Fuzzy Tracking Control for Markov Jump Systems With Mismatched Faults by Iterative Proportional–Integral Observers

Mouquan Shen, Yongsheng Ma, Ju H. Park, *Senior Member, IEEE*, and Qing-Guo Wang

***Abstract*—This article is devoted to the fuzzy fault-tolerant tracking control of Markov jump systems with unknown mismatched faults. To reconstruct the faults and system states, a sequence of proportional–integral observers are established via the system outputs. With the help of a structure separation technique, the proportional–integral gains and the observer gains are solved by a unified linear matrix inequality framework. Resorting to the rebuilt faults and states from an iterative estimation algorithm, a backstepping-based fuzzy fault-tolerant tracking control scheme against the mismatched faults is established to make the resultant closed-loop system be uniformly ultimately bounded. Simulations are provided to verify the effectiveness of the proposed methods.**

***Index Terms*—$\mathcal{H}_\infty$ control, linear matrix inequality (LMI), Markov jump systems (MJSs), Takagi–Sugeno (T-S) fuzzy systems.**

I. Introduction

Over the last few decades, Markov process has been deemed as a powerful tool for engineering systems to describe abrupt changes incurred by component aging, operation point shifting, and interconnection failure [1]. Some typical applications are commonly founded in power systems and biochemical processes [2]. As a consequence, theoretical studies on Markov jump systems (MJSs) are fruitful, such as stability and stabilization [3]–[6], $\mathcal{H}_2$ and $\mathcal{H}_\infty$ [7]–[11], sliding mode control [12], [13], quantization control [14], [15], asynchronous control [16], [17], and reliable control [18], just to mention a few.

Alternatively, owing to the powerful functional approximation ability, the Takagi–Sugeno (T-S) fuzzy model has been employed widely in the nonlinear fields [19]. Accordingly, interesting control issues for the T-S fuzzy systems have been reported in [20]–[26] and the references therein. Regarding the T-S fuzzy systems with Markov jumping parameters, a mode-independent fuzzy stabilization strategy for nonlinear MJSs in terms of linear matrix inequalities (LMIs) is obtained by alleviating the coupling between the stochastic Lyapunov matrix and the system matrices containing controller gains [27]. A coupled Lyapunov function based systematic technique is developed by [28] to get a stochastic fuzzy controller so that the required L_2 performance of the closed-loop MJSs can be ensured. Assumed transition probabilities to be known, uncertain, and unknown, a new controller synthesis method with less conservativeness for fuzzy MJSs is established in [29]. The work in [30] tackles the positive stabilization of fuzzy MJSs with two types of membership functions via parallel distributed compensator and switching strategies, respectively.

With the increasing demand of the safety and reliability requirement for practical systems, fault detection, fault estimation, and fault-tolerant control of T-S fuzzy systems have gained lots of research interests [31]–[37]. Along this topic to MJSs, an interval type-2 fuzzy approach is employed by [38] to discuss the quantized fault detection of fuzzy semi-MJSs under networked environments. By describing the intermittent data dropouts as Bernoulli process, the asynchronous fault detection of fuzzy MJSs is delivered in [39] by means of the dissipativity theory and robust techniques. The fault detection of uncertain fuzzy MJSs with state delays is delivered in [40] via an optimization algorithm to minimize the difference between the reference model and the designed robust fault detection filter. A rapid and accurate adaptive fault estimation approach for fuzzy time-delay MJSs is presented by [41] in terms of LMIs. Resorting to the cone complementarity linearization algorithm, the existence of fault detection filter for nonhomogeneous MJSs is developed in [42] to ensure the fault sensitivity and the robustness of the external disturbances. Additionally, using two sets of unrelated random variables to present actuator faults and their failures, [43] considers an event-triggered reliable control of fuzzy neural networked time-delay MJSs via the

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Lyapunov–Krasovskii functional technique. In contrast to these fault detection or estimation results fallen in the robust framework, [44] exploits a fuzzy logic system (FLS) based adaptive fault-tolerant compensation scheme to treat MJSs against unknown mismatched nonlinearity and simultaneous additive and multiplicative actuator faults. This FLS scheme integrated with the norm estimation approach is also employed by [45] to handle the adaptive fuzzy tracking control of strict-feedback MJSs with simultaneously unpredictable actuator failures, unknown nonlinearities, and unmodeled dynamics. Note that since faults in [38]–[45] are matched to input channels, the obtained methods cannot be applied to actuator faults occurred in batch processes [46] and microgrids [47] which are spanned out of the input space. Furthermore, the fault estimators in the aforementioned references belonging to traditional one-step strategy ignored the effect of input disturbances from the faults [34]. Although a *k*-step approach has been employed in [34] and [35], the integral of system outputs has not been adopted to improve the estimate accuracy. Besides, the conservativeness is introduced by the special form of Lyapunov matrix in [35]. Consequently, it is meaningful to improve the existing *k*-step approach with the better estimation and relax the special requirement for stability analysis, especially for fuzzy MJSs with mismatched faults.

Motivated by the aforementioned observations, the fuzzy tracking control of MJSs with mismatched faults is addressed in this article. A sequence of proportional–integral observers (PIOs) are constructed from system outputs to estimate the system states and faults with better estimation accuracy. To get the observer gains in terms of LMIs, a structural separation technique is employed to relax the special requirement on the Lyapunov matrix. Resorting to the stochastic stability of the observer error system, an iterative algorithm is supplied to get the estimated states and faults. Combining a matrix conversion technique to free the input matrix, the fault-tolerant tracking scheme for fuzzy MJSs is established via a backstepping approach to compensate the mismatched fault and ensure the uniform ultimate boundedness of the closed-loop system. The validity of the proposed estimation algorithm and fault-tolerant control scheme is verified by two numerical examples.

The rest of this article is organized as follows. In Section II, the formulation of the problem, some lemmas, and the proposed PIOs are provided. Section III addresses the stability of the proposed PIOs, the state and fault estimation algorithm, and the backstepping-like fault-tolerant control scheme. Two examples in Section IV show the effectiveness of the proposed schemes. Finally, Section V concludes this article

Notation: Throughout this article, $He(A)$ means $A+A^\top$. $A^\dagger$ denotes the right pseudoinverse of A. I_n is a unit matrix with n dimensions.

II. Problem Formulation

Consider the nonlinear MJSs as the following fuzzy system formulation:

Rule *i*: IF $\nu_1(t)$ is $\mathbb{F}_1^i$,..., and $\nu_h(t)$ is $\mathbb{F}_h^i$,

THEN

$$\begin{cases}\dot{x}(t)=A_i(s_t)x(t)+B_i(s_t)u(t)+F_i(s_t)f(t)+D_i(s_t)d(t)\\ y(t)=C(s_t)x(t)\end{cases} \tag{1}$$

where $x(t)=[x_1^\top(t)\ \ x_2^\top(t)]^\top$ with $x_1(t)\in\mathcal{R}^{n_{x_1}}$, and $x_2(t)\in\mathcal{R}^{n_{x_2}}$, $u(t)\in\mathcal{R}^{n_{x_1+x_2}}$, and $y(t)\in\mathcal{R}^{n_y}$ denote the system state vector, the control input, and the system output, respectively. $f(t)\in\mathcal{R}^{n_f}$ denotes unknown faults. $d(t)\in\mathcal{R}^{n_d}$ stands for exogenous disturbance with unknown bounds $\bar{\mathfrak{d}}$ ($\| d(t)\|\leq\bar{\mathfrak{d}}$) and belongs to $L_2[0,\infty)$. $\nu_1(t),\ldots,\nu_h(t)$ are available premise variables. $\mathbb{F}_q^i(i=1,\ldots,l,q=1,\ldots,h)$ represent the fuzzy rules. $A_i(s_t)$, $B_i(s_t)=[0\ \ B_{2i}^\top(s_t)]^\top(B_{2i}(s_t)\in\mathcal{R}^{n_{x_2}})$, $C(s_t)=[I_{n_{x_1}}\ \ 0]$, $F_i(s_t)$, $D_i(s_t)$ are known system matrices with approximate dimensions. s_t is a continuous-time discrete state Markov process taking values in a finite set $\mathbb{S}=\{1,2,\ldots,N\}$. The transition probability matrix $\prod=[\pi_{sp}]_{s,p\in\mathbb{S}}$ satisfies

$$\mathbb{P}(s_{t+\Delta t}=p|s_t=s)=\begin{cases}\pi_{sp}\Delta t+o(\Delta t), & s\neq p\\ 1+\pi_{sp}\Delta t+o(\Delta t), & s=p\end{cases}$$

where $\Delta t>0$, $\lim_{\Delta t\to 0}o(\Delta t)/\Delta t=0$ is the probability and $\pi_{sp}\geq 0$ for $s\neq p$, $-\pi_{ss}=\sum_{p=1,p\neq s}^N\pi_{sp}$.

Remark 1: In (1), due to the structure of $B_i(s_t)=[0\ \ B_{2i}^\top(s_t)]^\top$, it means that the faults are spanned out of the space of input channels, which can be found in batch processes [46] and microgrids [47].

