Finite-Time Convergence Adaptive Fuzzy Control for Dual-Arm Robot With Unknown Kinematics and Dynamics

Chenguang Yang, *Senior Member, IEEE*, Yiming Jiang, Jing Na, *Member, IEEE*, Zhijun Li, *Senior Member, IEEE*, Long Cheng, *Senior Member, IEEE*, and Chun-Yi Su, *Senior Member, IEEE*

***Abstract*—Due to strongly coupled nonlinearities of the grasped dual-arm robot and the internal forces generated by grasped objects, the dual-arm robot control with uncertain kinematics and dynamics raises a challenging problem. In this paper, an adaptive fuzzy control scheme is developed for a dual-arm robot, where an approximate Jacobian matrix is applied to address the uncertain kinematic control, while a decentralized fuzzy logic controller is constructed to compensate for uncertain dynamics of the robotic arms and the manipulated object. Also, a novel finite-time convergence parameter adaptation technique is developed for the estimation of kinematic parameters and fuzzy logic weights, such that the estimation can be guaranteed to converge to small neighborhoods around their ideal values in a finite time. Moreover, a partial persistent excitation property of the Gaussian-membership-based fuzzy basis function was established to relax the conventional persistent excitation condition. This enables a designer to reuse these learned weight values in the future without relearning. Extensive simulation studies have been carried out using a dual-arm robot to illustrate the effectiveness of the proposed approach.**

Manuscript received April 11, 2017; revised September 13, 2017 and April 18, 2018; accepted July 26, 2018. Date of publication August 10, 2018; date of current version February 27, 2019. This work was supported in part by the National Nature Science Foundation under Grant 61473120, Grant 61811530281, Grant 61573174, Grant 61873268, and Grant 61633016, in part by the Science and Technology Planning Project of Guangzhou under Grant 201607010006, in part by the State Key Laboratory of Robotics and System (HIT) under Grant SKLRS-2017-KF-13, in part by the Fundamental Research Funds for the Central Universities under Grant 2017ZD057, in part by the Research Fund for Young Top-Notch Talent of National Ten Thousand Talent Program, in part by the Beijing Municipal Natural Science Foundation under Grant 4162066, and in part by the Major Science and Technology Project of Beijing under Grant Z181100003118006. *(Corresponding author: Chenguang Yang.)*

C. Yang and Y. Jiang are with the Key Laboratory of Autonomous Systems and Networked Control, College of Automation Science and Engineering, South China University of Technology, Guangzhou 510640, China (e-mail: cyang@ieee.org; ym.jiang2015@gmail.com).

J. Na is with the Faculty of Mechanical and Electrical Engineering, Kunming University of Science and Technology, Kunming 650500, China (e-mail: najing25@163.com).

Z. Li is with the Department of Automation, University of Science and Technology of China, Hefei 230026, China (e-mail: zjli@ieee.org).

L. Cheng is with the Key Laboratory of Complex Systems and Intelligence Science, Institute of Automation, Chinese Academy of Sciences, Beijing 100190, China, and also with the School of Artificial Intelligence, University of Chinese Academy of Sciences, Beijing 100049, China (e-mail: long.cheng@ia.ac.cn).

C.-Y. Su is with the School of Automation, Guangdong University of Technology, Guangzhou 510006, China, on leave from Concordia University, Montreal, QC H3G 1M8, Canada (e-mail: cysu@alcor.concordia.ca).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TFUZZ.2018.2864940

***Index Terms*—Dual-arm robots, finite-time (FT) convergence, fuzzy logic system (FLS), uncertain kinematics.**

I. Introduction

In RECENT decades, there has been a pronounced tendency to focus the studies of coordinated dual-arm robots in robotics and automation communities [1]–[5]. With a bimanual cooperation, the dual-arm robots can accomplish complex tasks, such as cooperative load transporting and coordinate manipulation. But difficulties of the controller designs are also increased significantly, since an additional arm not only leads to a complicated structure and mechanism, but also adds strong coupled nonlinearities as well as the internal forces generated by the grasped object. To address these problems, coordinate control of dual-arm robots has been widely investigated [6]–[10]. Early work of the dual-arm coordinate position tracking control was presented in [1] and [2]. A decentralized adaptive control algorithm was proposed for multiple redundant cooperative manipulators to address the position tracking and internal forces regulation in [6]. In [8], the coordination problem of redundant robot systems was addressed by using the dual neural network with multicriteria to minimize the energy cost. In [11], an adaptive neural control for the humanoid robot was presented to deal with unknown output nonlinearities by employing a smooth adaptive inverse technique.

It is worth mentioning that many existing dual-arm robot coordinated control schemes were developed under the assumption that the robot kinematics are fully known. However, in realistic operational scenarios, kinematic uncertainties widely exist. For example, when a dual-arm robot equips with replaceable tools or grasps an unknown object, the system parameters may be changed and lead to unknown kinematic parameters. Recently, the robot tracking control with uncertain kinematics for single-arm robots has been reported [12]–[16]. An adaptive set-point control scheme of robots with uncertain kinematics and gravitational force was proposed in [14], where the exact knowledge of kinematics and Jacobian matrix was not required. This scheme has been extended to the robot tracking control [15], without using knowledge of robotic kinematics, dynamics, and actuator model. A neural network controller combined with the approximate Jacobian matrix (AJM) scheme was also presented to deal with the robot tracking in the absence of kinematics and dynamics [12]. The uncertain kinematics control of a single-arm robot was well addressed in the above-mentioned works, however, few research in the literature has investigated the control of the dual-arm robot, where accurate kinematic parameters are unavailable. Without precise kinematic information, the task space position cannot be accurately converted into joint space, and therefore, the control performance of the dual-arm robot may be greatly limited.

On the other hand, dynamic uncertainties of the dual-arm robot widely exist as well, let alone various unknown factors of the operating environment and the objects under manipulation. These uncertainties may cause degeneration of the control performance or even incur instable system states. To deal with this problem, especially the unstructured model uncertainties, model-free control design approaches have been extensively studied [17]–[24]. It is worth mentioning that, among these control approaches, fuzzy logic system (FLS) has been characterized as a powerful approximator by using linguistic knowledge representations and fuzzy rules and has been wildly used to deal with uncertainties [25]–[30], [31]. To address the strict-feedback control of single-input single-output systems, an FLS was constructed for the compensation of uncertain nonlinearities in [25]. To compensate for the unknown system dynamics of a class of nonlinear systems, a backstepping-based adaptive controller was developed by utilizing the universal approximation ability of the FLS [32]. In [33], to enhance the control performance of a humanoid robot in the presence of unknown actuator backlash and uncertain dynamics, a decentralized controller was designed by using the adaptive FLS and a smooth backlash inverse. In the above-mentioned works, however, only ultimate boundedness of the weights estimation error can be guaranteed, and the convergence of the weights was not analyzed. The FLS is required to online adjust the learned weights to transform the expert linguistic knowledge into adaptive learning ability of the control system. Without convergence, the weight values need to relearn every time when the FLS runs again even if repeating the same task. It is desirable to guarantee fast and accurate convergence of estimated parameters to improve the control performance [34].

In this paper, we develop a dual-arm robot control scheme by using the AJM technique and the adaptive FLS, such that the robot can be well controlled in the absence of robot dynamics and kinematics. The proposed control scheme can guarantee the convergence of the tracking errors in a finite time (FT), in addition to the parameter estimation. It has been reported that the estimation performance can be improved if the adaptation law contains the information of the estimation errors [35]. An adaptive estimation technique was proposed in [36], where the parameter estimation errors were used to ensure the convergence of the estimated parameters. In [37], an optimal control strategy was applied in a nonlinear system by using the dynamic programming algorithm to ensure the FT convergence. Inspired by the concept of "direct" parameter estimation [35]–[38], in this paper, an FT adaptive estimation algorithm is developed by introducing a leakage term driven by parameter estimation errors, such that the estimated kinematic parameters can converge to small neighborhoods of their optimal values in FT.

It should be noted that the persistent excitation (PE) condition is important to guarantee the convergence of estimated parameters [34]. However, for the conventional FLS, the requirement of the PE condition is restricted and not easy to be satisfied. In [39], a partial persistent excitation (PPE) is proposed to relax the PE condition of the neural network control, which proved that the PE condition of certain regression subvectors of radical basis functions (RBFs) along with a recurrent trajectory (e.g., periodic and periodic-like orbits) could be rigorously guaranteed [39]. Inspired by this work, in this paper, the PPE condition of the FLS is investigated to replace the conventionally used PE condition. Based on the PPE condition, an adaptive weight updating scheme is further developed by introducing an auxiliary framework to express the weights errors. Under the PPE condition, the FT convergence of partial FLS weight estimated and accurate approximation of unknown dynamics are guaranteed.

The main contributions of the proposed control scheme could be summarized as follows:

1) constructing an adaptive fuzzy logic control scheme for the coordinated robot arms with neither *a priori* knowledge of system dynamics nor information of the kinematic parameters;
2) designing a novel parameters adaptation framework by applying a set of auxiliary filtered matrices, such that the parameter estimation errors could be appropriately expressed without using the robot joint accelerations;
3) relaxing the PE condition by introducing the concepts of the PPE and spatially localized approximation (SLA) of the FLS, such that the weights could converge to their optimal values when tracking a periodic trajectory.

II. Modeling Procedure of the Robot and Preliminaries

A. Kinematics Modeling of the Dual-Arm Robot

As shown in Fig. 1, the dual-arm robot is commanded to grasp a common object to follow a reference trajectory. Assume that the end effector of each arm grasps the object rigidly so that no relative motion occurs between the end effectors and the object. Then, the object's position and orientation vector can be calculated by the joint variable of each arm as follows:

$$x = f_{\mathrm{kine}_i}(q_i), \quad i = 1, 2 \tag{1}$$

where $x \in \mathbb{R}^{N_0}$ is the position of the object, N_0 is the object's degree of freedom (DOF), q_i denotes the joint angle of the ith arm. The subscript "i" denotes the "left" and "right" robotic arms. Differentiating (1) with respect to time yields

$$\dot{x} = J_i(q_i, \phi_i)\dot{q}_i \tag{2}$$

where $\phi_i \in \mathbb{R}^{h_i}$ denotes the kinematic parameters of the ith arm, and h_i denotes the number of the D–H parameters. $J_i(q_i, \phi_i) \in \mathbb{R}^{N_o \times N_i}$ is the Jacobian matrix, which satisfies the following property.

Property 1 (see[12]): The Jacobian matrix $J_i(q_i, \phi_i)$ can be formulated with a linearly parameterized form as

$$J_i(q_i, \phi_i)\xi_i = R_i(q_i, \xi_i)\phi_i \tag{3}$$

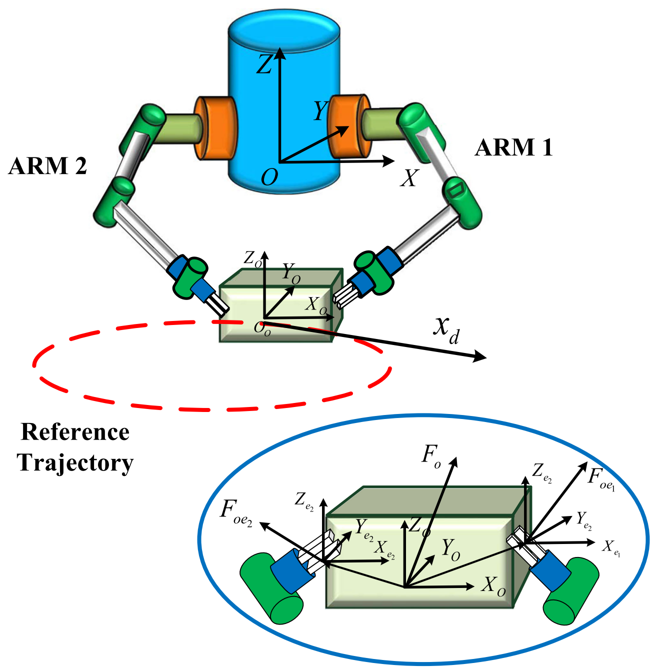


Fig. 1. Overlook of the dual-arm robot manipulating an object.

where $R_i(q_i, \xi_i) \in \mathbb{R}^{N_o \times h_i}$ is the regressor matrix of kinematics with respect to q_i and ξ_i, and ξ_i is a known vector. Without loss of generality, we assume that ϕ_i is bounded by known vectors $\underline{\phi_i}$ and $\overline{\phi_i}$, as $\underline{\phi_i} \geq \phi_i \geq \overline{\phi_i}$, where $\underline{\phi_i}$ and $\overline{\phi_i}$ denote the lower and upper bounds of ϕ_i, respectively.

From Property 1, we can derive that $J_i(q_i, \phi_i)\xi_i = J_i(q_i, \hat{\phi}_i)\xi_i + J_i(q_i, \tilde{\phi}_i)\xi_i$, where $\phi_i = \hat{\phi}_i + \tilde{\phi}_i$, $\hat{\phi}_i$ is the estimated kinematic parameters and $\tilde{\phi}_i$ is the estimation error. This property is useful in the identification of the kinematic parameters.

