trials of the Eindhoven University of Technology.

As a conclusion of their research, they determined that based on their experimental results, the variable-gain controller could switch gains depending on the patient's flow, so it was in fact capable of outperforming the linear counterpart in order to have a good machine-patient synchronization and a good breathing assistance [4].

Given these facts, the goal of this paper is that by applying a state-space control, there will be an increase in the response time and an error reduction prompting to design an efficient controller that monitors the dynamic variables inherent to the mechanical ventilator system in order to enhance its performance and reliability. This proposed controller is actually composed of two dependent controllers in cascade that will be working together: a mechatronic controller that will be in charge of controlling the angular speed, angular acceleration and torque of the ventilator and a bio-pneumatic controller in charge of controlling the air volume, flow and pressure supplied by the system to the patient.

On chapter 1, the importance of mechanical ventilators for pulmonary assistance is discussed and is also presented the approach used for a mechanical respiratory system designed by engineers from the Eindhoven University of Technology by regulating the system with a variable-gain control method.

On chapter 2, the theoretical foundations of the implemented methodology are discussed, giving a brief description of important concepts of control theory such as the state-variable representation, state-vectors, and the design of the overall control system.

Next, on chapter 3, each detail regarding the calculation and implementation of the mechatronic and the bio-pneumatic control of the system is thoroughly described. Later in chapter 4, the simulations and their graphs are analyzed in detail for comparing the variables behavior of the system in open-loop against closed-loop.

Finally, on chapter 5 it is concluded whether our hypothesis and objectives were fulfilled, and it is also given a comparison between our approach and the state of the art.

II. THEORETICAL FRAMEWORK

A. Model's Synthesis

Linear time invariant state models are a linear representation for dynamic system, in either continuous or discrete time [5]. Putting a model into state space system representation is the basis for control theory. It lets you solve all the limitation that the classical control theory, in where the transfer function is applied. State space describe the behavior of a dynamic system as a set of first order ordinary differential equations. If a dynamic model is described by a higher order ordinary differential equation, the same model can be described by a set of coupled first order ordinary differential equations.

The internal variables of a state space model are called state variables and they fully describe the dynamic system response. The number of the state variables of a state space model is defined by the highest order of the ordinary differential equation describing the dynamic system and for a given state space model, the number of variables is equal to the number of the initial condition needed to completely solve the system model [6].

A state space model is given by

$$\dot{x} = A \cdot x + B \cdot u$$
$$y = C \cdot x + D \cdot u$$

$$x = \begin{Bmatrix} x_1 \\ x_2 \\ \dots \\ x_n \end{Bmatrix}_{n \times 1}$$

$$u = \begin{Bmatrix} u_1 \\ u_2 \\ \dots \\ u_m \end{Bmatrix}_{m \times 1}$$

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix}_{n \times n}$$

$$B = \begin{bmatrix} b_{11} & b_{12} & \dots & b_{1m} \\ b_{21} & b_{22} & \dots & b_{2m} \\ \dots & \dots & \dots & \dots \\ b_{n1} & b_{n2} & \dots & b_{nm} \end{bmatrix}_{n \times m}$$

Where, x is the state vector and $\dot{x}$ its derivative. Therefore A would be the state transition matrix, B the input matrix and u the input vector. And,

$$y = \begin{Bmatrix} y_1 \\ y_2 \\ \dots \\ y_p \end{Bmatrix}_{p \times 1}$$

$$C = \begin{bmatrix} c_{11} & c_{12} & \dots & c_{1n} \\ c_{21} & c_{22} & \dots & c_{2n} \\ \dots & \dots & \dots & \dots \\ c_{p1} & c_{p2} & \dots & c_{pn} \end{bmatrix}_{p \times n}$$

$$D = \begin{bmatrix} d_{11} & d_{12} & \dots & d_{1m} \\ d_{21} & d_{22} & \dots & d_{2m} \\ \dots & \dots & \dots & \dots \\ d_{p1} & d_{p2} & \dots & d_{pm} \end{bmatrix}_{p \times m}$$

Where, y is the output vector, C the output matrix and D the direct transmission matrix, typically fill with zeros because the input usually does not affect the output directly [7].

