This article has been accepted for publication in IEEE Transactions on Fuzzy Systems. This is the author's version which has not been fully edited and content may change prior to final publication. Citation information: DOI 10.1109/TFUZZ.2024.3387429

Fuzzy Machine Learning: A Comprehensive Framework and Systematic Review

Jie Lu, *Fellow, IEEE,* Guangzhi Ma, *Student Member, IEEE,* and Guangquan Zhang

***Abstract*—Machine learning draws its power from various disciplines, including computer science, cognitive science, and statistics. Although machine learning has achieved great advancements in both theory and practice, its methods have some limitations when dealing with complex situations and highly uncertain environments. Insufficient data, imprecise observations, and ambiguous information/relationships can all confound traditional machine learning systems. To address these problems, researchers have integrate machine leaning from different aspects, and fuzzy techniques including fuzzy sets, fuzzy systems, fuzzy logic, fuzzy measures, fuzzy relations, and so on. This paper presents a systematic review of fuzzy machine learning, from theory, approach to application, with the overall objective of providing an overview of recent achievements in the field of fuzzy machine learning. To this end, the concepts and frameworks discussed are divided into five categories: (a) fuzzy classical machine learning; (b) fuzzy transfer learning; (c) fuzzy data stream learning; (d) fuzzy reinforcement learning; and (e) fuzzy recommender systems. The literature presented should provide researchers with a solid understanding of the current progress in fuzzy machine learning research and its applications.**

***Index Terms*—Machine learning, fuzzy sets and systems, fuzzy logic, transfer learning, data stream learning, recommender systems**

I. Introduction

In the dynamic realm of technology, machine learning has profoundly transformed various sectors. It leads innovation by decoding complex data patterns, driving advancements in artificial intelligence, and influencing how we engage with information and understand the capabilities of computational systems. However, with most existing machine learning methods, accuracy suffers in scenarios characterized by uncertainty, such as the only available observations are imprecise, or where the data are noisy or incomplete. Additionally, many real-world datasets contain uncertain relationships, and conventional machine learning methods generally find it difficult to identify or work with these structures. To address these issues, researchers have used fuzzy techniques to integrate into machine learning called *Fuzzy Machine Learning* [1] (FML) as a solution, since fuzzy techniques are successful to deal with uncertainties. FML systems fuse machine learning algorithms with fuzzy techniques, such as fuzzy sets [2], fuzzy systems [3], fuzzy clustering [4], fuzzy relations [5], fuzzy measures [6], fuzzy matching [7], fuzzy optimization [8], and so on, to build new models that are more robust to the many and varied types of uncertainty found in real-world problems.

The work presented in this paper was supported by the Australian Research Council (ARC) under FL190100149 and DP220102635.

Jie Lu, Guangzhi Ma and Guangquan Zhang are with the Australian Artificial Intelligence Institute, Faulty of Engineering and Information Technology, University of Technology Sydney, Sydney, NSW, 2007, Australia, e-mail: Guangzhi.Ma@student.uts.edu.au, {Jie.Lu; Guangquan.Zhang}@uts.edu.au.

FML stands out as an invaluable ally in the realm of complex, and dynamic (uncertain) environments, presenting substantial advantages that elevate its efficacy. Unlike traditional machine learning approaches, fuzzy techniques that generally based on the concept of fuzzy sets [9] and fuzzy theory [10] excel in capturing and navigating the nuanced shades of uncertainty inherent in dynamic scenarios. Their inherent ability to model uncertainty empowers it to gracefully adapt to the ever-changing patterns that characterize dynamic environments. In situations where traditional models might falter or struggle to keep pace, fuzzy techniques emerge as robust problem-solvers, providing a more accurate representation of the inherent fuzziness present in real-world data [11]. Furthermore, in the relentless quest for interpretability, fuzzy machine learning triumphs. Its models not only navigate complexity but also offer clear insights into decision-making processes. This interpretability proves to be a critical asset in dynamic environments where understanding the rationale behind model decisions is paramount. Next, we summarize some main successes of how fuzzy techniques can improve machine learning algorithms:

1) Fuzzy sets [2] can be used to represent vague or ambiguous concepts and data, such as that commonly found with linguistic variables, noisy or incomplete data, and interval-valued data. The fuzzy sets enhance the algorithm's ability to make decisions in uncertain and complex situations, which can be particularly useful in applications where real-world conditions can be unpredictable, such as robotics or autonomous vehicles.
2) Fuzzy rule-based systems [3] can provide a transparent and interpretable prediction frameworks. Fuzzy rule-based systems use linguistic rules to represent knowledge and, so, can be used to generate explanations for the decisions made by the system. This can be useful in applications like medical diagnosis.
3) Fuzzy clustering [4], which is a well-known approach to clustering, can improve machine learning algorithms by identifying patterns in data that traditional clustering methods may not easily identify. Fuzzy clustering not only allows for overlapping clusters, it can also handle data points that may not belong to any particular cluster with certainty. This can be useful in applications like image recognition.
4) Fuzzy relations [5] can provide a more flexible and nuanced representation of the relationships between variables or data points. They can also capture nonlinear relationships to enable more accurate and expressive machine learning models. In addition, fuzzy relations are useful when handling multi-modal data or data assembled

This article has been accepted for publication in IEEE Transactions on Fuzzy Systems. This is the author's version which has not been fully edited and content may change prior to final publication. Citation information: DOI 10.1109/TFUZZ.2024.3387429

from multiple sources because researchers can define fuzzy relations between the different modalities to result in a more comprehensive and accurate model.

In the past decade, there have been over $500,000$ articles in high-quality journals and conference proceedings containing the words "fuzzy" and "machine learning". However, none of these articles provide a comprehensive review of the recent literature on FML. Several previous surveys in the area only offered valuable insights into certain subfields of FML. For example, Baraldi *et al.* [12] provides a brief review of fuzzy clustering algorithms for pattern recognition, while Skrjanc *et al.* [13] summarizes models based on evolving fuzzy rules and neuro-fuzzy networks for clustering, regression, identification, and classification problems. Additionally, Zheng *et al.* [14] reviewed recent work on fusing deep learning models with fuzzy systems. Moreover, the last decade has witnessed the emergence of new subfields in FML, such as fuzzy transfer learning and fuzzy data stream learning. Providing an investigative report to outline these new subfields is significant. For these reasons, a new, more comprehensive, and more up-to-date survey of FML is warranted. This paper primarily targets researchers interested in employing fuzzy techniques to enhance the performance of machine learning methods, particularly in situations involving complex or uncertain factors.

The studies included in this survey were selected following three steps:

Step 1 Identify and determine an appropriate set of publication databases to search. We searched the well-known databases of Science Direct, ACM Digital Library, IEEE Xplore and SpringerLink. These provided a comprehensive bibliography of research papers on machine learning and FML.

Step 2 Preliminary screening of articles: The first search was based on keywords. The articles were then selected for inclusion in the review if they: 1) presented a new theory, algorithm or methodology in the area of FML; or 2) reported an application built around a FML algorithm.

Step 3 Filtering the results for presentation: The articles selected in Step 2 were then divided into five groups to be summarized in separate sections: (a) fuzzy classical machine learning; (b) fuzzy transfer learning; (c) fuzzy data stream learning; (d) fuzzy reinforcement learning; and (e) fuzzy recommender systems. At this point, we undertook one final screening of the articles, see Fig. 1. A study was retained if it demonstrated sufficient: (1) novelty, i.e., it had been published within the last decade; and (2) impact, i.e., it had been published in a high-quality journal/conference or having high citations.

The main contributions of this paper are as follows:

1) It survey comprehensively summarizes the developments and achievements in the field of FML. Work in this field is divided into five main categories for discussion.
2) The shortcomings of traditional machine learning methods in real-world scenarios are analyzed for each category, followed by an explanation of how FML has been used to address these issues. The insights provided are designed to help researchers understand the context of developments in FML research and its applications.
3) It provides a critical discussion of the state-of-the-art (SOTA) FML models and outlines directions for future research.

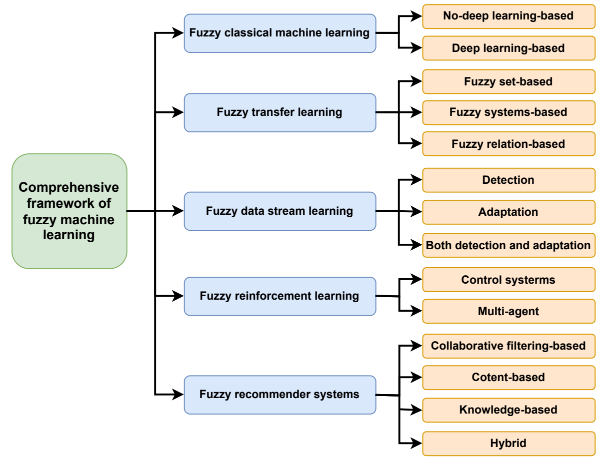


Fig. 1. The main framework of this survey.

The remainder of this paper is structured as follows. Section II provides some relevant mathematical concepts to illustrate how fuzzy logic can be integrated into machine learning. Sections III-VII discuss the five categories of FML respectively. Finally, Section IX summarizes the material covered and goals of this review and outlines future work.

II. Basic Concepts of Fuzzy Machine Learning

In this section, we briefly introduce some relevant mathematical concepts to illustrate how fuzzy logic can be integrated into transfer learning, data stream learning, reinforcement learning, and recommender systems. These concepts should help researchers to better understand the articles introduced in the following sections.

A. Fuzzy transfer learning

Transfer learning [15] tries to train a well-performed model in one domain (target) by leveraging knowledge from another domain (source) that has different distribution or learning tasks compared with the previous one. This section introduces two representative fuzzy transfer learning frameworks: 1) fuzzy rule-based [16]; 2) fuzzy equivalence-based [17].

Fuzzy rule-based transfer learning framework [16]: Let $\mathcal{S} = \{\mathcal{S}^1, \mathcal{S}^2, \cdots, \mathcal{S}^N\}$ denotes a set of source domains, where $\mathcal{S}^n = \{(\mathbf{x}_i^{S_n}, y_i^{S_n}) | \mathbf{x}_i^{S_n} \in \mathcal{X}^n, y_i^{S_n} \in \mathcal{Y}\}_{i=1}^{m_n}, n \in [N]$ and $(\mathbf{x}_i^{S_n}, y_i^{S_n})$ is the i-th input-output data pairs in the n-th source domain. Here, $\mathcal{X}^n \subset \mathbb{R}^p$ denotes the feature space of each source domain and $\mathcal{Y}$ is a response space ($\mathcal{Y} = \{1, 2, \cdots, K\}$ given a classification task, and $\mathcal{Y} \subset \mathbb{R}$ given a regression task). $\mathcal{T} = \{\mathbf{x}_i^T | \mathbf{x}_i^T \in \mathcal{X}^T\}_{i=1}^{m_t}$ is the unlabeled target domain (for unsupervised scenario), where $\mathcal{X}^T \subset \mathbb{R}^p$ is the feature space of the target domain. In homogeneous cases, $\mathcal{X}^1, \cdots, \mathcal{X}^N, \mathcal{X}^T$ have the same number of features, while they contain different number of features in heterogeneous cases.

We denote $\mathcal{R} = \{\mathcal{R}^1, \mathcal{R}^2, \cdots, \mathcal{R}^N\}$ as the constructed fuzzy rules space of $\mathcal{S}$, where $\mathcal{R}^n = \{r(v_l^{S_n}, a_l^{S_n})\}_{l=1}^{l_n}, n \in [N]$ is the n-th rule set of $\mathcal{S}^n$. Here, the rule $r(v_l^{S_n}, a_l^{S_n})$ is represented as:

$$\begin{aligned} &\text{if } \ \mathbf{x}_i^{S_n} \ \text{ is } \ A_l(\mathbf{x}_i^{S_n}, v_l^{S_n}), \\ &\text{then } \ \mathbf{y}_i^{S_n} \ \text{ is } \ P_l(\mathbf{x}_i^{S_n}, a_l^{S_n}), \\ &l = 1, 2, \cdots, l_n. \end{aligned} \tag{1}$$

Let $\mathcal{R}^T$ denote the obtained fuzzy rules of target domain $\mathcal{T}$.

Finally, $\Phi = \{\Phi^1, \Phi^2, \cdots, \Phi^N\}$ is denoted as the conclusion of $\mathcal{R}$ (eg., linear combination), where $\Phi^n(\mathcal{R}^n, \mu_n), n \in [N]$ is the n-th conclusion of $\mathcal{R}^n$. Hence, fuzzy rule-based transfer learning aims to use the knowledge from $\mathbf{D} = \{\mathcal{S}, \mathcal{R}, \Phi\}$ to fit the data in the target domain, i.e., obtain $\mathcal{R}^T$ and the conclusion of $\mathcal{R}^T$.

Fuzzy equivalence-based transfer learning framework [17]: Different from fuzzy rule-based transfer learning, this framework applies the fuzzy equivalence relations among features in source and target domains to replace the fuzzy rules. Let $\mathcal{U} = \{\mathcal{U}^1, \mathcal{U}^2, \cdots, \mathcal{U}^N\}$ denotes the membership function space of the features in $\mathcal{S}$, where $\mathcal{U}^n = \{\mu_1^{S_n}, \mu_2^{S_n}, \cdots, \mu_{m_n}^{S_n}\}, n \in [N]$ and $\mu_i^{S_n}, i \in [m_n]$ is the membership function of $\mathbf{x}_i^{S_n}$. $\mathbf{R}_{\mathcal{S}}^M = \{\mathbf{R}_1^M, \mathbf{R}_2^M, \cdots, \mathbf{R}_N^M\}$ denoted as the fuzzy equivalence relations space on $\mathcal{S}$, where $\mathbf{R}_n^M, n \in [N]$ is the fuzzy equivalence relation on $\mathcal{S}^n$. Here, $\mathbf{R}_n^M, n \in [N]$ is a $m_n \times m_n$ matrix (see [17], [18] for details),

$$(\mathbf{R}_n^M)_{ij} = \mathbf{R}_{S_n}(\mathbf{x}_i^{S_n}, \mathbf{x}_j^{S_n}; \mu_i^{S_n}, \mu_j^{S_n}), i, j \in [m_n], \tag{2}$$

where $\mathbf{R}_{S_n}$ is a fuzzy equivalence relation operator on $\mathcal{S}^n$.

Hence, fuzzy equivalence-based transfer learning framework aims to use the knowledge from $\mathbf{D} = \{\mathcal{S}, \mathcal{U}, \mathbf{R}_{\mathcal{S}}^M\}$ to fit the data in the target domain.

B. Fuzzy data stream learning

Data stream learning [19], [20], also known as stream mining, refers to a set of techniques and algorithms designed to handle and analyze data that arrives continuously over time in a streaming fashion. However, in real-world scenarios, the statistical properties of the data may change over time, making models and algorithms that were previously accurate less effective over time. This phenomenon is known as concept drift [21]–[23]. A formal definition of concept drift follows.

Definition 1 (Concept drift [23]): Consider a time period $[0, t]$ and a set of samples, denoted as $S_{0,t} = \{d_0, \cdots, d_t\}$, where $d_i = (X_i, y_i)$ is one observation (or one data instance). X_i is the feature vector, y_i is the label, and $S_{0,t}$ follows a certain distribution $\mathbb{F}_{0,t}(X, y)$. Concept drift occurs at timestamp $t+1$, if $\mathbb{F}_{0,t}(X, y) \neq \mathbb{F}_{t+1,\infty}(X, y)$, denoted as $\exists t : \mathbb{P}_t(X, y) \neq \mathbb{P}_{t+1}(X, y)$.

Hence, when a concept drift occurs at $t+1$, we aim to adapt the predictor $H_t = \arg\min_{h \in \mathcal{H}} \ell(h, X, y | (X, y) \in \mathbb{P}_t(X, y))$ to fit the new distribution $\mathbb{P}_{t+1}(X, y)$. Next, we briefly introduce a fuzzy clustering-based drift learning structure [24] to show how fuzzy logic can be integrated into data stream learning.

