Evaluating R&D Projects as Investments by Using an Overall Ranking From Four New Fuzzy Similarity Measure-Based TOPSIS Variants

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***Abstract*—Research and development (R&D) project ranking as investments is a well-known problem that is made difficult by incomplete and imprecise information about future project profitability. This paper shows how profitability results of R&D project evaluation with the fuzzy pay-off method can be ranked with four new variants of fuzzy TOPSIS each using a different fuzzy similarity measure. An overall project ranking that incorporates the four new variants' rankings with three different ideal solutions totaling 12 subrankings is presented. The implementation of the created methods is illustrated with a numerical example.**

***Index Terms*—Fuzzy pay-off method (FPOM), fuzzy similarity measures, fuzzy TOPSIS, R&D project selection.**

I. Introduction

MANAGEMENT of research and development (R&D) projects as a portfolio of investments is a forward looking procedure that is done under uncertainty about the future and under imprecise knowledge about the business benefits and the potential of each project [1]. A key challenge in the financial project profitability-based assessment is the difficulty of precisely estimating the size and the timing of future project cash-flows. A number of interconnected factors can affect the cash-flows causing complexity that creates imprecision in cash-flow estimation. In reality, managers are able to come up with good enough, or "satisficing" imprecise estimates at best, and managerial estimation is often the only credible way of gathering financial information about R&D projects. It is important to provide decision makers with a realistic representation of the projects as investments and this means including the perceived imprecision in the analysis. Fuzzy set theory is an appropriate tool for the representation of imprecise information and estimates in environments such as R&D investment analysis. Fuzzy logic provides a framework for handling imprecision and allows for an exact formulation of subjective managerial estimates of project criteria and of expected cash-flows.

R&D projects often require a relatively small initial investment that allows large companies to run a considerable number of these projects simultaneously. Usually, there are more potential projects than money and resources, making multiple criteria project ranking important; ranking of projects is a real problem in companies today. Many of the commonly used and available R&D project ranking methods are built on the assumption that managers are able to come up with precise estimates for the future attributes of the projects—an assumption that most often does not hold [2]. This calls for fuzzy versions of ranking methods, and indeed, there is existing literature on fuzzy selection of R&D projects (see, e.g., [3]–[6]).

Profitability basis in the selection and ranking of R&D projects usually means using discounted cash-flow (DCF)-based methods for the task, especially the use of the net present value (NPV) method. NPV is used also here. NPV is the most commonly used profitability analysis method in the industry [7]. Fuzzy NPV allows including the estimation inaccuracy in the profitability analysis (see, e.g., [8]). Using the NPV method alone in profitability analysis has been identified as a drawback [9], [10], because NPV tends to ignore managerial flexibility—the reality that managers can initiate a project and then decide whether or not to carry them to completion, is not considered. Not considering managerial flexibility in R&D project analysis may lead to a wrong selection of projects [11].

Real option valuation (ROV) can be used to overcome the problem of overlooking managerial flexibility because the ROV treats R&D projects as options. ROV is traditionally done with methods designed for financial option valuation that rely on assumptions, such as random walk, that may fit stock markets, but that definitely are a poor match with the reality of R&D project evaluation. In reality, the cash-flows of R&D projects are affected by management actions, and they are seldom random. New models designed for ROV, such as the Datar–Mathews method [12], [13], which is based on probability theory and uses simulation and the fuzzy pay-off method (FPOM) for ROV [14], are able to relax the ill-fitting assumptions of the traditional methods used for ROV. The FPOM can be used for the calculation of the real option value directly from a fuzzy project NPV, and it can be used when the existing project cash-flow data has a nonstatistical imprecision; and when the cash-flow estimates are based on subjective managerial expert opinions. The FPOM is used here in the calculation of the real option value for R&D projects.

One of the relevant ranking methods for R&D projects is the technique for order of preference by similarity to ideal solution (TOPSIS). There are also fuzzy versions of TOPSIS (see, e.g., [15] and [16]), and they are suitable for the ranking of objects under imprecise information. Four new variants of the

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fuzzy TOPSIS method that use similarity measures are presented in this paper. Fuzzy similarity measures offer a smart way of comparing the similarity of two fuzzy sets that is more advanced than just simply comparing single numbers extracted from two fuzzy sets. By using similarity measures, it is possible to reach a multiple criteria comparison that considers many aspects of the phenomenon of imprecision that a fuzzy set is able to include. The fuzzy similarity measures can take into consideration issues, such as geometric distance, perimeter, and area of fuzzy numbers (FNs), that other methods are not capable of considering. To the best of our knowledge, this is the first time the suitability of a fuzzy similarity-based TOPSIS is studied with regard to R&D project ranking.

Furthermore, an overall ranking is introduced that uses the result from all four fuzzy similarity-based methods in the formation of a holistic project ranking—this allows for multiple points of view to be taken into consideration simultaneously, under imprecise information. The overall ranking using the results from four similarity-based fuzzy TOPSIS variants is a new contribution. Fuzzy TOPSIS has been previously used for project selection in connection with, for example, real estate [17], telecommunications [18], and R&D [19].

Based on the above background, a decision support system for ranking R&D projects as investments under imprecise information is proposed. The system is based on using managerial cash-flow scenarios, which are able to account for the estimation inaccuracy and imprecision. The FPOM [14] is used in the construction of a simple pay-off distribution from the cash-flows that is then treated as an FN, and that is substantially the same as fuzzy NPV in [20]. The pay-off distribution and the ROV are used together with fuzzy project cost and fuzzy project labor intensity as inputs into the four new variants of the fuzzy TOPSIS method. An overall ranking of the four new fuzzy TOPSIS variants' rankings with three different ideal solutions is performed to gain a holistic ranking of the projects that can be used as decision support in the R&D portfolio selection process.

The system discussed here addresses the R&D project ranking and evaluation problem; however, the methods and observations made may be applicable also to patents and other intellectual property rights, as well as any types of projects and assets that share similar attributes with regard to availability of precise and complete information.

This paper is organized as follows. Section II shortly introduces the methods and concepts that we use in our analysis: the pay-off method, TOPSIS, and the four fuzzy similarity measures. In Section III, we present four new fuzzy TOPSIS variants using the four similarity measures and introduce an overall ranking that considers all four variants. Section IV concentrates on numerically demonstrating how this innovative combination of methods can be used in ranking R&D projects. Section V is devoted to a discussion about what we have learned and to the conclusions drawn from the experiments.

II. Methods

A. Fuzzy Pay-Off Method

The FPOM [14] is a practical tool for investment project profitability analysis and for ROV that is suitable also for cases where input information estimation accuracy is not necessarily very high. The FPOM is usable also in the valuation of compound real options [21] and for many different types of real assets [22]. These characteristics make the FPOM a good fit with the profitability analysis of R&D projects.

The FPOM can be used to calculate a real option value directly from a project pay-off distribution that can be simply constructed by using managerial cash-flow scenarios for a project. The procedure is explained in [14] and [22]. The pay-off distribution is treated as an FN. The pay-off distribution alone is a good aid in R&D decision support, but descriptive numbers, such as the ROV, can be calculated directly from it. ROV is an indicator of project potential and has been widely used in the analysis and selection of R&D projects (see, e.g., [4]–[6], [23], and [24]).

The ROV is calculated directly from the pay-off distribution as follows:

$$\mathrm{ROV} = \frac{\int_0^\infty A(x)\,dx}{\int_{-\infty}^\infty A(x)\,dx} \times E(A_+) \tag{1}$$

where A stands for the (fuzzy) NPV, $E(A_+)$ denotes the possibilistic mean value of the positive side of the NPV, $\int_{-\infty}^{\infty} A(x)\,dx$ computes the area below the whole NPV distribution A, and $\int_0^\infty A(x)\,dx$ computes the area below the positive part of A.

Because of their simplicity, the use of triangular or trapezoidal pay-off distributions (FNs) is suggested. This means that three or four cash-flow scenarios are required from the managers. Derivation of the fuzzy mean for the positive side of different types of distributions in this context is presented in [14] and [22], and the definition of the fuzzy mean is presented in [25].

