Model-Based Fuzzy $l_2 - l_\infty$ Filtering for Discrete-Time Semi-Markov Jump Nonlinear Systems Using Semi-Markov Kernel

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***Abstract*—This article concentrates on the model-based fuzzy $l_2 - l_\infty$ filtering problem of a discrete-time semi-Markov jump nonlinear system. The random jumps in the studied system are governed by the discrete-time semi-Markov process. Therefore, the storage characteristics of the transition probability between systems are fully considered. To analyze the σ-mean-square stability of the filtering error system, a mode-dependent filter, which is based on the discrete-time fuzzy semi-Markov jump model, is constructed to estimate the state of the system. Thereafter, based on the Lyapunov stability theory and the discrete-time semi-Markov kernel concept, a set of sufficient criteria to ensure the σ-mean-square stability and $l_2 - l_\infty$ performance are derived. In addition, by using the Takagi–Sugeno fuzzy model, the nonlinear problem is effectively solved. Two illustrative examples, including a numerical example and a tunnel diode circuit example, are demonstrated to reveal the practicability of the developed filtering strategy.**

***Index Terms*—$l_2 - l_\infty$ filtering issue, semi-Markov jump nonlinear systems, σ-mean-square stability (σ-MSS), Takagi–Sugeno (T-S) fuzzy model.**

I. Introduction

With the development of advanced control technology, the scale of the system will inevitably become larger and larger, and the modeling of the system will become more and more difficult. Moreover, in the actual operation process of the system, the internal parameters or structure of the system are susceptible to sudden changes in the external environment, which may result in random jumps of the system mode, and such jumps usually occur during system operation. In the engineering field, modeling and controlling of such systems are quite challenging. Thence, as an important approach to describe this kind of complex system, the Markov jump model has received widespread attention in both academia and industry, for instance, electric circuits [1], communication networks [2], and biological systems [3]. However, owing to the memoryless characteristics of the transition probability (TP) of the Markov chain, as pointed out in [4], Markov jump systems (MJSs) have a strict restriction on the sojourn-time probability density, which may lead to inapplicability for the Markov jump model in many practical scenarios. Therefore, researchers have turned to the semi-Markov process with more general distribution of the sojourn time while describing the stochastic process.

For the semi-Markov process, the sojourn time of each system mode is considered to be bounded. Among them, the TP of the semi-Markov jump systems (S-MJSs) depends on all the historical information of the past jumping sequence, that is, the TP is time-varying, which is the main difficulty in analysis and synthesis of S-MJSs. Compared with the MJSs, the semi-Markov model is more suitable for the analysis and reliability calculations of complex dynamic systems because of the "memory" characteristics. In recent years, the S-MJSs have stimulated the study interest of many researchers, and abundant research results have been produced, such as multibus systems [5], reward systems [6], motor speed control system [7], and so on. A series of discrete-time S-MJS stability problems were studied in [8].

Sometimes, the internal state of the system cannot be obtained directly, which brings difficulties to the control of the system. Therefore, the research on the filtering problem becomes more important. The purpose of the filtering problem is to reconstruct the unmeasurable state and filter the external noise. It has been proved to be useful for signal processing, target tracking, and control design [9]–[11]. This problem has attracted widespread attention in the past few years, and many filtration solutions have been developed, such as energy-to-peak ($l_2 - l_\infty$) filtering [12]–[14] and peak-to-peak (H_∞) filtering [15]–[17].

In addition, considering that most devices possess nonlinear characteristics in practical applications [18], [19], how to deal with nonlinear systems has become a challenging and meaningful task. Recently, the work on nonlinear control is a research hotspot, and a large number of research results have been obtained [20]–[23]. Compared with general nonfuzzy

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control and filtering methods, the Takagi–Sugeno (T-S) model can deal with more complex nonlinear functions with a small number of fuzzy rules, which can effectively reduce the number of fuzzy rules when dealing with multivariable systems, and thus has great advantages. Within the general framework of the T-S fuzzy model, numerous studies on stability analysis and performance design problems have been developed [24]–[28]. Moreover, the research on the problem of the filter design of the T-S fuzzy system has been active, and rich results also have been proposed [9], [15]. Literature [29] studied the model-based fuzzy $l_2 - l_\infty$ filtering problem. In [14] and [30], fuzzy H_∞ filtering of nonlinear systems was considered. As far as the authors know, the fuzzy mode-dependent filter design for discrete-time S-MJSs has not been studied, which motivates us to carry out this research.

Compared with [31], the S-MJS, whose transition probability matrix (TPM) possesses "memory" characteristics, is considered, and the filter design method based on the S-MJS is more realistic. Therefore, in the fuzzy $l_2 - l_\infty$ filtering problem for the discrete-time S-MJS, the following three basic difficulties should be solved. Question 1: How to design a reasonable model-based fuzzy $l_2 - l_\infty$ filter for discrete-time S-MJSs? Question 2: How to choose a Lyapunov function (LF) to effectively reduce the conservativeness of the results obtained? Question 3: How to deal with the coupling terms in matrix inequalities?

Inspired by the above statement, the purpose of this article is to solve the $l_2 - l_\infty$ filtering problem of discrete-time S-MJSs based on the T-S fuzzy model. Compared with the existing research results in the field of S-MJSs, the contributions and challenges of our work are mainly reflected in the following three aspects.

1) As the first attempt, the discrete-time T-S model-based fuzzy $l_2 - l_\infty$ filtering problem for S-MJSs is exploited. Different from [31], the random jumping phenomenon of the filtering error system is described by the semi-Markov process. With the help of the semi-Markov kernel (SMK) method, the filtering error system can maintain σ-mean-square stability (σ-MSS) with an $l_2 - l_\infty$ performance;
2) To make the calculation result more realistic in line with the actual situation, the LF in this article is not only related to the system mode but also related to the sojourn time of the mode.
3) In this article, by setting the switching ratio, the value of the LF is allowed to rise or fall in the subinterval, and the value of the LF at the next jump instant is less than the value of the LF at the previous jump instant. Compared with the switching ratio that only decreases monotonically, this result is less conservative.

Notations: $\mathbb{R}$ presents the set of real numbers, $\mathbf{Z}$ and $\mathbf{Z}_+$ signify the nonnegative integers and positive integers, respectively, $\mathbf{Z}_{[m,n]}$ represents the set $\{x \in \mathbf{Z} \,|m \le x \le n\}$, $\mathbb{R}^x$ denotes the x-dimensional Euclidean space, I and 0 stand for the identity matrix and zero matrix of appropriate dimensions, respectively, $P > 0$ ($P < 0$) means that the matrix P is positive (negative) definite, the superscripts "P^T" and "P^{-1}" represent transpose and inverse of matrix P, respectively, $\mathcal{E}\{\cdot\}|_y$ is used to denote the conditional expectation operator conditioned on y, diag$\{\cdots\}$ is used to depict the diagonal matrix, $\mu_{\max}(P)$ ($\mu_{\min}(P)$) denote the maximum (minimum) eigenvalues of matrix P, $\|\cdot\|$ is employed to signify the Euclidean vector norm, and $l_2[0,\infty)$ is the space of square-summable vector functions.

II. Preliminaries

A. *System Description and Filter Design*

Consider a probability space $(\Theta, \Gamma, \Pr)$, where the discrete-time fuzzy S-MJS is defined. Θ signifies the sample space, Γ stands for the σ-algebra of subsets of Θ, and $\Pr$ denotes the probability, which is measured on Γ. The discrete-time fuzzy S-MJS is shown as follows.

Plant Rule $\acute{g}$: **IF** $\eth_1(x(\iota))$ is $\epsilon_{\acute{g}1}$, $\eth_2(x(\iota))$ is $\epsilon_{\acute{g}2}, \ldots$, and $\eth_n(x(\iota))$ is $\epsilon_{\acute{g}n}$,**THEN**

$$x(\iota+1) = A_{\acute{g}}(\grave{r}(\iota))x(\iota) + B_{\acute{g}}(\grave{r}(\iota))\,\omega(\iota) \tag{1}$$

$$y(\iota) = C_{\acute{g}}(\grave{r}(\iota))x(\iota) + D_{\acute{g}}(\grave{r}(\iota))\,\omega(\iota) \tag{2}$$

$$z(\iota) = L_{\acute{g}}(\grave{r}(\iota))\,x(\iota) \tag{3}$$

where $\acute{g} \in \mathcal{F} \triangleq \{1, 2, \ldots, \mathbf{f}\}$ and $\mathbf{f}$ is the number of **IF–THEN** rules; $\eth(\iota) \triangleq \{\eth_1(x(\iota)), \eth_2(x(\iota)), \ldots, \eth_n(x(\iota))\}$ stands for the premise variables; the parameters $\epsilon_{\acute{g}1}, \epsilon_{\acute{g}2}, \ldots, \epsilon_{\acute{g}n}$ denote the fuzzy sets; $x(\iota) \in \mathbb{R}^{n_x}$ denotes the system state vector, which is unable to be detected directly; $y(\iota) \in \mathbb{R}^{n_y}$ represents the measurement output; $\omega(\iota) \in \mathbb{R}^{n_\omega}$ symbolizes the external interference input signal belonging to $l_2[0,\infty)$; $z(\iota) \in \mathbb{R}^{n_z}$ is the system output to be detected; the stochastic process $\{\grave{r}(\iota)\}_{\iota\in\mathbf{Z}_+}$ is the semi-Markov chain (SMC) and takes values in a finite state space $\mathbf{M} \triangleq \{1, 2, \ldots, \mathcal{M}\}$, for each $\grave{r}(\iota) \in \mathbf{M}$; and $A_{\acute{g}}(\grave{r}(\iota))$, $B_{\acute{g}}(\grave{r}(\iota)), C_{\acute{g}}(\grave{r}(\iota)), D_{\acute{g}}(\grave{r}(\iota))$, and $L_{\acute{g}}(\grave{r}(\iota))$ are system matrices of real constants with appropriate dimensions. Based on the above discrete-time fuzzy S-MJS, the dynamic equations are expressed as follows:

$$x(\iota+1) = \sum_{\acute{g}=1}^{\mathbf{f}} \alpha_{\acute{g}}(\eth(x(\iota)))(A_{\acute{g}}(\grave{r}(\iota))x(\iota) + B_{\acute{g}}(\grave{r}(\iota))\,\omega(\iota)) \tag{4}$$

$$y(\iota) = \sum_{\acute{g}=1}^{\mathbf{f}} \alpha_{\acute{g}}(\eth(x(\iota)))(C_{\acute{g}}(\grave{r}(\iota))x(\iota) + D_{\acute{g}}(\grave{r}(\iota))\,\omega(\iota)) \tag{5}$$

$$z(\iota) = \sum_{\acute{g}=1}^{\mathbf{f}} \alpha_{\acute{g}}(\eth(x(\iota)))(L_{\acute{g}}(\grave{r}(\iota))\,x(\iota)) \tag{6}$$

where

$$\alpha_{\acute{g}}(\eth(x(\iota))) \triangleq \frac{\Pi_{s=1}^{n} \epsilon_{s\acute{g}}(\eth_{\acute{g}}(x(\iota)))}{\sum_{\acute{g}=1}^{\mathbf{f}} \Pi_{s=1}^{n} \epsilon_{s\acute{g}}(\eth_{\acute{g}}(x(\iota)))},$$