In fuzzy clustering-based drift learning [24], fuzzy clustering is applied to learn how many patterns exist in the observed data instances and the membership degree of each instance belonging to each pattern during the process of learning the parameters for the predictor. Given $\{\mu_{tk}\}$ the membership of t-th instance belonging to k-th cluster, $\{C_k\}$ the k-th cluster centroid, $\{X_t\}$ the input variables at time step t and $\{\theta_t\}$ the parameters for the k-th predictor. Then, the purpose of fuzzy clustering-based drift learning is shown as follow:

$$\begin{aligned} \min_{\mu_{tk}, C_k, \theta_t} & \sum_{t=1}^{N} (\sum_{k=1}^{K} \mu_{tk} X_t \theta_k - y_t)^2 \\ & + \lambda_1 \sum_{k=1}^{K} \sum_{t=1}^{N} \mu_{tk}^2 \parallel X_t - C_k \parallel_2^2 + \lambda_2 \sum_{k=1}^{K} \parallel \theta_k \parallel_2^2 \\ & s.t. \sum_{k=1}^{K} \mu_{tk} = 1, t \in [N]. \end{aligned} \tag{3}$$

where λ_1, λ_2 are two pre-assigned parameters. Fuzzy clustering [25], [26] is utilize to optimize μ_{tk}, C_k.

C. Fuzzy reinforcement learning

Reinforcement learning (RL) [27] is the study of planning and learning in a scenario where a learner (called an agent) proactively interacts with the environment to achieve a certain goal. The agent's aim is to develop the optimal strategy for accumulating rewards. It does this by learning from the feedback it receives. Reinforcement learning has been successfully applied to a variety of real-world problems, such as robotics control [28], game playing [29], and autonomous driving [30]. In this section, we provide information of how fuzzy logic can be integrated into RL.

First, fuzzy sets can be used to represent uncertainty in state, action, or reward spaces in RL. For instance, fuzzy reward signals [31] represent the uncertainty or imprecision in the reward received by an agent. Additionally, fuzzy controllers [32] that use fuzzy logic to map inputs to control actions can be integrated into RL systems to handle uncertain or qualitative control decisions. Next, we give a general mathematical expression for fuzzy controller. Let $X_1, \cdots, X_n$ be the input variables to the fuzzy controller, and Y be the output variable representing the control action. The fuzzy sets associated with each variable are denoted as $A_1, \cdots, A_n$ for inputs and B for the output. Let $\mu_{A_i}(x_i)$ represent the membership function for the fuzzy set A_i of input X_i, and $\mu_B(y)$ represent the membership function for the fuzzy set B for output Y. Then, generic fuzzy rules that define the mapping from inputs to outputs can be expressed as:

$$\text{if } X_i \text{ is } A_{1j} \text{ and} \cdots \text{and } X_n \text{ is } A_{nj}, \text{then } Y \text{ is } B_j. \tag{4}$$

After applying the fuzzy rules, defuzzification is performed to obtain a crisp output value.

Further, a fuzzy inference system can be used to make decisions in RL [33], such as determining the next action to take based on fuzzy input signals representing uncertain states or rewards. For example, fuzzy Q-learning [34] extends Q-learning by incorporating fuzzy logic to handle uncertain and imprecise state-action pairs. Fuzzy rules and membership functions are applied to update the Q-values.

This article has been accepted for publication in IEEE Transactions on Fuzzy Systems. This is the author's version which has not been fully edited and content may change prior to final publication. Citation information: DOI 10.1109/TFUZZ.2024.3387429

D. Fuzzy recommender system

A recommender system [35], [36] is a type of information filtering system that analyzes user preferences or behavior to provide suggestions personalized to that particular user. In this section, we provide information of how fuzzy logic can be integrated into recommender systems.

Let $\mathcal{V} = \{v_1, v_2, \cdots, v_M\}$ denotes the item set and $\mathcal{U} = \{u_1, u_2, \cdots, u_N\}$ denotes the user set. In data preprocess, fuzzy set or linguistic variable can be used to represent item/user terms and user-item rating matrix $\mathbf{R}, (\mathbf{R})_{ij} = r_{u_i,v_j}, i \in [N], j \in [M]$ (r_{u_i,v_j} is a rating of a user for a item). Fuzzy set can help dealing with some types of uncertainty in the description of item features. For example, Yager [37] denotes a set of primitive assertions to describe items, denoted as $A = \{A_1, \cdots, A_n\}$. For one item v, we can view the item v as a fuzzy subset over the space A. If one item v satisfies assertion A_i, the assertion has validity equal to one otherwise zero. The membership degree on A_i in v is $v(A_i)$. Then, an item in a recommender system can be represented as a fuzzy set over an assertion set. In addition, linguistic variable is widely used to generate the user-item linguistic term-based rating matrix $\mathbf{R}$.

In fuzzy user preference/profile generation process, fuzzy rule-based system, such as Takagi–sugeno–kang fuzzy system (TSK-FS), is usually applied to model the uncertainty and imprecision inherent in users' preferences. Finaly, in order to obtain the final predicted ratings $\hat{r}_{u_i,v_j}$ for unrated items, fuzzy similarity is widely used for calculating the similarity between items and users. For instance, $S(v_i, v_j)$ [38] is a fuzzy similarity to measure the similarity between item v_i and v_j,

$$S(v_i, v_j) = \frac{\sum\limits_{u \in U_{ij}} \int_0^1 f([r_{u,v_i}]_\alpha, [r_{u,v_j}]_\alpha) d\alpha}{\sqrt{\sum\limits_{u \in U_{ij}} (\int_0^1 g([r_{u,v_i}]_\alpha) d\alpha)^2} \sqrt{\sum\limits_{u \in U_{ij}} (\int_0^1 h([r_{u,v_j}]_\alpha) d\alpha)^2}}, \tag{5}$$

where U_{ij} represents the set of users that both rated items v_i and v_j . $[r_{u,v_i}]_\alpha$ represent the α-cut of r_{u,v_i} (linguistic variable), and f, g, h are predefined functions.

III. FUZZY CLASSICAL MACHINE LEARNING

Classical machine learning algorithms, such as decision trees, support vector machines, and neural networks, have been responsible for remarkable achievements both theoretically and from a practical point of view. Numerous articles involve combining fuzzy techniques with classical machine learning algorithms to overcome different types of problems with uncertainty, such as incomplete information and imprecise observations. In this section, we summarize these works, dividing the techniques into two categories: 1) Non-deep learning-based method, 2) Deep learning-based method.

A. Non-deep learning-based method

The non-deep learning-based methods can be further divided into three main types: clustering, regression, and classification. Each is discussed in turn next.

1) Clustering: Fuzzy clustering has been widely researched over the last 40 years, and several survey papers have already been published summarizing prior work in this field [39], [40]. First, we summarize the main ascendancies of applying fuzzy techniques in clustering.

- **Soft Assignment of Data Points:** Traditional clustering algorithms assign each data point to a single cluster, resulting in a hard assignment. However, in many cases, some of the data points may have ambiguous relationships with the clusters or their memberships may overlap into multiple clusters. Fuzzy clustering allows for *soft assignment*, where a data point's membership in a cluster is not simply binary, but rather it is measured in degrees and can apply to multiple clusters.
- **Flexibility in Cluster Shape:** Unlike traditional hard clustering algorithms, such as K-means, which assume spherical clusters of equal size, fuzzy clustering allows for more flexible and irregular cluster shapes. Fuzzy logic allows researchers and analysts to model overlapping clusters, clusters of varying sizes and densities, and clusters with complex boundaries. Thus, fuzzy clustering is highly suitable for datasets with complex structures.
- **Handling Outliers and Noise:** Applying fuzzy logic makes clustering more robust to outliers and noisy data than traditional clustering methods. With fuzzy logic, a data point can have a low membership degree to a cluster, which effectively reduces the influence of outliers or noisy data points on the overall clustering results.
- **Interpretability and Granularity:** The fuzzy membership degrees assigned to data points offer a quantitative measure of their association with each cluster. This allows for a more nuanced understanding of the data and provides insights into the degree of similarity or dissimilarity between data points and clusters. Fuzzy logic also allows for the representation of gradual transitions, providing a more detailed and fine-grained view of the clustering.

One of the most powerful and well-known algorithms in fuzzy clustering analysis is Fuzzy C-means (FCM), developed by Dunn in 1973 [25] and further developed by Bezdek in 1984 [41]. In the intervening years, FCM has been widely used and revised many times to deal with different types of problems [42]–[46]. Among the most recent of these achievements, Ding *et al.* [42] proposed a novel kernel-based FCM clustering algorithm that uses genetic algorithm optimization to improve clustering performance. To enhance the robustness of image segmentation, Gao *et al.* [46] presented a new robust FCM clustering method that combines an elastic FCM with a smoothing method. This elastic FCM provides a sparser description for reliable points and a fuzzier description of the marginal points of clusters. Lei *et al.* [43] designed a more efficient and more robust FCM algorithm for fast and reliable image segmentation. Their variant is based on morphological reconstruction and membership filtering. Subsequently, Lei *et al.* [47] built a fuzzy clustering framework around the above implementation for image segmentation.

In research departing from FCM, Jiao *et al.* [48] developed a fuzzy clustering algorithm that relies on unsupervised fuzzy decision trees to improve model interpretability. To cluster multiple nominal data streams, Sangma *et al.* [49] proposed a fuzzy hierarchical clustering method that involves the clustering-by-variable approach. The method calculates the fuzzy affinity of data streams to different clusters using normalized cosine similarity and handles concept evolution by updating the hierarchical clustering structure.

2) Regression: Fuzzy regression models [50] perform a type of regression analysis that incorporates both possibility theory and fuzzy set theory [51]. They are particularly useful when precise data is lacking or when the relationships between input and output variables are complex and difficult to model using classical regression methods. In addition, fuzzy regression models are good at expressing nonlinear relationships and dealing with noisy data. Fuzzy logic handles the nonlinear relationships between variables. Noisy or incomplete data is handled by allowing for partial memberships and fuzzy sets. Thus, by assigning lower membership values to outliers or inconsistent data points, fuzzy regression models provide a mechanism for mitigating the impact of this type of uncertainty. Another technique for improving the performance of regression models has been to integrate fuzzy techniques with classical machine learning techniques, such as SVM [52]–[54] and neural networks [55]–[58]. Other solutions combine interval regression analysis [59] with machine learning methods [60]–[63].

The latest developments in fuzzy regression analysis include He *et al.* [64], [65], who developed a fuzzy nonlinear regression model using a random weight network that takes triangular fuzzy numbers as its inputs and outputs the same. Baser and Demirhan [66] proposed a new method that combines fuzzy regression models with SVM to estimate the yearly mean and daily values of horizontal global solar radiation. By applying fuzzy regression functions, their method is robust to outlier observations and problems with over-fitting. Chachi [67] designed a robust fuzzy regression modeling technique based on weighted least squares fuzzy regression to handle crisp input-fuzzy output data. Choi *et al.* [68] addressed issues with multicollinearity in fuzzy regression models by incorporating ridge regression. Naderkhani *et al.* [69] proposed an adaptive neuro-fuzzy inference system for analyzing and predicting nonparametric fuzzy regression functions with crisp-valued inputs and symmetric trapezoidal fuzzy outputs. Xia *et al.* [70] developed a novel regression model built around a Takagi-Sugeno fuzzy regression tree to address complex industrial modeling problems, while Zhang *et al.* [71] introduced an interpretable model based on graph community neural networks and time-series fuzzy decision trees for predicting the delays experienced by a high-speed train.

3) Classification: Numerous studies have combined fuzzy techniques with classical machine learning algorithms to address classification problems. The techniques used include fuzzy decision trees [72]–[76], neuro-fuzzy classification [77]–[79], and SVR-based fuzzy classification [80], [81]. Rabcan *et al.* [82], for example, recently introduced a new approach to signal classification that includes a fuzzification procedure in the transformation process and fuzzy decision trees to perform the classifications. Xue *et al.* [83] proposed an adaptive softmin model based on an enhanced TSK-FS to classify high-dimensional datasets. An adaptive softmin function overcomes the drawbacks of "numeric underflows" and "fake minimums" that frequently arise in existing fuzzy systems. However, although the enhanced TSK-FS maintains adequate rules, it does not grow the number of rules exponentially with features. Ma *et al.* [11], [84] put forward a novel framework for addressing multi-class classification problems with imprecise observations that provides a theoretical analysis of the problem based on fuzzy Rademacher complexity. The imprecise observations can be either fuzzy-valued or interval-valued, and the framework, which combines classical machine learning techniques like neural networks and SVM with a fuzzy membership-based defuzzification method, extracts crisp-valued information from these fuzzy- or interval-valued features.

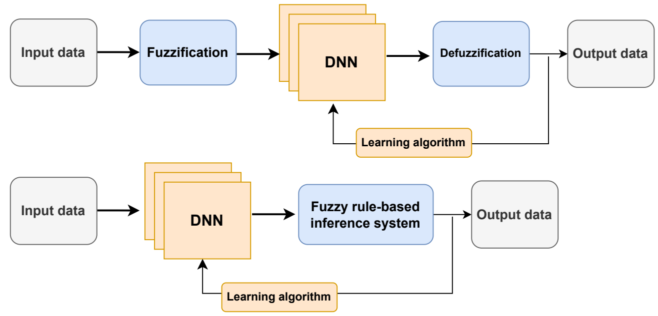


Fig. 2. Two different FNN structures.

B. Deep learning-based method

Fuzzy neural networks (FNNs), also known as neuro-fuzzy networks (NFN), are a type of hybrid neural network that combine fuzzy techniques with neural networks to improve the efficiency and interpretability of machine learning models. A standard FNN structure is illustrated in Fig. 2. The fuzzy logic component of FNNs allow them to handle imprecise or incomplete data and make decisions based on uncertain inputs. Additionally, using an FNN capitalizes on the many significant advances achieved through deep learning in fields such as computer vision, natural language processing, and robotics.

Many researchers have fused deep learning methods with fuzzy techniques to address different types of problems with uncertainty. The most commonly used deep learning models include deep belief networks (DBNs) [85], convolutional neural networks (CNNs) [86], and recurrent neural network (RNNs) [87]. In this section, we summarize the SOTA achievements in FNNs from 2020 to 2023. Earlier research successes can be found in prior surveys like [14], [88], [89].

Chen *et al.* [90] devised a fuzzy deep neural network with a sparse autoencoder as a way to try and predict human intentions. The model is based on human emotions and identification information. Lu *et al.* [91] constructed a novel hashing method that integrates deep neural networks and fuzzy logic to measure the similarity between pairwise images. Zadeh [51] introduced the concept of a type-2 fuzzy set as far back as 1975. These sets, whose membership level themselves are type-1 fuzzy sets, can be used when there is uncertainty about the membership function itself – for example, if one does not

This article has been accepted for publication in IEEE Transactions on Fuzzy Systems. This is the author's version which has not been fully edited and content may change prior to final publication. Citation information: DOI 10.1109/TFUZZ.2024.3387429

TABLE I
SUMMARY OF THE SOTA DEEP LEARNING-BASED FUZZY NEURAL NETWORKS ACHIEVEMENTS.

Year	Fuzzy technique	Deep learning algorithm		No.
		CNN	Others(eg., RNN)	
2020	Fuzzy clustering	[94] [107] [108] [109]	-	4
	Type-1 fuzzy systems	[94] [108] [110] [91] [111]	[101] [112] [113]	8
	Type-2 fuzzy systems	[107] [114] [115]	-	3
	Others	[116]	[116] [117] [118] [119]	4
2021	Type-1 fuzzy systems	[120] [121]	[122] [123] [124] [125]	6
	Type-2 fuzzy systems	-	[126] [127] [128]	3
	Fuzzy entropy	[129]	[129]	1
2022	Fuzzy clustering	-	[130] [131]	2
	Type-1 fuzzy systems	[132] [133] [134] [135] [136] [137]	[132] [130] [138] [139] [140] [141] [142] [143]	13
	Others	[144] [145]	-	2
2023	Fuzzy clustering	[146]	[147]	2
	Type-1 fuzzy systems	[148] [149] [146] [150] [151] [152]	[147] [153] [154] [155] [156] [157]	11

know the shape of the function or some of its parameters. The superior performance of type-2 fuzzy sets has seen them used in a range of machine learning tasks. For instance, to perform complex stock time series tasks, Cao *et al.* [92], [93] designed two multi-objective evolution models. Both combine interval type-2 fuzzy sets with rough FNNs.

Several fuzzy-based ensemble models have also been developed to address problems like load forecasting [94], image classification [95]–[98], and image fusion [99], [100]. For example, Khatter *et al.* [101] combined an RNN with fuzzy techniques and a web blog searching method to enhance classification performance, while Concepcion *et al.* [102] presented a theoretical analysis of why fuzzy-rough cognitive networks delivered better performance than the state-of-the-art classifiers. Long short-term memory models (LSTMs) have also been combined with fuzzy techniques. Some example works include [103]–[106]. Table I provides a summary of the SOTA literature related to deep learning with FNNs.