By using the standard fuzzy inference method [19], (1) is rewritten as

$$\begin{cases}\dot{x}(t)=\sum_{i=1}^l\nu_i(t)\{A_i(s_t)x(t)+B_i(s_t)u(t)+F_i(s_t)f(t)\\ \qquad +D_i(s_t)d(t)\}\\ y(t)=C(s_t)x(t)\end{cases} \tag{2}$$

in which $\lambda_i(t)=\prod_{q=1}^h\mathbb{F}_q^i$, $\nu_i(t)=\frac{\lambda_i(t)}{\sum_{i=1}^l\lambda_i(t)}$ and the fact that $\sum_{i=1}^l\nu_i(t)=1$, $0\leq\nu_i(t)\leq 1$, $i=1,2,\ldots,l$.

Before further proceeding, a definition, a lemma, and three assumptions are introduced first.

Definition 1. [3]: The autonomous system (2) ($u(t)\equiv 0$, $f(t)\equiv 0$, $d(t)\equiv 0$) is stochastically stable if

$$E\left\{\int_0^\infty x^\top(t)x(t)dt|x(0)\right\}<\infty \tag{3}$$

for every initial condition $x(0)$.

Lemma 1. [48]: For all $\mathscr{F}=\mathscr{F}^\top\in\mathcal{R}^{b\times b}$, if $\mathscr{B}\in\mathcal{R}^{c\times b}$ is given, then following statements are equivalent:

1) $\mathscr{V}^\top\mathscr{F}\mathscr{V}<0$, for all $\mathscr{V}\neq 0,\mathscr{B}\mathscr{V}=0$;

2) ${\mathscr{B}^\perp}^\top\mathscr{F}\mathscr{B}^\perp<0$;

3) $\exists\mathscr{S}\in\mathcal{R}^{b\times c}$ such that $\mathscr{F}+He(\mathscr{S}\mathscr{B})<0$.

Assumption 1: $\| f(t)\|\leq\tilde{a}$ ($\tilde{a}>0$), and $\dot{f}(t)$ belongs to $L_2[0,\infty)$.

Assumption 2: $rank(F_i(s_t))=n_{x_2}\leq n_{x_1}+n_{x_2}$ and $rank(C(s_t)F_i(s_t))=n_{x_2}$.

Assumption 3: $\kappa(t)$ and $\dot{\kappa}(t)$ are continuous and bounded, where $\kappa(t)$ is the preset tracking trajectory.

Remark 2: Assumption 1 means that the fault signal and its first-order derivative are bounded, which is reasonable in many practical applications, such as a class of continuous electroencephalography signals that occurred in the finite time range [49] and multiple faults in combinational networks [50]. Consequently, this assumption has also been commonly employed to study the fault estimation and fault tolerance in [35], [36], and [41]. Assumption 2 is adopted to ensure the feasibility of fault estimation and is vital for the later PIOs design.

Based on the above preliminaries, we construct iterative PIOs with n steps to estimate the system states and the mismatched faults as follows:

The 0th observer:

$$\begin{cases} \dot{\hat{x}}^{[0]}(t) = A_i(s)\hat{x}^{[0]}(t) + B_i(s)u(t) + F_i(s)\hat{f}^{[0]}(t) \\ \qquad + L_{1i}(s)e_y^{[0]}(t) + B_i(s)X_I^{[0]}(t) \\ \hat{y}^{[0]}(t) = C(s)\hat{x}^{[0]}(t), e_y^{[0]}(t) = \hat{y}^{[0]}(t) - y(t) \\ \dot{X_I}^{[0]}(t) = L_{2i}(s)e_y^{[0]}(t) \\ \dot{\hat{f}}^{[0]}(t) = E_i(s)e_y^{[0]}(t) - M(s)\dot{e}_y^{[0]}(t) \end{cases} \tag{4}$$

The j th observer ($j = 1, 2, \ldots, n-1$):

$$\begin{cases} \dot{\hat{x}}^{[j]}(t) = A_i(s)\hat{x}^{[j]}(t) + B_i(s)u(t) + F_i(s)\hat{f}^{[j]}(t) \\ \qquad + L_{1i}(s)e_y^{[j]}(t) + B_i(s)X_I^{[j]}(t) \\ \hat{y}^{[j]}(t) = C(s)\hat{x}^{[j]}(t), e_y^{[j]}(t) = \hat{y}^{[j]}(t) - y(t) \\ \dot{X_I}^{[j]}(t) = L_{2i}(s)e_y^{[j]}(t) \\ \dot{\hat{f}}^{[j]}(t) = E_i(s)e_y^{[j]}(t) - M(s)\dot{e}_y^{[j]}(t) + \dot{\hat{f}}^{[j-1]}(t) \end{cases} \tag{5}$$

The n th observer:

$$\begin{cases} \dot{\hat{x}}^{[n]}(t) = A_i(s)\hat{x}^{[n]}(t) + B_i(s)u(t) + F_i(s)\hat{f}^{[n]}(t) \\ \qquad + L_{1i}(s)e_y^{[n]}(t) + B_i(s)X_I^{[n]}(t) \\ \hat{y}^{[n]}(t) = C(s)\hat{x}^{[n]}(t), e_y^{[n]}(t) = \hat{y}^{[n]}(t) - y(t) \\ \dot{X_I}^{[n]}(t) = L_{2i}(s)e_y^{[0]}(t) \\ \dot{\hat{f}}^{[n]}(t) = E_i(s)e_y^{[n]}(t) - M(s)\dot{e}_y^{[n]}(t) + \dot{\hat{f}}^{[n-1]}(t) \end{cases} \tag{6}$$

where $\hat{x}^{[0]}(t) \in \mathcal{R}^{n_{x_1}+n_{x_2}}$, $\hat{x}^{[j]}(t) \in \mathcal{R}^{n_{x_1}+n_{x_2}}$, and $\hat{x}^{[n]}(t) \in \mathcal{R}^{n_{x_1}+n_{x_2}}$ indicate the 0th, jth, and nth estimation of $x(t)$, respectively, and so do the estimated faults $\hat{f}^{[0]}(t) \in \mathcal{R}^{n_f}$, $\hat{f}^{[j]}(t) \in\in \mathcal{R}^{n_f}$, $\hat{f}^{[n]}(t) \in \mathcal{R}^{n_f}$, the output errors $e_y^{[0]}(t)$, $e_y^{[j]}(t)$, $e_y^{[n]}(t)$, and the integrators $X_I^{[0]}(t)$, $X_I^{[j]}(t)$, $X_I^{[n]}(t)$. $L_{1i}(s) = \sum_{i=1}^{l} \nu_i(t)L_{1i}(s_t)$, $E_i(s) = \sum_{i=1}^{l} \nu_i(t)E_i(s_t)$, $M(s) = M(s_t)$, and $L_{2i}(s) = \sum_{i=1}^{l} \nu_i(t)L_{2i}(s_t)$ are the iterative observer gains to be designed.

Remark 3: Compared with [34]–[36], the iterative observers (4)–(6) contain integral terms related to system output. Due to these terms, the estimation accuracy of the states and faults can be further improved as shown in Section IV.

Following are the main objectives of this article:

1) for (2) and the proposed (4)–(6), seek conditions to make the observer error systems be stochastically stable;
2) with the estimated states and faults, construct the fault tolerant controller to ensure the closed-loop system to be uniformly ultimately bounded.

III. Main Results

A. Stability Analysis of the Observer Error Systems

In this section, the Lyapunov method is employed to give a solution for 1) as the below details.

Set $\mathcal{E}_x^{[n]}(t) = \hat{x}^{[n]}(t) - x(t)$, $\mathcal{E}_f^{[n]}(t) = \hat{f}^{[n]}(t) - f(t)$, and $e_f^{[-1]} = 0$ for all $n \in \mathbb{N}$, and then the nth error dynamics system is obtained

$$\begin{cases} \dot{\mathcal{E}_x}^{[n]}(t) = \dot{\hat{x}}(t) - \dot{x}(t) = (A_i(s) + L_{1i}(s)C(s))\mathcal{E}_x^{[n]}(t) \\ \qquad + F_i(s)\mathcal{E}_f^{[n]}(t) - D_i(s)d(t) + B_i(s)X_I^{[n]}(t) \\ \dot{\mathcal{E}_f}^{[n]}(t) = \dot{\hat{f}}(t) - \dot{f}(t) = -M(s)C(s)F_i(s)\cdot\mathcal{E}_f^{[n]}(t) \\ \qquad + (-M(s)C(s)A_i(s) + H_i(s)C(s))\mathcal{E}_x^{[n]}(t) \\ \qquad + M(s)C(s)D_i(s)d(t) - M(s)C(s)B_i(s)X_I^{[n]}(t) \\ \qquad + \dot{\mathcal{E}_f}^{[n-1]}(t) \\ \dot{X_I}^{[n]}(t) = L_{2i}(s)C(s)\mathcal{E}_x^{[n]} \end{cases} \tag{7}$$

where $H_i(s) = E_i(s) - M(s)C(s)L_{1i}(s)$.

For the above error system (7), set $\mathcal{E}^{[n]}(t) = [\mathcal{E}_x^{[n]\top}(t)\ \mathcal{E}_f^{[n]\top}(t)\ X_I^{[n]\top}(t)]^\top$ and $\varpi^{[n]}(t) = [d^\top(t)\ \dot{\hat{f}}^{[n-1]^\top}(t) - \dot{f}^\top(t)]^\top$, and then the main task is to get iterative observers (4)–(6) such that the nth dynamic error system (7) is stochastically stable with the $\mathcal{H}_\infty$ performance index γ defined by

$$E\left\{\int_0^\infty \parallel \mathcal{E}^{[n]}(t) \parallel^2 dt\right\} \le \gamma^2 E\left\{\int_0^\infty \parallel \varpi^{[n]}(t) \parallel^2 dt\right\}. \tag{8}$$

Resorting to the Lyapunov method and combining a structure separation technique, a solution for the aforementioned task is presented in Theorem 1.