The structure of the FPOM is in line with the option valuation logic of the classical option valuation methods, but is on a different level of usability for the day-to-day decision making connected to R&D project analysis. The FPOM has been previously used in the analysis of R&D projects in [5], [6], and [14]; these differ from the work presented here, among other things, with regard to the ranking methods used.

B. TOPSIS

TOPSIS is one of the well-known classical MCDM methods, and it is originally based on the concept that the highest ranking alternative should have the shortest distance from the positive ideal solution (PIS) and be farthest away from the negative ideal solution (NIS). The method was extended to the fuzzy environment in 2000 by Chen [15], where ratings and weights were considered to be triangular FNs. Later, the fuzzy extension of the method was enhanced to cover trapezoidal FNs (see [16]). Ashtiani *et al.* [26] further extended fuzzy TOPSIS to cover interval-valued fuzzy sets. These TOPSIS variants use the fuzzy distance in the calculation of the aforementioned distances from the PIS and the NIS. A similarity measure was used to determine the similarity between the alternatives and the PIS and the NIS for the first time by Luukka [27]. He also studied a selection of three different choices of fuzzy positive ideal solution (FPIS) and fuzzy negative ideal solution (FNIS). These two issues, i.e., using a fuzzy similarity measure as a similarity measure, and the choice of (fuzzy) ideal solutions within the framework of

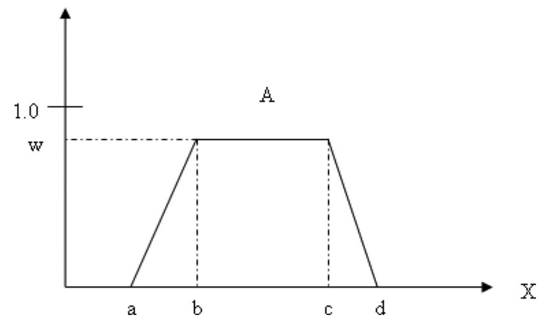

Fig. 1. Generalized trapezoidal FN A. The number is a normal trapezoidal FN when $w = 1.0$.

the TOPSIS method, is a fresh research subject with very few contributions so far. This study extends the research into these topics to cover the use of four different fuzzy similarity measures, the selection of an appropriate fuzzy similarity measure, and the choice of the (fuzzy) ideal solutions.

C. Fuzzy Similarity Measures

In this section, we review different fuzzy similarity measures that are applied in the fuzzy similarity-based TOPSIS R&D ranking system. Herein, four different similarities were selected on the basis that they are taking several different properties into account some individually and some combining them. For example, the fourth similarity measure combines the concepts of geometric distance, perimeters, and the area of generalized fuzzy numbers (GFN). The first one of the four similarity measures is defined for FN and the three last for GFNs, as introduced by Chen [28]. Let A be a trapezoidal GFN $A = (a, b, c, d, w)$, as shown in Fig. 1, where a, b, c, and d are real values, and $w \in (0, 1]$.

The first similarity measure used was originally presented by Hsieh and Chen [29] between trapezoidal FNs using graded mean integration representation distance, where the degree of similarity *S(A,B)* between the FNs A and B is calculated as follows:

$$S_1(A, B) = \frac{1}{1 + d(A, B)} \tag{2}$$

where $d(A, B) = |P_1(A) - P_1(B)|$, and $P_1(A)$ and $P_1(B)$ are the mean graded integration representation of A and B. For trapezoidal FNs, they are defined as follows:

$$P_1(A) = \frac{a_1 + 2a_2 + 2a_3 + a_4}{6}$$
$$P_1(B) = \frac{b_1 + 2b_2 + 2b_3 + b_4}{6}.$$

The second fuzzy similarity measure used is by Chen and Chen [30]. They represented the similarity for trapezoidal GFNs. Let $A = (a_1, a_2, a_3, a_4, w_a)$ and $B = (b_1, b_2, b_3, b_4, w_b)$ be two trapezoidal GFNs. The center of gravity (COG) points (X_a, Y_a) and (X_b, Y_b) of trapezoidal GFNs A and B are used. The COG point (X_a, Y_a) for GFN A is calculated as follows:

$$Y_a = \begin{cases} w_a \left(\dfrac{a_3 - a_2}{a_4 - a_1}\right), & \text{if } a_1 \neq a_4 \text{ and } 0 < w_a \leq 1 \\ \dfrac{w_a}{2}, & \text{if } a_1 = a_4 \text{ and } 0 < w_a \leq 1 \end{cases}$$
$$X_a = \frac{Y_a(a_3 + a_2) + (a_4 + a_1)(w_a - Y_a)}{2w_a}.$$

The degree of similarity $S(A,B)$ is calculated as follows:

$$S_2(A, B) = \left(1 - \frac{\sum_{i=1}^{4} |a_i - b_i|}{4}\right)(1 - |X_a - X_b|)^{\frac{S_a - S_b}{2}} \times \frac{\min(Y_a, Y_b)}{\max(Y_a, Y_b)} \tag{3}$$

where $S_2(A, B) \in [0, 1]$; $\frac{S_a - S_b}{2} = 0$ when $\frac{S_a - S_b}{2} = 0$; $\frac{S_a - S_b}{2} = 1$ when $0 < \frac{S_a - S_b}{2} \leq 1$, where $S_a = a_4 - a_1$ and $S_b = b_4 - b_1$.

The third similarity measure used was proposed by Wei and Chen [31] to calculate similarity $S(A,B)$ for trapezoidal GFNs as follows:

$$S_3(A, B) = \left(1 - \frac{\sum_{i=1}^{4} |a_i - b_i|}{4}\right) \times \left(\frac{\min(P_2(A), P_2(B)) + \min(w_a, w_b)}{\max(P_2(A), P_2(B)) + \max(w_a, w_b)}\right) \tag{4}$$

where $P_2(A)$ and $P_2(B)$ are the perimeters of trapezoidal GFN A and B, which are defined as

$$P_2(A) = \sqrt{(a_1 - a_2)^2 + w_a^2} + \sqrt{(a_3 - a_4)^2 + w_a^2} + (a_3 - a_2) + (a_4 - a_1)$$
$$P_2(B) = \sqrt{(b_1 - b_2)^2 + w_b^2} + \sqrt{(b_3 - b_4)^2 + w_b^2} + (b_3 - b_2) + (b_4 - b_1).$$

The fourth similarity measure under investigation is by Hejazi *et al.* [32], and it takes into account the geometric distance, the perimeter of the two trapezoidal GFNs, and the area of the two trapezoidal GFNs. This similarity measure is the latest of the four and is defined as

$$S_4(A, B) = \left(1 - \frac{\sum_{i=1}^{4} |a_i - b_i|}{4}\right)\left(\frac{\min(P_2(A), P_2(B))}{\max(P_2(A), P_2(B))}\right) \times \left(\frac{\min(\text{Area}(A), \text{Area}(B)) + \min(w_a, w_b)}{\max(\text{Area}(A), \text{Area}(B)) + \max(w_a, w_b)}\right) \tag{5}$$

where perimeters $P_2(A)$ and $P_2(B)$ are calculated as earlier, and areas of two trapezoidal GFNs are calculated as follows:

$$\text{Area}(A) = \frac{1}{2} w_a (a_3 - a_2 + a_4 - a_1)$$
$$\text{Area}(B) = \frac{1}{2} w_b (b_3 - b_2 + b_4 - b_1).$$

Even if three of the above four similarity measures are originally defined for GFNs, this paper uses normal FNs, with the height of one. This is of importance, because the FPOM is only defined for normal FNs.

In addition to the four similarity measures defined previously, there are also several others, for example, Wilbik and Keller [33] used a fuzzy measure similarity between sets of linguistic summaries in eldercare, similarity measures were used in a relevance-based learning model by Le Capitaine [34], and a similarity measure was also used in clustering by Hüllermeier *et al.* [35].