In summary, combining fuzzy techniques with classical machine learning algorithms is not only useful for solving uncertainty problems, like imprecise or noisy data, it can also improve the interpretability and robustness of the algorithms. Fuzzy sets are good at handling ambiguity and uncertainty and typically provide a more realistic representation of the inherent fuzziness and uncertainty present. Additionally, fuzzy logic tends to improve interpretability. These logics often rely on rule-based systems, where the rules express relationships between the input variables and the output decisions. The rules can either be derived from expert knowledge, or they can be learned from the data.

IV. FUZZY TRANSFER LEARNING

Notably, most current transfer learning [158]methods have limitations when handling real-world situations with uncertainty, such as when only a few labeled instances are available. To overcome these problems, many researchers have turned to fuzzy sets and fuzzy logic.

Existing studies on transfer learning can be divided into categories based on the type of knowledge that is being transferred. These knowledge categories include: instances [159], feature representations [160], model parameters [161], and relational knowledge [162]. Alternatively, in terms of the problem settings tackled, studies can be grouped into four categories: multi-task learning [163], domain adaptation [164], [165], cross-domain adaptation [166], and heterogeneous learning [167]. We have divided our summary of recent works (2015 to 2023) into three areas based on the fuzzy technique used. These are fuzzy sets, fuzzy systems, and fuzzy relations. Table II summarizes recent achievements in the field of fuzzy transfer learning.

A. Transfer learning based on fuzzy sets

Behbood *et al.* [168] proposed an innovative fuzzy-based transfer learning framework to predict long-term bank failures. The framework relies on fuzzy sets, as well as similarity and dissimilarity, to modify the labels of target instances predicted by an FNN classifier. Wu *et al.* [169] developed OwARR, a new algorithm that combines fuzzy sets with domain adaptation. The aim is to reduce the amount of object-specific calibration data so as to solve the important regression problem of estimating online drowsiness in drivers from EEG signals in brain-computer interfaces. Gargees *et al.* [170] proposed a transfer learning method for the possibilistic c-means clustering problem with insufficient data, overcoming a crucial problem for clustering tasks where the source and target domains have a different number of clusters. Based on the idea of fuzzy sets, the proposed algorithm employs historical cluster centers of the data in the source domain as a reference to guide the clustering of data in the target domain.

In terms of applying type-2 fuzzy sets to transfer learning models, Sun *et al.* [171] proposed a new transfer learning model to address the uncertainty caused by conflicting implications in text sequence recognition. The proposed model uses FCM to transform the correspondences among words into information granules. By integrating type-2 fuzzy sets into a hidden Markov model, this granular information can be used for sequence recognition. To reliably estimate GDP from only CO_2 emission data, Shukla *et al.* [172] proposed a new approach to a kernel extreme learning machine (KELM) that combines transfer learning with interval type-2 fuzzy sets. Interval type-2 fuzzy sets are used to improve the efficiency of the knowledge transfer. To consider the uncertainty in input datasets, Kumar *et al.* [173] presented a novel transfer learning approach that incorporates type-1 and interval type-2 fuzzy sets into a KELM framework. The aim is to predict GDP based on uncertain carbon emissions data.

In general, fuzzy sets have been widely applied to address uncertainty in data in transfer learning scenarios, and, experimentally, they have been shown to improve both the efficiency and accuracy of knowledge transfer in comparison to non-fuzzy methods.

B. Transfer learning based on fuzzy systems

Most existing transfer learning methods have a number of drawbacks. For instance, the performance of model-based

This article has been accepted for publication in IEEE Transactions on Fuzzy Systems. This is the author's version which has not been fully edited and content may change prior to final publication. Citation information: DOI 10.1109/TFUZZ.2024.3387429

transfer learning algorithms is heavily dependent on the selected classifier. Additionally, feature-based transfer learning methods can negatively impact the discriminant information and geometric properties of instances from both the source and target domains. Further, the lack of interpretability and an inability to handle uncertainty are two significant flaws. To address these issues, researchers have turned to fuzzy rule-based systems to improve interpretability and handle uncertainty. Notably, TSK-FS [3] has received significant attention in this regard.

Shell *et al.* [174] proposed FuzzyTL, a novel structure that combines transfer learning with a fuzzy rule-based system. This structure is designed to bridge the knowledge gap between contexts that lack prior direct contextual knowledge. Meher *et al.* [175] developed an interpretable domain adaptation method, named the rule-based fuzzy ELM classification model, that uses a fuzzy inference system to design an ELM architecture for remote sensing image classification. The model uses the maximum fuzzy membership grade of features, which is characterized by class-belonging fuzzification, to construct the fuzzy rules and two rule extraction matrices. Moreover, Deng *et al.* [176], [177] proposed two novel transfer learning approaches for regression tasks using the Mamdani-Larsen fuzzy system and TSK-FS coupled with a new fuzzy logic algorithm and its objective functions. However, they noticed that the antecedent parameters of the TSK-FS model constructed in the target domain were directly inherited from the source domain, which meant that they could not leverage enough knowledge from the source domain. To address this problem, Deng *et al.* [178] proposed a new transfer learning method that contains two knowledge-leveraging strategies to better learn the antecedent and consequent parameters in the TSK-FS model. First, they applied an FCM-based clustering transfer technique to the antecedent parameters, which means that the antecedent parameters can be learned from both the source and target domains. Second, they introduced an enhanced knowledge-leverage mechanism to learn the consequent parameters. Another knowledge-leverage term is then introduced to make more effective use of the knowledge in the source domain. Further, they applied and modified these methods so that they could be used for analysis in scenarios with insufficient data, such as recognizing EEG signals [179]–[184] or with situations involving multiple-source domains [183]. The aim of transfer representation learning is to learn a shared space that matches the distributions of instances from both domains. However, transfer representation learning based on kernels suffers from some shortcomings, such as a lack of interpretability and difficulties with selecting a kernel function. To overcome these issues, Xu *et al.* [185] proposed a new transfer representation learning method that uses TSK-FS instead of kernel functions to realize nonlinear transformations. In this approach, instances from both domains are transformed into a fuzzy feature space to minimize the differences between the distributions. Meanwhile, any discriminant information or geometric properties are preserved using latent Dirichlet allocation and principal component analysis.

Notably, Zuo *et al.* [186] devised a new way of constructing a TSK-FS model for regression tasks. This model uses data from the source domain to construct fuzzy rules and then modifies these rules using a nonlinear continuous function based on sigmoid functions to estimate values in the target domain. To address any significant difference in the label distribution between the source and target domains, Zuo *et al.* [187] developed some fuzzy system-based domain adaptation models for classification tasks. In [188], they applied granular computing techniques to transfer learning and proposed a comprehensive domain adaptation framework based on a Takagi-Sugeno fuzzy model to handle three different regression scenarios: one where the source and target domains share different conditions, one where they share different conclusions, and one where both apply. Moreover, they identified two issues in fuzzy transfer learning that had not yet been resolved: how to choose an appropriate source domain and how to efficiently select labeled data for the target domain when the target data structure is unbalanced. The solutions, which involve an innovative method again based on a Takagi-Sugeno fuzzy model [189], combine an infinite Gaussian mixture model with active learning to improve the performance and generalizability of the initial model. Li *et al.* [190] designed a new transfer learning model for multi-source domain adaptation that relies on a fuzzy-rule based deep neural network. To address the more challenging problem in multi-source domain adaptation where no source data is available, Li *et al.* [191] proposed a new model based on a deep neural network with fuzzy rules.

Importantly, all the domain adaptation studies mentioned so far only work when both domains have identical feature spaces and the same number of fuzzy rules, i.e., they are all methods of homogeneous domain adaptation. Zuo *et al.* [192], however, devised a novel approach to heterogeneous scenarios based on a Takagi-Sugeno fuzzy model. In this framework, fuzzy rules are constructed in the source domain and then transferred to the target domain using canonical correlation analysis so as to minimize the discrepancy between the feature spaces of the two domains. This was the first article to solve heterogeneous domain adaptation problems using a fuzzy rule-based system. Subsequently, Lu *et al.* [16] addressed the more challenging scenario of when the only available instances to build the model span multiple source domains. They proposed two novel transfer learning methods for regression tasks based on a Takagi-Sugeno fuzzy model – one for when the feature spaces are homogeneous and one for when the spaces are heterogeneous. In the former, knowledge from multiple source domains is merged in the form of fuzzy rules, while, in the latter, knowledge is merged in the form of both data and fuzzy rules. Che *et al.'s* [193] fuzzy transfer learning method addresses multi-output regression problems in both homogeneous and heterogeneous scenarios. Their approach applies fuzzy rules to accurately capture the commonalities and characteristics of multiple numerical output variables.

In summary, most of the above methods share a common model construction framework: they begin by constructing a fuzzy rule-based model on the source data (e.g., a TSK-FS) and subsequently modify the existing model (fuzzy rules) to establish a new fuzzy model for the target domain. Fuzzy rule-based systems provide a linguistic representation of knowledge, enabling generalization and adaptation, while also mak-

TABLE II
SUMMARY OF THE SOTA PAPERS IN FUZZY TRANSFER LEARNING.

Fuzzy techniques	Type		No.
	Regression	Classification	
Fuzzy sets	[169] [172] [173]	[168] [171] [197]	6
Fuzzy systems	[174] [176] [177] [186] [188] [189] [192] [16] [193]	[175] [178] [179] [180] [181] [182] [183] [184] [185] [187] [190] [191] [198] [199]	23
Fuzzy relations	-	[18] [196] [17] [200]	4

ing the model more robust to domain shift. Their power to transfer relevant knowledge also helps to improve a model's interpretability. All these characteristics make fuzzy rule-based systems well-suited to transfer learning tasks – particularly, the more challenging tasks, such as heterogeneous domain adaptation and source-free domain adaptation.

C. Transfer learning based on fuzzy relations

Most studies mentioned so far focus on supervised or semi-supervised transfer learning in homogeneous scenarios, where both the source and target domains have labeled instances and only their data distributions are different. However, it is not uncommon in the real-world for there to be no available labeled instances in the target domain. Further, the feature spaces of the source and target domains will usually be different. This scenario, which is characterized by a high degree of uncertainty, is commonly referred to as heterogeneous unsupervised domain adaptation (HeUDA). Recently, researchers have developed n-dimensional fuzzy geometry theory [194] and fuzzy equivalence relations [195] to analyze and handle such problems with uncertainty.

Liu *et al.'s* [18] solution to HeUDA problems, called F-HeUDA, is to use fuzzy geometry to measure the similarity of features between the source and target domains. Shared fuzzy equivalence relations are then introduced, which means both domains will share the same number of clustering categories. Hence, knowledge can be transferred from a heterogeneous source domain to a target domain with only unlabeled data. Using these techniques, F-HeUDA outperformed the SOTA models on four real datasets, and performed especially well when the target domain had very few instances. Moreover, Liu *et al.* [17], [196] focused on a more realistic problem called the multi-source HeUDA problem. Solving this problem involves transferring knowledge from several different source domains that have labeled data but heterogeneous dimensions and one target domain with unlabeled data. Their approach, called a shared fuzzy equivalence relations neural network, improves upon previous work in shared fuzzy equivalence relations to extract the shared fuzzy information contained in multiple heterogeneous domains.

In summary, because there is a high degree of uncertainty when transferring knowledge from a heterogeneous source domain to a target domain with only unlabeled data, non-fuzzy models will not usually perform well. Fuzzy relations offer a flexible, interpretable, and adaptable framework for representing and transferring knowledge between such domains. Hence, researchers tend to apply fuzzy relations to improve transfer efficiency in heterogeneous situations.

TABLE III
SUMMARY OF THE SOTA ACHIEVEMENTS IN FUZZY DATA STREAM LEARNING.

Fuzzy techniques	Type				No.
	Detection	Adaptation	Detection and adaptation	Others	
Fuzzy clustering	[201] [202]	[203] [204] [24] [205]	-	-	6
Fuzzy set theory	[206] [207]	-	-	-	2
Fuzzy systems	-	[208] [209] [210] [211]	[212] [213]	[214] [215] [216] [217] [218] [219]	12
Fuzzy time series	-	[220] [221]	-	-	2
Others	[222]	[223] [224]	[225] [226]	[227]	6

V. FUZZY DATA STREAM LEARNING

Learning from data streams [19], [20] involves developing algorithms and techniques to adaptively and incrementally process and learn from continuously arriving data. Unlike traditional machine learning scenarios where a static dataset is available for offline training, data stream learning deals with dynamic, evolving data streams that may not be stored entirely. However, data streams often exhibit concept drift, which refers to changes in the statistical properties of the data. Detecting and adapting to concept drift are two important challenges in data stream learning. One approach is to continuously monitor the data and update models or retrain them periodically to account for changes. Another approach is to use online learning techniques that can adapt to changes in the data stream in real-time. While concept drift often come with some uncertainty problems – for example, making predictions from data streams with mixed drift problems and detecting drift in data streams with missing values – researchers are considering the application of fuzzy techniques to address these challenges.

The aim of concept drift detection is to identify when concept drift has occurred so that appropriate measures can be taken to update or retrain the models in question. Several research teams have turned to FCM-based methods to detect concept drift [201], [202]. These two methods derive fuzzy membership functions from the data stream and use the membership results to mine concept drift patterns. Zhang *et al.* [206] designed a new drift detection model based on fuzzy set theory to address drift problems associated with user interests for recommender systems, while Dong *et al.* [207] developed a data distribution-based drift detection method for business intelligence and data-driven decision support systems that incorporates fuzzy set theory. In both these methods, fuzzy set theory is used to handle the challenging issue of where an item's features and its related information is usually incomplete and imprecise. Along these lines, Liu *et al.* [222] proposed a robust drift detection algorithm that can handle missing values. This algorithm comprises a masked distance learning algorithm to reduce the cumulative errors caused by missing values and a fuzzy-weighted frequency method to identify discrepancies in the data distribution.

Concept drift adaptation refers to the process of updating or modifying a machine learning model in response to concept drift so that it remains accurate and effective over time. Over

This article has been accepted for publication in IEEE Transactions on Fuzzy Systems. This is the author's version which has not been fully edited and content may change prior to final publication. Citation information: DOI 10.1109/TFUZZ.2024.3387429

the past five years, many new adaptation models that fuse fuzzy techniques with machine learning algorithms have been built to deal with the phenomenon of concept drift. The applied fuzzy techniques include fuzzy clustering algorithms [24], [203]–[205], fuzzy rule-based systems [208]–[211], and fuzzy time series [220], [221]. Song *et al.* proposed a series of kernel FCM-based adaptive models to handle data stream regression problems with concept drift [24], [203], [204]. In [203], [204], kernel FCM is used to determine the most relevant learning set, while, in [24], kernel FCM is used to measure the degree to which upcoming examples belong to different patterns. These fuzzy membership values are then embedded in the learning process to handle mixed drift data streams.

In terms of applying fuzzy rule-based systems, Garcia *et al.* [210] developed a modified evolving granular fuzzy-rule-based model that incorporates an incremental learning algorithm to simultaneously impute missing data as well as adapt the model's parameters and structure over time. García-Vico *et al.* [211] proposed an evolutionary fuzzy system to extract knowledge from data streams as a way to adapt to concept drift. Both these methods use type-1 fuzzy systems; however, by contrast, Pratama *et al.* [208] proposed an evolving type-2 recurrent FNN to simultaneously address three challenges: data uncertainty, temporal behavior, and system absence. Fuzzy time series (FTS) [228] is a mathematical framework that combines fuzzy logic and time series analysis to model and forecast uncertain and imprecise data over time. FTS is particularly useful in situations where the data has missing values, outliers, or noise, and where traditional time series models may not perform well. de Lima e Silva *et al.* [220] introduced a non-Stationary FTS, while Severiano *et al.* [221] introduced an evolving forecasting model based on FTS to deal with concept drift. Moreover, Liu *et al.* [223] proposed a new concept drift adaptation method based on a fuzzy windowing approach. Unlike traditional windowing methods, this approach employs sliding windows with an overlapping period to enable precise identification of the data instances that belong to different concepts. Focusing on multiple relevant data stream regression with concept drift, Song *et al.* [224] developed a new adaptation model based on fuzzy drift variance, where the variance is designed to measure the correlated drift patterns among streams.

