$\mathcal{H}_\infty$ Synchronization for Fuzzy Markov Jump Chaotic Systems With Piecewise-Constant Transition Probabilities Subject to PDT Switching Rule

Jing Wang, Jianwei Xia, Hao Shen, *Member, IEEE*, Mengping Xing, and Ju H. Park

***Abstract*—This article investigates the nonfragile $\mathcal{H}_\infty$ synchronization issue for a class of discrete-time Takagi–Sugeno (T–S) fuzzy Markov jump systems. With regard to the T–S fuzzy model, a novel processing method based on the matrix transformation is introduced to deal with the double summation inequality containing fuzzy weighting functions, which may be beneficial to obtain conditions with less conservatism. In view of the fact that the uncertainties may occur randomly in the execution of the actuator, a nonfragile controller design scheme is presented by virtue of the Bernoulli distributed white sequence. The main novelty of this article lies in that the transition probabilities of the Markov chain are considered to be piecewise time-varying, and whose variation characteristics are described by the persistent dwell-time switching regularity. Then, based on the Lyapunov stability theory, it is concluded that the resulting synchronization error system is mean-square exponentially stable with a prescribed $\mathcal{H}_\infty$ performance in the presence of actuator gain variations. Finally, an illustrative example about Lorenz chaotic systems is provided to show the effectiveness of the established results.**

***Index Terms*—Mean-square exponential stability, nonfragile $\mathcal{H}_\infty$ synchronization, persistent dwell-time (PDT) switching, Takagi–Sugeno (T–S) fuzzy Markov jump chaotic systems (TFMJCSs).**

I. Introduction

OVER the past few decades, hybrid systems (HSs) involving a set of subsystems and the interconnection of logic have received much attention due to the powerful capability of modeling practical systems displaying switching features [1]–[5]. Generally, the switching signal that can reveal the logic association of HSs can be modeled either in stochastic or deterministic framework depending on whether the switching rule contains stochastic statistical properties [6]. An extensively investigated class of stochastic HSs are Markov jump systems (MJSs) whose jumps are determined by Markov chains. On account of the probabilistic properties, MJSs are quite suitable for describing the phenomena of random changes caused by unexpected events, and fruitful results focusing on the stability and stabilization analysis for MJSs have emerged accordingly [7]–[11].

For MJSs, the transition probabilities (TPs) that are usually calculated through a large amount of statistical data occupy an important position in determining the dynamic behavior of the whole system. Most of the current researches on MJSs are based on an implicit assumption that the TPs are time invariant. Nevertheless, this assumption may not be consistent with the actual engineering applications since the data utilized for obtaining TPs may exhibit different features during different periods. Consequently, motivated by the increasing needs of more realistic stochastic models, abundant research efforts have been devoted to exploring suitable TPs that can better reveal the random property. Currently, an available strategy is the introduction of nonhomogeneous TPs, which can be usually described in two ways. One is to consider the TPs displaying memory property, i.e., semi-Markov chain, while the other is the consideration of piecewise-constant TPs, for which the TPs are time-varying but remain constant within a fixed interval, and the variation is governed by another high-level switching signal [12], [13]. Generally, the deterministic switching signal is a preponderant option [14]. However, due perhaps to the complexity of two-level switching mechanism, relevant researches on this issue are far from maturity, and the mechanism that characterizes the switching of TPs for MJSs is mainly confined to dwell time (DT) or average dwell-time (ADT) switching [15], [16]. To the best of the authors' knowledge, there exists few available literature on MJSs with persistent dwell-time (PDT) switched TPs. As the PDT switching has been verified to be more flexible than the DT and ADT switching mechanism [17]–[19], exploring an appropriate disposing method to deal with MJSs subject to PDT switching in the stochastic jumps is of great significance.

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Jing Wang and Hao Shen are with the School of Electrical and Information Engineering, Anhui University of Technology, Ma'anshan 243002, China (e-mail: jingwang08@126.com; haoshen10@gmail.com).

Jianwei Xia is with the School of Mathematical Sciences, Liaocheng University, Liaocheng 252059, China (e-mail: njustxjw@126.com).

Mengping Xing is with the School of Electrical and Information Engineering, Anhui University of Technology, Ma'anshan 243002, China, and also with the School of Information Science and Engineering, Chengdu University, Chengdu 610106, China (e-mail: 13856079203@163.com).

Ju H. Park is with the Department of Electrical Engineering, Yeungnam University, Kyongsan 38541, Korea (e-mail: jessie@ynu.ac.kr).

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On another research forefront, nonlinearity is an intrinsic feature that widely exists in a majority of actual physical systems, which is also an essential element that cannot be neglected in the process of system modeling [20]–[23]. It is recognized that the Takagi–Sugeno (T–S) fuzzy model proposed in [24] is a prevailingly adopted tool for the approximation of nonlinear systems [25]–[29] . Described by a series of local linear submodels, the T–S fuzzy model can approximate systems with nonlinear term accurately [30]–[35]. As a consequence, considerable attention has been attached to the investigation of T–S fuzzy MJSs (TFMJSs) since the proposal of the magnificent scheme [36]–[38]. It should be mentioned that the majority of the works on TFMJSs involve the processing of the double summation inequality containing membership function terms, and at present, there are mainly three types of widely utilized disposal methods, as presented in [8], [18], and [39]. Obviously, with respect to these methods, the conservatism of the results may increase with the accretion of the fuzzy rules. Therefore, seeking a more effective method to solve this issue deserves further investigation, which gives rise to our great interests.

Moreover, the synchronization of chaotic systems has captured attention and enthusiasm of many researchers during the past decades as the chaos phenomenon provides broader space for exploring the irregular and unpredictable behavior of natural nonlinear systems [39]. It is well known that a crucial character of chaotic systems is that they are particularly sensitive to initial conditions and system parameters, and a minor variation in initial state can lead to great changes in system response [40]. Therefore, exploring an effective control scheme to achieve precise synchronization of the chaotic master and slave systems is especially important, and an extraordinary volume of the literature has been published on relevant issues. To name a few, in allusion to a class of nonlinear chaotic systems, the fault-tolerant dissipative synchronization issue was investigated in [41], and the composite nonlinear feedback controller design issue was explored in [42]. What should be mentioned is that the actuator may work improperly during the operation of the systems when some unexpected errors occur, such as equipment failures or external interference. In response to this issue, we are interested in designing a resilient or nonfragile controller that possesses the advantage of effective implementation in the presence of gain variation in the controller parameters.

The aforementioned analyses motivate us to construct a nonfragile $\mathcal{H}_\infty$ controller for T–S fuzzy Markov jump chaotic systems (TFMJCSs) with PDT switched TPs to achieve the synchronization of the master and slave systems. The main contributions can be synthesized as the following three aspects.

1) As the first attempt, the discrete-time TFMJCSs with time-varying TPs is investigated, of which the variation of TPs are supposed to be governed by a deterministic switching signal. On account of the generality compared with the DT or ADT switching, the PDT switching regularity is introduced to describe the variation of TPs.
2) With regard to the double summation inequality containing membership function terms of the T–S fuzzy model, a novel processing method based on [43] is presented. Compared with the disposing approach given in [8], [18], and [39], it may be conducive to deriving conditions with less conservatism.
3) For ensuring the effective implementation of the designed controller, we consider that the gain uncertainty may occur in the controller parameters, and the Bernoulli distribution is employed to describe the randomness of uncertainty. Then, a nonfragile controller that can guarantee the mean-square exponential stability and $\mathcal{H}_\infty$ performance of the resulting synchronization error system is constructed.

Notations: The notations employed in this work are standard [15]. $\mathbb{Z}_{\geq a}$: The set of integers no less than a; $\mathcal{E}\{\cdot\}$: The expectation operator; sym$\{A\}$: $A+A^T$; $\lambda_{\max}\{A\}/\lambda_{\min}\{A\}$: The maximum or minimum eigenvalue of matrix A; and $P>0$: Matrix P is positive definite.

II. Problem Formulation

A. Markov Jump and PDT Switching Regularity

Given a fixed probability space $(\bar{\Omega}, \bar{F}, \mathrm{Pr})$, the right-continuous Markov chain $\{\vartheta(k), k\in\mathbb{Z}_{\geq 0}\}$ taking values in a finite set $\mathcal{M}=\{1,2,\ldots,M\}$ is employed to govern the stochastic jumping of the TFMJCS. As the TPs of the Markov chain are time-varying, the elements of the transition probability matrix (TPM) $\Pi^{\lambda(k)}=[\pi_{ij}^{\lambda(k)}]_{M\times M}$ are expressed as follows:

$$\pi_{ij}^{\lambda(k)} \triangleq \mathrm{Pr}\{\vartheta(k+1)=j|\vartheta(k)=i\} \tag{1}$$

where $0\leq \pi_{ij}^{\lambda(k)}\leq 1\ \forall i,j\in\mathcal{M}$, $k\in\mathbb{Z}_{\geq 0}$. For $\forall i\in\mathcal{M}$, it satisfies $\sum_{j=1}^{M}\pi_{ij}^{\lambda(k)}=1$. $\lambda(k)$ is used to denote the PDT switching signal. As a right-continuous piecewise constant function, $\lambda(k)$ takes values in a given set $\mathcal{H}=\{1,2,\ldots,H\}$. To illustrate the concept of the PDT switching regularity, the following definition is provided.

