A Transmission-Line-Based Cochlear Standing Wave Model To Elucidate Mechanism of Human Auditory System

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***Abstract*— How do people hear sounds? As a counterpart of Prof. G. V. Békésy's traveling wave theory, we have proposed resonance theory of outer hair cells and cochlear standing wave theory, respectively. Based on these proposals, this paper develops a transmission-line-based cochlear standing wave model. Since the macroscopic cochlear model is designed as it looks like, various auditory physiology can be explained. Transient analyses with pure-tone excitation and Gaussian pulse excitation are carried out, and Prof. D. Kemp's otoacoustic emission (OAE) is demonstrated successfully.**

***Clinical Relevance*— Our new model has a great potential to explain auditory physiology including structural inner disorders, hearing loss, and even tinnitus.**

I. Introduction

How do people hear sounds? The first to answer this grave question was Prof. G. V. Békésy of Harvard University. In his experiments, he thoroughly examined the ears of a dead body and discovered that different areas of the basilar membrane (BM) in the cochlear reacts to each frequencies of sound waves. This frequency-dependent displacement of the BM was considered to be a fundamental principle of the hearing mechanism. Later, his idea was named as "Traveling Wave Theory", and he got the Nobel Prize in 1961[1].

In those days, to explain the auditory system, numerous attempts were made from a macroscopic point of view, and various equivalent circuits were designed. In 1950, Peterson, Bogert[2] and Zwislocki[3] suggested a cochlear model using one-dimensional transmission line configuration. Scala vestibuli (SV) and scala tympani (ST) were expressed by a transmission line with a combination of series capacitors $C(x)$ and shunt inductors $L(x)$, iteratively. The oval and round windows located at the base of the SV and ST were expressed by the capacitors C_{OW} and C_{RW}, respectively. Following this trial, the SM, including the BM, was defined by an impedance $Z_{CP}(x)$ of serially connected capacitor $C_{CP}(x)$ (spring), inductor $L_{CP}(x)$ (mass) and resistor $R_{CP}(x)$ (buffer). Despite their effort however, actual medium constants of the SM do not change enough to explain the resonance from 20 Hz to 20000 Hz. Finally, this idea was considered to be impractical.

Two-dimensional[4] and three-dimensional[5] models were also proposed, although both lacked in non-linearity and were insufficient in explaining cochlear characteristics. One-dimensional model with non-linear terms $Z_{NL}(x)$ added to the shunt impedance $Z_{CP}(x)$ was also designed[6]. However, transmission line based models were gradually replaced by the mechanical non-linear models[7] and the cochlear transmission line models lost attention soon after.

Now let's get back to the starting point. This paper proposes a new cochlear transmission line model, with a similar structure to Peterson's one-dimensional model, but with an arrangement of *LC* resonant circuits of outer hair cells in the SM. Apart from Békésy's traveling wave theory, that is, apart from displacement of the BM, and instead, accepting our "resonance theory of outer hair cells[8-10], various auditory physiology can be explained easily and reasonably. In Sec. II, the reason why we have doubts on the conventional transmission line theory is explained. And then, our "resonance theory of outer hair cells" is shortly introduced as a counterpart. In Sec. III, a design procedure of our cochlear standing wave model is explained. And in Sec. IV, two kinds of transient analysis are reported; 1) a pure-tone continuous-wave excitation, and 2) a Gaussian pulse excitation to demonstrate D. Kemp's otoacoustic emissions.

II. Conventional and Novel Theory To Elucidate Auditory Mechanism

A. *Békésy's Traveling Wave Theory* [1]

According to G.V. Békésy, sound stimuli coming into the cochlear through the OW generates a traveling wave and makes frequency-dependent displacements on the BM. Recently, the protein-motor "prestin" was discovered on the OHC plasma membrane [11], and it is considered that the tiny initial displacement on the BM is largely amplified by the prestin and huge BM displacement is regenerated. However, we believe that it is impossible because the amplified output goes back to the input, and then the *positive feedback loop* is naturally formed and *catastrophic oscillation* will soon occur. Once oscillation had started in one OHC, we would suffer from painful *tinnitus*. The fact is that we have totally 20000 OHCs in our ears.