Additionally, several works simultaneously address concept drift detection and adaptation [212], [213], [225], [226]. For example, Dong *et al.* [225] introduced an adaptive ensemble algorithm based on fuzzy instance weighting to handle data streams involving concept drift. Yu *et al.* [213] presented an evolving fuzzy-neuro system for streaming data regression that employs an online topology learning algorithm to self-organize each layer of the proposed system. To effectively detect drift and adapt the learned model, Zhang *et al.* [226] proposed a novel approach that combines a dynamic intuitionistic fuzzy cognitive map scheme and a concept drift detection algorithm.

More recently, researchers have used fuzzy techniques to address data stream classification and regression problems. These techniques include evolving fuzzy systems [214]–[216], neuro-fuzzy systems [217], granular fuzzy rule-based systems [218], [219], and the fuzzy time-matching method [227]. Not only can these techniques help to improve the performance of streaming data classification and regression in uncertain environments, they can also be applied to handle the phenomenon of concept drift. Table III summarizes these recent achievements in the field of fuzzy data stream learning.

TABLE IV
SUMMARY OF THE SOTA ACHIEVEMENTS IN FUZZY REINFORCEMENT LEARNING.

Fuzzy techniques	Type			No.
	Control systems	Multi-agent RL	Others	
Fuzzy systems	[231] [232] [233]	[234] [235] [236]	[237] [238]	8
Others	[239]	-	[240]	2

In summary, fuzzy techniques are applied in data stream learning, especially to handle concept drift scenarios, owing to their capacity to handle uncertainty, adapt to changing patterns, and provide interpretable models. These features make fuzzy techniques valuable for detecting, understanding, and adapting to concept drift, leading to better performance than non-fuzzy methods.

VI. Fuzzy Reinforcement Learning

Reinforcement learning (RL) [27] represents a powerful paradigm in machine learning, where agents learn to make decisions through interaction with an environment, guided by a system of rewards or penalties. However, the traditional RL framework is not without its challenges, especially in scenarios where the training process is inherently slow due to complex and uncertain environments [229] or sparse reward signals [230]. Fuzzy RL emerges as a promising approach to address these limitations, leveraging fuzzy logic to enhance training efficiency and overcome the hurdles associated with slow reinforcement processes.

One of the primary advantages of fuzzy RL lies in its adaptability to dynamic (uncertain) environments. In traditional RL, slow training processes can be exacerbated by the challenges posed by dynamic scenarios where the optimal strategy may change rapidly. Fuzzy logic allows the system to gracefully adapt to these changes, incorporating fuzzy rules that capture the gradual transitions and uncertainties in the environment. Additionally, in many RL applications, the scarcity of meaningful rewards can impede the learning process, leading to slow convergence or even stagnation. Fuzzy RL introduces the concept of fuzzy rewards [31], enabling the system to consider partial or intermediate successes that may not be fully captured by binary reward signals. This approach helps mitigate the challenge of sparse rewards by providing a more nuanced and continuous feedback mechanism, allowing the agent to learn from a broader spectrum of experiences.

Fuzzy Sarsa Learning (FSL) [31] is a critic-only fuzzy RL algorithm that combines the Sarsa algorithm with fuzzy logic. In traditional Sarsa learning, an agent learns to take actions through trial and error that maximize a reward signal in a given environment. In FSL, the state and action spaces are represented as fuzzy sets, which allows for a more gradual transition between states and actions, rather than strict boundaries. The algorithm updates the Q-value function based on the fuzzy membership functions of the current state and action, as well as the fuzzy reward function. The use of fuzzy

This article has been accepted for publication in IEEE Transactions on Fuzzy Systems. This is the author's version which has not been fully edited and content may change prior to final publication. Citation information: DOI 10.1109/TFUZZ.2024.3387429

logic allows FSL to handle more complex environments with uncertain or imprecise information, while still maintaining a high level of performance. As an example, Fathinezhad *et al.* [237] proposed a novel method of robot navigation that combines supervised learning and FSL. Their method applies a zero order Takagi–Sugeno fuzzy controller with some candidate actions for each rule as the main module. Also, Hein *et al.* [238] developed a new particle swarm approach to RL based on fuzzy controllers. This approach builds fuzzy RL policies by training parameters on world models that simulate real system dynamics. Furthermore, Shi *et al.* [240] developed an adaptive fuzzy comprehensive evaluation method that integrates a fuzzy analytical hierarchy process, a Bayesian network, and RL. The authors successfully applied this method to a robot soccer system, which is a typical complex time-sequence decision-making system.

In the field of control systems, Zhang *et al.* [231] designed a new fault-tolerant control algorithm by combining RL with a fuzzy augmented model for partially unknown systems with actuator faults. The fuzzy augmented model was inspired by the well-known Takagi-Sugeno fuzzy model. With this algorithm, less information needs to be transmitted, which reduces computational loads during the learning process, even when dynamic matrices are partially unknown. Similarly, Zhang *et al.* [232] proposed a novel parallel tracking control optimization algorithm using fuzzy RL techniques for partially unknown fuzzy interconnected systems. This algorithm uses the pre-compensation technique to treat working feedback controls as reconstructed dynamics with virtual controls. This approach to building the model results in a new augmented and interconnected fuzzy tracking system where a valid performance index is guaranteed for optimal control. In the realm of traffic light control systems, Kumar *et al.* [233] proposed a novel system that is both dynamic and intelligent to overcome issues with long waiting times, fuel waste, and rising carbon emissions. Traditional traffic light systems operate on a fixed duration mode, whereas Kumar's proposed system uses a deep RL model to switch the lights and a fuzzy inference system to select one among three modes based on current traffic information. To mitigate frequency deviations caused by power fluctuations, Yin *et al.* [239] developed a fuzzy vector RL approach to control how much power a power system generates. The framework also considers flywheel energy storage systems.

Turning to large-scale multi-agent RL, Li *et al.* [234] introduced the concept of fuzzy agents to be used for training homogeneous agents. They also proposed a new RL method that uses fuzzy logic to learn abstract policies. In comparison to other simplification methods, their fuzzy agents both reduce the computing resources required to train a model and ensure that an effective policy is learned. Zhu *et al.* [235] devised a new control strategy based on RL and a fuzzy wavelet network. The aim here is to improve the stability of the hybrid system's buffer compliance control. To reduce the negative effects of noisy information in communication channels on multi-agent RL, Fang *et al.* [236] developed a two-stream fused fuzzy deep neural network by applying fuzzy a inference module and a DNN module. Experiments with two large-scale traffic signal control environments demonstrate the proposed method's superior performance. Table IV summarizes these recent achievements in the field of fuzzy RL.

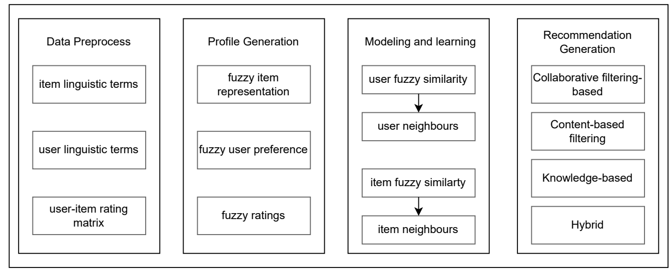


Fig. 3. A fuzzy recommender system framework.

In general, by combining RL and fuzzy logic, fuzzy reinforcement learning can handle the complexity of real-world environments that involve uncertain information and imprecise data, making it a promising technique for solving problems in fields such as robotics, control systems, and game theory.

VII. Fuzzy Recommender Systems

In real-world recommender systems, descriptions of user preferences and item features, item values, and business knowledge are often vague, imprecise, and plagued with uncertainty. And, further, these issues can occur across the entire recommendation process from collecting the data to generating the recommendations. Other key problems that can occur with recommender systems include sparsely populated user-item matrices and problems with measuring the similarity of items and users (see Fig. 3). Commonly used fuzzy techniques to deal with these issues include intuitionistic fuzzy sets [241], fuzzy user profiles [242], fuzzy rule-based systems [243], and fuzzy similarity [244]. This section provides a summary of recent articles focused on these techniques.

Collaborative filtering is a key approach to recommender systems. However, traditional collaborative filtering methods, such as unsupervised clustering, are quite sensitive to uncertainty and therefore often experience high error rates. Once again, FCM or modified FCM algorithms have been implemented to eliminate these issues [245]–[247]. For example, FCM has been used to classify the users in a dataset according to the similarity of their item ratings. To improve the quality of recommender systems with sparse datasets, Nilashi *et al.* [248] designed a hybrid item similarity model that combines an adjusted Google similarity with an intuitionistic Kullback–Leibler similarity based on fuzzy sets. This approach essentially makes a trade-off between prediction accuracy and efficiency.

In terms of content-based filtering recommender systems, Yera *et al.'s* [249] solution uses a fuzzy decision tree to match the most appropriate function in the individual recommendation aggregation step. Other researchers have also relied on fuzzy rule-based systems to extract relevant knowledge from uncertain data to improve the performance of knowledge-based recommender systems [250]–[252].

Hybrid recommender systems are another area of research progress. Here, Walek *et al.* [253] designed a new hybrid recommender system that combines a collaborative filtering

TABLE V
SUMMARY OF THE SOTA FUZZY TECHNIQUES-BASED RECOMMENDER SYSTEMS ACIEVEMENTS.

Fuzzy techniques	Type				No.
	Collaborative	Content-Based	Knowledge-Based	Hybrid	
Fuzzy systems	[248] [254] [255]	[256]	[257] [250] [251] [258] [259] [260]	[261] [262]	12
Intuitionistic fuzzy set	[263] [264]	-	-	-	2
Fuzzy clustering	[245] [246] [247]	-	-	-	3
Fuzzy profile	[265] [266]	-	-	-	2
Others	[267] [265]	[268] [249]	[269] [270] [271] [271] [272]	[273] [274] [275] [253]	13

system, a content-based filtering system, and a fuzzy expert system to enhance recommendation performance. The fuzzy expert system is used to evaluate the importance of the recommended products with vague information and rank them appropriately for users. Table V summarizes these recent studies in the field of fuzzy-based recommender systems.

In summary, fuzzy techniques provide a rich spectrum of methods for managing uncertainty, vagueness, and imprecision in data both during the learning process and when making recommendations. Particularly, fuzzy techniques are well suited to handling imprecise user preference descriptions (e.g. in linguistic terms), knowledge description, and the gradual accumulation of user preference profiles. Therefore, applying fuzzy techniques in recommender system can bring more efficient and accurate performance than non-fuzzy models.

VIII. FUTURE RESEARCH DIRECTIONS

So far we have summarized recent achievements of FML. In this section, we aim to give further discussion of FML's current research trends and share some insights on future research directions.

A. Fuzzy classic machine learning

Most current research in fuzzy classic machine learning is mainly focusing on the following aspects: 1) handling noisy or incomplete data; 2) addressing imbalanced datasets; and 3) enhancing algorithms' interpretability and robustness. Analyzing imprecise data (fuzzy-valued or interval-valued) [84] has not received widespread attention. However, in many real-world scenarios, we will inevitably encounter this kind of data. Therefore, it would be a promising direction to investigate how to apply fuzzy logic to analyze imprecise data.

Moreover, deep learning [86] has made significant strides, but it still faces several challenges. Deep neural networks, particularly complex architectures like deep convolutional or recurrent networks, are often viewed as black boxes. Understanding how these models arrive at specific decisions is crucial, especially in applications where interpretability is essential, such as healthcare and finance. In addition, deep learning models are vulnerable to adversarial attacks [276], where small, carefully crafted perturbations to input data can lead to misclassification. Fuzzy techniques are potential tools to overcome these challenges. We suggest future work that uses fuzzy techniques to overcome these challenges.

B. Fuzzy transfer learning

Recent fuzzy transfer learning works [191], [277] mainly focus on applying fuzzy rule-based systems to model the uncertainty and variability between different source and target domains, enabling more effective adaptation of knowledge from the source to the target domain. However, open-set problems [165], [278] has gain more and more attention in transfer learning, where target domain contain private categories. Detecting unknown classes is an challenge problem that contain a large degree of uncertainty. We believe applying fuzzy techniques to address this challenge problem is worth investigating for future work.

C. Fuzzy data stream learning

A couple of recent works [222], [224] in fuzzy data stream learning are focused on developing adaptive fuzzy models that can effectively handle concept drift in data streams. Learning from multiple stream [20] is a crucial and challenge problem in data stream learning, especially when streams have different rates, arrive asynchronously, or experience delays. Streams may vary in terms of data types, formats, and modalities. Additionally, there is an uncertain relationship between each pair of streams. Traditional machine learning algorithms face difficulty in addressing these challenges. Therefore, we recommend that researchers use fuzzy techniques in future studies to tackle these issues.

D. Fuzzy reinforcement learning

Recent research [233], [237] is mainly focus on integrating fuzzy systems with RL for improved performance in complex and dynamic environments. Research is exploring the integration of fuzzy logic into Q-learning algorithms to handle uncertainties in estimating state-action values. Further, fuzzy logic is applied to model and handle uncertain or imprecise reward signals in RL. However, in multi-agent RL [234], capturing complex relationships and dependencies between agents while maintaining a scalable and efficient learning process is a key challenge. Additionally, agents in a multi-agent system may have diverse capabilities, objectives, or learning speeds. Coordinating heterogeneous agents and ensuring fair and effective collaboration is another challenging problem. We believe it would be a promising direction to investigate how to apply fuzzy techniques to address these challenges.

E. Fuzzy recommender systems

Fuzzy techniques are mainly used to handling imprecise user preference descriptions (e.g. in linguistic terms), knowledge description, and the gradual accumulation of user preference profiles in fuzzy recommender systems [260], [264]. Cross-domain recommendations [279], where recommendations are made across different domains or platforms, present several challenges due to the diversity and heterogeneity between domains. Different domains may have distinct characteristics, user behaviors, and item features. Moreover, there will be uncertain relationships between different domains. Applying fuzzy techniques to address these challenges in cross-domain recommendations is promising in future work.

IX. Summary

In this paper, we reviewed recent developments across the five main research streams of FML. Our review shows that fuzzy techniques can significantly improve machine learning algorithms by providing a way to handle different uncertainty situations. The main improvements are reflected in the following five aspects: 1) enhancing the representation of the inputs; 2) improving the learning process of different machine learning algorithms; 3) enhancing measurement accuracy and reliability; 4) improving the accuracy of the matching function; 5) enhancing the performance (e.g., accuracy, robustness and interpretability) of the output results.

In future research, several new directions in the field of FML warrant thorough consideration. For instance, applying fuzzy techniques to address open-set transfer learning problems, where the target domain encompasses classes that are unknown in the source domain. In addition, multi-stream learning, multi-agent RL, and cross-domain recommendations are three challenge problems that are far from being solved. They all involve intricate relationships and heterogeneous information, posing difficulties for traditional machine learning algorithms. Fuzzy techniques emerge as promising tools for investigating and addressing these complex problems.

We believe that this survey can provide researchers with SOTA knowledge on machine learning based on fuzzy techniques and give a guide on future research directions in the field of FML.

References

1 Dubois, D. J., *Fuzzy sets and systems: theory and applications*. Academic press, 1980, vol. 144.

2 Wu, H. C., "Probability density functions of fuzzy random variables," *Fuzzy Sets and Systems*, vol. 105, no. 1, pp. 139–158, 1999.

3 Takagi, T. and Sugeno, M., "Fuzzy identification of systems and its applications to modeling and control," *IEEE Transactions on Systems, Man, and Cybernetics*, vol. 15, no. 1, pp. 116–132, 1985.

4 Lin, Zhu, Fulai, Chung, Shitong, and Wang, "Generalized fuzzy c-means clustering algorithm with improved fuzzy partitions." *IEEE Transactions on Systems, Man Cybernetics: Part B*, vol. 39, no. 3, pp. 578–591, 2009.

5 Czogala, E., Drewniak, J., and Pedrycz, W., "Fuzzy relation equations on a finite set," *Fuzzy Sets and systems*, vol. 7, no. 1, pp. 89–101, 1982.

6 Sugeno, M., "Fuzzy measure and fuzzy integral," *Transactions of the Society of Instrument and Control Engineers*, vol. 8, no. 2, pp. 218–226, 1972.

7 Wen, W., Li, J., Lin, S., Chen, J., and Chang, S., "A fuzzy-matching model with grid reduction for lithography hotspot detection," *IEEE Transactions on Computer-Aided Design of Integrated Circuits and Systems*, vol. 33, no. 11, pp. 1671–1680, 2014.