Definition 1: [19] Given positive integers τ_P (the persistent dwel time) and T_P (the period of persistent), $\lambda(k)$ complies PDT switching regularity if the constraints presented in the following are satisfied.

1) For a series of nonadjacent intervals of length greater or equal to τ_P, $\lambda(k)$ is a constant in each of these intervals.
2) The intervals mentioned previously are separated by segments no longer than T_P, $\lambda(k)$ can take different values in these segments as long as the duration of each value is less than τ_P.

Remark 1: To facilitate the subsequence analysis, Fig. 1 presents the possible variation trends of the conceived Lyapunov function under the Markov jump sequence $\vartheta(k)$ and PDT switching sequence $\lambda(k)$. With regard to the PDT switching signal, the value of the Lyapunov function can increase within a certain range at the switching instant while is required to decrease at the nonswitching instant. To ensure the function as a whole tends to decay, the range that the function can rise at the switching instant should be confined. Therefore, we consider that the function value at the beginning of the current stage is less than that at the beginning of the previous stage. As the jumping of the investigated system is governed by the Markov chain, the mode jumping exhibits memoryless characteristics, and whether the

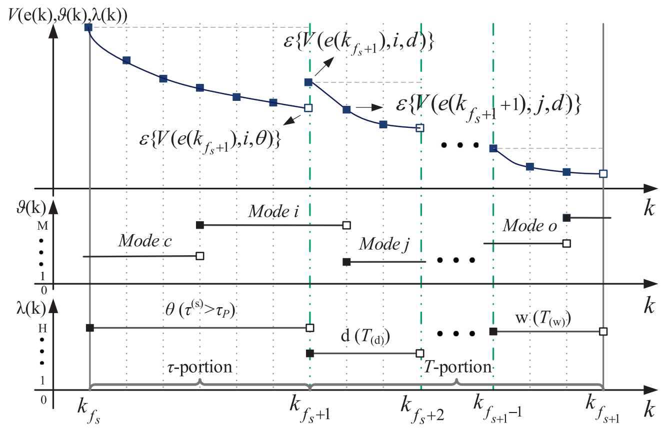

Fig. 1. Possible variation trends of the conceived Lyapunov function, Markov jump sequence, and PDT switching sequence in the sth stage.

system jumps or not is irrelevant with the previous mode. Furthermore, the sth stage of the PDT switching signal $\lambda(k)$ consists of two portions: the τ-portion (actual length $\tau^{(s)}$, $\tau^{(s)} \geq \tau_P$) and the T-portion (actual length $T^{(s)} = T_{(d)} + T_{(g)} + \cdots + T_{(w)}$, $T^{(s)} \leq T_P$). k_{f_s}, k_{f_s+1}, …, $k_{f_{s+1}-1}$ and $k_{f_{s+1}}$ represent the switching instants of the TPs.

B. System Description

We consider the following nonlinear Markov jump chaotic system:

$$x(k+1) = f(x(k), \vartheta(k), k)$$

where $x(k) \in \mathbb{R}^{n_x}$ is the state vector; and $f(\cdot)$ is a nonlinear function. Then, with the employing of the T–S fuzzy method, the ath rule of the TFMJCS is expressed as follows.

Rule a: IF $\eta_1(k)$ is $\mathcal{S}_{a1}, \ldots, \eta_h(k)$ is $\mathcal{S}_{ah}$, THEN

$$\begin{cases} x(k+1) = A_{a\vartheta(k)}x(k) + I(k) \\ z(k) = B_{a\vartheta(k)}x(k) \end{cases} \tag{2}$$

where $z(k) \in \mathbb{R}^{n_z}$ signifies the system output and $I(k) \in \mathbb{R}^{n_x}$ means the external input. Consider that the total number of IF–THEN rules is N, and a belongs to the set $\mathcal{N} = \{1, 2, \ldots, N\}$. Besides, $\eta(k) = [\eta_1(k), \eta_2(k), \ldots, \eta_h(k)]$ is utilized to denote the premise variable. $\mathcal{S}_{at} (t = 1, 2, \ldots, h)$ is the fuzzy sets. $A_{a\vartheta(k)}$ and $B_{a\vartheta(k)}$ are given constant matrices and their dimensions are suitable. For convenience, we denote $A_{a\vartheta(k)} \triangleq A_{ai}$ and $B_{a\vartheta(k)} \triangleq B_{ai}$ $\forall \vartheta(k) = i \in \mathcal{M}$. Other relevant symbols are defined in a similar way. Subsequently, by fuzzy blending the aforementioned submodels, the global model corresponding to the ith subsystem is presented as

$$\begin{cases} x(k+1) = \sum_{a=1}^{N} r_a(\eta(k)) \left[A_{ai}x(k) + I(k)\right] \\ z(k) = \sum_{a=1}^{N} r_a(\eta(k)) B_{ai}x(k) \end{cases} \tag{3}$$

where $r_a(\eta(k))$ is the fuzzy weighting function satisfying

$$r_a(\eta(k)) = \frac{\prod_{t=1}^{h} \mathcal{S}_{at}(\eta_t(k))}{\sum_{a=1}^{N} \prod_{t=1}^{h} \mathcal{S}_{at}(\eta_t(k))}$$

with $\mathcal{S}_{at}(\eta_t(k))$ being the membership degree of $\eta_t(k)$ in $\mathcal{S}_{at}$. As $\prod_{t=1}^{h} \mathcal{S}_{at}(\eta_t(k)) \geq 0$ $(a \in \mathcal{N})$, then one can get

$$r_a(\eta(k)) \geq 0, \sum_{a=1}^{N} r_a(\eta(k)) = 1. \tag{4}$$

For the purpose of simplification, in the following, we use $r_a^{(k)}$ to denote $r_a(\eta(k))$.

The system (3) is the master system, based on which the following controlled fuzzy slave system can be established by using the similar method as aforementioned:

$$\begin{cases} y(k+1) = \sum_{a=1}^{N} r_a^{(k)} \left[A_{ai}y(k) + I(k) + u(k) + E_{ai}\omega(k)\right] \\ \tilde{z}(k) = \sum_{a=1}^{N} r_a^{(k)} \left[B_{ai}y(k) + F_{ai}u(k)\right] \end{cases} \tag{5}$$

where $y(k) \in \mathbb{R}^{n_x}$, $\omega(k) \in \mathbb{R}^{n_\omega}$, $u(k) \in \mathbb{R}^{n_x}$, and $\tilde{z}(k) \in \mathbb{R}^{n_z}$ are used to denote the state vector, the external disturbance belonging to $l_2[0, \infty)$, the control input vector, and the output vector of the slave system, respectively.

In the following, we denote $e(k) \triangleq y(k) - x(k)$ and $\bar{z}(k) \triangleq \tilde{z}(k) - z(k)$ with $e(k)$ being the synchronization error, and the resulting synchronization error system (Σ) can, thereby, be given as

$$\begin{cases} e(k+1) = \sum_{a=1}^{N} r_a^{(k)} \left[A_{ai}e(k) + u(k) + E_{ai}\omega(k)\right] \\ \bar{z}(k) = \sum_{a=1}^{N} r_a^{(k)} \left[B_{ai}e(k) + F_{ai}u(k)\right]. \end{cases} \tag{6}$$

One of the main objectives of this article is to design a nonfragile controller that can guarantee the stability and prescribed performance of the synchronization error system. To achieve this purpose, a mode-dependent fuzzy controller is constructed as follows.