$\alpha_{\acute{g}}(\eth(x(\iota))) \geq 0$ is the fuzzy basis function, and $\epsilon_{s\acute{g}}(\eth_{\acute{g}}(x(\iota)))$ indicates the membership grade of $\eth(x(\iota))$ in $\epsilon_{s\acute{g}}$. In addition, we can get that $\sum_{\acute{g}=1}^{\mathbf{f}} \alpha_{\acute{g}}(\eth(x(\iota))) = 1$.

Filter Rule $\acute{h}$: **IF** $\eth_1(\tilde{x}(\iota))$ is $\epsilon_{\acute{h}1}$, $\eth_2(\tilde{x}(\iota))$ is $\epsilon_{\acute{h}2}, \ldots$, and $\eth_n(\tilde{x}(\iota))$ is $\epsilon_{\acute{h}n}$, **THEN**

$$\tilde{x}(\iota+1) = A_{f,\acute{h}}(\grave{r}(\iota))\,\tilde{x}(\iota) + B_{f,\acute{h}}(\grave{r}(\iota))\,y(\iota) \tag{7}$$

$$\tilde{z}(\iota) = C_{f,\acute{h}}(\grave{r}(\iota))\,\tilde{x}(\iota) \tag{8}$$

where $\tilde{x}(\iota) \in R^{n_{\tilde{x}}}$ and $\tilde{z}(\iota) \in R^{n_{\tilde{z}}}$ are the filter state vector and the output vector, respectively. The fuzzy basis function for the $\acute{h}$th rule is defined as

$$\alpha_{\acute{h}}(\eth(\tilde{x}(\iota))) \triangleq \frac{\Pi_{s=1}^{n}\epsilon_{s\acute{h}}(\eth_{\acute{h}}(\tilde{x}(\iota)))}{\sum_{\acute{h}=1}^{\mathbf{f}}\Pi_{s=1}^{n}\epsilon_{s\acute{h}}(\eth_{\acute{h}}(\tilde{x}(\iota)))},$$

$\alpha_{\acute{h}}(\eth(\tilde{x}(\iota))) \geq 0$, and $\sum_{\acute{h}=1}^{\mathbf{f}}\alpha_{\acute{h}}(\eth(x(\iota))) = 1$. For each $\grave{r}(\iota) \in \mathbf{M}$, the matrices $A_{f,\acute{h}}(\grave{r}(\iota))$, $B_{f,\acute{h}}(\grave{r}(\iota))$, and $C_{f,\acute{h}}(\grave{r}(\iota))$ are the filter gains. For brevity, we use $\alpha_{\acute{h}}$ to represent $\alpha_{\acute{h}}(\eth(\tilde{x}(\iota)))$. Furthermore, the above fuzzy filter can be rewritten as

$$\tilde{x}(\iota+1) = \sum_{\acute{h}=1}^{\mathbf{f}}\alpha_{\acute{h}}\Big(A_{f,\acute{h}}(\grave{r}(\iota))\,\tilde{x}(\iota) + B_{f,\acute{h}}(\grave{r}(\iota))\,y(\iota)\Big) \quad (9)$$

$$\tilde{z}(\iota) = \sum_{\acute{h}=1}^{\mathbf{f}}\alpha_{\acute{h}}C_{f,\acute{h}}(\grave{r}(\iota))\,\tilde{x}(\iota). \quad (10)$$

Before ending this subsection, a necessary definition needs to be provided, which is essential to acquire the subsequent results. Define $\bar{x}(\iota) \triangleq [\,x^T(\iota)\ \tilde{x}^T(\iota)\,]^T$ and the filtering error $e(\iota) \triangleq z(\iota) - \tilde{z}(\iota)$. In addition, for simplicity, we sign $A_{\acute{g}}(\grave{r}(\iota))$ as $A_{\acute{g}v}$, and the others are the same signed. Then, one can deduce the fuzzy filtering error system (Σ) from (4)–(6), (9), and (10)

$$\bar{x}(\iota+1) = \sum_{\acute{g}=1}^{\mathbf{f}}\sum_{\acute{h}=1}^{\mathbf{f}}\alpha_{\acute{g}}\alpha_{\acute{h}}\Big(\bar{A}_{\acute{g}\acute{h}v}\bar{x}(\iota) + \bar{B}_{\acute{g}\acute{h}v}\omega(\iota)\Big) \quad (11)$$

$$e(\iota) = \sum_{\acute{g}=1}^{\mathbf{f}}\sum_{\acute{h}=1}^{\mathbf{f}}\alpha_{\acute{g}}\alpha_{\acute{h}}\bar{C}_{\acute{g}\acute{h}v}\bar{x}(\iota) \quad (12)$$

where

$$\bar{A}_{\acute{g}\acute{h}v} \triangleq \begin{bmatrix} A_{\acute{g}v} & 0 \\ B_{f,\acute{h}v}C_{\acute{g}v} & A_{f,\acute{h}v} \end{bmatrix}, \quad \bar{B}_{\acute{g}\acute{h}v} \triangleq \begin{bmatrix} B_{\acute{g}v} \\ B_{f,\acute{h}v}D_{\acute{g}v} \end{bmatrix}$$

$$\bar{C}_{\acute{g}\acute{h}v} \triangleq \begin{bmatrix} L_{\acute{g}v} & -C_{f,\acute{h}v} \end{bmatrix}.$$

For the semi-Markov process, some basic concepts need to be emphasized. Hence, before going any further, let us make the following provisions and introduce some relevant definitions and an assumption.

Assumption 1: ι_m is the instant at the mth jump and $\iota_0 = 0$; ϑ_m is the mode index of the system at the mth jump; and S_m is the sojourn time of mode ϑ_m between the mth and $(m+1)$th jump, and it satisfies $S_m = \iota_{m+1} - \iota_m$.

Definition 1 (see [32]): Consider a stochastic process $\{(\vartheta_m, \iota_m)\}_{m\in\mathbf{Z}}$, which is a discrete-time homogeneous Markov renewal chain (HMRC) if $\forall v, w \in \mathbf{M},\ v \neq w,\ \forall d \in \mathbf{Z}_+$ and $\forall m \in \mathbf{Z}$

$$\begin{aligned}&\Pr(\vartheta_{m+1} = w, S_m = d \mid \iota_0, \vartheta_0; \iota_1, \vartheta_1, \ldots, \iota_m, \vartheta_m = v)\\ &= \Pr(\vartheta_{m+1} = w, S_m = d \mid \vartheta_m = v)\\ &= \Pr(\vartheta_1 = w, S_0 = d \mid \vartheta_0 = v).\end{aligned}$$

Definition 2 (see [32]): Given an HMRC $\{(\vartheta_m, \iota_m)\}_{m\in\mathbf{Z}}$, the stochastic process $\{\grave{r}(\iota)\}_{\iota\in\mathbf{Z}}$ is referred to the embedded Markov chain of the HMRC. The TPs of $\{\grave{r}(\iota)\}_{\iota\in\mathbf{Z}}$ are defined by

$$\theta_{vw} \triangleq \Pr(\vartheta_{m+1} = w \mid \vartheta_m = v)$$

with $0 \leq \theta_{vw} \leq 1\ \forall v, w \in \mathbf{M}$; specifically, we assign $\theta_{vv} \triangleq 0\ \forall v \in \mathbf{M}$ and $\sum_{w=1}^{\mathcal{M}}\theta_{vw} = 1$. The corresponding TPM is defined as $\Theta \triangleq [\theta_{vw}]_{v,w\in\mathbf{M}}$.