B. Dynamics Modeling of the Dual-Arm Robot

The dynamics model of the ith robotic arm is given by the Lagrangian method as follows:

$$H_i(q_i)\ddot{q}_i + D_i(q_i, \dot{q}_i)\dot{q}_i + G_i(q_i) = \tau_i + J_{e_i}^T(q_i)F_{e_i} \quad (4)$$

where $H_i(q_i) \in \mathbb{R}^{N_i \times N_i}$, $D_i(q_i, \dot{q}_i) \in \mathbb{R}^{N_i \times N_i}$, and $G_i(q_i) \in \mathbb{R}^{N_i}$ are the inertial matrix, Coriolis matrix, and gravity term, respectively. N_i denotes the DOF of the ith robotic arm. $J_{e_i}^T(q_i)$ is the Jacobian matrix of the ith robotic arm, while $\tau_i \in \mathbb{R}^{N_i}$ is the joint torque and $F_{e_i} \in \mathbb{R}^{N_0}$ is the external force exerted at the ith end effector. The dynamics of the object can be described as follows:

$$H_o(x)\ddot{x} + D_o(x, \dot{x})\dot{x} + G_o(x) = F_o \quad (5)$$

where $H_o(x) \in \mathbb{R}^{N_0 \times N_0}$ denotes the inertial matrix of the object, $D_o(x, \dot{x}) \in \mathbb{R}^{N_0 \times N_0}$ is the Coriolis and centrifugal matrix, $G_o(x) \in \mathbb{R}^{N_0 \times N_0}$ is the object's gravity term, and $F_o \in \mathbb{R}^{N_0}$ denotes the result force exerted on the object. As shown in Fig. 1, the force F_o can be represented by two force vectors as follows:

$$F_o = -F_{oe_1} - F_{oe_2} \quad (6)$$

where $F_{oe_i} \in \mathbb{R}^{N_0}$ is the interaction force applied by the object on the ith end effector. The relationship between F_{oe_i} and F_{e_i} is described as

$$F_{oe_i} = J_{oe_i}^T(x)F_{e_i} \quad (7)$$

where $J_{oe_i}(x) \in \mathbb{R}^{N_0 \times N_0}$ is the manipulated Jacobian matrix from the ith manipulator's end effector to the mass center of the object. The force F_{oe_i} can be decomposed into an external force and an internal force, such that

$$F_{oe_i} = f_i + f_{o_i} \quad (8)$$

where the external forces f_{oi} only contribute to the motion of the object, while the internal forces f_i satisfy $f_1 + f_2 = 0_{[n]}$. The combination of (5), (6), and (8) yields

$$H_o(x)\ddot{x} + D_o(x, \dot{x})\dot{x} + G_o(x) = -f_{o_1} - f_{o_2}. \quad (9)$$

The external force f_{o_i} could be represented as follows [33]:

$$f_{o_i} = -C_i(t)\Big(H_o(x)\ddot{x} + D_o(x, \dot{x})\dot{x} + G_o(x)\Big) \quad (10)$$

where $C_i(t)$ is a positive-definite $N_0 \times N_0$ diagonal matrix denoting the object load distribution onto the ith robotic arm, with $C_1(t) + C_2(t) = I_{N_0}$; $I_{N_0} \in \mathbb{R}^{N_0 \times N_0}$ is an identity matrix. Substituting (9) and (10) into (8), we have

$$f_i = F_{oe_i} - C_i(t)(f_{o_1} + f_{o_2}). \quad (11)$$

Let us combine (4)–(6), (8), (11), and the kinematic equations (1), (2), the dynamics of the ith robotic arm could be reformulated as follows:

$$\tau_i = \mathcal{H}_i(q_i)\ddot{q}_i + \mathcal{D}_i(q_i, \dot{q}_i)\dot{q}_i + \mathcal{G}_i(q_i) - J_i^T(q_i, \phi_i)f_i \quad (12)$$

where $\mathcal{H}_i = H_i + C_i\mathcal{H}_o$, $\mathcal{H}_o = J_i^T M_o J_i$, $\mathcal{D}_i = D_i + C_i(H_D + \mathcal{D}_o)$, $\mathcal{D}_o = J_i^T D_o J_i$, $H_D = J_i^T \mathcal{H}_o \dot{J}_i$, $\mathcal{G}_i = G_i + C_i\mathcal{G}_o$, $\mathcal{G}_o = J_i^T G_o$, and $J_{ei} = J_{oe_i} J_i$. To facilitate the analysis, a number of useful properties and assumptions are given as follows.

Property 2 (see [7]): The matrix $2\mathcal{D}_i(q_i, \dot{q}_i) - [\dot{\mathcal{H}}_i(q_i) - \dot{C}_i(t)\mathcal{H}_o(q_i, \dot{q}_i)]$ is skew-symmetric and satisfies that

$$\partial^T \Big\{ \big(2\mathcal{D}_i(q_i, \dot{q}_i) - \dot{\mathcal{H}}_i(q_i)\big) - \dot{C}_i(t)\mathcal{H}_o(q_i, \dot{q}_i) \Big\} \partial = 0 \quad \forall \partial$$

Property 3 (see[7]): The term $\dot{C}_i(t)\mathcal{H}_0(q_i)$ is uniformly continuous and bounded by a known constant as

$$\|\dot{C}_i(t)\mathcal{H}_0(q_i)\| \leq 2\varrho \quad \forall t \geq 0$$

where ϱ is a positive constant.

Assumption 1: The Jacobian matrices of the robotic arms are of full rank during the operation.

Assumption 2: The robot grasps the object tightly such that no relative motion or rotation occurs between the object and the grasper. In addition, the rigid object would not be deformed by the applied forces.

Assumption 3: The reference task space trajectory x_d and its time derivative $\dot{x}_d$ are continuous and bounded.

C. Preliminaries

1) FLS: The FLS has been widely applied in the control of nonlinear systems by its powerful ability to approximate complex systems and no requirement of *a priori* experience of system dynamics. Considering the unknown nonlinear function $F(z) \in \mathbb{R}^m$, and the measurable scalar input $z \in \mathbb{R}^n$, with n and m being the dimensions of the input and output, respectively, we employ a multi-input multi-output FLS to approximated an unknown nonlinear system $F(z)$ with the following steps [40].

1) *Fuzzification:* Fuzzification maps a real scalar input z into fuzzy linguistic terms by using membership functions.
2) *Fuzzy IF–THEN rules:* The fuzzy IF–THEN rules are adopted to relate an input set to an output set with the Mamdani min-implication as follows.
   Rule l, $(l = 1, 2, \ldots, L)$: If z_1 is A_1^l, z_2 is A_2^l, $\ldots$, z_n is A_n^l, *then*
   y_1 is B_1^l, y_2 is B_2^l, $\ldots$, y_m is B_m^l, where $z = [z_1, \ldots, z_n] \in \mathbb{R}^n$ and $y = [y_1, \ldots, y_m] \in \mathbb{R}^m$ are the premise variables with respect to the input and output, A_i^l and B_j^l are fuzzy sets, $i = 1, \ldots, n$, $j = 1, \ldots, m$, and L is the number of the rules.
3) *Fuzzy inference engine and defuzzification:* Combining the singleton fuzzifier, sum-product inference, and center-average defuzzifier, the defuzzification can be performed as
$$y_j(z) = \frac{\sum_{l=1}^{L} \Phi_j^l \prod_{i=1}^{n} \mu_{A_i^l}(z_i)}{\sum_{l=1}^{L} \prod_{i=1}^{n} \mu_{A_i^l}(z_i)} \tag{13}$$
   where $\mu_{A_i^l}(z_i)$ is the membership function of the linguistic variable $A_i^l(z_i)$, and $\Phi_j^l = \max_{y_j(z)\in\mathbb{R}}\{\mu_{B_j^l}(\Phi_l)\}$ is the point at which $\mu_{B_j^l}(\Phi_j^l)$ achieves its maximum value. Let $p_l(z) = \frac{\prod_{i=1}^{n} \mu_{A_i^l}(z_i)}{\sum_{l=1}^{L} \prod_{i=1}^{n} \mu_{A_i^l}(z_i)}$, $P(z) = [p_1(z), p_2(z), \ldots, p_L(z)]^T \in \mathbb{R}^L$, $W_j = [\Phi_j^1, \Phi_j^2, \ldots, \Phi_j^L]^T \in \mathbb{R}^L$, $W = [W_1, W_2, \ldots, W_m] \in \mathbb{R}^{L\times m}$. Then, the FLSs can be rewritten as follows:
$$y(z) = W^T P(z). \tag{14}$$

Lemma 1 (see[31]): Let $f_j(z) \in \mathbb{R}$ be a continuous function defined on a compact set Ω_j; then, for any given constant ϵ_j, there exists an FLS such that
$$\sup_{f_j(z)\in\Omega_j} |f_j(z) - y_j(z)| \le \epsilon_j \tag{15}$$
where $y_j = W_j^T P(z)$.

In terms of Lemma 1, $F(z) = [f_1(z), f_2(z), \ldots, f_m(z)] \in \mathbb{R}^m$ can be formulated as
$$F(z) = W^T P(z) + \varepsilon \tag{16}$$
where $W \in \mathbb{R}^{L\times m}$ is the optimal weight matrix, $P \in \mathbb{R}^L$ is the fuzzy basis vector, $\varepsilon \in \mathbb{R}^m$ is the approximation error, and L is the number of fuzzy rules.

2) SLA [41]: The SLA means that for any bounded trajectory z that remains in a compact set Ω, the unknown function $f(z)$ can be approximated by a limited number of fuzzy rules in a local region (close to the trajectory z) as
$$f(z) = W_\xi^{*T} P_\xi(z) + \varepsilon_\xi(z) \tag{17}$$
where $P_\xi(z) = [p_{1_\xi}(z), \ldots, p_{L_\xi}(z)]^T \in \mathbb{R}^{L_\xi}$ denotes a subvector of P, with $L_\xi < L$, $|p_{l\xi}| > \varsigma$ with ς being a small positive constant, $W_\xi^* = [W_{j1}^{*T}, \ldots, W_{j\xi}^{*T}]^T$ represents the corresponding weight matrix, and ε_ξ is the construction error, which satisfies $||\varepsilon_\xi(z) - \varepsilon(z)|| \le \sigma$, with σ being a small value.

3) PPE Condition: In the parameter estimation, the PE condition is important to ensure the convergence of estimated parameters. The definition of the PE condition could be described as follows.

Definition 1 (see[36]): A vector or matrix $P(t)$ is called persistently excited if there exist two constants $T > 0$ and $\varpi > 0$, such that
$$\int_t^{t+T} P^T(r)P(r)\, dr \ge \varpi I \quad \forall t \ge 0.$$

Note that for FLS, the PE condition is relatively strict and not easy to be satisfied. To relax the PE condition, we introduce a PPE condition for the FLS with the Gaussian membership function, which is established by the following theorem.

Theorem 1: Consider a periodic trajectory $Z(t)$, which is continuous on a compact set Ω, and $\dot{Z}(t)$ is bounded. For an FLS $W^T P(Z)$ with $P(Z)$ chosen to be Gaussian fuzzy membership functions, and the centers placed on a regular lattice (large enough to cover the compact set Ω), the regressor subvector $P_\xi(Z(t))$ could satisfy the PE condition.

Proof: See the Appendix.

Remark 1: In the previous studies [39], [41], [42], the SLA of the RBF-based neural network was well established, such that the accurate approximation can be achieved by a limited number of neural nodes. This property has been widely employed in the identification of system dynamics such as robotic manipulators [39], unmanned surface vessels [42], fault detection system [43], etc. Inspired by the above discussion, in this paper, we investigate the PPE property of the FLS. We have proved that the PPE condition of FLS holds if the inputs trajectory is periodic, which means that the PPE condition of the FLS could be more easy to satisfy.

Remark 2: Theorem 1 shows that for the GFBFs whose centers are close to the periodic trajectory, the PE condition could be relaxed to PPE condition, and hence, the convergence of the estimated weights is more easy to satisfy. On the other hand, for the GFBFs whose centers are far away from the tracking trajectory, the weights are only slightly updated.

III. Control Design

A. Control Design for the Closed-Loop Robot System

Step 1: Define the tracking error as $e_x = x(t) - x_d(t)$. Taking its differentiation with respect to time, we have
$$\dot{e}_x = \dot{x}(t) - \dot{x}_d(t) = J_i(q_i, \phi_i)\dot{q}_i - \dot{x}_d \tag{18}$$

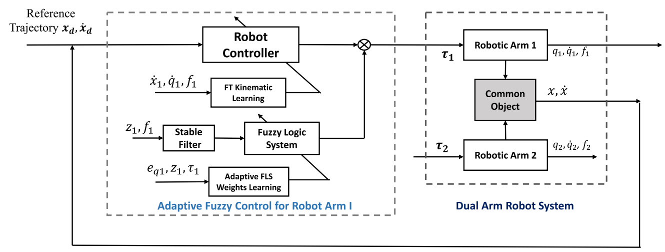


Fig. 2. Control strategy of the dual-arm robot with uncertain dynamics and kinematics.

where $\dot{x} = J_i(q_i, \phi_i)\dot{q}_i$ holds according to (2). Then, an auxiliary controller $\dot{q}_{d_i}$ is designed as follows:

$$\dot{q}_{d_i} = J_i^+(q_i, \hat{\phi}_i)(\dot{x}_d - K_e e_x - \alpha_{f_i}) \tag{19}$$

where $J_i^+(q_i, \hat{\phi}_i)$ is the Moore–Penrose inverse of $J_i(q_i, \hat{\phi}_i)$, K_e is the control gain, α_{f_i} is an auxiliary term defined by $\alpha_{f_i} = \beta_{f_i}\Lambda_{F_i}$, with β_{f_i} being a positive constant, and Λ_{F_i} is a leakage term, which will be designed later. The combination of (3), (18), and (19) yields

$$\dot{e}_x = -K_e e_x + J_i(q_i, \phi_i)e_{q_i} + R_i(q_i, \dot{q}_{d_i})\tilde{\phi}_i - \alpha_{f_i} \tag{20}$$

where $e_{q_i} = \dot{q}_i - \dot{q}_{d_i}$.

Step 2: Substituting (18) and (19) into (12), we have

$$\begin{aligned}\mathcal{H}_i(q_i)\dot{e}_{q_i} + \mathcal{D}_i(q_i, \dot{q}_i)e_{q_i} &= \tau_i - \mathcal{H}_i(q_i)\ddot{q}_{d_i} - \mathcal{D}_i(q_i, \dot{q}_i)\dot{q}_{d_i} \\ &\quad - \mathcal{G}_i(q_i) + J_i(q_i, \phi_i)f_i.\end{aligned} \tag{21}$$

Let us define

$$F_i(z_i) = \mathcal{H}_i(q_i)\ddot{q}_{d_i} + \mathcal{D}_i(q_i, \dot{q}_i)\dot{q}_{d_i} + \mathcal{G}_i(q_i) \tag{22}$$

where $z_i = [\ddot{q}_{d_i}^T, \dot{q}_{d_i}^T, q_i^T, \dot{q}_i^T]^T$. Since $\mathcal{H}_i(q_i)$, $\mathcal{D}_i(q_i, \dot{q}_i)$, and $\mathcal{G}_i(q_i)$ are not available, we use an FLS to approximate (22) as

$$F_i(z_i) = W_{F_i}^T P_{F_i} + \varepsilon_{F_i} \tag{23}$$

where $W_{F_i} \in \mathbb{R}^{\sigma_{F_i} \times N_i}$ is the optimal FLS weight matrix, and $P_{F_i}(z_i) = [P_{F_{i_1}}(z_i), P_{F_{i_2}}(z_i), \ldots, P_{F_{i\sigma_{F_i}}}(z_i)]^T \in \mathbb{R}^{\sigma_{F_i}}$ is the fuzzy basis vector, σ_{F_i} is the number of fuzzy rules, and $\varepsilon_{F_i} \in \mathbb{R}^{N_i}$ is the construction error.

Then, the controller τ_i can be designed as follows:

$$\begin{aligned}\boldsymbol{\tau_i} = \hat{F}_i(e_{q_i}) - K_i e_{q_i} - J_i^T(q_i, \hat{\phi}_i)(e_x - K_{f_i}\Lambda_{f_i} + f_{d_i}) \\ + \Upsilon_i(e_{q_i})\end{aligned} \tag{24}$$

where K_i is the selected control gain, $\hat{F}_i(z_i) = \hat{W}_{F_i}^T P_{F_i}(z_i)$, and $\hat{W}_{F_i} \in \mathbb{R}^{\sigma_{F_i} \times N_i}$ is the estimation of W_{F_i}; f_{di} is the desired internal force. The robust term $\Upsilon_i(e_{q_i})$ is designed as $\Upsilon_i(e_{q_i}) = \begin{cases} -\beta_i \frac{e_{q_i}}{\|e_{q_i}\|}, & e_{q_i} \neq 0 \\ 0, & \text{otherwise} \end{cases}$, and β_i is a selected positive constant. And Λ_{f_i} is defined as $\Lambda_{f_i} = \int_0^t \tilde{f}_i dt$; $\tilde{f}_i = f_i - f_{d_i}$. The overall control scheme is shown in Fig. 2.