Controller rule b: IF $\eta_1(k)$ is $\mathcal{S}_{b1}, \ldots, \eta_h(k)$ is $\mathcal{S}_{bh}$, THEN

$$u(k) = (K_{bi} + \varsigma(k)\Delta K_{bi})e(k) \tag{7}$$

where $\varsigma(k) \in \{0, 1\}$ is the Bernoulli white sequence, which is independent of the Markov chain and satisfies the following probability distribution law:

$$\Pr\{\varsigma(k) = 1\} = \varsigma, \Pr\{\varsigma(k) = 0\} = 1 - \varsigma$$
$$\mathcal{E}\{\varsigma(k)\} = \varsigma, \mathcal{E}\{|\varsigma(k) - \varsigma|^2\} = \varsigma(1 - \varsigma).$$

Besides, K_{bi} denotes the controller gain that needs to be determined. ΔK_{bi} means the gain variation of the controller, whose form is as follows:

$$\Delta K_{bi} \triangleq U_{bi}\mathcal{F}_{bi}(k)V_{bi}, \mathcal{F}_{bi}^T(k)\mathcal{F}_{bi}(k) \leq I \; \forall k \geq k_0 \tag{8}$$

and U_{bi} and V_{bi} are known constant matrices. By fuzzy blending the individual linear controller (7), the overall controller can be given as

$$u(k) = \sum_{b=1}^{N} r_b^{(k)} \left[K_{bi} + \varsigma(k)\Delta K_{bi}\right] e(k). \tag{9}$$

Synthesize the aforementioned, the synchronization error system $(\bar{\Sigma})$ can finally be expressed as

$$\begin{cases} e(k+1) = \sum_{a=1}^{N} \sum_{b=1}^{N} r_a^{(k)} r_b^{(k)} \left[\bar{A}_{abi}e(k) + E_{ai}\omega(k)\right] \\ \bar{z}(k) = \sum_{a=1}^{N} \sum_{b=1}^{N} r_a^{(k)} r_b^{(k)} \bar{B}_{abi}e(k) \end{cases} \tag{10}$$

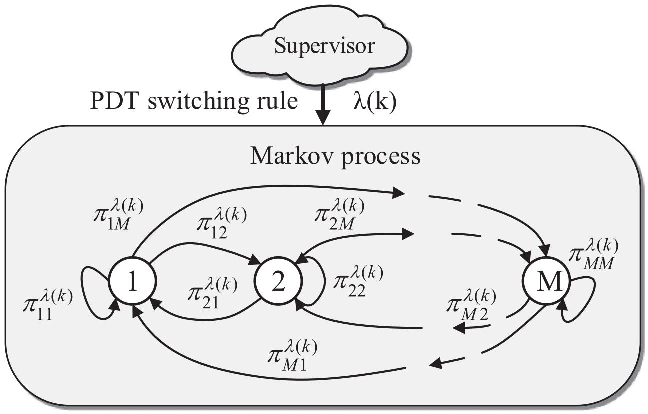

Fig. 2. HSs governed by the Markov process with PDT switched TPs.

where

$$\bar{A}_{abi} \triangleq A_{ai} + K_{bi} + \varsigma(k)\Delta K_{bi}$$
$$\bar{B}_{abi} \triangleq B_{ai} + F_{ai}(K_{bi} + \varsigma(k)\Delta K_{bi}).$$

Remark 2: For better illustrating the HSs with stochastic and deterministic switching properties, Fig. 2 is presented, where the HSs are composed of M stochastic modes, and the jumping of theses modes is governed by the Markov chain. Meanwhile, the decision of the supervisor will have an effect on the TPs, which means the TPs are time-varying. Our purpose is to construct an appropriate control scheme, such that the stability and performance of the overall system can be ensured when the supervisor complies with the PDT switching constraint. The advantage of this constraint lies in that the PDT switching rule can describe the switching with both fast and slow characteristics, and it is more flexible than DT or ADT switching.

C. Necessary Lemmas and Definitions

In order to investigate the stabilization and performance issue of the synchronization error system, the following lemmas, and definitions are provided.

Definition 2: [13] The synchronization error system $(\bar{\Sigma})$ is mean-square exponentially stable (MSES), if under the condition of $\omega(l) \equiv 0$, there exist $\kappa > 0$, $0 < \varphi < 1$ such that for any $e(0) \in \mathbb{R}^{n_x}$, $\vartheta(0) \in \mathcal{N}$, $\lambda(0) \in \mathcal{H}$, the inequality presented in the following holds:

$$\mathcal{E}\{\|e(k)\|^2\} \leq \kappa\varphi^{k-k_0}\mathcal{E}\{\|e(k_0)\|^2\} \,\forall k \geq k_0. \tag{11}$$

Definition 3: [44] The synchronization error system (10) is MSES with a prescribed $\mathcal{H}_\infty$ performance level $\tilde{\sigma}$, if the system (10) is MSES, and there exists a scalar $\tilde{\sigma} > 0$ such that $\forall\omega(t) \in l_2[0,\infty)$, the following condition is satisfied under zero-initial conditions:

$$\sum\nolimits_{t=0}^{\infty} \mathcal{E}\{\bar{z}^T(t)\bar{z}(t)\} \leq \tilde{\sigma}^2 \sum\nolimits_{t=0}^{\infty} \mathcal{E}\{\omega^T(t)\omega(t)\}. \tag{12}$$

Lemma 1: [45] U, V, and $\mathcal{F}(k)$ are matrices with appropriate dimensions with $\mathcal{F}(k)$ satisfying $\mathcal{F}^T(k)\mathcal{F}(k) \leq I$. Then, for any positive scalar ϵ, the following condition holds:

$$U\mathcal{F}^T(k)V + V^T\mathcal{F}(k)U^T \leq \epsilon^{-1}UU^T + \epsilon V^TV. \tag{13}$$

Lemma 2: [43] Let Θ_{ab} $(a, b \in \mathcal{N})$ be matrices with proper dimensions. If there exist matrices Ψ_{ab} $(a, b \in \mathcal{N}, a \neq b)$ such that

$$\Theta_{ab} + \Theta_{ba} < \Psi_{ab} + \Psi_{ab}^T \,\forall a, b \in \mathcal{N}, a < b \tag{14}$$

$$\bar{\Theta} \triangleq \begin{bmatrix} \Theta_{11} & \Psi_{12} & \cdots & \Psi_{1N} \\ \Psi_{12}^T & \Theta_{22} & \cdots & \Psi_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \Psi_{1N}^T & \Psi_{2N}^T & \cdots & \Theta_{NN} \end{bmatrix} < 0 \tag{15}$$

then, the following inequality holds:

$$\sum\nolimits_{a=1}^{N} \sum\nolimits_{b=1}^{N} r_a^{(k)} r_b^{(k)} \Theta_{ab} < 0. \tag{16}$$

Proof: As $r_a^{(k)}$ $(a \in \mathcal{N})$ is the fuzzy weighting function satisfying $r_a^{(k)} \geq 0$ and $\sum_{a=1}^{N} r_a^{(k)} = 1$, it can be deduced from (14) that

$$\begin{aligned} &\sum\nolimits_{a=1}^{N} \sum\nolimits_{b=1}^{N} r_a^{(k)} r_b^{(k)} \Theta_{ab} \\ <\ &\sum\nolimits_{a=1}^{N} \left[(r_a^{(k)})^2 \Theta_{aa} + \sum\nolimits_{b>a}^{N} r_a^{(k)} r_b^{(k)} (\Psi_{ab} + \Psi_{ab}^T) \right] \\ =\ &(\bar{r}^{(k)})^T \bar{\Theta} \bar{r}^{(k)} \end{aligned}$$

where $\bar{r}^{(k)} \triangleq [\, r_1^{(k)} \cdots r_N^{(k)} \,]^T$. As (15) means $\bar{\Theta} < 0$, one can easily get (16). This completes the proof. ■

Remark 3: For T–S fuzzy models, the generally utilized processing methods for ensuring the establishment of $\sum_{a=1}^{N} \sum_{b=1}^{N} r_a^{(k)} r_b^{(k)} \Theta_{ab} < 0$ are presented as the following three types: 1) $\Theta_{ab} < 0 \,\forall a, b \in \mathcal{N}$ [18]; 2) $\Theta_{aa} < 0 \,\forall a \in \mathcal{N}$ and $\Theta_{ab} + \Theta_{ba} < 0 \,\forall a, b \in \mathcal{N}, a < b$ [8]; and 3) $\Theta_{aa} < 0 \,\forall a \in \mathcal{N}$ and $\frac{2}{N-1}\Theta_{aa} + \Theta_{ab} + \Theta_{ba} < 0 \,\forall a, b \in \mathcal{N}, a \neq b$ [39]. It can be noted that, if the value of N is large, these three methods may lead to relatively conservative results. Therefore, inspired by [43], Lemma 2 is introduced to deal with the double summation inequality containing fuzzy weighting function terms. Although the calculation burden may increase with regard to the form of (15), the method presented here may be beneficial for obtaining conditions with less conservatism.

III. Main Results

In this section, the attention is focused on deriving sufficient conditions that can guarantee the mean-square exponential stability and $\mathcal{H}_\infty$ performance of the resulting synchronization error system (10). Then, the concrete form of the desired nonfragile fuzzy controller gains can be obtained based on the established criteria.

A. Stabilization and Performance Analysis

Before presenting further, some notations are given

$$\begin{aligned} \bar{\varepsilon}_1 &\triangleq \lambda_{\min_{\forall a \in \mathcal{M}, \theta \in \mathcal{H}}}\{(Q_a^\theta)^{-1}\}, \bar{\varsigma} \triangleq \varsigma(1-\varsigma) \\ \bar{\varepsilon}_2 &\triangleq \lambda_{\max_{\forall a \in \mathcal{M}, \theta \in \mathcal{H}}}\{(Q_a^\theta)^{-1}\}, \bar{\kappa} \triangleq \bar{\varepsilon}_2/\bar{\varepsilon}_1 \\ \bar{\alpha} &\triangleq \alpha\beta^{(T_P+1)/(T_P+\tau_P)}, \bar{\beta} \triangleq \beta^{(T_P+1)(1/(T_P+\tau_P)+1)}. \end{aligned}$$

Besides, we denote $M(k) \triangleq \max\{f_s + n | k \geq k_{f_s+n}\}$, which signifies the mark of the nearest switching from k.