8 Luhandjula, M., "Fuzzy optimization: an appraisal," *Fuzzy sets and systems*, vol. 30, no. 3, pp. 257–282, 1989.

9 Zadeh, L. A., "Fuzzy sets," *Information and Control*, vol. 8, no. 3, pp. 338–353, 1965.

10 Zimmermann, H., *Fuzzy set theory—and its applications*. Springer Science & Business Media, 2011.

11 Ma, G., Lu, J., Liu, F., Fang, Z., and Zhang, G., "Multiclass classification with fuzzy-feature observations: Theory and algorithms," *IEEE Transactions on Cybernetics*, vol. 54, no. 2, pp. 1048–1061, 2022.

12 Baraldi, A. and Blonda, P., "A survey of fuzzy clustering algorithms for pattern recognition. i," *IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics)*, vol. 29, no. 6, pp. 778–785, 1999.

13 Škrjanc, I., Iglesias, J. A., Sanchis, A., Leite, D., Lughofer, E., and Gomide, F., "Evolving fuzzy and neuro-fuzzy approaches in clustering, regression, identification, and classification: A survey," *Information Sciences*, vol. 490, pp. 344–368, 2019.

14 Zheng, Y., Xu, Z., and Wang, X., "The fusion of deep learning and fuzzy systems: A state-of-the-art survey," *IEEE Transactions on Fuzzy Systems*, vol. 30, no. 8, pp. 2783–2799, 2021.

15 Pan, S. J. and Yang, Q., "A survey on transfer learning," *IEEE Transactions on Knowledge and Data Engineering*, vol. 22, no. 10, pp. 1345–1359, 2010.

16 Lu, J., Zuo, H., and Zhang, G., "Fuzzy multiple-source transfer learning," *IEEE Transactions on Fuzzy Systems*, vol. 28, no. 12, pp. 3418–3431, 2020.

17 Liu, Feng and Zhang, Guangquan and Lu, Jie, "Multi-source heterogeneous unsupervised domain adaptation via fuzzy-relation neural networks," *IEEE Transactions on Fuzzy Systems*, 2020.

18 Liu, F., Lu, J., and Zhang, G., "Unsupervised heterogeneous domain adaptation via shared fuzzy equivalence relations," *IEEE Transactions on Fuzzy Systems*, vol. 26, no. 6, pp. 3555–3568, 2018.

19 Sun, Y., Tang, K., Minku, L. L., Wang, S., and Yao, X., "Online ensemble learning of data streams with gradually evolved classes," *IEEE Transactions on Knowledge and Data Engineering*, vol. 28, no. 6, pp. 1532–1545, 2016.

20 Zhou, M., Lu, J., Song, Y., and Zhang, G., "Multi-stream concept drift self-adaptation using graph neural network," *IEEE Transactions on Knowledge and Data Engineering*, 2023.

21 Schlimmer, J. C. and Granger, R. H., "Incremental learning from noisy data," *Machine learning*, vol. 1, pp. 317–354, 1986.

22 Gama, J., Žliobaitė, I., Bifet, A., Pechenizkiy, M., and Bouchachia, A., "A survey on concept drift adaptation," *ACM computing surveys*, vol. 46, no. 4, pp. 1–37, 2014.

23 Lu, J., Liu, A., Dong, F., Gu, F., Gama, J., and Zhang, G., "Learning under concept drift: A review," *IEEE transactions on knowledge and data engineering*, vol. 31, no. 12, pp. 2346–2363, 2018.

24 Song, Y., Lu, J., Lu, H., and Zhang, G., "Fuzzy clustering-based adaptive regression for drifting data streams," *IEEE Transactions on Fuzzy Systems*, vol. 28, no. 3, pp. 544–557, 2020.

25 Dunn, J. C., "A fuzzy relative of the isodata process and its use in detecting compact well-separated clusters," *Journal of Cybernetics*, vol. 3, no. 3, pp. 32–57, 1973.

26 Yang, X., Zhang, G., Lu, J., and Ma, J., "A kernel fuzzy c-means clustering-based fuzzy support vector machine algorithm for classification problems with outliers or noises," *IEEE Transactions on Fuzzy Systems*, vol. 19, no. 1, pp. 105–115, 2010.

27 Sutton, R. S. and Barto, A. G., *Reinforcement learning: An introduction*. MIT press, 2018.

28 Kormushev, P., Calinon, S., and Caldwell, D. G., "Reinforcement learning in robotics: Applications and real-world challenges," *Robotics*, vol. 2, no. 3, pp. 122–148, 2013.

29 Goldwaser, A. and Thielscher, M., "Deep reinforcement learning for general game playing," in *Proceedings of the AAAI Conference on Artificial Intelligence*, vol. 34, no. 02, 2020, pp. 1701–1708.

30 Kiran, B. R., Sobh, I., Talpaert, V., Mannion, P., Al Sallab, A. A., Yogamani, S., and Pérez, P., "Deep reinforcement learning for autonomous driving: A survey," *IEEE Transactions on Intelligent Transportation Systems*, vol. 23, no. 6, pp. 4909–4926, 2021.

31 Er, M. J. and Deng, C., "Online tuning of fuzzy inference systems using dynamic fuzzy q-learning," *IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics)*, vol. 34, no. 3, pp. 1478–1489, 2004.

32 Ma, X., Sun, Z., and He, Y., "Analysis and design of fuzzy controller and fuzzy observer," *IEEE Transactions on fuzzy systems*, vol. 6, no. 1, pp. 41–51, 1998.

33 Jouffe, L., "Fuzzy inference system learning by reinforcement methods," *IEEE Transactions on Systems, Man, and Cybernetics, Part C (Applications and Reviews)*, vol. 28, no. 3, pp. 338–355, 1998.

34 Glorennec, P. Y. and Jouffe, L., "Fuzzy q-learning," in *Proceedings of 6th international fuzzy systems conference*, vol. 2, 1997, pp. 659–662.

35 Bobadilla, J., Ortega, F., Hernando, A., and Gutiérrez, A., "Recommender systems survey," *Knowledge-based systems*, vol. 46, pp. 109–132, 2013.

36 Lu, J., Wu, D., Mao, M., Wang, W., and Zhang, G., "Recommender system application developments: a survey," *Decision Support Systems*, vol. 74, pp. 12–32, 2015.

37 Yager, R. R., "Fuzzy logic methods in recommender systems," *Fuzzy Sets and Systems*, vol. 136, no. 2, pp. 133–149, 2003.

38 Yera, R. and Martinez, L., "Fuzzy tools in recommender systems: A survey," *International Journal of Computational Intelligence Systems*, vol. 10, no. 1, p. 776, 2017.

39 Yang, M., "A survey of fuzzy clustering," *Mathematical and Computer Modelling*, vol. 18, no. 11, pp. 1–16, 1993.

40 Ruspini, E. H., Bezdek, J. C., and Keller, J. M., "Fuzzy clustering: A historical perspective," *IEEE Computational Intelligence Magazine*, vol. 14, no. 1, pp. 45–55, 2019.

This article has been accepted for publication in IEEE Transactions on Fuzzy Systems. This is the author's version which has not been fully edited and content may change prior to final publication. Citation information: DOI 10.1109/TFUZZ.2024.3387429

41 Bezdek, J. C., Ehrlich, R., and Full, W., "Fcm: The fuzzy c-means clustering algorithm," *Computers & Geosciences*, vol. 10, no. 2-3, pp. 191–203, 1984.

42 Ding, Y. and Fu, X., "Kernel-based fuzzy c-means clustering algorithm based on genetic algorithm," *Neurocomputing*, vol. 188, pp. 233–238, 2016.

43 Lei, T., Jia, X., Zhang, Y., He, L., Meng, H., and Nandi, A. K., "Significantly fast and robust fuzzy c-means clustering algorithm based on morphological reconstruction and membership filtering," *IEEE Transactions on Fuzzy Systems*, vol. 26, no. 5, pp. 3027–3041, 2018.

44 Mishro, P. K., Agrawal, S., Panda, R., and Abraham, A., "A novel type-2 fuzzy c-means clustering for brain mr image segmentation," *IEEE Transactions on Cybernetics*, vol. 51, no. 8, pp. 3901–3912, 2020.

45 Zhou, J., Pedrycz, W., Yue, X., Gao, C., Lai, Z., and Wan, J., "Projected fuzzy c-means clustering with locality preservation," *Pattern Recognition*, vol. 113, p. 107748, 2021.

46 Gao, Y., Wang, Z., Xie, J., and Pan, J., "A new robust fuzzy c-means clustering method based on adaptive elastic distance," *Knowledge-Based Systems*, vol. 237, p. 107769, 2022.

47 Lei, T., Liu, P., Jia, X., Zhang, X., Meng, H., and Nandi, A. K., "Automatic fuzzy clustering framework for image segmentation," *IEEE Transactions on Fuzzy Systems*, vol. 28, no. 9, pp. 2078–2092, 2019.

48 Jiao, L., Yang, H., Liu, Z., and Pan, Q., "Interpretable fuzzy clustering using unsupervised fuzzy decision trees," *Information Sciences*, vol. 611, pp. 540–563, 2022.

49 Sangma, J. W., Pal, V., Kumar, N., Kushwaha, R. *et al.*, "Fhc-nds: Fuzzy hierarchical clustering of multiple nominal data streams," *IEEE Transactions on Fuzzy Systems*, vol. 31, no. 3, pp. 786–798, 2022.

50 Chukhrova, N. and Johannssen, A., "Fuzzy regression analysis: systematic review and bibliography," *Applied Soft Computing*, vol. 84, p. 105708, 2019.

51 Zadeh, L. A., "The concept of a linguistic variable and its application to approximate reasoning—i," *Information Sciences*, vol. 8, no. 3, pp. 199–249, 1975.

52 Hong, D. H. and Hwang, C., "Support vector fuzzy regression machines," *Fuzzy Sets and Systems*, vol. 138, no. 2, pp. 271–281, 2003.

53 Lu, J., Yang, X., and Zhang, G., "Support vector machine based approach in situation assessment," *Dynamics of Continuous Discrete and Impulsive Systems-series B-applications & Algorithms*, 2006.

54 Lu, J. and Yang, X. and Zhang, G., "Support vector machine-based multi-source multi-attribute information integration for situation assessment," *Expert Systems with Applications*, vol. 34, no. 2, pp. 1333–1340, 2008.

55 Dunyak, J. P. and Wunsch, D., "Fuzzy regression by fuzzy number neural networks," *Fuzzy Sets and Systems*, vol. 112, no. 3, pp. 371–380, 2000.

56 Zhang, D., Deng, L., Cai, K., and So, A., "Fuzzy nonlinear regression with fuzzified radial basis function network," *IEEE Transactions on Fuzzy Systems*, vol. 13, no. 6, pp. 742–760, 2005.

57 Mosleh, M., Otadi, M., and Abbasbandy, S., "Fuzzy polynomial regression with fuzzy neural networks," *Applied Mathematical Modelling*, vol. 35, no. 11, pp. 5400–5412, 2011.

58 Roh, S., Ahn, T., and Pedrycz, W., "Fuzzy linear regression based on polynomial neural networks," *Expert Systems with Applications*, vol. 39, no. 10, pp. 8909–8928, 2012.

59 Tanaka, H. and Lee, H., "Interval regression analysis by quadratic programming approach," *IEEE Transactions on Fuzzy Systems*, vol. 6, no. 4, pp. 473–481, 1998.

60 Hong, D. and Hwang, C., "Interval regression analysis using quadratic loss support vector machine," *IEEE Transactions on Fuzzy Systems*, vol. 13, no. 2, pp. 229–237, 2005.

61 Hwang, C., Hong, D., and Seok, K., "Support vector interval regression machine for crisp input and output data," *Fuzzy Sets and Systems*, vol. 157, no. 8, pp. 1114–1125, 2006.

62 Hao, P., "Interval regression analysis using support vector networks," *Fuzzy Sets and Systems*, vol. 160, no. 17, pp. 2466–2485, 2009.

63 Huang, C., "A reduced support vector machine approach for interval regression analysis," *Information Sciences*, vol. 217, pp. 56–64, 2012.

64 He, Y., Wang, X., and Huang, J. Z., "Fuzzy nonlinear regression analysis using a random weight network," *Information Sciences*, vol. 364, pp. 222–240, 2016.

65 He, Y., Wei, C., Long, H., Ashfaq, R. A. R., and Huang, J. Z., "Random weight network-based fuzzy nonlinear regression for trapezoidal fuzzy number data," *Applied Soft Computing*, vol. 70, pp. 959–979, 2018.

66 Baser, F. and Demirhan, H., "A fuzzy regression with support vector machine approach to the estimation of horizontal global solar radiation," *Energy*, vol. 123, pp. 229–240, 2017.

67 Chachi, J., "A weighted least squares fuzzy regression for crisp input-fuzzy output data," *IEEE Transactions on Fuzzy Systems*, vol. 27, no. 4, pp. 739–748, 2018.

68 Choi, S. H., Jung, H., and Kim, H., "Ridge fuzzy regression model," *International Journal of Fuzzy Systems*, vol. 21, no. 7, pp. 2077–2090, 2019.

69 Naderkhani, R., Behzad, M., Razzaghnia, T., and Farnoosh, R., "Fuzzy regression analysis based on fuzzy neural networks using trapezoidal data," *International Journal of Fuzzy Systems*, vol. 23, pp. 1267–1280, 2021.

70 Xia, H., Tang, J., Yu, W., Cui, C., and Qiao, J., "Takagi–sugeno fuzzy regression trees with application to complex industrial modeling," *IEEE Transactions on Fuzzy Systems*, 2022.

71 Zhang, D., Xu, Y., Peng, Y., Du, C., Wang, N., Tang, M., Lu, L., and Liu, J., "An interpretable station delay prediction model based on graph community neural network and time-series fuzzy decision tree," *IEEE Transactions on Fuzzy Systems*, 2022.

72 Yuan, Y. and Shaw, M. J., "Induction of fuzzy decision trees," *Fuzzy Sets and Systems*, vol. 69, no. 2, pp. 125–139, 1995.

73 Suárez, A. and Lutsko, J. F., "Globally optimal fuzzy decision trees for classification and regression," *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol. 21, no. 12, pp. 1297–1311, 1999.

74 Chiang, I. and Hsu, J. Y., "Fuzzy classification trees for data analysis," *Fuzzy Sets and Systems*, vol. 130, no. 1, pp. 87–99, 2002.

75 Wang, X., Liu, X., Pedrycz, W., and Zhang, L., "Fuzzy rule based decision trees," *Pattern Recognition*, vol. 48, no. 1, pp. 50–59, 2015.

76 Segatori, A., Marcelloni, F., and Pedrycz, W., "On distributed fuzzy decision trees for big data," *IEEE Transactions on Fuzzy Systems*, vol. 26, no. 1, pp. 174–192, 2017.

77 Nauck, D. and Kruse, R., "A neuro-fuzzy method to learn fuzzy classification rules from data," *Fuzzy sets and Systems*, vol. 89, no. 3, pp. 277–288, 1997.

78 Cpalka, K., "A new method for design and reduction of neuro-fuzzy classification systems," *IEEE Transactions on Neural Networks*, vol. 20, no. 4, pp. 701–714, 2009.

79 Ghosh, A., Shankar, B. U., and Meher, S. K., "A novel approach to neuro-fuzzy classification," *Neural Networks*, vol. 22, no. 1, pp. 100–109, 2009.

80 Juang, C. and Hsieh, C.-D., "Ts-fuzzy system-based support vector regression," *Fuzzy Sets and Systems*, vol. 160, no. 17, pp. 2486–2504, 2009.

81 Chen, L., Wu, M., Zhou, M., Liu, Z., She, J., and Hirota, K., "Dynamic emotion understanding in human–robot interaction based on two-layer fuzzy svr-ts model," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 50, no. 2, pp. 490–501, 2017.

82 Rabcan, J., Levashenko, V., Zaitseva, E., Kvassay, M., and Subbotin, S., "Application of fuzzy decision tree for signal classification," *IEEE Transactions on Industrial Informatics*, vol. 15, no. 10, pp. 5425–5434, 2019.

83 Xue, G., Chang, Q., Wang, J., Zhang, K., and Pal, N. R., "An adaptive neuro-fuzzy system with integrated feature selection and rule extraction for high-dimensional classification problems," *IEEE Transactions on Fuzzy Systems*, 2022.

84 Ma, G., Liu, F., Zhang, G., and Lu, J., "Learning from imprecise observations: An estimation error bound based on fuzzy random variables," in *2021 IEEE International Conference on Fuzzy Systems*, 2021, pp. 1–8.

