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A Brief History of Computational Electromagnetics in Microwave Engineering—A Personal Retrospective

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ABSTRACT As the MTT-Society is celebrating seventy years of existence – an age approaching the average human lifespan – the number of members who witnessed the early years of microwave engineering is slowly declining. While their achievements and contributions are well documented, less is known about their perceptions and personal experiences of living through the evolutionary stages of our Society. Yet, this experience is valuable in understanding how and why new ideas and methodologies evolve, survive initial rejection, achieve acceptance, and then either persist or give way to new developments. This personal and hence, subjective retrospective of computational electromagnetics in microwave engineering reflects on the synergy between the evolution of field-theoretical and algorithmic developments, computing science and technology, and microwave technology, particularly during the first decades of the MTT-Society. It also highlights several personal experiences and interactions that have spawned new ideas and initiatives, and thus contributed to the present state of the art in computational electromagnetics.

INDEX TERMS Computational electromagnetics, electromagnetic field theory, numerical methods, frequency domain modeling, time domain modeling, microwave circuits, computer-aided design, multi-level system design, MTT 70th Anniversary Special Issue.

I. INTRODUCTION

Computational electromagnetics is generally defined as the theory and practice of solving Maxwell's equations or equations derived from them, on digital computers. This definition describes what it is today but provides little insight into its origins and its evolution. In fact, it was conceived by scientists, articulated by mathematicians, powered by computer engineers, and instrumentalized by electrical engineers. Looking back at the history of computational electromagnetics we realize that it could not have evolved without the complex interplay between these protagonists.

When I was planning to contribute a paper on the history of numerical techniques and modeling to this special MTT anniversary issue, my first thought was to write a chronological review of computational electromagnetics. On second thought I realized that such a paper would be too extensive and merely repeat what my distinguished and knowledgeable colleagues had already said and written. Hence, I decided to prepare instead an account of my personal journey through the world of microwaves with a focus on the evolution and impact of computational electromagnetics. While subjective, it reveals some of the context in which concepts, capabilities and design tools evolved, how new ideas emerged, and how they morphed into practical engineering tools while facing challenges of acceptance by the microwave community, and sometimes lively rivalry between champions of "competing" methods. It also provides an opportunity to reflect on the synergy between numerical modeling, the evolution of microwave technology from waveguide through planar and quasi-planar to monolithic integrated circuits, and the meteoric rise of computer technology.

Since the origin of computational electromagnetics goes back further than the human lifespan, we rely on the vast literature in physics and mathematics rather than our personal memories to explore its early evolution. I will thus briefly retrace the concepts and ideas that still form the essence of computational electromagnetics today, up to the early 1950s. Since I am not much older than the MTT-Society, I shall relate how I personally experienced some defining early developments during my own journey through the past seventy years, and I will share some personal memories of special colleagues who are no longer with us, but who had a lasting impact on my professional and personal life.

II. ESSENTIAL CONCEPTS OF COMPUTATIONAL ELECTROMAGNETICS

The first name that comes to mind when contemplating computational electromagnetics is James Clerk Maxwell [1] who synthesized what was known about electricity and magnetism at the time, into the famous equations named after him. It aims to solve these equations as re-stated by Oliver Heaviside [2], or to find solutions to equations derived from them, such as the wave equation named after Hermann Helmholtz [3].

For the mathematician, this task amounts to solving differential or integral equations, subject to given boundary, material, and initial conditions. Ideally the solutions would be expressions in elegant, closed form that can be evaluated with minimal computational effort. However, such solutions exist only for a limited number of canonical structures having rectangular, cylindrical, or spherical geometries.

For most electromagnetic problems of practical interest to the engineer, a solution $u(\boldsymbol{r}, t)$ can only be found *approximately*, with an error that is generally reduced at increasing computational expense. The common strategy is to approximate $u(\boldsymbol{r}, t)$ by a trial function $\tilde{u}(\boldsymbol{r}, t)$ which is a sum of known basis or expansion functions $\varphi_n(\boldsymbol{r}, t)$ with unknown coefficients:

$$u(\boldsymbol{r}, t) \approx \tilde{u}(\boldsymbol{r}, t) = \sum_n a_n \varphi_n(\boldsymbol{r}, t). \tag{1}$$

The coefficients a_n must then be determined such that $\tilde{u}(\boldsymbol{r}, t)$ approaches the exact solution as closely as possible. (A perfect approximation would usually require an infinite set of expansion functions, but in practice, n can only be finite. A well-known example of such an expansion is the Fourier series). This strategy is known as projective approximation in a linear function space or inner product space; it amounts to a discretization of the problem and forms an extensive mathematical framework developed during the early 1900s. Virtually all numerical methods in electromagnetics can be formulated in terms of this general approach, above all the method of moments and its variants, but also the finite element, finite difference, and transmission line matrix methods, even though they were originally derived differently. The difference between them boils down to the choice of the expansion functions φ_n and to the strategy employed in finding the coefficients a_n. This strategy involves the choice of a second set of so-called *weighting functions* and is known as the *method of weighted residuals*. It leads to a matrix equation which is a linear system of equations for determining the expansion coefficients a_n. The approach seems quite straightforward, but as usual, the devil is in the details [4].