Substituting (23) and (24) into (21), the closed-loop error dynamics of the dual-arm robot can be rewritten as

$$\begin{aligned}\mathcal{H}_i(q_i)\dot{e}_{q_i} + \mathcal{D}_i(q_i, \dot{q}_i)e_{q_i} &= -K_i e_{q_i} - J_i^T(q_i, \hat{\phi}_i)e_x \\ &+ \Upsilon_i(e_{q_i}) + J_i^T(q_i, \phi_i)f_i - J_i^T(q_i, \hat{\phi}_i)(f_{d_i} - K_{f_i}\Lambda_{f_i}) \\ &+ \varepsilon_{F_i} + \tilde{W}_{F_i}^T S_{F_i}(z_i)\end{aligned} \tag{25}$$

where $\tilde{W}_{F_i} = \hat{W}_{F_i} - W_{F_i}$.

B. FT Kinematic Parameter Estimation Design

In this section, we will preform the parameter estimation design of the unknown system parameters.

1) Finite-Time Convergence Parameter Adaptation (FCPA) Design of the Kinematic Parameters: In order to derive the parameter estimation of $\hat{\phi}_i$, we define an auxiliary matrix $\mathcal{U}_i \in \mathbb{R}^{h_i \times h_i}$ and two vectors $\mathcal{T}_i \in \mathbb{R}^{N_i}$, $\mathcal{P}_i \in \mathbb{R}^{N_i}$ as

$$\begin{cases} \dot{\mathcal{U}}_i = -\zeta_i \mathcal{U}_i + R_i(q_i, \dot{q}_i)^T R_i(q_i, \dot{q}_i), & \mathcal{U}_i(0) = 0 \\ \dot{\mathcal{T}}_i = -\zeta_i \mathcal{T}_i + R_i(q_i, \dot{q}_i)^T \dot{x}, & \mathcal{T}_i(0) = 0 \\ \mathcal{P}_i = \mathcal{T}_i - \mathcal{U}_i\hat{\phi}_i \end{cases} \tag{26}$$

where ζ_i is a positive constant that introduces a forgetting factor for the filter matrix, which can be designed to make a tradeoff of the robustness and the convergence rate.

By integrating on both sides of (26) with respect to time, the solution of $\mathcal{U}_i$ and $\mathcal{T}_i$ can be derived as

$$\begin{cases} \mathcal{U}_i(t) = \int_0^t e^{-\zeta_i(t-r)} R_i(q_i(r), \dot{q}_i(r))^T R_i(q_i(r), \dot{q}_i(r))dr \\ \mathcal{T}_i(t) = \int_0^t e^{-\zeta_i(t-r)} R_i(q_i(r), \dot{q}_i(r))^T \dot{x}(r)dr. \end{cases} \tag{27}$$

Remark 3: From the analysis in Section II, we have $\dot{x} = R(q_i, \dot{q}_i)\phi_i$. Comparing the structure between $\mathcal{T}_i$ and $\mathcal{U}_i$ in (27), we can derive that $\mathcal{T}_i = \mathcal{U}_i\phi_i$. Then, $\mathcal{P}_i$ can be represented by $\mathcal{P}_i = \mathcal{U}_i\phi_i - \mathcal{U}_i\hat{\phi}_i = \mathcal{U}_i\tilde{\phi}_i$. In this sense, the term $\mathcal{P}_i$ contains the information of estimation error of ϕ_i. This is important to improve the estimation performance.

Then, the updating law for $\hat{\phi}_i$ is designed by using the following projection algorithm:

$$(\dot{\hat{\phi}}_i)_k = \begin{cases} -\lambda_i\left(R_i^T(q_i, \dot{q}_{d_i})e_x + \kappa_i\frac{\mathcal{U}_i^T\mathcal{P}_i}{\|\mathcal{P}_i\|} + \mathcal{N}_i\right)_k, \\ \text{if} \quad (\underline{\phi}_i)_k \le (\hat{\phi}_i)_k \le (\overline{\phi}_i)_k \\ \text{or if} \quad (\hat{\phi}_i)_k = (\underline{\phi}_i)_k \quad \text{and} \quad (\phi_{i_F})_k \le 0 \\ \text{or if} \quad (\hat{\phi}_i)_k = (\overline{\phi}_i)_k \quad \text{and} \quad (\phi_{i_F})_k \ge 0 \\ 0, \quad \text{otherwise} \end{cases} \tag{28}$$

where λ_i and κ_i are positive constants to be specified by the designer, $(\cdot)_k$ denotes the kth element of the vector $(\cdot)$, $\mathcal{N}_i$ is a leakage term which will be designed later, $\phi_{i_F} = -R_i^T(q_i, \dot{q}_{d_i})e_x - \kappa_i\frac{\mathcal{U}_i^T\mathcal{P}_i}{\|\mathcal{P}_i\|} - \mathcal{N}_i$, and $\overline{\phi}_i$ and $\underline{\phi}_i$ are the known lower and upper bounds of the real value of ϕ_i, respectively.

2) FCPA Design of the FLS Weights: In order to achieve the FT estimation of the FLS weight parameters without using the joint acceleration $\ddot{q}_i$, the following functions have been designed:

$$\begin{cases} \Xi_i(z_{c_i}) = \mathcal{H}_i e_{q_i} \\ \Psi_i(z_{d_i}) = -\dot{\mathcal{H}}_i e_{q_i} + \mathcal{D}_i(q_i, \dot{q}_i) e_{q_i} - J_i^T(q_i, \phi_i) f_i \end{cases} \tag{29}$$

where $z_{c_i} = [q_i^T, \dot{q}_i^T, \dot{q}_{d_i}^T]^T$, $z_{d_i} = [q_i^T, \dot{q}_i^T, \dot{q}_{d_i}^T, f_i^T]^T$. By combining (22) and (29), and considering that $\dot{\Xi}(z_{c_i}) = \mathcal{H}_i \dot{e}_{q_i} + \dot{\mathcal{H}}_i e_{q_i}$, the closed-loop dual-arm robot system (12) can be represented as

$$F_i(z_i) + \dot{\Xi}_i(z_{c_i}) + \Psi_i(z_{d_i}) = \tau_i. \tag{30}$$

Since $\Xi_i(z_{c_i})$ and $\Psi_i(z_{d_i})$ are not available, the following FLS is constructed for Ξ_i and Ψ_i as

$$\begin{cases} \Xi_i(z_{c_i}) = W_{\Xi_i}^T P_{\Xi_i} + \varepsilon_{\Xi_i} \\ \Psi_i(z_{d_i}) = W_{\Psi_i}^T P_{\Psi_i} + \varepsilon_{\Psi_i} \end{cases} \tag{31}$$

where $W_{\Xi_i} \in \mathbb{R}^{\sigma_{\Xi} \times N_i}$ and $W_{\Psi_i} \in \mathbb{R}^{\sigma_{\Psi_i} \times N_i}$ are the optimal weight matrices, $P_{\Xi_i} \in \mathbb{R}^{\sigma_{\Xi_i}}$ and $P_{\Psi_i} \in \mathbb{R}^{\sigma_{\Psi_i}}$ are the fuzzy basis vectors, and ε_{Ψ_i} and ε_{Ξ_i} are the construction errors.

Inspired by the work in [36], we introduce a stable linear filter $(\cdot)_f = \frac{1}{b_i s+1}(\cdot)$, $b_i > 0$ on both sides of (30) as follows:

$$\begin{aligned} F_{if}(z_i) + \dot{\Xi}_{if}(z_{c_i}) + \Psi_{if}(z_{d_i}) &= W_{F_i}^T P_{F_i f} + W_{\Xi_i}^T \dot{P}_{\Xi_i f} \\ &+ W_{\Psi_i}^T P_{\Psi_i f} + \varepsilon_{F_i f} + \dot{\varepsilon}_{\Xi_i f} + \varepsilon_{\Psi_i f} = \tau_{if} \end{aligned} \tag{32}$$

where $P_{F_i f} \in \mathbb{R}^{\sigma_{F_i}}$, $P_{\Xi_i f} \in \mathbb{R}^{\sigma_{\Xi_i}}$, $P_{\Psi_i f} \in \mathbb{R}^{\sigma_{\Psi_i}}$, $\varepsilon_{F_i f} \in \mathbb{R}^{N_i}$, $\varepsilon_{\Xi_i f} \in \mathbb{R}^{N_i}$, $\varepsilon_{\Psi_i f} \in \mathbb{R}^{N_i}$, and $\tau_{if} \in \mathbb{R}^{N_i}$ are the filtered version of P_{Ξ_i}, P_{Ψ_i}, P_{F_i}, ε_{F_i}, ε_{Ξ_i}, ε_{Ψ_i}, and τ_i, respectively, and given as follows:

$$\begin{cases} b_i \dot{P}_{F_i f} + P_{F_i f} = P_{F_i} & b_i \dot{\varepsilon}_{F_i f} + \varepsilon_{F_i f} = \varepsilon_{F_i} \\ b_i \dot{P}_{\Xi_i f} + P_{\Xi_i f} = P_{\Xi_i} & b_i \dot{P}_{\Psi_i f} + P_{\Psi_i f} = P_{\Psi_i} \\ b_i \dot{\varepsilon}_{\Xi_i f} + \varepsilon_{\Xi_i f} = \varepsilon_{\Xi_i} & b_i \dot{\varepsilon}_{\Psi_i f} + \varepsilon_{\Psi_i f} = \varepsilon_{\Psi_i} \\ b_i \dot{\tau}_{if} + \tau_{if} = \tau_i \end{cases} \tag{33}$$

where b_i is a positive constant, $P_{F_i}(0) = 0$, $\varepsilon_{F_i}(0) = 0$, $P_{\Xi_i f}(0) = 0$, $P_{\Psi_i f}(0) = 0$, $\varepsilon_{\Xi_i f}(0) = 0$, $\varepsilon_{\Psi_i f}(0) = 0$, and $\tau_i(0) = 0$. Then, the filtered dynamics can be rewritten as

$$\tau_{if} = W_i^T S_i(Z_i) + \varepsilon_i \tag{34}$$

where $W_i = [W_{F_i}^T, \psi_i^T]^T \in \mathbb{R}^{\sigma_i \times N_i}$ and $\psi_i = [W_{\Xi_i}^T, W_{\Psi_i}^T]^T \in \mathbb{R}^{\sigma_{Y_i} \times N_i}$ are the optimal weight matrices of the FLS, $S_i(Z_i) = [P_{F_{if}}^T(z_i), P_{Y_i}^T(c_i)]^T \in \mathbb{R}^{\sigma_i}$, and $P_{Y_i}(c_i) = [\frac{P_{\Xi_i}^T - P_{\Xi_i f}^T}{b_i}, P_{\Psi_i f}^T]^T \in \mathbb{R}^{\sigma_{Y_i}}$ is the fuzzy basis function, $\sigma_i = \sigma_{F_i} + \sigma_{Y_i}$ with σ_{Y_i} being the number of fuzzy rules; $\varepsilon_i = \varepsilon_{F_{if}} + \varepsilon_{Y_i} \in \mathbb{R}^{N_i}$ and $\varepsilon_{Y_i} = \frac{\varepsilon_{\Xi_i} - \varepsilon_{\Xi_i f}}{b_i} + \varepsilon_{\Psi_i f} \in \mathbb{R}^{\Psi_i}$ are the approximation errors. To derive the parameter estimation of W_i, we design the auxiliary filter matrices $\mathcal{L}_i \in \mathbb{R}^{\sigma_{Y_i} \times \sigma_{Y_i}}$, $\mathcal{Q}_i \in \mathbb{R}^{\sigma_{Y_i} \times N_i}$, and $\mathcal{V}_i \in \mathbb{R}^{\sigma_{Y_i} \times N_i}$ as

$$\begin{cases} \dot{\mathcal{L}}_i = -\delta_i \mathcal{L}_i + S_i(z_i) S_i^T(z_i), & \mathcal{L}_i(0) = 0 \\ \dot{\mathcal{Q}}_i = -\delta_i \mathcal{Q}_i + S_i(z_i) \tau_{if}^T, & \mathcal{Q}_i(0) = 0 \\ \mathcal{V}_i = \mathcal{L}_i \hat{W}_i - \mathcal{Q}_i \end{cases} \tag{35}$$

where δ_i is a positive constant. The solutions of $\mathcal{L}_i$ and $\mathcal{Q}_i$ are derived by integrating on both sides of (35) as

$$\begin{cases} \mathcal{L}_i(t) = \int_0^t e^{-\delta_i(t-r)} S_i(z_i(r)) S_i(z_i(r))^T dr \\ \mathcal{Q}_i(t) = \int_0^t e^{-\delta_i(t-r)} S_i(z_i(r))^T \tau_i dr. \end{cases} \tag{36}$$

From the definition of $\mathcal{V}_i$, $\mathcal{L}_i$, and $\mathcal{Q}_i$ in (35) and (36), we have

$$\mathcal{V}_i = \mathcal{L}_i \hat{W}_i - \mathcal{L}_i W_i + \chi_i = \mathcal{L}_i \tilde{W}_i + \chi_i \tag{37}$$

where $\chi_i = -\int_0^t \exp(-\delta_i(t-r)) S_i(z_i(r)) \varepsilon_i(r) dr$. Since ε_i and the basis function $S_i(z_i)$ are bounded, the χ_i is also bounded and satisfies that $\chi_i \leq \bar{\chi}_i$, where $\bar{\chi}_i$ is the upper bound of χ_i. Then, the FLS weight updating law $\dot{\hat{W}}_i$ is designed as follows:

$$\dot{\hat{W}}_i = -\Gamma_i \left(M_i S_{F_i}(z_i) e_{q_i}^T + \gamma_i \frac{\mathcal{L}_i^T \mathcal{V}_i}{||\mathcal{V}_i||} \right) \tag{38}$$

where $M_i = \left[\begin{smallmatrix} I_{\sigma_{F_i} \times \sigma_{F_i}} \\ 0_{\sigma_{Y_i} \times \sigma_{F_i}} \end{smallmatrix}\right]$ and γ_i is a positive constant.

C. Stability Analysis

The stability analysis of the proposed control algorithm is established by the following theorem.

Theorem 2: Consider the dual-arm robot system grasping a common object in (12). Assume that the regressor matrices $R_i(q_i, \dot{q}_i)$ and $P_i(Z_i)$ satisfy the PE condition. Then, the adaptive control input τ_{d_i} developed in (24) with the auxiliary controllers $\dot{q}_{d_i}$ in (19), as well as the parameter adaptation law (28) and the FLS weight adaptation law in (38), can guarantee that all the signals in the closed-loop system are uniformly ultimately bounded, the tracking error e_x converges to a small residual set around zero, and the estimate error $\tilde{\phi}_i$ and $\tilde{W}_i$ could converge to a small neighborhood around zero.

Proof: Consider the following Lyapunov function for the dual-arm robot:

$$V_i = V_{i1} + V_{i2} + V_{i3} \tag{39}$$

where

$$V_{i1} = \frac{1}{2} e_x^T e_x + \frac{1}{2\lambda_i} \tilde{\phi}_i^T \tilde{\phi}_i \tag{40}$$

$$V_{i2} = \frac{1}{2} e_{q_i}^T \mathcal{H}_i e_{q_i} + \frac{1}{2} \beta_i \Lambda_{f_i}^T \Lambda_{f_i} \tag{41}$$

$$V_{i3} = \frac{1}{2} tr\left(\tilde{W}_i^T \Gamma_i^{-1} \tilde{W}_i \right) \tag{42}$$

and $\tilde{\phi}_i = \phi_i - \hat{\phi}_i$, β_i is a positive constant.