B. *Resonance Theory of Outer Hair Cells* [8-10]

Fig. 1 shows an illustration of Organ of Colti and a close-up of the OHC. The OHC has many tiny holes penetrating the

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cuticular plates, called "rootlet". The rootlet is connected to the stereocilia, and K^+ ions accepted at the MET channels at the tip of the stereocilia are carried into the OHC body through the rootlets. These rootlets are acoustically attractive because they work like an inductor L. That is, the sound wave traveling through the rootlet suffers 90-degree phase delay and high-frequency sounds hardly pass it through.

On the other hand, the OHC cell membrane has three layered architecture configured by plasma membrane, cortical lattice, and subsurface cisternae. Generally, membranes work as a capacitor C because the sound wave traveling through the membrane suffers 90-degree phase advance and low-frequency sounds hardly pass it through. Therefore, when the sound enters the OHC from the rootlet side and goes out through the cell membrane side, the OHC works as an *LC series resonant circuit*. In addition, unlike any other cell membranes, the OHC cell membrane is quite tough due to the actin-fiber-based cortical lattice. As a result, the OHC membranes provide appropriate higher Young modulus so that the LC circuit can resonate at the audible frequency, 20 ~ 20000 Hz.

When the OHC resonates, the sound energy is stored in the cell and the inner pressure swings largely (the inner pressure swing can be observed by the voltage V_{out} in Fig. 1). We believe that such an enhanced sound pressure helps to take K+ ions into the OHC more efficiently.

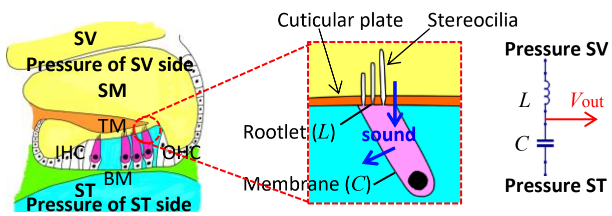


Figure 1. An illustration of Organ of Colti and a close-up of the OHC. Based on [8-10], an LC series resonance circuit is formed in the OHC and the OHC inner pressure is largely enhanced when the OHC is resonating for the incoming sound stimuli.

III. Design of Cochlear Standing Wave Models

Fig. 2(a) shows the configuration of human auditory system. The sound stimuli given to pinna, is guided to eardrum as vibration of air, and then transformed into mechanical vibration of ossicular in middle ear. This sound wave is finally received by oval window (OW), an entrance of the cochlear, as vibration of liquid (lymph). Fig. 1(b) shows an illustration of unrolled cochlear. The cochlear is configured by three kinds of ducts called scalar vestibuli (SV), scala media (SM), and scalar tympani (ST). The sound stimuli received at the OW travels in the SV heading to the apex of the cochlear, and goes down to the ST through helicotrema, a small opening connecting between the SV and ST at the apex. Continuously, the sound wave travels in the ST toward the base, and finally it is reflected by the round window (RW) at the base of the ST. The important thing is that the sound wave is reflected mostly at the RW with conditions that the pressure becomes zero and the displacement of the medium becomes maximum at the boarder. This is because the outside of the RW is the light air region of the middle ear cavity, and the inside of the RW is the heavy liquid region of the cochlear, resulting in significant impedance mismatch. The reflected sound wave travels back to the same way and makes interaction with the incidental one. As a result, a standing wave is formed in the SV and ST.

We human hear sounds from 20 Hz to 20000 Hz. This means that our ears can detect sounds with the wavelengths from 75 mm (20000 Hz) to 75 m (20 Hz) when the sound speed is assumed to be 1500 m/s in liquid. Comparing with the wavelength of all audible frequencies, the length of our cochlear (35 mm) is much less than the wavelengths of any audible frequencies. This means that we see only a small part of the standing waves in the cochlear.