The monumental work accomplished by scientists and mathematicians in the 17th to the 20th century clearly forms the experimental and field-theoretical foundation of modern computational electromagnetics. As early as 1690, Christiaan Huygens [5] articulated the essence of today's numerical methods in his work on the nature and properties of light, which Maxwell discovered later to be an electromagnetic wave. The famous principle named after him is illustrated in Figs. 1 to 3 which he published in 1690 in his *Traité de la Lumière (Treatise of light)*.

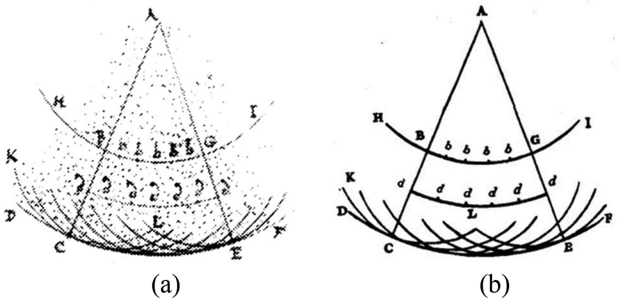

FIGURE 1. Huygens' principle as shown (a) in the manuscript, and (b) in the printed edition of his Traité de la Lumière. The ether particles, visible in the manuscript but omitted in print, clearly emphasize the discretized nature of Huygens' model. (After [5]).

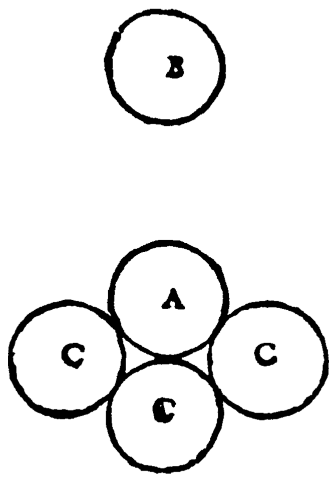

FIGURE 2. Huygens' elementary scattering event in which an ether particle B hits a particle cluster that spreads the incoming energy in all directions in the form of a spherical wave. (After [5]).

Fig. 1(a) shows the representation of his famous principle as it appears in his original manuscript, while Fig. 1(b) shows the printed version which conspicuously omits the dots that Huygens drew to mark the particles of his fictitious *ether*. However, these are significant since they illustrate the *space-discrete* nature of his model. Unlike Newton's corpuscular model in which the light particles travel at high velocity through space, Huygens' oscillating ether particles remain stationary, transmitting light in the form of shock waves from particle to particle, like sound is propagated through the air. The elementary process is explained in Fig. 2 in which a particle B hits a cluster of particles that spread the incoming energy.

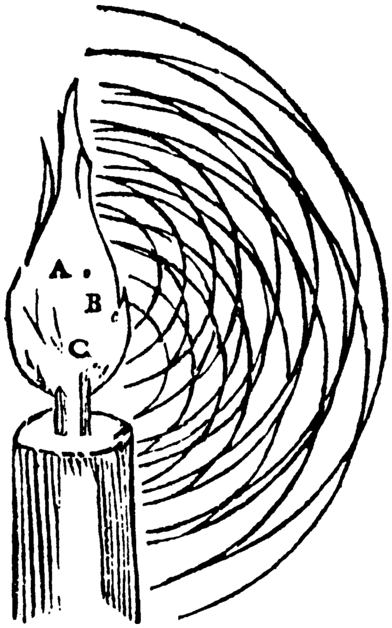

FIGURE 3. Huygens' candle, representing the emitted light as superposition of spherical waves emanating from point sources in the source region. It foreshadows the concept of a field as the convolution of a Green function with a source distribution. (after [5]).

Huygens thus suggests that light is a superposition of spherical wavelets, which amounts to its representation by spherical basis or expansion functions, as formulated in (1) and illustrated by the circular wavefronts in Fig. 1(a) and (b). Fig. 3 goes one step further by explaining that the radiation from a distributed source – a candle flame – is the sum or superposition of spherical waves emanating from discrete point sources.

Huygens explains: "In the flame of this candle we can distinguish points A, B and C; the concentric circles drawn around each point represent the waves that originate from them. We must imagine that all other points on the surface and inside the flame also emit such waves. But since the percussions in the center of these waves are quite irregular, do not imagine that the wave fronts follow each other at equal distances. I have drawn equidistant circles simply to mark the progress of a single wavefront in equal time steps, rather than to represent several wave fronts emerging from a common source point." (Translation by the author.)

In mathematical language, the circles in Fig. 3 are snapshots of the free-space Green function, pictured at equal time intervals, and the total field represents its convolution with the source (candle flame) function. Hence, the drawing depicts a space- and time-discrete model of wave propagation.

While Huygens provides many ingenious geometrical constructions that explain and demonstrate properties of light, such as reflection, refraction, and birefringence, he does not offer a single formula in his Treatise. In fact, Huygens' mechanical model, while fictitious, correctly predicted the behavior of electromagnetic fields before the mathematical formulations that we owe to Maxwell and Heaviside, which enable us to solve electromagnetic field problems and to design microwave circuits and systems. Fortunately, Maxwell's theory did not require a revision in the wake of Einstein's theory of relativity because the magnetic field is a manifestation of the relativistic change of the electric field of moving charges. (This is most likely the reason why my first course in electricity and magnetism did not include the theory of relativity.)