Definition 3 (see [33]): Consider an HMRC $\{(\vartheta_m, \iota_m)\}_{m\in\mathbf{Z}}$; the stochastic process $\{\grave{r}(\iota)\}_{\iota\in\mathbf{Z}}$ is an SMC corresponding to the HMRC $\{(\vartheta_m, \iota_m)\}_{m\in\mathbf{Z}}$, if $\forall \iota \in \mathbf{Z}_+,\ \grave{r}(\iota) = \vartheta_{\mathbb{Z}(\iota)}$ with $\mathbb{Z}(\iota) \triangleq \max\{m \in \mathbf{Z} \mid \iota \geq \iota_m\}$. The SMK of the SMC is defined by $\pi_{vw}(d) \triangleq \Pr(\vartheta_{m+1} = w, S_m = d \mid \vartheta_m = v)$ with $\pi_{vw}(d) \in [0,1]\ \forall v, w \in \mathbf{M}$. Then, define the sojourn-time probability density function (STPDF) as $F(d) \triangleq [f_{vw}(d)]$, where $f_{vw}(d) \triangleq \Pr(S_m = d \mid \vartheta_{m+1} = w, \vartheta_m = v),\quad 0 \leq f_{vw}(d) \leq 1\ \ \forall v, w \in \mathbf{M},\ \ \forall d \in \mathbf{Z}_+$. Specifically, we set $f_{vv}(d) = 0\ \forall v \in \mathbf{M},\ \forall d \in \mathbf{Z}_+$. Then, we can get the SMK definition, which is shown as

$$\pi_{vw}(d) \triangleq \theta_{vw}f_{vw}(d) \quad \forall v, w \in \mathbf{M}, \forall d \in \mathbf{Z}_+.$$

Remark 1: In the Markov process, the sojourn time in each state obeys the exponential distribution. Due to the memoryless characteristic of the exponential distribution, any time t is an update point, that is, any time possesses Markov properties. However, in the semi-Markov process, the sojourn time in each state is generally distributed, so not all moments but only these state update points satisfy the Markov property.

Remark 2: Compared with [31], the TPM of the MJS is considered to be constant, which seems very strict in practice. Therefore, in this article, the considered TP of the system mode not only depends on the current system mode, but also depends on the sojourn time of the mode, which is more common in practice. Because of its special form of TP, a semi-Markov model to represent the mode information of the error system is chosen in this article.

Definition 4 (see [34]): For given an integer $\mathbb{L}$, the system is said to be mean-square stochastically stable with a prescribed $l_2 - l_\infty$ performance level γ, if for any nonzero $\omega(\iota) \in l_2[0,\infty)$, the following inequality simultaneously hold:

$$\sup_{0\leq\iota\leq\mathbb{L}}\mathcal{E}\left\{e^T(\iota)\,e(\iota)\right\} \leq \gamma^2\sum_{\iota=0}^{\mathbb{L}}\omega^T(\iota)\,\omega(\iota) \quad (13)$$

under zero initial conditions.

Definition 5 (see [35]): Given a sojourn-time upper bound (STUB) $\check{d}_v \in \mathbf{Z}_+$ for mode $v(\forall v \in \mathbf{M})$, the system (Σ) is said to be σ-MSS if for $\omega(\iota) \equiv 0$, scalars $\lambda > 0,\ 0 < \kappa < 1$, and the following condition simultaneously hold under any nonzero initial conditions $x(0) \in \mathbb{R}^{n_x}$, $\grave{r}(0) \in \mathbf{M}$:

$$\begin{aligned}&\mathcal{E}\left\{\|x(\iota)\|^2\right\}_{x(0),\grave{r}(0)\left(S_m\in\mathbf{Z}_{[1,\check{d}_v]}\mid\vartheta_m=v\right)}\\ &\quad\leq \lambda\kappa^{\iota-\iota_0}\mathcal{E}\left\{\|x(\iota_0)\|^2\right\} \quad \forall \iota \geq \iota_0. \quad (14)\end{aligned}$$

Definition 6 (see [36]): Letting $\ddot{\Omega}_{\acute{g}\acute{h}}$ be the matrices with appropriate dimensions, we have $\sum_{\acute{g}=1}^{\mathbf{f}}\sum_{\acute{h}=1}^{\mathbf{f}}\alpha_{\acute{g}}(\theta(\iota))\alpha_{\acute{h}}(\theta(\iota))\ddot{\Omega}_{\acute{g}\acute{h}} < 0$, assuming that the following inequalities hold:

$$\ddot{\Omega}_{\acute{g}\acute{g}} < 0 \quad \forall \acute{g} = 1, 2, \ldots, \mathbf{f},$$

$$\frac{2}{\hbar - 1}\ddot{\Omega}_{\acute{g}\acute{g}} + \ddot{\Omega}_{\acute{g}\acute{h}} + \ddot{\Omega}_{\acute{h}\acute{g}} < 0 \quad \forall \acute{g}, \acute{h} = 1, 2, \ldots, \mathbf{f}, \acute{g} \neq \acute{h}.$$

Remark 3: In the actual semi-Markov process, the sojourn time of the system mode is generally regarded as bounded. If the given upper bound is large enough, from the perspective of probability, the probability of the sojourn time for any mode, which exceeds the upper bound, is very small, even if the probability of the sojourn time d tends to infinity, the sojourn time of the system is still regarded as $\breve{d}_{\max}$. Zhang *et al.* [8] point out that, even if the sojourn time is set unbounded, the MSS of the jump system can be guaranteed when the sojourn time owns upper bounded. In [8], it is also explained that the selection of sojourn time has a significant impact on the stability of the system, so when analyzing the MSS of S-MJSs, the upper bound of the sojourn time should be considered.

Remark 4: Local linearization is a powerful means of approximating nonlinear systems. Nevertheless, the local linearization model is vulnerable to environmental constraints, which has been widely recognized [37]. However, fuzzy models can be used well to describe nonlinearities [27], [28], [38]–[40]. Compared with local linearization, the T-S fuzzy model can achieve ideal approximation accuracy through fewer fuzzy rules when approximating nonlinear systems. Therefore, the T-S fuzzy model possesses great strengths [41], [42].

III. Main Results

In this part, the filter gains will be obtained by calculating a set of inequalities. A set of terseness stability and stabilization criteria for fuzzy discrete-time S-MJSs is presented via the following theorems. Based on the proposed condition, the σ-MSS with an $l_2 - l_\infty$ performance of the closed-loop system can be guaranteed.

A. σ-MSS Criteria

Lemma 1 (see [36]): For given a bounded sojourn time $\breve{d}_{\grave{r}(\iota_m)}$, consider a discrete-time nonlinear S-MJS $x(\iota+1) = f_{\grave{r}(\iota)}(x(\iota))$, where $\grave{r}(\iota)$ and $x(\iota) \in \mathbb{R}^{n_x}$ represent the system mode index and the system state, respectively. In addition, ι_0, $\iota_1, \ldots, \iota_m, \ldots$, represent the switching instants with $\iota_0 = 0$. If there exist scalars $\varphi_1 > 0, \varphi_2 > 0$, an LF $V(x(\iota), \grave{r}(\iota), \iota - \iota_m)$: $\mathbb{R}^{n_x} \to \mathbb{R}$ with $\iota \in \mathbf{Z}_{[\iota_m, \iota_{m+1})}$, such that positive scalars $\rho_{\grave{r}(\iota_m)}$, $l_{\grave{r}(\iota_m)}$, $\grave{r}(\iota_m) \in \mathbf{M}$, and the following inequalities hold under any initial conditions $x(0) \in \mathbb{R}^{n_x}$, $\grave{r}(0) \in \mathbf{M}$:

$$\bar{\mu}_{\grave{r}(\iota_m)} \triangleq \begin{cases} l_{\grave{r}(\iota_m)} \rho_{\grave{r}(\iota_m)}^{\breve{d}_{\grave{r}(\iota_m)}} < 1, \text{when } \rho_{\grave{r}(\iota_m)} > 1 \\ l_{\grave{r}(\iota_m)} \rho_{\grave{r}(\iota_m)} < 1, \text{when } 0 < \rho_{\grave{r}(\iota_m)} \le 1 \end{cases} \tag{15}$$

$$\varphi_1 \|x(\iota)\|^2 \le V(x(\iota), \grave{r}(\iota), \iota - \iota_m) \le \varphi_2 \|x(\iota)\|^2 \tag{16}$$

$$\begin{aligned} &\rho_{\grave{r}(\iota)} V(x(\iota-1), \grave{r}(\iota-1), \iota - \iota_m - 1) \\ &\ge V(x(\iota), \grave{r}(\iota), \iota - \iota_m) \quad \forall \iota \in \mathbf{Z}_{[\iota_m+1, \iota_{m+1})} \end{aligned} \tag{17}$$

$$\begin{aligned} &\mathcal{E}\left\{ l_{\grave{r}(\iota_m)} V(x(\iota_{m+1}), \grave{r}(\iota_m), \iota_{m+1} - \iota_m) \right\} |_{x(\iota_m), \grave{r}(\iota_m)} \\ &\ge \mathcal{E}\{V(x(\iota_{m+1}), \grave{r}(\iota_{m+1}), 0)\} \end{aligned} \tag{18}$$

then we consider that the system is σ-MSS.

Proof: Consider that the STUB for switching mode v ($\forall v \in \mathbf{M}$) is $\breve{d}_v$ and define a σ-algebra generated by $\{\phi_\varkappa \triangleq (x(\iota_\varkappa), \grave{r}(\iota_\varkappa), \varkappa \in \mathbf{Z}_{[0,m]})\}$ and $\Xi_m \triangleq \sigma\{\phi_0, \phi_1, \ldots, \phi_m\}$. First, from (15), (17), and (18), the following inequality holds:

$$\begin{aligned} &\mathcal{E}\{V(x(\iota_{m+1}), \grave{r}(\iota_{m+1}), 0)\} |_{\phi_m} \\ &\le \mathcal{E}\left\{ l_{\grave{r}(\iota_m)} \rho_{\grave{r}(\iota)}^{S_m} V(x(\iota_m), \grave{r}(\iota_m), 0) \right\} |_{\phi_m} \\ &\le \bar{\mu}_{\grave{r}(\iota_m)} V(x(\iota_m), \grave{r}(\iota_m), 0). \end{aligned} \tag{19}$$

Then, recalling the property of conditional expectations [43], the following inequality holds from (19):

$$\begin{aligned} 0 &\le \mathcal{E}\left\{ \bar{\mu}_{\grave{r}(\iota_m)} V(x(\iota_m), \grave{r}(\iota_m), 0) \right. \\ &\quad \left. - \mathcal{E}\{V(x(\iota_{m+1}), \grave{r}(\iota_{m+1}), 0)\} |_{\phi_m} \right\} |_{\phi_0} \\ &= \mathcal{E}\left\{ \bar{\mu}_{\grave{r}(\iota_m)} V(x(\iota_m), \grave{r}(\iota_m), 0) \right\} |_{\phi_0} \\ &\quad - \mathcal{E}\{V(x(\iota_{m+1}), \grave{r}(\iota_{m+1}), 0)\} |_{\phi_0} \end{aligned} \tag{20}$$

which further means

$$\begin{aligned} &\mathcal{E}\{V(x(\iota_{m+1}), \grave{r}(\iota_{m+1}), 0)\} |_{\phi_0} \\ &\le \bar{\mu} \mathcal{E}\{V(x(\iota_m), \grave{r}(\iota_m), 0)\} |_{\phi_0} \end{aligned} \tag{21}$$

with $\bar{\mu} \triangleq \max_{\forall \grave{r}(\iota_m) \in \mathbf{M}} \{\bar{\mu}_{\grave{r}(\iota_m)}\}$.