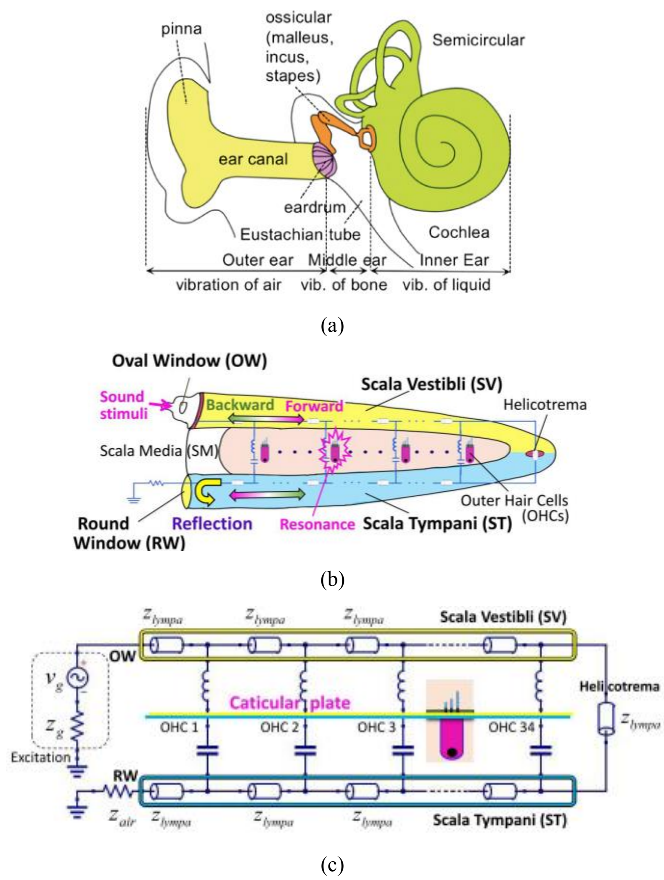


Figure 2. Design of a cochlear standing wave model. (a) Configuration of human auditory system. (b) An unrolled cochlear. (c) A cochlear standing-wave model.

The OHCs are regularly arranged in the SM region so as to detect high-frequency sounds near the base and low-frequency sounds near the apex according to the cochlear anatomy.

A cochlear standing wave model is designed based on transmission line theory. A sound pressure P in the cochlear is expressed by a node voltage V in the circuit. The SV and ST are treated as a transmission line with a characteristic impedance z_{lympa}=1.5 MΩ and a length of 35 mm each. An excitation circuit including a voltage source $v_g(t)$ and an internal impedance z_g=1.5 MΩ is connected to the OW portion. In addition, a terminating resistor with z_{air}=440 Ω is given to the RW portion to express the effect of an air-filled middle ear cavity.

Following to the frame design of the SV and ST, the OHCs in the SM are designed by a series *LC* resonant circuit [12]. Here, the characteristic frequency (CF) of the OHCs is given by geometric progression formula

$$f(n) = a r^{n-1} \quad (1)$$

where $n = 1, 2, ..., 3400$, $a = 20000$, $r = 0.99797$. For example,

- maximum audible frequency : $f(1)$=20000 Hz,
- minimum audible frequency : $f(3400)$=20 Hz,

respectively. In the real cochlear, 3400 OHCs with different CFs are lined at equal intervals along the BM. However, in the design of the cochlear standing wave model, 34 OHCs were arranged at 1.0 mm intervals for simplicity.

Resonant frequency of the OHC can be determined easily by the equation;

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \quad (2)$$

However, it is important to balance the 34 resonators to have the same quality factor. In the actual *LC* design, we evaluated transmission parameters "S_{21}" for various *L* and C, separately and stored the results. Then, we chose *L* and *C* whose S_{21} becomes -20 dB at the CF. Finally, 34 OHCs were designed as shown in TABLE I.

