Fuzzy Optimal Tracking Control of Hypersonic Flight Vehicles via Single-Network Adaptive Critic Design

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***Abstract*—Optimal performance is extremely important for hypersonic flight control. Different from most existing methodologies, which only consider basic control performance including stability, robustness, and transient performance, this article deals with the design of nearly optimal tracking controllers for hypersonic flight vehicles (HFVs). First, main controllers are developed for the velocity subsystem and the altitude subsystem of HFVs via concise fuzzy approximations. Then, optimal controllers are nearly implemented utilizing single-network adaptive critic design. Moreover, the stability of closed-loop systems and the convergence of optimal controllers are theoretically proved. Finally, compared simulation results are given to verify the superiority. The special contribution is the application of a low-complex control structure owing to the critic-only network and advanced learning laws developed for fuzzy approximations, which is expected to guarantee satisfied real-time performance.**

***Index Terms*—Fuzzy approximations, hypersonic flight vehicles (HFVs), optimal performance, real-time performance, single-network adaptive critic design.**

I. Introduction

Hypersonic flight vehicles (HFVs) have potential to serve as long-range transports and carriers of rapid and accurate attack weapons [1]–[5]. The design of control systems for HFVs continues to be a topic of important research interest, and it is inherently difficult due to the fact that the vehicle dynamics are nonlinear, coupled, and uncertain. Furthermore, hypersonic flight controllers must handle narrow flight envelopes, rapidly time-varying flight circumstances, and notable flexible effects. In addition to ensuring control authority over the entire flight envelope, an optimal index is also necessary for HFVs' control systems to accomplish miscellaneous missions.

Hypersonic flight control has received worldwide attention in recent years because of the interesting and hard-to-handle flight conditions connected with high Mach numbers. Thereby, considerable efforts have been made by researchers to exploit advanced controllers for HFVs for the purpose of attaining stable tracking of reference trajectories [6]–[9]. In the literature, stability performance of the closed-loop control system is a primarily considered index for HFVs by incorporating baseline controllers with additional terms [10]–[13]. In [12], a robust control method is studied for HFVs to maintain control stability and reject parametric perturbations. First, dynamic inversion is combined with back-stepping to develop baseline controllers for the velocity subsystem and the altitude subsystem. And then, sliding-mode switching terms together with neural estimators are applied to increase the tolerance of closed-loop control systems to external disturbances and model uncertainties. An alternative method, which is available for resisting system uncertainties and external disturbances of HFVs, is the active disturbance rejection control (ADRC) approach [14]. The difference from [12] is that it [14] only considers the attitude control issue. A common problem, which arises in the hypersonic control domain, is actuator faults/saturations [15]–[17]. This may result in tracking performance reducing even instability. A possible solution to this problem is to add auxiliary systems to baseline controllers. The compensation signals generated by auxiliary systems can stabilize hypersonic flight control systems in the presence of actuator faults/saturations. Unlike the abovementioned disturbance-compensation methodologies, a new offset-free control approach is proposed in [18] to make the control system resistant to disturbances and unknown dynamics. Besides, other estimators such as disturbance observers [8] and intelligent approximations [19]–[21] also are usually used to resist disturbances. On the other hand, except for stabilization, transient performance is widely considered to be a very significant index for hypersonic flight control systems. Prescribed performance control (PPC) [9], [22] has been shown to efficiently guarantee transient performance. The key point of PPC is to devise performance functions, which are used to impose funnel constraints on tracking errors. And then, the desired prescribed performance is realized owing to the boundedness of transformed errors [9], [22].

Most of the methodologies mentioned above, however, are aimed at guaranteeing steady-state performance and transient performance for hypersonic flight control systems under different actual conditions including uncertainties, disturbances, and actuator saturations/faults. Unfortunately, only such basic performance is not enough for hypersonic flight control, and instead we must further seek some optimal performance indexes [23]–[27]. Adaptive critic design (ACD) is a newly emerging methodology to solve optimal control problems, and it has many possible uses in the optimal control of simple dynamic systems. Its typical structure is the dual-network framework containing a critic network and an action network. This is also called the critic–actor structure inspired by reinforcement learning. The main superiority of ACD in comparison with traditional optimal controllers is that its critic network can generate strength signals based on the current control quality to further improve the action

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movement [28]–[34]. This finally minimizes cost functions and enhances control performance. Despite this advantage, a well-known problem with the dual-network structure is that it results in high-computation burdens caused by the fact that both the critic network and the action network should be approximately estimated by neural or fuzzy approximations. However, another popular framework called single-network ACD (SACD) exhibits a better application prospect, because it has simpler structure compared with dual-network ACD. Even so, the existing SACD cannot be directly employed to hypersonic flight control because of the following serious defects. First, most of SACD strategies only focus on the stabilization problem, while hypersonic flight control belongs to a trajectory tracking control issue, and secondly the existing studies require that the control input must eventually converge to zero, which is unrealistic for hypersonic flight control systems.

This article considers nearly optimal trajectory tracking control designing for HFVs with uncertain dynamics. The main contributions are summarized as follows.

1) The control structure complexity is low via concise SACD for the sake of guaranteeing real-time performance. Different from the existing studies [27], [28], [33], this article proposes a single-adaptive-parameter-based strategy to construct the critic network, which has only one adaptive parameter.
2) The previous methods [29], [34] focus on the stabilization problem, that is, system outputs must converge to zero. The addressed optimal controller in this article is extended to the tracking control issue and it enables system outputs to track given reference commands.
3) The optimal tracking methodologies developed in [30]–[32] are only applicable to special dynamic systems whose control inputs must eventually converge to zero. Such restriction is released in this article and the control inputs are allowed to converge to nonzero constants.

II. Problem Formulation and Preliminaries

A. Vehicle Model

The considered vehicle model consists of five rigid-body states (velocity V, altitude h, flight-path angle γ, pitch angle θ, and pitch rate Q) and two flexible states η_1 and η_2. The motion equations are described as [35] follows:

$$\dot{V} = \frac{1}{m}\left[\cos(\theta-\gamma)T - D\right] - g\sin\gamma \tag{1}$$

$$\dot{h} = V\sin\gamma \tag{2}$$

$$\dot{\gamma} = \frac{1}{V}\left[\frac{L}{m} + \sin(\theta-\gamma)\frac{T}{m} - g\cos\gamma\right] \tag{3}$$

$$\dot{\theta} = Q \tag{4}$$

$$\dot{Q} = \frac{1}{I_{yy}}\left(M + \tilde{\psi}_1\ddot{\eta}_1 + \tilde{\psi}_2\ddot{\eta}_2\right) \tag{5}$$

$$\ddot{\eta}_1 = -\frac{2\zeta_1\omega_1\dot{\eta}_1}{k_1} - \frac{\omega_1^2\eta_1}{k_1} + \frac{N_1}{k_1} - \frac{\tilde{\psi}_1}{k_1 I_{yy}}\left(M + \tilde{\psi}_2\ddot{\eta}_2\right) \tag{6}$$

$$\ddot{\eta}_2 = -\frac{2\zeta_2\omega_2\dot{\eta}_2}{k_2} - \frac{\omega_2^2\eta_2}{k_2} + \frac{N_2}{k_2} - \frac{\tilde{\psi}_2}{k_2 I_{yy}}\left(M + \tilde{\psi}_1\ddot{\eta}_1\right) \tag{7}$$

where T, D, L, M, N_1, and N_2 stand for trust force, drag force, lift force, pitching moment, and generalized forces, respectively. These forces are functions of system states and control inputs, given by

$$\begin{aligned}
T &\approx \alpha^3(\beta_1\Phi + \beta_2) + \alpha^2(\beta_3\Phi + \beta_4) \\
&\quad + \alpha(\beta_5\Phi + \beta_6) + \beta_7\Phi + \beta_8 \\
D &\approx 0.5e^{\frac{h_0-h}{h_s}}\rho_0 V^2 S\left(C_D^{\alpha^2}\alpha^2 + C_D^{\delta_e^2}\delta_e^2 + C_D^{\alpha}\alpha + C_D^{\delta_e}\delta_e + C_D^0\right) \\
L &\approx 0.5e^{\frac{h_0-h}{h_s}}\rho_0 V^2 S\left(C_L^{\alpha}\alpha + C_L^{\delta_e}\delta_e + C_L^0\right) \\
M &\approx z_T T + 0.5e^{\frac{h_0-h}{h_s}}\rho_0 V^2 S\bar{c} \\
&\quad \times\left[C_{M,\alpha}^{\alpha^2}\alpha^2 + C_{M,\alpha}^{\alpha}\alpha + C_{M,\alpha}^0 + c_e\delta_e\right] \\
N_1 &= N_1^{\alpha^2}\alpha^2 + N_1^{\alpha}\alpha + N_1^0, N_2 = N_2^{\alpha^2}\alpha^2 + N_2^{\alpha}\alpha \\
&\quad + N_2^{\delta_e}\delta_e + N_2^0.
\end{aligned}$$

The control inputs include the elevator angular deflection δ_e and the fuel equivalence ratio Φ. All the definitions of coefficients and variables in the abovementioned equations can be seen in [35]. We should mention that only rigid-body states can be actually measured such that they are available for state feedbak, while the flexible states are often suppressed as disturbances. Then, the control objective is to devise Φ and δ_e such that $V \to V_{\text{ref}}$ and $h \to h_{\text{ref}}$, where V_{ref} and h_{ref} are chosen reference trajectories for velocity and altitude.

B. Fuzzy Approximation

In this section, we recall the basic principle of fuzzy approximations, which will be applied to stabilize the unknown dynamics of HFVs' model via an estimation-compensation approach.

The fuzzy approximation of a continuous function $f(\mathbf{x}_f)$ is formulated as the following input-to-output mapping [36]:

$$f(\mathbf{x}_f) = \boldsymbol{\zeta}_f^T\boldsymbol{\varpi}_f(\mathbf{x}_f) + \varepsilon_f \tag{8}$$

where $f(\mathbf{x}_f)$ is the function of $\mathbf{x}_f$ and $\mathbf{x}_f = [x_1^f, x_2^f, \ldots, x_n^f]^{\mathrm{T}} \in \Re^n$ is the input vector of fuzzy system (8). $\boldsymbol{\zeta}_f = [\zeta_1^f, \zeta_2^f, \ldots, \zeta_N^f]^T \in \Re^N$ is a weight vector. $\boldsymbol{\varpi}_f(\mathbf{x}_f) = [\varpi_1^f(\mathbf{x}_f), \varpi_2^f(\mathbf{x}_f), \ldots, \varpi_N^f(\mathbf{x}_f)]^T$ is a basis function vector with $\varpi_j^f(\mathbf{x}_f) = \prod_{i=1}^n \mu_{\Psi_i^j}(x_i^f)/(\sum_{j=1}^N [\prod_{i=1}^n \mu_{\Psi_i^j}(x_i^f)])$, $i = 1, 2, \ldots, n; j = 1, 2, \ldots, N$, where $\mu_{\Psi_i^j}(x_i^f)$ is a Gaussian-function-based fuzzy membership function. ε_f means the fuzzy approximation error. It has been proved that there exists a constant $\varepsilon_f^M \in \Re^+$ such that $\sup_{\mathbf{x}_f\in\Omega_{\mathbf{x}_f}} |f(\mathbf{x}_f) - \boldsymbol{\zeta}_f^T\boldsymbol{\varpi}_f(\mathbf{x}_f)| = \varepsilon_f \le \varepsilon_f^M$, where $\Omega_{\mathbf{x}_f}$ is a compact set.