Consider that $\iota \in [\iota_m + 1, \iota_{m+1})$, and it follows from (17) and (20) that

$$\begin{aligned} &\mathcal{E}\{V(x(\iota), \grave{r}(\iota), \iota - \iota_m)\} |_{\phi_0} \\ &\le \bar{\mu}^m \mathcal{E}\left\{ \rho_{\grave{r}(\iota)}^{\iota - \iota_m} V(x(\iota_0), \grave{r}(\iota_0), 0) \right\} |_{\phi_0} \\ &\le \bar{\lambda} \kappa^{\iota - \iota_0} \mathcal{E}\{V(x(\iota_0), \grave{r}(\iota_0), 0)\} |_{\phi_0} \end{aligned} \tag{22}$$

with $\bar{\lambda} \triangleq \max_{\forall \grave{r}(\iota_m) \in \mathbf{M}} \{\rho_{\grave{r}(\iota_m)}^{\breve{d}_{\grave{r}(\iota_m)}}, 1\}$, $\kappa \triangleq \bar{\mu}^{m/(\iota - \iota_0)}$, with $\kappa \in (0, 1)$; then, combining with (16), we can get

$$\mathcal{E}\left\{ \|x(\iota)\|^2 \right\} |_{\rho_0} \le \frac{\phi_2}{\phi_1} \bar{\lambda} \nu^{\iota - \iota_0} \mathcal{E}\left\{ \|x(\iota_0)\|^2 \right\}. \tag{23}$$

Thereafter, the proof is complete.

Remark 5: In Lemma 1, we derive the σ-MSS criterion of the filtering error system. It should be noted that the sojourn time is upper bounded. If the sojourn time is unlimited, Theorem 1 will reduce to the correlative criterion on MSS. Nevertheless, since the number of inequalities is unlimited, it is difficult to verify the obtained criterion through numerical examples.

Remark 6: In this article, the value of the LF does not necessarily decrease monotonically, which makes the obtained results less conservative. Meanwhile, in the derivation process, the switching rate of the LF between sampling instants $x(\iota_m)$ and $x(\iota_m + 1)$ and switching instants $x(\iota_m)$ and $x(\iota_m + S_m)$ needs to be determined.

B. Stabilization and Performance Analysis

In this part, the stabilization criterion of the filtering error system with an $l_2 - l_\infty$ performance will be given. Before presenting further, we give the following notations:

$$\tilde{\mu} \triangleq \min_{\forall v \in \mathbf{M}} \{\tilde{\mu}_v\}, \tilde{\mu}_v \triangleq \begin{cases} l_v \rho_v^{\breve{d}_v}, \text{when } 0 < \rho_v \le 1 \\ l_v \rho_v, \text{when } \rho_v > 1 \end{cases}$$

$$\tilde{\rho} \triangleq \min_{\forall v \in \mathbf{M}} \{\rho_v\}, \rho_{\max} \triangleq \max_{\forall v \in \mathbf{M}} \{\bar{\rho}_v, 1\}, \bar{\mu} \triangleq \min_{\forall v \in \mathbf{M}} \{\bar{\mu}_v\}.$$

Theorem 1: The system (11), (12) is σ-MSS with a prescribed $l_2 - l_\infty$ performance index $\bar{\gamma} = \gamma\sqrt{\frac{\rho_{\max}^{\check{d}_{\max}}}{\tilde{\rho}}}$, if there exist scalars $\rho_v > 0$, $l_v > 0$, $\check{d}_v \geq 1$, $v \in \mathbf{M}$, and $\gamma > 0$, and symmetrical positive-definite matrix $P_v(d)$, for $\forall v, w \in \mathbf{M}$, $\forall d \in \mathbf{Z}_{[1,\check{d}_v]}$, and the following inequalities hold:

$$\bar{\mu}_v \triangleq \begin{cases} l_v \rho_v^{\check{d}_v} < 1, \text{when } \rho_v > 1 \\ l_v \rho_v < 1, \text{when } 0 < \rho_v \leq 1 \end{cases} \tag{24}$$

$$\begin{bmatrix} -\rho_v P_v(d-1) & 0 & \bar{A}^T_{\acute{g}\acute{h}v} \\ * & -\gamma^2 I & \bar{B}^T_{\acute{g}\acute{h}v} \\ * & * & -P_v^{-1}(d) \end{bmatrix} < 0 \tag{25}$$

$$\begin{bmatrix} -P_v(d) & \bar{C}^T_{\acute{g}\acute{h}v} \\ * & -I \end{bmatrix} < 0 \tag{26}$$

$$P_w(0) - l_v P_v(d) < 0. \tag{27}$$

Proof: Construct the following LF:

$$V(x(\iota), \grave{r}(\iota), d) \triangleq x^T(\iota) P_v(d) x(\iota) \tag{28}$$

where $d = \iota - \iota_m$ is the sojourn time; $P_v(d)$ is a positive-definite unknown matrix and is related to sojourn time. By using the Schur complement to (25), it is readily obtained that

$$\begin{aligned} &V(x(\iota_m + d), \grave{r}(\iota_m), d) \\ &- \rho_v V(x(\iota_m + d - 1), \grave{r}(\iota_m), d-1) \\ &- \gamma^2 \omega^T(\iota_m + d - 1)\omega(\iota_m + d - 1) \\ \leq{}& 0 \quad \forall v \in \mathbf{M}, \forall d \in \mathbf{Z}_{[1,S_m]}. \end{aligned} \tag{29}$$

First the σ-MSS of the filtering error system (11), (12) under $\omega(\iota) \equiv 0$ is proved.

From (28), we can get

$$\lambda_1 \|x(\iota)\|^2 \leq V(x(\iota), \grave{r}(\iota), d) \leq \lambda_2 \|x(\iota)\|^2 \tag{30}$$

with $\lambda_1 \triangleq \min_{\forall \grave{r}(\iota) \in \mathbf{M}, d \in \mathbf{Z}_{[1,\check{d}_v]}} (P_{\grave{r}(\iota)}(d))$ and $\lambda_2 \triangleq \max_{\forall v \in \mathbf{M}, d \in \mathbf{Z}_{[1,\check{d}_v]}} (P_v(d))$; thus, (16) is satisfied.

Combining with (28), (29), and $\omega(\iota) \equiv 0$, we can get

$$\begin{aligned} &\rho_v V(x(\iota_m + d - 1), v, d-1) \\ &\geq V(x(\iota_m + d), v, d) \quad \forall d \in \mathbf{Z}_{[1,S_m]} \end{aligned} \tag{31}$$

which means (17) is guaranteed.

Furthermore, from (27), one can deduce that

$$\begin{aligned} &\mathcal{E}\{V(x(\iota_{m+1}), w, 0) - l_v V(x(\iota_{m+1}), v, d)\}|_{\rho_m} \\ &= \sum_{d=1}^{\check{d}_v} \sum_{w \in \mathbf{M}} \frac{\pi_{vw}(d)}{\eta_v} x^T(\iota_m + d) \\ &\quad \times [P_w(0) - l_v P_v(S_m)] x(\iota_m + d) \\ &\leq 0 \end{aligned} \tag{32}$$

where $\eta_v \triangleq \sum_{d=1}^{\check{d}_v} \sum_{w \in \mathbf{M}} \pi_{vw}(d)$, which means (18) is satisfied. Therefore, from Lemma 1, the σ-MSS of the filtering error system (11), (12) is guaranteed.

In the following part, the $l_2 - l_\infty$ performance of the filtering error system is proved.

For $\iota \in [\iota_m + 1, \iota_{m+1})$, the following inequalities hold from (15), (17), (18), and (28):

$$\begin{aligned} &\mathcal{E}\{V(x(\iota), v, d)\} \\ &\leq \mathcal{E}\Bigg\{\rho_{\max}^{\check{d}_{\max}} V(x(\iota_m), v, 0) \\ &\qquad + \gamma^2 \sum_{\ell=\iota_m}^{\iota-1} \rho_v^{\iota-\ell-1} \omega^T(\ell)\omega(\ell)\Bigg\} \end{aligned} \tag{33}$$

and

$$\begin{aligned} &\mathcal{E}\{V(x(\iota_m), \grave{r}(\iota_m), 0)\} \\ &\leq \mathcal{E}\Bigg\{\bar{\mu} V(x(\iota_{m-1}), \grave{r}(\iota_{m-1}), 0) \\ &\qquad + \gamma^2 \sum_{\ell=\iota_{m-1}}^{\iota_m - 1} l_{\grave{r}(\iota_{m-1})} \rho_{\grave{r}(\iota_{m-1})}^{\iota_m - \ell - 1} \omega^T(\ell)\omega(\ell)\Bigg\}. \end{aligned} \tag{34}$$