Theorem 1: For a prescribed positive scalar γ and a given matrix Y, if there exist symmetric and positive definite matrices $P_1(s)$, $P_2(s)$, $P_3(s)$, $P_4(s)$, $P_5(s)$ and matrices $V_{1i}(s)$, $V_{2i}(s)$, $V_3(s)$, and $V_{4i}(s)$ ($i = 1, 2, ..., l$, $s = 1, 2, ..., N$) such that the following LMIs hold:

$$\begin{bmatrix} \Pi_{i11}^s & * & * & * & * & * \\ \Pi_{i21}^s & \Pi_{i22}^s & * & * & * & * \\ \Pi_{i31}^s & \Pi_{i32}^s & \Pi_{i33}^s & * & * & * \\ \Pi_{i41}^s & \Pi_{i42}^s & \Pi_{i43}^s & \Pi_{i44}^s & * & * \\ 0 & \Pi_{i52}^s & 0 & 0 & \Pi_{i55}^s & * \\ \Pi_{i61}^s & 0 & \Pi_{i63}^s & 0 & 0 & \Pi_{i66}^s \end{bmatrix} < 0 \tag{9}$$

where

$$\Pi_{i11}^s = He(P_1(s)A_i(s) + V_{1i}(s)C(s) - YV_{2i}(s)C(s))$$

$$+\sum_{p=1}^{N}\pi_{sp}P_1(p)+I,$$

$$\Pi^s_{i21}=F_i^\top(s)P_1(s)-V_3(s)C(s)A_i(s)+V_{4i}(s)C(s)$$

$$\Pi^s_{i22}=-He(V_3(s)C(s)F_i(s))+\sum_{p=1}^{N}\pi_{sp}P_2(p)+I$$

$$\Pi^s_{i31}=Y^\top P_1(s)A_i(s)+Y^\top V_{1i}(s)C(s)-V_{2i}(s)C(s)$$

$$+B_i^\top(s)P_1(s)+\sum_{p=1}^{N}\pi_{sp}Y^\top P_1(p)$$

$$\Pi^s_{i32}=Y^\top P_1(s)F_i(s)-B_i^\top(s)C(s)V_3^\top(s)$$

$$\Pi^s_{i33}=He(Y^\top P_1(s)B_i(s))+\sum_{p=1}^{N}\pi_{sp}P_3(p)+I$$

$$\Pi^s_{i41}=-D_i^\top(s)P_1(s),\ \Pi^s_{i42}=D_i^\top(s)C^\top(s)V_3^\top(s)$$

$$\Pi^s_{i43}=-D_i^\top P_1(s)Y,\ \Pi^s_{i44}=\sum_{p=1}^{N}\pi_{sp}P_4(p)-\gamma^2 I$$

$$\Pi^s_{i52}=P_2(s),\ \Pi^s_{i55}=\sum_{p=1}^{N}\pi_{sp}P_5(p)-\gamma^2 I$$

$$\Pi^s_{i61}=-bV_{2i}(s)C(s)-Y^\top P_1(s)-W_i(s)Y^\top$$

$$\Pi^s_{i63}=-P_3(s)-W_i(s),\ \Pi^s_{i66}=-b(W_i(s)+W_i^\top(s))^\top$$

and

$$P(s)=\begin{bmatrix}P_1(s) & * & * & * & *\\ 0 & P_2(s) & * & * & *\\ Y^\top P_1(s) & 0 & P_3(s) & * & *\\ 0 & 0 & 0 & P_4(s) & *\\ 0 & 0 & 0 & 0 & P_5(s)\end{bmatrix}>0 \qquad (10)$$

then the observer error system (7) is stochastically stable with the required $\mathcal{H}_\infty$ performance index γ. Moreover, $L_{1i}(s)=P_1(s)^{-1}V_{1i}(s)$, $M(s)=P_3(s)^{-1}V_3(s)$, and $E_i(s)=P_2(s)^{-1}V_{4i}(s)+M(s)C(s)L_{1i}(s)$.

Proof: Setting $\theta=[\mathcal{E}_x^{[n]^\top}(t)\ \ \mathcal{E}_f^{[n]^\top}(t)\ \ X_I^{[n]^\top}(t)\ \ d^\top(t)\ \ \Delta f^\top(t)]^\top$, (7) is rewritten as

$$\dot\theta=\bar A_i^s\theta$$

where

$$\bar A_i^s=\begin{bmatrix}\mathfrak{G}^s_{i11} & F_i(s) & B_i(s) & -D_i(s) & 0\\ \mathfrak{G}^s_{i21} & \mathfrak{G}^s_{i22} & \mathfrak{G}^s_{i23} & \mathfrak{G}^s_{i24} & I\\ L_{2i}C(s) & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\end{bmatrix} \qquad (11)$$

in which $\mathfrak{G}^s_{i11}=A_i(s)+L_{1i}(s)C(s)$, $\mathfrak{G}^s_{i21}=M(s)C(s)A_i(s)+H_i(s)C$, $\mathfrak{G}^s_{i22}=-M(s)C(s)F_i(s)$, $\mathfrak{G}^s_{i23}=-M(s)C(s)B_i(s)$, and $\mathfrak{G}^s_{i24}=M(s)C(s)D_i(s)$.

Choose the candidate Lyapunov function as

$$V(t)=\theta^\top P(s)\theta \qquad (12)$$

where

$$P(s)=\begin{bmatrix}P_1(s) & * & * & * & *\\ 0 & P_2(s) & * & * & *\\ Y^\top P_1(s) & 0 & P_3(s) & * & *\\ 0 & 0 & 0 & P_4(s) & *\\ 0 & 0 & 0 & 0 & P_5(s)\end{bmatrix}. \qquad (13)$$

Recalling the tasks (I) and (II), calculating the derivative operation to (8) and (12) yields

$$E\left\{\dot V+\mathcal{E}^{[n]\top}(t)\mathcal{E}^{[n]}(t)-\gamma^2\varpi^{[n]\top}\varpi^{[n]}(t)\right\}=\theta^\top\Phi\theta$$

where

$$\Phi_i^s=\begin{bmatrix}\Psi^s_{i11} & * & * & * & *\\ \Psi^s_{i21} & \Psi^s_{i22} & * & * & *\\ \Psi^s_{i31} & \Psi^s_{i32} & \Psi^s_{i33} & * & *\\ \Psi^s_{i41} & \Psi^s_{i42} & \Psi^s_{i43} & \Psi^s_{i44} & *\\ 0 & \Psi^s_{i52} & 0 & 0 & \Psi^s_{i55}\end{bmatrix}$$

in which

$$\Psi^s_{i11}=He(P_1(s)A_i(s)+P_1(s)L_{1i}(s)C(s)$$

$$-P_1(s)YL_{2i}(s)C(s))+\sum_{p=1}^{N}\pi_{sp}P_1(p)+I$$

$$\Psi^s_{i21}=F_i^\top(s)P_1(s)-P_2(s)M_3(s)C(s)A_i(s)$$

$$+P_2(s)H_i(s)C(s)$$

$$\Psi^s_{i22}=-He(P_2(s)M_3(s)C(s)F_i(s))+\sum_{p=1}^{N}\pi_{sp}P_2(p)+I$$

$$\Psi^s_{i31}=Y^\top P_1(s)A_i(s)+Y^\top P_1(s)L_{1i}(s)C(s)$$

$$+P_3(s)L_{2i}(s)C(s)+B_i^\top(s)P_1(s)$$

$$+\sum_{s=1}^{N}\pi_{sp}Y^\top P_1(p)$$

$$\Psi^s_{i32}=Y^\top P_1(s)F_i(s)-B_i^\top(s)C^\top(s)M^\top(s)P_2(s)$$

$$\Psi^s_{i33}=He(Y^\top P_1(s)B_i(s))+\sum_{p=1}^{N}\pi_{sp}P_3(p)+I$$

$$\Psi^s_{i41}=-D_i^\top(s)P_1(s),\Psi^s_{i42}=D_i^\top(s)C^\top(s)M(s)^\top(s)P_2(s)$$

$$\Psi^s_{i43}=-D_i^\top P_1(s)Y,\ \Psi^s_{i44}=\sum_{p=1}^{N}\pi_{sp}P_4(p)-\gamma^2 I$$

$$\Psi^s_{i52}=P_2(s),\ \Psi^s_{i55}=\sum_{p=1}^{N}\pi_{sp}P_5(p)-\gamma^2 I.$$

To ensure the required H_∞ performance (8) in (II), the main task is to prove the fact $\Phi<0$.