Remark 1: Though ε_f can be made infinitesimal when choosing sufficiently large dimensions for $\boldsymbol{\zeta}_f$ and $\boldsymbol{\varpi}_f(\mathbf{x}_f)$, the unknown term ε_f still leads to a problem related to the implementations of control laws, which is usually tackled using the adaptive strategy by online updating $\boldsymbol{\zeta}_f$. This also results in high computational costs owing to too many elements of $\boldsymbol{\zeta}_f$. For an arbitrarily unknown function, we can obtain a low-computational approximation by directly turning $||\boldsymbol{\zeta}_f||^2$ instead of its elements. This is very significant for HFVs to guarantee their control systems with excellent real-time performance.

Based on SACD, function approximations are necessary for critic network designs. Unfortunately, the abovementioned fuzzy system (8) is not suitable for HFVs to devise critic networks. According to the gradient descend method, we must adjust all the elements of $\boldsymbol{\zeta}_f$ via developing adaptive laws based on Lyapunov theory. Undoubtedly, the control real-time performance cannot meet the requirement of hypersonic flight control if we use fuzzy formulation (8) to develop critic networks. For this reason, we give another form of fuzzy approximations, called fuzzy Hyperbolic model (FHM) [37], to construct the critic network subsequently.

FHM can be utilized to reconstruct an arbitrary dynamic system

$$\dot{\varphi}_x = \mathbf{W}_A^{\varphi_x}\mathbf{tanh}(\mathbf{K}_{\varphi_x}\varphi_x) + \mathbf{W}_B^{\varphi_x}\mathbf{tanh}(\mathbf{K}_\upsilon \upsilon) \tag{9}$$

where $\varphi_x = [\varphi_1^x, \varphi_2^x, \ldots, \varphi_n^x]^T \in \Re^n$ is a system state vector and $\upsilon = [\upsilon_1, \upsilon_2, \ldots, \upsilon_m] \in \Re^m$ an input vector. $\mathbf{W}_A^{\varphi_x} \in \Re^{n\times n}$ and $\mathbf{W}_B^{\varphi_x} \in \Re^{n\times m}$ are weight vectors. $\mathbf{tanh}(\mathbf{K}_{\varphi_x}\varphi_x) = [\tanh(k_1^{\varphi_x}\varphi_1^x), \tanh(k_2^{\varphi_x}\varphi_2^x), \ldots, \tanh(k_n^{\varphi_x}\varphi_n^x)]^T$ and $\mathbf{tanh}(\mathbf{K}_\upsilon \upsilon) = [\tanh(k_1^\upsilon \upsilon_1), \tanh(k_2^\upsilon \upsilon_2), \ldots, \tanh(k_m^\upsilon \upsilon_m)]^T$ are Hyperbolic basis function vectors.

Remark 2: In what follows, we use FHM to design critic networks, which helps to reduce computation loads and ensure real-time performance because φ_x and $\mathbf{W}_A^{\varphi_x}$ degenerate into scalars in the design process of each critic network.

III. Controller Design

A. Vehicle Controller Design

This subsystem presents the design process of a fuzzy optimal controller for velocity subsystem (1) to make $V\to V_{\mathrm{ref}}$ and minimize the cost function (20).

We represent velocity subsystem (1) as follows:

$$\dot{V} = a_V + \Phi_s + \Phi^* \tag{10}$$

where $\Phi = \Phi_s + \Phi^*$, Φ_s is a main controller and Φ^* is an optimal controller, which is used to optimize performance index (20). $a_V = T\cos(\theta-\gamma)/m - D/m - g\sin\gamma - \Phi$ is an unknown but continuous function. Note that a_V is a function of states and control inputs, and control inputs are functions of states since they are computed based on state-feedback controllers. Hence, we can use fuzzy system (8) to estimate a_V.

$$a_V = \boldsymbol{\zeta}_V^T \boldsymbol{\varpi}(\boldsymbol{X}) + \varepsilon_V \tag{11}$$

where $\boldsymbol{\zeta}_V = [\zeta_1^V, \zeta_2^V, \ldots, \zeta_n^V]^{\mathrm{T}} \in \Re^n$ is a weight vector and $\boldsymbol{X} = [V, h, \gamma, \theta, Q]^T \in \Re^5$ is an input vector. The estimation error ε_V satisfies $|\varepsilon_V| \le \varepsilon_V^M$ with a positive constant ε_V^M. $\boldsymbol{\varpi}(\boldsymbol{X})$ has the same formulation as $\boldsymbol{\varpi}_f(\mathbf{x}_f)$.

We define

$$\tilde{V} = V - V_{\mathrm{ref}}. \tag{12}$$

Invoking (10), we obtain $\dot{\tilde{V}}$

$$\dot{\tilde{V}} = a_V + \Phi_s + \Phi^* - \dot{V}_{\mathrm{ref}}. \tag{13}$$

We design Φ_s as follows:

$$\Phi_s = -k_{V1}\tilde{V} - k_{V2}\int_0^t \tilde{V}d\tau - 0.5\tilde{V}\hat{\varphi}_V\boldsymbol{\varpi}^T(\boldsymbol{X})\boldsymbol{\varpi}(\boldsymbol{X}) + \dot{V}_{\mathrm{ref}} \tag{14}$$

where $k_{V1} \in \Re^+$ and $k_{V2} \in \Re^+$ are constants, and $\hat{\varphi}_V$ is the estimation of $\varphi_V = ||\boldsymbol{\zeta}_V||^2$. We devise the following learning law for $\hat{\varphi}_V$:

$$\dot{\hat{\varphi}}_V = 0.5 l_V \tilde{V}^2 \boldsymbol{\varpi}^T(\boldsymbol{X})\boldsymbol{\varpi}(\boldsymbol{X}) - 2k_{V1}\hat{\varphi}_V \tag{15}$$

where $l_V \in \Re^+$ is a constant.

The Lyapunov function is defined as follows:

$$\mathbf{L}_{\mathbf{V},1} = 0.5\tilde{V}^2 + 0.5k_{V2}\left(\int_0^t \tilde{V}\mathrm{d}\tau\right)^2 + 0.5\tilde{\varphi}_V^2/l_V \tag{16}$$

with $\tilde{\varphi}_V = \hat{\varphi}_V - \varphi_V$.

Utilizing (11) and (13)–(15), $\dot{\mathbf{L}}_{\mathbf{V},1}$ is reduced as

$$\begin{aligned}\dot{\mathbf{L}}_{\mathbf{V},1} = & -k_{V1}\tilde{V}^2 + \tilde{V}\boldsymbol{\zeta}_V^T\boldsymbol{\varpi}(\boldsymbol{X}) - 0.5\tilde{V}^2\hat{\varphi}_V\boldsymbol{\varpi}^T(\boldsymbol{X})\boldsymbol{\varpi}(\boldsymbol{X})\\ & + 0.5\tilde{V}^2\tilde{\varphi}_V\boldsymbol{\varpi}^T(\boldsymbol{X})\boldsymbol{\varpi}(\boldsymbol{X}) - 2k_{V1}\hat{\varphi}_V\tilde{\varphi}_V/l_V\\ & + \tilde{V}\varepsilon_V + \tilde{V}\Phi^* \end{aligned} \tag{17}$$

Because of $2k_{V1}\tilde{\varphi}_V\hat{\varphi}_V \ge -k_{V1}\varphi_V^2$, $\tilde{V}\varepsilon_V \le 0.5\tilde{V}^2 + 0.5(\varepsilon_V^M)^2$ and $\tilde{V}\boldsymbol{\zeta}_V^T\boldsymbol{\varpi}(\boldsymbol{X}) \le 0.5\tilde{V}^2\varphi_V\boldsymbol{\varpi}^T(\boldsymbol{X})\boldsymbol{\varpi}(\boldsymbol{X}) + 0.5$, $\dot{\mathbf{L}}_{\mathbf{V},1}$ becomes

$$\begin{aligned}\dot{\mathbf{L}}_{\mathbf{V},1} \le & \underbrace{-(k_{V1}-0.5)\tilde{V}^2 + 0.5 + k_{V1}\varphi_1^2/l_V + 0.5\left(\varepsilon_V^M\right)^2}_{\triangleq \sum_V^1}\\ & + \underbrace{\tilde{V}\Phi^*}_{\triangleq\sum_V^2}. \end{aligned} \tag{18}$$

The existing work [8] shows that $\sum_V^1$ can be stabilized. For $\sum_V^2$, we will develop an optimal controller based on SACD.

We give the following dynamics:

$$\dot{\tilde{V}} = \Phi^*. \tag{19}$$

We define the following cost function:

$$J_V = \int_0^\infty \left[Q_V(\tilde{V}) + \vartheta_V(\Phi^*)\right] dt \tag{20}$$

where $Q_V(\tilde{V}) = q_V\tilde{V}^2$ and $\vartheta_V(\Phi) = r_V(\Phi^*)^2$ with positive constants q_V and r_V.

We further obtain the Hamilton–Jacobi–Bellman (HJB) equation

$$H_{\tilde{V}}(\tilde{V}, \Phi^*, J_V) = Q_V(\tilde{V}) + \vartheta_V(\Phi^*) + \nabla J_V\Phi^* \tag{21}$$

with $\nabla J_V = \partial J_V/\partial\tilde{V}$.

We design Φ^* as follows:

$$\Phi^* = -0.5r_V^{-1}\nabla J_V. \tag{22}$$

Since ∇J_V is an unknown term, we use FHM to construct the critic network via adaptively estimating J_V as follows:

$$J_V = A_{\tilde{V}}\tanh(K_{\tilde{V}}\tilde{V}) + \varepsilon_{\tilde{V}} \tag{23}$$

where $A_{\tilde{V}} \in \Re$, $K_{\tilde{V}} \in \Re^+$, and $\varepsilon_{\tilde{V}}$ is an estimation error.