The differentiation of (40) with respect to time gives us

$$\dot{V}_{i1} = e_x^T \dot{e}_x + \frac{1}{\lambda_i} \tilde{\phi}_i^T \dot{\tilde{\phi}}_i. \tag{43}$$

Substituting (20) and the updating law (28) into (43) yields

$$\begin{aligned}\dot{V}_{i1} &= e_x^T\Big(-K_e e_x + J_i(q_i,\phi_i)e_{q_i} + R_i(q_i,\dot{q}_{di})\tilde{\phi}_i - \alpha_{f_i}\Big)\\&\quad + \frac{1}{\lambda_i}\tilde{\phi}_i^T\dot{\hat{\phi}}_i\\&= -e_x^T K_e e_x + e_x^T J_i(q_i,\hat{\phi}_i)e_{q_i} + e_x^T J_i(q_i,\tilde{\phi}_i)e_{q_i} - e_x^T\alpha_{f_i}\\&\quad - \tilde{\phi}_i^T\Big(R_i^T(q_i,\dot{q}_i)e_x + \kappa_i\frac{\mathcal{U}_i^T\mathcal{P}_i}{||\mathcal{P}_i||} + \mathcal{N}_i\Big) + e_x^T R_i(q_i,\dot{q}_{di})\tilde{\phi}_i.\end{aligned} \tag{44}$$

As mentioned above, $\mathcal{P}_i$ can be represented by $\mathcal{P}_i = \mathcal{U}_i\tilde{\phi}_i$. Then, (44) can be rewritten as

$$\begin{aligned}\dot{V}_{i1} &= -e_x^T K_e e_x + e_x^T J_i(q_i,\hat{\phi}_i)e_{q_i} - \kappa_i\tilde{\phi}_i^T\frac{\mathcal{U}_i^T\mathcal{U}_i\tilde{\phi}_i}{||\mathcal{U}_i\tilde{\phi}_i||} - \tilde{\phi}_i^T\mathcal{N}_i\\&\quad - e_x^T\alpha_{f_i}.\end{aligned} \tag{45}$$

Taking the derivative of V_{i2} with respect to time yields

$$\dot{V}_{i2} = e_{q_i}^T\mathcal{H}_i\dot{e}_{q_i} + \frac{1}{2}e_{q_i}^T\dot{\mathcal{H}}_i e_{q_i} + \beta_i\Lambda_{f_i}^T\tilde{f}_i. \tag{46}$$

Substituting the error dynamics of the closed-loop system (25) into (46), we have

$$\begin{aligned}\dot{V}_{i2} &= e_{q_i}^T\Big(-K_i e_{q_i} + \Upsilon_i(e_{q_i}) + \tilde{W}_{F_i}^T S_{F_i}(z_i) + \varepsilon_{F_i}\Big)\\&\quad + e_{q_i}^T\Big(-J_i^T(q_i,\hat{\phi}_i)(e_x + f_{d_i} - K_{f_i}\Lambda_{f_i}) + J_i^T(q_i,\phi_i)f_i\Big)\\&\quad + \frac{1}{2}e_{q_i}^T\dot{\mathcal{H}}_i e_{q_i} - e_{q_i}^T\mathcal{D}_i(q_i,\dot{q}_i)e_{q_i} + \beta_i\Lambda_{f_i}^T\tilde{f}_i.\end{aligned} \tag{47}$$

In terms of Properties 2 and 3 of the dual-arm robot, and employing the inequality $e_{q_i}^T\big(2\mathcal{D}_i(q_i,\dot{q}_i) - \dot{\mathcal{H}}_i(q_i)\big)e_{q_i} \leq \varrho_i e_{q_i}^T e_{q_i}$, (47) can be rewritten as

$$\begin{aligned}\dot{V}_{i2} \leq &-\Big(K_i - \varrho_i\Big)e_{q_i}^T e_{q_i} + e_{q_i}^T\tilde{W}_{F_i}^T S_{F_i}(z_i) + e_{q_i}^T\varepsilon_{F_i}\\&- e_{q_i}^T J_i^T(q_i,\hat{\phi}_i)e_x + e_{q_i}^T J_i^T(q_i,\phi_i)f_i - \beta_i||e_{q_i}||\\&- e_{q_i}^T J_i^T(q_i,\hat{\phi}_i)\Big(f_{d_i} - K_{f_i}\Lambda_{f_i}\Big) + \beta_{f_i}\Lambda_{f_i}^T\tilde{f}_i.\end{aligned} \tag{48}$$

Note that $J_i(q_i,\phi_i)e_{q_i} = \dot{e}_x + K_e e_x - R_i(q_i,\dot{q}_{d_i})\tilde{\phi}_i + \alpha_{f_i}$ and $J_i(q_i,\hat{\phi}_i)e_{q_i} = \dot{e}_x + K_e e_x - R_i(q_i,\dot{q}_i)\tilde{\phi}_i + \alpha_{f_i}$ hold according to (20); then, the combination of (45) and (48) yields

$$\begin{aligned}&\dot{V}_{i1} + \dot{V}_{i2}\\&\leq -e_x^T K_e e_x - \Big(K_i - \varrho_i\Big)e_{q_i}^T e_{q_i} - \beta_i||e_{q_i}|| - \kappa_i||\mathcal{U}_i\tilde{\phi}_i||\\&- \tilde{\phi}_i^T\mathcal{N}_i + \Big(\dot{e}_x + K_e e_x + \alpha_{f_i}\Big)^T(f_i - f_{di} + K_{f_i}\Lambda_{f_i})\\&- \tilde{\phi}_i^{\ T}R_i^T(q_i,\dot{q}_{d_i})f_i + \tilde{\phi}_i^{\ T}R_i^T(q_i,\dot{q}_i)f_{di} + e_{q_i}^T\tilde{W}_{F_i}^T S_{F_i}(z_i)\\&- \tilde{\phi}_i^{\ T}R_i^T(q_i,\dot{q}_i)K_{f_i}\Lambda_{f_i} + e_{q_i}^T\varepsilon_{F_i} + \beta_{f_i}\Lambda_{f_i}^T\tilde{f}_i - e_x^T\beta_{f_i}\Lambda_{F_i}.\end{aligned} \tag{49}$$

Let us design $\mathcal{N}_i$ to be $\mathcal{N}_i = R_i^T(q_i,\dot{q}_i)f_{di} - R_i^T(q_i,\dot{q}_{d_i})f_i - R_i^T(q_i,\dot{q}_i)K_{f_i}\Lambda_{f_i}$, and substituting it into (49) yields

$$\begin{aligned}&\dot{V}_{i1} + \dot{V}_{i2}\\&\quad\leq -e_x^T K_e e_x - \Big(K_i - \varrho_i\Big)e_{q_i}^T e_{q_i} - \beta_i||e_{q_i}|| - \kappa_i||\mathcal{U}_i\tilde{\phi}_i||\\&\quad + \Big(\dot{e}_x + K_e e_x + \alpha_{f_i}\Big)^T(\tilde{f}_i + K_{f_i}\Lambda_{f_i}) + e_{q_i}^T\tilde{W}_{F_i}^T S_{F_i}(z_i)\\&\quad + e_{q_i}^T\varepsilon_{F_i} + \beta_{f_i}\Lambda_{f_i}^T\tilde{f}_i - e_x^T\beta_{f_i}\Lambda_{F_i}.\end{aligned} \tag{50}$$

Then, let us consider the convergence of the FLS weight errors. Taking the derivative of the V_{i3} in (42), we have

$$\begin{aligned}\dot{V}_{i3} &= \mathrm{tr}\Big(\tilde{W}_i^T\Gamma_i^{-1}\dot{\hat{W}}_i\Big) = -e_{q_i}^T\tilde{W}_{F_i}^T S_{F_i}(z_i) - \mathrm{tr}\Big(\gamma_i\tilde{W}_i^T\frac{\mathcal{L}_i^T\mathcal{V}_i}{||\mathcal{V}_i||}\Big)\\&= -e_{q_i}^T\tilde{W}_{F_i}^T S_{F_i}(z_i) - \gamma_i\frac{||\mathcal{L}_i||^2}{||\mathcal{V}_i||}||\tilde{W}_i||^2 - \gamma_i\mathcal{C}_i\end{aligned} \tag{51}$$

where $\mathcal{C}_i = \mathrm{tr}(\tilde{W}_i^T\mathcal{L}_i^T\chi_i/||\mathcal{V}_i||)$ is a bounded term.

Combining $\dot{V}_{i1}$, $\dot{V}_{i2}$, and $\dot{V}_{i3}$, and according to the definition of V_i in (39), we can obtain the time derivation of V_i as follows:

$$\begin{aligned}\dot{V}_i &= \dot{V}_{i1} + \dot{V}_{i2} + \dot{V}_{i3}\\&\leq -e_x^T K_e e_x - \Big(K_i - \varrho_i\Big)e_{q_i}^T e_{q_i} - \beta_i||e_{q_i}||\\&\quad - \kappa_i||\mathcal{U}_i\tilde{\phi}_i|| + \Big(\dot{e}_x + K_e e_x + \alpha_{f_i}\Big)^T(\tilde{f}_i + K_{f_i}\Lambda_{f_i})\\&\quad + e_{q_i}^T\tilde{W}_{F_i}^T S_{F_i}(z_i) + e_{q_i}^T\varepsilon_{F_i} + \beta_{f_i}\Lambda_{f_i}^T\tilde{f}_i - e_x^T\beta_{f_i}\Lambda_{F_i}\\&\quad - e_{q_i}^T\tilde{W}_{F_i}^T S_{F_i}(z_i) - \gamma_i\frac{||\mathcal{L}_i||^2}{||\mathcal{V}_i||}||\tilde{W}_i||^2 - \gamma_i\mathcal{C}_i.\end{aligned} \tag{52}$$

Let us consider the following inequalities:

$$e_{q_i}^T\varepsilon_{F_i} \leq \frac{1}{2}e_{q_i}^T e_{q_i} + \frac{1}{2}\varepsilon_{F_i}^T\varepsilon_{F_i} \tag{53}$$

$$-e_x^T\beta_{f_i}\Lambda_{F_i} \leq \frac{\beta_{f_i}}{2}e_x^T e_x + \frac{\beta_{f_i}}{2}\Lambda_{F_i}^T\Lambda_{F_i}. \tag{54}$$

Substituting (53) and (54) into (52), we have

$$\begin{aligned}\dot{V}_i &\leq -e_x^T\Big(K_e - \frac{\beta_{f_i}}{2}\Big)e_x - \Big(K_i - \varrho_i - \frac{1}{2}\Big)e_{q_i}^T e_{q_i} - \beta_i||e_{q_i}||\\&- \kappa_i||\mathcal{U}_i\tilde{\phi}_i|| + \Big(\dot{e}_x + K_e e_x + \alpha_{f_i}\Big)^T(\tilde{f}_i + K_{f_i}\Lambda_{f_i}) - \gamma_i\mathcal{C}_i\\&+ \frac{1}{2}\varepsilon_{F_i}^T\varepsilon_{F_i} + \beta_{f_i}\Lambda_{f_i}^T\tilde{f}_i + \frac{\beta_{f_i}}{2}\Lambda_{f_i}^T\Lambda_{f_i} - \gamma_i\frac{||\mathcal{L}_i||^2}{||\mathcal{V}_i||}||\tilde{W}_i||^2.\end{aligned} \tag{55}$$

Let $\alpha_{f_i} = -\beta_{f_i}\Lambda_{f_i}$, and $K_{f_1} = K_{f_2} = K_f$ and $\beta_{f_1} = \beta_{f_2} = \beta_f$, with K_f and β_f being a positive constants. Then, considering the property of internal forces errors $\tilde{f}_1 + \tilde{f}_2 = 0$, $\Lambda_1 + \Lambda_2 = 0$, and taking the Lyapunov function $V = V_1 +$

$V_2 = \sum_{i=1}^{2}(V_{i1} + V_{i2} + V_{i3})$, we have

$$\begin{aligned}\dot{V} =& \sum_{i=1}^{2}\Big(\dot{V}_{i1} + \dot{V}_{i2} + \dot{V}_{i3}\Big)\\ \leq& -e_x^T(2K_e - \beta_f)e_x - \sum_{i=1}^{2}\Big(K_i - \varrho_i - \frac{1}{2}\Big)e_{q_i}^T e_{q_i}\\ &+\sum_{i=1}^{2}\Big(-\kappa_i\|\mathcal{U}_i\tilde{\phi}_i\| - \beta_i\|e_{q_i}\| + \beta_{f_i}\Lambda_{f_i}^T\tilde{f}_i\Big)\\ &+\sum_{i=1}^{2}\Big(-\beta_{f_i}\Lambda_{f_i}^T\tilde{f}_i - \beta_{f_i}\Lambda_{f_i}^T K_f\Lambda_{f_i} + \frac{\beta_{f_i}}{2}\Lambda_{f_i}^T\Lambda_{f_i}\Big)\\ &-\sum_{i=1}^{2}\Big(\gamma_i\frac{\|\mathcal{L}_i\|^2}{\|\mathcal{V}_i\|}\|\tilde{W}_i\|^2 + \gamma_i\mathcal{C}_i - \frac{1}{2}\varepsilon_{F_i}^T\varepsilon_{F_i}\Big)\\ \leq& -e_x^T(2K_e - \beta_{f_i})e_x - \sum_{i=1}^{2}\Big(K_i - \varrho_i - \frac{1}{2}\Big)e_{q_i}^T e_{q_i}\\ &+\sum_{i=1}^{2}\Big(-\beta_{f_i}\Lambda_{f_i}^T\Big(K_f - \frac{1}{2}\Big)\Lambda_{f_i}\Big)\\ &-\sum_{i=1}^{2}\Big(\gamma_i\frac{\|\mathcal{L}_i\|^2}{\|\mathcal{V}_i\|}\|\tilde{W}_i\|^2 - \frac{1}{2}\varepsilon_{F_i}^T\varepsilon_{F_i} + \gamma_i\mathcal{C}_i\Big). \end{aligned} \tag{56}$$

If the gains K_i and K_e are selected to satisfy $K_i \geq \varrho_i + \frac{1}{2}$ and $K_e \geq \frac{1}{2}\beta_f$, γ_i is chosen to be $\gamma_i \geq \|(\mathcal{L}_i^{-1})^T\Gamma_i^{-1}\omega_i\|$, then, from (56) and the definition of V in (39) and (40)–(42), we can derive that

$$\dot{V} \leq -\eta V + \mu \tag{57}$$

where $\eta = \min[\lambda_{\max}(2K_e - \beta_{f_i}), \lambda_{\max}(\frac{K_i - \varrho_i - I}{\mathcal{H}_i}), \gamma_i\frac{\|\mathcal{L}_i\|^2}{\|\mathcal{V}_i\|}, \beta_{f_i}(K_f - \frac{1}{2})]$, $\mu = \frac{1}{2}\sum_{i=1}^{2}(\varepsilon_{F_i}^T\varepsilon_{F_i} + \gamma_i\mathcal{C}_i)$, $i = 1, 2$. According to the results in [36], we can derive that the tracking errors and estimation errors could converge to a small neighborhood near zero. This completes the proof. ■