According to (24), $\tilde{\mu} \leq \bar{\mu} < 1$ and $\tilde{\mu} \leq l_{\grave{r}(\iota_m)} \rho_{\grave{r}(\iota_m)}^{\check{d}_m} \leq \bar{\mu}$, $\forall \grave{r}(\iota_m) \in \mathbf{M}$, $\check{d}_{\grave{r}(\iota_m)} \in \mathbf{Z}_{[1,S_{\grave{r}(\iota_m)}]}$, are guaranteed. By iterating on inequalities (33) and (34), one can deduce that

$$\begin{aligned} &\mathcal{E}\{V(x(\iota), \grave{r}(\iota), \iota - \iota_m)\} \\ &\leq \mathcal{E}\left\{\rho_{\max}^{\check{d}_{\max}} \bar{\mu}^m V(x(\iota_0), \grave{r}(\iota_0), 0)\right\} \\ &\quad + \gamma^2 \rho_{\max}^{\check{d}_{\max}} \mathcal{E}\Bigg\{\bar{\mu}^{m-1} \sum_{\ell=\iota_0}^{\iota_1 - 1} l_{\grave{r}(\iota_0)} \rho_{\grave{r}(\iota_0)}^{\iota_1 - \ell - 1} \omega^T(\ell)\omega(\ell) \\ &\quad + \bar{\mu}^{m-2} \sum_{\ell=\iota_1}^{\iota_2 - 1} l_{\grave{r}(\iota_1)} \rho_{\grave{r}(\iota_1)}^{\iota_2 - \ell - 1} \omega^T(\ell)\omega(\ell) + \\ &\quad \cdots + \bar{\mu}^0 \sum_{\ell=\iota_{m-1}}^{\iota_m - 1} l_{\grave{r}(\iota_{m-1})} \rho_{\grave{r}(\iota_{m-1})}^{\iota_m - \ell - 1} \omega^T(\ell)\omega(\ell)\Bigg\} \\ &\quad + \gamma^2 \sum_{\ell=\iota_m}^{\iota-1} \rho_v^{\iota-\ell-1} \omega^T(\ell)\omega(\ell). \end{aligned} \tag{35}$$

Then, under zero initial conditions, one has

$$\begin{aligned}&\mathcal{E}\left\{V\left(x\left(\iota\right),v,\iota-\iota_m\right)\right\}\\&\leq\gamma^2\rho_{\max}^{\breve{d}_{\max}}\mathcal{E}\left\{\bar{\mu}^{m-1}\sum_{\ell=\iota_0}^{\iota_1-1}l_{\grave{r}(\iota_0)}\rho_{\grave{r}(\iota_0)}^{\iota_1-\ell-1}\omega^T(\ell)\omega(\ell)\right.\\&+\bar{\mu}^{m-2}\sum_{\ell=\iota_1}^{\iota_2-1}l_{\grave{r}(\iota_1)}\rho_{\grave{r}(\iota_1)}^{\iota_2-\ell-1}\omega^T(\ell)\omega(\ell)\\&+\cdots+\bar{\mu}^{0}\sum_{\ell=\iota_{m-1}}^{\iota_m-1}l_{\grave{r}(\iota_{m-1})}\rho_{\grave{r}(\iota_{m-1})}^{\iota_m-\ell-1}\omega^T(\ell)\omega(\ell)\\&\left.+\gamma^2\sum_{\ell=\iota_m}^{\iota-1}\rho_v^{\iota-\ell-1}\omega^T(\ell)\omega(\ell)\right\}\end{aligned}\tag{36}$$

which further means

$$\begin{aligned}&\mathcal{E}\left\{V\left(x\left(\iota\right),v,\iota-\iota_m\right)\right\}\\&\leq\gamma^2\rho_{\max}^{\breve{d}_{\max}}\mathcal{E}\left\{\bar{\mu}^{m-1}\sum_{\ell=\iota_0}^{\iota_1-1}\omega^T(\ell)\omega(\ell)\right.\\&\left.+\bar{\mu}^{m-2}\sum_{n=\iota_1}^{\iota_2-1}\omega^T(\ell)\omega(\ell)+\ldots+\bar{\mu}^{0}\sum_{n=\iota_{m-1}}^{\iota_m-1}\omega^T(\ell)\omega(\ell)\right\}\\&+\gamma^2\sum_{\ell=\iota_m}^{\iota-1}\rho_v^{\iota-\ell-1}\omega^T(\ell)\omega(\ell).\end{aligned}\tag{37}$$

Denoting $Y(t,\iota)$ as the total jumping times on the interval $(t,\iota]$, it is easy to get that

$$\frac{\iota-t}{\breve{d}_{\max}}-1\leq Y\left(t,\iota\right)\leq\iota-t.\tag{38}$$

From (38), one attains that

$$\begin{aligned}&\mathcal{E}\left\{V\left(x\left(\iota\right),v,\iota-\iota_m\right)\right\}\\&\leq\frac{\rho_{\max}^{\breve{d}_{\max}}}{\tilde{\rho}}\gamma^2\mathcal{E}\left\{\sum_{\ell=\iota_0}^{\iota-1}\bar{\mu}^{\frac{\iota-\ell-1}{\breve{d}_{\max}}}\omega^T(\ell)\omega(\ell)\right\}\\&\leq\bar{\gamma}^2\mathcal{E}\left\{\sum_{\ell=\iota_0}^{\iota-1}\omega^T(\ell)\omega(\ell)\right\}\end{aligned}\tag{39}$$

where $\bar{\gamma}=\gamma\sqrt{\frac{h_{\max}^{\breve{d}_{\max}}}{\tilde{h}}}$.

Then, by using the Schur complement to (26), it is easy to get

$$\bar{C}_{\acute{g}\acute{h}v}^T\bar{C}_{\acute{g}\acute{h}v}<P_v(d).\tag{40}$$

Thereafter, combining inequalities (39) and (40), it can be easily deduced that

$$\begin{aligned}e^T(\iota)e(\iota)&\leq V\left(x\left(\iota\right),\grave{r}\left(\iota\right),\iota-\iota_m\right)\\&\leq\bar{\gamma}^2\sum_{\ell=\iota_0}^{\mathbb{L}}\omega^T(\ell)\omega(\ell).\end{aligned}\tag{41}$$

Therefore, for any $\omega(\ell)\in l_2[0,\infty)$, we have that condition (13) is satisfied; therefore, the proof is complete.

Theorem 2: With regard to the finite positive scalars $\bar{\gamma}$, ρ_v, and l_v, if there exist matrices $\tilde{A}_{f,\acute{h}v}$, $\tilde{B}_{f,\acute{h}v}$, $\tilde{C}_{f,\acute{h}v}$, $\tilde{\Lambda}_v\triangleq\left[\begin{smallmatrix}\Lambda_{1v} & n_1\Lambda_v\\ \Lambda_{2v} & n_2\Lambda_v\end{smallmatrix}\right]$, and $\Re\triangleq\mathrm{diag}\{\Re_1,\Re_2\}$ and symmetric matrix $P_v(d)\triangleq\left[\begin{smallmatrix}P_{1v}(d) & P_{2v}(d)\\ * & P_{3v}(d)\end{smallmatrix}\right]>0$, such that (24), (26), (27), and the following conditions hold $\forall v,w\in\mathbf{M},d\in[0,\breve{d}_v]$:

$$\Psi_{\acute{g}\acute{g}v}<0\tag{42}$$

$$\frac{2}{\hbar-1}\Psi_{\acute{g}\acute{g}v}+\Psi_{\acute{g}\acute{h}v}+\Psi_{\acute{h}\acute{g}v}<0\tag{43}$$

$$\Psi_{\acute{g}\acute{h}v}\triangleq\begin{bmatrix}\Psi_{1\acute{g}\acute{h}v} & \Psi_{2\acute{g}\acute{h}v}\\ * & \Psi_{3\acute{g}\acute{h}v}\end{bmatrix}<0\tag{44}$$

where

$$\Psi_{3\acute{g}\acute{h}v}\triangleq\begin{bmatrix}\Phi_1 & \Phi_2\\ * & \Phi_3\end{bmatrix}$$

$$\Psi_{1\acute{g}\acute{h}v}\triangleq\begin{bmatrix}-\rho_vP_{1v}\left(d-1\right) & -\rho_vP_{2v}\left(d-1\right)\\ * & -\rho_vP_{3v}\left(d-1\right)\end{bmatrix}$$

$$\Psi_{2\acute{g}\acute{h}v}\triangleq\begin{bmatrix}A_{\acute{g}i}^T\Lambda_{1v}^T+n_1C_{\acute{g}v}^T\tilde{B}_{f,\acute{h}v}^T & A_{\acute{g}v}^T\Lambda_{2v}^T+n_2C_{\acute{g}v}^T\tilde{B}_{f,\acute{h}v}^T\\ n_1\tilde{A}_{f,\acute{h}v}^T & n_2\tilde{A}_{f,\acute{h}v}^T\end{bmatrix}$$

and

$$\begin{aligned}\Phi_1&\triangleq\Re_1P_{1v}(d)\Re_1^T-\Re_1\Lambda_{1v}^T-\Re_1^T\Lambda_{1v}\\\Phi_2&\triangleq\Re_1P_{2v}(d)\Re_2^T-\Re_1\Lambda_{2v}^T-n_1\Lambda_v\Re_2^T\\\Phi_3&\triangleq\Re_2P_{3v}(d)\Re_2^T-n_2\Re_2\Lambda_v^T-n_2\Lambda_v\Re_2^T\end{aligned}$$

then the system (Σ) is σ-MSS with a prescribed l_2-l_∞ performance level $\bar{\gamma}$; moreover, the filter gains can be expressed as

$$A_{f,\acute{h}v}=\Lambda_v^{-1}\tilde{A}_{f,\acute{h}v},B_{f,\acute{h}v}=\Lambda_v^{-1}\tilde{B}_{f,\acute{h}v},C_{f,\acute{h}v}=\tilde{C}_{f,\acute{h}v}.\tag{45}$$

Proof: First, from

$$\left(\Re P_v(d)\Re^T-\Re\tilde{\Lambda}_v\right)P_v^{-1}(d)\left(\Re P_v(d)\Re^T-\Re\tilde{\Lambda}_v\right)^T\geq0$$

the following inequality holds $\forall v,w\in\mathbf{M},d\in[1,\breve{d}_v]$

$$-\tilde{\Lambda}_vP_v^{-1}(d)\tilde{\Lambda}_v^T\leq\Re P_v(d)\Re^T-\Re\tilde{\Lambda}_v^T-\tilde{\Lambda}_v^T\Re^T.$$

Then, perform congruence transformations to (44) by $\mathrm{diag}\{I,I,\tilde{\Lambda}_v^{-1}\}$ and its transpose. Then, let $\tilde{A}_{f,\acute{h}v}\triangleq\Lambda_vA_{f,\acute{h}v}$, $\tilde{B}_{f,\acute{h}v}\triangleq\Lambda_vB_{f,\acute{h}v}$, and $\tilde{C}_{f,\acute{h}v}\triangleq C_{f,\acute{h}v}$, by using the Schur complement and Lemma 1. From (44), we can easily find that (25) is guaranteed. This completes the proof.