III. FROM FIELD MODELS TO CIRCUIT MODELS

The solution of a three-dimensional electromagnetic field problem always depends on the geometry and properties of the boundaries and media that contain the field. This suggests that every new geometry must be solved *"from scratch"*. This view is typically reflected in most early textbooks that present a collection of electromagnetic problems which can be solved analytically or semi-analytically. From the engineering point of view this approach is labor-intensive and time-consuming. For the engineer the electromagnetic field solution is not a final goal, but a means to obtain the system properties of a structure, such as radiation characteristics of an antenna, scattering parameters of a coupler or filter, or the transfer function of a communication system. However, the goal of the microwave designer is to create or synthesize structures having electromagnetic properties that satisfy desired specifications, which is the inverse of analysis. Since field-based synthesis is more challenging than analysis for several reasons, the current design practice is to employ optimization strategies in combination with repeated analysis. This makes it even more important to develop field-based methods that lead to fast, yet accurate models if one wants to use them in concert with optimization.

The early development of microwave technology was driven by the importance of radar in World War II. To obtain sufficient resolution and directivity with antenna structures of compact size, signals at wavelengths in the centimeter range were required. The development of the klystron and the magnetron made it possible to generate such signals. However, the electronic circuit technology used at lower frequencies was unsuitable for the microwave range and was replaced by waveguide technology based on metal pipes of rectangular or circular cross-section, also known as microwave plumbing. From the electromagnetic point of view, these are structures with boundaries conformal with the axes of an orthogonal coordinate system, in which Maxwell's equations can be analytically solved using the *separation of variables*. The solutions, or eigenmodes, are trigonometric functions in the rectangular, and Bessel functions in the circularly cylindrical case.

The separation of a waveguide mode into a longitudinal and a transverse solution yields a one-dimensional transmission line model characterized by a frequency-dispersive phase velocity and characteristic impedance. The field solution becomes more complex in the presence of discontinuities, such as conducting or dielectric obstacles, apertures, or junctions.

The field perturbation caused by a discontinuity is then expressed in terms of propagating and evanescent modes of the waveguide. The evanescent modes embody reactive energy stored at the discontinuity that can be modeled by an equivalent reactance. The field-theoretical description of a waveguide structure is thus reduced to an approximate circuit-theoretical problem that can be solved using Kirchhoff's laws or the Smith Chart. In fact, the first microwave simulators did not solve field models but circuit models, as pointed out in Section V below.

A more fundamental and general method for turning Maxwell's field equations into a network model has been proposed in 1944 by Kron [6]. At the same time, Whinnery and Ramo [7] published several two-dimensional examples of Kron's network, realized by a square mesh of series inductances and shunt capacitances, to experimentally model the fields in both rectangular and circular cylindrical waveguides. The difference equations known from the FDTD method are already given in cylindrical coordinates in [7]. The PEEC method is also based on a network model, as well as the TLM algorithm which can be directly obtained by replacing the LC-sections of Krohn's model by short sections of transmission lines. The work by Kron, Whinnery, and Ramo thus foreshadowed the dominant time domain methods of computational electromagnetics by more than twenty years.

IV. MICROWAVES AT THE DAWN OF THE MTT-SOCIETY

A. COMPUTATIONAL TOOLS

When the MTT-Society came into being, the physical and mathematical foundations of electromagnetics were clearly established, and waveguide technology was well developed and documented. The three principal computational tools available to microwave practitioners and students during the early 1950s were the slide rule, the Smith Chart, and tables of functions, including roots, exponentials, logarithms, and higher functions that occurred in the solutions of differential and integral equations in various coordinate systems. However, the type of digital computers we use today for solving general electromagnetic scenarios were still in their early stages of development.

1) THE SLIDE RULE

At the dawn of the MTT-Society I was just eleven years old and knew little about electromagnetics or microwaves. However, I remember receiving my first slide rule for my birthday and enjoying the ability to multiply and divide numbers, compute roots, powers, and trigonometric functions literally by sleight of hand. It came in handy when I entered the electrical engineering program at the Technical University of Aachen in 1959. At that time, the slide rule, conceived in the 17th century, was still the predominant calculation tool of scientists and engineers. In fact, one could easily pick out engineers in a crowd by the slide rule that stuck out of their vest pocket. Engineering students also used to hold popular contests where several competitors drew their slide rules upon the unveiling of a complicated formula, striving to solve it in the shortest time and with highest accuracy. I still own my first slide rule and a historical user manual by Pickworth [8] published in 1919.

2) THE SMITH CHART

Another ubiquitous computing device used in microwave engineering at the time was the Smith Chart [9]. It was the dominant tool for graphically solving problems involving transmission lines, matching, and feeding networks. Note that the Smith Chart can only be used when the microwave problem is cast in the form of a transmission line circuit problem. To transform an electromagnetic field scenario into an equivalent circuit, one had to either solve the field problem analytically and evaluate the resulting expression with the aid of tables of higher mathematical functions or the slide rule. Several field solution methods for the equivalent circuits of discontinuities in rectangular and circular waveguides had been presented by Schwinger and Saxon [10] in the 1940s in a series of lectures, later published as a monograph. The Waveguide Handbook by Marcuvitz [11], first published in 1950 as part of the MIT Radiation Laboratory Series, was a treasure trove for circuit models of waveguide discontinuities, junctions, transitions, and composite structures in the form of equivalent networks together with analytical expressions for the network elements, and normalized graphs of their numerical values.