Theorem 2 shows that the tracking error e_{q_i} could converge to a small neighborhood of zero. For the estimated parameters $\hat{W}_i$, however, they may not converge to their optimal values since the PE condition of the FLS is not easy to satisfy. According to SLA in (17), the FLS in (23) and (31) can be expressed using localized Gaussian fuzzy basis functions as

$$\begin{cases} F_{ij\xi}(z_i) = W_{F_{ij\xi}}^T P_{F_{ij\xi}} + \varepsilon_{F_{ij\xi}}\\ \Xi_{ij\xi}(z_{c_i}) = W_{\Xi_{ij\xi}}^T P_{\Xi_{ij\xi}} + \varepsilon_{\Xi_{ij\xi}}\\ \Psi_{ij\xi}(z_{d_i}) = W_{\Psi_{ij\xi}}^T P_{\Psi_{ij\xi}} + \varepsilon_{\Psi_{ij\xi}} \end{cases} \tag{58}$$

where $P(\cdot)_{ij\xi}$ and $W(\cdot)_{ij\xi}$ are the subvectors of $P_{(\cdot)}$ and $W_{(\cdot)}$, whose centers are close to the reference trajectory $\varphi_{i\xi} = \{z_{i\xi}, z_{c_i\xi}, z_{d_i\xi}\}$. Then, (34) can be rewritten as $\tau_{ij} = W_{ij\xi}^T P_{ij\xi}(Z_i) + \varepsilon_{ij\xi}$, where $W_{ij\xi} = [W_{F_{ij\xi}}^T, W_{\Xi_{ij\xi}}^T, W_{\Psi_{ij\xi}}^T]^T$ and $P_{ij\xi}(Z_i) = [P_{F_{ijf}\xi}^T, (P_{\Xi_{ij\xi}}^T - P_{\Xi_{ijf}\xi}^T)/2, P_{\Psi_{ijf}\xi}^T]^T$. Since the elements in $P(\cdot)_{ij\xi}$ are activated, we can obtain that the regressors $P_{F_{ij}\xi}$, $P_{\Xi_{ij}\xi}$, $P_{\Psi_{ij}\xi}$, and $P_{ij\xi}$ satisfy the PE condition. For the subvector of FLS weight $\hat{W}_{ij\xi}$, its adaptation law can be designed as

$$\dot{\hat{W}}_{ij\xi} = -\Gamma_{ij\xi}\left(P_{F_{ij\xi}}e_{q_{ij}}^T + \gamma_i\frac{\mathcal{L}_{ij\xi}^T\mathcal{V}_{ij\xi}}{\|\mathcal{V}_{ij\xi}\|}\right) \tag{59}$$

where $P_{F_{ij\xi}}$, $\mathcal{V}_{ij\xi}$, and $\mathcal{L}_{ij\xi}$ are elements of the "subvector" version of P_{F_i}, $\mathcal{V}_i$, and $\mathcal{L}_i$. Similar to (35) and (36), we have

$$\begin{aligned}\mathcal{L}_{ij\xi}(t) &= \int_0^t e^{-\delta_i(t-r)}P_{ij\xi}(Z_i(r))P_{ij\xi}^T(Z_i(r))dr\\ \mathcal{Q}_{ij\xi}(t) &= \int_0^t e^{-\delta_i(t-r)}P_{ij\xi}^T(Z_i(r))\tau_{ij}dr\\ \mathcal{V}_{ij\xi}(t) &= \mathcal{L}_{ij\xi}\tilde{W}_{ij\xi} + \chi_{ij\xi}\end{aligned} \tag{60}$$

where $\chi_{ij\xi} = -\int_0^t \exp(-\delta_i(t-r))P_{ij\xi}(Z_i(r))\varepsilon_{ij\xi}(r)dr$.

Theorem 3: Consider the dual-arm robot system in (12) and provide the auxiliary controllers $\dot{q}_{d_i}$ in (20). Then, for any recurrent trajectory $\varphi_\xi(Z_i)$ starting from the initial condition $Z_i(0) \in \Omega_0$ (where Ω_0 is a properly chosen compact set), the adaptive control input τ_i developed in (24) with the parameter adaptation law in (28) and the FLS weight adaptation law in (59) can guarantee that the tracking error e_x exponentially converges to a small residual set around zero in FT, the estimation error $\tilde{\phi}_1$, $\tilde{\phi}_2$, could converge to a small neighborhood around zero in FT, and the estimated parameters $\hat{W}_i$ converge to small neighborhoods of their optimal values in FT.

Proof: Let us consider the following Lyapunov function for the robotic arm i:

$$L_i = V_{i1} + V_{i2} + V_{i3\xi} \tag{61}$$

where V_{i1} and V_{i2} are defined in (40) and (41) in the proof of Theorem 2, and

$$V_{i3_\xi} = \frac{1}{2}\sum_{j=1}^{N_i}\mathcal{V}_{ij\xi}^T(\mathcal{L}_{ij\xi}^{-1})^T\Gamma_{ij\xi}^{-1}\mathcal{L}_{ij\xi}^{-1}\mathcal{V}_{ij\xi} \tag{62}$$

where $\Gamma_{i\xi}$ is the "subvector" version of Γ_i.

Then, considering the $\mathcal{L}_{ij\xi}^{-1}\mathcal{V}_{ij\xi}$ in $V_{i3\xi}$, the derivation with respect to time gives

$$\frac{\partial\mathcal{L}_{ij\xi}^{-1}\mathcal{V}_{ij\xi}}{\partial t} = \dot{\hat{W}}_{ij\xi} - \mathcal{L}_{ij\xi}^{-1}\dot{\mathcal{L}}_{ij\xi}\mathcal{L}_{ij\xi}^{-1}\chi_{ij\xi} + \mathcal{L}_{ij\xi}^{-1}\dot{\chi}_{ij\xi}. \tag{63}$$

Since $P_{ij\xi}(Z_i)$ is PE and Z_i is bounded, we can obtain that $\mathcal{L}_{ij\xi}^{-1}$ and $\dot{\mathcal{L}}_{ij\xi}^{-1}$ are bounded; then, $\mathcal{L}_{ij\xi}^{-1}$ is also bounded. Then, taking the derivative of $V_{i3\xi}$ with respect to time and substituting (63) in it, we obtain

$$\dot{V}_{i3\xi} = \sum_{j=1}^{N_i}\mathcal{V}_{ij\xi}^T\mathcal{L}_{ij\xi}^{-1}\Gamma_{ij\xi}^{-1}(\dot{\hat{W}}_{ij\xi} + \omega_{ij}) \tag{64}$$

where $\omega_{ij} = -\mathcal{L}_{ij\xi}^{-1}\dot{\mathcal{L}}_{ij\xi}\mathcal{L}_{ij\xi}^{-1}\chi_{ij\xi} + \mathcal{L}_{ij\xi}^{-1}\dot{\chi}_{ij\xi}$.

Then, substituting the updating law (59) into (64), and considering $\mathcal{V}_{ij\xi} = \mathcal{L}_{ij\xi}\tilde{W}_{ij\xi} + \chi_{ij\xi}$, we have

$$\begin{aligned}
\dot{V}_{i3\xi} &= \sum_{j=1}^{N_i} \mathcal{V}_{ij\xi}^T (\mathcal{L}_{ij\xi}^{-1})^T \left(- P_{Fij\xi}^T e_{q_{ij}} - \gamma_i \frac{\mathcal{L}_{ij\xi}^T \mathcal{V}_{ij\xi}}{||\mathcal{V}_{ij\xi}||} + \Gamma_{ij\xi}^{-1}\omega_{ij} \right) \\
&\le \sum_{j=1}^{N_i} \left(- \mathcal{V}_{ij\xi}^T (\mathcal{L}_{ij\xi}^{-1})^T P_{Fij\xi} e_{q_{ij}} + \mathcal{V}_{ij\xi}^T \left(\mathcal{L}_{ij\xi}^{-1}\right)^T \Gamma_{ij\xi}^{-1}\omega_{ij} \right) \\
&+ \sum_{j=1}^{N_i} \left(- \gamma_i \frac{\mathcal{V}_{ij\xi}^T (\mathcal{L}_{ij\xi}^{-1})^T \mathcal{L}_{ij\xi} \mathcal{V}_{ij\xi}}{||\mathcal{V}_{ij\xi}||} \right) \\
&\le \sum_{j=1}^{N_i} \left(- \tilde{W}_{ij\xi}^T P_{F_{ij\xi}} e_{q_{ij}} + \rho_{ij} ||e_{q_{ij}}|| \right) \\
&- \sum_{j=1}^{N_i} \left((\gamma_i - ||(\mathcal{L}_{ij\xi}^{-1})^T \Gamma_{ij\xi}^{-1}\omega_{ij}||) ||\mathcal{V}_{ij\xi}|| \right)
\end{aligned} \tag{65}$$

where ρ_{ij} is a positive parameter defined by $\rho_{ij} = ||\chi_{ij\xi}^T (\mathcal{L}_{ij\xi}^{-1})^T P_{F_{ij\xi}}^T(z_i)||$. According to the definition of $\chi_{ij\xi}$, it can be proved that $\chi_{ij\xi}$ and $\dot{\chi}_{ij\xi}$ are all bounded in terms of the boundness of $P_{ij\xi}(Z_i)$ and ε_{ij}, which have been established in Theorem 2. $\mathcal{L}_{ij\xi}^{-1}$ is also bounded according to the satisfaction of the PPE condition of $P_{ij\xi}(Z_i)$. Therefore, we can derive that ρ_{ij} and ω_{ij} are bounded; hence, the term $||(\mathcal{L}_{ij\xi}^{-1})^T \Gamma_{ij\xi}^{-1}\omega_{ij}||$ is also bounded.

Combining $\dot{V}_{i1}$, $\dot{V}_{i2}$, and $\dot{V}_{i3\xi}$, we can obtain the time derivation of the Lyapunov function $L = L_1 + L_2$ as follows:

$$\begin{aligned}
\dot{L} &= \sum_{i=1}^{2} \left(\dot{V}_{i1} + \dot{V}_{i2} + \dot{V}_{i3\xi} \right) \\
&\le -e_x^T (2K_e - \beta_{f_i}) e_x - \sum_{i=1}^{2} \left(K_i - \varrho_i - \frac{1}{2} \right) e_{q_i}^T e_{q_i} \\
&+ \sum_{i=1}^{2} \left(- \kappa_i ||\mathcal{U}_i \tilde{\phi}_i|| \right) - \sum_{i=1}^{2} \left((\beta_i - \rho_i) ||e_{q_i}|| \right) \\
&+ \sum_{i=1}^{2} \left(- \beta_{f_i} \Lambda_{f_i}^T K_f \Lambda_{f_i} + \frac{\beta_{f_i}}{2} \Lambda_{f_i}^T \Lambda_{f_i} + \frac{1}{2} \varepsilon_{F_i}^T \varepsilon_{F_i} \right) \\
&- \sum_{i=1}^{2} \sum_{j=1}^{N_i} \left((\gamma_i - ||(\mathcal{L}_{ij\xi}^{-1})^T \Gamma_{ij\xi}^{-1}\omega_{ij}||) ||\mathcal{V}_{ij\xi}|| \right) \\
&\le \sum_{i=1}^{2} \left(- \kappa_i ||\mathcal{U}_i \tilde{\phi}_i|| \right) - \sum_{i=1}^{2} \left((\beta_i - \rho_i) ||e_{q_i}|| \right) \\
&+ \sum_{i=1}^{2} \left(- \sum_{j=1}^{N_i} (\gamma_i - ||\mathcal{L}_{ij\xi}^{-1T} \Gamma_{ij\xi}^{-1}\omega_{ij}||) ||\mathcal{V}_{ij\xi}|| + \frac{1}{2} \varepsilon_{F_i}^T \varepsilon_{F_i} \right) \\
&\le -\eta_1 \sqrt{L} + \mu_1
\end{aligned} \tag{66}$$

where $\rho_i = \max(\rho_{i1}, \rho_{i2}, \ldots, \rho_{iN_i})$, γ_i is chosen to be $\gamma_i \ge ||\mathcal{L}_{ij\xi}^{-1T} \Gamma_{ij\xi}^{-1}\omega_{ij}||$, β_i is chosen to be $\beta_i \ge \rho_i$, $\eta_1 = \sqrt{2}\min [\lambda_{\max}(\frac{\gamma_i - ||\mathcal{L}_{i\xi}^{-1T}\Gamma_{i\xi}^{-1}\omega_i||}{\sqrt{\lambda_{\min}(\Gamma_{i\xi}^{-1})}}), (\beta_i - \rho_i)/\sqrt{\lambda_{\min}(\mathcal{H}_i)}, \quad \kappa_i ||\mathcal{U}_i|| \sqrt{\lambda_i}]$, and $\mu_1 = \frac{1}{2}\sum_{i=1}^{2}(\varepsilon_{F_i}^T \varepsilon_{F_i})$, $i = 1, 2$. Following the results in [44] and according to (66), the tracking errors and the parameter estimation errors could converge to a small neighborhood near zero in FT. This completes the proof.