B. Design of the Control

The state space model is stable if all eigenvalues are negative real numbers or have a negative real part to complex eigenvalues. If all its real parts are negative values then the system is stable, meaning that any initial condition converge exponential to a stable attracting point. If the eigenvalue's real parts are zero, the system will not converge in no point. If the eigenvalues are positive, the system is unstable and will exponentially diverge. In a state feedback design, the states are feedback to the input side to place the closed poles at the desire locations.

Considering the following state space:

$$\dot{x} = A \cdot x(t) + B \cdot u(t)$$

$$y(t) = C \cdot x(t)$$

If we want the states to approach zero starting from any arbitrary initial state, the design problem es known as regulation, where the internal stability of the system with the desire a parameter is achieve by implementing:

$$u(t) = -k \cdot x(t)$$

By substituting this last equation, the close loop system becomes:

$$\dot{x} = (A - Bk) \cdot x(t)$$

Where k is the state feedback gain matrix designed to place the poles of the closed loop system in the desired locations [5].

III. METHODOLOGY

To give stability to both the mechatronic and bio-pneumatic systems, first, it is necessary to obtain their dynamic models for categorizing them according to their respective damping, which is achieved by the analysis of the characteristics of the open-loop poles, mainly to detect the stability level and if the systems have an oscillatory frequency. This categorization is done according to the next table:

TABLE I. DAMPING CATEGORIZATION

Category	**Characteristics of the open-loop poles**
Overdamped	Different among them and with only real values
Critically damped	Equal among them and with only real values
Underdamped	With real and imaginary values

Having properly categorized them, the stability is given by the sign of the real part of the open-loop pole, so that if it is positive, then the system is unstable, but if it is negative, then it is stable. However, the degree of stability or instability is given by the number following the sign, thus, the higher positive the pole is, the more unstable it is, in contrast, the higher negative it is, the more stable it is.

Knowing the values of the open-loop poles, some new values must be proposed for improving the stability of the system in a close-loop with a state-space controller. This technique is chosen as it allows to control more than one variable simultaneously in contrast to a traditional PID which only works with 1 input and 1 output. As a matter of fact, the mechatronic system has 1 input (voltage) and 3 outputs (angular acceleration, angular speed, and torque), while the bio-pneumatic has 3 inputs and 3 outputs (in both cases respectively: air volume, flow and pressure).

After designing the proper controller, it will return the proper control gains that will be implemented in a Simulink simulation which will be used for comparing the signal graphs in open-loop against the ones in closed-loop to compare if the system stability was improved.

A. Mechatronic Control

First, having the differential equations (1), (2) and (3) that model the mechatronic system dynamics of the respirator, the equation (4) was constructed in order to get the state-space representation where the states vector is formed by angular acceleration [a(t)], angular speed [w(t)] and torque [T(t)] being the 3 variables time dependent. Also, the model considers the electric resistance [R], the electric inductance [L], the damping [B_θ], the polar moment of inertia [J_θ], the gears ratio [k_e], the voltage-speed ratio [k_{m1}], the current-torque ratio [k_{m2}], the voltage input [$v(t)$], and the feedback state gains for each of the 3 variables of the states vector: k_a, k_w and k_T respectively.

$$\frac{d\alpha(t)}{dt} = \frac{1}{k_e}\omega(t) \tag{1}$$

$$\frac{d\omega(t)}{dt} = -\frac{K_\theta k_e}{J_\theta}\alpha(t) - \frac{B_\theta}{J_\theta}\omega(t) + \frac{1}{J_\theta}T(t) \tag{2}$$

$$\frac{dT(t)}{dt} = \frac{1}{Lk_{m_1}}v(t) - \frac{k_{m_2}}{Lk_{m_1}}\omega(t) - \frac{R}{L}T(t) \tag{3}$$