The Smith Chart was then, and still is, an indispensable teaching tool in microwave engineering courses. I vividly recall my first microwave course as an undergraduate in 1962. A large Smith Chart, beautifully graved into a square-shaped blackboard, dominated the stage of the classroom, and the professor drew colorful lines and circles on it like an astrologer foretelling the future.

Today, most electromagnetic simulators and network analyzers still include a Smith Chart option for the display of results. Finally, the Smith Chart remains a favorite identity symbol of the microwave community, gracing the logo of many microwave events, companies, and institutions, including the header of the mtt.org website.

3) TABLES OF HIGHER FUNCTIONS AND INTEGRALS

When the accuracy of the slide rule, which is about three decimal significant digits, was insufficient for the computation of a transcendental or an exponential function, one needed a table of functions, such as the classical volume by Jahnke and Emde [12] which tabulates the most frequently encountered functions with typically five significant digits. Many of the tables were calculated using mechanical computing machines. In the 1960s, tables with higher resolution became available, such as the Handbook of Mathematical Functions by Abramovitz and Stegun [13], featuring up to ten significant digits. This high accuracy could be achieved thanks to the emerging mainframe digital computers. Solutions of all imaginable integrals were available in book form as well. I remember using two beautiful volumes titled “Unbestimmte Integrale” *(Indefinite Integrals)* and “Bestimmte Integrale” *(Definite Integrals)* by Gröbner and Hofreiter [14], [15] to solve some discontinuity problems. Two unusual features made a lasting impression on me. Firstly, the integrals in both books were not typeset but reproduced in beautiful facsimile handwriting. Secondly, Hofreiter acknowledges his spouse, Dr. Margarete Hofreiter, for having carefully verified the correctness of each and every integral in both books – a true labor of love!

4) DIGITAL COMPUTERS

Some programmable electronic digital computers were already available in the early days of the MTT-Society, but they were built with traditional circuits and electron tubes and could hardly solve real-world electromagnetic problems. The first transistorized digital computers emerged in 1953 and became commercially available. Research institutions, companies, and universities began to acquire large mainframe computers during the early 1960s. Installed in a climate-controlled computing center, they were often shared between the administration and researchers. To illustrate what it was like to access this precious resource in these days, here is one of my first attempts to use it as a student. I wanted to compute the characteristic impedance and the phase velocity of a ferrite-loaded coaxial line which required the calculation of Bessel and Neumann functions. I punched holes into a stack of cards, each card containing one command line of my FORTRAN program, and brought it to the Computing Center, expecting to get the result the next day on a z-folded ream of printer paper. But instead of the results, I found only my stack of cards with the note: “Job rejected, requires more than 1.2 kilobytes of memory”. This sounds incredible today, but thus began my journey in computational electromagnetics.

B. EMERGING MICROWAVE TECHNOLOGIES

Several technological developments have driven the evolution of numerical modeling of microwave circuits and systems in the 1950s, 1960s, and 1970s. Among them, two have been the focus of my own research during my doctoral studies in Grenoble, France, and my early years as a faculty member at the University of Ottawa, Canada. The first was the development of microwave ferrites, and the second was the rise of planar and quasi-planar circuit technology. Initial formulae for stripline and microstrip design were mostly based on approximate expressions obtained by curve-fitting of measured results, perturbation solutions, or approximate models such as the parallel-plate waveguide model that had closed-form field solutions. As computers became more accessible and powerful, computer-aided design (CAD) began to supersede traditional design practices.

1) FERRITES

Ferrites are ferrimagnetic materials having low electric conductivity. They are thus transparent at microwave frequencies. This property enables a strong interaction of the spin magnetic moment with an applied microwave field, resulting in a precession of the magnetization vector in the material. The natural precession frequency depends on an externally applied dc magnetic field. This gyromagnetic interaction can be exploited to build non-reciprocal microwave components such as circulators, isolators, Faraday rotators, as well as magnetically tunable filters and resonators.

The research objective of my doctoral thesis was to develop a method for measuring the magnetic resonant bandwidth of high-Q gyromagnetic YIG spheres, fabricated at the French *Centre National de Recherche Scientifique (CNRS)* near Paris. The experimental technique was based on the field analysis of coupling between an electromagnetic cavity and a YIG sphere of very narrow bandwidth, modeled by an equivalent circuit. It is described in my first paper [16], published in 1967 in the Transactions of the French Academy of Sciences, under the sponsorship of Louis Néel, Academician and Nobel Prize in Physics. Naturally, I was very proud of this paper and the prestigious circumstances of its publication.