In the next section, a new variant of the TOPSIS method that uses fuzzy similarity measures in determining the similarity between the studied alternative (project) and the FPIS and the FNIS is presented.

III. Overall Ranking of R&D Projects Based on Four New Variants of Fuzzy TOPSIS That Use Fuzzy Similarity Measures

A situation is considered where a finite set of projects $P = \{P_i | i = 1, 2, \ldots, m\}$ need to be evaluated by a committee of decision makers $D = \{D_l | l = 1, 2, \ldots, k\}$, by considering a finite set of given criteria $C = \{C_j | j = 1, 2, \ldots, n\}$. Decision matrix representation of performance ratings of each project P_i, with respect to each criterion C_j, is presented as follows:

$$X = \begin{bmatrix} x_{11} & \cdots & x_{1n} \\ \vdots & \ddots & \vdots \\ x_{m1} & \cdots & x_{mn} \end{bmatrix} \quad (6)$$

where m rows represent m possible projects, n columns represents n relevant criteria, and x_{ij} represents the performance rating of the ith project with respect to jth criterion C_j. These ratings are trapezoidal FNs with height equaling one for all cases. The weight w_j of criterion C_j are assumed to be $W = [w_1, w_2, \ldots, w_n]$, the weights are trapezoidal FNs. The aforementioned fuzzy decision matrix is formed for each decision maker D_l. To aggregate the fuzzy decision matrices from each decision maker to one single decision matrix, an aggregation, which is proposed in [16], is used, where an aggregated trapezoidal FN $R = (a,b,c,d)$ is calculated using $a = \min\{a_l\}_l, b = \frac{1}{k}\sum_{l=1}^{k} b_l, c = \frac{1}{k}\sum_{l=1}^{k} c_l$, and $d = \max\{d_l\}_l$. The aggregated fuzzy ratings x_{ij} of projects with respect to each criterion are now $x_{ij} = (a_{ij}, b_{ij}, c_{ij}, d_{ij})$, where $a_{ij} = \min_l \{a_{ijl}\}$, $b_{ij} = \frac{1}{k}\sum_{l=1}^{k} b_{ijl}, c_{ij} = \frac{1}{k}\sum_{l=1}^{k} c_{ijl}$, and $d_{ij} = \max_l \{d_{ijl}\}$. The aggregated weights over the decision makers are calculated in the same way. For more details, see [16]. The next step for the decision matrix is to form a linear scale transformation to transform the various criteria scales into comparable scales. The criteria set can be divided into a benefit criteria (the larger the rating, the greater the preference) and a cost criteria (the smaller the rating, the greater the preference). Therefore, the normalized fuzzy decision matrix can be represented as

$$R = (r_{ij})_{m \times n} \quad (7)$$

where B and C are the sets of benefit criteria and cost criteria, respectively, and

$$r_{ij} = \left(\frac{a_{ij}}{d_j^{\oplus}}, \frac{b_{ij}}{d_j^{\oplus}}, \frac{c_{ij}}{d_j^{\oplus}}, \frac{d_{ij}}{d_j^{\oplus}}\right) j \in B$$

$$r_{ij} = \left(\frac{a_j^{\ominus}}{d_{ij}}, \frac{a_j^{\ominus}}{c_{ij}}, \frac{a_j^{\ominus}}{b_{ij}}, \frac{a_j^{\ominus}}{a_{ij}}\right) j \in C$$

where $d_j^{\oplus} = \max_i (d_{ij}), j \in B$, and $a_j^{\ominus} = \min_i (a_{ij}), j \in C$. After the normalization, importance weights are applied to get our weighted normalized fuzzy decision matrix

$$V = (v_{ij})_{m \times n} \quad (8)$$

where $v_{ij} = r_{ij} \cdot w_j$.

The next step is to obtain FPIS and FNIS. There are several possible alternatives on how to choose them, as pointed out by Luukka [27]. What is common to the choices is that a weighted normalized decision matrix is used to form them; ideal solutions are vectors, where there is always one FN for each criterion that represents the particular ideal solution for a particular criterion. FPIS $A^{\oplus}$ and FNIS $A^{\ominus}$ are given as follows:

$$A^{\oplus} = \left[v_1^+, v_2^+, \ldots, v_n^+\right] \quad (9)$$

$$A^{\ominus} = \left[v_1^-, v_2^-, \ldots, v_n^-\right]. \quad (10)$$

Chen *et al.* [16] proposed the use of the maxima and the minima from weighted normalized values as follows:

$$v_j^+ = \max_i v_{ij4} \quad (11)$$

$$v_j^- = \min_i v_{ij1}. \quad (12)$$

Two other ways for selecting FPIS $A^{\oplus}$ and FNIS $A^{\ominus}$ were given in [27] as follows: 1) $A^{\oplus}$ and $A^{\ominus}$ are considered as vectors of ones and zeros leading to $v_j^+ = (1, 1, 1, 1)$ and $v_j^- = (0, 0, 0, 0)$; 2) it is considered that every element of $A^{\oplus}$ is the maximum for all i weighted normalized values, and every element of $A^{\ominus}$ is the minimum for all i weighted values

$$v_j^+ = \left(\max_i v_{ij1}, \max_i v_{ij2}, \max_i v_{ij3}, \max_i v_{ij4}\right) \quad (13)$$

$$v_j^- = \left(\min_i v_{ij1}, \min_i v_{ij2}, \min_i v_{ij3}, \min_i v_{ij4}\right). \quad (14)$$

Notice that the last choice of fuzzy ideal solutions is the only one that truly is a trapezoidal FN, whereas the other two choices are crisp numbers that are presented as trapezoidal FNs. In the last choice, considerably more information is included (within the "true" FN) compared with the other two choices. To take advantage of this "extra" information, fuzzy similarity measures are capable of handling much more of this type of information than earlier-used fuzzy distance measures; this is the main reason why we are considering similarity measures here. The next step in the process is to calculate fuzzy similarities (to the FPIS and FNIS) for each project and $A^{\oplus}$ and $A^{\ominus}$. They are calculated as

$$S_i^{\oplus} = \sum_{j=1}^{n} S_v\left(v_{ij}, v_j^+\right) i = 1, 2, \ldots, m \quad (15)$$

$$S_i^{\ominus} = \sum_{j=1}^{n} S_v\left(v_{ij}, v_j^-\right) i = 1, 2, \ldots, m \quad (16)$$