$$\frac{d}{dt}\begin{bmatrix}\alpha(t)\\ \omega(t)\\ T(t)\end{bmatrix} = \begin{bmatrix}0 & \frac{1}{k_e} & 0\\ -\frac{K_\theta k_e}{J_\theta} & -\frac{B_\theta}{J_\theta} & \frac{1}{J_\theta}\\ 0 & -\frac{k_{m_2}}{Lk_{m_1}} & -\frac{R}{L}\end{bmatrix}\begin{bmatrix}\alpha(t)\\ \omega(t)\\ T(t)\end{bmatrix} + \begin{bmatrix}0\\ 0\\ \frac{1}{Lk_{m_1}}\end{bmatrix}v(t) \tag{4}$$

Next, by obtaining the open-loop poles of the state-space representation, the system could be classified according to Table 1. To obtain those values, from equation (4) the 3x3 matrix (matrix A) is extracted and subtracted to a lambda identity matrix (where lambda is an open-loop pole) so that this new matrix could be populated with the proper numerical data of the mechatronic system for solving its determinant:

$$det(\lambda I - A) = det\begin{bmatrix}\lambda & -\frac{1}{k_e} & 0\\ \frac{K_\theta k_e}{J_\theta} & \lambda + \frac{B_\theta}{J_\theta} & -\frac{1}{J_\theta}\\ 0 & \frac{k_{m_2}}{Lk_{m_1}} & \lambda + \frac{R}{L}\end{bmatrix} = det\begin{bmatrix}\lambda & -\frac{1}{3} & 0\\ 0.0012 & \lambda + 0.08 & -0.04\\ 0 & \frac{2.5}{82} & \lambda + \frac{5}{0.82}\end{bmatrix} \tag{5}$$

By solving the determinant of the equation (5) and equaling it to zero, the next open-loop poles were obtained:

$$\lambda_1 = -0.00534$$
$$\lambda_2 = -0.07485$$
$$\lambda_3 = -6.09735$$

Based on these results, the mechatronic system was considered overdamped as the open-loop poles do not have an imaginary part; however, in terms of stability, the system in appearance presented a tendency to it as the poles have negative sign, but the values were pretty near to zero, denoting no stability at all. In consequence, higher negative poles no close to zero were proposed for the closed-loop controller that ensured the stability of the system by multiplying each open-loop pole for a power of 10 so that each one could reach the range of the tens; as a result of these operations, the next closed-loop poles were proposed:

$$s_1 = -54$$
$$s_2 = -75$$
$$s_3 = -61$$

Having the open and closed-loop poles, equation (4) was reduced by substituting $v(t)$ with the proper gains matrix (matrix K_C), that ensured the correct dimensions of 3x3 after being multiplied by the vector B in order to be added with the first addend of the sum which was also subtracted to a lambda identity matrix as in the calc of the open-loop poles; these procedures are presented in equation (6):

$$(\lambda I - A) + BK_c = \begin{bmatrix}\lambda & -\frac{1}{k_e} & 0\\ \frac{K_\theta k_e}{J_\theta} & \lambda + \frac{B_\theta}{J_\theta} & -\frac{1}{J_\theta}\\ 0 & \frac{k_{m_2}}{Lk_{m_1}} & \lambda + \frac{R}{L}\end{bmatrix} + \frac{1}{Lk_{m_1}}\cdot\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ k_\alpha & k_\omega & k_T\end{bmatrix} \tag{6}$$

Then, equation (6) was reduced, again populated with the proper numerical data of the mechatronic system, and equaled to a control polynomial with the proposed closed-loop poles in equation (7):

$$det(\lambda I - A + BK_c) = det\begin{bmatrix}\lambda & -\frac{1}{3} & 0\\ 0.0012 & \lambda + 0.08 & -0.04\\ \frac{k_\alpha}{82} & \frac{2.5 + k_\omega}{82} & \lambda + \frac{5}{0.82} + \frac{k_T}{82}\end{bmatrix} = (\lambda + 54)(\lambda + 75)(\lambda + 61) \tag{7}$$