2) PLANAR AND QUASI-PLANAR CIRCUITS

The invention of the stripline [17] and the microstrip [18] in the early 1950s ushered in three decades of planar and quasi-planar technology development. It saw the evolution from waveguide technology to smaller, lighter, and low-cost printed circuits. The significance of this development has often been likened to that of the invention of the printing press. While the technological advantages of planar circuits over waveguide technology for low-cost and light-weight systems were significant, the theoretical treatment and the need for highly accurate layout design brought new challenges. Waveguide technology was well developed and documented (i.e., the Waveguide Handbook by Marcuvitz [11]), but the electromagnetic field in planar and quasi-planar structures was more difficult to analyze and called for new mathematical approaches. The main reasons were the hybrid nature of the modes of propagation due to the inhomogeneous cross-sections of these transmission media, and the singular behavior of fields at the edges and corners of conducting strips. The earlier models of the new transmission media were quasistatic equivalent waveguide approximations, or empirical expressions for the propagation constant and impedance of the line sections [19], combined with equivalent circuits or S-parameters for discontinuities and devices. One versatile semi-analytical approach that was frequently used to characterize inhomogeneous waveguides was the transverse resonance method [20] where a short section of inhomogeneous waveguide is short- or open-circuited at both ends to form a $\lambda_g/2$ resonator, and the structure is analyzed in transverse direction by considering it to be a piecewise homogeneous waveguide carrying longitudinal-section electric (LSE) and longitudinal-section magnetic (LSM) modes. The resonant frequencies were the solutions of a transcendental equation. If the structure contained longitudinal metal strips, such as metal fins, they were treated as transverse waveguide discontinuities, many of which could be found in the Waveguide Handbook [11] and included as equivalent lumped elements into the transverse resonant circuit.

V. EMERGING NUMERICAL METHODS AND TOOLS

Waveguide models did not account for the radiation effects that occurred in open transmission lines, such as microstrip. In fact, the radiation from microstrip patches made them suitable for printable antennas and arrays. At the time of the invention of stripline and microstrip, the first transistorized programmable electronic computers became available and opened new possibilities for computing the wave properties of planar and coplanar structures including surface modes and radiation.

A. THE RISE OF THE PRINCIPAL NUMERICAL METHODS

The period from 1965 to 1980 saw the emergence of the principal computational methods that require digital computers to be of practical use. Among the most popular algorithms that came into being during that period, and remain reliable workhorses of electromagnetic simulators, are the *Eigenmode Expansion* or *Mode Matching Method* already described by Schwinger and Saxon [10], *the Finite Difference Time Domain* (FDTD) method formulated by Yee [21] in 1966 and instrumentalized by Taflove [22] in 1980, the electromagnetic formulation of the *Finite Element Method* (FEM) by Silvester [23] in 1969, the *Method of Moments* (MOM) by Harrington [24], [25] in 1969, *the Transmission Line Matrix (TLM) Method* by Johns and Beurle [26], [27] in 1971, the *Spectral Domain Method* (SDM) by Itoh and Mittra [28], [29] in 1973, the *Partial Element Equivalent Circuit* (PEEC) method by Ruehli [30] in 1974, and the *Finite Integration Technique* (FIT) by Weiland [31] in 1977. Numerous other related solution methods for integral and differential forms of Maxwell's equations in the frequency and time domains were proposed during that period, most of which are described in books, papers, and selected reprint volumes referenced in Section IX.

It is remarkable that the first publication of Yee's FDTD algorithm in 1966 and Johns' TLM algorithm in 1971 remained largely unknown in the electromagnetics community for several years. I believe that the main reason was the lack of computing power at the time, but also the focus on traditional analytical approaches. It took almost ten years before Yee's algorithm was resurrected by Taflove and Brodwin [32], and it took several years for Johns' algorithm to spread beyond his research group. One attractive feature of Johns' TLM and Yee's FDTD was that they could generate dynamic electromagnetic solutions in both space and time, opening the door to life-like transient simulation and visualization that could even be reversed in time on the computer. Furthermore, a single simulation run with an impulse excitation yielded a wide-band frequency response after Fourier transform. The possibilities and the innovative potential of these methods were unprecedented. One could even argue that they are discrete alternatives to Maxwell's continuous model, like the mechanical model proposed by Huygens [5].

B. THE EVOLUTION OF COMPUTER-AIDED DESIGN

While these numerical methods were being developed, most microwave design continued to employ semi-analytical approaches and approximate design formulas based on numerical simulations and experiments. As computers became more powerful and more accessible, microwave design entered the realm of computer-aided design. This approach originated with the seminal work by Besser in the late 1960s [33] and published in 1971 [34]. It featured an S-parameter database of circuit elements and components, such as transmission line sections, discontinuities, and transistors, that could be concatenated using a BASIC program to form a microwave circuit model and to simulate it. While Besser's first generation program SPEEDY (1970) lacked optimization capability, his second-generation program COMPACT included optimization and noise analysis. It was followed by the third generation SuperCOMPACT in 1981. This program was very successful and was acquired by Ansoft in 1997, then by Ansys in 2008, and is now called DesignerRF.

In 1983, Childs and Abronson founded EEsof and developed a competing software TouchStone which was specifically designed to run on the newly available IBM PC. EEsof was acquired by Hewlett-Packard in 1993 and became part of Agilent Technologies and then of Keysight Technologies. Both simulators went through several updates and are still widely used today as part of microwave design suites.

A crucial component of these and similar tools is the optimization feature which has been pioneered by Bandler and his associates [35], notably the Space Mapping approach which relates the parameters of a fine model to those of a coarse model that can be analyzed and hence, optimized with much higher speed and less computational resources. This is a decisive requirement for the design of complex microwave circuits, such as MMICs, and makes it possible to employ electromagnetic field models in an optimization loop. In 1983 Bandler founded his company Optimization Systems Associates (OSA) which was acquired by Hewlett-Packard 1997 and became part of HP EEsof.