Remark 7: Specifically, even for a simple S-MJS $x(\iota+1)=A_vx(\iota)$, the coupling term A_v^τ will be introduced, which will bring great difficulties to the decoupling process. If there exist nonlinear items or interference items in the system, the correlation analysis will be hard to proceed. Fortunately, the method proposed in this article can effectively solve this problem, so it can be applied to a wider range of systems.

IV. Simulation Results

In this part, two examples are given to illustrate the effectiveness of the proposed methods.

Example 1: Consider a discrete-time S-MJS with three modes, two fuzzy rules, and the following parameters:

$$A_{11} = \begin{bmatrix} 0.0465 & -0.2575 \\ 0.0374 & -0.2592 \end{bmatrix}, A_{12} = \begin{bmatrix} 0.0402 & -0.1907 \\ 0.0604 & -0.2501 \end{bmatrix}$$

$$A_{13} = \begin{bmatrix} 0.0469 & -0.1846 \\ 0.0296 & -0.2179 \end{bmatrix}, A_{21} = \begin{bmatrix} 0.0412 & -0.2892 \\ 0.1602 & -0.4984 \end{bmatrix}$$

$$A_{22} = \begin{bmatrix} 0.0535 & -0.2259 \\ 0.1144 & -0.3129 \end{bmatrix}, A_{23} = \begin{bmatrix} 0.0355 & -0.2799 \\ 0.0904 & -0.2475 \end{bmatrix}$$

$$B_{11} = \begin{bmatrix} -0.7 & -0.4 \end{bmatrix}^T, B_{12} = \begin{bmatrix} 0.6 & 0.2 \end{bmatrix}^T$$

$$B_{13} = \begin{bmatrix} -0.6 & -0.1 \end{bmatrix}^T, B_{21} = \begin{bmatrix} -0.4 & -0.2 \end{bmatrix}^T$$

$$B_{22} = \begin{bmatrix} -0.1 & -0.1 \end{bmatrix}^T, B_{23} = \begin{bmatrix} -0.5 & -0.3 \end{bmatrix}^T$$

$$C_{11} = \begin{bmatrix} 0.4 & -0.4 \\ 0.6 & -0.15 \end{bmatrix}, C_{12} = \begin{bmatrix} 0.12 & -0.3 \\ 0.21 & -0.2 \end{bmatrix}$$

$$C_{13} = \begin{bmatrix} 0.6 & -0.14 \\ 0.33 & -0.51 \end{bmatrix}, C_{21} = \begin{bmatrix} 0.11 & -0.24 \\ 0.15 & 0.15 \end{bmatrix}$$

$$C_{22} = \begin{bmatrix} -0.12 & 0.12 \\ 0.11 & -0.25 \end{bmatrix}, C_{23} = \begin{bmatrix} 0.58 & 0.6 \\ 0.11 & 0.7 \end{bmatrix}$$

$$D_{11} = \begin{bmatrix} 1.26 & 0.56 \end{bmatrix}^T, D_{12} = \begin{bmatrix} 1.96 & -0.42 \end{bmatrix}^T$$

$$D_{13} = \begin{bmatrix} 1.82 & 1.12 \end{bmatrix}^T, D_{21} = \begin{bmatrix} 1.12 & 0.70 \end{bmatrix}^T$$

$$D_{22} = \begin{bmatrix} 1.82 & 0.98 \end{bmatrix}^T, D_{23} = \begin{bmatrix} 1.54 & 0.84 \end{bmatrix}^T$$

$$L_{11} = \begin{bmatrix} 0.36 & 0.09 \\ 0.066 & -0.045 \end{bmatrix}, L_{12} = \begin{bmatrix} 0.27 & -0.09 \\ 0.15 & -0.033 \end{bmatrix}$$

$$L_{13} = \begin{bmatrix} 0.09 & 0.15 \\ 0.03 & -0.033 \end{bmatrix}, L_{21} = \begin{bmatrix} 0.153 & 0.24 \\ 0.06 & 0.081 \end{bmatrix}$$

$$L_{22} = \begin{bmatrix} 0.18 & 0.27 \\ 0.03 & 0.117 \end{bmatrix}, L_{23} = \begin{bmatrix} -0.27 & 0.03 \\ 0.057 & -0.057 \end{bmatrix}.$$

In addition, the jumping process among the three modes is governed by an SMC with the STUB chosen as $\breve{d}_1 = 10$, $\breve{d}_2 = 8$, and $\breve{d}_3 = 6$; the SMK of the SMC is computed by Definition 3 with $\Theta \triangleq [\theta_{vw}]_{v,w\in\mathbf{M}}$ and STPDF matrix $F(d) \triangleq [f_{vw}(d)]_{\forall v,w\in\mathbf{M}}$, which are given as follows:

$$\Theta = \begin{bmatrix} 0 & 0.8 & 0.2 \\ 0.7 & 0 & 0.3 \\ 0.6 & 0.4 & 0 \end{bmatrix}$$

$$F(d) = \begin{bmatrix} 0 & \frac{0.6^d 0.4^{10-d} 10!}{(10-d)!d!} & 0.2 \cdot 8^{d-1} \\ 0.9^{(d-1)^2} & 0 & \frac{0.5^8 \cdot 8!}{(8-d)!d!} \\ 0.4 \cdot 0.6^{d-1} & 0.3^{(d-1)^{0.8}} - 0.3^{d^{0.8}} & 0 \end{bmatrix}.$$

Besides, for the variation of the LF, we choose $\rho_1 = \rho_2 = \rho_3 = 0.8$, and $l_1 = l_2 = l_3 = 0.7$; scalars $n_1 = 2$, $n_2 = 3.5$, and $\hbar = 2$; the $l_2 - l_\infty$performance index $\gamma = 0.1$; and diagonal matrix $\Re = \text{diag}\ \{3, 3\}$. Through setting the above parameters, the desired filter gains can be obtained by solving a series of linear matrix inequalities derived in Theorem 2, which are shown as follows:

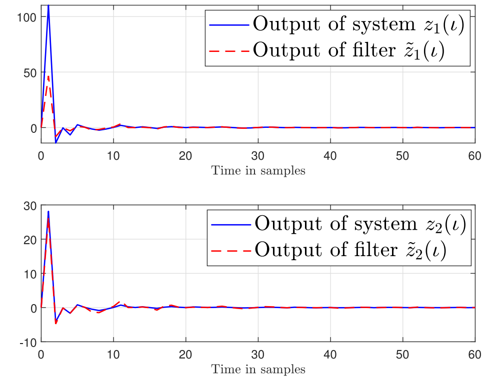

Fig. 1. Output trajectories of the system and the filter.

$$A_{f11} = \begin{bmatrix} 0.0313 & -0.1806 \\ -0.1435 & -0.2291 \end{bmatrix}, A_{f12} = \begin{bmatrix} -0.0214 & -0.0162 \\ -0.0143 & -0.0167 \end{bmatrix}$$

$$A_{f13} = \begin{bmatrix} 0.0952 & -0.0200 \\ 0.0027 & -0.0438 \end{bmatrix}, A_{f21} = \begin{bmatrix} -0.0952 & -0.1455 \\ -0.1160 & -0.2897 \end{bmatrix}$$

$$A_{f22} = \begin{bmatrix} -0.0494 & -0.0376 \\ -0.0327 & -0.0408 \end{bmatrix}, A_{f23} = \begin{bmatrix} 0.1161 & 0.0587 \\ 0.0626 & 0.0127 \end{bmatrix}$$

$$B_{f11} = \begin{bmatrix} 0.2393 & 0.2254 \\ 0.1960 & -0.1802 \end{bmatrix}, B_{f12} = \begin{bmatrix} -0.1548 & -0.0249 \\ -0.0433 & -0.0284 \end{bmatrix}$$

$$B_{f13} = \begin{bmatrix} 0.1529 & 0.0188 \\ 0.0387 & -0.0155 \end{bmatrix}, B_{f21} = \begin{bmatrix} -0.0757 & 0.1536 \\ 0.1233 & 0.0904 \end{bmatrix}$$

$$B_{f22} = \begin{bmatrix} 0.0256 & 0.0201 \\ 0.0236 & -0.0129 \end{bmatrix}, B_{f23} = \begin{bmatrix} 0.1448 & 0.0416 \\ 0.0464 & 0.0566 \end{bmatrix}$$

$$C_{f11} = \begin{bmatrix} -0.2708 & -0.1661 \\ -0.1661 & -0.1783 \end{bmatrix}, C_{f12} = \begin{bmatrix} -0.2530 & -0.1284 \\ -0.1284 & -0.0930 \end{bmatrix}$$

$$C_{f13} = \begin{bmatrix} -0.2784 & -0.0728 \\ -0.0728 & 0.0032 \end{bmatrix}, C_{f21} = \begin{bmatrix} -0.2708 & -0.1661 \\ -0.1661 & -0.1783 \end{bmatrix}$$

$$C_{f22} = \begin{bmatrix} -0.2530 & -0.1284 \\ -0.1284 & -0.0930 \end{bmatrix}, C_{f23} = \begin{bmatrix} -0.2784 & -0.0728 \\ -0.0728 & 0.0032 \end{bmatrix}.$$

According to the obtained filter gains, and combining with the initial conditions $x(0) = [0.3\ -0.1]^T$ and $\tilde{x}(0) = [0\ 0]^T$, the external interference is chosen as $\omega(\iota) = 60\sin(0.3\iota\pi)/(0.03 + 0.04\iota^2)$. Then, the output trajectories of the system and the filter and the filtering error response are plotted in Figs. 1 and 2, respectively. It can be seen from Fig. 1 that the filter output response curve gradually approaches the system output response curve, and the error response of the system shown in Fig. 2 gradually tends to 0, which indicates that the proposed methods are effective.