Remark 4: Note that for parameter estimation, the PE (or PPE) condition should be fulfilled to guarantee the convergence of the estimation errors. Also, in practical robot control implementations, some unknown factors such as friction, actuator nonlinearities, nonrigid grasping, and external disturbances may exist. These factors could affect the parameter estimation performance and may degrade the control performance. Further investigation together with experimental studies shall be made in our future work. ■

IV. Simulation Study

To illustrate the effectiveness of the proposed control scheme, simulation studies are carried out based on a model of a dual-arm robot with three joints for each arm and in the scenario that a common object is firmly held in between the arms. The robot dyanmics model for each arm is described as follows [45]:

$$H_i(q_i)\ddot{q}_i + D_i(q_i, \dot{q}_i)\dot{q}_i + G_i(q_i) = \tau_i + J_{e_i}^T(q_i) F_{e_i}$$

where the manipulator inertial matrix $H_i(q_i)$ and the Coriolis matrix $D(q_i \dot{q}_i)$ are described as

$$H_i(q_i) = H_i^T(q_i) = \begin{bmatrix} H_{11} & * & * \\ H_{21} & H_{22} & * \\ H_{31} & H_{32} & H_{33} \end{bmatrix}$$

$$D_i(q_i) = \begin{bmatrix} D_{11} & D_{12} & D_{13} \\ D_{21} & D_{22} & D_{23} \\ D_{31} & D_{32} & D_{33} \end{bmatrix}, \; G_i(q_i) = \begin{bmatrix} G_{11} \\ G_{21} \\ G_{31} \end{bmatrix}$$

with $H_{11} = p_1 + 2p_4 l_1 c_2 + 2p_5(l_2 c_3 + l_1 c_{23})$, $H_{21} = p_2 + p_4 l_1 c_2 + p_5(2l_2 c_3 + l_1 c_{23})$, $H_{22} = p_2 + 2p_5 l_2 c_3$, $H_{31} = p_3 + p_5 (l_2 c_3 + l_1 c_{23})$, $H_{32} = p_3 + p_5 l_2 c_3$, $H_{33} = p_3$, $D_{11} = l_1(p_4 s_2 + p_5 s_{12})$, $D_{12} = l_2 p_5 s_3 - l_1 p_4 s_2$, $D_{13} = -p_5(l_2 s_3 + l_1 s_{12})$, $D_{21} = l_1(p_4 s_2 + p_5 s_{12})$, $D_{22} = l_2 p_5 s_3$, $D_{23} = -l_2 p_5 s_3$, $D_{31} = -l_1 p_5 s_{12}$, $D_{32} = V$, $D_{33} = 0$, $G_{11} = 0$, $G_{21} = 0$, $G_{31} = 0$. And $s_j = \sin(q_{ij})$, $c_j = \cos(q_{ij})$, $s_{j_1 j_2} = \sin(q_{ij_1} + q_{ij_2})$, $c_{j_1 j_2} = \cos(q_{ij_1} + q_{ij_2})$, $s_{123} = \sin(q_{i1} + q_{i2} + q_{i2})$, $c_{123} = \cos(q_{i1} + q_{i2} + q_{i2})$, $p_1 = I_1 + I_2 + I_3 + l_1^2(m_1 + m_2 + m_3) + 2l_1 m_1 l_{c2} + l_2^2(m_2 + m_3) + 2l_2 m_2 l_{c2}$, $p_2 = I_2 + I_3 + l_2^2(m_1 + m_2) + 2l_2 m_2 l_{c2}$, $p_3 = I_3$, $p_4 = m_3 l_{c2} + l_2(m_2 + m_3)$, and $p_5 = m_3 l_{c3}$. The motion dynamics of the object is described as

$$H_o \ddot{x} + D_o \dot{x} + G_o(x) = F_o \tag{67}$$

where

$$H_o = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0.1 \end{bmatrix} \text{ and } C_o = G_o = \mathbf{0}.$$

The kinematics and inertia parameters of each link of the robot are given in Table I. The parameters of the grasped object are given as $l_o = 0.05$ m, $m_o = 0.1$ kg, and $m_o = 0.1$ kg·m^2.

TABLE I
LINK PARAMETERS OF THE ROBOT ARM

Parameters	Link 1	Link 2	Link 3
link length (m)	$l_1 = 0.35$	$l_2 = 0.31$	$l_3 = 0.1$
link mass (kg)	$m_1 = 2$	$m_2 = 0.85$	$m_3 = 0.3$
Inertia (kgm^2)	$I_1 = 1.1$	$I_2 = 0.3$	$I_3 = 0.3$
link distance (m)	$l_{c_1} = 0.18$	$I_{c_2} = 0.155$	$l_{c_3} = 0.05$

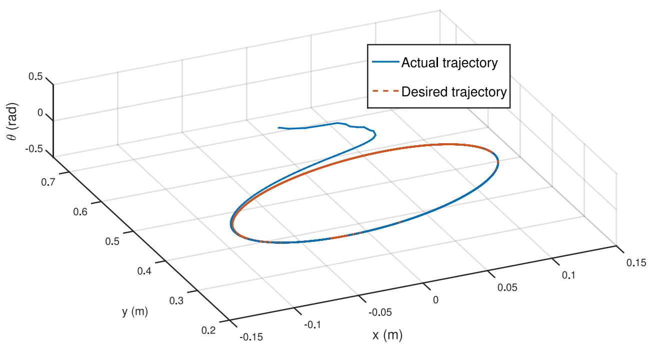


Fig. 3. Trajectory tracking performance in phase plane.

The dual-arm robot is commanded to grasp an object to follow a desired periodic circular trajectory as follows:

$$\begin{pmatrix} x \\ y \\ \theta \end{pmatrix} = \begin{pmatrix} 0.1\cos(0.5t) \\ 0.38 + 0.1\sin(0.5t) \\ 0.2\sin(0.5 * t) \end{pmatrix}.$$

The initial position and velocity of the manipulated object are chosen as $x(0) = [-0.03, 0.52, 0]$ and $\dot{x}(0) = [0, 0, 0]$, respectively. The estimated kinematic parameters are initialized as $\hat{\phi}_1(0) = [0.2, 0.2, 0, 0]^T$, and its upper and lower bounds are given by $\bar{\phi}_1 = [0.5, 0.5, 0.2, 0.1]^T$ and $\underline{\phi}_1 = [0.1, 0.1, 0, -0.05]^T$, respectively. For the FLS, Gaussian membership functions are selected for each input dimension. The Gaussian membership functions are continuously differentiable and have the advantage for the theoretical analysis of the FLS. We choose parameter -3, -2, -1, 0.5, 1.5, 1, and 2 for the central points and π for the standard deviations for the FLS. The initial FLS weights are chosen to be $\hat{W}_1(0) = 0$, $\hat{W}_2(0) = 0$, which will be updated with the FT estimated law (59). The adaptation gains are chosen to be $\zeta_1 = \zeta_2 = 5$, $\delta_1 = \delta_2 = 1.5$, $\kappa_1 = \kappa_2 = 0.05$, $\lambda_1 = \lambda_2 = 5$, $\Gamma_1 = \Gamma_2 = 1.5$, and $b_1 = b_2 = 0.001$. And the control gains are selected to be positive-definite matrix as $K_e = \text{diag}\{5, 5, 3\}$ and $K_i = \text{diag}\{20, 10, 9\}$.

We employ the proposed controller (24) into the dual-arm robot system to achieve the control goal without using *a priori* knowledge of both the kinematics and dynamics of the robot. The simulation results are performed in Figs. 3–17. An overlook of the tracking performance is shown in Fig. 3, where the grasped object is controlled to follow the periodic tracking trajectories in the phase plane. Figs. 4–6 and 7–9 depict the tracking performance and tracking errors. As we can see, the actual trajectories in all dimensions (red line "-") have successfully followed the reference trajectory (blue dashed line "–") and all tracking errors converge to zeros, even in the presence of kinematic and dynamic uncertainties. To verify the superiority of the proposed control method, comparison studies have been further carried out based on a model-based controller [6] and a conventional proportional derivative (PD) controller. The comparative results are depicted in Figs. 7–9. From the figures, we can find that the proposed controller (24) obtained the best control performance, and tracking errors are much smaller than the other two methods.

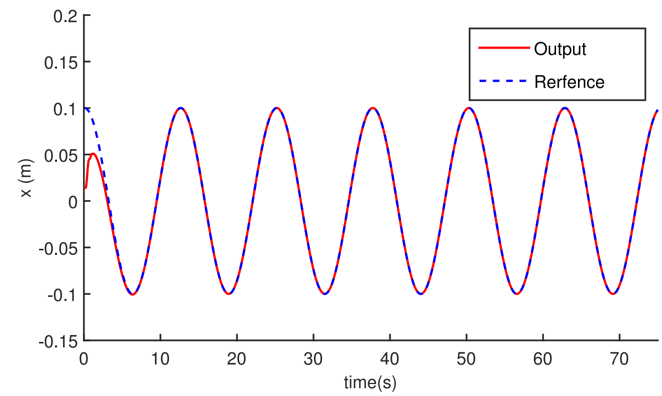


Fig. 4. Trajectory tracking performance in the x-direction.

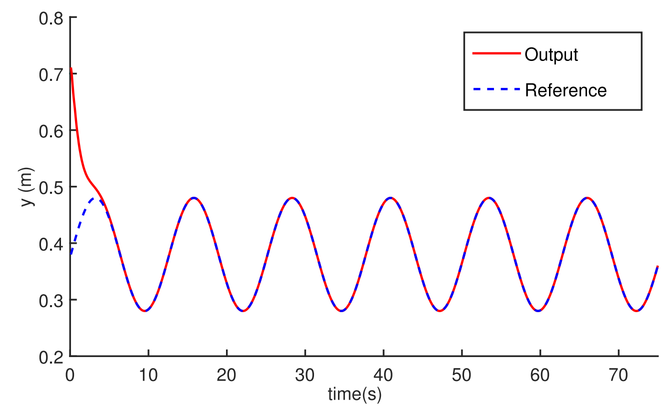


Fig. 5. Trajectory tracking performance in the y-direction.

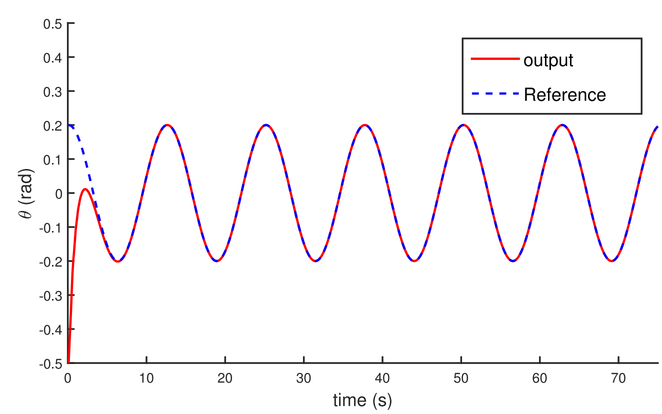


Fig. 6. Trajectory tracking performance in the θ-direction.

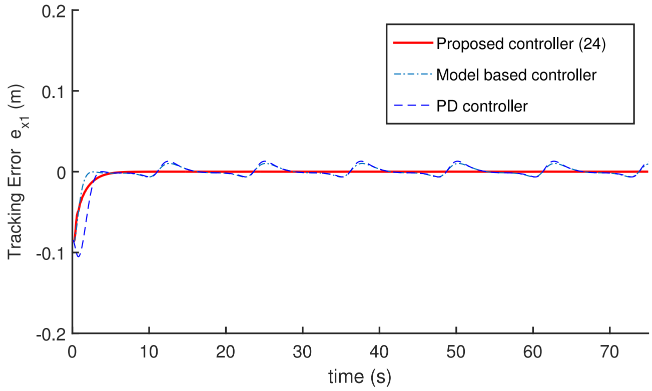


Fig. 7. Trajectory tracking errors e_{x1}.

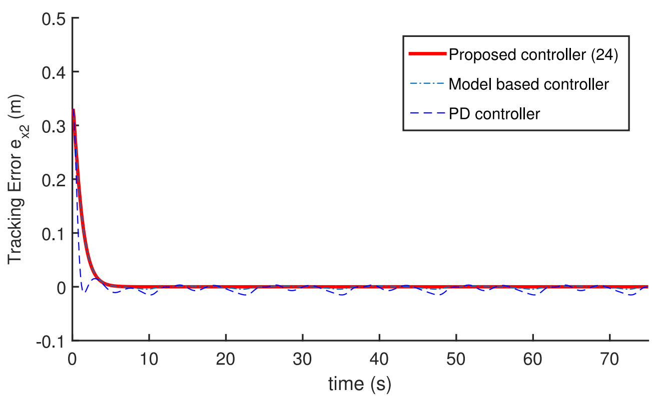


Fig. 8. Trajectory tracking errors e_{x2}.

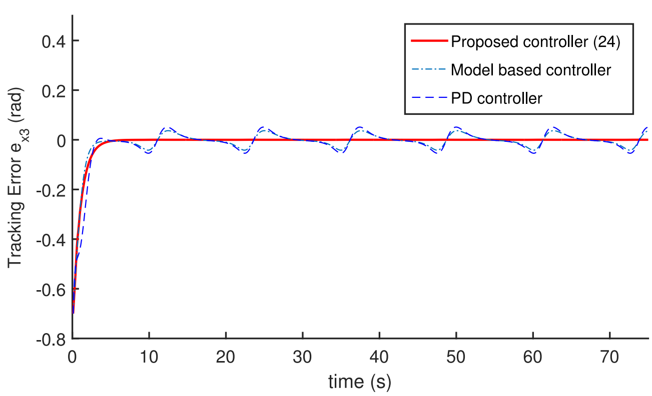


Fig. 9. Trajectory tracking errors e_{x3}.

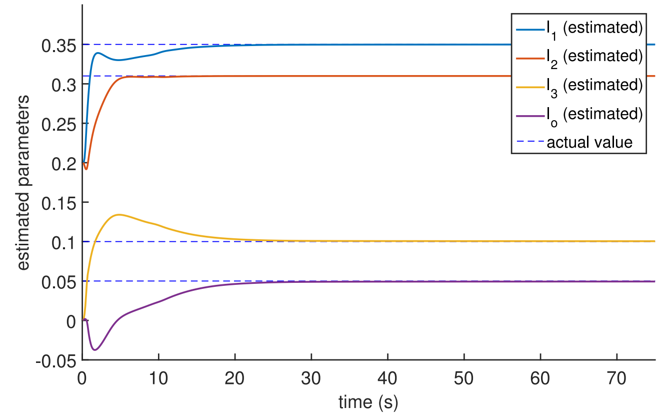


Fig. 10. Estimation performance of the kinematic parameters l_{11}, l_{12}, l_{13}, and l_o.

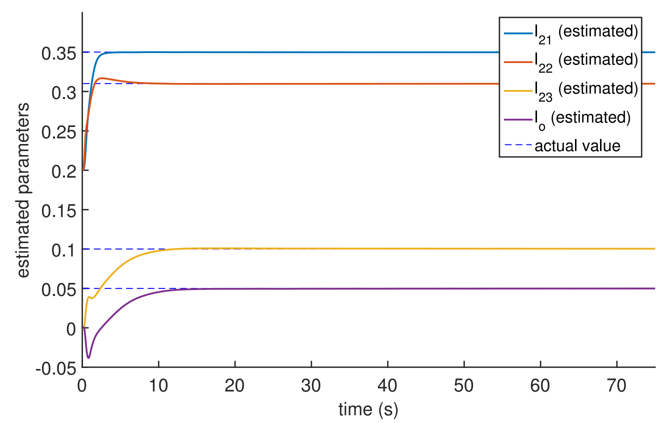


Fig. 11. Estimation performance of the kinematic parameters l_{21}, l_{22}, l_{23}, and l_o.

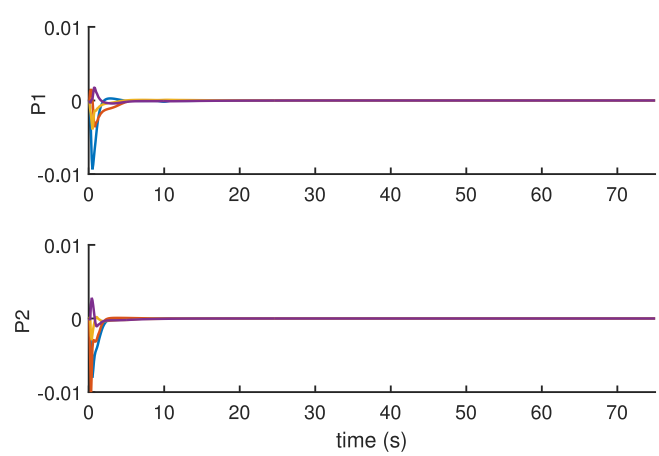


Fig. 12. Profile of auxiliary vectors $\mathcal{P}_1$ and $\mathcal{P}_2$.