TABLE I
FUZZY PRESENT VALUE OF COSTS/REVENUES FOR 20 CANDIDATE PROJECTS

Projects	phase 1 cost	phase 2 cost	phase 3 cost	phase 3 revenue
P1	(2, 2.2, 0.3, 0.3)	(30, 33, 4.5, 4.5)	(30, 33, 4.5, 4.5)	(50, 55, 7.5, 7.5)
P2	(3, 3.3, 0.45, 0.45)	(50, 55, 7.5, 7.5)	(45, 49.5, 6.75, 6.75)	(100, 110, 15, 15)
P3	(10,11 1.5, 1.5,)	(75, 82.5, 11.25, 11.25)	(100, 110, 15, 15)	(200, 220, 30, 30)
P4	(5, 5.5, 0.75, 0.75)	(65, 71.5, 9.75, 9.75)	(170, 187, 25.5, 25.5)	(200, 220, 30, 30)
P5	(20, 22, 3, 3)	(85, 93.5, 12.75, 12.75)	(200, 220, 30, 30)	(600, 660, 90, 90)
P6	(15, 16.5, 2.25, 2.25)	(40, 44, 6, 6)	(45, 49.5, 6.75, 6.75)	(100, 110, 15, 15)
P7	(7, 7.7, 1.05, 1.05)	(35, 38.5, 5.25, 5.25)	(30, 33, 4.5, 4.5)	(80, 88, 12, 12)
P8	(5, 5.5, 0.75, 0.75)	(55, 60.5, 8.25, 8.25)	(50, 55, 7.5, 7.5)	(100, 110, 15, 15)
P9	(10, 11, 1.5, 1.5)	(75, 82.5, 11.25, 11.25)	(80, 88, 12, 12)	(180, 198, 27, 27)
P10	(18, 19.8, 2.7, 2.7)	(85, 93.5, 12.75, 12.75)	(120, 132, 18, 18)	(380, 418, 57, 57)
P11	(5, 5.5, 0.75, 0.75)	(35, 38.5, 5.25, 5.25)	(30, 33, 4.5, 4.5)	(80, 88, 12, 12)
P12	(7, 7.7, 1.05, 1.05)	(40, 44, 6, 6)	(60, 66, 9, 9)	(100, 110, 15, 15)
P13	(15, 16.5, 2.25, 2.25)	(95, 104.5, 14.25, 14.25)	(180, 198, 27, 27)	(40, 44, 6, 6)
P14	(35, 38.5, 5.25, 5.25)	(120, 132, 18, 18)	(280, 308, 42, 42)	(700, 770, 105, 105)
P15	(25, 27.5, 3.75, 3.75)	(70, 77, 10.5, 10.5)	(100, 110, 15, 15)	(500, 550, 75, 75)
P16	(15, 16.5, 2.25, 2.25)	(95, 104.5, 14.25, 14.25)	(150, 165, 22.5, 22.5)	(300, 330, 45, 45)
P17	(17, 18.7, 2.55, 2.55)	(80, 88, 12, 12)	(180, 198, 27, 27)	(350, 385, 52.5, 52.5)
P18	(20, 22, 3, 3)	(90, 99, 13.5, 13.5)	(220, 242, 33, 33)	(550, 605, 82.5, 82.5)
P19	35, 38.5, 5.25, 5.25)	(120, 132, 18, 18)	(250, 275, 37.5,37.5)	(800, 880, 120, 120)
P20	(50, 55, 7.5, 7.5)	(130, 143, 19.5, 19.5)	(350, 385, 52.5, 2.5)	(1150, 1265, 172.5, 172.5)

FNs presented in their original format from [6].

where $S_v(\cdot,\cdot)$ is now chosen from similarity measures, which are denoted in (1)–(4) and presented above. The last step of this process is the calculation of the closeness coefficients. A closeness coefficient is defined to determine the ranking order of all possible projects, once $S_i^{\oplus}$ and $S_i^{\ominus}$ have been calculated for each project. The closeness coefficient in this case takes into account the similarity of the project to the FPIS and the FNIS simultaneously, by calculating the relative closeness to the FPIS. The closeness coefficient (CCS_i) for each project is calculated as

$$CCS_i = \frac{S_i^{\oplus}}{S_i^{\ominus} + S_i^{\ominus}},\ i = 1, 2, \ldots, m. \tag{17}$$

Closeness coefficients CCS_i are used here to form rankings for the projects RP_i, as usual. This ranking RP_i for project i clearly depends on the choice of the ideal solution, and on the choice of the similarity measure. For example, the additional information studied here includes the COG point, the perimeter, and the area of FNs in the similarity measures. In addition, all three types of selections for the ideal solution have their merits. The total number of possible combinations of how the rankings can be done with this type of TOPSIS is 12, with each one having their own merits and possibly a differing ranking result. This means that a project will have vector ranks RP_{ij}, where $j \in 1, 2, \ldots, 12$. The vector is used to fuse this information into one single triangular FN $\widehat{RP}_i$ by following $\widehat{RP}_i = (a_{1i}, a_{2i}, a_{3i})$, where $a_{1i} = \min_j RP_{ij}, a_{2i} = \text{mean}_j RP_{ij}, a_{3i} = \max_j RP_{ij}$. In this way the information from all of the cases is taken into consideration by forming fuzzy triangular numbers from the individual rankings. These FNs are then ranked. For this process, the method for ranking of triangular FNs, which is introduced by Kaufmann and Gupta [36], is used.

The motivation for this procedure is to find a total order or a linear order, for FNs, where all FNs and fuzzy intervals are

TABLE II
REQUIRED LABOR REPRESENTED BY FNS FOR 20 CANDIDATE PROJECTS

projects	fuzzy development resource (in working months)		
	phase 1	phase 2	phase 3
P1	(6, 6, 0.6, 0.6)	(72, 72, 7.2, 7.2)	(50, 50, 5, 5)
P2	(12, 12, 1.2, 1.2)	(80, 80, 8, 8)	(48, 48, 4.8, 4.8)
P3	(24, 24, 2.4, 2.4)	(95, 95, 9.5, 9.5)	(70, 70, 7, 7)
P4	(12, 12, 1.2, 1.2)	(100, 100, 10, 10)	(70, 70, 7, 7)
P5	(32, 32, 3.2, 3.2)	(120, 120, 12, 12)	(80, 80, 8, 8)
P6	(26, 26, 2.6, 2.6)	(105, 105, 10.5, 10.5)	(75, 75, 7.5, 7.5)
P7	(20, 20, 2, 2)	(85, 85, 8.5, 8.5)	(52, 52, 5.2, 5.2)
P8	(12, 12, 1.2, 1.2)	(110, 110, 11, 11)	(75, 75, 7.5, 7.5)
P9	(24, 24, 2.4, 2.4)	(150, 150, 15, 15)	(90, 90, 9, 9)
P10	(30, 30, 3, 3)	(155, 155, 15.5, 15.5)	(100, 100, 10, 10)
P11	(14, 14, 1.4, 1.4)	(90, 90, 9, 9)	(60, 60, 6, 6)
P12	(15, 15, 1.5, 1.5)	(75, 75, 7.5, 7.5)	(70, 70, 7, 7)
P13	(30, 30, 3, 3)	(180, 180, 18, 18)	(120, 120, 12, 12)
P14	(45, 45, 4.5, 4.5)	(200, 200, 20, 20)	(130, 130, 13, 13)
P15	(40, 40, 4, 4)	(160, 160, 16, 16)	(110, 110, 11, 11)
P16	(35, 35, 3.5, 3.5)	(190, 190, 19, 19)	(125, 125, 12.5, 12.5)
P17	(36, 36, 3.6, 3.6)	(190, 190, 19, 19)	(120, 120, 12, 12)
P18	(38, 38, 3.8, 3.8)	(200, 200, 20, 20)	(130, 130, 13, 13)
P19	(36, 36, 3.6, 3.6)	(220, 220, 22, 22)	(150, 150, 15, 15)
P20	(48, 48, 4.8, 4.8)	(230, 230, 23, 23)	(160, 160, 16, 16)

FNs presented in their original format from [6].

TABLE III
REAL OPTION VALUES CALCULATED USING POM FOR 20 CANDIDATE PROJECTS

projects	fuzzy NPV	ROV
P1	(-18.2, -7, 16.8, 16.8)	0.06
P2	(-7.8, 12, 29.7, 29.7)	5.94
P3	(-3.5, 35, 57.75, 57.75)	18.00
P4	(-64, -20, 66, 66)	0.54
P5	(264.5, 355, 135.75, 135.75)	309.75
P6	(-10, 10, 30, 30)	5.00
P7	(0.8, 16, 22.8, 22.8)	8.51
P8	(-21, 0, 31.5, 31.5)	1.58
P9	(-1.5, 33, 51.75, 51.75)	17.15
P10	(134.7, 195, 90.45, 90.45)	164.85
P11	(3, 18, 22.5, 22.5)	10.03
P12	(-17.7, 3, 31.05, 31.05)	2.39
P13	(-279, -246, 49.5, 49.5)	0.00
P14	(221.5, 335, 170.25, 170.25)	278.25
P15	(285.5, 355, 104.25, 104.25)	320.25
P16	(14, 70, 84, 84)	39.66
P17	(45.3, 108, 94.05, 94.05)	72.48
P18	(187, 275, 132, 132)	231.00
P19	(354.5, 475, 180.75, 180.75)	414.75
P20	(567, 735, 252, 252)	651.00

FNs presented in their original format from [6].