In addition, the corresponding semi-Markov mode is presented in Fig. 3. The $l_2 - l_\infty$ performance level $\bar{\gamma}$ is shown in Fig. 4.

Furthermore, the influence to minimal $l_2 - l_\infty$ performance index $\bar{\gamma}_{\min}$ of the value of LF variation ρ_v is explored and shown in Table I and Fig. 5. First, we fixed the value of attenuation rate $l_v = 1$ and, then, change the value of LF variation ρ_v. From

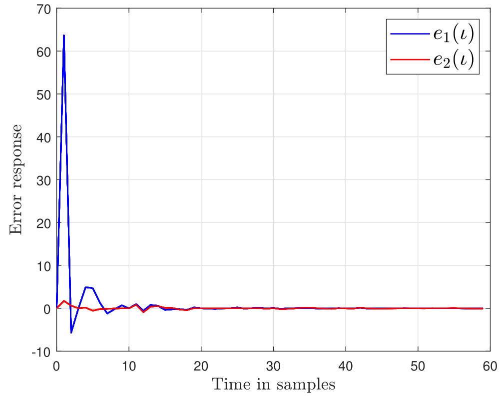

Fig. 2. Filtering error response.

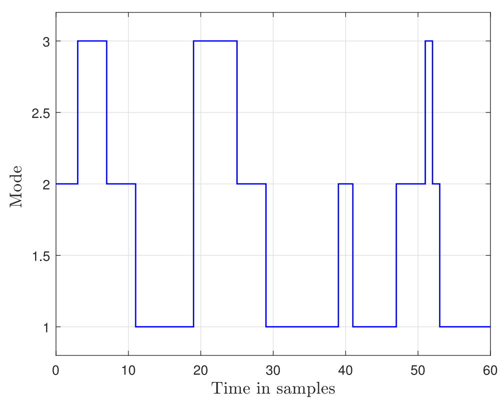

Fig. 3. Filtering error system mode.

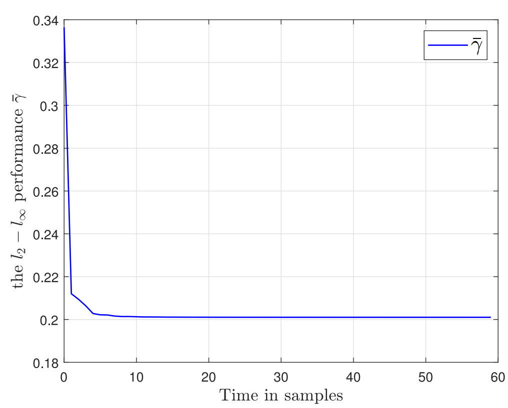

Fig. 4. $l_2 - l_\infty$ performance index $\bar{\gamma}$.

Table I and Fig. 5, one can discover that the minimal $l_2 - l_\infty$ performance index $\bar{\gamma}_{\min}$ is decreasing when the LF variation ρ_v increases within a certain range.

Example 2: In this section, a tunnel diode circuit model, which is shown in Fig. 6, modified from [44] is adopted to prove the validity of the presented method. The tunnel diode is characterized by

$$i_{\mathcal{D}}(\jmath) = 0.002\mathcal{V}_{\mathcal{D}}(\jmath) + 0.01\mathcal{V}_{\mathcal{D}}^3(\jmath). \tag{46}$$

TABLE I

OPTIMAL $l_2 - l_\infty$ PERFORMANCE INDEX $\bar{\gamma}_{\min}$ FOR DIFFERENT ρ_v

ρ_v	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2
$\bar{\gamma}_{\min}$	infeasible	0.1058	0.0762	0.0671	0.0624	0.0591	0.0574	0.0557

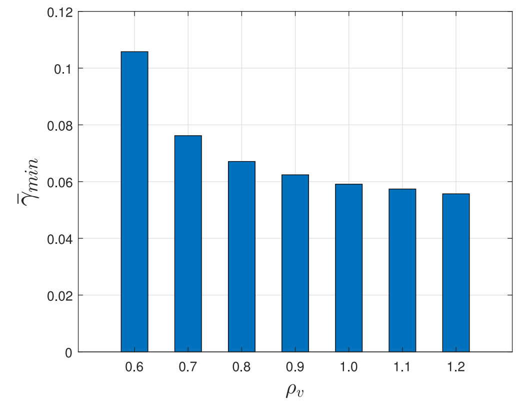

Fig. 5. $l_2 - l_\infty$ performance index $\bar{\gamma}_{\min}$ for different ρ_v.

TABLE II

RELATED PARAMETERS OF THE DIODE CIRCUIT MODEL

V_C	i_C	V_L	i_L	V_D	i_D	S
capacitor voltage	capacitor current	inductance voltage	inductance current	diode voltage	diode current	circuit switch

According to the corresponding system model (1)–(3), let the state variable $x_1(\jmath)$ represent the capacitor voltage $\mathcal{V}_C(\jmath)$ and the state variable $x_2(\jmath)$ denote the inductor current $i_L(\jmath)$. Here, the definitions of some circuit parameters are listed in Table II. Besides, the switch S is subject to an SMC. According to Kirchhoff's law, the circuit model is constructed as follows:

$$\begin{cases} \dot{x}_1(\jmath) = \frac{-0.002}{C}x_1(\jmath) - \frac{0.01}{C}x_1^3(\jmath) + \frac{1}{C}x_2(\jmath) \\ \dot{x}_2(\jmath) = -\frac{1}{L}x_1(\jmath) - \frac{R_m}{L}x_2(\jmath) \\ y(\jmath) = \check{\mathcal{C}}_{\acute{g}v}x(\jmath) \\ z(\jmath) = \check{\mathcal{L}}_{\acute{g}v}x(\jmath) \end{cases} \tag{47}$$

where $\acute{g} = 1, 2$ and $v = 1, 2, 3$. The parameters are given as $C = 0.02$ F, $L = 0.1$ H, $R_1 = 1\Omega$, $R_2 = 5\Omega$, and $R_3 = 10\Omega$; then, (47) can be rewritten as

$$\begin{cases} \dot{x}_1(\jmath) = -0.1x_1(\jmath) - 0.5x_1^3(\jmath) + 50x_2(\jmath) \\ \dot{x}_2(\jmath) = -10x_1(\jmath) - \frac{R_m}{0.1}x_2(\jmath) \\ y(\jmath) = \check{\mathcal{C}}_{\acute{g}v}x(\jmath) \\ z(\jmath) = \check{\mathcal{L}}_{\acute{g}v}x(\jmath). \end{cases} \tag{48}$$

To deal with the nonlinear term $0.01x_1^3(\jmath)$, two local linear models are introduced, and consider that $x_1(\jmath) \in [-3, 3]$. Then, assume that $|x(\jmath)| \leq 3$, and $\tilde{x}(\jmath)$ satisfies the same fuzzy basis function. The nonlinear network system can be approximated by the following T-S fuzzy model.

Plant Rule 1: **IF** $|x_1(\jmath)|$ is 1, **THEN**

$$\begin{cases} \dot{x}(\jmath) = \check{A}_{1v}x(\jmath) + \check{B}_{1v}\omega(\jmath) \\ y(\jmath) = \check{\mathcal{C}}_{1v}x(\jmath) + \check{\mathcal{D}}_{1v}\omega(\jmath) \\ z(\jmath) = \check{\mathcal{L}}_{1v}x(\jmath). \end{cases}$$

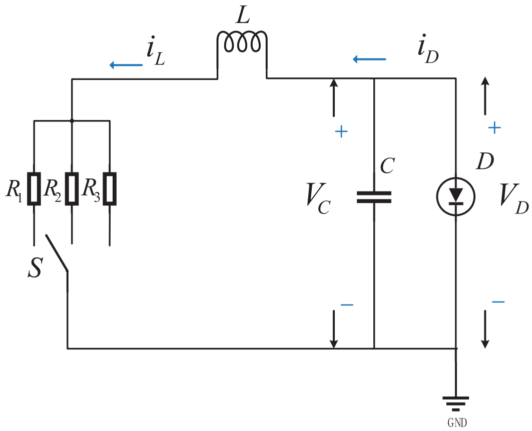

Fig. 6. Modified tunnel diode circuit system model.

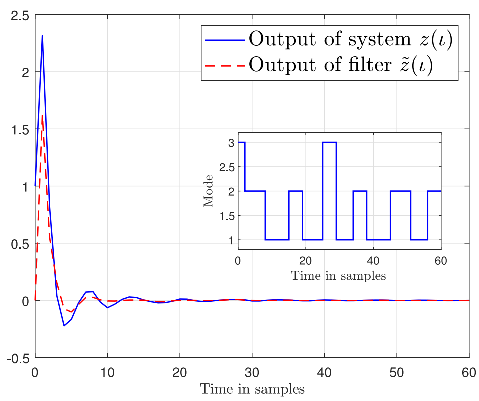

Fig. 7. Output trajectories and jumping modes.