By solving equation (7), the equations (8), (9) and (10) were obtained with the gains in terms of angular acceleration, angular speed, and torque; thus, by considering these 3 equations as a 3x3 equation system, the gains could be finally determined:

$$0.01219k_T + 6.17756 = 190 \quad (8)$$
$$0.00048k_\omega + 0.00097k_T + 0.48942 = 11919 \quad (9)$$
$$0.00016k_\alpha + 4.87505x10^{-6}k_T + 0.00243 = 247050 \quad (10)$$

After solving the 3x3 equation system, the feedback state gains were obtained and represented in equation (11) as a vector:

$$k_\alpha = 1{,}544{,}062{,}025$$
$$k_\omega = 24{,}799{,}756.67$$
$$k_T = 15{,}079.77358$$

$$k = [1{,}544{,}062{,}025 \;\; 24{,}799{,}756.67 \;\; 15{,}079.77358] \quad (11)$$

With the feedback state gains vector, the control simulations in Simulink could be adjusted and run obtaining the graphs from Fig. 2 to Fig. 5:

Fig. 2. Mechatronic error without control

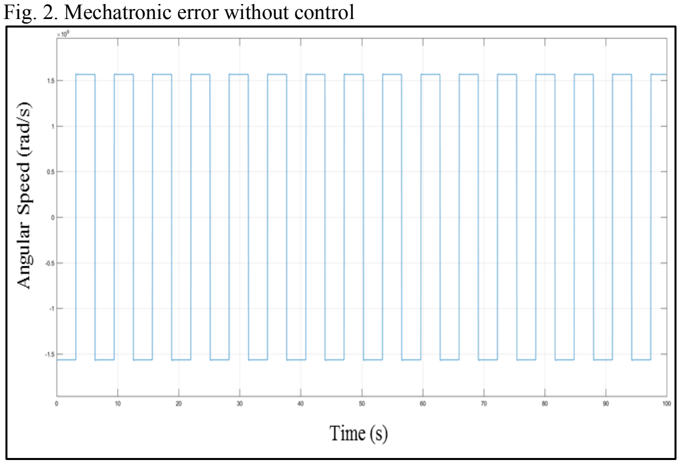


Fig. 3. Mechatronic error with control

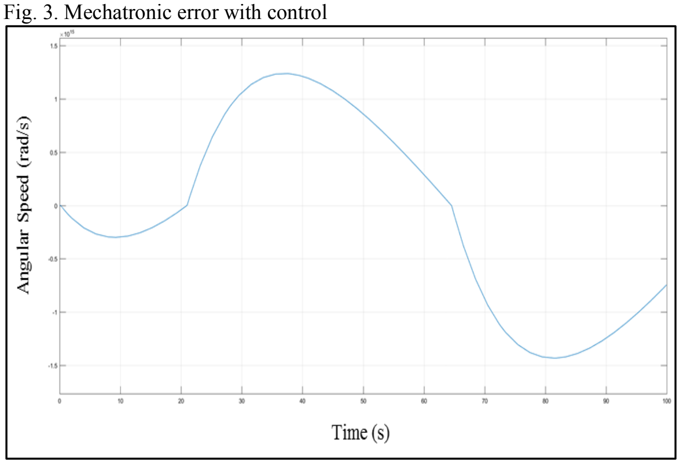


Fig. 4. Mechatronic behavior without control

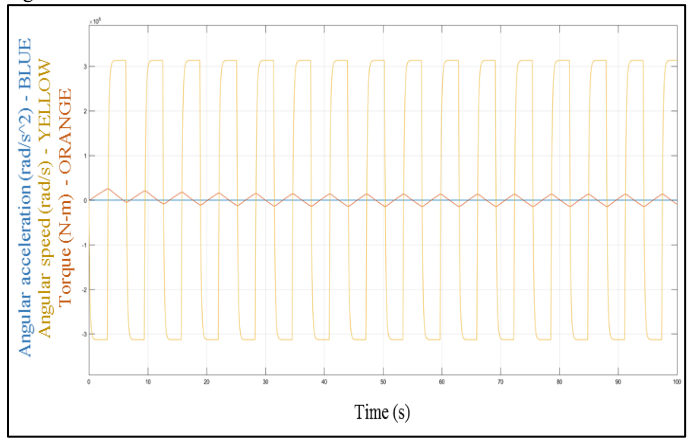
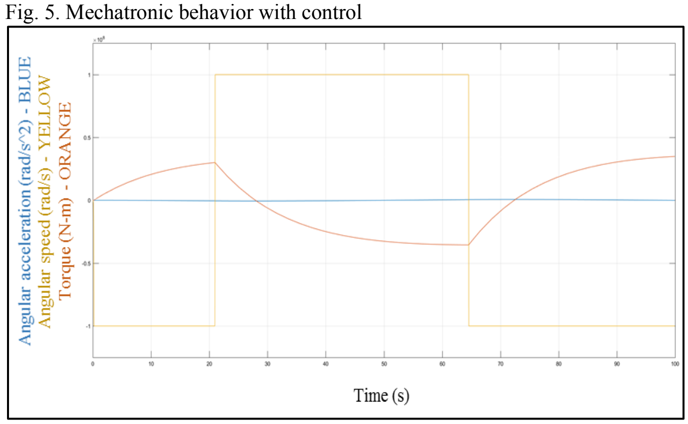


Fig. 5. Mechatronic behavior with control



B. Bio-pneumatic Control

For this system, also the space-state representation was constructed in equation (12), but in this case, the differential equations that represent the bio-pneumatic system of the ventilator were not used as the math would be really complex, so instead, an alternative method was implemented in which the coefficients of both 3x3 matrixes were manually calculated by splitting each coefficient of the representation in 5 differential equations that after being solved returned the value of the respective coefficient. The states vector is formed by air volume, flow, and pressure, being the 3 variables time dependent.