Theorem 1: For given scalars $\alpha \in (0,1)$, $\beta \in (1,\infty)$, $0<\varepsilon_1<\varepsilon_2$ and $\sigma>0$, if there exist Lyapunov functions $V(e(k),\vartheta(k),\lambda(k))$, such that for $\forall\vartheta(k)\in\mathcal{M}$, $\lambda(k)\in\mathcal{H}$, the following conditions hold:

$$\varepsilon_1\|e(k)\|^2 \le V(e(k),\vartheta(k),\lambda(k)) \le \varepsilon_2\|e(k)\|^2 \tag{17}$$

$$\begin{aligned}&\mathcal{E}\left\{V(e(k+1),\vartheta(k+1),\lambda(k_{M(k)}))\right\}\\ &\quad\le \mathcal{E}\left\{\alpha V(e(k),\vartheta(k),\lambda(k_{M(k)}))+\Gamma(k)\right\}\end{aligned} \tag{18}$$

$$\begin{aligned}&\mathcal{E}\left\{V(e(k_{f_q+n}),\vartheta(k_{f_q+n}),\lambda(k_{f_q+n}))\right\}\\ &\quad\le \mathcal{E}\left\{\beta V(e(k_{f_q+n}),\vartheta(k_{f_q+n}),\lambda(k_{f_q+n-1}))\right\}\end{aligned} \tag{19}$$

$$\beta^{T_P+1}\alpha^{T_P+\tau_P}<1 \tag{20}$$

where

$$\Gamma(k)\triangleq -\bar{z}^T(k)\bar{z}(k)+\sigma^2\omega^T(k)\omega(k).$$

Then, the synchronization error system (10) is MSES with a prescribed $\mathcal{H}_\infty$ performance index

$$\bar{\sigma}\triangleq\sigma\sqrt{\bar{\beta}(1-\alpha)/(1-\bar{\alpha})}. \tag{21}$$

Proof: Step 1: First of all, we prove that the synchronization error system (10) is MSES under $\omega(k)\equiv 0$.

Obviously, the following inequality can be derived from (18) easily as $\omega(k)\equiv 0$

$$\begin{aligned}&\mathcal{E}\left\{V(e(k+1),\vartheta(k+1),\lambda(k_{M(k)}))\right\}\\ &\quad\le \alpha\mathcal{E}\left\{V(e(k),\vartheta(k),\lambda(k_{M(k)}))\right\}.\end{aligned} \tag{22}$$

For the interval $(a_1,a_2]$, we use $\Omega(a_1,a_2)$ to signify the switching number of the PDT switching signal. Apparently, one can get $\Omega(k_{f_s},k_{f_{s+1}})\le T^{(s)}+1$. Then, combining with (19) and (22), one can deduce

$$\begin{aligned}&\mathcal{E}\left\{V(e(k_{f_{s+1}}),\vartheta(k_{f_{s+1}}),\lambda(k_{f_{s+1}}))\right\}\\ &\le\beta\mathcal{E}\left\{V(e(k_{f_{s+1}}),\vartheta(k_{f_{s+1}}),\lambda(k_{f_{s+1}-1}))\right\}\\ &\le\beta\alpha\mathcal{E}\left\{V(e(k_{f_{s+1}}-1),\vartheta(k_{f_{s+1}}-1),\lambda(k_{f_{s+1}-1}))\right\}\\ &\cdots\\ &\le\beta^{\Omega(k_{f_s},k_{f_{s+1}})}\alpha^{k_{f_{s+1}}-k_{f_s}}\mathcal{E}\left\{V(e(k_{f_s}),\vartheta(k_{f_s}),\lambda(k_{f_s}))\right\}\\ &\le\beta^{T^{(s)}+1}\alpha^{\tau_P+T^{(s)}}\mathcal{E}\left\{V(e(k_{f_s}),\vartheta(k_{f_s}),\lambda(k_{f_s}))\right\}\\ &\le\delta\mathcal{E}\left\{V(e(k_{f_s}),\vartheta(k_{f_s}),\lambda(k_{f_s}))\right\}\end{aligned} \tag{23}$$

where $\delta\triangleq\max_{\forall s\in\mathbb{Z}_{\ge1}}\{\beta^{T^{(s)}+1}\alpha^{\tau_P+T^{(s)}}\}$.

If $\alpha\beta\ge1$, one can get from (20) that

$$0<\delta\le(\alpha\beta)^{T_P}\beta\alpha^{\tau_P}<1.$$

If $0<\alpha\beta<1$, one can deduce $\beta^{T^{(s)}+1}\alpha^{\tau_P+T^{(s)}}=(\alpha\beta)^{T^{(s)}+1}\alpha^{\tau_P-1}<1$ directly, which means $0<\delta<1$. Then, combining the case of $\alpha\beta\ge1$ with $0<\alpha\beta<1$, one can obtain that $0<\delta<1$ always holds under the condition of $\alpha\in(0,1)$, $\beta\in(1,\infty)$ and (20).

Considering $k\in[k_{f_s},k_{f_{s+1}})$, $s\in\mathbb{Z}_{\ge1}$, it can be inferred from $\Omega(l,k)<(\frac{k-l}{\tau_P+T_P}+1)(T_P+1)$, (22) and (23) that

$$\begin{aligned}&\mathcal{E}\left\{V(e(k),\vartheta(k),\lambda(k))\right\}\\ &\le\alpha^{k-k_{f_s}}\beta^{\Omega(k_{f_s},k)}\delta^{s-1}\mathcal{E}\left\{V(e(k_{f_1}),\vartheta(k_{f_1}),\lambda(k_{f_1}))\right\}\\ &\le\beta^{T_P+1}\delta^{s-1}\mathcal{E}\left\{V(e(k_{f_1}),\vartheta(k_{f_1}),\lambda(k_{f_1}))\right\}\end{aligned}$$

which means

$$\begin{aligned}&\mathcal{E}\left\{V(e(k),\vartheta(k),\lambda(k))\right\}\\ &\le\ \beta^{T_P+1}\delta^{-1}\bar{\delta}^{k-k_0+1}\mathcal{E}\left\{V(e(k_0),\vartheta(k_0),\lambda(k_0))\right\}\end{aligned}$$

with $k_0\triangleq k_{f_1}$ and there exists $\bar{\delta}\triangleq\max_{\forall s\in\mathbb{Z}_{\ge1},k\ge k_0}\{\delta^{s/(k-k_0+1)}\}$. As $0<s/(k-k_0+1)<1$ and $\delta\in(0,1)$, it implies $\bar{\delta}\in(0,1)$. Then, combining with (17), one can get $\forall k\ge k_0$

$$\mathcal{E}\{\|e(k)\|^2\}\le\beta^{T_P+1}\bar{\delta}\varepsilon_2/(\delta\varepsilon_1)\bar{\delta}^{k-k_0}\mathcal{E}\{\|e(k_0)\|^2\}.$$

Step 2: Under zero-initial conditions, the $\mathcal{H}_\infty$ performance for the synchronization error system (10) is proved in the following.

For $k\in[k_{f_s},k_{f_{s+1}})$, one can get from (18) and (19) that

$$\begin{aligned}&\mathcal{E}\left\{V(e(k),\vartheta(k),\lambda(k))\right\}\\ &\le\beta^{\Omega(k_0,k)}\alpha^{k-k_0}\mathcal{E}\left\{V(e(k_0),\vartheta(k_0),\lambda(k_0))\right\}\\ &\quad+\sum\nolimits_{l=k_0}^{k-1}\beta^{\Omega(l,k)}\alpha^{k-l-1}\mathcal{E}\left\{\Gamma(l)\right\}.\end{aligned} \tag{24}$$

Moreover, as $0<\Omega(l,k)<(\frac{k-l}{\tau_P+T_P}+1)(T_P+1)$, then, under zero-initial conditions, it is obvious that

$$\begin{aligned}&\sum\nolimits_{l=k_0}^{k-1}\alpha^{k-l-1}\mathcal{E}\{\bar{z}^T(l)\bar{z}(l)\}\\ &\le\sigma^2\bar{\beta}\sum\nolimits_{l=k_0}^{k-1}\bar{\alpha}^{k-l-1}\mathcal{E}\{\omega^T(l)\omega(l)\}\end{aligned}$$

which means

$$\begin{aligned}&\sum\nolimits_{k=k_0+1}^{\infty}\sum\nolimits_{l=k_0}^{k-1}\mathcal{E}\{\alpha^{k-l-1}\bar{z}^T(l)\bar{z}(l)\}\\ &\le\ \sigma^2\bar{\beta}\sum\nolimits_{k=k_0+1}^{\infty}\sum\nolimits_{l=k_0}^{k-1}\mathcal{E}\{\bar{\alpha}^{k-l-1}\omega^T(l)\omega(l)\}.\end{aligned}$$

Subsequently, (12) can be obtained by exchanging the summation order together with the utilization of equal ratio summation formula.