From (23), we have ∇J_V as follows:

$$\nabla J_V = \Lambda_{\tilde{V}}A_{\tilde{V}} + \partial\varepsilon_{\tilde{V}}/\partial\tilde{V} \tag{24}$$

with $\Lambda_{\tilde{V}} = \partial\tanh(K_{\tilde{V}}\tilde{V})/\partial\tilde{V}$.

Substituting (24) into (21), we have

$$H_{\tilde{V}}(\tilde{V},\Phi^*,J_V)=A_{\tilde{V}}\sigma_{\tilde{V}}+Q_V(\tilde{V})+\vartheta_V(\Phi^*)-\varepsilon_{\rm HJB}^V \quad (25)$$

where $\sigma_{\tilde{V}}=\Lambda_{\tilde{V}}\Phi^*$ and $\varepsilon_{\rm HJB}^V=\Phi^*\partial\varepsilon_{\tilde{V}}/\partial\tilde{V}$ means the residual error of HJB. The sustained incentive condition can ensure the boundedness of $\varepsilon_{\rm HJB}$, that is, $||\varepsilon_{\rm HJB}^V||\le\varepsilon_{\rm HJB}^{V,M}$, where $\varepsilon_{\rm HJB}^{V,M}$ is a positive constant.

Then, the optimal control Φ^* becomes

$$\Phi^*=-0.5r_V^{-1}\left(\Lambda_{\tilde{V}}A_{\tilde{V}}+\partial\varepsilon_{\tilde{V}}/\partial\tilde{V}\right). \quad (26)$$

Because $\Lambda_{\tilde{V}}$ is an unknown variable, we define its estimation as $\hat{A}_{\tilde{V}}$. Furthermore, we use $\hat{\Lambda}_{\tilde{V}}$ to replace $\Lambda_{\tilde{V}}$, leading to

$$\hat{J}_V=\hat{A}_{\tilde{V}}\tanh(K_{\tilde{V}}\tilde{V}) \quad (27)$$

$$\nabla\hat{J}_V=\Lambda_{\tilde{V}}\hat{A}_{\tilde{V}}. \quad (28)$$

Finally, we obtain Φ^* as follows:

$$\hat{\Phi}^*=-0.5r_V^{-1}\left[K_{\tilde{V}}-K_{\tilde{V}}\tanh^2(K_{\tilde{V}}\tilde{V})\right]\hat{A}_{\tilde{V}}. \quad (29)$$

The HJB equation becomes

$$e_{\tilde{V}}\triangleq H_{\tilde{V}}(\tilde{V},\Phi^*,\hat{J}_V)=\Sigma_{\tilde{V}}+Q_V(\tilde{V})+\vartheta_V(\Phi^*) \quad (30)$$

with $\Sigma_{\tilde{V}}=\Lambda_{\tilde{V}}\hat{A}_{\tilde{V}}\Phi^*$.

Defining $E_{\tilde{V}}=e_{\tilde{V}}^2/2$, we develop the following regulation law to minimize $E_{\tilde{V}}$:

$$\begin{aligned}\dot{\hat{A}}_{\tilde{V}}&=-\alpha_{\tilde{V}}\partial E_{\tilde{V}}/\partial\hat{A}_{\tilde{V}}\\&=-\alpha_{\tilde{V}}\sigma_{\tilde{V}}\left[\sigma_{\tilde{V}}\hat{A}_{\tilde{V}}+Q_V(\tilde{V})+\vartheta_V(\Phi^*)\right]\end{aligned} \quad (31)$$

with $\sigma_{\tilde{V}}=\Lambda_{\tilde{V}}\Phi^*=[K_{\tilde{V}}-K_{\tilde{V}}\tanh^2(K_{\tilde{V}}\tilde{V})]\Phi^*$ and $\alpha_{\tilde{V}}\in\Re^+$.

Defining $\tilde{A}_{\tilde{V}}=\hat{A}_{\tilde{V}}-A_{\tilde{V}}$ and using (25), we get

$$\dot{\tilde{A}}_{\tilde{V}}=-\alpha_{\tilde{V}}\sigma_{\tilde{V}}\left(\sigma_{\tilde{V}}\tilde{A}_{\tilde{V}}+\varepsilon_{\rm HJB}^V\right). \quad (32)$$

We chose the Lyapunov function candidate as follows:

$$L_{V,\mathbf{2}}=\tilde{A}_{\tilde{V}}^2/(2\alpha_{\tilde{V}})+\tilde{V}^2+2\Gamma_VJ_V. \quad (33)$$

$\dot{L}_{V,\mathbf{2}}$ is derived as follows:

$$\begin{aligned}\dot{L}_{V,\mathbf{2}}=&-\sigma_{\tilde{V}}^2\tilde{A}_{\tilde{V}}^2-2\frac{\alpha_{\tilde{V}}}{\sqrt{2\alpha_{\tilde{V}}}}\sigma_{\tilde{V}}\tilde{A}_{\tilde{V}}\frac{1}{\sqrt{2\alpha_{\tilde{V}}}}\varepsilon_{\rm HJB}^V+2\tilde{V}\Phi^*\\&+2\Gamma_V\left[-Q_V(\tilde{V})-\vartheta_V(\Phi^*)\right].\end{aligned} \quad (34)$$

Using $-2\frac{\alpha_{\tilde{V}}}{\sqrt{2\alpha_{\tilde{V}}}}\sigma_{\tilde{V}}\tilde{A}_{\tilde{V}}\frac{1}{\sqrt{2\alpha_{\tilde{V}}}}\varepsilon_{\rm HJB}^V\le\frac{\alpha_{\tilde{V}}}{2}\sigma_{\tilde{V}}^2\tilde{A}_{\tilde{V}}^2+\frac{1}{2\alpha_{\tilde{V}}}(\varepsilon_{\rm HJB}^{V,M})^2$, (34) becomes

$$\begin{aligned}\dot{L}_{V,\mathbf{2}}\le&-\left(\sigma_{\tilde{V}}^2-\frac{\alpha_{\tilde{V}}}{2}\sigma_{\tilde{V}}^2\right)\tilde{A}_{\tilde{V}}^2+\frac{1}{2\alpha_{\tilde{V}}}\left(\varepsilon_{\rm HJB}^{V,M}\right)^2\\&+[1-2\Gamma_V\lambda_{\min}(Q_V)]\tilde{V}^2+[1-2\Gamma_V\lambda_{\min}(\vartheta_V)](\Phi^*)^2.\end{aligned} \quad (35)$$

Combining (16) and (33), we have the total Lyapunov function

$$L_V=L_{V,\mathbf{1}}+L_{V,\mathbf{2}}. \quad (36)$$

We further have $\dot{L}_V$

$$\begin{aligned}\dot{L}_V\le&-k_{V1}\tilde{V}^2-\left(\sigma_{\tilde{V}}^2-\alpha_{\tilde{V}}\sigma_{\tilde{V}}^2/2\right)\tilde{A}_{\tilde{V}}^2\\&+[2-2\Gamma_V\lambda_{\min}(Q_V)]\tilde{V}^2\\&+[1.5-2\Gamma_V\lambda_{\min}(\vartheta_V)](\Phi^*)^2+\Psi_V\end{aligned} \quad (37)$$

with $\Psi_V=0.5+k_{V1}\varphi_1^2/l_1+0.5(\varepsilon_1)^2+0.5(\varepsilon_{\rm HJB}^{V,M})^2/\alpha_{\tilde{V}}$.

Let $\Gamma_V>\max\{1/\lambda_{\min}(Q_V),3/4\lambda_{\min}(\vartheta_V)\}$, $k_{V1}>0$, and $\alpha_{\tilde{V}}<2$. Then, (36) becomes

$$\dot{L}_V\le-k_{V1}\tilde{V}^2-\left(\sigma_{\tilde{V}}^2-\alpha_{\tilde{V}}\sigma_{\tilde{V}}^2/2\right)\tilde{A}_{\tilde{V}}^2+\Psi_V. \quad (38)$$

It can be seen that $\tilde{V}$ and $\tilde{A}_{\tilde{V}}$ are convergent, that is, $\tilde{V}\to\Omega_{\tilde{V}}$ and $\tilde{A}_{\tilde{V}}\to\Omega_{\tilde{A}_{\tilde{V}}}$ when $t\to\infty$, where $\Omega_{\tilde{A}_{\tilde{V}}}=\{\tilde{A}_{\tilde{V}}||\tilde{A}_{\tilde{V}}|\le\sqrt{\Psi_V/(\sigma_{\tilde{V}}^2-0.5\alpha_{\tilde{V}}\sigma_{\tilde{V}}^2)}\}$ and $\Omega_{\tilde{V}}=\{\tilde{V}||\tilde{V}|\le\sqrt{\Psi_V/k_{V1}}\}$.

Remark 3: The sustained incentive condition leads to that $\|\partial\varepsilon_{\tilde{V}}/\partial\tilde{V}\|\le\Delta\varepsilon_{\tilde{V},M}$ with a constant $\Delta\varepsilon_{\tilde{V},M}\in\Re^+$. From (22) and (26), we know $|\hat{\Phi}^*-\Phi^*|=|0.5r_V^{-1}(\Lambda_{\tilde{V}}\tilde{A}_{\tilde{V}}-\partial\varepsilon_{\tilde{V}}/\partial\tilde{V})|\le0.5r_V^{-1}[|\Lambda_{\tilde{V}}|(\tilde{A}_{\tilde{V}}^{\rm M})^2+(\Delta\varepsilon_{\tilde{V},M})^2]$, where $\tilde{A}_{\tilde{V}}^{\rm M}$ is the upper bound of $\tilde{A}_{\tilde{V}}$.

B. Altitude Controller Design

In this subsystem, we will develop a fuzzy optimal controller for altitude subsystem (2)–(5) via back-stepping such that $h\to h_{\rm ref}$ and makes cost function (64) minimal.

We define $\gamma_{\rm d}=\arcsin(-k_{h1}\tilde{h}/V-k_{h2}\int_0^t\tilde{h}d\tau/V+\dot{h}_{\rm ref}/V)$, where $\tilde{h}=h-h_{\rm ref}$ is the altitude tracking error, $k_{h1}\in\Re^+$ and $k_{h2}\in\Re^+$ are constants, and $\gamma_{\rm d}$ is the command of γ. Then, we can conclude that $\tilde{h}\to0$ (when $t\to\infty$) if $\gamma\to\gamma_{\rm d}$ since $\dot{\tilde{h}}\tilde{h}=-k_{h1}\tilde{h}^2-k_{h2}\int_0^t\tilde{h}^2d\tau\le0$ and $\dot{\tilde{h}}\tilde{h}=0$ only when $\tilde{h}=0$. The subsequent design goal becomes $\gamma\to\gamma_{\rm d}$.