TABLE IV
CLOSENESS COEFFICIENTS AND RANKING ORDER FOR 20 CANDIDATE PROJECTS USING THREE POSSIBLE IDEAL SOLUTIONS AND FIRST SIMILARITY MEASURE

Projects	CCS with FPIS and FNIS 1	Ranking	CCS with FPIS and FNIS 2	Ranking	CCS with FPIS and FNIS 3	Ranking
P1	0.4555	2	0.4461	2	0.4904	2
P2	0.4324	8	0.4250	8	0.4627	7
P3	0.4068	15	0.4022	15	0.4329	15
P4	0.4020	16	0.3975	16	0.4279	16
P5	0.4415	4	0.4382	4	0.4678	6
P6	0.4121	13	0.4071	13	0.4396	12
P7	0.4376	5	0.4303	6	0.4691	3
P8	0.4104	14	0.4054	14	0.4377	14
P9	0.3954	17	0.3924	17	0.4202	17
P10	0.4135	12	0.4111	12	0.4383	13
P11	0.4369	6	0.4297	7	0.4683	4
P12	0.4213	10	0.4150	10	0.4500	9
P13	0.3707	20	0.3691	20	0.3932	20
P14	0.4247	9	0.4236	9	0.4498	10
P15	0.4352	7	0.4329	5	0.4607	8
P16	0.3883	19	0.3867	19	0.4118	19
P17	0.3937	18	0.3922	18	0.4174	18
P18	0.4161	11	0.4149	11	0.4405	11
P19	0.4420	3	0.4409	3	0.4678	5
P20	0.4811	1	0.4788	1	0.5102	1

comparable. Linear orders can be found by giving different emphases to different properties of fuzzy sets. Here, an importance order is given to the criteria. If the first criterion does not give a unique linear order, then the second criterion should be used. One continues in this way as long as it is needed. The three different criteria used here are described in the following.

1) *First criterion, the removal:* Let us consider an ordinary number $k \in R$ and an FN A. The left side removal of A with respect to k, which is denoted by $R_l(A, k)$, is defined as the area bounded by k and the left side of the FN A.

Similarly, the right side removal $R_r(A, k)$ is defined. The removal of the FN A with respect to k is defined as the mean of $R_l(A, k)$ and $R_r(A, k)$. Thus

$$R(A, k) = \frac{1}{2}\left(R_l(A, k) + R_r(A, k)\right). \qquad (18)$$

The position of k can be located anywhere on the x-axis including $k = 0$. The areas, by definition, are positive quantities, but here they are evaluated by integration, taking into account the position (negative, zero, or positive) of the variable x; therefore, $R(A, k)$ can be positive, negative, or zero.

The first criterion, therefore, will be this removal with respect to k. However, two different FNs can have the same removal with respect to the same k. In fact, this criterion decomposes a set of FNs into classes having the same removal number. If the origin 0 is conveniently moved to the left, it is possible in this case that all of the FNs will have positive removal numbers. Hence, the removal numbers become positive if k is correctly chosen. The removal number with respect to a given k, therefore, can be taken as a measure of distances, and can thus be used for ordering the FNs. The removal number $R(A,k)$ defined in this criterion, which is relocated to $k = 0$, is equivalent to an "ordinary representative" of the FN. In the case of a triangular FN, this ordinary representative is given by

$$\tilde{A} = \frac{a_1 + 2a_2 + a_3}{4} \qquad (19)$$

where $A = (a_1, a_2, a_3)$.

TABLE V
CLOSENESS COEFFICIENTS AND RANKING ORDER FOR 20 CANDIDATE PROJECTS USING THREE POSSIBLE IDEAL SOLUTIONS AND SECOND SIMILARITY MEASURE

Projects	CCS with FPIS and FNIS 1	Ranking	CCS with FPIS and FNIS 2	Ranking	CCS with FPIS and FNIS 3	Ranking
P1	0.7392	1	0.7763	1	0.8740	1
P2	0.5175	5	0.5180	5	0.7680	5
P3	0.2629	12	0.2282	11	0.6195	13
P4	0.2832	10	0.2544	10	0.6215	12
P5	0.2680	11	0.2136	12	0.6424	10
P6	0.3297	8	0.2985	8	0.6728	8
P7	0.5646	3	0.5744	3	0.7973	3
P8	0.3206	9	0.2896	9	0.6649	9
P9	0.1640	16	0.1196	15	0.5572	16
P10	0.1470	17	0.0963	17	0.5569	17
P11	0.5608	4	0.5699	4	0.7956	4
P12	0.4215	6	0.4077	6	0.7190	6
P13	0.1104	18	0.0739	18	0.4874	20
P14	0.2541	13	0.1962	13	0.6218	11
P15	0.2048	14	0.1473	14	0.6007	14
P16	0.1026	20	0.0593	20	0.5048	19
P17	0.1091	19	0.0640	19	0.5144	18
P18	0.1741	15	0.1185	16	0.5722	15
P19	0.3612	7	0.3074	7	0.6733	7
P20	0.6820	2	0.6706	2	0.8055	2

TABLE VI
CLOSENESS COEFFICIENTS AND RANKING ORDER FOR 20 CANDIDATE PROJECTS USING THREE POSSIBLE IDEAL SOLUTIONS AND THIRD SIMILARITY MEASURE

Projects	CCS with FPIS and FNIS 1	Ranking	CCS with FPIS and FNIS 2	Ranking	CCS with FPIS and FNIS 3	Ranking
P1	0.3754	2	0.3344	2	0.4708	2
P2	0.3254	8	0.2766	8	0.4139	8
P3	0.2675	15	0.2098	15	0.3515	15
P4	0.2554	16	0.1958	16	0.3408	16
P5	0.3598	4	0.3172	4	0.4326	4
P6	0.2809	13	0.2254	13	0.3678	12
P7	0.3385	6	0.2918	6	0.4293	5
P8	0.2768	14	0.2206	14	0.3633	14
P9	0.2418	17	0.1802	17	0.3258	17
P10	0.2887	12	0.2347	12	0.3660	13
P11	0.3368	7	0.2900	7	0.4279	6
P12	0.3010	10	0.2485	10	0.3883	10
P13	0.1815	20	0.1103	20	0.2695	20
P14	0.3209	9	0.2724	9	0.3974	9
P15	0.3457	5	0.3011	5	0.4180	7
P16	0.2259	19	0.1619	19	0.3090	19
P17	0.2397	18	0.1779	18	0.3213	18
P18	0.2984	11	0.2461	11	0.3750	11
P19	0.3661	3	0.3250	3	0.4401	3
P20	0.4570	1	0.4309	1	0.5305	1

2) *Second criterion, the mode:* In each class of FNs, one should look for the mode, and these modes will generate subclasses. If the FNs under consideration have a nonunique mode, one takes the mean position of the modal values. It must be noted that this is only one way of obtaining subclasses, and one may need the following third divergence criterion for further subclassification.

TABLE VII
CLOSENESS COEFFICIENTS AND RANKING ORDER FOR 20 CANDIDATE PROJECTS USING THREE POSSIBLE IDEAL SOLUTIONS AND FOURTH SIMILARITY MEASURE

Projects	CCS with FPIS and FNIS 1	Ranking	CCS with FPIS and FNIS 2	Ranking	CCS with FPIS and FNIS 3	Ranking
P1	0.3429	5	0.2950	5	0.4739	2
P2	0.3075	9	0.2545	9	0.4060	7
P3	0.2608	15	0.2013	15	0.3309	15
P4	0.2479	16	0.1864	16	0.3189	16
P5	0.3583	3	0.3153	3	0.4199	6
P6	0.2694	13	0.2112	13	0.3510	12
P7	0.3164	7	0.2649	7	0.4246	4
P8	0.2659	14	0.2071	14	0.3457	13
P9	0.2376	18	0.1749	18	0.3006	17
P10	0.2884	11	0.2341	11	0.3439	14
P11	0.3147	8	0.2631	8	0.4230	5
P12	0.2868	12	0.2309	12	0.3755	10
P13	0.1786	20	0.1066	20	0.2374	20
P14	0.3167	6	0.2678	6	0.3784	9
P15	0.3465	4	0.3019	4	0.4027	8
P16	0.2243	19	0.1598	19	0.2803	19
P17	0.2384	17	0.1763	17	0.2936	18
P18	0.2970	10	0.2446	10	0.3529	11
P19	0.3599	2	0.3184	2	0.4264	3
P20	0.4378	1	0.4100	1	0.5275	1

3) *Third criterion, the, divergence:* If the divergence around the mode for each subclass is considered, subsubclasses are obtained, and this criterion may be sufficient to obtain the final linear ordering of FNs. When one orders FNs to size order, the procedure is that the above presented criterion is applied in the order 1)–2)–3), such that, if the unique linear order is not obtained, then one moves to the next criterion.