Thus, from Definitions 2 and 3, one can conclude that the system (10) is MSES with a prescribed $\mathcal{H}_\infty$ performance index $\bar{\sigma}$. ■

B. Controller Design

In the rest part of this section, by virtue of proper matrix processing methods, a nonfragile fuzzy controller is constructed based on Theorem 1.

Theorem 2: Given scalars $\alpha\in(0,1)$, $\beta\in(1,\infty)$, $\varsigma\in[0,1]$ and $\sigma>0$, if there exist symmetrical positive definite matrices Q_i^θ, $i\in\mathcal{M}$, $\theta\in\mathcal{H}$, matrices J_i, $i\in\mathcal{M}$, matrices $\Psi_{ab}^{i\theta}\triangleq[\psi_{ab}^{i\theta}(u,v)]_{u,v\in[1,6]}$, and positive scalars ϵ_{bi}, $b\in\mathcal{N}$, $i\in\mathcal{M}$, such that $\forall a,b\in\mathcal{N}$, $i\in\mathcal{M}$, and $\theta,\delta_1,\delta_2\in\mathcal{H}$, (20) and the following inequalities hold:

$$\begin{bmatrix} \Upsilon_{ab}^{i\theta} & \Lambda_{1bi} & \Lambda_{1ai} \\ * & -\epsilon_{bi}I & 0 \\ * & * & -\epsilon_{ai}I \end{bmatrix} < 0, a < b \tag{25}$$

$$\begin{bmatrix} \hat{\Upsilon}^{i\theta} & \hat{\Lambda}_i \\ * & \hat{\Xi}_i \end{bmatrix} < 0 \tag{26}$$

$$Q_i^{\delta_1} < \beta Q_i^{\delta_2}, \delta_1 \neq \delta_2 \tag{27}$$

where

$$\begin{aligned}
\Upsilon_{ab}^{i\theta} &\triangleq \bar{\Theta}_{ab}^{i\theta} + \bar{\Theta}_{ba}^{i\theta} - \Psi_{ab}^{i\theta} - \Psi_{ab}^{i\theta T} + \epsilon_{bi}\Lambda_{2abi}\Lambda_{2abi}^T \\
&\quad + \epsilon_{ai}\Lambda_{2bai}\Lambda_{2bai}^T \\
\Lambda_{1ai}^T &\triangleq \begin{bmatrix} V_{ai}J_i \ 0\ 0\ 0\ 0\ 0 \end{bmatrix} \\
\Lambda_{2abi}^T &\triangleq U_{bi}^T \begin{bmatrix} 0\ 0\ \varsigma L_i^\theta\ \sqrt{\bar{\varsigma}}L_i^\theta\ \varsigma F_{ai}^T\ \sqrt{\bar{\varsigma}}F_{ai}^T \end{bmatrix} \\
\hat{\Upsilon}^{i\theta} &\triangleq \begin{bmatrix} \hat{\Theta}_{11}^{i\theta} & \Psi_{12}^{i\theta} & \cdots & \Psi_{1N}^{i\theta} \\ * & \hat{\Theta}_{22}^{i\theta} & \cdots & \Psi_{2N}^{i\theta} \\ \vdots & \vdots & \ddots & \vdots \\ * & * & \cdots & \hat{\Theta}_{NN}^{i\theta} \end{bmatrix} \\
\hat{\Lambda}_i &\triangleq \mathrm{diag}\{\Lambda_{11i}, \Lambda_{12i}, \ldots, \Lambda_{1Ni}\} \\
\hat{\Xi}_i &\triangleq \mathrm{diag}\{-\epsilon_{1i}I, -\epsilon_{2i}I, \ldots, -\epsilon_{Ni}I\}
\end{aligned}$$

with

$$\begin{aligned}
\bar{\Theta}_{ab}^{i\theta} &\triangleq \begin{bmatrix} \tilde{Q}_i^\theta & 0 & \hat{A}_{abi}^\theta & 0 & \hat{B}_{abi}^\theta & 0 \\ * & -\sigma^2 I & E_{ai}^T L_i^\theta & 0 & 0 & 0 \\ * & * & -\bar{Q}^\theta & 0 & 0 & 0 \\ * & * & * & -\bar{Q}^\theta & 0 & 0 \\ * & * & * & * & -I & 0 \\ * & * & * & * & * & -I \end{bmatrix} \\
\hat{\Theta}_{aa}^{i\theta} &\triangleq \bar{\Theta}_{aa}^{i\theta} + \epsilon_{ai}\Lambda_{2aai}\Lambda_{2aai}^T
\end{aligned}$$

and

$$\begin{aligned}
\tilde{Q}_i^\theta &\triangleq \alpha(Q_i^\theta - J_i - J_i^T), \bar{Q}^\theta \triangleq \mathrm{diag}\{Q_1^\theta, \ldots, Q_M^\theta\} \\
L_i^\theta &\triangleq \begin{bmatrix} \sqrt{\pi_{i1}^\theta} I\ \sqrt{\pi_{i2}^\theta} I\ \ldots\ \sqrt{\pi_{iM}^\theta} I \end{bmatrix} \\
\hat{A}_{abi}^\theta &\triangleq J_i^T A_{ai}^T L_i^\theta + \bar{K}_{bi}^T L_i^\theta, \hat{B}_{abi}^\theta \triangleq J_i^T B_{ai}^T + \bar{K}_{bi}^T F_{ai}^T.
\end{aligned}$$

Then, the synchronization error system (10) is MSES with a prescribed $\mathcal{H}_\infty$ performance index $\bar{\sigma}$. Furthermore, the expected controller gains of (9) can be given as

$$K_{bi} = \bar{K}_{bi} J_i^{-1}. \tag{28}$$

Proof: The Lyapunov function is constructed as

$$V(e(k), \vartheta(k), \lambda(k)) = e^T(k)(Q_{\vartheta(k)}^{\lambda(k)})^{-1} e(k) \tag{29}$$

from which one can get

$$\bar{\varepsilon}_1 \|e(k)\|^2 \leq V(e(k), \vartheta(k), \lambda(k)) \leq \bar{\varepsilon}_2 \|e(k)\|^2$$

which means the condition (17) is satisfied. For convenience, we consider that $\vartheta(k) = i$, $\lambda(k_{M(k)}) = \theta$, $i \in \mathcal{M}$ and $\theta \in \mathcal{H}$.

Denote $\tilde{\Theta}_{ab}^{i\theta} \triangleq \bar{\Theta}_{ab}^{i\theta} + \epsilon_{bi}^{-1}\Lambda_{1bi}\Lambda_{1bi}^T + \epsilon_{bi}\Lambda_{2abi}\Lambda_{2abi}^T$. For inequality (25), it can be inferred from the Schur complement that

$$\tilde{\Theta}_{ab}^{i\theta} + \tilde{\Theta}_{ba}^{i\theta} < \Psi_{ab}^{i\theta} + \Psi_{ab}^{i\theta T}, a < b. \tag{30}$$

Similarly, (26) is equivalent to

$$\tilde{\Upsilon}^{i\theta} \triangleq \begin{bmatrix} \tilde{\Theta}_{11}^{i\theta} & \Psi_{12}^{i\theta} & \cdots & \Psi_{1N}^{i\theta} \\ * & \tilde{\Theta}_{22}^{i\theta} & \cdots & \Psi_{2N}^{i\theta} \\ \vdots & \vdots & \ddots & \vdots \\ * & * & \cdots & \tilde{\Theta}_{NN}^{i\theta} \end{bmatrix} < 0. \tag{31}$$

Then, by virtue of Lemma 2, it can be elicited from (30) and (31) that

$$\sum_{a=1}^N \sum_{b=1}^N r_a^{(k)} r_b^{(k)} \tilde{\Theta}_{ab}^{i\theta} < 0 \tag{32}$$

which combining with Lemma 1 and (8) means

$$\mathcal{R}^{i\theta}(k) \triangleq \sum_{a=1}^N \sum_{b=1}^N r_a^{(k)} r_b^{(k)} R_{ab}^{i\theta}(k) < 0 \tag{33}$$

with $R_{ab}^{i\theta}(k) \triangleq \bar{\Theta}_{ab}^{i\theta} + \mathrm{sym}\{\Lambda_{1bi}\mathcal{F}_{bi}^T(k)\Lambda_{2abi}^T\}$.