The parameter estimation performance is shown in Figs. 10 and 11. From the figures, we can observe that the estimated kinematic parameters of both arms converge to their real values (blue dashed line "–") in a short period of time (less than a periodic). Additionally, the profile of the auxiliary terms $\mathcal{P}_1$ and $\mathcal{P}_2$ and the minimum eigenvalue of the auxiliary matrices $\mathcal{U}_1$ and $\mathcal{U}_2$ are also depicted in Figs. 12 and 13, respectively. It is shown that both $\sigma_1 = \lambda_{\min}(\mathcal{U}_1)$ and $\sigma_2 = \lambda_{\min}(\mathcal{U}_2)$ are great than zero. This implies that the PE condition is satisfied during the estimation, which has been further verified in Fig. 12, where both $\mathcal{P}_1$ and $\mathcal{P}_2$ (which contain the information of estimation errors) have approached to zero. Therefore, the validity of the kinematics identification is demonstrated. In comparison to the work in [12], our proposed method ensures not only the

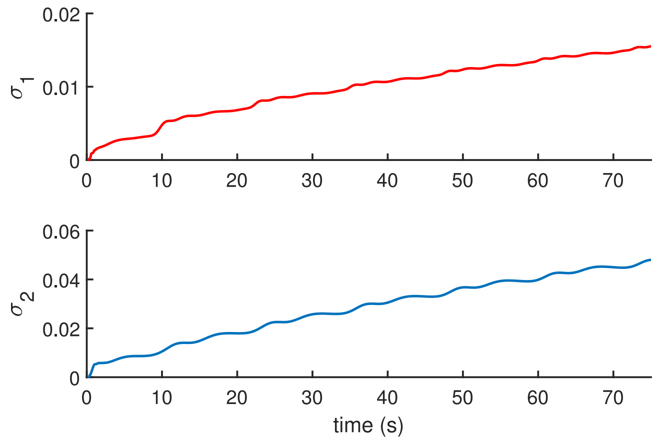


Fig. 13. Profile of minimum eigenvalues of $\mathcal{U}_1$ and $\mathcal{U}_2$.

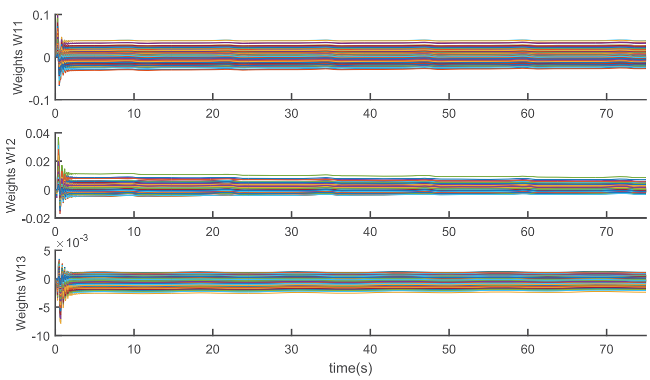


Fig. 14. FLS weight W_1 with FT adaptation.

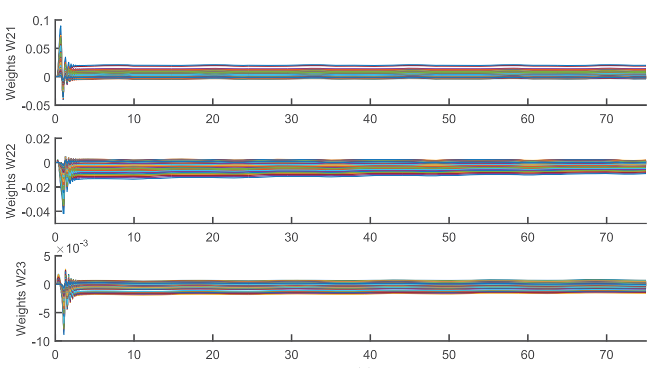


Fig. 15. FLS weight W_2 with FT adaptation.

convergence of the tracking errors, but also the FT convergence of the estimated parameters.

Simulation results of the proposed fuzzy logic control are depicted in Figs. 14–17. The weights of the FLS are shown in Figs. 14 and 15. From the figures, we can see that the weights of the FLSs of both arms are converged with a fast rate. It should also be emphasized that not all of the FLS weights have converged to a relatively larger value, and a number of weights only converge to the small neighborhood around zero. This is

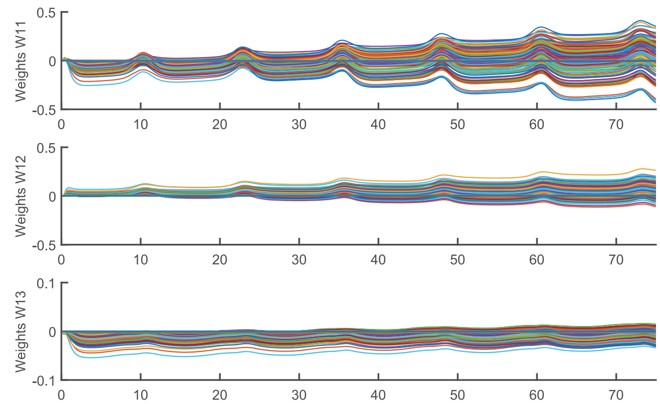


Fig. 16. FLS weight W_1 with the conventional adaptation law.

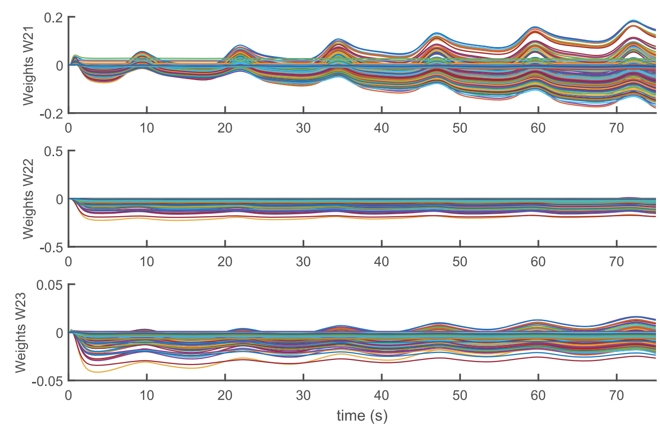


Fig. 17. FLS weight W_2 with the conventional adaptation law.

consistent with the PPE condition, and only part of the fuzzy weights are activated and updated by the adaptive law, while for the Gaussian membership functions, whose centers are far away from the FLS inputs, the corresponding FLS weights are not activated and hence will remain in their initial conditions. A comparative study of the FLS weight estimation using gradient descent adaptation is shown in Figs. 16 and 17. It can be observed that some weights diverge in the estimation, and not all the weights are converged to their optimal values.

The simulation results have illustrated that our proposed controller can successfully track the desired trajectory in the presence of uncertain kinematics and dynamics with the guarantee of FT convergence of the estimated weights.

V. Conclusion

In this paper, we have developed an adaptive fuzzy control scheme for the coordinated robot arms in the presence of system uncertainties. To guarantee that the estimated parameters converge to the optimal values, the PPE property of the fuzzy basis function has been investigated. An FCPA technique is developed for the estimation of both the unknown kinematics parameters and FLS weights. We have shown that, under the PPE condition, the estimated parameters are able to converge to a small neighborhood of their actual values in an FT, such that they could be reused next time with an improved computational efficiency and control performance. Extensive simulation studies have been carried out to illustrate that the manipulated object is able to track well a desired trajectory in the presence of uncertain kinematics and dynamics.

APPENDIX

Before proceeding to prove Theorem 1, let us introduce some useful definitions and lemmas as follows.

Definition 2 (see[39]): A uniformly bounded piecewise continuous vector function $P \in \mathbb{R}^m$ is said to be PE if there exist positive constants α_1, α_2, and T_0 such that $\alpha_1 \leq \int_{t_0}^{t_0+T_0} |P(\tau)^T c|^2 d\tau \leq \alpha_2$ holds for all unit vectors $c \in \mathbb{R}^m$.

Other than Definition 1, Definition 2 introduces an alternative expression of the PE condition, which is in the scalar form. The following lemma holds for the RBF [41].

Lemma 2 (see[41]): Consider any continuous recurrent trajectory $Z(t)$, and $Z(t)$ remains in a bounded compact set Ω_Z; then, for the localized RBF $S(Z)$ in the form of $S(Z) = [s(||Z(t) - \xi_1||), \ldots, ||s(Z(t) - \xi_L)||]^T$,

$$s(||Z(t) - \xi_l||) = \exp\Big[-\frac{(Z-\xi_l)^T(Z-\xi_l)}{\varsigma^2}\Big] \tag{68}$$

where $\xi_1, \ldots, \xi_L$ are the centers placed on a regular lattice (large enough to cover the compact set Ω_i), and ς is a positive constant, the regressor subvector $S_\xi(Z(t)) \in \mathbb{R}^{N_\xi}$ is persistent exiting, i.e., for any constant vector $c \in \mathbb{R}^{L_\xi}$, there exists

$$\alpha_1||c||^2 \leq \int_{t_0}^{t_0+T_0} |S_\xi(Z(t))^T c|^2 d\mu(\tau) \leq \alpha_2||c||^2 \tag{69}$$

where α_1 and α_2 are positive constants.

Proof of Theorem 1: As defined in (17), the subvector of the fuzzy basis function of the FLS $P_\xi(Z) = [p_1(Z), p_2(Z), \ldots, p_L(Z)]^T \in \mathbb{R}^L$ is given by

$$p_l(Z) = \frac{\prod_{i=1}^N \mu_{A_i^l}(z_i)}{\sum_{l=1}^L \prod_{i=1}^n \mu_{A_i^l}(z_i)} \tag{70}$$

with L being the number of the active fuzzy rules of the $P_\xi(Z)$, $Z = [z_1, z_2, \ldots, z_N]^T$. Without loss of generality, the fuzzy membership function $\mu_{A_i^l}(z_i)$ is chosen to be the Gaussian fuzzy membership function as $\mu_{A_i^l}(z_i) = \exp[-(\frac{z_i - \xi_{li}}{\rho})^2]$, where ρ is a positive constant, and ξ_{li} are distinct points of the membership function functions. Then, $\prod_{i=1}^N \mu_{A_i^l}(z_i) = \varphi_l(Z)$ can be rewritten as

$$\varphi_l(Z) = \prod_{i=1}^N \exp\Big[-\Big(\frac{z_i-\xi_{li}}{\rho}\Big)^2\Big]$$

$$= \exp\Big[-\sum_{i=1}^N \Big(\frac{z_i-\xi_{li}}{\rho}\Big)^2\Big] = \exp\Big[-\frac{(Z-\xi_l)^T(Z-\xi_l)}{\rho^2}\Big]$$

where $\xi_l = [\xi_{l1}, \xi_{l2}, \ldots, \xi_{lN}]^T$. And (70) can be rewritten as

$$p_l(Z) = \frac{\varphi_l(Z)}{\varphi_l + \varphi_2 + \cdots + \varphi_L}. \tag{71}$$

In comparison to (68) and (71), if ρ and ς are chosen to be the same, we can deduce that φ_l is functional equivalent to an RBF. Then, according to Lemma 2, for the RBF $\varphi = [\varphi_1, \varphi_2, \ldots, \varphi_L]^T$, there exist an arbitrary selected constant vector $c \in \mathbb{R}^L$ and positive constants α_1' and α_2', such that

$$\alpha_1'||c||^2 \leq \int_{t_0}^{t_0+T_0} |\varphi(Z(t))^T c|^2 d\mu(\tau) \leq \alpha_2'||c||^2. \tag{72}$$

φ_l is bounded with the boundedness of Z according to (71). And, from (72), we can also easily obtain that there exists a $\varphi_l(Z(t))$ among $\varphi_1, \ldots, \varphi_L$, such that $\varphi_l(Z(t)) > 0$ in the time interval $[t_0, t_0 + T_0]$. Let $\varphi_u = \varphi_1 + \cdots + \varphi_L$; then, we have $\bar{\varphi}_u \leq \varphi_u \leq \underline{\varphi}_u$, where $\bar{\varphi}_u$ and $\underline{\varphi}_u$ are the upper and lower bounds of φ_u, which are positive constants.

Then, combining (71), (72), and the definition of P_ξ, we have

$$\int_{t_0}^{t_0+T_0} |P_\xi(Z(\tau))^T c|^2 d\mu(\tau) = \int_{t_0}^{t_0+T_0} \frac{|\varphi(Z(\tau))^T c|^2}{\varphi_u^2} d\mu(\tau)$$

$$\leq \int_{t_0}^{t_0+T_0} \frac{|\varphi(Z(\tau))^T c|^2}{\underline{\varphi}_u^2} d\mu(\tau) \leq \alpha_{P2}||c||^2 \tag{73}$$

and

$$\int_{t_0}^{t_0+T_0} |P_\xi(Z(\tau))^T c|^2 d\mu(\tau)$$

$$\geq \int_{t_0}^{t_0+T_0} \frac{|\varphi(Z(\tau))^T c|^2}{\bar{\varphi}_u^2} d\mu(\tau) \geq \alpha_{P1}||c||^2 \tag{74}$$

with $\alpha_{P1} = \alpha_1'/\bar{\varphi}_u^2$ and $\alpha_{P2} = \alpha_2'/\underline{\varphi}_u^2$.

Therefore, from (73) and (74), for any constant vector $c \in R^L$, we have

$$\alpha_{P1}||c||^2 \leq \int_{t_0}^{t_0+T_0} |P_\xi(Z(\tau))^T c|^2 d\mu(\tau) \leq \alpha_{P2}||c||^2. \tag{75}$$

From the above analysis and Definition 2, we can conclude that for any periodic trajectory $Z(t)$, the corresponding regressor subvector of the FLS $P_\xi(Z(t))$ is persistently exciting. This completes the proof.

REFERENCES

[1] J.-H. Jean and L.-C. Fu, "An adaptive control scheme for coordinated multimanipulator systems," *IEEE Trans. Robot. Automat.*, vol. 9, no. 2, pp. 226–231, Apr. 1993.

[2] W.-H. Zhu and J. D. Schutter, "Control of two industrial manipulators rigidly holding an egg," *IEEE Control Syst.*, vol. 19, no. 2, pp. 24–30, Apr. 1999.

[3] J. Huang, W. Huo, W. Xu, S. Mohammed, and Y. Amirat, "Control of upper-limb power-assist exoskeleton using a human-robot interface based on motion intention recognition," *IEEE Trans. Automat. Sci. Eng.*, vol. 12, no. 4, pp. 1257–1270, Oct. 2015.

[4] C. Yang, X. Wang, Z. Li, Y. Li, and C.-Y. Su, "Teleoperation control based on combination of wave variable and neural networks," *IEEE Trans. Syst., Man, Cybern.: Syst.*, vol. 47, no. 8, pp. 2125–2136, Aug. 2017.

[5] C. Yang, K. Huang, H. Cheng, Y. Li, and C.-Y. Su, "Haptic identification by elm-controlled uncertain manipulator," *IEEE Trans. Syst., Man, Cybern.: Syst.*, vol. 47, no. 8, pp. 2398–2409, Aug. 2017.

[6] Y.-H. Liu and S. Arimoto, "Decentralized adaptive and nonadaptive position/force controllers for redundant manipulators in cooperations," *Int. J. Robot. Res.*, vol. 17, no. 3, pp. 232–247, 1998.