On the other hand, according to Lemma 1, $\Phi<0$ could be guaranteed once the following inequality holds:

$$\mathscr{F}_s^i+He(\mathscr{S}_s^i\mathscr{B}_s^i)<0 \qquad (14)$$

where

$$\mathscr{B}_s^{i\perp} = \begin{bmatrix} I & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 \\ 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & I \\ -L_{2i}(s)C(s) & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathscr{F}_s^i = \begin{bmatrix} \Psi_{i11}^s & * & * & * & * & * \\ \Psi_{i21}^s & \Psi_{i22}^s & * & * & * & * \\ \Psi_{i31}^s & \Psi_{i32}^s & \Psi_{i33}^s & * & * & * \\ \Psi_{i41}^s & \Psi_{i42}^s & \Psi_{i43}^s & \Psi_{i44}^s & * & * \\ 0 & \Psi_{i52}^s & 0 & 0 & \Psi_{i55}^s & * \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathscr{S}_s^i = [(P_1(s)Y + YW_i(s))^\top \;\; 0 \;\; (P_3(s) + W_i(s))^\top \;\; 0 \;\; 0 \;\; (bW_i(s))^\top]^\top$$

$$\mathscr{B}_s^i = [\,-L_{2i}(s)C(s) \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; -I\,].$$

Choosing $V_{1i}(s) = P_1(s)L_{1i}(s)$, $V_{2i}(s) = W_i(s)L_{2i}(s)$, $V_3(s) = P_2(s)M(s)$, and $V_{4i}(s) = P_2(s)H_i(s)$, (14), with some matrix operations, is rewritten as

$$\begin{bmatrix} \Pi_{i11}^s & * & * & * & * & * \\ \Pi_{i21}^s & \Pi_{i22}^s & * & * & * & * \\ \Pi_{i31}^s & \Pi_{i32}^s & \Pi_{i33}^s & * & * & * \\ \Pi_{i41}^s & \Pi_{i42}^s & \Pi_{i43}^s & \Pi_{i44}^s & * & * \\ 0 & \Pi_{i52}^s & 0 & 0 & \Pi_{i55}^s & * \\ \Pi_{i61}^s & 0 & \Pi_{i63}^s & 0 & 0 & \Pi_{i66}^s \end{bmatrix} < 0 \tag{15}$$

which is just (9). Namely, once (9) holds, it readily gets the fact

$$E\left\{\dot{V}^{[n]}(t) + \mathcal{E}^{[n]\top}(t)\mathcal{E}^{[n]}(t)(t) - \gamma^2\varpi^{[n]\top}(t)\varpi^{[n]}(t)\right\} < 0. \tag{16}$$

Therefore, the stochastic stability of the nth error system (7) could be guaranteed by (16) when $\varpi^{[n]}(t) = 0$.

Integrating both sides of (16) from 0 to ∞ gives

$$\int_0^\infty E(\dot{V})^{[n]}(t)dt + \int_0^\infty \bar{\mathcal{E}}^\top(t)\bar{\mathcal{E}}(t)dt - \gamma^2\int_0^\infty \varpi^{[n]\top}(t)\varpi^{[n]}(t)dt < 0$$

which is also

$$\int_0^\infty \bar{\mathcal{E}}^\top(t)\bar{\mathcal{E}}(t)dt - \gamma^2\int_0^\infty \varpi^{[n]\top}(t)\varpi^{[n]}(t)dt < -\int_0^\infty E(\dot{V})^{[n]}(t)dt.$$

Under the zero initial condition, one has

$$\int_0^\infty \bar{\mathcal{E}}^\top(t)\bar{\mathcal{E}}(t)dt - \gamma^2\int_0^\infty \varpi^{[n]\top}(t)\varpi^{[n]}(t)dt < 0$$

which is just the index defined in (II). ■

Remark 4: In contrast to [36] with special forms on the Lyapunov variables, a relaxed strategy is employed in Theorem 1 and the introduced nonlinearity is conquered by a structure separation technique based on Lemma 1. Consequently, the conditions given in Theorem 1 are less conservative than those in [36].

Remark 5: In Theorem 1, the number of LMI decision variables $\mathscr{N}_{dv}$ and the number of LMI rows $\mathscr{N}_r$ are $[(\frac{(n_{x_1}+n_{x_2})(n_{x_1}+n_{x_2}+1)}{2} + n_f(n_f + n_{x_2} + 1) + \frac{n_d(n_d+1)}{2}) + (n_{x_1} + (2 + n_f)n_{x_2} + 1)\cdot i]\cdot s + 1$ and $(n_{x_1} + n_{x_2} + n_f + 4)\cdot s\cdot i$, respectively. Referring to [24], the computation times of LMIs in (9) are in polynomial with proportional to $\mathscr{N}_{\mathscr{C}} = \mathscr{N}_{dv}^3\mathscr{N}_r$. Fortunately, these LMIs are solved offline.

Remark 6: Guaranteed by the stochastic stability established in Theorem 1, as [34] and [36], the unknown system states and the mismatched faults are reconstructed by

$$\begin{aligned} \hat{x}_1(t) &= \frac{1}{n+1}(\hat{x}_1^{[1]}(t) + \cdots + \hat{x}_1^{[n]}(t)) \\ \hat{x}_2(t) &= \frac{1}{n+1}(\hat{x}_2^{[1]}(t) + \cdots + \hat{x}_2^{[n]}(t)) \\ \hat{f}(t) &= \frac{1}{n+1}(\hat{f}^{[1]}(t) + \cdots + \hat{f}^{[n]}(t)) \end{aligned} \tag{17}$$

where $\hat{x}_1^{[n]}(t)$, $\hat{x}_2^{[n]}(t)$, and $\hat{f}^{[n]}(t)$ are estimated by Algorithm 1 as follows.

Algorithm 1:

(S1) Obtain the values of 0th observer ($\hat{x}^{[0]}(t)$, $\hat{f}^{[0]}(t)$) by performing (4) and then take $n = 1$.

(S2) Get $\hat{x}^{[n]}(t)$ and $\hat{f}^{[n]}(t)$ from (6), and calculate

$$\rho^{[n+1]}(t) = \frac{\hat{f}^{[0]}(t) + \hat{f}^{[1]}(t) + \cdots + \hat{f}^{[n]}(t)}{n+1}$$

$$\sigma^{[n+1]}(t) = \frac{\hat{x}^{[0]}(t) + \hat{x}^{[1]}(t) + \cdots + \hat{x}^{[n]}(t)}{n+1}.$$

(S3) For $t_{\text{final}} = K > 0$, select a sufficient small scalar $\varepsilon > 0$, if

$$\Omega_{n+1} = \sum_{t=0}^{K} \| \varrho^{[n+1]}(t) - \varrho^{[n]}(t) \| > \varepsilon$$

where $\varrho^{[n]}(t) = [\,\rho^{[n]\top}(t)\ \sigma^{[n]\top}(t)\,]^\top$. After that, set $n = n + 1$ and come back to S2. Otherwise, output the following estimation of $x(t)$ and $f(t)$:

$$\hat{f}(t) = \rho^{[n+1]}(t),\ \hat{x}(t) = \sigma^{[n+1]}(t).$$

Remark 7: According to the estimated $\hat{x}^{[n]}(t)$ and $\hat{f}^{[n]}(t)$, the upper bounds of $\mathcal{E}_x$ and $\mathcal{E}_f$ are individually selected as $\bar{\mathcal{E}}_x = \| \frac{1}{n+1}\sum_{i=0}^{n}\mathcal{E}_x^{[i]}(t) \|$ and $\bar{\mathcal{E}}_f = \| \frac{1}{n+1}\sum_{i=0}^{n}\mathcal{E}_f^{[i]}(t) \|$ so that the system (22) with the designed fault tolerant controller (27) in Section III-B is uniformly ultimately bounded.

Deleting the integral part of (4)–(6), the observers for the system states and the mismatched faults are the same as those in [36] which are also given in the following:

The 0th observer:

$$\begin{cases}\dot{\hat{x}}^{[0]}(t) = A_i(s)\hat{x}^{[0]}(t) + B_i(s)u(t)\\ \qquad\qquad + F_i(s)\hat{f}^{[0]}(t) + L_{1i}(s)e_y^{[0]}(t)\\ \hat{y}^{[0]}(t) = C(s)\hat{x}^{[0]}(t), e_y^{[0]}(t) = \hat{y}^{[0]}(t) - y(t)\\ \dot{\hat{f}}^{[0]}(t) = E_i(s)e_y^{[0]}(t) - M(s)\dot{e}_y^{[0]}(t)\end{cases} \tag{18}$$

The *j* th observer ($j = 1, 2, \ldots, n-1$):

$$\begin{cases}\dot{\hat{x}}^{[j]}(t) = A_i(s)\hat{x}^{[j]}(t) + B_i(s)u(t)\\ \qquad\qquad + F_i(s)\hat{f}^{[j]}(t) + L_{1i}(s)e_y^{[j]}(t)\\ \hat{y}^{[j]}(t) = C(s)\hat{x}^{[j]}(t), e_y^{[j]}(t) = \hat{y}^{[j]}(t) - y(t)\\ \dot{\hat{f}}^{[j]}(t) = E_i(s)e_y^{[j]}(t) - M(s)\dot{e}_y^{[j]}(t) + \dot{\hat{f}}^{[j-1]}(t)\end{cases} \tag{19}$$

The *n* th observer:

$$\begin{cases}\dot{\hat{x}}^{[n]}(t) = A_i(s)\hat{x}^{[n]}(t) + B_i(s)u(t)\\ \qquad\qquad + F_i(s)\hat{f}^{[n]}(t) + L_{1i}(s)e_y^{[n]}(t)\\ \hat{y}^{[n]}(t) = C(s)\hat{x}^{[n]}(t), e_y^{[n]}(t) = \hat{y}^{[n]}(t) - y(t)\\ \dot{\hat{f}}^{[n]}(t) = E_i(s)e_y^{[n]}(t) - M(s)\dot{e}_y^{[n]}(t) + \dot{\hat{f}}^{[n-1]}(t).\end{cases} \tag{20}$$

To ensure the stochastic stability with the required $\mathcal{H}_\infty$ performance for (18)–(20), deleting some rows and columns of (9) yields the following corollary.