We formulate (3)–(5) as follows:

$$\dot{x}_\gamma=a_\gamma+x_\theta,\dot{x}_\theta=x_Q,x_Q=a_Q+\delta_e \quad (39)$$

with $a_\gamma=L/(mV)+T\sin(\theta-\gamma)/(mV)-g\cos\gamma/V-\theta$, $a_Q=(M+\tilde{\psi}_1\ddot{\eta}_1+\tilde{\psi}_2\ddot{\eta}_2)/I_{yy}-\delta_e$, $x_\gamma=\gamma$, $x_\theta=\theta$, and $x_Q=Q$.

Similarly, we use the fuzzy system (8) to approximate unknown functions a_γ and a_Q.

$$\begin{cases}a_\gamma=\boldsymbol{\zeta}_\gamma^T\boldsymbol{\varpi}(\boldsymbol{X})+\varepsilon_\gamma,|\varepsilon_\gamma|\le\varepsilon_\gamma^M\\a_Q=\boldsymbol{\zeta}_Q^T\boldsymbol{\varpi}(\boldsymbol{X})+\varepsilon_Q,|\varepsilon_Q|\le\varepsilon_Q^M\end{cases} \quad (40)$$

where $\boldsymbol{\zeta}_\gamma=[\zeta_1^\gamma,\zeta_2^\gamma,\ldots,\zeta_n^\gamma]^{\rm T}\in\Re^n$ and $\boldsymbol{\zeta}_Q=[\zeta_1^Q,\zeta_2^Q,\ldots,\zeta_n^Q]^{\rm T}\in\Re^n$ are weight vectors; ε_γ^M and ε_Q^M are the upper bounds of ε_γ and ε_Q, respectively.

Step 1: Define

$$e_\gamma=x_\gamma-\gamma_{\rm d}. \quad (41)$$

Utilizing (39), $\dot{e}_\gamma$ is given by the following:

$$\dot{e}_\gamma=a_\gamma+x_\theta-\dot{\gamma}_d=a_\gamma+e_\theta+s_\theta+\bar{\theta}_s+\bar{\theta}^*-\dot{\gamma}_d \quad (42)$$

where $\bar{\theta}_s$ is a main virtual controller and $\bar{\theta}^*$ is an optimal virtual controller. e_θ and s_θ will be defined subsequently.

We define $\bar{\theta}_s$ as follows:

$$\bar{\theta}_s = -k_{\gamma 1} e_\gamma - k_{\gamma 2} \int_0^t e_\gamma d\tau - \frac{1}{2} e_\gamma \hat{\varphi}_\gamma \boldsymbol{\varpi}^T(\boldsymbol{X}) \boldsymbol{\varpi}(\boldsymbol{X}) + \dot{\gamma}_d \tag{43}$$

where $k_{\gamma 1} \in \Re^+$ and $k_{\gamma 2} \in \Re^+$ are design constants. $\hat{\varphi}_\gamma$ denotes the estimate of $\varphi_\gamma = ||\boldsymbol{\zeta}_\gamma||^2$. We select the following adaptive law for $\hat{\varphi}_\gamma$ as follows:

$$\dot{\hat{\varphi}}_\gamma = 0.5 l_\gamma e_\gamma^2 \boldsymbol{\varpi}^T(\boldsymbol{X}) \boldsymbol{\varpi}(\boldsymbol{X}) - 2k_{\gamma 1} \hat{\varphi}_\gamma \tag{44}$$

where $l_\gamma \in \Re^+$ is a design parameter.

We define x_θ^d as the estimate of $\bar{\theta}$ and introduce the following filter:

$$\tau_\theta \dot{x}_\theta^d = \bar{\theta} - x_\theta^d, \bar{\theta}(0) = x_\theta^d(0) \tag{45}$$

where $\tau_\theta \in \Re^+$ is a constant.

Step 2: Define

$$e_\theta = x_\theta - x_\theta^d. \tag{46}$$

Based on (39), $\dot{e}_\theta$ is given by the following:

$$\dot{e}_\theta = x_Q - \dot{x}_\theta^d = e_Q + s_Q + \bar{Q}_s + \bar{Q}^* - \dot{x}_\theta^d \tag{47}$$

where e_Q and s_Q will be defined subsequently. $\bar{Q}_s$ is a main virtual controller and $\bar{Q}^*$ is an optimal virtual controller.

Define $\bar{Q}_s$ as follows:

$$\bar{Q}_s = -k_{\theta 1} e_\theta - k_{\theta 2} \int_0^t e_\theta d\tau - e_\gamma + \dot{x}_\theta^d \tag{48}$$

where $k_{\theta 1} \in \Re^+$ and $k_{\theta 2} \in \Re^+$ are chosen constants.

To obtain the time derivative of $\bar{Q}_s$, we give the following filter:

$$\tau_Q \dot{x}_Q^d = \bar{Q} - x_Q^d, \bar{Q}(0) = x_Q^d(0) \tag{49}$$

where x_Q^d is the estimation of $\bar{Q}_s$.

Step 3: Define

$$e_Q = x_Q - x_Q^d. \tag{50}$$

Invoking (39), $\dot{e}_Q$ is formulated as follows:

$$\dot{e}_Q = a_Q + \delta_e - \dot{x}_Q^d = a_Q + \delta_e^s + \delta_e^* - \dot{x}_Q^d \tag{51}$$

where δ_e^s is a main altitude controller and δ_e^* is an optimal altitude controller.

We design δ_e^s as follows:

$$\begin{aligned} \delta_e^s = & -k_{Q1} e_Q - k_{Q2} \int_0^t e_Q d\tau - 0.5 e_Q \hat{\varphi}_Q \boldsymbol{\varpi}^T(\boldsymbol{X}) \boldsymbol{\varpi}(\boldsymbol{X}) \\ & - e_\theta + \dot{x}_Q^d \end{aligned} \tag{52}$$

where $k_{Q1} \in \Re^+$ and $k_{Q2} \in \Re^+$ are constants. $\hat{\varphi}_Q$ is the estimate of $\varphi_Q = ||\boldsymbol{\zeta}_Q||^2$, and its learning law is developed as follows:

$$\dot{\hat{\varphi}}_Q = 0.5 l_Q e_Q^2 \boldsymbol{\varpi}^T(\boldsymbol{X}) \boldsymbol{\varpi}(\boldsymbol{X}) - 2k_{Q1} \hat{\varphi}_Q \tag{53}$$

with the constant $l_Q \in \Re^+$.

Define

$$s_\theta = \theta_d - \bar{\theta}, s_Q = Q_d - \bar{Q}. \tag{54}$$

According to (45), (49), and (54), we get

$$\dot{s}_\theta = -s_\theta/\tau_\theta - \dot{\bar{\theta}}, \dot{s}_Q = -s_Q/\tau_Q - \dot{\bar{Q}}. \tag{55}$$

The previous study [8] proves that there exist constants B_1^M and B_2^M such that $|-\dot{\bar{\theta}}| \leq B_\theta^M$ and $|-\dot{\bar{Q}}| \leq B_Q^M$.

We further define

$$\tilde{\varphi}_\gamma = \hat{\varphi}_\gamma - \varphi_\gamma, \tilde{\varphi}_Q = \hat{\varphi}_Q - \varphi_Q. \tag{56}$$

Select the Lyapunov function as follows:

$$L_{h,1} = L_{h,1}^\gamma + L_{h,1}^\theta + L_{h,1}^Q \tag{57}$$

with

$$L_{h,1}^\gamma = 0.5 e_\gamma^2 + 0.5 k_{\gamma 2} \left(\int_0^t e_\gamma \mathrm{d}\tau \right)^2 + \frac{\tilde{\varphi}_\gamma^2}{2 l_\gamma} + 0.5 s_\theta^2$$

$$L_{h,1}^\theta = 0.5 e_\theta^2 + 0.5 k_{\theta 2} \left(\int_0^t e_\theta \mathrm{d}\tau \right)^2 + 0.5 s_Q^2$$

$$L_{h,1}^Q = 0.5 e_Q^2 + 0.5 k_{Q2} \left(\int_0^t e_Q \mathrm{d}\tau \right)^2 + \frac{\tilde{\varphi}_Q^2}{2 l_Q}.$$

Using (42)–(44), (47), (48), and (50)–(56), we have

$$\begin{aligned} \dot{L}_{h,1}^\gamma = & -k_{\gamma 1} e_\gamma^2 + e_\gamma \boldsymbol{\zeta}_\gamma^T \boldsymbol{\varpi}(\boldsymbol{X}) + e_\gamma \varepsilon_\gamma + e_\gamma e_\theta \\ & + e_\gamma s_\theta - 0.5 e_\gamma^2 \hat{\varphi}_\gamma \boldsymbol{\varpi}^T(\boldsymbol{X}) \boldsymbol{\varpi}(\boldsymbol{X}) + e_\gamma \bar{\theta}^* \\ & + 0.5 e_\gamma^2 \tilde{\varphi}_\gamma \boldsymbol{\varpi}^T(\boldsymbol{X}) \boldsymbol{\varpi}(\boldsymbol{X}) - 2k_{\gamma 1} \tilde{\varphi}_\gamma \hat{\varphi}_\gamma / l_\gamma \\ & - s_\theta^2/\tau_\theta - s_\theta \dot{\bar{\theta}} \end{aligned} \tag{58}$$

$$\begin{aligned} \dot{L}_{h,1}^\theta = & -k_{\theta 1} e_\theta^2 + e_\theta e_Q + e_\theta s_Q - e_\theta e_\gamma + e_\theta \bar{Q}^* - s_Q^2/\tau_Q \\ & - s_Q \dot{\bar{Q}} \end{aligned} \tag{59}$$

$$\begin{aligned} \dot{L}_{h,1}^Q = & e_Q \boldsymbol{\zeta}_Q^T \boldsymbol{\varpi}(\boldsymbol{X}) + e_Q \varepsilon_Q - k_{Q1} e_Q^2 - 0.5 e_Q^2 \hat{\varphi}_Q \boldsymbol{\varpi}^T(\boldsymbol{X}) \\ & \times \boldsymbol{\varpi}(\boldsymbol{X}) - e_Q e_\theta + e_Q \delta_e^* + 0.5 \tilde{\varphi}_Q e_Q^2 \boldsymbol{\varpi}^T(\boldsymbol{X}) \boldsymbol{\varpi}(\boldsymbol{X}) \\ & - 2k_{Q1} \tilde{\varphi}_Q \hat{\varphi}_Q / l_Q. \end{aligned} \tag{60}$$