Since the early 1980, CAD tools that included single and hybrid field simulation engines became available, such as *Sonnet* and *Momentum* based on the method of moments, *HFSS* based on the finite element method, *MAFIA* based on the finite integration method (now *CST Studio Suite* including also TLM, finite element, multilevel fast multipole, and particle-in-cell methods), FEKO based on the method of moments, finite elements and FDTD, *XFdtd* and *Empire XPU* based on the

FDTD method, and µ*Wave Wizard* based on mode matching and finite element methods, to name only a few.

C. CHALLENGES FACING NUMERICAL METHODS

A simple timetable of the evolution of numerical methods reveals little about the reception they received when they were first presented to the professional community, and the difficulties they had to overcome before they were generally accepted into the mainstream of engineering design practice.

One objection many proponents of computational methods had to face, was the disdain of colleagues who specialized in analytical solutions of electromagnetic problems employing advanced mathematical concepts. I vividly recall a conversation I had in 1975 with a colleague from the Department of Mathematics about solving Maxwell's equations with one of the emerging time- and space-discrete algorithms, such as FDTD or TLM. His reaction was almost predictable: he felt that such methods were nothing but *mindless number-crunching*.

Scientists and engineers are generally very conservative when it comes to established and time-proven practices. Consider the fact that virtually all microwave concepts are based on the time-harmonic case. There are many good reasons for this, both mathematical and technical. However, the complex notation in both circuit and field formulations, albeit unphysical, has become a virtual reality in the mind of microwave engineers. This can become a challenge for someone trying to promote a time-domain numerical technique to a microwave audience.

One such experience dates to the 1988 MTT-S Intl. Microwave Symposium in New York where I took part in a workshop discussion on the suitability of time domain numerical methods for microwave applications. Several participants were wondering about the wastefulness of solving microwave field problems in the time domain because the results were only valid at one frequency when the problems involved lossy frequency-dispersive media and boundaries with complex constitutive parameters and impedances. This would invalidate the claim that time-domain simulations yield accurate wide-band solutions in a single run. The question strongly resonated with the MTT community, leading Fred Gardiol to ask an "Open question to time-domain experts" in the Summer 1988 IEEE MTT-S Newsletter [36]. Answers to the open questions appeared in two subsequent newsletters [37], [38], confirming that the problem was being addressed by several research groups. Various publications appeared shortly afterwards, describing successful implementations of dispersive behavior in time domain, such as recursive convolution algorithms and time domain diakoptics.

This experience suggests that what may initially be perceived as a challenge to a good idea, is in hindsight a valuable evolutionary test that, if overcome, acts as a stimulus and incentive to make the idea prevail and flourish. I always thought of this when occasionally a paper containing some non-conventional idea got rejected.

VI. ENTERING THE MAINSTREAM OF MICROWAVE ENGINEERING

By 1980 many numerical methods we use today were known and validated, but they had not yet entered the mainstream of microwave engineering. However, that rapidly changed when smaller and more powerful computers became available, such as the VAX, the RISC workstation, and the Personal Computer. Massively Parallel Computers, such as the Connection Machine and the DECmpp 12000, and special computer graphics hardware allowed for the parallelization of time domain algorithms with considerable reduction of run-time.

The period from 1980 to 2000 were the golden years of computational electromagnetics in which the field grew from the pioneering work of the early researchers into a major academic and industrial effort to enable the modeling and design of electromagnetic components and systems that were performing "right the first time", to paraphrase a well-known slogan coined during that period. This activity encompassed the IEEE MTT, AP, and EMC societies, each emphasizing different types of electromagnetic analysis and design scenarios. MTT members pursued mainly research relevant to computer-aided engineering of microwave/millimeter-wave circuits and systems, AP focused more on scattering, propagation, and radiation scenarios, while EMC concentrated on shielding, interference, discharges, and signal integrity. Societies, such as the *Union Radio-Scientifique Internationale (URSI)*, the *European Microwave Association (EuMA)* and the *Applied Computational Electromagnetics Society (ACES)*, as well as numerous national organizations, also played major roles in the advancement of electromagnetic theory, modeling, and CAD applications by organizing international conferences, workshops, and short courses focused on the subject.

In the late 1980s and early 1990s, the first field-based simulators implemented on workstations and personal computers began to appear at these conferences, mostly as part of the exhibition. While these simulators were able to demonstrate the capabilities of the algorithms that formed their numerical engine, developers needed to focus on user-centered issues, such as the input of the geometry and material constitutive parameters of a structure, the excitation of the field, and the output and visualization of the simulation results. A Graphical User Interface or GUI allows the user to visually enter a structure using a pointing device and a selection of icons on the computer screen rather than a command line entered via the keyboard. It determines the usability and user-friendliness of a simulator and allows users to solve electromagnetic problems without abstract mathematics. Most simulators had their own proprietary GUI. As specialized computer programs such as *AutoCad* for designing geometric object shapes, also known as *Computer-Aided Geometric Design (CAGD)* became available, they were interfaced with field simulators via a compatible data file to input 2D or 3D structure models for electromagnetic analysis.