Corollary 1: For a positive scalar γ, if there exist symmetric and positive definite matrices $P(s)$ and $Q(s)$ and matrices $W_{1i}(s)$, $W_2(s)$, and $W_{3i}(s)$ such that the LMIs given in the following hold:

$$\begin{bmatrix}\Omega_{i11}^s & * & * & *\\ \Omega_{i21}^s & \Omega_{i22}^s & * & *\\ -D_i(s)P(s) & W_2(s)C(s)D_i(s) & -\gamma^2 I & *\\ 0 & Q(s) & 0 & -\gamma^2 I\end{bmatrix} < 0 \tag{21}$$

for $i = 1, 2, ..., l$, $s = 1, 2, ..., N$, where $\Omega_{i11}^s = He(P(s)A_i(s) + W_{1i}(s)C(s)) + \sum_{p=1}^{l}\pi_{sp}P(p) + I$, $\Omega_{i21}^s = F_i^\top(s)P(s) - W_2(s)C(s)A_i(s) + W_{3i}(s)C(s)$, and $\Omega_{i22}^s = -He(W_2(s)C(s)F_i(s)) + \sum_{p=1}^{l}\pi_{sp}Q(p) + I$. Then, the dynamic error system (7) composed of the sequence iterative observers in (18)–(20) without integral term is stochastically stable with the required $\mathcal{H}_\infty$ performance. Moreover, $L_{1i}(s) = P(s)^{-1}W_{1i}(s)$, $M(s) = Q(s)^{-1}W_2(s)$, and $E_i(s) = Q(s)^{-1}W_{3i}(s) + M(s)C(s)L_{1i}(s)$.

Remark 8: The conditions given in Corollary 1 are different from those in [36] where the Lyapunov variables for the estimated fault error parts are constant. Additionally, a structure separation technique is employed to get the observer gains. Therefore, the proposed method is less conservative than that in [36], which is verified via the numerical example in Section IV.

B. Control Scheme Design

Based on the estimated system states and faults in (17), a backstepping-like approach is exploited to establish the fault-tolerant controller in this part.

Before proceeding, (2) is rewritten as

$$\begin{cases}\dot{x}_1(t) = A_{11i}(s)x_1(t) + A_{12i}(s)x_2(t) + F_{1i}(s)f(t)\\ \qquad + D_{1i}(s)d(t)\\ \dot{x}_2(t) = A_{21i}(s)x_1(t) + A_{22i}(s)x_2(t) + B_{2i}(s)u(t)\\ \qquad + F_{2i}(s)f(t) + D_{2i}(s)d(t).\end{cases} \tag{22}$$

Adopting matrix conversion technique in [23], (22) is equivalently rewritten as

$$\begin{cases}\dot{x}_1(t) = A_{11i}(s)x_1(t) + (\bar{A}_{12} + Z_1U_{1i}(s)T_1)x_2(t)\\ \qquad + F_{1i}(s)f(t) + D_{1i}(s)d(t)\\ \dot{x}_2(t) = A_{21i}(s)x_1(t) + A_{22i}(s)x_2(t)\\ \qquad + (\bar{B}_2 + Z_2U_{2i}(s)T_2)u(t) + F_{2i}(s)f(t)\\ \qquad + D_{2i}(s)d(t)\end{cases} \tag{23}$$

where

$$\bar{A}_{12} = \frac{1}{l}\sum_{i=1}^{l}A_{12i}(s), \bar{B}_2 = \frac{1}{l}\sum_{i=1}^{l}B_{2i}(s)$$

$$Z_1 = \frac{1}{2}[\ \bar{A}_{12}\ \cdots\ \bar{A}_{12} - A_{l12}\ ]$$

$$Z_2 = \frac{1}{2}[\ \bar{B}_2\ \cdots\ \bar{B}_{12} - B_{l12}\ ]$$

$$U_{1i}(s) = \text{diag}[\ (1-2\nu_1)I\ \cdots\ (1-2\nu_l)I\ ]$$

$$U_{2i}(s) = \text{diag}[\ (1-2\nu_1)I\ \cdots\ (1-2\nu_l)I\ ]$$

$$T_1 = [\ I\ \cdots\ I\ ]^\top, T_2 = [\ I\ \cdots\ I\ ].^\top$$

Remark 9: Like [36], $(\bar{A}_{12} + Z_1U_{1i}(s)T_1)^\dagger$ should be full row rank and $\bar{B}_2 + Z_2U_{2i}(s)T_2$ is nonsingular so that the adaptive fault-tolerant tracking controller could be developed by means of the backstepping-like approach.

On the basis of the above preparations, the fault-tolerant controller is designed by two steps as follows:

Step 1: Assuming $\zeta_1(t) = x_1(t) - \kappa(t)$ and combining $\dot{x}_1(t)$ in (23) yields

$$\begin{aligned}\dot{\zeta}_1(t) &= \dot{x}_1(t) - \dot{\kappa}(t)\\ &= A_{11i}(s)x_1(t) + (\bar{A}_{12} + Z_1U_{1i}(s)T_1)x_2(t)\\ &\quad + F_{1i}(s)f(t) + D_{1i}(s)d(t) - \dot{\kappa}(t).\end{aligned} \tag{24}$$

Choose the Lyapunov function for (24) as

$$V_1 = \frac{1}{2}\zeta_1^\top(t)\zeta_1(t).$$

On the other hand, set $\zeta_2(t) = \hat{x}_2(t) - \eta_1(t) - (\bar{A}_{12} + Z_1U_{1i}(s)T_1)^\dagger\dot{\kappa}(t)$ and $\eta_1(t)$ is selected as

$$\begin{aligned}\eta_1(t) = &-(\bar{A}_{12} + Z_1U_{1i}(s)T_1)^\dagger(k_1\zeta_1(t) + A_{11i}(s)x_1(t)\\ &+ F_{1i}(s)\hat{f}(t) + \frac{1}{2z}\Lambda\zeta_1(t))\end{aligned}$$

in which k_1 is a designed constant, z is a known given constant, and

$$\Lambda = (\bar{A}_{12} + Z_1U_{1i}(s)T_1)I_2I_2^\top(\bar{A}_{12} + Z_1U_{1i}(s)T_1)^\top$$

$$+ F_{1i}(s)F_{1i}^{\top}(s) + D_{1i}(s)D_{1i}^{\top}(s)$$

where $I_2 = [\, 0 \; I_{n_{x_2}} \,]$, i.e., $\hat{x}_2(t) = I_2\hat{x}(t)$.

Consequently, calculating $\dot{V}_1$ with (24) gives

$$\begin{aligned}\dot{V}_1 &= \zeta_1(t)\dot{\zeta}_1(t)\\ &= -k_1 \parallel \zeta_1(t) \parallel^2 + \zeta_1^{\top}(t)(\bar{A}_{12} + Z_1U_{1i}(s)T_1)\zeta_2(t)\\ &\quad - \zeta_1^{\top}(t)(\bar{A}_{12} + Z_1U_{1i}(s)T_1)I_2\mathcal{E}_x(t)\\ &\quad - \zeta_1^{\top}(t)F_{1i}(s)\mathcal{E}_f(t) + \zeta_1^{\top}(t)D_{1i}(s)d(t)\\ &\quad - \frac{1}{2z}\zeta_1^{\top}(t)\Lambda\zeta_1(t). \end{aligned}\tag{25}$$

On the other hand, employing the fact $\mathscr{X}^T\mathscr{Y} + \mathscr{Y}^T\mathscr{X} \leq \mathscr{X}^T\mathscr{X} + \mathscr{Y}\mathscr{Y}^T$ in [48], one has the following inequalities:

$$\begin{cases} -\zeta_1^{\top}(t)(\bar{A}_{12} + Z_1U_{1i}(s)T_1)I_2\mathcal{E}_x(t)\\ \leq \dfrac{1}{2z}\zeta_1^{\top}(t)(\bar{A}_{12} + Z_1U_{1i}(s)T_1)I_2I_2^{\top}\\ (\bar{A}_{12} + Z_1U_{1i}(s)T_1)^{\top}(r)\zeta_1(t) + \dfrac{z}{2}\bar{\mathcal{E}_x}^2\\ -\zeta_1^{\top}(t)F_{1i}(s)\mathcal{E}_f(t)\\ \leq \dfrac{1}{2z}\zeta_1^{\top}(t)F_{1i}(s)F_{1i}^{\top}(s)\zeta_1(t) + \dfrac{z}{2}\bar{\mathcal{E}_f}^2\\ -\zeta_1^{\top}(t)D_{1i}(s)d(t)\\ \leq \dfrac{1}{2z}\zeta_1^{\top}(t)D_{1i}(s)D_{1i}^{\top}(s)\zeta_1(t) + \dfrac{z}{2}\bar{\mathfrak{d}}^2. \end{cases}\tag{26}$$