In the next section, a numerical example is used to show how the output from the FPOM, that is, the pay-off distributions for each project, and the ROV can be used as inputs into a ranking performed by the four new variants of fuzzy TOPSIS. In addition, we use the required amount of labor and total project cost as fuzzy inputs into the evaluation.

IV. NUMERICAL EXAMPLE

This numerical example uses the R&D project data that were also used in [6]. A pharmaceutical company can select among 20 R&D projects. Each project has three phases: drug discovery, testing, and market introduction. For simplicity, we assume that the first and second phases of all projects take three and seven years, respectively. It is assumed that costs, revenues, and budgets are discounted to the beginning of the planning horizon. The preferred development budgets for phases 1, 2, and 3 are presented in terms of FNs (in thousands) (271.2, 271.2, 271.2, 311.2), (984.9, 984.9, 984.9, 1184.9), and (1975.8, 1975.8, 1975.8, 2225.8), respectively. Similarly, available labor (in working months) to staff projects for these phases are (in working months) (374.5, 374.5, 374.5, 424.5), (1964.9,

TABLE VIII
AVERAGE RANKINGS FOR THE PROJECTS FROM 12 STUDIED CASES

Projects	Average with S_1	Average with S_2	Average with S_3	Average with S_4	Overall average rankings
P1	2.0000	4.0000	2.0000	1.0000	2.25
P2	8.0000	8.3333	7.6667	5.0000	7.25
P3	15.0000	15.0000	15.0000	12.0000	14.25
P4	16.0000	16.0000	16.0000	10.6667	14.67
P5	4.0000	4.0000	4.6667	11.0000	5.9167
P6	12.6667	12.6667	12.6667	8.0000	11.5
P7	5.6667	6.0000	4.6667	3.0000	4.83
P8	14.0000	13.6667	14.0000	9.0000	12.67
P9	17.0000	17.6667	17.0000	15.6667	16.83
P10	12.3333	12.0000	12.3333	17.0000	13.42
P11	6.6667	7.0000	5.6667	4.0000	5.83
P12	10.0000	11.3333	9.6667	6.0000	9.25
P13	20.0000	20.0000	20.0000	18.6667	19.67
P14	9.0000	7.0000	9.3333	12.3333	9.42
P15	5.6667	5.3333	6.6667	14.0000	7.92
P16	19.0000	19.0000	19.0000	19.6667	19.17
P17	18.0000	17.3333	18.0000	18.6667	18
P18	11.0000	10.3333	11.0000	15.3333	11.92
P19	3.0000	2.3333	3.6667	7.0000	4
P20	1.0000	1.0000	1.0000	2.0000	1.25

TABLE IX
AVERAGE RANKINGS w.r.t. CHOSEN IDEAL SOLUTIONS

projects	Averages w.r.t criteria 1	Averages w.r.t criteria 2	Averages w.r.t criteria 3
P1	2.5000	2.5000	1.7500
P2	7.5000	7.5000	6.7500
P3	14.2500	14.0000	14.5000
P4	14.5000	14.5000	15.0000
P5	5.5000	5.7500	6.5000
P6	11.7500	11.7500	11.0000
P7	5.2500	5.5000	3.7500
P8	12.7500	12.7500	12.5000
P9	17.0000	16.7500	16.7500
P10	13.0000	13.0000	14.2500
P11	6.2500	6.5000	4.7500
P12	9.5000	9.5000	8.7500
P13	19.5000	19.5000	20.0000
P14	9.2500	9.2500	9.7500
P15	7.5000	7.0000	9.2500
P16	19.2500	19.2500	19.0000
P17	18.0000	18.0000	18.0000
P18	11.7500	12.0000	12.0000
P19	3.7500	3.7500	4.5000
P20	1.2500	1.2500	1.2500

1964.9, 1964.9, 2214.9), and (1319.5, 1319.5, 1319.5, 1479.5), respectively.

Table I lists the fuzzy costs and revenues discounted to the beginning of phase 1 with interest rates of 4% and 15%. It is clear that the discount rate may significantly affect the final portfolio decision and, in practice, it is essential that the used discount rates are determined by means of effective methods available from financial engineering literature, or by otherwise in sufficient detail.

Table II lists the R&D staff resources required for the three phases in terms of FNs. Moreover, the candidate R&D projects can be classified into three strategic types: new drug (S1 = (#13, #14, #17, #18, #19, #20)), derivates of existing drug (S2 = (#5, #6, #8, #9, #10, #15, #16)), and incremental improvement to existing drugs (S3 = (#1, #2, #3, #4, #7, #11, #12)). It is further assumed that the target balances across three R&D strategies are 40–70%, 20–40%, and 10–30%, respectively.

The NPV of each project is first calculated, and then, the FPOM is used for the calculation of the ROV for the candidate projects, as shown in Table III.

To get the needed fuzzy decision matrix from this data, fuzzy addition is used to sum the three phases into one single FN for each project. This is done for Table I costs and for Table II fuzzy development resources and by doing this gaining two criteria to consider, the third criteria is the revenue from Table I, and the fourth criteria in the decision matrix is the ROV calculated using the FPOM. For importance weights for the criteria in the fuzzy TOPSIS, the same linguistic assessment was used for importance of all the four criteria, i.e., we use only one and the same weight. No other weighting schemes are considered, while it is acknowledged that weighting schemes are an issue for further research. The result is a decision matrix consisting of four criteria and 20 projects that need ranked.

In Table IV, one can see the calculated closeness coefficients for each project and the ranking order using similarity (2). Results are calculated using the given three possible choices for the ideal vectors. Results for the other three similarity measures are reported in Tables V–VII. As can be seen from the results, there are several individual ranking order differences among the results, depending on which similarity measure and ideal solutions are used.

In general, what can be observed is that the best projects always get relatively high ranking, and the inferior projects are always in the bottom of the ranking, although the individual

TABLE X
RANKINGS FOR THE PROJECTS FROM THE 12 STUDIED CASES

Projects	S_1	S_1	S_1	S_2	S_2	S_2	S_3	S_3	S_3	S_4	S_4	S_4
P1	2	2	2	5	5	2	2	2	2	1	1	1
P2	8	8	8	9	9	7	8	8	7	5	5	5
P3	15	15	15	15	15	15	15	15	15	12	11	13
P4	16	16	16	16	16	16	16	16	16	10	10	12
P5	4	4	4	3	3	6	4	4	6	11	12	10
P6	13	13	12	13	13	12	13	13	12	8	8	8
P7	6	6	5	7	7	4	5	6	3	3	3	3
P8	14	14	14	14	14	13	14	14	14	9	9	9
P9	17	17	17	18	18	17	17	17	17	16	15	16
P10	12	12	13	11	11	14	12	12	13	17	17	17
P11	7	7	6	8	8	5	6	7	4	4	4	4
P12	10	10	10	12	12	10	10	10	9	6	6	6
P13	20	20	20	20	20	20	20	20	20	18	18	20
P14	9	9	9	6	6	9	9	9	10	13	13	11
P15	5	5	7	4	4	8	7	5	8	14	14	14
P16	19	19	19	19	19	19	19	19	19	20	20	19
P17	18	18	18	17	17	18	18	18	18	19	19	18
P18	11	11	11	10	10	11	11	11	11	15	16	15
P19	3	3	3	2	2	3	3	3	5	7	7	7
P20	1	1	1	1	1	1	1	1	1	2	2	2