TABLE I. OHCs For Cochlear standing wave Model

Position from the base of the cochlear (mm)	Inductance L(H)	Capacitance C(pF)	Resonance f(Hz)
1	291	0.32	16456
2	354	0.39	13509
3	432	0.48	11089
4	526	0.58	9103
5	641	0.71	7472
6	781	0.86	6134
7	951	1.05	5035
8	1158	1.28	4133
9	1411	1.56	3393
10	1719	1.90	2785
11	2094	2.31	2286
12	2551	2.82	1877
13	3108	3.43	1541
14	3786	4.18	1265
15	4612	5.10	1038
16	5618	6.21	852
17	6844	7.56	699
18	8337	9.21	574
19	10156	11.2	471
20	12372	13.7	387
21	15071	16.7	318
22	18360	20.3	261
23	22366	24.7	214
24	27246	30.1	176
25	33191	36.7	144
26	40432	44.7	118
27	49255	54.4	94.2
28	60002	66.3	79.8
29	73094	80.8	65.5
30	89042	98.4	53.8
31	98042	119.9	44.1
32	108470	146.0	36.2
33	132138	177.9	29.7
34	160970	216.7	24.4

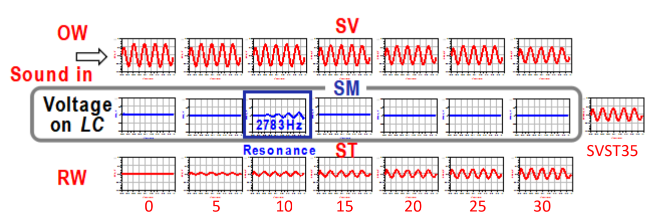
To connect the *LC* resonators to the frame model, the transmission lines of the SV and ST are divided into small pieces with length of 1.0 mm/each. Then, the resonators are connected as shown in Fig. 2(c) in accordance with the cochlear anatomy. For convenience, the OHCs and their nodes are numbered from the base of the cochlear. For example, the OHC located at 10 mm distance from the base is named as "OHC10", and the nodes on the SV, between *LC* elements, and on the ST are named as "SV10", "SM10", and "ST10", respectively. In the later section, the expressions "SVn", "SMn", and "STn" are used to show the node voltages. Here, it should be noted that the node voltage "SMn" is related to the inner pressure of the OHC. When the voltage SMn is enhanced, this means that the corresponding OHC is resonating to the sound stimuli and detecting sounds sensitively.

IV. Demonstrations

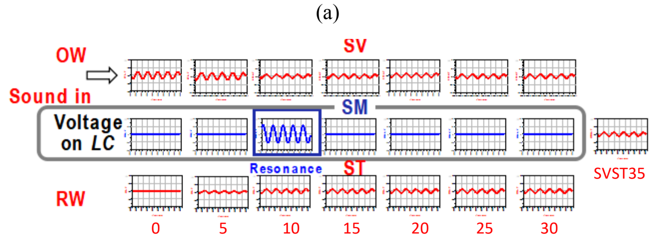
To show the basic behaviors of the cochlear standing wave model, two types of transient analysis are carried out; one is a pure-tone continuous-wave excitation and the other is a Gaussian pulse excitation. The configuration and circuit parameters of the simulation model are the same as shown in Fig. 2(c) and TABLE I. For simulations, a commercial circuit simulator Advanced Design System (ADS) 2020 provided by Keysight Technologies Co. Ltd. was used.

A. Transient Analysis

A 2785 Hz pure-tone, continuous sound wave with peak-peak voltage of 1.0 V is given to the voltage source $v_g(t)$, and then, the representative node voltages, SVn, SMn and STn, where n=0, 5, 10, 15, 20, 25, and 30, are observed as shown in Fig. 3. Fig. 3(a) shows an initial state when t=0 ms ~ 1.74 ms, and Fig. 3(b) presents a stable state when t=100 ms ~ 101.74 ms. The graphs on the first line show the node voltages of SV0, SV5, ..., SV30 from the left. Similarly, the second line shows SM0, SM5, ..., SM30, and the third line shows ST0, ST5, ..., ST30, respectively. SVST35 on the second line presents a node voltage at helicotrema.