$$\frac{d}{dt}\begin{bmatrix} Vs \\ Fs \\ Ps \end{bmatrix} = \begin{bmatrix} a11 & a12 & a13 \\ a21 & a22 & a23 \\ a31 & a32 & a33 \end{bmatrix}\begin{bmatrix} Vs \\ Fs \\ Ps \end{bmatrix} + \begin{bmatrix} b11 & b12 & b13 \\ b21 & b22 & b23 \\ b31 & b32 & b33 \end{bmatrix}\begin{bmatrix} Ve \\ Fe \\ Pe \end{bmatrix} \quad (12)$$

Next, by obtaining the open-loop poles of the state-space representation, the system could be classified according to Table 1. To obtain those values, from equation (12) the first 3x3 matrix (matrix A) is extracted and subtracted to a lambda identity matrix (where lambda is an open-loop pole) so that this new matrix could be populated with the proper calculated coefficients for solving its determinant:

$$det(\lambda I - A) = det\begin{bmatrix} \lambda - a11 & -a12 & -a13 \\ -a21 & \lambda - a22 & -a23 \\ -a31 & -a32 & \lambda - a33 \end{bmatrix} = det\begin{bmatrix} \lambda + 279.8462 & -0.0692 & -0.8276 \\ -0.3294 & \lambda + 424.28 & -0.9890 \\ -0.00398 & -0.0847 & \lambda + 948.4374 \end{bmatrix} \quad (13)$$

By solving the determinant of the equation (13) and equaling it to zero, the next open-loop poles were obtained:

$$\lambda_1 = -279.8462$$
$$\lambda_2 = -424.28$$
$$\lambda_3 = -948.4374$$

Based on these results, the bio-pneumatic system was also considered overdamped as the open-loop poles do not have an imaginary part and as the values following the sign are far from zero, the system is quite stable, but, in order to ensure that stability, higher negative poles were proposed for the closed-loop controller system by multiplying each open-loop pole for 10 so that each one could reach the range of the thousands; as a result of these operations, the next closed-loop poles were proposed:

$$s_1 = -2800$$
$$s_2 = -4250$$
$$s_3 = -9500$$

Having the open and closed-loop poles, equation (12) was reduced by substituting the input vector from the right with the proper gains matrix (matrix K_C), that ensured the correct dimensions of 3x3 after being multiplied by the second 3x3 matrix (matrix B) in order to be added with the first addend of the sum which was also subtracted to a lambda identity matrix as in the calc of the open-loop poles; these procedures are presented in equation (14):

$$\lambda I - A + BK_c = \begin{bmatrix} \lambda - a11 & -a12 & -a13 \\ -a21 & \lambda - a22 & -a23 \\ -a31 & -a32 & \lambda - a33 \end{bmatrix} + \begin{bmatrix} b11 & b12 & b13 \\ b21 & b22 & b23 \\ b31 & b32 & b33 \end{bmatrix}\begin{bmatrix} k_{V_S} & 0 & 0 \\ 0 & k_{F_S} & 0 \\ 0 & 0 & k_{P_S} \end{bmatrix} \quad (14)$$

Then, equation (14) was reduced, again populated with the proper calculated coefficients, and equaled to a control polynomial with the proposed closed-loop poles in equation (15):

$$det(\lambda I - A + BK_c) = det\begin{bmatrix} \lambda + 279.8462 + 255 \cdot k_{V_S} & -0.0692 + 0.2425 \cdot k_{F_S} & -0.8276 + 0.8892 \cdot k_{P_S} \\ -0.3294 + 0.2905 \cdot k_{V_S} & \lambda + 424.28 + 113.28 \cdot k_{F_S} & -0.9890 + 99.4880 \cdot k_{P_S} \\ -0.00398 + 0.0082 \cdot k_{V_S} & -0.0847 + 28.6451 \cdot k_{F_S} & \lambda + 948.4374 + 936.06 \cdot k_{P_S} \end{bmatrix} = (\lambda + 2800)(\lambda + 4250)(\lambda + 9500) \quad (15)$$

By solving equation (15), the equations (16), (17) and (18) were obtained with the gains in terms of air volume (a), flow (b) and pressure (c); thus, by solving these 3 equations with the Excel Solver Add-in, the gains could be finally determined:

$$255a + 113.38b + 936.06c + 1652.5636 = 16550 \quad (16)$$

$$350042.96388a + 139291.20445b + 28911.82955ab + 103280.63909bc + 659112.80094c + 238695.29270ac + 786552.65827 = 78875000 \quad (17)$$

$$102612770.72372a + 27428278.48287ab + 26336503.79685abc + 101275806.32086ac + 30100945.61305b + 28902761.07026bc + 111143686.77577c + 112610909.55341 = 113050000000 \quad (18)$$

After the calculations of the Excel Solver Add-in, the feedback state gains were obtained and represented in equation (19) as a vector:

$$a = k_{V_S} \approx 36.15740537$$
$$b = k_{F_S} \approx 32.40663888$$
$$c = k_{P_S} \approx 2.139855211$$

$$k = [36.15740537 \quad 32.40663888 \quad 2.139855211] \quad (19)$$

With the feedback state gains vector, the control simulations in Simulink could be adjusted and run obtaining the graphs from Fig. 6 to Fig. 9:

Fig. 6. Bio-pneumatic error without control

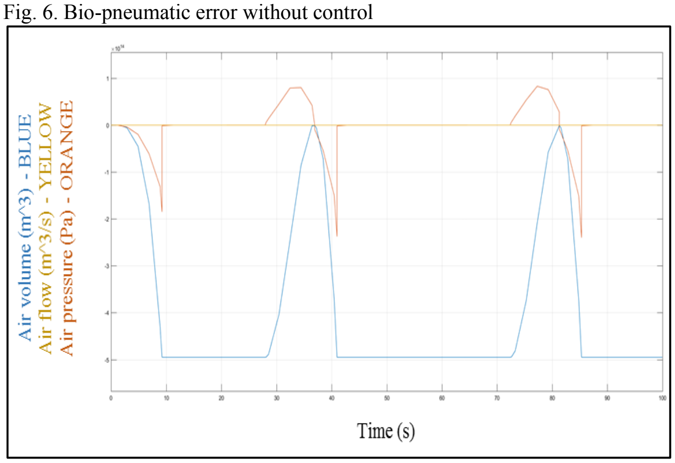


Fig. 7. Bio-pneumatic error with control

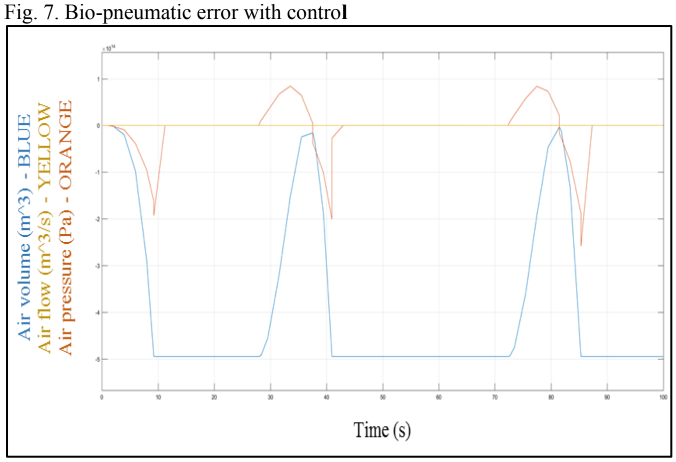


Fig. 8. Bio-pneumatic behavior without control

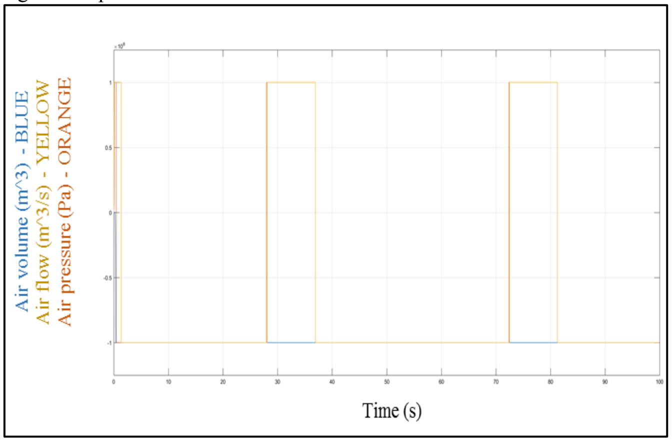
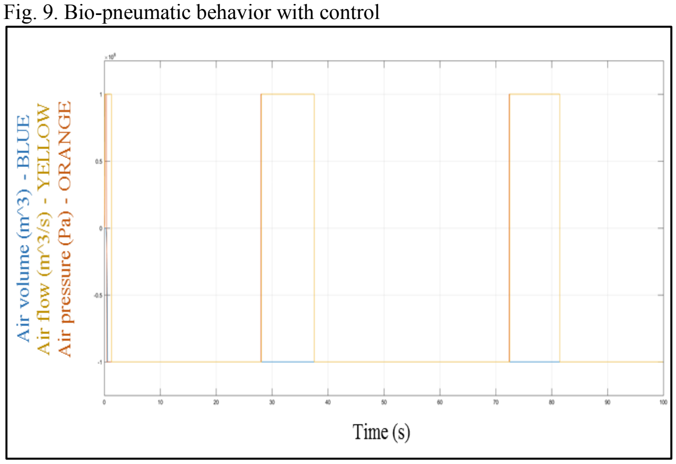


Fig. 9. Bio-pneumatic behavior with control



IV. RESULTS

A. Mechatronic Control

The closed-loop poles are very large compared to the eigenvalues because, although they are not positive, their negativity is minimal since 2 of them are practically 0 and the other is very close to 0, so the instability is still latent, so 10,000, 1,000 and 10 times larger poles respectively are proposed so that they are all values between -10 and -100. Having such large closed-loop poles forces the control gains to be too large to force the system to relocate its unstable eigenvalues to the stable control condition that is required of the system. Derived from the above, the control can be performed but with a cost, the increase in the magnitude of the error. When comparing both error graphs, it is denoted that in the same time interval the oscillation of being a quadratic pulse train, with the control it became a single wave with a frequency much lower than the previous one, however, although the oscillation of the error decreased, its magnitude increased from the order of 10 ^ 9 to 10 ^ 15. Analogously, the same behavior is observed in the system output signals:

- The angular speed (yellow) went from being a rounded semi-quadratic pulse train to a single rectangular pulse.
- In the case of angular acceleration (blue), the change is negligible because, as a priori and posteriori the control, the angular speeds were stable and did not show changes in time more than momentarily, therefore, the derivative is clearly 0 in the entire interval.
- The torque (orange) went from being a train of triangular pulses to a semi-triangular wave with rounding.
- In the case of angular speed and torque, it is observed that the operating range increased from 10 ^ 6 to 10 ^ 8; therefore, it is concluded that although the control was successful by considerably decreasing the oscillation, the magnitude of the error will increase by forcing the poles of the system to diverge greatly from their eigenvalues.

B. Bio-pneumatic control

Very different from the previous mechatronic control scenario, the eigenvalues of this system are already quite stable since they are in the range of -100 to -1,000, so it is viable in this case to apply a control where the closed loop poles are 10 times larger than the same eigenvalues by locating them in the range of -1,000 to -10,000. It is denoted that by comparing the graphs there is not a considerable change a priori against a posteriori of the control because, as it was already explained, the initial system is already quite stable, however there are some slight changes that are worth commenting on:

- For errors, at least for volume (blue) and pressure (orange), a smoothing in the direction changes of the signal is observed, making it less aggressive and promoting a more progressive change not so abrupt.
- In the behavior of volume, force and pressure, there is no considerable change other than the fact that it is almost negligible that the time at which the force signal is arisen slightly increases. These very small changes are therefore due to the fact that the control gains (-32, -36 and -2 respectively) are really low compared to their own values and the closed-loop poles; consequently, it is denoted that the a priori and a posteriori control operation ranges remain the same at 10 ^ 14 for the error and 10 ^ 8 for the behavior of the 3 bio-pneumatic variables.

V. CONCLUSIONS

- The implemented control has provided an increase in the response time and has reduced the error, thus, theoretically, both controls must provide stability and a higher performance to the ventilator prompting an appropriate assistance to the hospitalized victims of the COVID-19.
- Even though the system is now more reliable because of the control it is still unable to be used as an emergency ventilator since it requires tests to analyze the real performance of the machine with this control parameters.

- Comparing the obtained results with the ones from the variable-gain control ventilator of the Eindhoven University of Technology, it was confirmed that a state space control is suitable for a faster response time and an error reduction; however, in the simulations this reduction is better appreciated in the mechatronic control than in the bio-pneumatic one, thus, real testing must be conducted in order to assure an homogeneous error reduction throughout this cascade vectorial controller.
- The cost of the error reduction with our proposal is a momentaneous initial rise of the error magnitude, due to the high control gains; however, it was demonstrated that after that instant, the error becomes a wave with a very low frequency compared to the open-loop one that presented a pulse train form with a high frequency. Besides this situation, the computational cost of the algorithm has been low compared to the results achieved regarding the error, making transcendental this contribution as it was proved that even with minimal calculations it is possible to accomplish this reduction.
- The state space control has many advantages over the classic control since it permits more internal variables, which helps to obtain more reliable outputs in any system.
- The nearest the eigenvalues are to "0", the highest the control gains must be in order to give stability to the system and relocate them.
- The association of different systems require the full understanding of all the involved variables to determine its influence and impact within a complex environment.

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It is important to clarify that the all information used in this work does not include any modification from the open source design available for the mechanical ventilator [6], neither on hardware not on software, because this work scope just has the educational intention to verify how this kind of medical devices reacts when use a Kalman filter; therefore, any authors rights from [6] were not affected.

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