85 Liu, P., Han, S., Meng, Z., and Tong, Y., "Facial expression recognition via a boosted deep belief network," in *Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition*, 2014, pp. 1805–1812.

86 Gu, J., Wang, Z., Kuen, J., Ma, L., Shahroudy, A., Shuai, B., Liu, T., Wang, X., Wang, G., Cai, J. *et al.*, "Recent advances in convolutional neural networks," *Pattern Recognition*, vol. 77, pp. 354–377, 2018.

87 Schuster, M. and Paliwal, K. K., "Bidirectional recurrent neural networks," *IEEE transactions on Signal Processing*, vol. 45, no. 11, pp. 2673–2681, 1997.

88 Shihabudheen, K. and Pillai, G. N., "Recent advances in neuro-fuzzy system: A survey," *Knowledge-Based Systems*, vol. 152, pp. 136–162, 2018.

89 De Campos Souza, P. V., "Fuzzy neural networks and neuro-fuzzy networks: A review the main techniques and applications used in the literature," *Applied Soft Computing*, vol. 92, p. 106275, 2020.

90 Chen, L., Su, W., Wu, M., Pedrycz, W., and Hirota, K., "A fuzzy deep neural network with sparse autoencoder for emotional intention understanding in human–robot interaction," *IEEE Transactions on Fuzzy Systems*, vol. 28, no. 7, pp. 1252–1264, 2020.

This article has been accepted for publication in IEEE Transactions on Fuzzy Systems. This is the author's version which has not been fully edited and content may change prior to final publication. Citation information: DOI 10.1109/TFUZZ.2024.3387429

91 Lu, H., Zhang, M., Xu, X., Li, Y., and Shen, H. T., "Deep fuzzy hashing network for efficient image retrieval," *IEEE Transactions on Fuzzy Systems*, vol. 29, no. 1, pp. 166–176, 2020.

92 Cao, B., Zhao, J., Lv, Z., Gu, Y., Yang, P., and Halgamuge, S. K., "Multiobjective evolution of fuzzy rough neural network via distributed parallelism for stock prediction," *IEEE Transactions on Fuzzy Systems*, vol. 28, no. 5, pp. 939–952, 2020.

93 Cao, B., Zhao, J., Liu, X., Arabas, J., Tanveer, M., Singh, A. K., and Lv, Z., "Multiobjective evolution of the explainable fuzzy rough neural network with gene expression programming," *IEEE Transactions on Fuzzy Systems*, vol. 30, no. 10, pp. 4190–4200, 2022.

94 Sideratos, G., Ikonomopoulos, A., and Hatziargyriou, N. D., "A novel fuzzy-based ensemble model for load forecasting using hybrid deep neural networks," *Electric Power Systems Research*, vol. 178, p. 106025, 2020.

95 Banerjee, A., Singh, P. K., and Sarkar, R., "Fuzzy integral-based cnn classifier fusion for 3d skeleton action recognition," *IEEE Transactions on Circuits and Systems for Video Technology*, vol. 31, no. 6, pp. 2206–2216, 2020.

96 Hsu, M., Chien, Y., Wang, W., and Hsu, C., "A convolutional fuzzy neural network architecture for object classification with small training database," *International Journal of Fuzzy Systems*, vol. 22, pp. 1–10, 2020.

97 Ge, C., Liu, Z., Fang, L., Ling, H., Zhang, A., and Yin, C., "A hybrid fuzzy convolutional neural network based mechanism for photovoltaic cell defect detection with electroluminescence images," *IEEE Transactions on Parallel and Distributed Systems*, vol. 32, no. 7, pp. 1653–1664, 2020.

98 Sharma, T., Verma, N. K., and Masood, S., "Mixed fuzzy pooling in convolutional neural networks for image classification," *Multimedia Tools and Applications*, vol. 82, no. 6, pp. 8405–8421, 2023.

99 Bhalla, K., Koundal, D., Sharma, B., Hu, Y., and Zaguia, A., "A fuzzy convolutional neural network for enhancing multi-focus image fusion," *Journal of Visual Communication and Image Representation*, vol. 84, p. 103485, 2022.

100 Bhalla, K., Koundal, D., Bhatia, S., Rahmani, M. K. I., and Tahir, M., "Fusion of infrared and visible images using fuzzy based siamese convolutional network," *Computers, Materials & Continua*, vol. 70, no. 3, pp. 5503–5518, 2022.

101 Khatter, H. and Ahlawat, A. K., "An intelligent personalized web blog searching technique using fuzzy-based feedback recurrent neural network," *Soft Computing*, vol. 24, pp. 9321–9333, 2020.

102 Concepción, L., Nápoles, G., Grau, I., and Pedrycz, W., "Fuzzy-rough cognitive networks: Theoretical analysis and simpler models," *IEEE Transactions on Cybernetics*, vol. 52, no. 5, pp. 2994–3005, 2020.

103 Li, R., Hu, Y., and Liang, Q., "T2f-lstm method for long-term traffic volume prediction," *IEEE Transactions on Fuzzy Systems*, vol. 28, no. 12, pp. 3256–3264, 2020.

104 Bilgili, M., Yildirim, A., Ozbek, A., Celebi, K., and Ekinci, F., "Long short-term memory (lstm) neural network and adaptive neuro-fuzzy inference system (anfis) approach in modeling renewable electricity generation forecasting," *International Journal of Green Energy*, vol. 18, no. 6, pp. 578–594, 2021.

105 Xue, M., Yan, H., Wang, M., Shen, H., and Shi, K., "Lstm-based intelligent fault detection for fuzzy markov jump systems and its application to tunnel diode circuits," *IEEE Transactions on Circuits and Systems II: Express Briefs*, vol. 69, no. 3, pp. 1099–1103, 2021.

106 Li, Y., Tong, Z., Tong, S., and Westerdahl, D., "A data-driven interval forecasting model for building energy prediction using attention-based lstm and fuzzy information granulation," *Sustainable Cities and Society*, vol. 76, p. 103481, 2022.

107 Shen, T., Wang, J., Gou, C., and Wang, F., "Hierarchical fused model with deep learning and type-2 fuzzy learning for breast cancer diagnosis," *IEEE Transactions on Fuzzy Systems*, vol. 28, no. 12, pp. 3204–3218, 2020.

108 Kalaiselvi, S. and Gomathi, V., "α-cut induced fuzzy deep neural network for change detection of sar images," *Applied Soft Computing*, vol. 95, p. 106510, 2020.

109 Özyurt, F., Sert, E., and Avcı, D., "An expert system for brain tumor detection: Fuzzy c-means with super resolution and convolutional neural network with extreme learning machine," *Medical Hypotheses*, vol. 134, p. 109433, 2020.

110 Puchkov, A., Dli, M., and Kireyenkova, M., "Fuzzy classification on the base of convolutional neural networks," in *Advances in Artificial Systems for Medicine and Education II 2*, 2020, pp. 379–391.

111 Kang, C., Yu, X., Wang, S., Guttery, D. S., Pandey, H. M., Tian, Y., and Zhang, Y., "A heuristic neural network structure relying on fuzzy logic for images scoring," *IEEE Transactions on Fuzzy Systems*, vol. 29, no. 1, pp. 34–45, 2020.

112 Cunha Sergio, G. and Lee, M., "Emotional video to audio transformation using deep recurrent neural networks and a neuro-fuzzy system," *Mathematical Problems in Engineering*, vol. 2020, pp. 1–15, 2020.

113 Wang, G., Zhou, T., Choi, K., and Lu, J., "A deep-ensemble-level-based interpretable takagi–sugeno–kang fuzzy classifier for imbalanced data," *IEEE Transactions on Cybernetics*, vol. 52, no. 5, pp. 3805–3818, 2020.

114 Ghasemi, M., Kelarestaghi, M., Eshghi, F., and Sharifi, A., "T2-fdl: a robust sparse representation method using adaptive type-2 fuzzy dictionary learning for medical image classification," *Expert Systems with Applications*, vol. 158, p. 113500, 2020.

115 Sharma, T. and Verma, N. K., "Estimating depth and global atmospheric light for image dehazing using type-2 fuzzy approach," *IEEE Transactions on Emerging Topics in Computational Intelligence*, vol. 6, no. 1, pp. 93–102, 2020.

116 Asadi, R. and Regan, A. C., "A spatio-temporal decomposition based deep neural network for time series forecasting," *Applied Soft Computing*, vol. 87, p. 105963, 2020.

117 Han, H., Zhang, H., and Qiao, J., "Robust deep neural network using fuzzy denoising autoencoder," *International Journal of Fuzzy Systems*, vol. 22, pp. 1356–1375, 2020.

118 Altameem, T., "Fuzzy rank correlation-based segmentation method and deep neural network for bone cancer identification," *Neural Computing and Applications*, vol. 32, pp. 805–815, 2020.

119 Wang, F., Pedrycz, W., Herrera, F., and Su, S., "Fuzzy logic and artificial intelligence: A special issue on emerging techniques and their applications," *IEEE Transactions on Fuzzy Systems*, vol. 28, no. 12, pp. 3063–3064, 2020.

120 Reddy, T. K., Arora, V., Gupta, V., Biswas, R., and Behera, L., "Eeg-based drowsiness detection with fuzzy independent phase-locking value representations using lagrangian-based deep neural networks," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 52, no. 1, pp. 101–111, 2021.

121 Alzubi, J. A., Jain, R., Nagrath, P., Satapathy, S., Taneja, S., and Gupta, P., "Deep image captioning using an ensemble of cnn and lstm based deep neural networks," *Journal of Intelligent & Fuzzy Systems*, vol. 40, no. 4, pp. 5761–5769, 2021.

122 Asghar, M. Z., Subhan, F., Ahmad, H., Khan, W. Z., Hakak, S., Gadekallu, T. R., and Alazab, M., "Senti-esystem: a sentiment-based esystem-using hybridized fuzzy and deep neural network for measuring customer satisfaction," *Software: Practice and Experience*, vol. 51, no. 3, pp. 571–594, 2021.

123 Wang, G. and Qiao, J., "An efficient self-organizing deep fuzzy neural network for nonlinear system modeling," *IEEE Transactions on Fuzzy Systems*, vol. 30, no. 7, pp. 2170–2182, 2021.

124 Pourmeidani, H. and Demara, R. F., "High-accuracy deep belief network: Fuzzy neural networks using mram-based stochastic neurons," *IEEE Journal on Exploratory Solid-State Computational Devices and Circuits*, vol. 7, no. 2, pp. 125–131, 2021.

125 Sharma, R., Goel, T., Tanveer, M., Dwivedi, S., and Murugan, R., "Faf-drvfl: Fuzzy activation function based deep random vector functional links network for early diagnosis of alzheimer disease," *Applied Soft Computing*, vol. 106, p. 107371, 2021.

126 Qasem, S. N. and Mohammadzadeh, A., "A deep learned type-2 fuzzy neural network: Singular value decomposition approach," *Applied Soft Computing*, vol. 105, p. 107244, 2021.

127 Wang, Y. and Luo, C., "An intelligent quantitative trading system based on intuitionistic-gru fuzzy neural networks," *Applied Soft Computing*, vol. 108, p. 107471, 2021.

128 Qazani, M. R. C., Asadi, H., Al-Ashmori, M., Mohamed, S., Lim, C. P., and Nahavandi, S., "Time series prediction of driving motion scenarios using fuzzy neural networks:* motion signal prediction using fnns," in *2021 IEEE International Conference on Mechatronics*, 2021, pp. 1–6.

129 Sun, J., Cao, R., Zhou, M., Hussain, W., Wang, B., Xue, J., and Xiang, J., "A hybrid deep neural network for classification of schizophrenia using eeg data," *Scientific Reports*, vol. 11, no. 1, pp. 1–16, 2021.

130 Dong, L., Jiang, F., Wang, M., and Li, X., "Fuzzy deep wavelet neural network with hybrid learning algorithm: application to electrical resistivity imaging inversion," *Knowledge-Based Systems*, vol. 242, p. 108164, 2022.

131 Yu, J., Kim, J., Li, X., Jong, Y., Kim, K., and Ryang, G., "Water quality forecasting based on data decomposition, fuzzy clustering and deep learning neural network," *Environmental Pollution*, vol. 303, p. 119136, 2022.