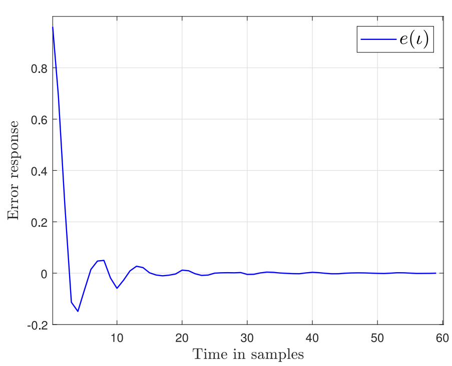

Fig. 8. Filtering error response.

Plant Rule 2: **IF** $|x_1(\jmath)|$ is 4, **THEN**

$$\begin{cases} \dot{x}(\jmath) = \breve{A}_{2v}x(\jmath) + \breve{B}_{2v}\omega(\jmath) \\ y(\jmath) = \breve{\mathcal{C}}_{2v}x(\jmath) + \breve{\mathcal{D}}_{2v}\omega(\jmath) \\ z(\jmath) = \breve{\mathcal{L}}_{2v}x(\jmath) \end{cases}$$

by choosing the membership functions as

$$\alpha_1(x_1(\jmath)) = \begin{cases} 1 - (x_1(\jmath))^2/9, -3 \le x_1(\jmath) \le 3 \\ 0, \text{otherwise} \end{cases}$$

$$\alpha_2(x_1(\jmath)) = 1 - \alpha_1(x_1(\jmath))$$

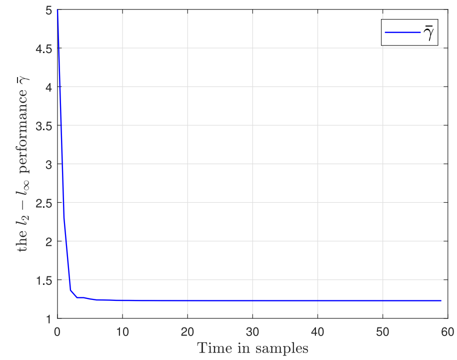

Fig. 9. $l_2 - l_\infty$ performance index $\bar{\gamma}$.

where $y(\jmath)$ signifies the measurement output, $\omega(\jmath)$ denotes the external interference input, and $z(\jmath)$ represents the system output. $\breve{A}_{\acute{g}v}$, $\breve{B}_{\acute{g}v}$, $\breve{\mathcal{C}}_{\acute{g}v}$, $\breve{\mathcal{L}}_{\acute{g}v}$ $\breve{\mathcal{D}}_{\acute{g}v}$, $\acute{g} = 1, 2$; $v = 1, 2, 3$, are the constant matrices; among them, $\breve{\mathcal{C}}_{gv}$ is the sensor matrix, and the necessary objective system matrices can be obtained as

$$\breve{A}_{11} = \begin{bmatrix} -0.1 & 10 \\ -10 & -10 \end{bmatrix}, \breve{A}_{21} = \begin{bmatrix} -4.6 & 10 \\ -10 & -10 \end{bmatrix}$$

$$\breve{A}_{12} = \begin{bmatrix} -0.1 & 10 \\ -10 & -50 \end{bmatrix}, \breve{A}_{22} = \begin{bmatrix} -4.6 & 10 \\ -10 & -50 \end{bmatrix}$$

$$\breve{A}_{13} = \begin{bmatrix} -0.1 & 10 \\ -10 & -100 \end{bmatrix}, \breve{A}_{23} = \begin{bmatrix} -4.6 & 10 \\ -10 & -100 \end{bmatrix}.$$

Then, selecting the sampling time as $T = 0.2$ s, it is easy to obtain the discrete-time fuzzy model after discretizing the system as follows.

Plant Rule 1: **IF** $|x_1(\iota)|$ is 1, **THEN**

$$\begin{cases} x(\iota+1) = \breve{A}_{1v}x(\iota) + \breve{B}_{1v}\omega(\iota) \\ y(\iota) = \breve{\mathcal{C}}_{1v}x(\iota) + \breve{\mathcal{D}}_{1v}\omega(\iota) \\ z(\iota) = x(\iota). \end{cases}$$

Plant Rule 2: **IF** $|x_1(\iota)|$ is 4, **THEN**

$$\begin{cases} x(\iota+1) = \breve{A}_{2v}x(\iota) + \breve{B}_{2v}\omega(\iota) \\ y(\iota) = \breve{\mathcal{C}}_{2v}x(\iota) + \breve{\mathcal{D}}_{2v}\omega(\iota) \\ z(\iota) = x(\iota) \end{cases}$$

where

$$\breve{A}_{11}=\begin{bmatrix} 0.1441 & 0.4133 \\ -0.4133 & -0.2651 \end{bmatrix}, \breve{A}_{21}=\begin{bmatrix} -0.01964 & 0.2262 \\ -0.2262 & -0.1418 \end{bmatrix}$$

$$\breve{A}_{12}=\begin{bmatrix} 0.6746 & 0.1411 \\ -0.1411 & -0.02944 \end{bmatrix}, \breve{A}_{22}=\begin{bmatrix} 0.2648 & 0.06145 \\ -0.06145 & -0.01419 \end{bmatrix}$$

$$\breve{A}_{13}=\begin{bmatrix} 0.809 & 0.08181 \\ -0.08181 & -0.008273 \end{bmatrix}, \breve{A}_{23}=\begin{bmatrix} 0.3261 & 0.03456 \\ -0.03456 & -0.003663 \end{bmatrix}$$

$$\breve{B}_{\acute{g}v} = \begin{bmatrix} 0.5 \\ 0 \end{bmatrix}, \breve{\mathcal{C}}_{gv} = \breve{\mathcal{L}}_{\acute{g}v} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}^T, \breve{D}_{\acute{g}v} = 0.5$$

$$\acute{g} = 1, 2, v = 1, 2, 3.$$

Then, according to the fuzzy filter model (7), (8), we construct a filtering error system as same as (11), (12). After the construction of the filtering error system is finished, we let the SMK be the same as in Example 1; then, choose scalars $\rho_1 = \rho_2 = \rho_3 = 0.56$; $l_1 = l_2 = l_3 = 0.8$; $n_1 = 2$, $n_2 = 3.5$, $\hbar = 2$; the $l_2 - l_\infty$performance index $\gamma = 1$; and diagonal matrix $\Re = \text{diag}\{3, 3\}$. Through solving the conditions derived in Theorem 2, the desired filter gains can be calculated as

$$A_{f11} = \begin{bmatrix} -0.0280 & 0.1845 \\ 0.1147 & -0.0179 \end{bmatrix}, A_{f12} = \begin{bmatrix} -0.0156 & -0.0681 \\ -0.0492 & -0.0519 \end{bmatrix}$$

$$A_{f13} = \begin{bmatrix} -0.0459 & -0.1074 \\ -0.0537 & -0.0070 \end{bmatrix}, A_{f21} = \begin{bmatrix} -0.1489 & 0.0042 \\ 0.0302 & 0.0120 \end{bmatrix}$$

$$A_{f22} = \begin{bmatrix} -0.2038 & -0.0139 \\ -0.0740 & -0.0115 \end{bmatrix}, A_{f23} = \begin{bmatrix} -0.2495 & -0.0585 \\ -0.0516 & 0.0009 \end{bmatrix}$$

$$B_{f11} = \begin{bmatrix} -0.1544 \\ 0.1630 \end{bmatrix}, B_{f12} = \begin{bmatrix} -0.4186 \\ -0.0496 \end{bmatrix}, B_{f13} = \begin{bmatrix} -0.4879 \\ -0.0251 \end{bmatrix}$$

$$B_{f21} = \begin{bmatrix} -0.0862 \\ 0.0301 \end{bmatrix}, B_{f22} = \begin{bmatrix} -0.2953 \\ -0.0660 \end{bmatrix}, B_{f23} = \begin{bmatrix} -0.3012 \\ -0.0278 \end{bmatrix}$$

$$C_{f11} = \begin{bmatrix} -0.7033 \\ -0.1426 \end{bmatrix}^T, C_{f12} = \begin{bmatrix} -1.1260 \\ -0.2269 \end{bmatrix}^T, C_{f13} = \begin{bmatrix} -1.5497 \\ 0.0870 \end{bmatrix}^T$$

$$C_{f21} = \begin{bmatrix} -0.7033 \\ -0.1426 \end{bmatrix}^T, C_{f22} = \begin{bmatrix} -1.1260 \\ -0.2269 \end{bmatrix}^T, C_{f23} = \begin{bmatrix} -1.5497 \\ 0.0870 \end{bmatrix}^T.$$

Let the initial states of the system and filter are set as $x_0 = [1\ 3]^T$ and $\tilde{x}_0 = [0\ 0]^T$; the external disturbance is chosen as $0.3\cos(0.3\pi\iota)/(0.02 + 0.04 * \iota^2)$. Then, the output trajectories of the system and error responses are obtained and shown in Figs. 7 and 8; the $l_2 - l_\infty$ performance index is shown in Fig. 9. It is easy to see from Figs. 7 and 8 that the filter output response can track the system output well, and the filtering error eventually tends to 0, which means that the filter designed based on the method proposed in this article is effective.

V. Conclusion

In this article, the model-based fuzzy $l_2 - l_\infty$ filtering issue for discrete-time S-MJSs has been considered. Due to the existence of nonlinear characteristics of the system, in this article, the T-S fuzzy model has been introduced to deal with nonlinear terms. In addition, considering that the TP of the mode may depend on the previous historical information, the semi-Markov model has been used to describe random jump mode of the system. By using the SMK method and solving a series of restricted conditions, the filter gains can be calculated; the coupling terms in the inequalities can be eliminated through the method of contract transformation. Finally, two examples have been introduced to illuminate the validity of the obtained theoretical results. Furthermore, considering that the information in the proposed filter design method may be incomplete in the actual process, the design of fuzzy discrete-time filters with incomplete probability information based on S-MJSs is a meaningful question to further explore in the future.

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