Substituting (26) into (25) supplies

$$\begin{aligned}\dot{V}_1 \leq\; & -k_1 \parallel \zeta_1(t) \parallel^2 + \zeta_1^{\top}(t)(\bar{A}_{12} + Z_1U_{1i}(s)T_1)\zeta_2(t)\\ & + \frac{1}{2z}\zeta_1^{\top}(t)(\bar{A}_{12} + Z_1U_{1i}(s)T_1)I_2I_2^{\top}(\bar{A}_{12}\\ & + Z_1U_{1i}(s)T_1)^{\top} + F_{1i}(s)F_{1i}^{\top}(s) + D_{1i}(s)D_{1i}^{\top}(s)\cdot\\ & \zeta_1(t) + \frac{z}{2}(\bar{\mathcal{E}_x}^2 + \bar{\mathcal{E}_f}^2 + \bar{\mathfrak{d}}^2) - \frac{1}{2z}\zeta_1^{\top}(t)\Lambda\zeta_1(t)\\ =\; & -k_1 \parallel \zeta_1(t) \parallel^2 + \zeta_1^{\top}(t)(\bar{A}_{12} + Z_1U_{1i}(s)T_1)\zeta_2(t)\\ & + \frac{z}{2}(\bar{\mathcal{E}_x}^2 + \bar{\mathcal{E}_f}^2 + \bar{\mathfrak{d}}^2).\end{aligned}$$

Step 2: Recalling $\zeta_2(t)$ in Step 1, the derivative of $\zeta_2(t)$ with $\dot{x}_2(t)$ in (23) is

$$\begin{aligned}\dot{\zeta}_2 &= \dot{\hat{x}}_2 - \dot{\eta}_1 - (\bar{A}_{12} + Z_1U_{1i}(s)T_1)^{\dagger}\ddot{\kappa}(t)\\ &= I_2\dot{\hat{x}} - \frac{\partial\eta_1}{\partial\zeta_1}\dot{\zeta}_1 - \frac{\partial\eta_1}{\partial x_1}\dot{x}_1(t) - \frac{\partial\eta_1}{\partial\hat{f}}\dot{\hat{f}}(t)\\ &\quad - (\bar{A}_{12} + Z_1U_{1i}(s)T_1)^{\dagger}\ddot{\kappa}(t).\end{aligned}$$

Select the Lyapunov function as

$$V_2 = \frac{1}{2}\zeta_2^{\top}(t)\zeta_2(t)$$

and the controller $u(t)$ is designed as

$$\begin{aligned}u(t) = &-(\bar{B}_2 + Z_2U_{2i}(s)T_2)^{\dagger}(k_2\zeta_2(t)\\ &+ A_{21i}(s)\hat{x}_1(t) + (\bar{A}_{12} + Z_1U_{1i}(s)T_1)\hat{x}_2(t)\\ &+ F_{2i}(s)\hat{f}(t) + I_2L_{1i}(s)(\hat{y}(t) - y(t))\\ &+ I_2B_i(s)X_I(t)\\ &- \frac{\partial\eta_1}{\partial\zeta_1}\dot{\zeta}_1 - \frac{\partial\eta_1}{\partial x_1}\dot{x}_1(t) - \frac{\partial\eta_1}{\partial\hat{f}}\dot{\hat{f}}(t))\end{aligned}\tag{27}$$

where $\hat{x}_1(t)$, $\hat{x}_2(t)$, and $\hat{f}(t)$ are provided in (17) and k_2 is a given constant. Then, one has

$$\begin{aligned}\dot{V}_2 &= \zeta_2^{\top}(t)(-k_2\zeta_2(t) - (\bar{A}_{12} + Z_1U_{1i}(s)T_1)^{\dagger}\zeta_1(t))\\ &= -k_2 \parallel \zeta_2(t) \parallel - \zeta_2^{\top}(t)(\bar{A}_{12} + Z_1U_{1i}(s)T_1)^{\dagger}\zeta_1(t).\end{aligned}$$

Set $V = V_1 + V_2$, then

$$\begin{aligned}\dot{V} &= \dot{V}_1 + \dot{V}_2\\ &\leq -k_1 \parallel \zeta_1(t) \parallel - k_2 \parallel \zeta_2(t) \parallel\\ &\quad + \frac{z}{2}(\bar{\mathcal{E}_x}^2 + \bar{\mathcal{E}_f}^2 + \bar{\mathfrak{d}}^2).\end{aligned}\tag{28}$$

Summing up two steps and combining the designed controller (27) supply the following theorem to guarantee the uniform ultimate boundedness of (2).

Theorem 2: For (2) under Assumptions 1–3, the resultant closed-loop system is uniformly ultimately bounded with the designed observer (4)–(6) and controller (27).

Proof: Integrating (28) with Assumptions 1–3 provides

$$V(t) \leq V(0)e^{-kt} + \frac{\mathscr{E}}{k}\tag{29}$$

where $k = \min\{2k_1, 2k_2\}$ and $\mathscr{E} = \frac{z}{2}(\bar{\mathcal{E}_x}^2 + \bar{\mathcal{E}_f}^2 + \bar{\mathfrak{d}}^2)$, and then the uniform ultimate boundedness is obtained. ■

IV. Numerical Examples

In this section, two examples are supplied to show the validity of the proposed methods over those in [36].

Example 1: A tunnel diode (TD) circuit borrowed from [2] is shown in Fig. 1 and the property of TD is

$$i_{\mathrm{TD}}(t) = 0.5v_{\mathrm{TD}} + \alpha(t)v_{\mathrm{TD}}^3(t)\tag{30}$$

where $\alpha(t)$ is the characteristic parameter and has two modes as follows:

$$\alpha(t) = \begin{cases} 0.04, & \text{Mode 1}\\ 0.05, & \text{Mode 2}.\end{cases}$$

The transition probability matrix is $\begin{bmatrix} -0.5 & 0.5\\ 0.3 & -0.3\end{bmatrix}$.

Selecting $x_1(t) = v_C(t)$ and $x_2(t) = i_L(t)$, the state-space representation for Fig. 1 with (30) is

$$\begin{cases} C\dot{x}_1(t) = -0.5x_1(t) - \alpha x_1^3(t) + x_2(t) + f(t)\\ L\dot{x}_2(t) = -x_1(t) - Rx_2(t) + u(t) + d(t)\\ y(t) = Jx(t)\end{cases}\tag{31}$$

where $x(t) = [\, x_1^{\top}(t) \;\; x_2^{\top}(t) \,]^{\top}$, $u(t)$ is the input voltage source, $f(t)$ is the arc current fault caused by loose cable joints, poor

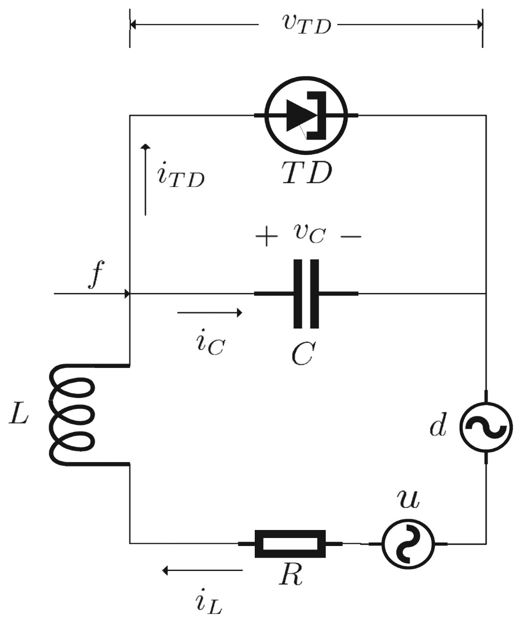

Fig. 1. Tunnel diode circuit

contacts, and broken wire insulations, and $d(t)$ denotes the noise voltage incurred by radio waves, respectively. J is the sensor output matrix.

In this example, the detailed system parameters are $C = 10\,\mathrm{F}$, $L = 10$ H, $R = 10\,\Omega$, and $J = [1 \quad 0]$. Moreover, supposing $x_1(t) \in [-1 \quad 1]$ and selecting the fuzzy membership functions as

$$\begin{cases} \nu_1(x_1(t)) = 1 - x_1^2(t) \\ \nu_2(x_1(t)) = x_1^2(t) \end{cases}$$

to (31) yields

Plant rule i: IF $x_1(t)$ is $\nu_i(x_1(t))$, THEN

$$\begin{cases} \dot{x}(t) = A_i(s)x_1(t) + B_i u(t) + F_i(s)f(t) + D_i(s)d(t) \\ y(t) = C(s)x(t), \quad i, s = 1, 2 \end{cases}$$

where

$$A_1(1) = \begin{bmatrix} -0.05 & 0.1 \\ -0.1 & -1.0 \end{bmatrix} A_1(2) = \begin{bmatrix} -0.05 & 0.1 \\ -0.1 & -1.0 \end{bmatrix}$$

$$A_2(1) = \begin{bmatrix} -0.0504 & 0.1 \\ -0.1 & -1.0 \end{bmatrix}$$

$$A_2(2) = \begin{bmatrix} -0.0505 & 0.1 \\ -0.1 & -1.0 \end{bmatrix}$$

$$B_1(1) = B_1(2) = B_2(1) = B_2(2) = [0 \; 0.1]^\top$$

$$D_1(1) = D_1(2) = D_2(1) = D_2(2) = [0 \; 0.1]^\top$$

$$F_1(1) = F_1(2) = F_2(1) = F_2(2) = [0.1 \; 0]^\top$$

$$C(1) = C(2) = [1 \; 0]^\top.$$

TABLE I
γ FOR DIFFERENT METHODS

Methods	γ
Theorem 1	0.0141
Corollary 1	0.0733
[36]	/

Solving the conditions given in Theorem 1, Corollary 1, and [36], respectively, the H_∞ performance indices are listed in Table I.

Moreover, the corresponding observer gains for the former two are obtained as follows.