(a)



(b)

Figure 3. Transient analysis of the cochlear standing wave model with parameters in Table I. Continuous wave at frequency of 2783 Hz is given. (a) Initial state from t=0 ms to 1.74 ms. (b) Stable state from 100 ms to 101.74 mm.

It can be read from the initial state of Fig. 3(a) that a standing wave is generated at first so as to become zero at the RW. At this moment, the node voltage of the target OHC (SM15) is not increasing, and the SV15 and ST15 on both sides of the target OHC have different node voltages. However, with time, strong resonance can be confirmed on SM15, and the node voltages SV15 and ST15 become equal due to the series *LC* resonance. Thus, the standing wave is formed at first in the SV and ST ducts in the initial state, and then, the *LC* resonance is gradually formed.

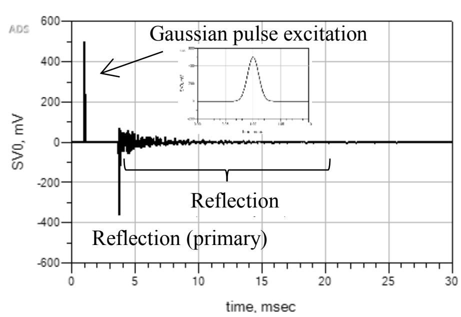
B. *Prof. D. Kemp's Otoacoustic Emissions*

In 1978, Prof. D. Kemp reported that when a click sound was given to the healthy ears, two types of reflections were observed; one was the primary wave that is reflected very quickly, and the other was the secondary wave whose amplitude is small and gradually decayed with time[13]. However, the secondary wave could not be observed when the inner ears had some troubles.

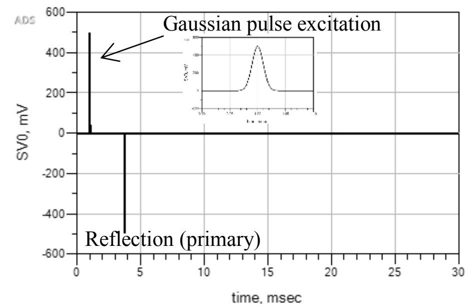
In this demonstration, as a click sound, a Gaussian pulse is given to the voltage source $v_g(t)$ of the cochlear standing wave model, and the node voltage SV0, corresponding to the voltage at the oval window (OW), is observed. When the *LC* circuits are working well, the response is obtained as shown in Fig. 4(a). As can be seen, the secondary wave was observed soon after the primary wave reflection and lasting more than 15 ms. However, as presented in Fig. 4(b), the secondary wave is not seen when the *LC* resonators are removed completely. This fact indicates that the secondary wave is caused by the sound energy storage and release mechanism of the *LC* resonators. In other words, the *LC* resonators in the inner ears are inevitable and indispensable to generate the secondary wave.

V. CONCLUSION

The RF front-end is the most important part of the radio receiving system. This is completely the same as ears. To hear mosquitoes flapping, ears need an excellent performance to pick-up super small sounds with extremely high efficiency and reasonable signal-to-noise ratio. (a) The OHC resonant theory and (b) the cochlear standing wave theory are both developed from microscopic and macroscopic points of view, and are applicable for various auditory physiology including structural inner disorders, hearing loss, and even tinnitus. Further, our linear model can be easily extended to the nonlinear model. We believe that it is important to return to the origin.



(a)



(b)

Figure 4. Demonstration of Prof. D. Kemp's Otoacoustic Emissions[13]. A Gaussian pulse is given as a click sound and a node voltage SV0 is observed (at the oval window, OW). (a) The *LC* resonant circuits are all working well in the cochlear standing wave model. (b) The *LC* resonant circuits are all removed from the cochlear standing wave model.

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