Based on (58)–(60), we further obtain

$$\begin{aligned} \dot{L}_{h,1} = & -k_{\gamma 1} e_\gamma^2 - k_{\theta 1} e_\theta^2 - k_{Q1} e_Q^2 - s_\theta^2/\tau_\theta - s_Q^2/\tau_Q \\ & + e_\gamma \boldsymbol{\zeta}_\gamma^T \boldsymbol{\varpi}(\boldsymbol{X}) + 0.5 e_\gamma^2 \tilde{\varphi}_\gamma \boldsymbol{\varpi}^T(\boldsymbol{X}) \boldsymbol{\varpi}(\boldsymbol{X}) \\ & - 0.5 e_\gamma^2 \hat{\varphi}_\gamma \boldsymbol{\varpi}^T(\boldsymbol{X}) \boldsymbol{\varpi}(\boldsymbol{X}) + e_\gamma \varepsilon_\gamma + e_Q \varepsilon_Q + e_\gamma s_\theta \\ & + e_\theta s_Q - s_\theta \dot{\bar{\theta}} - s_Q \dot{\bar{Q}} - 2k_{\gamma 1} \tilde{\varphi}_\gamma \hat{\varphi}_\gamma / l_\gamma - 2k_{Q1} \tilde{\varphi}_Q \hat{\varphi}_Q / l_Q \\ & + e_Q \boldsymbol{\zeta}_Q^T \boldsymbol{\varpi}(\boldsymbol{X}) - 0.5 e_Q^2 \hat{\varphi}_Q \boldsymbol{\varpi}^T(\boldsymbol{X}) \boldsymbol{\varpi}(\boldsymbol{X}) \\ & + 0.5 \tilde{\varphi}_Q e_Q^2 \boldsymbol{\varpi}^T(\boldsymbol{X}) \boldsymbol{\varpi}(\boldsymbol{X}) + e_\gamma \bar{\theta}^* + e_\theta \bar{Q}^* + e_Q \delta_e^*. \end{aligned} \tag{61}$$

Since $e_\gamma \boldsymbol{\zeta}_\gamma^T \boldsymbol{\varpi}(\boldsymbol{X}) \leq 0.5 e_\gamma^2 \varphi_\gamma \boldsymbol{\varpi}^T(\boldsymbol{X}) \boldsymbol{\varpi}(\boldsymbol{X}) + 0.5$, $e_Q \boldsymbol{\zeta}_Q^T \boldsymbol{\varpi}(\boldsymbol{X}) \leq 0.5 e_Q^2 \varphi_Q \boldsymbol{\varpi}^T(\boldsymbol{X}) \boldsymbol{\varpi}(\boldsymbol{X}) + 0.5$, $2k_{\gamma 1} \tilde{\varphi}_\gamma \hat{\varphi}_\gamma \geq -k_{\gamma 1} \varphi_\gamma^2$, $2k_{Q1} \tilde{\varphi}_Q \hat{\varphi}_Q \geq -k_{Q1} \varphi_Q^2$, $e_\gamma \varepsilon_\gamma \leq 0.5 \varepsilon_\gamma^M e_\gamma^2 + 0.5 \varepsilon_\gamma^M$, $e_Q \varepsilon_Q \leq 0.5 \varepsilon_Q^M e_Q^2 + 0.5 \varepsilon_Q^M$, $e_\gamma s_\theta \leq 0.5 e_\gamma^2 + 0.5 s_\theta^2$, $e_\theta s_Q \leq 0.5 e_\theta^2 + 0.5 s_Q^2$, $-s_\theta \dot{\bar{\theta}} \leq 0.5 B_\theta^M s_\theta^2 + 0.5 B_\theta^M$, and $-s_Q \dot{\bar{Q}} \leq$

$0.5B_Q^M s_Q^2 + 0.5B_Q^M$, (61) becomes

$$\dot{L}_{h,1} \leq -\left(k_{\gamma 1} - 0.5\varepsilon_\gamma^M - 0.5\right) e_\gamma^2 - \left(k_{\theta 1} - 0.5\right) e_\theta^2 - \left(k_{Q1} - 0.5\varepsilon_Q^M\right) e_Q^2 - \left(\frac{1}{\tau_\theta} - 0.5 - 0.5B_\theta^M\right) s_\theta^2 - \left(\frac{1}{\tau_Q} - 0.5 - 0.5B_Q^M\right) s_Q^2 + \Xi_h + \mathbf{Z}_h^T \boldsymbol{U}_h^* \quad (62)$$

with $\Xi_h = 0.5\varepsilon_\gamma^M + 0.5\varepsilon_Q^M + 0.5B_\theta^M + 0.5B_Q^M + k_{\gamma 1}\varphi_\gamma^2/l_\gamma + k_{Q1}\varphi_Q^2/l_Q$, $\mathbf{Z}_h = [e_\gamma, e_\theta, e_Q]^T$, and $\boldsymbol{U}_h^* = [\bar{\theta}^*, \bar{Q}^*, \delta_e^*]^T$.

In what follows, we will devise an optimal controller to handle the last term in (62).

$$\dot{\mathbf{Z}}_h = \boldsymbol{U}_h^*. \quad (63)$$

The cost function is given by the following:

$$J_h = \int_0^\infty \left[Q_h(\mathbf{Z}_h) + \vartheta_V(\boldsymbol{U}_h^*)\right] dt \quad (64)$$

with $Q_h(\mathbf{Z}_h) = \mathbf{Z}_h^T \mathbf{q}_h \mathbf{Z}_h$ and $\vartheta_V(\boldsymbol{U}_h^*) = (\boldsymbol{U}_h^*)^T \boldsymbol{r}_h \boldsymbol{U}_h^*$, where $\mathbf{q}_h^T = \mathbf{q}_h$ and $\boldsymbol{r}_h^T = \boldsymbol{r}_h$ are positive definite.

We obtain the HJB equation as follows:

$$H_{\mathbf{Z}_h}(\mathbf{Z}_h, \boldsymbol{U}_h^*, J_h) = Q_h(\mathbf{Z}_h) + \vartheta_V(\boldsymbol{U}_h^*) + \nabla J_h \boldsymbol{U}_h^* \quad (65)$$

with $\nabla J_h = \partial J_h / \partial \mathbf{Z}_h$.

The optimal control law is as follows:

$$\boldsymbol{U}_h^* = -0.5\boldsymbol{r}_h^{-1} \nabla J_h. \quad (66)$$

To deal with the unknown term ∇J_h, we apply FHM to approximate the critic network

$$J_h = \mathbf{A}_{\mathbf{Z}_h}^T \tanh(\mathbf{K}_{\mathbf{Z}_h} \mathbf{Z}_h) + \varepsilon_{\mathbf{Z}_h} \quad (67)$$

where $\mathbf{A}_{\mathbf{Z}_h}$ is a matrix with unknown elements and $\varepsilon_{\mathbf{Z}_h}$ is the estimate error.

It is derived from (67) that

$$\nabla \boldsymbol{J}_h = \boldsymbol{\Lambda}_{\mathbf{Z}_h}^T \boldsymbol{A}_{\mathbf{Z}_h} + \partial \varepsilon_{\mathbf{Z}_h} / \partial \mathbf{Z}_h \quad (68)$$

with $\boldsymbol{\Lambda}_{\mathbf{Z}_h} = \partial \tanh(\mathbf{K}_{\mathbf{Z}_h} \mathbf{Z}_h) / \partial \mathbf{Z}_h$.

Then, the HJB equation (65) becomes

$$H_{\mathbf{Z}_h}(\mathbf{Z}_h, \boldsymbol{U}_h^*, J_h) = \mathbf{A}_{\mathbf{Z}_h}^T \boldsymbol{\sigma}_{\mathbf{Z}_h} + Q_h(\mathbf{Z}_h) + \vartheta_V(\boldsymbol{U}_h^*) - \varepsilon_{\mathrm{HJB}}^h \quad (69)$$

with $\boldsymbol{\sigma}_{\mathbf{Z}_h} = \boldsymbol{\Lambda}_{\mathbf{Z}_h}^T \boldsymbol{U}_h^*$, $\varepsilon_{\mathrm{HJB}}^h = (\partial \boldsymbol{\varepsilon}_{\mathbf{Z}_h} / \partial \mathbf{Z}_h)^T \boldsymbol{U}_h^*$ and $|\varepsilon_{\mathrm{HJB}}^h| \leq \varepsilon_{\mathrm{HJB}}^{h,M}$, where $\varepsilon_{\mathrm{HJB}}^{h,M}$ is a positive constant.

Equation (66) becomes

$$\boldsymbol{U}_h^* = -0.5\left(\mathbf{r}_h^{-1}\right)^T \left(\boldsymbol{\Lambda}_{\mathbf{Z}_h}^T \boldsymbol{A}_{\mathbf{Z}_h} + \partial \boldsymbol{\varepsilon}_{\mathbf{Z}_h} / \partial \mathbf{Z}_h\right). \quad (70)$$

Define $\hat{\boldsymbol{A}}_{\mathbf{Z}_h}$ as the estimation of $\boldsymbol{A}_{\mathbf{Z}_h}$, and we obtain

$$\hat{\boldsymbol{J}}_h = \hat{\boldsymbol{A}}_{\mathbf{Z}_h}^T \tanh(\mathbf{K}_{\mathbf{Z}_h} \mathbf{Z}_h) \quad (71)$$

$$\nabla \hat{\mathbf{J}}_h = \boldsymbol{\Lambda}_{\mathbf{Z}_h}^T \hat{\boldsymbol{A}}_{\mathbf{Z}_h}. \quad (72)$$

Finally, the optimal altitude controller is given by the following:

$$\hat{\boldsymbol{U}}_h^* = -0.5\left(\mathbf{r}_h^{-1}\right)^T \left[\mathbf{K}_{\mathbf{Z}_h} - \mathbf{K}_{\mathbf{Z}_h} \tan h^2(\mathbf{K}_{\mathbf{Z}_h} \mathbf{Z}_h)\right] \hat{\mathbf{A}}_{\mathbf{Z}_h}. \quad (73)$$

Substituting (72) into (65) yields

$$e_{\mathbf{Z}_h} \triangleq H_{\mathbf{Z}_h}(\mathbf{Z}_h, \boldsymbol{U}_h^*, \hat{J}_h) = \Sigma_{\mathbf{Z}_h} + Q_h(\mathbf{Z}_h) + \vartheta_V(\boldsymbol{U}_h^*) \quad (74)$$

with $\Sigma_{\mathbf{Z}_h} = (\boldsymbol{\Lambda}_{\mathbf{Z}_h}^T \boldsymbol{A}_{\mathbf{Z}_h})^T \boldsymbol{U}_h^*$.