Gains for Theorem 1:

$$L_{11}(1) = \begin{bmatrix} 0.1991 \\ 1.1255 \end{bmatrix}, \; L_{11}(2) = \begin{bmatrix} -1.1909 \\ 0.9759 \end{bmatrix}$$

$$L_{12}(1) = \begin{bmatrix} 0.0946 \\ 1.1255 \end{bmatrix}, \; L_{12}(2) = \begin{bmatrix} -1.2954 \\ 0.9759 \end{bmatrix}$$

$$L_{21}(1) = 0.1255, \; L_{21}(2) = -0.0965, \; L_{22}(1) = 0.1255$$

$$L_{22}(2) = -0.0965, \; E_1(1) = 0.1089, \; E_1(2) = -0.4954$$

$$E_2(1) = 0.1089, \; E_2(2) = -0.4954, \; M(1) = 0.7406$$

$$M(2) = 0.3986.$$

Gains for Corollary 1:

$$L_{11}(1) = \begin{bmatrix} -0.0626 \\ 0.2696 \end{bmatrix}, \; L_{11}(2) = \begin{bmatrix} 10.7736 \\ 2.4843 \end{bmatrix}$$

$$L_{12}(1) = \begin{bmatrix} -0.0622 \\ 0.2696 \end{bmatrix}, \; L_{12}(2) = \begin{bmatrix} 10.7741 \\ 2.4843 \end{bmatrix}$$

$$E_1(1) = -0.4581, \; E_1(2) = -1.4187, \; E_2(1) = -0.4581$$

$$E_2(2) = -1.4187, \; M(1) = 4.0670, \; M(2) = -0.1323.$$

In the viewpoint of solution, the proposed methods of Theorem 1 and Corollary 1 are more effective than those in [36] where the Lyapunov matrix for stability analysis has a special structure. Furthermore, the H_∞ obtained from Theorem 1 is smaller than that of Corollary 1 since the former contains not just system output but also its integral.

For the simulation purpose, it is assumed that $d(t) = 0.1\sin(t)$, ($t \in [6s \; 8s]$); otherwise, $d(t) = 0$. $f(t) = 5\sin(t)$, ($t \in [12s \; 15s]$; otherwise, $f(t) = 0$. Meanwhile, choose $k_1 = 1$, $k_2 = 10$, $z = 0.5$, and $x_1(0) = 0$, $x_2(0) = 0.1$.

With the obtained gains and the initial parameters, some curves are obtained in the framework of Theorem 1 for different iterative times. Namely, Figs. 2–4 show the estimation comparisons of system states and fault for one iteration (normal), four iterations, and ten iterations, respectively. Figs. 5–8 shows the estimation comparisons of system states, the fault, and the tracking error on four iterations for Theorem 1 and Corollary 1.

According to Figs. 2–4, it is seen that the estimates of the iterative observers for four iterations and ten iterations are almost the same but better than those of one iteration. To make the tradeoff between the real-time requirement and the implementation cost, Figs. 5–8 exhibit the comparisons of the estimated system states

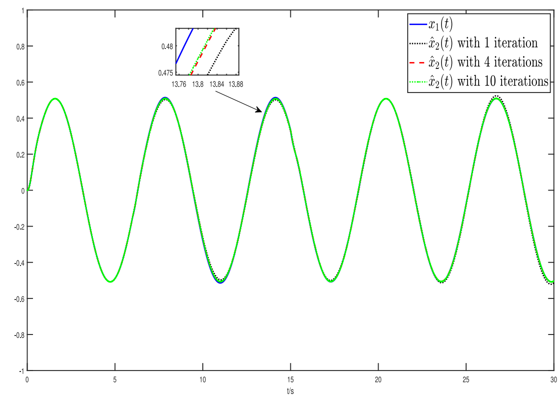

Fig. 2. Curves of $\hat{x}_1(t)$ for different iterations.

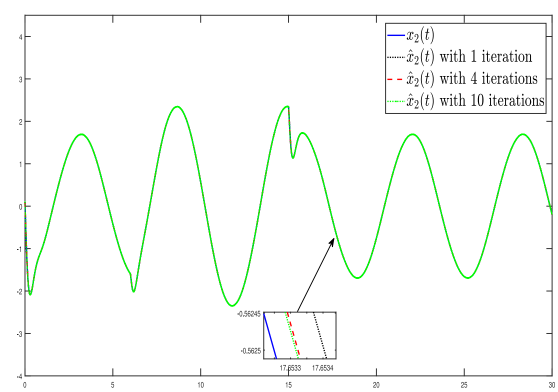

Fig. 3. Curves of $\hat{x}_2(t)$ for different iterations.

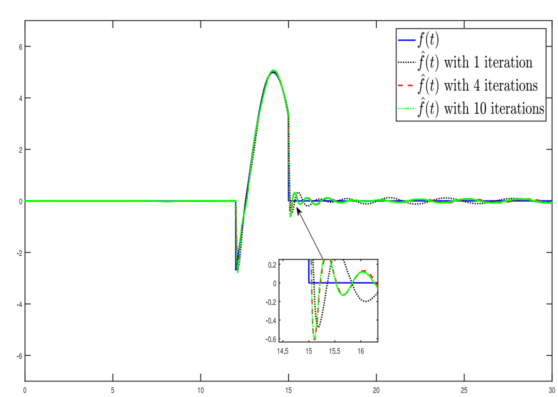

Fig. 4. Curves of $\hat{f}(t)$ for different iterations.

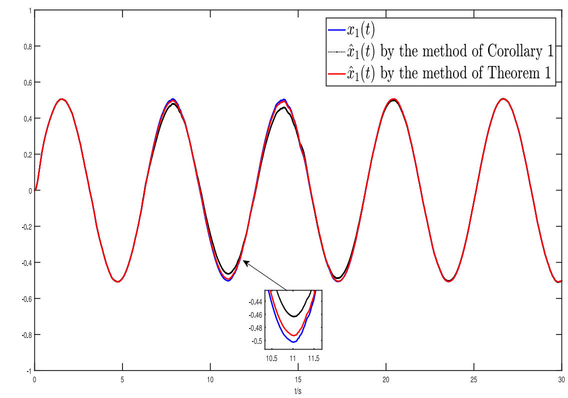

Fig. 5. Curves of $\hat{x}_1(t)$ for different methods with four iterations.

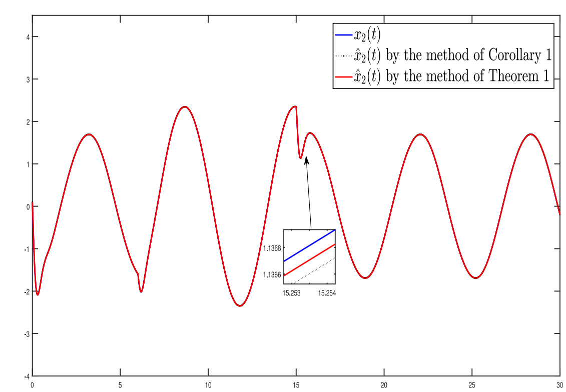

Fig. 6. Curves of $\hat{x}_2(t)$ for different methods with four iterations.

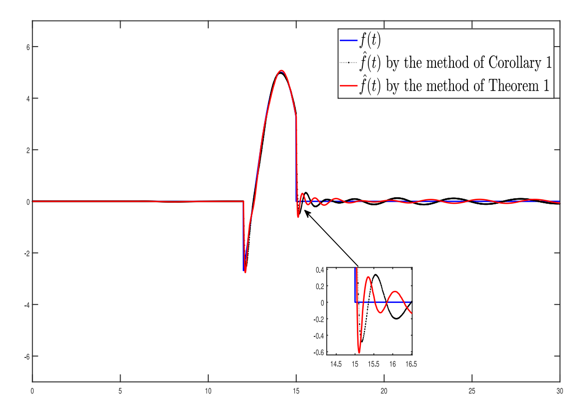

Fig. 7. Curves of $\hat{f}(t)$ for different methods with four iterations.

and fault for Theorem 1 and Corollary 1 under four iterations. From these comparisons, it is not difficult to conclude that the iterative observers with the additional integral term (Theorem 1) are more exact than those without the term (Corollary 1).

In contrast to no solution for [36] in the first example, the full comparison for all methods is given in the following example.

Example 2: Consider (1) with the following parameters:

$$A_1(1) = \begin{bmatrix} -1.2 & 0.5 \\ -0.3 & -0.7 \end{bmatrix}, \; A_1(2) = \begin{bmatrix} -0.8 & -0.1 \\ -0.4 & -1.5 \end{bmatrix}$$

$$A_2(1) = \begin{bmatrix} 1.2 & -0.3 \\ -0.3 & -1.0 \end{bmatrix}, \; A_2(2) = \begin{bmatrix} 0.2 & -0.8 \\ -0.1 & -0.6 \end{bmatrix}$$

$$B_1(1) = B_1(2) = B_2(1) = B_2(2) = \begin{bmatrix} 0 & 0.1 \end{bmatrix}^\top$$

$$D_1(1) = D_1(2) = D_2(1) = D_2(2) = \begin{bmatrix} 0.1 & 0.1 \end{bmatrix}^\top$$

$$F_1(1) = F_1(2) = F_2(1) = F_2(2) = \begin{bmatrix} 0.1 & 0.1 \end{bmatrix}^\top$$

$$C(1) = C(2) = \begin{bmatrix} 1 & 0 \end{bmatrix}^\top.$$

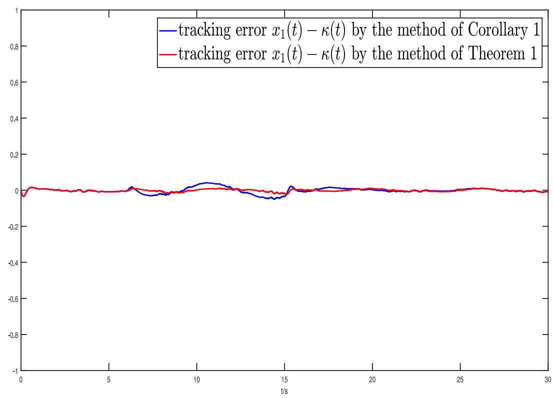

Fig. 8. Curves of $x_1(t) - \kappa(t)$ for different methods with four iterations.