At the same time, the quest for greater numerical efficiency and shorter cpu-time became another focus of modeling research and development. The solution of grand-challenge

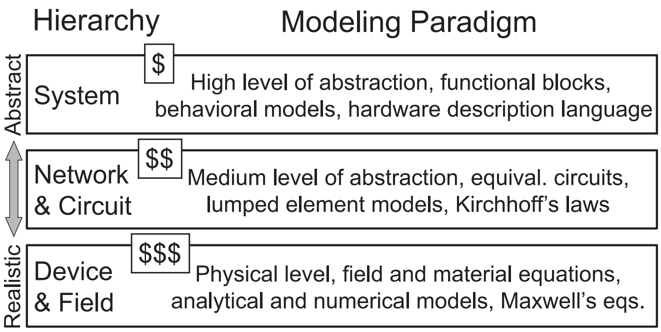

FIGURE 4. **Hierarchy of modeling in multi-level systems.**

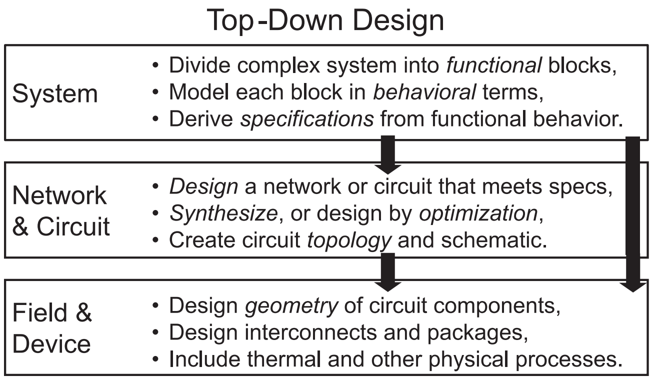

FIGURE 5. **Top-down design of a multi-level system.**

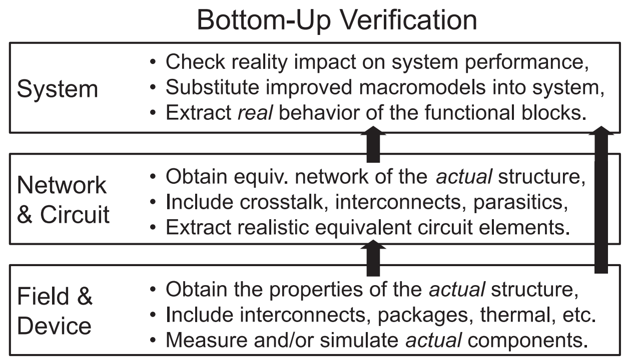

FIGURE 6. **Bottom-up verification of a multi-level system.**

problems, such as modeling the impact of lightning on the electrical system of an airplane, designing a monolithic microwave integrated circuit, or optimizing a complex structure by recursive analysis, called for highly efficient and fast solvers. Considerable research efforts thus focused on techniques to accelerate electromagnetic computations, such as SVD, FFT, multipoles, preconditioning, and Krylov-based iterative methods [39]. The general approach to reducing computational complexity is to replace it with a reduced order model while keeping the approximation error small and conserving the properties of the full order model. This can take many forms, including equivalent circuit extraction [11], space mapping [35], or training of an equivalent neural network [40].

The innovative contributions to the rapidly growing arsenal of numerical techniques and their implementation are too numerous to mention in this short retrospective, and the reader may thus refer to the selected bibliography given in Section IX at the end of this paper.

VII. MULTI-LEVEL MICROWAVE AND HIGH-SPEED CIRCUIT MODELING

Modern communication systems that include both analog microwave and high-speed digital components are too complex to be modeled in a global electromagnetic simulation. Their design, production and test require the integration of field- and circuit-based modeling into a multi-level system design environment. This involves the creation and interfacing of models at the behavioral, circuit and field/device levels. Hierarchy, design, and verification of a complex system are illustrated in Figs. 4, 5, and 6. Field-based modeling is the most physically realistic but also the most expensive part of the design process.

As the demand for more advanced and complex communication and data systems is ever increasing, and international competition imposes a high innovation rate and short design cycles, development, production and testing require seamless integration of system, circuit, field, and device modeling. Computational electromagnetics plays a fundamental role in this process at the Field & Device level.

VIII. SOME PERSONAL ENCOUNTERS AND MEMORIES

A. PETER B. JOHNS

The key event that sparked my interest in time-domain modeling was my first encounter with Peter B. Johns at the 1975 IEEE International Microwave Symposium in Palo Alto, CA. His presentation titled "Three-dimensional numerical analysis of microwave cavities using the TLM method" [18] was truly inspirational. Johns' new model of electromagnetic field propagation, published first in 1971 [41], was an incredibly simple concept that could potentially solve the most complex electromagnetic problems if one had access to a sufficiently powerful computer.

In 1987, Peter B. Johns proposed to John Wiley & Sons to create a new Journal called the International Journal of Numerical Modelling: Electronic Networks, Devices and Fields (IJNM) as a communication vehicle for interdisciplinary exchange on numerical modeling rather than abstract numerical mathematics. I will never forget the excitement and enthusiasm in his voice when he first told me about his plans for such a Journal and asked me to be his North American co-editor. In the spring of 1987, we met in Paris to finalize our editorial board and policy, and we celebrated on the *Champs-Elysées* the birth of the new Journal. Peter lived only to see the first issue which appeared in March 1988, but his spirit continues to live to this day through this journal he initiated, and his pioneering contributions to numerical modeling.