To minimize $E_{\mathbf{Z}_h} = 0.5\mathbf{Z}_h^T \mathbf{Z}_h$, we devise the following adaptive law for $\hat{\boldsymbol{A}}_{\mathbf{Z}_h}$:

$$\dot{\hat{\mathbf{A}}}_{\mathbf{Z}_h} = -\alpha_{\mathbf{Z}_h} \frac{\partial E_{\mathbf{Z}_h}}{\partial \hat{\mathbf{A}}_{\mathbf{Z}_h}} = -\alpha_{\mathbf{Z}_h} \boldsymbol{\sigma}_{\mathbf{Z}_h} \left[\boldsymbol{\sigma}_{\mathbf{Z}_h}^T \hat{\mathbf{A}}_{\mathbf{Z}_h} + Q_h(\mathbf{Z}_h) + \vartheta_V(\boldsymbol{U}_h^*)\right] \quad (75)$$

with $\boldsymbol{\sigma}_{\mathbf{Z}_h} = \Lambda_{\tilde{V}} \Phi^* = [K_{\tilde{V}} - K_{\tilde{V}} \tan h^2(K_{\tilde{V}} \tilde{V})] \boldsymbol{U}_h^*$, $\alpha_{\mathbf{Z}_h} \in \Re^+$, and $\Lambda_{\tilde{V}} = \mathbf{K}_{\mathbf{Z}_h} - \mathbf{K}_{\mathbf{Z}_h} \tan h^2(\mathbf{K}_{\mathbf{Z}_h} \mathbf{Z}_h)$.

Defining $\tilde{\mathbf{A}}_{\mathbf{Z}_h} = \hat{\mathbf{A}}_{\mathbf{Z}_h} - \mathbf{A}_{\mathbf{Z}_h}$ and using (69), we have

$$\dot{\tilde{\mathbf{A}}}_{\mathbf{Z}_h} = -\alpha_{\mathbf{Z}_h} \sigma_{\mathbf{Z}_h} \left[\boldsymbol{\sigma}_{\mathbf{Z}_h}^T \hat{\mathbf{A}}_{\mathbf{Z}_h} + Q_h(\mathbf{Z}_h) + \vartheta_V(\boldsymbol{U}_h^*)\right] = -\alpha_{\mathbf{Z}_h} \sigma_{\mathbf{Z}_h} \left(\sigma_{\mathbf{Z}_h}^T \tilde{\mathbf{A}}_{\mathbf{Z}_h} + \varepsilon_{\mathrm{HJB}}^h\right). \quad (76)$$

Define the Lyapunov function as follows:

$$L_{h,\mathbf{2}} = tr\left(\tilde{\mathbf{A}}_{\mathbf{Z}_h}^T \tilde{\mathbf{A}}_{\mathbf{Z}_h}\right) / (2\alpha_{\mathbf{Z}_h}) + \mathbf{Z}_h^T \mathbf{Z}_h + 2\Gamma_h J_h. \quad (77)$$

$\dot{L}_{h,\mathbf{2}}$ is given by the following:

$$\dot{L}_{h,\mathbf{2}} = -\tilde{\mathbf{A}}_{\mathbf{Z}_h}^T \sigma_{\mathbf{Z}_h} \sigma_{\mathbf{Z}_h}^T \tilde{\mathbf{A}}_{\mathbf{Z}_h} - 2\frac{\alpha_{\mathbf{Z}_h}}{\sqrt{2\alpha_{\mathbf{Z}_h}}} \sigma_{\mathbf{Z}_h}^T \tilde{\mathbf{A}}_{\mathbf{Z}_h} \frac{1}{\sqrt{2\alpha_{\mathbf{Z}_h}}} \varepsilon_{\mathrm{HJB}}^h + 2\mathbf{Z}_h^T \boldsymbol{U}_h^* + 2\Gamma_h \left[-Q_h(\mathbf{Z}_h) - \vartheta_V(\boldsymbol{U}_h^*)\right]. \quad (78)$$

Because $-2\frac{\alpha_{\mathbf{Z}_h}}{\sqrt{2\alpha_{\mathbf{Z}_h}}} \sigma_{\mathbf{Z}_h}^T \tilde{\mathbf{A}}_{\mathbf{Z}_h} \frac{1}{\sqrt{2\alpha_{\mathbf{Z}_h}}} \varepsilon_{\mathrm{HJB}}^h \leq \frac{\alpha_{\mathbf{Z}_h}}{2} \|\sigma_{\mathbf{Z}_h}^T \tilde{\mathbf{A}}_{\mathbf{Z}_h}\|^2 + \frac{1}{2\alpha_{\mathbf{Z}_h}} (\varepsilon_{\mathrm{HJB}}^{h,M})^2$, (78) becomes

$$\dot{L}_{h,\mathbf{2}} \leq -\left(\|\sigma_{\mathbf{Z}_h}\|^2 - \alpha_{\mathbf{Z}_h} \|\sigma_{\mathbf{Z}_h}\|^2 / 2\right) \left\|\tilde{\mathbf{A}}_{\mathbf{Z}_h}\right\|^2 + \left(\varepsilon_{\mathrm{HJB}}^{h,M}\right)^2 / (2\alpha_{\mathbf{Z}_h}) + \left[1 - 2\Gamma_h \lambda_{\min}(Q_h)\right] \|\mathbf{Z}_h\|^2 + \left[1 - 2\Gamma_h \lambda_{\min}(\vartheta_h)\right] \|\boldsymbol{U}_h^*\|^2. \quad (79)$$

The total Lyapunov function is formulated as follows:

$$L_h = L_{h,\mathbf{1}} + L_{h,\mathbf{2}}. \quad (80)$$

The time derivative of L_h is described as follows:

$$\dot{L}_h \leq -\left(k_{\gamma 1} - \frac{1}{2}\varepsilon_\gamma^M - \frac{1}{2}\right) e_\gamma^2 - \left(k_{\theta 1} - \frac{1}{2}\right) e_\theta^2 - \left(k_{Q1} - \frac{1}{2}\varepsilon_Q^M\right) e_Q^2 - \left(\frac{1}{\tau_\theta} - \frac{1}{2} - \frac{B_\theta^M}{2}\right) s_\theta^2 - \left(\frac{1}{\tau_Q} - \frac{1}{2} - \frac{B_Q^M}{2}\right) s_Q^2 + \Psi_h + \left[2 - 2\Gamma_h \lambda_{\min}(Q_h)\right] \|\mathbf{Z}_h\|^2 + \left[\frac{3}{2} - 2\Gamma_h \lambda_{\min}(\vartheta_h)\right] \|\boldsymbol{U}_h^*\|^2 - \left(\|\sigma_{\mathbf{Z}_h}\|^2 - \frac{\alpha_{\mathbf{Z}_h}}{2} \|\sigma_{\mathbf{Z}_h}\|^2\right) \left\|\tilde{\mathbf{A}}_{\mathbf{Z}_h}\right\|^2 \quad (81)$$

with $\Psi_h = \Xi_h + (\varepsilon_{\mathrm{HJB}}^{h,M})^2 / (2\alpha_{\mathbf{Z}_h})$.

Let $k_{h1} > 0$, $\alpha_{\mathbf{Z}_h} < 2$, and $\Gamma_h > \max\{1/\lambda_{\min}(Q_h), 3/(4\lambda_{\min}(\vartheta_h))\}$. Then, (81) becomes

$$\dot{L}_h \leq -\left(k_{\gamma 1} - 0.5\varepsilon_\gamma^M - 0.5\right) e_\gamma^2 - \left(k_{\theta 1} - 0.5\right) e_\theta^2$$

$$-\left(k_{Q1}-0.5\varepsilon_Q^M\right)e_Q^2-\left(1/\tau_\theta-0.5-0.5B_\theta^M\right)s_\theta^2+\Psi_h$$
$$-\left(1/\tau_Q-0.5-0.5B_Q^M\right)s_Q^2$$
$$-\left(\|\sigma_{\mathbf{Z}_h}\|^2-0.5\alpha_{\mathbf{Z}_h}\|\sigma_{\mathbf{Z}_h}\|^2\right)\left\|\tilde{\mathbf{A}}_{\mathbf{Z}_h}\right\|^2. \quad (82)$$

We conclude that $e_\gamma \to \Omega_{e_\gamma}$, $e_\theta \to \Omega_{e_\theta}$, $e_Q \to \Omega_{e_Q}$, $s_\theta \to \Omega_{s_\theta}$, $s_Q \to \Omega_{s_Q}$, and $\tilde{\mathbf{A}}_{\mathbf{Z}_h} \to \Omega_{\tilde{\mathbf{A}}_{\mathbf{Z}_h}}$ when $t \to \infty$, where

$$\Omega_{e_\gamma}=\{e_\gamma||e_\gamma|\le\sqrt{\Psi_h/(k_{\gamma1}-0.5\varepsilon_\gamma^M-0.5)}\}$$
$$\Omega_{e_\theta}=\{e_\theta||e_\theta|\le\sqrt{\Psi_h/(k_{\theta1}-0.5)}\}$$
$$\Omega_{e_Q}=\{e_Q||e_Q|\le\sqrt{\Psi_h/(k_{Q1}-0.5\varepsilon_Q^M)}\}$$
$$\Omega_{s_\theta}=\{s_\theta||s_\theta|\le\sqrt{\Psi_h/(1/\tau_\theta-0.5-0.5B_\theta^M)}\}$$
$$\Omega_{s_Q}=\{s_Q||s_Q|\le\sqrt{\Psi_h/(1/\tau_Q-0.5-0.5B_Q^M)}\}$$
$$\Omega_{\tilde{\mathbf{A}}_{\mathbf{Z}_h}}=\{\tilde{\mathbf{A}}_{\mathbf{Z}_h}|\|\tilde{\mathbf{A}}_{\mathbf{Z}_h}\|\le\sqrt{\Psi_h/(\|\sigma_{\mathbf{Z}_h}\|^2-0.5\alpha_{\mathbf{Z}_h}\|\sigma_{\mathbf{Z}_h}\|^2)}\}.$$

Remark 4: The sustained incentive condition guarantees that $\|\partial\varepsilon_{\mathbf{Z}_h}/\partial\mathbf{Z}_h\|\le\Delta\varepsilon_{\mathbf{Z}_h,M}$ with the constant $\Delta\varepsilon_{\mathbf{Z}_h,M}\in\Re^+$. Based on (70) and (73), we have $\|\hat{\boldsymbol{U}}_h^*-\boldsymbol{U}_h^*\|=\|-0.5(\mathbf{r}_h^{-1})^T(\boldsymbol{\Lambda}_{\mathbf{Z}_h}^T\tilde{\boldsymbol{A}}_{\mathbf{Z}_h}-\partial\boldsymbol{\varepsilon}_{\mathbf{Z}_h}/\partial\mathbf{Z}_h)\|\le 0.5\|(\mathbf{r}_h^{-1})\|[\|\boldsymbol{\Lambda}_{\mathbf{Z}_h}^T\|(\tilde{A}_{\mathbf{Z}_h}^{\mathrm{M}})^2+(\Delta\varepsilon_{\mathbf{Z}_h,M})^2]$, where $\tilde{A}_{\mathbf{Z}_h}^{\mathrm{M}}$ is the upper bound of $||\tilde{\mathbf{A}}_{\mathbf{Z}_h}||$.