Denote $\bar{K}_{bi} \triangleq K_{bi}J_i$. Then, pre- and postmultiply $\mathcal{R}^{i\theta}(k)$ by $\mathrm{diag}\{(J_i^{-1})^T, I, I, I, I, I\}$ and $\mathrm{diag}\{J_i^{-1}, I, I, I, I, I\}$, together with the utilizing of inequality $-J_i^T(Q_i^\theta)^{-1}J_i \leq Q_i^\theta - J_i - J_i^T$ and Schur complement, one can get

$$\Phi_{i\theta} \triangleq \begin{bmatrix} \phi_{i\theta} - \alpha(Q_i^\theta)^{-1} & \check{A}_i^T \mathbb{Q}_i^\theta \check{E}_i \\ * & \check{E}_i^T \mathbb{Q}_i^\theta \check{E}_i - \sigma^2 I \end{bmatrix} < 0 \tag{34}$$

where

$$\begin{aligned}
\phi_{i\theta} &\triangleq \check{A}_i^T \mathbb{Q}_i^\theta \check{A}_i + \bar{\varsigma}\Delta\check{K}_i^T \left(\mathbb{Q}_i^\theta + \check{F}_i^T \check{F}_i\right)\Delta\check{K}_i + \check{B}_i^T \check{B}_i \\
\mathbb{Q}_i^\theta &\triangleq \sum_{j=1}^M \pi_{ij}^\theta (Q_j^\theta)^{-1}, \check{E}_i \triangleq \sum_{a=1}^N r_a^{(k)} E_{ai} \\
\check{A}_i &\triangleq \sum_{a=1}^N \sum_{b=1}^N r_a^{(k)} r_b^{(k)} [A_{ai} + K_{bi} + \varsigma\Delta K_{bi}] \\
\check{B}_i &\triangleq \sum_{a=1}^N \sum_{b=1}^N r_a^{(k)} r_b^{(k)} [B_{ai} + F_{ai}K_{bi} + \varsigma F_{ai}\Delta K_{bi}] \\
\check{F}_i &\triangleq \sum_{a=1}^N r_a^{(k)} F_{ai}, \Delta\check{K}_i \triangleq \sum_{b=1}^N r_b^{(k)} \Delta K_{bi}.
\end{aligned}$$

By virtue of (10) and (29), it can be calculated from (34) that

$$\begin{aligned}
&\mathcal{E}\{V(e(k+1), \vartheta(k+1), \theta) - \alpha V(e(k), i, \theta) - \Gamma(k)\} \\
&= \xi^T(k)\Phi_{i\theta}\xi(k) < 0
\end{aligned}$$

where $\xi(k) \triangleq [\, e^T(k)\ \omega^T(k) \,]^T$. It means (18) is satisfied.

Furthermore, it can be inferred from (27) that

$$\mathcal{E}\left\{ e^T(k_{f_q+n})((Q_i^{\delta_2})^{-1} - \beta(Q_i^{\delta_1})^{-1}) e(k_{f_q+n}) \right\} \leq 0.$$

Therefore, for any $i \in \mathcal{M}$, and $\delta_1, \delta_2 \in \mathcal{H}$, one has

$$\mathcal{E}\left\{V(e(k_{f_q+n}), i, \delta_2)\right\} \leq \mathcal{E}\left\{\beta V(e(k_{f_q+n}), i, \delta_1)\right\}$$

which implies (19) holds.

In addition, it can be inferred from (26) that $\hat{\Upsilon}^{i\theta} < 0$, which implies $\hat{\Theta}_{aa}^{i\theta} < 0$. Since $\hat{\Theta}_{aa}^{i\theta} \triangleq \bar{\Theta}_{aa}^{i\theta} + \epsilon_{ai}\Lambda_{2aai}\Lambda_{2aai}^T$ and $\epsilon_{ai}\Lambda_{2aai}\Lambda_{2aai}^T \geq 0$, one can obtain $\tilde{Q}_i^\theta < 0$. Then, $J_i < 0$ can be derived from $Q_i^\theta > 0$. Thus, J_i is a nonsingular matrix and the expected controller gains can be calculated by (28). This completes the proof. ■

Remark 4: The conditions derived in Theorem 2 have certain conservatism. This mainly stems from the following aspects.

1) The Lyapunov function constructed does not cover all characteristic information of the system, and some effective information may not be fully utilized. One of the possible improvements is the introduction of the fuzzy Lyapunov function.
2) During the processing of gain uncertainty and the inverse of unknown matrices such as $(Q_i^\theta)^{-1}$, some inequalities have been utilized.
3) Inequality scaling is introduced to eliminate fuzzy weighting functions in the condition. As illustrated in Remark 3, the inequality scaling conditions employed in Lemma 2 may be conducive to reducing conservatism.

However, the computational complexity, especially the dimension and number of matrices, will increase rapidly with the increase of $\mathcal{N}$. Besides, it can be observed from (25)–(27) that the computation burden is also raising with the increase of jump modes and the PDT switching rules. Therefore, it is of great significance to find a suitable strategy that can effectively lower the computational complexity while reducing the conservatism of the obtained results.

Remark 5: In order to get conditions in terms of the parameter-independent linear matrix inequality, the fuzzy weighting function in (32) should be disposed of. It can be noted from Remark 3 that $\tilde{\Theta}_{ab}^{i\theta} < 0$ $(\forall a, b \in \mathcal{N})$ utilized in [18] can also ensure the establishment of (32), which means the mean-square exponential stability and $\mathcal{H}_\infty$ performance of the system (10) can be also obtained under (20) and (27) and the following inequality:

$$\begin{bmatrix} \bar{\Theta}_{ab}^{i\theta} + \epsilon_{bi}\Lambda_{2abi}\Lambda_{2abi}^T & \Lambda_{1bi} \\ * & -\epsilon_{bi} I \end{bmatrix} < 0. \tag{35}$$

Obviously, the computational complexity of (35) may be less than (25) and (26). However, this method will inevitably bring about great conservatism. To show the superiority of the utilized method of the article in deriving conditions with less conservatism, some comparison results will be provided in the numerical example part.

IV. Illustrative Example

In this section, a Lorenz chaotic system with jumping parameters is employed to demonstrate the effectiveness of the proposed synchronization control scheme. The considered chaotic system modified from [46] is described as follows:

$$\begin{cases} \dot{x}_1(t) = -10x_1(t) + 10x_2(t) \\ \dot{x}_2(t) = b_i x_1(t) - x_2(t) - x_1(t)x_3(t) \\ \dot{x}_3(t) = x_1(t)x_2(t) - 8/3x_3(t) \end{cases}$$

where $b_i, i \in \{1, 2\}$ is a jumping parameter governed by a Markov chain with piecewise constant TPs. The variation of TPs is considered to be subject to the PDT switching rule. It is well known that the value of b_i has great influence on the chaos phenomenon of the system. Without loss of generality, we consider $b_1 = 46$ and $b_2 = 32$. Then, based on the T–S fuzzy theory (consider two fuzzy rules) and Euler's discretization approach (sampling time $\hat{T} = 0.01$ s), together with the consideration of the output of the master system, one can obtain the following discrete-time fuzzy master system:

$$\begin{cases} x(k+1) = \sum_{a=1}^{2} r_a(x_1(k)) A_{ai} x(k) \\ z(k) = \sum_{a=1}^{2} r_a(x_1(k)) B_{ai} x(k) \end{cases}$$

where

$$A_{ai} = \begin{bmatrix} 0.9000 & 0.1000 & 0 \\ \rho_i & 0.9900 & -\nu_a \\ 0 & \nu_a & 0.9733 \end{bmatrix}$$

$$\rho_1 = 0.4600, \rho_2 = 0.3200, \nu_1 = 0.3500, \nu_2 = -\nu_1$$

$$B_{11} = B_{21} = \begin{bmatrix} 0.72 & -0.09 & 0.27 \\ 0.09 & 0.63 & 0.18 \\ 0.36 & -0.27 & 0.81 \end{bmatrix}$$

$$B_{12} = B_{22} = \begin{bmatrix} 0.88 & -0.11 & 0.33 \\ 0.11 & 0.77 & 0.22 \\ 0.44 & -0.33 & 0.99 \end{bmatrix}$$

with the fuzzy weighting function chosen as

$$r_1(x_1(k)) = \begin{cases} 0.5(1 + \frac{x_1(k)}{35}), & |x_1(k)| \leq 35 \\ 0, & |x_1(k)| > 35 \end{cases}$$

$$r_2(x_1(k)) = 1 - r_1(x_1(k)).$$

Correspondingly, the parameters of the slave system are presented as follows:

$$E_{11} = \begin{bmatrix} 0.36 \\ 0.18 \\ 0.72 \end{bmatrix}, F_{11} = \begin{bmatrix} 0.36 & 0.18 & -0.09 \\ 0.27 & 0.36 & 0.54 \\ -0.18 & 0.09 & 0.45 \end{bmatrix}$$

$$E_{21} = \begin{bmatrix} 0.42 \\ 0.21 \\ 0.84 \end{bmatrix}, F_{21} = \begin{bmatrix} 0.32 & 0.16 & -0.08 \\ 0.24 & 0.32 & 0.48 \\ 0.16 & 0.08 & 0.40 \end{bmatrix}$$

$$E_{12} = \begin{bmatrix} 0.30 \\ 0.15 \\ 0.60 \end{bmatrix}, F_{12} = \begin{bmatrix} 0.52 & 0.26 & -0.13 \\ 0.39 & -0.52 & 0.78 \\ 0.26 & 0.13 & 0.65 \end{bmatrix}$$

$$E_{22} = \begin{bmatrix} 0.24 \\ 0.12 \\ 0.48 \end{bmatrix}, F_{12} = \begin{bmatrix} 0.48 & 0.24 & -0.12 \\ 0.36 & -0.48 & 0.72 \\ -0.24 & 0.12 & 0.60 \end{bmatrix}.$$

Furthermore, the gain variation matrices of the controller are considered to be

$$U_{11} = U_{21} = \nu \times \begin{bmatrix} 0.06 \; 0 \; 0.05 \end{bmatrix}^T$$

$$U_{12} = U_{22} = \nu \times \begin{bmatrix} 0.04 \; -0.02 \; 0 \end{bmatrix}^T$$

$$V_{11} = V_{21} = \nu \times \begin{bmatrix} 0.03 \; 0.01 \; 0.05 \end{bmatrix}$$

$$V_{12} = V_{22} = \nu \times \begin{bmatrix} 0.02 \; 0.04 \; 0.03 \end{bmatrix}$$

$$\mathcal{F}_{ai}(k) = 0.8\sin(k), a = 1, 2, i = 1, 2$$

with $\nu = 1$ and the occurrence expectation being $\varsigma = 0.8$.