[7] Y. Jiang, Z. Liu, C. Chen, and Y. Zhang, "Adaptive robust fuzzy control for dual arm robot with unknown input deadzone nonlinearity," *Nonlinear Dyn.*, vol. 81, no. 3, pp. 1301–1314, 2015.

[8] Z.-G. Hou, L. Cheng, and M. Tan, "Multicriteria optimization for coordination of redundant robots using a dual neural network," *IEEE Trans. Syst., Man, Cybern. B, Cybern.*, vol. 40, no. 4, pp. 1075–1087, Aug. 2010.

[9] Z.-G. Hou, L. Cheng, and M. Tan, "Decentralized robust adaptive control for the multiagent system consensus problem using neural networks," *IEEE Trans. Syst., Man, Cybern. B, Cybern.*, vol. 39, no. 3, pp. 636–647, Jun. 2009.

[10] C. Yang, Y. Jiang, Z. Li, W. He, and C.-Y. Su, "Neural control of bimanual robots with guaranteed global stability and motion precision," *IEEE Trans. Ind. Inform.*, vol. 13, no. 3, pp. 1162–1171, Jun. 2017.

[11] Z. Liu, C. Chen, Y. Zhang, and C. P. Chen, "Adaptive neural control for dual-arm coordination of humanoid robot with unknown nonlinearities in output mechanism," *IEEE Trans. Cybern.*, vol. 45, no. 3, pp. 507–518, Mar. 2015.

[12] L. Cheng, Z.-G. Hou, and M. Tan, "Adaptive neural network tracking control for manipulators with uncertain kinematics, dynamics and actuator model," *Automatica*, vol. 45, no. 10, pp. 2312–2318, 2009.

[13] C. C. Cheah, S. P. Hou, Y. Zhao, and J. J. E. Slotine, "Adaptive vision and force tracking control for robots with constraint uncertainty," *IEEE/ASME Trans. Mechatronics*, vol. 15, no. 3, pp. 389–399, Jun. 2010.

[14] C. C. Cheah, M. Hirano, S. Kawamura, and S. Arimoto, "Approximate jacobian control for robots with uncertain kinematics and dynamics," *IEEE Trans. Robot. Automat.*, vol. 19, no. 4, pp. 692–702, Aug. 2003.

[15] C.-C. Cheah, C. Liu, and J. Slotine, "Adaptive Jacobian tracking control of robots with uncertainties in kinematic, dynamic and actuator models," *IEEE Trans. Autom. Control*, vol. 51, no. 6, pp. 1024–1029, Aug. 2006.

[16] J. Huang, P. Di, T. Fukuda, and T. Matsuno, "Robust model-based online fault detection for mating process of electric connectors in robotic wiring harness assembly systems," *IEEE Trans. Control Syst. Technol.*, vol. 18, no. 5, pp. 1207–1215, Sep. 2010.

[17] Y. Huang, J. Na, X. Wu, and G. Gao, "Approximation-free control for vehicle active suspensions with hydraulic actuator," *IEEE Trans. Ind. Electron.*, vol. 65, no. 9, pp. 7258–7267, Sep. 2018.

[18] W. He, Y. Dong, and C. Sun, "Adaptive neural impedance control of a robotic manipulator with input saturation," *IEEE Trans. Syst., Man, Cybern.: Syst.*, vol. 46, no. 3, pp. 334–344, Mar. 2016.

[19] J. Na, Y. Huang, X. Wu, G. Gao, G. Herrmann, and J. Z. Jiang, "Active adaptive estimation and control for vehicle suspensions with prescribed performance," *IEEE Trans. Control Syst. Technol.*, 2017. doi: 10.1109/TCST.2017.2746060.

[20] C. Chen, Z. Liu, Y. Zhang, C. L. P. Chen, and S. Xie, "Actuator backlash compensation and accurate parameter estimation for active vibration isolation system," *IEEE Trans. Ind. Electron.*, vol. 63, no. 3, pp. 1643–1654, Mar. 2016.

[21] J. Huang, T. Fukuda, and T. Matsuno, "Model-based intelligent fault detection and diagnosis for mating electric connectors in robotic wiring harness assembly systems," *IEEE/ASME Trans. Mechatronics*, vol. 13, no. 1, pp. 86–94, Feb. 2008.

[22] N. Sun, Y. Fang, H. Chen, and B. He, "Adaptive nonlinear crane control with load hoisting/lowering and unknown parameters: Design and experiments," *IEEE/ASME Trans. Mechatronics*, vol. 20, no. 5, pp. 2107–2119, Oct. 2015.

[23] N. Sun, Y. Fang, and H. Chen, "A new antiswing control method for underactuated cranes with unmodeled uncertainties: Theoretical design and hardware experiments," *IEEE Trans. Ind. Electron.*, vol. 62, no. 1, pp. 453–465, Jan. 2015.

[24] C. Yang, C. Zeng, P. Liang, Z. Li, R. Li, and C.-Y. Su, "Interface design of a physical human–robot interaction system for human impedance adaptive skill transfer," *IEEE Trans. Automat. Sci. Eng.*, vol. 15, no. 1, pp. 329–340, Jan. 2018.

[25] B. Chen, X. Liu, K. Liu, and C. Lin, "Direct adaptive fuzzy control of nonlinear strict-feedback systems," *Automatica*, vol. 45, no. 6, pp. 1530–1535, 2009. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S0005109809000946

[26] J. Huang, W. Xu, S. Mohammed, and Z. Shu, "Posture estimation and human support using wearable sensors and walking-aid robot," *Robot. Auton. Syst.*, vol. 73, pp. 24–43, 2015.

[27] Y. Liu, Y. Gao, S. Tong, and Y. Li, "Fuzzy approximation-based adaptive backstepping optimal control for a class of nonlinear discrete-time systems with dead-zone," *IEEE Trans. Fuzzy Syst.*, vol. 24, no. 1, pp. 16–28, Feb. 2016.

[28] W. Ning and J. E. Meng, "Direct adaptive fuzzy tracking control of marine vehicles with fully unknown parametric dynamics and uncertainties," *IEEE Trans. Control Syst. Technol.*, vol. 24, no. 5, pp. 1845–1852, Sep. 2016.

[29] W. Ning, M. J. Er, J. C. Sun, and Y. C. Liu, "Adaptive robust online constructive fuzzy control of a complex surface vehicle system," *IEEE Trans. Cybern.*, vol. 46, no. 7, pp. 1511–1523, Jul. 2016.

[30] J. Huang, M. Ri, D. Wu, and S. Ri, "Interval type-2 fuzzy logic modeling and control of a mobile two-wheeled inverted pendulum," *IEEE Trans. Fuzzy Syst.*, vol. 26, no. 4, pp. 2030–2038, Aug. 2018.

[31] C. L. P. Chen, Y. J. Liu, and G. X. Wen, "Fuzzy neural network-based adaptive control for a class of uncertain nonlinear stochastic systems," *IEEE Trans. Cybern.*, vol. 44, no. 5, pp. 583–593, May 2014.

[32] Y.-J. Liu, S.-C. Tong, and W. Wang, "Adaptive fuzzy output tracking control for a class of uncertain nonlinear systems," *Fuzzy Sets Syst.*, vol. 160, no. 19, pp. 2727–2754, 2009. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S0165011408005836

[33] Z. Liu, C. Chen, and Y. Zhang, "Decentralized robust fuzzy adaptive control of humanoid robot manipulation with unknown actuator backlash," *IEEE Trans. Fuzzy Syst.*, vol. 23, no. 3, pp. 605–616, Jun. 2015.

[34] V. Adetola and M. Guay, "Finite-time parameter estimation in adaptive control of nonlinear systems," *IEEE Trans. Autom. Control*, vol. 53, no. 3, pp. 807–811, Apr. 2008.

[35] P. M. Patre, W. Mackunis, M. Johnson, and W. E. Dixon, "Composite adaptive control for Euler–Lagrange systems with additive disturbances," *Automatica*, vol. 46, no. 1, pp. 140–147, 2010.

[36] J. Na, M. N. Mahyuddin, G. Herrmann, X. Ren, and P. Barber, "Robust adaptive finite-time parameter estimation and control for robotic systems," *Int. J. Robust Nonlinear Control*, vol. 25, no. 16, pp. 3045–3071, 2015.

[37] Y. Lv, J. Na, Q. Yang, X. Wu, and Y. Guo, "Online adaptive optimal control for continuous-time nonlinear systems with completely unknown dynamics," *Int. J. Control*, vol. 89, no. 1, pp. 99–112, 2016.

[38] J. Na, G. Herrmann, and K. Zhang, "Improving transient performance of adaptive control via a modified reference model and novel adaptation," *Int. J. Robust Nonlinear Control*, vol. 27, no. 8, pp. 1351–1372, 2017.

[39] C. Wang and D. J. Hill, *Deterministic Learning Theory for Identification, Recognition, and Control*. Boca Raton, FL, USA: CRC Press, 2009.

[40] A. M. Smith, C. Yang, H. Ma, P. Culverhouse, A. Cangelosi, and E. Burdet, "Novel hybrid adaptive controller for manipulation in complex perturbation environments," *Plos One*, vol. 10, no. 6, 2015, Art. no. e0129281.

[41] C. Wang and D. J. Hill, "Deterministic learning and rapid dynamical pattern recognition," *IEEE Trans. Neural Netw.*, vol. 18, no. 3, pp. 617–630, May 2007.

[42] S. L. Dai, M. Wang, and C. Wang, "Neural learning control of marine surface vessels with guaranteed transient tracking performance," *IEEE Trans. Ind. Electron.*, vol. 63, no. 3, pp. 1717–1727, Mar. 2016.

[43] C. Wang and T. Chen, "Rapid detection of small oscillation faults via deterministic learning," *IEEE Trans. Neural Netw.*, vol. 22, no. 8, pp. 1284–1296, Aug. 2011.

[44] M. N. Mahyuddin, J. Na, G. Herrmann, X. Ren, and P. Barber, "Adaptive observer-based parameter estimation with application to road gradient and vehicle mass estimation," *IEEE Trans. Ind. Electron.*, vol. 61, no. 6, pp. 2851–2863, Jun. 2014.

[45] C. R. Carnigan, "Adaptive tracking for complex systems using reduced-order models," in *Proc. IEEE Int. Conf. Robot. Autom.*, 1990, vol. 3, pp. 2078–2083.

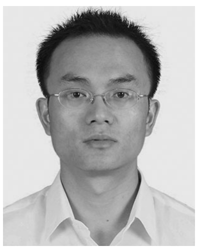
Chenguang Yang (M'10–SM'16) received the B.Eng. degree in measurement and control from Northwestern Polytechnical University, Xi'an, China, in 2005, and the Ph.D. degree in control engineering from the National University of Singapore, Singapore, in 2010.

He received postdoctoral training with Imperial College London, London, U.K. His research interests include robotics and automation.

Dr. Yang was a recipient of the Best Paper Award in the IEEE TRANSACTIONS ON ROBOTICS and a number of international conferences.

Yiming Jiang received the B.S. degree in automation from Hunan University, Changsha, China, in 2011, and the M.S. degree in control theory and engineering from the School of Automation, Guangdong University of Technology, Guangzhou, China, in 2015. He is currently pursuing the Ph.D. degree with the School of Control Science and Engineering, South China University of Technology, Guangzhou.

His research interests include robotics, intelligent control, and human–robot interaction.



Jing Na (M'15) received the B.Sc. and Ph.D. degrees in automation from the Beijing Institute of Technology, Beijing, China, in 2004 and 2010, respectively.

From 2011 to 2013, he was a Monaco/ITER Postdoctoral Fellow with the ITER Organization, Saint-Paul-lés-Durance, France. From 2015 to 2017, he was a Marie Curie Intra-European Fellow with the Department of Mechanical Engineering, University of Bristol, Bristol, U.K. Since 2010, he has been with the Faculty of Mechanical and Electrical Engineering, Kunming University of Science and Technology, Kunming, China, where he became a Professor in 2013. His research interests include intelligent control, adaptive parameter estimation, nonlinear control, and applications.

Dr. Na was a recipient of the Best Application Paper Award of the third IFAC International Conference on Intelligent Control and Automation Science 2013, and the 2017 Hsue-Shen Tsien Paper Award.

Zhijun Li (M'07–SM'09) received the Ph.D. degree in mechatronics from Shanghai Jiao Tong University, Shanghai, China, in 2002.

From 2012 to 2017, he was a Professor with the College of Automation Science and Engineering, South China University of Technology, Guangzhou, China. Since 2017, he has been a Professor with the Department of Automation, University of Science and Technology of China, Hefei, China. His research interests include service robotics, teleoperation systems, nonlinear control, and neural network optimization.

Dr. Li is an Editor-at-Large of the *Journal of Intelligent and Robotic Systems*, an Associate Editor for the IEEE TRANSACTIONS ON NEURAL NETWORKS AND LEARNING SYSTEMS, the IEEE TRANSACTIONS ON SYSTEMS, MAN AND CYBERNETICS: SYSTEMS, and the IEEE TRANSACTIONS ON AUTOMATION SCIENCE AND ENGINEERING. He was the General Chair of 2016 IEEE Conference on Advanced Robotics and Mechatronics, Macau, China.

Long Cheng (SM'14) received the B.S. (Hons.) degree in control engineering from Nankai University, Tianjin, China, in 2004, and the Ph.D. (Hons.) degree in control theory and control engineering from the Institute of Automation, Chinese Academy of Sciences, Beijing, China, in 2009.

In 2010, he was a Postdoctoral Research Fellow with the Department of Mechanical Engineering, University of Saskatchewan, Saskatoon, SK, Canada. From 2010 to 2011, he was a Postdoctoral Research Fellow with the Department of Mechanical and Industrial Engineering, Northeastern University, Boston, MA, USA. From 2013 to 2014, he was a Visiting Scholar with the Department of Electrical and Computer Engineering, University of California at Riverside, Riverside, CA, USA. He is currently a Professor with the Laboratory of Complex Systems and Intelligent Science, Institute of Automation, Chinese Academy of Sciences. He has authored or coauthored more than 50 technical papers in peer-refereed journals and prestigious conference proceedings. His research interests include intelligent control of smart materials, coordination of multiagent systems, neural networks, and their applications to robotics.

Chun-Yi Su (SM'99) received the Ph.D. degree in control engineering from the South China University of Technology, Guangzhou, China, in 1990.

He joined Concordia University, Montreal, QC, Canada, in 1998, after a seven-year stint with the University of Victoria, Victoria, BC, Canada. He is currently with the College of Automation Science and Engineering, South China University of Technology, on leave from Concordia University. He has authored or coauthored more than 300 publications in journals, book chapters, and conference proceedings. His research interests include the application of automatic control theory to mechanical systems. He is particularly interested in control of systems involving hysteresis nonlinearities.

Dr. Su has served as an Associate Editor for the IEEE TRANSACTIONS ON AUTOMATIC CONTROL, the IEEE TRANSACTIONS ON CONTROL SYSTEMS TECHNOLOGY, and the *Journal of Control Theory and Applications*. He has been on the Editorial Board of 18 journals, including the *IFAC Journal of Control Engineering Practice and Mechatronics*.