TABLE XI
STATISTICS FROM RANKINGS FOR THE PROJECTS, MINIMUM, MEAN, AND MAXIMUM RANKINGS

Projects	Minimum	Mean	Maximum
P1	1	2,25	5
P2	5	7,25	9
P3	11	14,25	15
P4	10	14,67	16
P5	3	5,92	12
P6	8	11,50	13
P7	3	4,83	7
P8	9	12,67	14
P9	15	16,83	18
P10	11	13,42	17
P11	4	5,83	8
P12	6	9,25	12
P13	18	19,67	20
P14	6	9,42	13
P15	4	7,92	14
P16	19	19,2	20
P17	17	18	19
P18	10	11,92	16
P19	2	4	7
P20	1	1,25	2

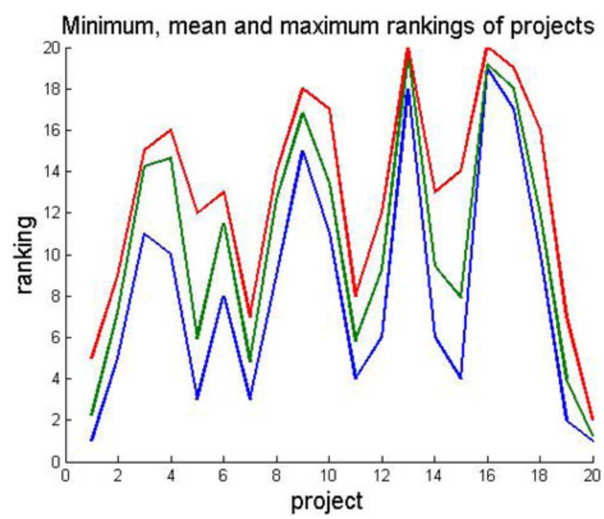

Fig. 2. Graphical representation of the ranking of the projects' statistics, minimum, mean, and maximum used.

ranking order may vary. For example, projects 1, 20, and 19, are within the top five rankings almost always, no matter which similarity measure is used, with the exception of project number 19 getting ranked to seventh place with the fourth similarity measure. For the bottom five choices, the results are even clearer. Projects 9, 13, 16, and 17 are always within the worst five choices. Table VIII shows the average ranking results from the 12 rankings that were made. One can clearly see that using this type of ranking, the top five projects would be projects 20, 1, 19, 7, and 11, and the worst five projects would be projects 13, 16, 17, 9, and 4.

Next, a closer look is given to the differences between the chosen similarity measures and the criteria for the computation of the ideal solutions. In Table VIII, next to the overall average ranking, one can also see the average rankings with respect to the similarity measure for the three possible ideal solutions. The first three similarity measures give similar ranking results, but the fourth similarity measure (the one that also considers the area of the FN) results in a slightly different ranking: the ranking is different for projects 4, 6, 8, 10, and 15. When the mean rankings are observed with regard to the criteria for the ideal solutions, the third criteria result in slightly varying (different) results, but using the first two criteria shows results that are very close to each other. This is likely due to the fact that the third ideal solutions' criterion is a "genuine" (trapezoidal) FN and includes more information. Of the projects that got "middle" rankings, the biggest differences were clearly caused by the chosen similarity measure. If the average rankings are compared with regard to

TABLE XII
OVERALL RANKINGS OF PROJECTS BY USING FNS

Ranking	Project	Removal value	Mode	divergence
1	20	1,375	1,25	1
2	1	2,625	2,25	4
3	19	4,25	4	5
4	7	4,917	4,833	4
5	11	5,917	5,833	4
6	5	6,708	5,917	9
7	2	7,125	7,25	4
8	15	8,458	7,917	10
9	12	9,125	9,25	6
10	14	9,458	9,4167	7
11	6	11	11,5	5
12	8	12,083	12,667	5
13	18	12,458	11,917	6
14	3	13,625	14,25	4
15	10	13,708	13,417	6
16	4	13,833	14,667	6
17	9	16,667	16,833	3
18	17	18	18	2
19	16	19,333	19,167	1
20	13	19,333	19,667	2

criteria, as done in Table IX, it can be seen that for the projects that got middle rankings, the averages are quite close to each other. Comparing average rankings for the different similarity measures, one can see that the fourth similarity measure gives a different ranking for the middle projects, indicating that in this case, the choice of the similarity measure has had more impact on the results, than the choice of the ideal solution. The fourth similarity measure is the only measure that takes into account the area of FNs, and this additional information seems to be causing the deviation in rankings when using this measure, compared with the other three measures. Table X summarizes all the rankings.

It is clear from Tables IV–X that different rankings are obtained, based on which similarity measure or ideal solution is used. In Table XI, statistics are collected from the rankings gained from the experiments, these can also be seen in Fig. 2, where minimum, mean, and maximum rankings for all projects are plotted. Table XI shows that different similarity measures focus on different "issues" in the data and result in different rankings. In order to find an overall ranking for the projects, fuzzy triangular numbers are created from rankings by using minimum, mean, and maximum ranking for each project: project number one gets the FN $P_1 = (1, 2.25, 5)$ and so on. Then, the FNs are processed by using ranking of triangular FNs to gain an overall ranking for the projects. For this procedure, a three-step process, which is introduced by Kaufmann and Gupta [36], is used.

Based on the removal number, mode, and divergence, we get the overall ranking, which is given in Table XII.

The overall ranking can now be used in supporting the portfolio selection process of R&D projects as investments.

V. DISCUSSION AND CONCLUSION

A new system was presented that allows the intuitive inclusion of estimation imprecision in the analysis and ranking of R&D projects as investments. The system represents imprecision with FNs, and uses them in the ranking of projects with a set of new similarity measure-based fuzzy variants of the well-known TOPSIS method. The fuzzy TOPSIS method was enhanced by replacing the fuzzy distance measure commonly used in fuzzy TOPSIS with fuzzy similarity measures; this is a new contribution. Three different criteria for the FPIS and FNIS needed in the fuzzy TOPSIS method were also considered. All in all, 12 different possible combinations of fuzzy similarity measures and ideal solutions for fuzzy TOPSIS were presented.

To illustrate and study the resulting differences in rankings of these 12 combinations, a numerical example was used, where the four similarity measures and three different types of ideal solutions were tested. It was found that the lowest ranking projects are always in the bottom ranks and that the best projects are getting high rankings. This is not obvious from the get-go, as there was no previous information about how the different measures and the different ideal solution combinations would affect the results. The results of the limited numerical illustration show that the individual rankings differ, depending on which fuzzy similarity measure and ideal solution is used. This shows that, even if the general picture seems to be such that the results are in line with each other, there are individual differences in the results from the different measure/ideal solution combinations. These differences indicate that if only one combination is used, it is relevant to select the measure used correctly.

In the numerical example, the largest deviation in final project rankings occurred when the third criterion for the selection of the ideal solutions and the fourth fuzzy similarity measure were used. This is most likely due to the fact that the third criterion is a truly fuzzy trapezoidal number, and the other two criteria are fuzzy representations of crisp numbers; there seems to be a difference in the included information. The fourth fuzzy similarity measure is able to take into consideration the most information of the four, with regard to the similarity of two FNs. There seems to be an indication of a potential benefit in using criteria and measures that are able to "carry" more information—information that can perhaps be used to, for example, contextualize analysis to better suit the R&D project selection situation.

An overall ranking was performed for the rankings that resulted from the four new fuzzy TOPSIS variants with the three different ideal solutions each. The ranking is able to include and take into consideration the variability of the 12 combinations that is caused by the different "point of view" that each ranking (combination) may represent. The presented overall ranking of subrankings as FNs is a new approach for R&D project ranking.