TABLE II
γ FOR DIFFERENT METHODS

Methods	γ
Theorem 1	0.2712
Corollary 1	0.8893
[36]	0.9123

Additionally, the membership functions, the transition probability matrix, the initial values of faults, and disturbance are chosen as same as Example 1.

Solving the conditions given in Theorem 1, Corollary 1, and [36], the H_∞ performance indices are listed in Table II.

The resultant observer gains are obtained.

Gains for Theorem 1:

$$L_{11}(1) = \begin{bmatrix} 1.2419 \\ 0.3956 \end{bmatrix}, \ L_{11}(2) = \begin{bmatrix} -6.4899 \\ -0.1074 \end{bmatrix}$$

$$L_{12}(1) = \begin{bmatrix} -1.1580 \\ 0.3956 \end{bmatrix}, \ L_{12}(2) = \begin{bmatrix} -7.4899 \\ -0.4074 \end{bmatrix}$$

$$E_1(1) = 0.0658, \ E_1(2) = -0.4438, \ E_2(1) = 0.0658$$

$$E_2(2) = -0.443, \ M(1) = 1.5684, \ M(2) = 0.0608.$$

Gains for Corollary 1:

$$L_{11}(1) = \begin{bmatrix} 1.3579 \\ 0.4265 \end{bmatrix}, \ L_{11}(2) = \begin{bmatrix} -0.5674 \\ 0.3762 \end{bmatrix}$$

$$L_{12}(1) = \begin{bmatrix} -1.0420 \\ 0.4265 \end{bmatrix}, \ L_{12}(2) = \begin{bmatrix} -1.5674 \\ 0.0762 \end{bmatrix}$$

$$E_1(1) = 0.1229, \ E_1(2) = -0.0795, \ E_2(1) = 0.1229$$

$$E_2(2) = -0.0795, \ M(1) = 0.7785, \ M(2) = 0.0582.$$

Gains for [36]:

$$L_{11}(1) = \begin{bmatrix} 1.6742 \\ 0.4334 \end{bmatrix}, \ L_{11}(2) = \begin{bmatrix} -0.7240 \\ 0.1678 \end{bmatrix}$$

$$L_{12}(1) = \begin{bmatrix} 0.9742 \\ 0.4334 \end{bmatrix}, \ L_{12}(2) = \begin{bmatrix} -2.3240 \\ -0.1321 \end{bmatrix}$$

$$E_1(1) = 0.0914, \ E_1(2) = -0.0132, \ E_2(1) = 0.0914$$

$$E_2(2) = -0.0132, \ M(1) = 0.5250, \ M(2) = 0.0068.$$

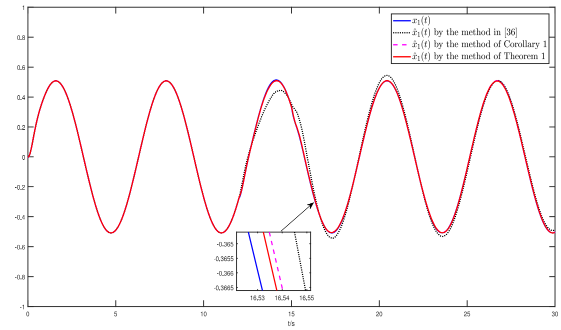

Fig. 9. Curves of $\hat{x}_1(t)$ for different methods.

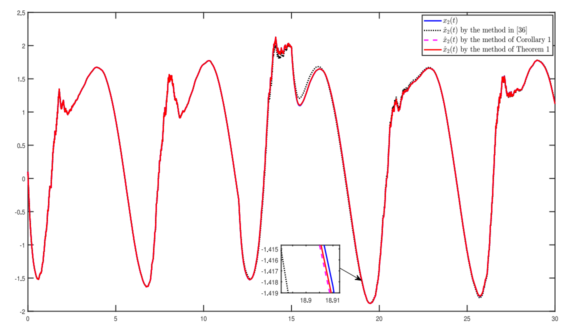

Fig. 10. Curves of $\hat{x}_2(t)$ for different methods.

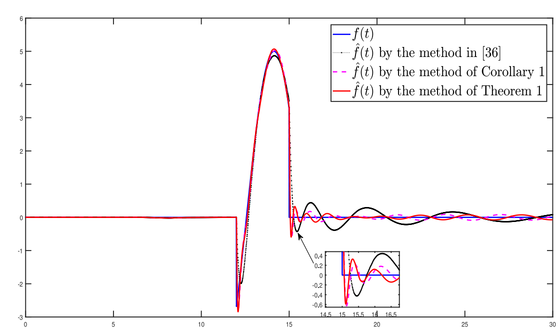

Fig. 11. Curves of $\hat{f}(t)$ for different methods.

Employing the above obtained gains and the prescribed initial parameters, comparison curves are depicted in Figs. 9–15 to show the estimated states and the faults, as well as the corresponding observed errors, for different methods with four iterations.

From Figs. 9–11, the transient and the steady performance of the estimated system states and the mismatched fault for Theorem 1 are better than those of Corollary 1 and [36]. Besides, the observer errors and the tracking error in Figs. 12–15 for Theorem 1 is the smallest. Consequently, Table II and Figs. 9–15 verified the effectiveness of the proposed methods over those in [36].

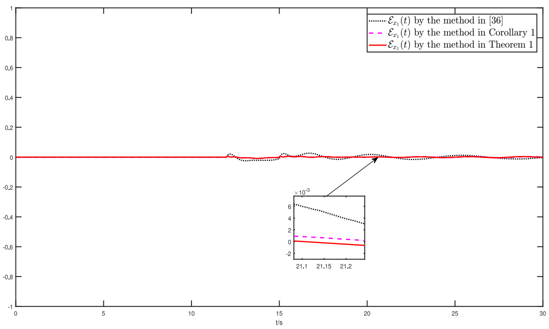

Fig. 12. Curves of $\mathcal{E}_{x_1}$ for different methods.

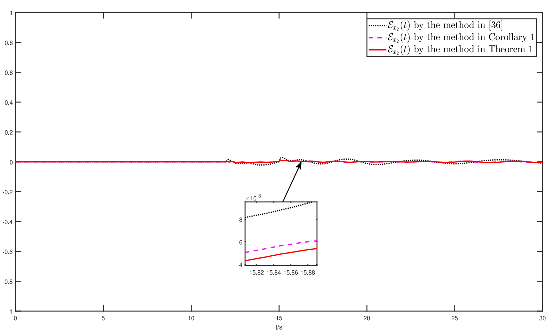

Fig. 13. Curves of $\mathcal{E}_{x_2}$ for different methods.

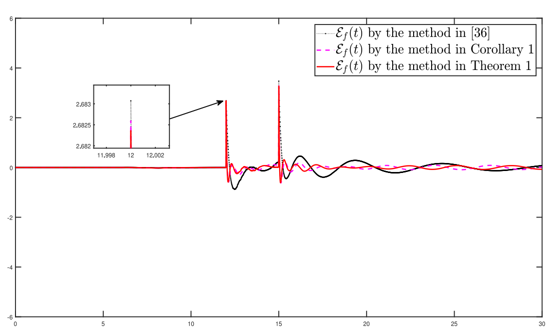

Fig. 14. Curves of $\mathcal{E}_f$ for different methods.

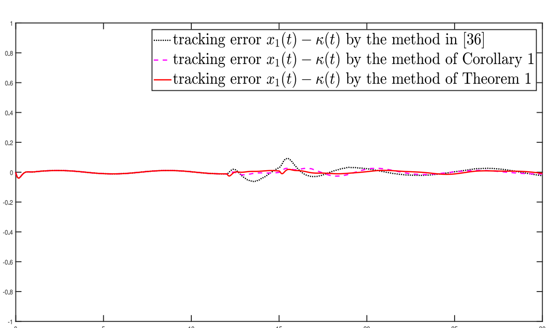

Fig. 15. Curves of tracking error $x_1(t) - \kappa(t)$ for different methods.

V. Conclusion

The mismatched fault-tolerant tracking control of Markov T-S fuzzy systems was discussed in this article. A set of novel iterative observers were constructed via making full use of system outputs to improve the estimation accuracy. Resorting to a structural separation technique, the stochastic stability of the observer error system with the prescribed H_∞ performance was established in terms of LMIs. Based on the established stability, the mismatched fault and the original system states were estimated by an iterative algorithm. A backstepping approach to realize fault-tolerant tracking control was supplied to guarantee the uniform ultimate boundedness of the closed-loop system, which is verified by two examples. Note that the established estimate and control scheme depended on the known premise variable; therefore, how to overcome new challenges caused by the unknown case will be explored in the future work.

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