Remark 5: In (11) and (40), each fuzzy approximation only contains one online learning parameter. Undoubtedly, the computational load is low. Moreover, the developed critic network also has only one adaptive parameter. This guarantees satisfactory real-time performance.

Remark 6: Traditionally, to ensure that the cost function is finite, the steady-state value of control input must be zero [30]–[32]. Unfortunately, most of actual systems do not satisfy this condition. In this article, we decompose the controller into two parts, namely, the main controller and the optimal controller. Only the optimal controller ultimately converges to zero, while the total control input converges to a constant instead of zero.

Remark 7: In this article, the vehicle dynamics is assumed to be unknown and it is approximated by a fuzzy system. Hence, the addressed method has the possibility of extending it to time-varying dynamic systems since the unknown vehicle dynamics can be time-varying or time-invariant.

IV. Simulation Study

The proposed controller is compared with a neural-approximation-based back-stepping control (NBC) [38] to validate its effectiveness and superiority. The values of model coefficients and parameters adopted in the simulation are referenced from [35]. The initial trim condition for HFVs is chosen as: $V = 7700$ ft/s, $h = 85\ 000$ ft, $\gamma = 0°$, $\theta = 1.62°$, $Q = 0°/\mathrm{s}^2$, $\eta_1 = 0.97$ and $\eta_2 = 0.80$. Design parameters are chosen as: $k_{V1} = 10$, $k_{V2} = 5.5$, $l_V = 0.05$, $q_V = 5$, $r_V = 1$, $K_{\tilde{V}} = 0.2$, $\alpha_{\tilde{V}} = 0.5$, $k_{h1} = 35$, $k_{h2} = 1.5$, $k_{\gamma1} = 1.9$, $k_{\gamma2} = 0.2$, $l_\gamma = 0.05$, $\tau_\theta = 0.2$, $k_{\theta1} = 25$, $k_{\theta2} = 1.2$, $\tau_Q = 0.2$, $k_{Q1} = 45$, $k_{Q2} = 15$, $l_Q = 0.05$, $\mathbf{K}_{\mathbf{Z}_h} = [0.2, 0.2, 0.2]$, $\mathbf{q}_h = \mathrm{diag}\{5, 5, 5\}$ and $\boldsymbol{r}_h = \mathrm{diag}\{1, 1, 1\}$.

The obtained simulation results are presented in Figs. 1–8. Velocity tracking performance and altitude tracking performance, depicted in Figs. 1 and 2, reveal that the proposed controller can provide better tracking of reference commands in comparison with NBC (see Table I). Especially, the responses of $\int_0^t|\tilde{V}|d\tau$

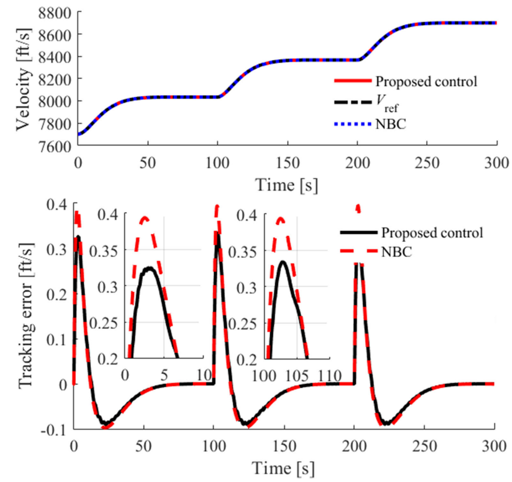

Fig. 1. Velocity tracking performance.

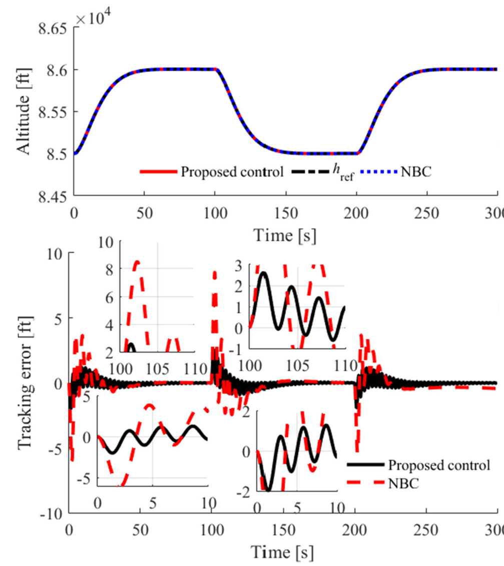

Fig. 2. Altitude tracking performance.

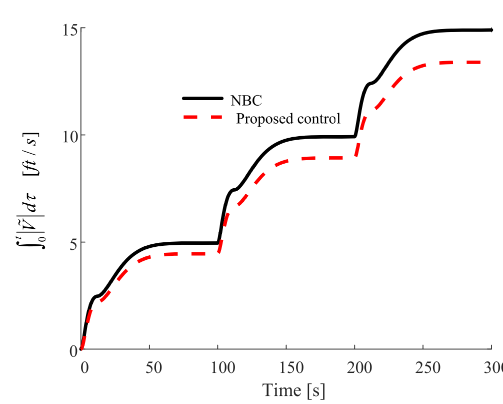

Fig. 3. Response of $\int_0^t|\tilde{V}|d\tau$.

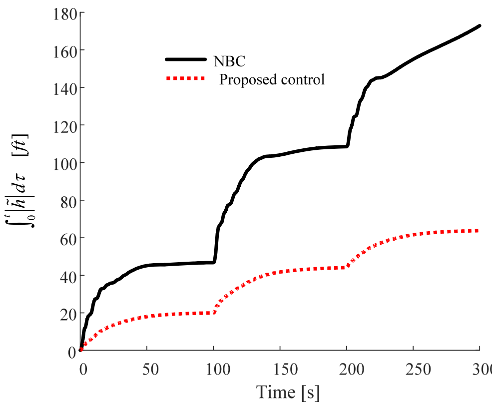

Fig. 4. Response of $\int_0^t |\tilde{h}| d\tau$.

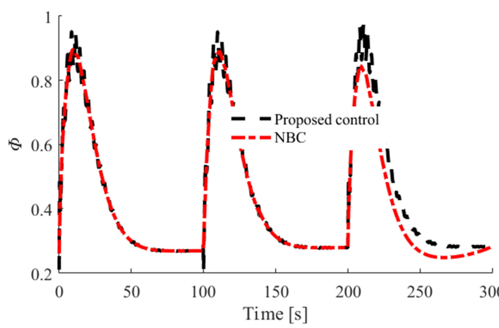

Fig. 5. Response of Φ.

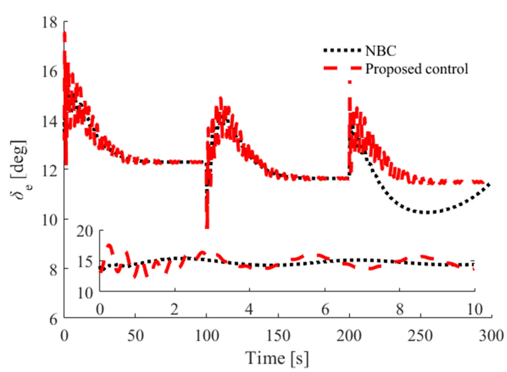

Fig. 6. Response of δ_e.

TABLE I
COMPARISON OF TRACKING ERRORS

Tracking error	NBC	Proposed method
$\tilde{V}$	$-1\,ft/s < \tilde{V} < 0.4\,ft/s$	$-1\,ft/s < \tilde{V} < 0.35\,ft/s$
$\tilde{h}$	$-6\,ft < \tilde{h} < 10\,ft$	$-2\,ft < \tilde{h} < 3\,ft$

and $\int_0^t |\tilde{h}| d\tau$ obviously show that the developed control approach's tracking precision is higher compared with NBC (see Figs. 3 and 4). Moreover, it can be seen from Figs. 5 and 6 that the control inputs of both methods are smooth and their values are reasonable. Finally, all the critic weights are convergent, as shown in Figs. 7 and 8.

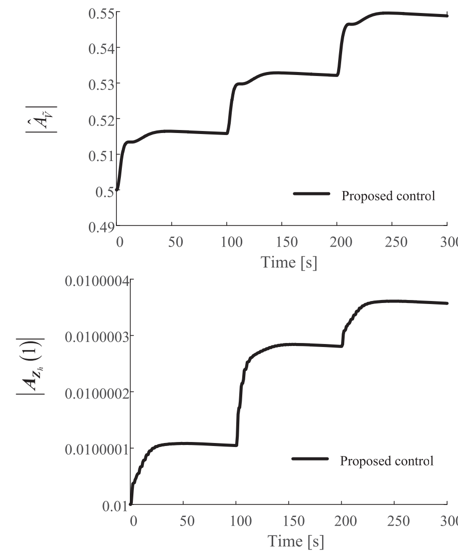

Fig. 7. Responses of $|\hat{A}_{\tilde{V}}|$ and $|\hat{\mathbf{A}}_{\mathbf{Z}_h}(1)|$.