132 Ghosal, S., Sarkar, M., and Sarkar, R., "Nofed-net: Nonlinear fuzzy ensemble of deep neural networks for human activity recognition," *IEEE Internet of Things Journal*, vol. 9, no. 18, pp. 17 526–17 535, 2022.
133 He, L., Chen, Y., and Wu, K., "Fuzzy granular deep convolutional network with residual structures," *Knowledge-Based Systems*, vol. 258, p. 109941, 2022.
134 Zheng, W., Yan, L., Gou, C., and Wang, F., "Fuzzy deep forest with deep contours feature for leaf cultivar classification," *IEEE Transactions on Fuzzy Systems*, vol. 30, no. 12, pp. 5431–5444, 2022.
135 Vega, C. F., Quevedo, J., Escandón, E., Kiani, M., Ding, W., and Andreu-Perez, J., "Fuzzy temporal convolutional neural networks in p300-based brain–computer interface for smart home interaction," *Applied Soft Computing*, vol. 117, p. 108359, 2022.
136 Kumar, A., Sangwan, S. R., Arora, A., and Menon, V. G., "Depress-dcnf: a deep convolutional neuro-fuzzy model for detection of depression episodes using iomt," *Applied Soft Computing*, vol. 122, p. 108863, 2022.
137 Zhou, W., Liu, M., and Xu, Z., "The dual-fuzzy convolutional neural network to deal with handwritten image recognition," *IEEE Transactions on Fuzzy Systems*, vol. 30, no. 12, pp. 5225–5236, 2022.
138 Zaremarjal, A. Y., Yiltas-Kaplan, D., and Lazemi, S., "Emotion extraction from text using fuzzy-deep neural network," in *Intelligent and Fuzzy Techniques for Emerging Conditions and Digital Transformation: Proceedings of the INFUS 2021 Conference, held August 24-26, 2021. Volume 2*, 2022, pp. 329–338.
139 Muthusamy, D. and Rakkimuthu, P., "Trilateral filterative hermitian feature transformed deep perceptive fuzzy neural network for finger vein verification," *Expert Systems with Applications*, vol. 196, p. 116678, 2022.
140 Wang, J., Chang, Q., Gao, T., Zhang, K., and Pal, N. R., "Sensitivity analysis of takagi–sugeno fuzzy neural network," *Information Sciences*, vol. 582, pp. 725–749, 2022.
141 Rafiei, H. and Akbarzadeh-T, M., "Reliable fuzzy neural networks for systems identification and control," *IEEE Transactions on Fuzzy Systems*, 2022.
142 Tu, Q., Zhang, Q., Zhang, Z., Gong, D., and Tang, M., "A deep spatiotemporal fuzzy neural network for subway passenger flow prediction with covid-19 search engine data," *IEEE Transactions on Fuzzy Systems*, vol. 31, no. 2, pp. 394–406, 2022.
143 Nasiri, H. and Ebadzadeh, M. M., "Mfrfnn: Multi-functional recurrent fuzzy neural network for chaotic time series prediction," *Neurocomputing*, vol. 507, pp. 292–310, 2022.
144 Thakare, K. V., Sharma, N., Dogra, D. P., Choi, H., and Kim, I., "A multi-stream deep neural network with late fuzzy fusion for real-world anomaly detection," *Expert Systems with Applications*, vol. 201, p. 117030, 2022.
145 Dey, S., Roychoudhury, R., Malakar, S., and Sarkar, R., "An optimized fuzzy ensemble of convolutional neural networks for detecting tuberculosis from chest x-ray images," *Applied Soft Computing*, vol. 114, p. 108094, 2022.
146 Tan, D., Huang, Z., Peng, X., Zhong, W., and Mahalec, V., "Deep adaptive fuzzy clustering for evolutionary unsupervised representation learning," *IEEE Transactions on Neural Networks and Learning Systems*, 2023.
147 Wang, Y., Ishibuchi, H., Er, M. J., and Zhu, J., "Unsupervised multilayer fuzzy neural networks for image clustering," *Information Sciences*, vol. 622, pp. 682–709, 2023.
148 Wang, C., Lv, X., Shao, M., Qian, Y., and Zhang, Y., "A novel fuzzy hierarchical fusion attention convolution neural network for medical image super-resolution reconstruction," *Information Sciences*, vol. 622, pp. 424–436, 2023.
149 Zhang, C., Lin, Y., Chen, C. P., Yao, H., Cai, H., and Fang, W., "Fuzzy representation learning on graph," *IEEE Transactions on Fuzzy Systems*, 2023.
150 Ma, X., Chen, L., Deng, Z., Xu, P., Yan, Q., Choi, K., and Wang, S., "Deep image feature learning with fuzzy rules," *IEEE Transactions on Emerging Topics in Computational Intelligence*, 2023.
151 Lin, C. and Yang, T., "A fusion-based convolutional fuzzy neural network for lung cancer classification," *International Journal of Fuzzy Systems*, vol. 25, no. 2, pp. 451–467, 2023.
152 Raz, N. R., Akbarzadeh-T, M., and Setayeshi, S., "Influence-based nano fuzzy swarm oxygen deficiency detection and therapy," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, 2023.
153 Yin, R., Pan, X., Zhang, L., Yang, J., and Lu, W., "A rule-based deep fuzzy system with nonlinear fuzzy feature transform for data classification," *Information Sciences*, 2023.
154 Pham, P., Nguyen, L. T., Nguyen, N. T., Kozma, R., and Vo, B., "A hierarchical fused fuzzy deep neural network with heterogeneous network embedding for recommendation," *Information Sciences*, vol. 620, pp. 105–124, 2023.
155 Aghaeipoor, F., Sabokrou, M., and Fernández, A., "Fuzzy rule-based explainer systems for deep neural networks: From local explainability to global understanding," *IEEE Transactions on Fuzzy Systems*, 2023.
156 Zhang, L., Shi, Y., Chang, Y., and Lin, C., "Robust fuzzy neural network with an adaptive inference engine," *IEEE Transactions on Cybernetics*, 2023.
157 Sindhuraj, I. C. G. L. and Patrick, A. J., "Loan eligibility prediction using adaptive hybrid optimization driven-deep neuro fuzzy network," *Expert Systems with Applications*, p. 119903, 2023.
158 Lu, J., Behbood, V., Hao, P., Zuo, H., Xue, S., and Zhang, G., "Transfer learning using computational intelligence: A survey," *Knowledge-Based Systems*, vol. 80, pp. 14–23, 2015.
159 Zhang, X., Zhang, X., Liu, H., and Liu, X., "Multi-task clustering through instances transfer," *Neurocomputing*, vol. 251, pp. 145–155, 2017.
160 Venkateswara, H., Chakraborty, S., and Panchanathan, S., "Deep-learning systems for domain adaptation in computer vision: Learning transferable feature representations," *IEEE Signal Processing Magazine*, vol. 34, no. 6, pp. 117–129, 2017.
161 Duan, L., Tsang, I. W., and Xu, D., "Domain transfer multiple kernel learning," *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol. 34, no. 3, pp. 465–479, 2012.
162 Dietterich, T. G., Domingos, P., Getoor, L., Muggleton, S., and Tadepalli, P., "Structured machine learning: the next ten years," *Machine Learning*, vol. 73, no. 1, p. 3, 2008.
163 Liu, Q., Liao, X., Carin, H. L., Stack, J. R., and Carin, L., "Semisupervised multitask learning," *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol. 31, no. 6, pp. 1074–1086, 2009.
164 Zuo, H., Zhang, G., Behbood, V., Lu, J., and Meng, X., "Transfer learning in hierarchical feature spaces," in *2015 10th International Conference on Intelligent Systems and Knowledge Engineering*, 2015, pp. 183–188.
165 Fang, Z., Lu, J., Liu, F., Xuan, J., and Zhang, G., "Open set domain adaptation: Theoretical bound and algorithm," *IEEE transactions on neural networks and learning systems*, vol. 32, no. 10, pp. 4309–4322, 2020.
166 Yang, J., Yan, R., and Hauptmann, A. G., "Cross-domain video concept detection using adaptive svms," in *Proceedings of the 15th ACM international conference on Multimedia*, 2007, pp. 188–197.
167 Fang, Z., Lu, J., Liu, F., and Zhang, G., "Semi-supervised heterogeneous domain adaptation: Theory and algorithms," *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol. 45, no. 1, pp. 1087–1105, 2022.
168 Behbood, V., Lu, J., Zhang, G., and Pedrycz, W., "Multistep fuzzy bridged refinement domain adaptation algorithm and its application to bank failure prediction," *IEEE Transactions on Fuzzy Systems*, vol. 23, no. 6, pp. 1917–1935, 2015.
169 Wu, D., Lawhern, V. J., Gordon, S., Lance, B. J., and Lin, C., "Driver drowsiness estimation from eeg signals using online weighted adaptation regularization for regression (owarr)," *IEEE Transactions on Fuzzy Systems*, vol. 25, no. 6, pp. 1522–1535, 2017.
170 Gargees, R., Keller, J. M., and Popescu, M., "Tlpcm: Transfer learning possibilistic c-means," *IEEE Transactions on Fuzzy Systems*, 2020.
171 Sun, S., Yun, J., Lin, H., Zhang, N., Abraham, A., and Liu, H., "Granular transfer learning using type-2 fuzzy hmm for text sequence recognition," *Neurocomputing*, vol. 214, pp. 126–133, 2016.
172 Shukla, A. K., Kumar, S., Jagdev, R., Muhuri, P. K., and Lohani, Q. D., "Interval type-2 fuzzy weighted extreme learning machine for gdp prediction," in *2018 International Joint Conference on Neural Networks*, 2018, pp. 1–8.
173 Kumar, S., Shukla, A. K., Muhuri, P. K., and Lohani, Q. D., "Transfer learning based gdp prediction from uncertain carbon emission data," in *2019 IEEE International Conference on Fuzzy Systems*, 2019, pp. 1–6.
174 Shell, J. and Coupland, S., "Fuzzy transfer learning: methodology and application," *Information Sciences*, vol. 293, pp. 59–79, 2015.
175 Meher, S. K. and Kothari, N. S., "Interpretable rule-based fuzzy elm and domain adaptation for remote sensing image classification," *IEEE Transactions on Geoscience and Remote Sensing*, 2020.
176 Deng, Z., Jiang, Y., Chung, F., Ishibuchi, H., and Wang, S., "Knowledge-leverage-based fuzzy system and its modeling," *IEEE Transactions on Fuzzy Systems*, vol. 21, no. 4, pp. 597–609, 2013.
177 Deng, Z., Jiang, Y., Choi, K., Chung, F., and Wang, S., "Knowledge-leverage-based tsk fuzzy system modeling," *IEEE Transactions on Neural Networks and Learning Systems*, vol. 24, no. 8, pp. 1200–1212, 2013.

178 Deng, Z., Jiang, Y., Ishibuchi, H., Choi, K., and Wang, S., "Enhanced knowledge-leverage-based tsk fuzzy system modeling for inductive transfer learning," *ACM Transactions on Intelligent Systems and Technology*, vol. 8, no. 1, pp. 1–21, 2016.

179 Yang, C., Deng, Z., Choi, K., and Wang, S., "Takagi–sugeno–kang transfer learning fuzzy logic system for the adaptive recognition of epileptic electroencephalogram signals," *IEEE Transactions on Fuzzy Systems*, vol. 24, no. 5, pp. 1079–1094, 2016.

180 Jiang, Y., Wu, D., Deng, Z., Qian, P., Wang, J., Wang, G., Chung, F., Choi, K., and Wang, S., "Seizure classification from eeg signals using transfer learning, semi-supervised learning and tsk fuzzy system," *IEEE Transactions on Neural Systems and Rehabilitation Engineering*, vol. 25, no. 12, pp. 2270–2284, 2017.

181 Deng, Z., Xu, P., Xie, L., Choi, K., and Wang, S., "Transductive joint-knowledge-transfer tsk fs for recognition of epileptic eeg signals," *IEEE Transactions on Neural Systems and Rehabilitation Engineering*, vol. 26, no. 8, pp. 1481–1494, 2018.

182 Xie, L., Deng, Z., Xu, P., Choi, K., and Wang, S., "Generalized hidden-mapping transductive transfer learning for recognition of epileptic electroencephalogram signals," *IEEE Transactions on Cybernetics*, vol. 49, no. 6, pp. 2200–2214, 2018.

183 Jiang, Y., Gu, X., Ji, D., Qian, P., Xue, J., Zhang, Y., Zhu, J., Xia, K., and Wang, S., "Smart diagnosis: A multiple-source transfer tsk fuzzy system for eeg seizure identification," *ACM Transactions on Multimedia Computing, Communications, and Applications*, vol. 16, no. 2s, pp. 1–21, 2020.

184 Jiang, Y., Zhang, Y., Lin, C., Wu, D., and Lin, C., "Eeg-based driver drowsiness estimation using an online multi-view and transfer tsk fuzzy system," *IEEE Transactions on Intelligent Transportation Systems*, 2021.

185 Xu, P., Deng, Z., Wang, J., Zhang, Q., Choi, K., and Wang, S., "Transfer representation learning with tsk fuzzy system," *IEEE Transactions on Fuzzy Systems*, 2019.

186 Zuo, H., Zhang, G., Pedrycz, W., Behbood, V., and Lu, J., "Fuzzy regression transfer learning in takagi–sugeno fuzzy models," *IEEE Transactions on Fuzzy Systems*, vol. 25, no. 6, pp. 1795–1807, 2017.

187 Zuo, H., Zhang, G., Lu, J., and Pedrycz, W., "Fuzzy rule-based transfer learning for label space adaptation," in *2017 IEEE International Conference on Fuzzy Systems*, 2017, pp. 1–6.

188 Zuo, H., Zhang, G., Pedrycz, W., Behbood, V., and Lu, J., "Granular fuzzy regression domain adaptation in takagi–sugeno fuzzy models," *IEEE Transactions on Fuzzy Systems*, vol. 26, no. 2, pp. 847–858, 2018.

189 Zuo, H., Lu, J., Zhang, G., and Liu, F., "Fuzzy transfer learning using an infinite gaussian mixture model and active learning," *IEEE Transactions on Fuzzy Systems*, vol. 27, no. 2, pp. 291–303, 2019.

190 Li, K., Lu, J., Zuo, H., and Zhang, G., "Multi-source domain adaptation with fuzzy-rule based deep neural networks," in *2021 IEEE International Conference on Fuzzy Systems*, 2021, pp. 1–6.

191 Li, K., Lu, J., Zuo, H., and Zhang, G., "Source-free multi-domain adaptation with fuzzy rule-based deep neural networks," *IEEE Transactions on Fuzzy Systems*, 2023.

192 Zuo, H., Lu, J., Zhang, G., and Pedrycz, W., "Fuzzy rule-based domain adaptation in homogeneous and heterogeneous spaces," *IEEE Transactions on Fuzzy Systems*, vol. 27, no. 2, pp. 348–361, 2019.

193 Che, X., Zuo, H., Lu, J., and Chen, D., "Fuzzy multioutput transfer learning for regression," *IEEE Transactions on Fuzzy Systems*, vol. 30, no. 7, pp. 2438–2451, 2021.

194 Pal, S. K. and Ghosh, A., "Fuzzy geometry in image analysis," *Fuzzy Sets and Systems*, vol. 48, no. 1, pp. 23–40, 1992.

195 Yang, J., Wang, G., and Zhang, Q., "Knowledge distance measure in multigranulation spaces of fuzzy equivalence relations," *Information Sciences*, vol. 448, pp. 18–35, 2018.

196 Liu, F., Zhang, G., and Lu, J., "A novel fuzzy neural network for unsupervised domain adaptation in heterogeneous scenarios," in *2019 IEEE International Conference on Fuzzy Systems*, 2019, pp. 1–6.

197 Ma, G., Lu, J., and Zhang, G., "Multi-source domain adaptation with interval-valued target data via fuzzy neural networks," *IEEE Transactions on Fuzzy Systems*, 2024.

198 Li, K., Lu, J., Zuo, H., and Zhang, G., "Attention-bridging ts fuzzy rules for universal multi-domain adaptation without source data," in *2023 IEEE International Conference on Fuzzy Systems*, 2023, pp. 1–6.

199 Ma, G., Lu, J., Liu, F., Fang, Z., and Zhang, G., "Domain adaptation with interval-valued observations: Theory and algorithms," *IEEE Transactions on Fuzzy Systems*, 2024.

200 Ma, G., Lu, J., and Zhang, G., "Interval-valued observations-based multi-source domain adaptation using fuzzy neural networks," in *2023 IEEE International Conference on Fuzzy Systems*, 2023.

201 Zhang, B., Qin, S., Wang, W., Wang, D., and Xue, L., "Data stream clustering based on fuzzy c-mean algorithm and entropy theory," *Signal Processing*, vol. 126, pp. 111–116, 2016.

202 Hong, T., Chen, C., Li, Y., and Wu, M., "Using fuzzy c-means to discover concept-drift patterns for membership functions," *Transactions on Fuzzy Sets and Systems*, vol. 1, no. 2, pp. 21–31, 2022.

203 Song, Y., Zhang, G., Lu, J., and Lu, H., "A fuzzy kernel c-means clustering model for handling concept drift in regression," in *2017 IEEE International Conference on Fuzzy Systems*, 2017, pp. 1–6.

204 Song, Y., Zhang, G., Lu, H., and Lu, J., "A noise-tolerant fuzzy c-means based drift adaptation method for data stream regression," in *2019 IEEE International Conference on Fuzzy Systems*, 2019, pp. 1–6.

205 Bechini, A., Marcelloni, F., and Renda, A., "Tsf-dbscan: a novel fuzzy density-based approach for clustering unbounded data streams," *IEEE Transactions on Fuzzy Systems*, vol. 30, no. 3, pp. 623–637, 2022.

206 Zhang, Q., Wu, D., Zhang, G., and Lu, J., "Fuzzy user-interest drift detection based recommender systems," in *2016 IEEE International Conference on Fuzzy Systems*, 2016, pp. 1274–1281.

207 Dong, F., Zhang, G., Lu, J., and Li, K., "Fuzzy competence model drift detection for data-driven decision support systems," *Knowledge-Based Systems*, vol. 143, pp. 284–294, 2018.

208 Pratama, M., Lu, J., Lughofer, E., Zhang, G., and Er, M. J., "An incremental learning of concept drifts using evolving type-2 recurrent fuzzy neural networks," *IEEE Transactions on Fuzzy Systems*, vol. 25, no. 5, pp. 1175–1192, 2017.

209 Song, Y., Zhang, G., Lu, H., and Lu, J., "A self-adaptive fuzzy network for prediction in non-stationary environments," in *2018 IEEE International Conference on Fuzzy Systems*, 2018, pp. 1–8.

210 Garcia, C., Leite, D., and Škrjanc, I., "Incremental missing-data imputation for evolving fuzzy granular prediction," *IEEE Transactions on Fuzzy Systems*, vol. 28, no. 10, pp. 2348–2362, 2020.

211 García-Vico, Á. M., Carmona, C. J., Gonzalez, P., Seker, H., and del Jesus, M. J., "Fepds: A proposal for the extraction of fuzzy emerging patterns in data streams," *IEEE Transactions on Fuzzy Systems*, vol. 28, no. 12, pp. 3193–3203, 2020.

212 Lughofer, E. and Angelov, P., "Handling drifts and shifts in on-line data streams with evolving fuzzy systems," *Applied Soft Computing*, vol. 11, no. 2, pp. 2057–2068, 2011.

213 Yu, H., Lu, J., and Zhang, G., "Topology learning-based fuzzy random neural networks for streaming data regression," *IEEE Transactions on Fuzzy Systems*, vol. 30, no. 2, pp. 412–425, 2020.

214 Ge, D. and Zeng, X.-J., "Learning data streams online—an evolving fuzzy system approach with self-learning/adaptive thresholds," *Information sciences*, vol. 507, pp. 172–184, 2020.

215 Gu, X. and Shen, Q., "A self-adaptive fuzzy learning system for streaming data prediction," *Information Sciences*, vol. 579, pp. 623–647, 2021.

216 Gu, X., "An explainable semi-supervised self-organizing fuzzy inference system for streaming data classification," *Information Sciences*, vol. 583, pp. 364–385, 2022.

217 Ferdaus, M. M., Pratama, M., Anavatti, S. G., and Garratt, M. A., "Palm: An incremental construction of hyperplanes for data stream regression," *IEEE Transactions on Fuzzy Systems*, vol. 27, no. 11, pp. 2115–2129, 2019.

218 Leite, D., Andonovski, G., Škrjanc, I., and Gomide, F., "Optimal rule-based granular systems from data streams," *IEEE Transactions on Fuzzy Systems*, vol. 28, no. 3, pp. 583–596, 2019.