B. TATSUO ITOH

I remember first meeting Tatsuo Itoh in 1975 at the IMS in Palo Alto, and subsequently at many international microwave events. In 1984, Tatsuo organized a workshop at the IMS in San Francisco on "Critical Inspection of Field-Theoretical Methods for Microwave Problems" and invited me to contribute a talk on TLM. The event highlighted aspects of fundamental importance and broad perspective, attracting considerable interest in the MTT community. In response, a special issue of the MTT Transactions [MTT-33 no. 10] containing 30 papers was devoted to the topic. This special issue was subsequently expanded into a book under Itoh's [42] editorship. It remains one of the most comprehensive collections of chapters describing the state-of-the-art in numerical modeling at that time.

Beyond Tatsuo's prodigious accomplishments, I look back at many inspiring personal interactions and joint activities over the last 35 years of his life.

C. ROBERTO SORRENTINO

I became aware of Roberto Sorrentino's research while he was collaborating with Tatsuo Itoh between 1983 and 1986. His original work on the application of transverse resonance methods to the characterization of quasi-planar transmission lines and discontinuities caught my attention. We first met in 1984 at the 14th European Microwave Conference in Liège, Belgium, and soon realized that we shared many interests beyond electromagnetics and microwaves. My research collaboration with Roberto over the next thirty years focused on electromagnetic modeling and led to many mutual visits and several joint publications on time domain diakoptics and its application to discontinuities.

One instance remains very vivid in my mind. Roberto had come to Ottawa for a short research visit. He suddenly dropped in at my office, visibly animated: "Did you ever notice that the 2D-TLM impulse scattering matrix is equal to its inverse?!" "Sure" I said, "it has this property by virtue of the spatial symmetry of the TLM node." "Take another look at the TLM node because there is more to it than spatial symmetry," Roberto replied, "We also have symmetry in time and thus can compute the cause from its effect without having to change the TLM algorithm! We immediately asked Poman So who was my research associate at that time, and asked him if he could implement this time reversal feature in our existing TLM simulator [43]. The next day we could all observe the time reversal unfold on the computer screen as predicted by Roberto. We presented his idea and our implementation at the 21st European Microwave Conference in Stuttgart [44]. Roberto left us much too soon, leaving a rich personal, professional, and cultural legacy.

D. PETER PEET SILVESTER

Among the outstanding pioneers of computational electromagnetics who left a defining and influential legacy I remember Peter Silvester, whose name is synonymous with the Finite Element formulation of electromagnetics which he pioneered with great vision and conviction. Peter's ideas changed forever the way in which electromagnetic field problems are solved on digital computers. His work has spawned many software tools that empower researchers, students, and practitioners around the world. He was one of the founding editorial board members of the IJNM. After an illustrious career at McGill University in Montreal he retired in Victoria and became an Adjunct Professor at my university. I was looking forward in anticipation of a joint research project we had been planning, when he passed away in October 1996, just after three new titles bearing his name, *Finite elements for electrical engineers*, 3rd edition (author, with R. L. Ferrari) [45], *Software for electrical engineering analysis and design* (editor) [46] and *Finite element software for microwave engineering* (editor, with T. Itoh and G. Pelosi) [47], had just appeared. Peter left me a collection of his books on mathematical physics as well as his Royal Society of Canada and IEEE Fellow pins, two precious souvenirs of a great protagonist of computational electromagnetics.

IX. ADDITIONAL SOURCES OF INFORMATION

The plethora of important contributions to computational electromagnetics makes it impossible to do justice to all of them by individually referencing them in this paper. However, the following books and papers include relevant information on the theory and application of computational electromagnetics and contain most seminal and pioneering literature references:

Electromagnetics	[48], [49], [50], [51], [52], [53], [54], [55].
Computational methods	[56], [57], [58], [59], [60], [61], [62].
Mode Matching	[63], [64], [65].
Method of Moments	[24], [25], [66], [67].
Finite Elements	[45], [47], [68].
FDTD	[69], [70], [71].
TLM	[72], [73], [74].
Spectral Domain	[75], [76], [77].
Miscellaneous	[78], [79], [80].

X. CONCLUSION

Over the seventy years of the MTT Society's existence, we have witnessed a dramatic evolution of the modeling and applications of electromagnetic fields. While the theoretical foundations and the governing Maxwell equations have endured, the way we solve and apply them has evolved at a breathtaking pace. We can identify three major dynamics that have driven this evolution. The first is the meteoric rise of computer hardware and software, the second is the advent of new technologies to generate, transmit, process, and detect electromagnetic signals and information, the third – which to a great extent enables the two previous dynamics – is the progress made in the science of materials, their processing, and their transformation into devices. At the seventieth anniversary of the MTT society, computational electromagnetics has reached a level of relative maturity and is firmly established as an indispensable part of microwave engineering. However, quantum computing is on the horizon, promising new developments and challenges to come.

It has been a tremendous privilege and joy to participate in this exciting worldwide endeavor, not only because of my fascination with the ideas and their engineering realizations, but also for the tremendous pleasure of working and interacting with bright and inspiring people.

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