Further avenues of research include considering the moments of FNs as possible criteria in R&D project selection as investments, and the inclusion of profitability information directly into the similarity measures. From the side of fuzzy TOPSIS, the different weighting for different criteria of the decision matrix merits more study. We have considered that all criteria have the same importance, but it is possible that decision makers may consider that different criteria are of different importance. In addition, extending the method to allow different criteria to use a different fuzzy similarity measure seems an interesting way to go—this would also lead to the need to investigate the suitability of different fuzzy similarity measures for different types of criteria.

REFERENCES

[1] M. Heikkilä, "R&D investment decisions with real options—Profitability and decision support," Department of Information Systems, Faculty Technol. Abo Akademi Univ., Turku, Finland, 2009.

[2] E. Karsak, "A generalized fuzzy optimization framework for R&D project selection using real options valuation," in *Computational Science and Its Applications*, M. E. A. Gavrilova, Ed. Berlin, Germany: Springer-Verlag, 2006, pp. 918–927.

[3] O. Pereira and D. Junior, "The R&D project selection problem with fuzzy coefficients," *Fuzzy Sets Syst.*, vol. 26, pp. 299–316, 1988.

[4] J. Wang and W.-L. Hwang, "A fuzzy set approach for R&D portfolio selection using a real options valuation model," *Omega*, vol. 35, pp. 247–257, 2007.

[5] F. Hassanzadeh, M. Collan, and M. Modarres, "A technical note on "A fuzzy set approach for R&D portfolio selection using a real options valuation model" by Wang and Hwang (2007)," *Omega*, vol. 39, pp. 464–465, 2011.

[6] F. Hassanzadeh, M. Collan, and M. Modarres, "A practical approach to R&D portfolio selection using fuzzy set theory," *IEEE Trans. Fuzzy Syst.*, vol. 20, no. 4, pp. 615–622, Aug. 2012.

[7] P. Ryan and G. Ryan, "Capital budgeting practices of the fortune 1000: How have things changed?" *J. Bus. Manage.*, vol. 8, pp. 355–365, 2002.

[8] D. Kuchta, "Fuzzy capital budgeting," *Fuzzy Sets Syst.*, vol. 111, pp. 367–385, 2000.

[9] E. Bas and C. Kahraman, "Fuzzy capital rationing model," *J. Comput. Appl. Math.*, vol. 224, pp. 628–645, 2009.

[10] N. Kulatilaka and A. Marcus, "Project valuation under uncertainty: When does DCF fail?" *J. Appl. Corporate Finance*, vol. 5, pp. 92–100, 1992.

[11] J. E. Smith and R. F. Nau, "Valuing risky projects: Option pricing theory and decision analysis," *Manage. Sci.*, vol. 41, pp. 795–816, 1995.

[12] V. Datar and S. Mathews, "European real options: An intuitive algorithm for the Black Scholes formula," *J. Appl. Finance*, vol. 14, pp. 7–13, 2004.

[13] S. Mathews and J. Salmon, "Business engineering: A practical approach to valuing high-risk, high-return projects using real options," in *Tutorials Operations Research*, P. Gray, Ed. Hanover, MA, USA: INFORMS, 2007.

[14] M. Collan, R. Fuller, and J. Mezei, "Fuzzy pay-off method for real option valuation," *J. Appl. Math. Decision Syst.*, vol. 2009, pp. 238196-1–238196-14, 2009.

[15] C. T. Chen, "Extensions of the TOPSIS for group decision-making under fuzzy environment," *Fuzzy Sets Syst.*, vol. 114, pp. 1–9, 2000.

[16] C. T. Chen, C. T. Lin, and S. F. Huang, "A fuzzy approach for supplier evaluation and selection in supply chain management," *Int. J. Prod. Economics*, vol. 102, pp. 289–301, 2006.

[17] G. Hu and J. Tan, "Investment decision-making method of real estate project based on grey correlation and TOPSIS," in *Proc. Int. Conf. E-Business E-Government*, Guangzhou, China, 2010, pp. 1546–1549.

[18] J. Dogangeh, M. Mojahed, and R. M. Yusuff, "Best project selection by using group TOPSIS method," in *Proc. Int. Assoc. Comput. Sci. Inf. Technol. Spring Conf.*, Singapore, 2009, pp. 50–53.

[19] C. Tolga, "Fuzzy multicriteria R&D project selection with a real options valuation model," *J. Intell. Fuzzy Syst.*, vol. 19, pp. 359–371, 2008.

[20] D. Kuchta, "A fuzzy model for R&D project selection with benefit: Outcome and resource," *Eng. Economics*, vol. 46, pp. 164–180, 2001.

[21] M. Collan, R. Fuller, and J. Mezei, "Compound real options with the fuzzy pay-off method: Three-stage case illustration," presented at the 13th Annu. Int. Conf. Real Options, Braga & Santiago, Portugal, 2009.

[22] M. Collan, *The Pay-Off Method: Re-Inventing Investment Analysis*. Charleston, NC, USA: CreateSpace, 2012.

[23] T. Faulkner, "Applying options thinking to R&D valuation," *Res. Technol. Manage.*, vol. 19, pp. 50–56, 1996.

[24] S. Mathews, V. Datar, and B. Johnson, "A practical method for valuing real options: The Boeing approach," *J. Appl. Corporate Finance*, vol. 19, pp. 95–104, 2007.

[25] C. Carlsson and R. Fullér, "On possibilistic mean value and variance of fuzzy numbers," *Fuzzy Sets Syst.*, vol. 122, pp. 315–326, 2001.

[26] B. Ashtiani, F. Haghighirad, A. Makui, and G. A. Montazer, "Extension of fuzzy TOPSIS method based on interval-valued fuzzy sets," *Appl. Soft Comput.*, vol. 9, pp. 457–461, 2009.

[27] P. Luukka, "Fuzzy similarity in multicriteria decision-making problem applied to supplier evaluation and selection in supply chain management," *Adv. Artif. Intell.*, vol. 2011, art. ID 353509, 9 pp., 2011. DOI: 10.1155/2011/353509.

[28] S. H. Chen, "Operations on fuzzy numbers with functional principal," *Tamkang J. Manage. Sci.*, vol. 6, pp. 13–25, 1985.

[29] C. H. Hsieh and S. H. Chen, "Similarity of generalized fuzzy numbers with graded mean integration representation," in *Proc. 8th Int. Fuzzy Syst. Assoc. World Congr.*, Taipei, Taiwan, 1999, pp. 551–555.

[30] S.-J. Chen and S.-M. Chen, "A new method to measure the similarity between fuzzy numbers," in *Proc. IEEE Int. Fuzzy Syst. Conf.*, Melbourne, Australia, 2001, pp. 1123–1126.

[31] S. H. Wei and S. M. Chen, "A new approach for fuzzy risk analysis based on similarity measures of generalized fuzzy numbers," *Expert Syst. Appl.*, vol. 36, pp. 589–598, 2009.

[32] S. R. Hejazi, A. Doostparast, and S. M. Hosseini, "An improved fuzzy risk analysis based on new similarity measures of generalized fuzzy numbers," *Expert Syst. Appl.*, vol. 38, pp. 9179–9185, 2011.

[33] A. Wilbik and J. M. Keller, "A fuzzy measure similarity between sets of linguistic summaries," *IEEE Trans. Fuzzy Syst.*, vol. 21, no. 1, pp. 183–189, Feb. 2013.

[34] H. Le Capitaine, "A relevance-based learning model of fuzzy similarity measures," *IEEE Trans. Fuzzy Syst.*, vol. 20, no. 1, pp. 57–68, Feb. 2012.

[35] E. Hullermeier, M. Rifqi, S. Henzgen, and R. Senge, "Comparing fuzzy partitions: A generalization of the rand index and related measures," *IEEE Trans. Fuzzy Syst.*, vol. 20, no. 3, pp. 546–556, Jun. 2012.

[36] M. Kaufmann and M. Gupta, *Fuzzy Mathematical Models in Engineering and Management Science*. New York, NY, USA: Elsevier, 1988.

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