219 Liu, Y., Zhao, J., Wang, W., and Pedrycz, W., "Prediction intervals for granular data streams based on evolving type-2 fuzzy granular neural network dynamic ensemble," *IEEE Transactions on Fuzzy Systems*, vol. 29, no. 4, pp. 874–888, 2020.

220 e Silva, P. C. d. L., Junior, C. A. S., Alves, M. A., Silva, R., Cohen, M. W., and Guimarães, F. G., "Forecasting in non-stationary environments with fuzzy time series," *Applied Soft Computing*, vol. 97, p. 106825, 2020.

221 Severiano, C. A., e Silva, P. C. d. L., Cohen, M. W., and Guimarães, F. G., "Evolving fuzzy time series for spatio-temporal forecasting in renewable energy systems," *Renewable Energy*, vol. 171, pp. 764–783, 2021.

222 Liu, A., Lu, J., and Zhang, G., "Concept drift detection: dealing with missing values via fuzzy distance estimations," *IEEE Transactions on Fuzzy Systems*, vol. 29, no. 11, pp. 3219–3233, 2020.

223 Liu, A., Zhang, G., and Lu, J., "Fuzzy time windowing for gradual concept drift adaptation," in *2017 IEEE International Conference on Fuzzy Systems*, 2017, pp. 1–6.

224 Song, Y., Zhang, G., Lu, H., and Lu, J., "A fuzzy drift correlation matrix for multiple data stream regression," in *2020 IEEE International Conference on Fuzzy Systems*, 2020, pp. 1–6.

225 Dong, F., Lu, J., Zhang, G., and Li, K., "Active fuzzy weighting ensemble for dealing with concept drift," *International Journal of Computational Intelligence Systems*, vol. 11, no. 1, pp. 438–450, 2018.

226 Zhang, N., Yao, X., and Luo, C., "The prediction of online time series with concept drift based on dynamic intuitionistic fuzzy cognitive map," *Intelligent Data Analysis*, vol. 25, no. 4, pp. 949–972, 2021.

227 Yu, H., Lu, J., Liu, A., Wang, B., Li, R., and Zhang, G., "Real-time prediction system of train carriage load based on multi-stream fuzzy learning," *IEEE Transactions on Intelligent Transportation Systems*, vol. 23, no. 9, pp. 15 155–15 165, 2022.

228 Song, Q. and Chissom, B. S., "Fuzzy time series and its models," *Fuzzy sets and systems*, vol. 54, no. 3, pp. 269–277, 1993.

229 Lütjens, B., Everett, M., and How, J. P., "Safe reinforcement learning with model uncertainty estimates," in *2019 International Conference on Robotics and Automation*, 2019, pp. 8662–8668.

230 Devidze, R., Kamalaruban, P., and Singla, A., "Exploration-guided reward shaping for reinforcement learning under sparse rewards," *Advances in Neural Information Processing Systems*, vol. 35, pp. 5829–5842, 2022.

231 Zhang, H., Zhang, K., Cai, Y., and Han, J., "Adaptive fuzzy fault-tolerant tracking control for partially unknown systems with actuator faults via integral reinforcement learning method," *IEEE Transactions on Fuzzy Systems*, vol. 27, no. 10, pp. 1986–1998, 2019.

232 Zhang, K., Zhang, H., Mu, Y., and Liu, C., "Decentralized tracking optimization control for partially unknown fuzzy interconnected systems via reinforcement learning method," *IEEE Transactions on Fuzzy Systems*, vol. 29, no. 4, pp. 917–926, 2020.

233 Kumar, N., Rahman, S. S., and Dhakad, N., "Fuzzy inference enabled deep reinforcement learning-based traffic light control for intelligent transportation system," *IEEE Transactions on Intelligent Transportation Systems*, vol. 22, no. 8, pp. 4919–4928, 2020.

234 Li, J., Shi, H., and Hwang, K., "Using fuzzy logic to learn abstract policies in large-scale multiagent reinforcement learning," *IEEE Transactions on Fuzzy Systems*, vol. 30, no. 12, pp. 5211–5224, 2022.

235 Zhu, A., Ai, H., and Chen, L., "A fuzzy logic reinforcement learning control with spring-damper device for space robot capturing satellite," *Applied Sciences*, vol. 12, no. 5, p. 2662, 2022.

236 Fang, B., Zheng, C., Wang, H., and Yu, T., "Two stream fused fuzzy deep neural network for multi-agent learning," *IEEE Transactions on Fuzzy Systems*, 2022.

237 Fathinezhad, F., Derhami, V., and Rezaeian, M., "Supervised fuzzy reinforcement learning for robot navigation," *Applied Soft Computing*, vol. 40, pp. 33–41, 2016.

238 Hein, D., Hentschel, A., Runkler, T., and Udluft, S., "Particle swarm optimization for generating interpretable fuzzy reinforcement learning policies," *Engineering Applications of Artificial Intelligence*, vol. 65, pp. 87–98, 2017.

239 Yin, L. and Li, Y., "Fuzzy vector reinforcement learning algorithm for generation control of power systems considering flywheel energy storage," *Applied Soft Computing*, vol. 125, p. 109149, 2022.

240 Shi, H., Lin, Z., Zhang, S., Li, X., and Hwang, K., "An adaptive decision-making method with fuzzy bayesian reinforcement learning for robot soccer," *Information Sciences*, vol. 436, pp. 268–281, 2018.

241 Atanassov, K., "Intuitionistic fuzzy sets," *Fuzzy Sets and Systems*, vol. 20, pp. 87–96, 1986.

242 Lu, J., Zhu, Y., Zeng, X., Koehl, L., Ma, J., and Zhang, G., "A linguistic multi-criteria group decision support system for fabric hand evaluation," *Fuzzy Optimization and Decision Making*, vol. 8, pp. 395–413, 2009.

243 Ishibuchi, H. and Nakashima, T., "Effect of rule weights in fuzzy rule-based classification systems," *IEEE Transactions on Fuzzy Systems*, vol. 9, no. 4, pp. 506–515, 2001.

244 Cornelis, C., Lu, J., Guo, X., and Zhang, G., "One-and-only item recommendation with fuzzy logic techniques," *Information sciences*, vol. 177, no. 22, pp. 4906–4921, 2007.

245 Katarya, R. and Verma, O. P., "An effective web page recommender system with fuzzy c-mean clustering," *Multimedia Tools and Applications*, vol. 76, pp. 21 481–21 496, 2017.

246 Katarya, R. and Verma, O., "Recommender system with grey wolf optimizer and fcm," *Neural Computing and Applications*, vol. 30, pp. 1679–1687, 2018.

247 Selvi, C. and Sivasankar, E., "A novel optimization algorithm for recommender system using modified fuzzy c-means clustering approach," *Soft Computing*, vol. 23, pp. 1901–1916, 2019.

248 Nilashi, M., Ibrahim, O. B., Ithnin, N., and Zakaria, R., "A multi-criteria recommendation system using dimensionality reduction and neuro-fuzzy techniques," *Soft Computing*, vol. 19, pp. 3173–3207, 2015.

249 Yera, R., Alzahrani, A. A., and Martínez, L., "A fuzzy content-based group recommender system with dynamic selection of the aggregation functions," *International Journal of Approximate Reasoning*, vol. 150, pp. 273–296, 2022.

250 Sulthana, A. R. and Ramasamy, S., "Ontology and context based recommendation system using neuro-fuzzy classification," *Computers & Electrical Engineering*, vol. 74, pp. 498–510, 2019.

251 Patro, S. G. K., Mishra, B. K., Panda, S. K., Kumar, R., Long, H. V., and Tuan, T. M., "Knowledge-based preference learning model for recommender system using adaptive neuro-fuzzy inference system," *Journal of Intelligent & Fuzzy Systems*, vol. 39, no. 3, pp. 4651–4665, 2020.

252 Zhang, W., Zhang, X., and Chen, D., "Causal neural fuzzy inference modeling of missing data in implicit recommendation system," *Knowledge-Based Systems*, vol. 222, p. 106678, 2021.

253 Walek, B. and Fajmon, P., "A hybrid recommender system for an online store using a fuzzy expert system," *Expert Systems with Applications*, vol. 212, p. 118565, 2023.

254 Kuanr, M., Rath, B. K., and Mohanty, S. N., "Crop recommender system for the farmers using mamdani fuzzy inference model," *International Journal of Engineering & Technology*, vol. 7, no. 4.15, pp. 277–280, 2018.

255 Castro, J., Yera, R., and Martinez, L., "A fuzzy approach for natural noise management in group recommender systems," *Expert Systems with Applications*, vol. 94, pp. 237–249, 2018.

256 Rutkowski, T., Romanowski, J., Woldan, P., Staszewski, P., Nielek, R., and Rutkowski, L., "A content-based recommendation system using neuro-fuzzy approach," in *2018 IEEE International Conference on Fuzzy Systems*, 2018, pp. 1–8.

257 Almohammadi, K., Hagras, H., Yao, B., Alzahrani, A., Alghazzawi, D., and Aldabbagh, G., "A type-2 fuzzy logic recommendation system for adaptive teaching," *Soft Computing*, vol. 21, pp. 965–979, 2017.

258 Nagaraj, P. and Deepalakshmi, P., "An intelligent fuzzy inference rule-based expert recommendation system for predictive diabetes diagnosis," *International Journal of Imaging Systems and Technology*, vol. 32, no. 4, pp. 1373–1396, 2022.

259 Karthik, R. and Ganapathy, S., "A fuzzy recommendation system for predicting the customers interests using sentiment analysis and ontology in e-commerce," *Applied Soft Computing*, vol. 108, p. 107396, 2021.

260 Yan, S., Pirooznia, S., Heidari, A., Navimipour, N. J., and Unal, M., "Implementation of a product-recommender system in an iot-based smart shopping using fuzzy logic and apriori algorithm," *IEEE Transactions on Engineering Management*, 2022.

261 Kermany, N. R. and Alizadeh, S. H., "A hybrid multi-criteria recommender system using ontology and neuro-fuzzy techniques," *Electronic Commerce Research and Applications*, vol. 21, pp. 50–64, 2017.

262 Manogaran, G., Varatharajan, R., and Priyan, M. K., "Hybrid recommendation system for heart disease diagnosis based on multiple kernel learning with adaptive neuro-fuzzy inference system," *Multimedia tools and applications*, vol. 77, pp. 4379–4399, 2018.

263 Thong, N. T. *et al.*, "Intuitionistic fuzzy recommender systems: an effective tool for medical diagnosis," *Knowledge-Based Systems*, vol. 74, pp. 133–150, 2015.

264 Guo, J., Deng, J., and Wang, Y., "An intuitionistic fuzzy set based hybrid similarity model for recommender system," *Expert Systems with Applications*, vol. 135, pp. 153–163, 2019.

265 Yera, R., Castro, J., and Martínez, L., "A fuzzy model for managing natural noise in recommender systems," *Applied Soft Computing*, vol. 40, pp. 187–198, 2016.

266 Wang, P., Wang, Y., Zhang, L. Y., and Zhu, H., "An effective and efficient fuzzy approach for managing natural noise in recommender systems," *Information Sciences*, vol. 570, pp. 623–637, 2021.

267 Saravanan, B., Mohanraj, V., and Senthilkumar, J., "A fuzzy entropy technique for dimensionality reduction in recommender systems using deep learning," *Soft Computing*, vol. 23, pp. 2575–2583, 2019.

268 Zenebe, A. and Norcio, A. F., "Representation, similarity measures and aggregation methods using fuzzy sets for content-based recommender systems," *Fuzzy sets and systems*, vol. 160, no. 1, pp. 76–94, 2009.

269 Shang, F., Liu, Y., Cheng, J., and Yan, D., "Fuzzy double trace norm minimization for recommendation systems," *IEEE Transactions on Fuzzy Systems*, vol. 26, no. 4, pp. 2039–2049, 2017.

270 Mezei, J. and Nikou, S., "Fuzzy optimization to improve mobile health and wellness recommendation systems," *Knowledge-Based Systems*, vol. 142, pp. 108–116, 2018.

271 Huitzil, I., Alegre, F., and Bobillo, F., "Gimmehop: A recommender system for mobile devices using ontology reasoners and fuzzy logic," *Fuzzy Sets and Systems*, vol. 401, pp. 55–77, 2020.

272 Abbasi-Moud, Z., Hosseinabadi, S., Kelarestaghi, M., and Eshghi, F., "Cafob: Context-aware fuzzy-ontology-based tourism recommendation system," *Expert Systems with Applications*, vol. 199, p. 116877, 2022.

273 Zhang, Z., Lin, H., Liu, K., Wu, D., Zhang, G., and Lu, J., "A hybrid fuzzy-based personalized recommender system for telecom products/services," *Information Sciences*, vol. 235, pp. 117–129, 2013.

274 Wu, D., Lu, J., and Zhang, G., "A fuzzy tree matching-based personalized e-learning recommender system," *IEEE transactions on fuzzy systems*, vol. 23, no. 6, pp. 2412–2426, 2015.

275 Martinez-Cruz, C., Porcel, C., Bernabé-Moreno, J., and Herrera-Viedma, E., "A model to represent users trust in recommender systems using ontologies and fuzzy linguistic modeling," *Information Sciences*, vol. 311, pp. 102–118, 2015.

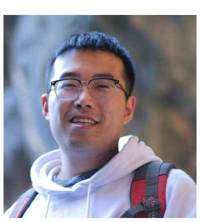
276 Gao, R., Wang, J., Zhou, K., Liu, F., Xie, B., Niu, G., Han, B., and Cheng, J., "Fast and reliable evaluation of adversarial robustness with minimum-margin attack," in *International Conference on Machine Learning*, 2022, pp. 7144–7163.

277 Zuo, H., Zhang, G., and Lu, J., "Fuzzy transfer learning in heterogeneous space using takagi-sugeno fuzzy models," in *International Fuzzy Systems Association World Congress*, 2019, pp. 752–763.

278 Fang, Z., Lu, J., Liu, A., Liu, F., and Zhang, G., "Learning bounds for open-set learning," in *International conference on machine learning*, 2021, pp. 3122–3132.

279 Zang, T., Zhu, Y., Liu, H., Zhang, R., and Yu, J., "A survey on cross-domain recommendation: taxonomies, methods, and future directions," *ACM Transactions on Information Systems*, vol. 41, no. 2, pp. 1–39, 2022.

Jie Lu (F'18) is an Australian Laureate Fellow, IFSA Fellow, Distinguished Professor and the Director of Australian Artificial Intelligence Institute (AAII) at the University of Technology Sydney, Australia. She received a PhD degree from Curtin University in 2000. Her main research expertise is in transfer learning, concept drift, fuzzy systems, decision support systems and recommender systems. She has published over 500 papers in IEEE Transactions and other leading journals and conferences. She is the recipient of two IEEE Transactions on Fuzzy Systems Outstanding Paper Awards (2019 and 2022), NeurIPS2022 Outstanding Paper Award, Australia's Most Innovative Engineer Award (2019), Australasian Artificial Intelligence Distinguished Research Contribution Award (2022), Australian NSW Premier's Prize on Excellence in Engineering or Information & Communication Technology (2023), and the Officer of the Order of Australia (AO) 2023.



Guangzhi Ma received his M.S. degree in probability and statistics from the School of Mathematics and Statistics, Lanzhou University, Lanzhou, China, in 2020. He is now a PhD candidate with the Faculty of Engineering and Information Technology, University of Technology Sydney, Australia. He is a Member of the Decision Systems and e-Service Intelligence Lab, Australia Artificial Intelligence Institute, University of Technology Sydney. His research interests include fuzzy transfer learning, domain adaptation and interval-valued data analysis. He has published several paper related to interval-valued data and transfer learning in FUZZ-IEEE, TFS, TCYB.

Guangquan Zhang is an Australian Research Council (ARC) QEII Fellow, Associate Professor and the Director of the Decision Systems and e-Service Intelligent (DeSI) Research Laboratory at the Australian Artificial Intelligence Institute, University of Technology Sydney, Australia. He received his PhD in applied mathematics from Curtin University, Australia, in 2001. From 1993 to 1997, he was a full Professor in the Department of Mathematics, Hebei University, China. His main research interests lie in fuzzy multi-objective, bilevel and group decision making, fuzzy measures, transfer learning and concept drift adaptation. He has published six authored monographs and over 500 papers including some 300 articles in leading international journals. He has supervised 40 PhD students to completion and mentored 15 Postdoc fellows. Prof Zhang has won ten very competitive ARC Discovery grants and many other research projects. His research has been widely applied in industries.