For the Markov chain, the TPM is given as

$$\Pi^1 = \begin{bmatrix} 0.65 & 0.35 \\ 0.15 & 0.85 \end{bmatrix}, \Pi^2 = \begin{bmatrix} 0.42 & 0.58 \\ 0.33 & 0.67 \end{bmatrix}$$

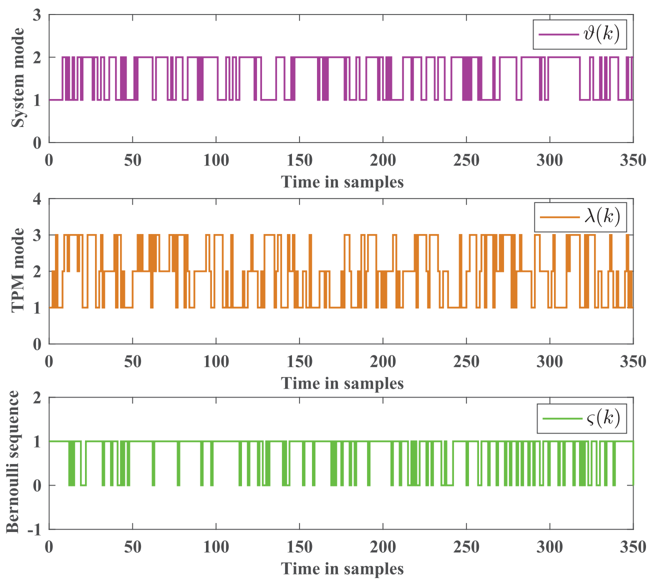

Fig. 3. Evolution of system mode, the PDT sequence determining the variation of TPs and the Bernoulli sequences relevant to $\varsigma(k)$.

$$\Pi^3 = \begin{bmatrix} 0.84 & 0.16 \\ 0.32 & 0.68 \end{bmatrix}.$$

In addition, the parameters relevant to the PDT switching regularity are provided as

$$T_P = 8, \tau_P = 6, \alpha = 0.9, \beta = 1.1.$$

Then, under the prescribed performance index $\bar{\sigma} = 2.8930$, the following controller gains can be calculated by virtue of Theorem 2:

$$K_{11} = \begin{bmatrix} -0.8394 & -0.1436 & -0.0245 \\ -0.4475 & -0.9089 & 0.3184 \\ -0.0279 & -0.2188 & -0.9881 \end{bmatrix}$$

$$K_{21} = \begin{bmatrix} -1.0270 & -0.0070 & -0.0841 \\ -0.4331 & -1.0818 & -0.3030 \\ 0.1144 & 0.1930 & -0.8827 \end{bmatrix}$$

$$K_{12} = \begin{bmatrix} -1.0725 & 0.0207 & -0.2662 \\ -0.4289 & -0.6992 & 0.1270 \\ -0.0082 & -0.3639 & -0.8457 \end{bmatrix}$$

$$K_{22} = \begin{bmatrix} -0.9395 & -0.1813 & -0.0680 \\ -0.5600 & -0.8741 & -0.5183 \\ -0.2798 & -0.0336 & -1.0483 \end{bmatrix}.$$

Given a set of PDT switching sequences, based on which the evolution of the Markov chain can be derived with the aid of Algorithm 1, as presented in Fig. 3, and a set of Bernoulli sequences relevant to $\varsigma(k)$ are also given there (For additional clarity and emphasis, only part of the sequences are shown there). Then, with disturbance $\omega(k) = 12\cos(5\,k)/(1+\exp(0.2\,k))$, the trajectories of the chaotic master and slave systems are plotted in Figs. 4 and 5 under the initial conditions of $x(0) = [15.8\ -12.48\ 15.64]^T$ and $y(0) = [10\ 6\ 13]^T$, and the state responses of the synchronization error system are provided in Fig. 6. It can be observed that the synchronization error curves

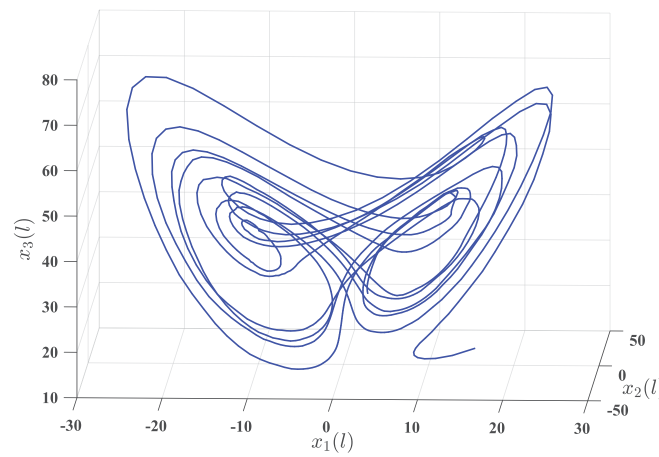

Fig. 4. State trajectory of the master system.

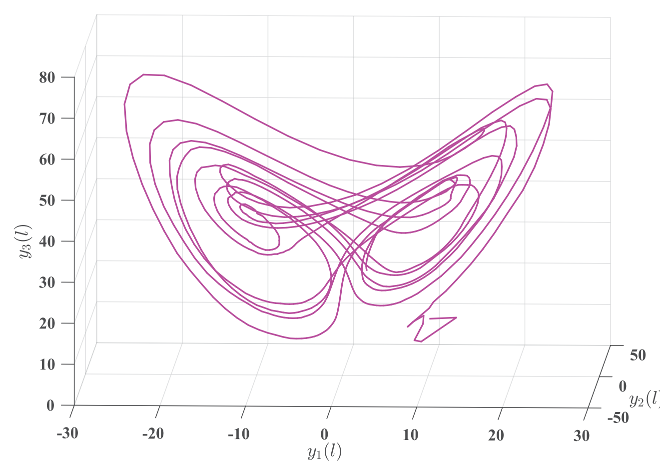

Fig. 5. State trajectory of the slave system.

are divergent when the controller gains are set as zeros, while converges to zero when the controller works, which indicates the validity of the developed design method.

Furthermore, under zero-initial conditions, the $\mathcal{H}_\infty$ performance is examined as follows:

$$\sqrt{\frac{\sum_{l=0}^{1000} \mathcal{E}\{\bar{z}^T(l)\bar{z}(l)\}}{\sum_{l=0}^{1000} \mathcal{E}\{\omega^T(l)\omega(l)\}}} = 0.4158 < \bar{\sigma} = 2.8930$$

which means the performance requirement in Definition 3 is satisfied.

In what follows, the influence of the stochastic variable $\varsigma(k)$, Lyapunov variation rates α and β on the performance of the fuzzy chaotic system is investigated. The optimal performance index corresponding to different parameters is denoted as $\bar{\sigma}_{\min}$. We set $\nu = 10$ and other parameters are the same as aforementioned. Then, by adjusting the value of ς, α, and β, the corresponding value of $\bar{\sigma}_{\min}$ can be calculated, as listed in Table I and Fig. 7. Particularly, the optimal performance index obtained on account of Lemma 2 in this article (corresponding to conditions presented in Theorem 2) and $\tilde{\Theta}_{ab}^{i\theta} < 0$ utilized in [18] (corresponding to conditions presented in Remark 5) are denoted as Cases I and II, respectively. Then, it can be observed from Table I that the

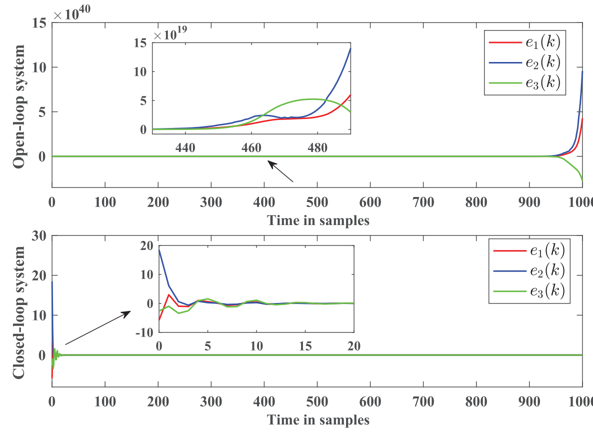

Fig. 6. Synchronization error of the open-loop system and the closed-loop system.