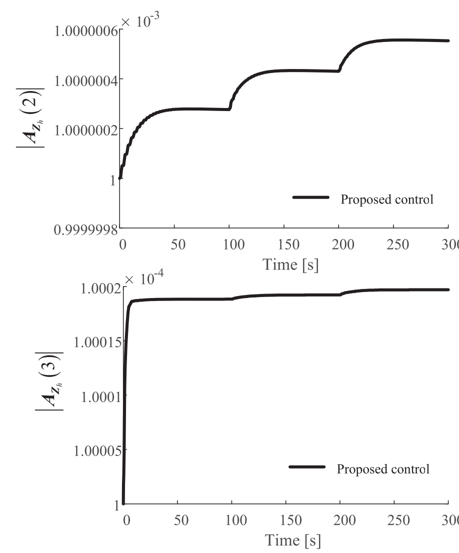

Fig. 8. Responses of $|\hat{\mathbf{A}}_{\mathbf{Z}_h}(2)|$ and $|\hat{\mathbf{A}}_{\mathbf{Z}_h}(3)|$.

V. CONCLUSION

This article investigated an optimal tracking control problem of HFVs subject to unknown dynamics. The vehicle dynamics consisted of the velocity subsystem and the altitude subsystem. Fuzzy approximations were applied to develop a robust tracking controller for the velocity subsystem via SACD. Furthermore, a back-stepping-based nearly optimal controller was devised for altitude dynamics. Advanced regulation algorithms were exploited for fuzzy approximations to construct a low-complexity control framework. Finally, the effectiveness and advantage of the addressed method were verified by simulation results.

REFERENCES

[1] J. Liu, H. An, Y. Gao, C. Wang, and L. Wu, "Adaptive control of hypersonic flight vehicles with limited angle-of-attack," *IEEE/ASME Trans. Mechatronics*, vol. 23, no. 2, pp. 883–894, Apr. 2018.

[2] J. Sun, C. Li, Y. Guo, C. Wang, and P. Li, "Adaptive fault tolerant control for hypersonic vehicle with input saturation and state constraints," *Acta Astronautica*, vol. 167, pp. 302–313, Feb. 2020.

[3] X. Bu, Y. Xiao, and H. Lei, "An adaptive critic design-based fuzzy neural controller for hypersonic vehicles: Predefined behavioral nonaffine control," *IEEE/ASME Trans. Mechatronics*, vol. 24, no. 4, pp. 1871–1881, Aug. 2019.

[4] J. Sun, Z. Pu, J. Yi, and Z. Liu, "Fixed-time control with uncertainty and measurement noise suppression for hypersonic vehicles via augmented sliding mode observers," *IEEE Trans. Ind. Informat.*, vol. 16, no. 2, pp. 1192–1203, Oct. 2019.

[5] Z. Guo, J. Guo, J. Zhou, and J. Chang, "Robust tracking for hypersonic reentry vehicles via disturbance estimation-triggered control," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 56, no. 2, pp. 1279–1289, Apr. 2020.

[6] X. Yu, P. Li, and Y. Zhang, "The design of fixed-time observer and finite-time fault-tolerant control for hypersonic gliding vehicles," *IEEE Trans. Ind. Electron.*, vol. 65, no. 5, pp. 4135–4144, May 2018.

[7] X. Shao and H. Wang, "Active disturbance rejection based trajectory linearization control for hypersonic reentry vehicle with bounded uncertainties," *ISA Trans.*, vol. 54, pp. 27–38, 2015.

[8] X. Bu, X. Wu, R. Zhang, and J. Huang, "Tracking differentiator design for the robust backstepping control of a flexible air-breathing hypersonic vehicle," *J. Franklin Inst.*, vol. 352, no. 4, pp. 1739–1765, 2015.

[9] X. Bu, "Air-breathing hypersonic vehicles funnel control using neural approximation of non-affine dynamics," *IEEE/ASME Trans. Mechatronics*, vol. 23, no. 5, pp. 2099–2108, Oct. 2018.

[10] Y. Ma and Y. Cai, "A fuzzy model predictive control based upon adaptive neural network disturbance observer for a constrained hypersonic vehicle," *IEEE Access*, vol. 6, pp. 5927–5938, Dec. 2017.

[11] X. Tang, D. Zhai, and X. Li, "Adaptive fault-tolerance control based finite-time backstepping for hypersonic flight vehicle with full state constrains," *Inf. Sci.*, vol. 507, pp. 53–66, Jan. 2020.

[12] J. Zhang, C. Sun, R. Zhang, and C. Qian, "Adaptive sliding mode control for re-entry attitude of near space hypersonic vehicle based on backstepping design," *IEEE/CAA J. Automatica Sinica*, vol. 2, no. 1, pp. 94–101, Jan. 2015.

[13] C. Liu, C. Dong, Z. Zhou, and Z. Wang, "Barrier Lyapunov function based reinforcement learning control for air-breathing hypersonic vehicle with variable geometry inlet," *Aerosp. Sci. Technol.*, vol. 96, Jan. 2020, Art. no. 105537.

[14] Y. Yu, H. Wang, N. Li, H. Zhang, Z. Su, and X. Shao, "Finite-time model-assisted active disturbance rejection control with a novel parameters optimizer for hypersonic reentry subject to multiple disturbances," *Aerosp. Sci. Technol.*, vol. 79, pp. 588–600, 2018.

[15] Y. Meng, B. Jiang, R. Qi, and J. Liu, "Fault-tolerant anti-windup control for hypersonic vehicle in reentry based on ISMDO," *J. Franklin Inst.*, vol. 355, pp. 2067–2090, 2018.

[16] X. Bu, X. Wu, M. Tian, J. Huang, R. Zhang, and Z. Ma, "High-order tracking differentiator based adaptive neural control of a flexible air-breathing hypersonic vehicle subject to actuators constraints," *ISA Trans.*, vol. 58, pp. 237–247, 2015.

[17] X. Bu, X. Wu, D. Wei, and J. Huang, "Neural-approximation-based robust adaptive control of flexible air-breathing hypersonic vehicles with parametric uncertainties and control input constraints," *Inf. Sci.*, vol. 346, pp. 29–43, 2016.

[18] H. Gao, J. Zhang, and W. Tang, "Offset-free trajectory tracking control for hypersonic vehicle under external disturbance and parametric uncertainty," *J. Franklin Inst.*, vol. 355, pp. 997–1017, 2018.

[19] Q. Zhou, H. Li, C. Wu, L. Wang, and C. K. Ahn, "Adaptive fuzzy control of nonlinear systems with unmodeled dynamics and input saturation using small-gain approach," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 47, no. 8, pp. 1979–1989, Aug. 2017.

[20] N. A. Sofianos and Y. S. Boutalis, "Stable indirect adaptive switching control for fuzzy dynamical systems based on T-S multiple models," *Int. J. Syst. Sci.*, vol. 44, no. 8, pp. 1546–1565, 2013.

[21] N. A. Sofianos and Y. S. Boutalis, "Robust adaptive multiple models based fuzzy control of nonlinear systems," *Neurocomputing*, vol. 173, no. 3, pp. 1733–1742, Jan. 2016.

[22] X. Bu, Y. Xiao, and K. Wang, "A prescribed performance control approach guaranteeing small overshoot for air-breathing hypersonic vehicles via neural approximation," *Aerosp. Sci. Technol.*, vol. 71, pp. 485–498, 2017.

[23] L. Hu, R. Li, T. Xue, and Y. Liu, "Neuro-adaptive tracking control of a hypersonic flight vehicle with uncertainties using reinforcement synthesis," *Neurocomputing*, vol. 285, pp. 141–153, 2018.

[24] X. Long, L. Lv, P. Liu, X. Liu, and G. Huang, "A novel adaptive gauss pseudospectral method for nonlinear optimal control of constrained hypersonic re-entry vehicle problem," *Int. J. Adaptive Control Signal Process.*, vol. 32, pp. 1243–1258, 2018.

[25] R. Xia, M. Chen, Q. Wu, and Y. Wang, "Neural network based integral sliding mode optimal flight control of near space hypersonic vehicle," *Neurocomputing*, vol. 379, pp. 41–52, 2020.

[26] C. Mu, Z. Ni, C. Sun, and H. He, "Air-breathing hypersonic vehicle tracking control based on adaptive dynamic programming," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 28, no. 3, pp. 584–598, Mar. 2017.

[27] R. Li and Y. Shi, "The fuel optimal control problem of a hypersonic aircraft with periodic cruising mode," *Math. Comput. Modelling*, vol. 55, pp. 2141–2150, 2012.

[28] N. Dhar, N. Verma, and L. Behera, "Adaptive critic-based event-triggered control for HVAC systems," *IEEE Trans. Ind. Informat.*, vol. 14, no. 1, pp. 178–188, Jan. 2018.

[29] B. Luo, D. Liu, and H. Wu, "Adaptive constrained optimal control design for data-based nonlinear discrete-time systems with critic-only structure," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 29, no. 6, pp. 2099–2111, Jun. 2018.

[30] D. Wang and C. Mu, "Adaptive-critic-based robust trajectory tracking of uncertain dynamics and its application to a spring-mass-damper system," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 65, no. 1, pp. 654–663, Jan. 2018.

[31] D. Wang, C. Mu, X. Yang, and D. Liu, "Event-based constrained robust control of affine systems incorporating and adaptive critic mechanism," *IEEE Trans. Syst., Man, Cybern, Syst.*, vol. 47, no. 7, pp. 1602–1612, Jul. 2017.

[32] D. Wang and D. Liu, "Learning and guaranteed cost control with event-based adaptive critic implementation," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 29, no. 12, pp. 6004–6014, Dec. 2018.

[33] Z. Wang, L. Liu, Y. Wu, and H. Zhang, "Optimal fault-tolerant control for discrete-time nonlinear strict-feedback systems based on adaptive critic design," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 29, no. 6, pp. 2179–2191, Jun. 2018.

[34] H. Jiang, H. Zhang, G. Xiao, and X. Cui, "Data-based approximate optimal control for nonzero-sum games of multi-player systems using adaptive dynamic programming," *Neurocomputing*, vol. 275, pp. 192–199, 2018.

[35] J. Parker, A. Serrani, S. Yurkovich, M. Bolender, and D. Doman, "Control-oriented modeling of an air-breathing hypersonic vehicle," *J. Guid., Control Dyn.*, vol. 30, pp. 856–869, 2007.

[36] B. Chen, X. Liu, and K. Liu, "Direct adaptive fuzzy control of nonlinear strict-feedback systems," *Automatica*, vol. 45, pp. 1530–1535, 2009.

[37] H. Zhang and Y. Quan, "Modeling, identification, and control of a class of nonlinear systems," *IEEE Trans. Fuzzy Syst.*, vol. 9, no. 2, pp. 349–354, Apr. 2001.

[38] X. Bu, X. Wu, Z. Ma, R. Zhang, and J. Huang, "Novel auxiliary error compensation design for the adaptive neural control of a constrained flexible air-breathing hypersonic vehicle," *Neurocomputing*, vol. 171, pp. 313–324, 2016.

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