TABLE I
OPTIMAL PERFORMANCE INDEX $\bar{\sigma}_{\min}$ CORRESPONDING TO DIFFERENT ς UNDER DIFFERENT CASES

$\bar{\sigma}_{\min}$	$\varsigma=0$	$\varsigma=0.2$	$\varsigma=0.6$	$\varsigma=0.8$	$\varsigma=1.0$
Case I	1.7103	1.8810	2.4461	2.8019	3.2086
Case II	2.0664	2.3260	3.0567	3.5358	4.1016

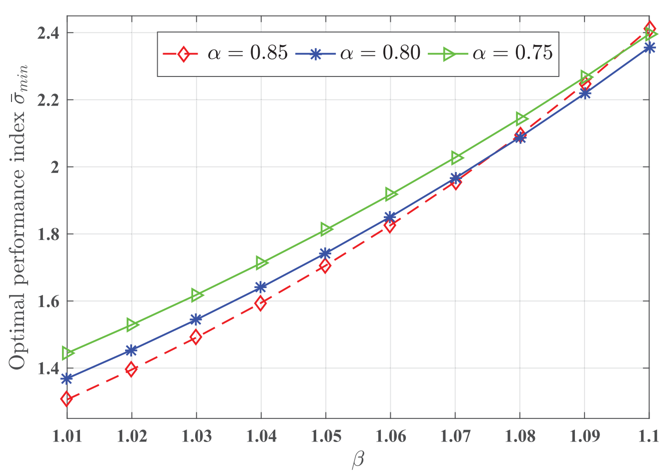

Fig. 7. Optimal performance index $\bar{\sigma}_{\min}$ for different α and β.

value of $\bar{\sigma}_{\min}$ grows larger with the increase of ς, which means the occurrence of the controller gain variation may have adverse effects on the system performance. Besides, compared with the method of the disposing fuzzy weighting function in [18], the processing method in this article can get smaller $\bar{\sigma}_{\min}$. Thus, the method employed in this article may be beneficial in deriving conditions with less conservatism. Moreover, Fig. 7 indicates that the selection of different pair (α, β) will also affect the performance index of the synchronization error system. Thus, choosing proper Lyapunov function variation rate to ensure the prescribed performance index of the investigated system is significant.

TABLE II
OBTAIN SYSTEM MODE SEQUENCE WITH $M=2$.

Algorithm 1: Obtain System Mode Sequence with $M=2$.

Input: $\mathcal{H}, T_P, \tau_P, Length, \Pi^i, i \in \mathcal{H}, InitM$;
Output: System mode sequence: $sMode$.
Generate a PDT switching sequence $\lambda(k)$ with given T_P, τ_P;
$k=1$; $sMode(1) = InitM$;
for $k = 1 : Length$ **do**
  Generate a random number $P \in [0,1]$;
  if $sMode(k) = 1$ **then**
    if $P < \Pi^{\lambda(k)}(1,2)$ **then**
      $k{+}{+}$; $sMode(k) = 2$; $InitM = 2$;
    else
      $k{+}{+}$; $sMode(k) = 1$; $InitM = 1$;
  else
    if $P < \Pi^{\lambda(k)}(2,1)$ **then**
      $k{+}{+}$; $sMode(k) = 1$; $InitM = 1$;
    else
      $k{+}{+}$; $sMode(k) = 2$; $InitM = 2$;

V. CONCLUSION

The nonfragile $\mathcal{H}_\infty$ controller design issue for a class of discrete-time nonlinear hybrid chaotic systems was addressed in this article, where the random occurrence of gain variation was described by a set of Bernoulli-distributed white sequences. Moreover, the stochastic jump of system parameters was supposed to be described by a Markov chain with time-varying TPM subject to a deterministic switching mechanism, i.e., the PDT switching. The nonlinearity of the systems was disposed by the T–S fuzzy model for which a novel disposing method for the double summation inequality containing fuzzy weighting function terms was introduced to derive criteria with less conservatism. Furthermore, an applicable synchronization controller that can ensure the mean-square exponential stability and $\mathcal{H}_\infty$ performance of the fuzzy chaotic MJSs with PDT switched TPs was constructed via the Lyapunov stability theory. Finally, the Lorenz chaotic system with jumping parameters was provided to illustrate the validity of the proposed control scheme. An interesting future work is to extend the proposed scheme to the adaptive tracking control of the interval type-2 T–S fuzzy switched model. Furthermore, constructing more suitable Lyapunov functions that are beneficial for deriving less conservative conditions is also a meaningful issue deserved to be further explored.

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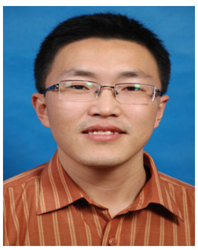
Jing Wang received the Ph.D. degree in power system and automation from Hohai University, Nanjing, China, in 2019.

She is currently an Associate Professor with the School of Electrical and Information Engineering, Anhui University of Technology, Ma'anshan, China. Her current research interests include Markov jump nonlinear systems, singularly perturbed systems, power systems, and nonlinear control.



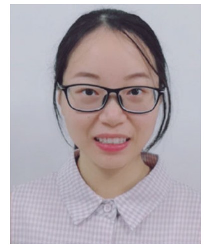
Jianwei Xia received the Ph.D. degree in automatic control from the Nanjing University of Science and Technology, Nanjing, China, in 2007.

From 2010 to 2012, he worked as a Postdoctoral Research Associate with the School of Automation, Southeast University, Nanjing. From 2013 to 2014, he worked as a Postdoctoral Research Associate with the Department of Electrical Engineering, Yeungnam University, Kyongsan, Korea. He is currently a Professor with the School of Mathematics Science, Liaocheng University, Liaocheng, China. His current research interests include robust control, stochastic systems, delay systems, and neural networks.

Hao Shen (Member, IEEE) received the Ph.D. degree in control theory and control engineering from the Nanjing University of Science and Technology, Nanjing, China, in 2011.

From February 2013 to March 2014, he was a Postdoctoral Fellow with the Department of Electrical Engineering, Yeungnam University, Kyongsan, Korea. Since 2011, he has been with Anhui University of Technology, Ma'anshan, China, where he is currently a Professor. He is a Postdoctoral Fellow with the School of Mathematics, Southeast University, Nanjing. His current research interests include stochastic hybrid systems, complex networks, fuzzy systems and control, and nonlinear control.

Dr. Shen has served on the Technical Program Committee for several international conferences. He is an Associate Editor/Guest Editor for several international journals, including *Journal of The Franklin Institute*, *Applied Mathematics and Computation*, IEEE ACCESS, *Neural Processing Letters*, and *Transactions of the Institute Measurement and Control*. He was the recipient of the Highly Cited Researcher Award by Clarivate Analytics (formerly, Thomson Reuters) in 2019.



Mengping Xing received the B.Sc. degree in electrical engineering and automation from Liaoning Technical University, Fuxin, China, in 2016, and the M.S. degree in electrical engineering from the School of Electrical and Information Engineering, Anhui University of Technology, Ma'anshan, China, in 2020.

Her current research interests include persistent dwell-time switched systems, singularly perturbed systems, robust control, and filtering.

Ju H. Park received the Ph.D. degree in electronics and electrical engineering from the Pohang University of Science and Technology (POSTECH), Pohang, Korea, in 1997.

From May 1997 to February 2000, he was a Research Associate with Engineering Research Center-Automation Research Center, POSTECH. He joined Yeungnam University, Kyongsan, Korea, in March 2000, where he is currently the Chuma Chair Professor. He is a Coauthor of the monographs: *Recent Advances in Control and Filtering of Dynamic Systems with Constrained Signals* (New York, NY, USA: Springer-Nature, 2018) and *Dynamic Systems with Time Delays: Stability and Control* (New York, NY, USA: Springer-Nature, 2019) and is the Editor of an edited volume: *Recent Advances in Control Problems of Dynamical Systems and Networks* (New York, NY, USA: Springer-Nature, 2020). He has published a number of papers in the areas of his current research interests, which include robust control and filtering, neural/complex network, fuzzy systems, multiagent systems, and chaotic systems.

Prof. Park serves as an Editor for the *International Journal of Control, Automation and Systems*. He is also a Subject Editor/Advisory Editor/Associate Editor/Editorial Board member for several international journals, including *IET Control Theory and Applications*, *Applied Mathematics and Computation*, *Journal of The Franklin Institute*, *Nonlinear Dynamics, Engineering Reports*, *Cogent Engineering*, IEEE TRANSACTIONS ON FUZZY SYSTEMS, IEEE TRANSACTIONS ON NEURAL NETWORKS AND LEARNING SYSTEMS, IEEE TRANSACTIONS ON CYBERNETICS, and so on. Since 2015, he has been the recipient of the Highly Cited Researcher Award by Clarivate Analytics (formerly, Thomson Reuters) and listed in three fields, engineering, computer sciences, and mathematics in 2019. He is a Fellow of the Korean Academy of Science and Technology.