Investigating Formation and Containment Problem for Nonlinear Multiagent Systems by Interval Type-2 Fuzzy Sliding Mode Tracking Approach

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***Abstract*—A fuzzy sliding mode (FSM) tracking approach is proposed by interval type-2 (IT-2) Takagi–Sugeno fuzzy model (T-SFM) to complete formation and containment (F-and-C) tasks for nonlinear multiagent systems (NMAS) systems with uncertainties and disturbances. Using an imperfect premise matching (IPM) approach, the IT-2 F-and-C fuzzy controllers (FCs) are designed in accordance with the IT-2 T-SFM. Different from the existing research, the individual tracking FC for each leader is proposed to complete the formation tasks and assignment of whole system dynamics. The stability analysis process is also simplified to ensure the formation purpose. Benefiting from the advantage, the FSM control is conveniently combined into the FC design to upgrade the performance of leaders. For followers, the analysis methods of linear multiagent (M-A) systems are extended to solve the containment analysis problem without the additional assumption by virtue of the IT-2 T-SFM representation. To deal with the disturbance effect, the passive constraints are combined into the analysis process. Finally, the proposed F-and-C FCs design method is applied to a nonlinear multiship system (NMSS) to illustrate the advantage.**

***Index Terms*—Formation and containment (F-and-C) control, interval type-2 (IT-2) Takagi–Sugeno fuzzy model (T-SFM), nonlinear multiagent systems (NMASs), sliding mode control (SMC), uncertainties and disturbances.**

I. Introduction

By INFORMATION communication, the multiagent (M-A) control system has been developed to solve various control issues more efficiently over the past decades. According to the purpose of missions, control problems are divided into consensus, formation, and containment. Owing to the broad range of applications, the formation control issue has been extensively discussed. To allocate the mission, the leader–follower structure has been proposed for M-A systems [1]. Generally, the more functional and expensive devices are only required to be equipped on leaders, which efficiently reduces the application costs. As an extension of consensus with multiple leaders, containment control has been introduced [2]. Nowadays, the containment control approach has also contributed to the situation that followers are required to be protected by leaders with higher capacity such as evacuation of crowds [3] and protection of important aircraft carriers with multiple armed ships. It has been witnessed that formation or containment control possesses many potential applications in both military and civilian spheres.

Notably, one of the important factors pushing forward the control issue of M-A systems is the successful progress of autonomous aerial, surface, and ground vehicles. However, the working environments of these vehicles often have many external disturbances, which may lead to the mission failure of formation and containment (F-and-C). Many researchers have contributed their efforts to confronting the disturbance in nonlinear M-A systems (NMASs). Extending the concept of passivity, the passive constraint has been developed to deal with disturbance effects [4], [5]. The passivity theory has been verified as a useful tool for analyzing electrical networks and nonlinear systems. By satisfying the condition that the storage energy of control systems is always smaller than the supply energy from outside, the system is called passive and the dissipative ability is possessed [6]. Moreover, the passive constraint, which regards disturbances as external inputs, can provide a general form for different performance requirements by the proper setting of the power supply function [4], [5].

As the increasingly sophisticated industrial systems, the nonlinear systems are required to be employed to describe their dynamic behaviors. Unfortunately, the controller design process also becomes complicated. By the representation of the Takagi–Sugeno fuzzy model (T-SFM), the nonlinear control problems can be recast to linear problems [7]. However, the dynamic of practical applications may not be accurately described and obtained by nonlinear systems [8]. The equipment will also have aging, wear, and so on due to long-term work. The factors which degrade control performances can be thought of as uncertain problems. The type-1 T-SFM based control approach such as in [4] and [5] did not have enough capacity to deal with uncertainties. Therefore, the interval type-2 (IT-2) T-SFM and fuzzy controller (FC) have been developed to represent and control uncertain nonlinear systems [9], [10]. By the IT-2

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TABLE I
COMPARISON WITH EXISTING RESEARCH

	[18], [19], [20]	[21]	[22], [23]	This research
1) System	Linear	Nonlinear	Nonlinear	Nonlinear
2) Controller design	Linear	Nonlinear	Linear	Linear
3) Additional controller	Required	Required	Unnecessary	Unnecessary
4) Additional assumption	Required	Required	Unnecessary	Unnecessary
5) Communication between leaders	Required	Required	Unnecessary	Unnecessary
6) IT-2 T-SFM	Without	Without	With (Containment)	With (Containment and Formation)

T-SFM, researchers have also successfully developed the IT-2 membership function (MF) dependent filter design for the disturbance problem [11].

By leveraging the advantage of being insensitive to the effect of uncertainties and disturbances, the sliding mode control (SMC) still has been frequently utilized nowadays. With type-1 T-SFM, the fuzzy sliding mode (FSM) controller has been developed for the control of nonlinear systems [12]. There are many researchers successfully using the SMC to enhance the performance of tracking control (TC) for practical applications [13]. In solving the formation problem of NMASs, some SMC approaches have been developed [14]. However, the SMC inputs consist of nonlinear form, which will increase the difficulties in the controller design of practical systems. For a single autonomous ship, some researchers have verified the effectiveness of the FSM controller in the dynamic control problem [15]. However, it is limited by the capacity of type-1 T-SFM in describing the uncertain factors. With the IT-2 T-SFM, the SMC not only can be designed with a simpler construction but also can deal with uncertainties and disturbances more completely.

Motivated by these reasons, an IT-2 F-and-C FSM controller design approach is proposed for the NMASs under the effect of uncertainties and disturbances in this research. Based on the IT-2 T-SFM and imperfect premise matching (IPM) concept [10], the IT-2 F-and-C FCs are developed, whose MFs can be designed with different rules and forms from the T-SFM. Then, Lyapunov theory is selected to implement the stability analysis. Due to the homogeneity, a simpler analysis process is proposed for the formation purpose. Consequently, the SMC approach is conveniently utilized for the IT-2 FC design to further improve the formation performance of leaders. Moreover, the passive constraint in [4] and [5] is extended into the IT-2 F-and-C FC design to solve the disturbance problem. By the linear analysis methods in [16] and [17], the containment analysis problem is solved. To evidence the advantage of the designed IT-2 F-and-C FCs with SMC, a nonlinear multiship system (NMSS) is considered for the simulation.

Compared with the related control issue in the existing research, the main contribution and novelty of this research are provided in Table I and the following statements.

1) For most of applications using M-A systems such as autonomous aerial and surface vehicles, their dynamics consist of nonlinearities. The F-and-C control for linear M-A systems in [18], [19], and [20] may not be appropriate.

2) The F-and-C control approach has been developed for NMASs in [21]. However, the F-and-C controllers are designed with nonlinear forms. This feature will increase the difficulty in practical applications.

3) and 4) Including but not limited to [18], [19], [20], and [21], the additional item in the F-and-C controllers has been established to specify the dynamic of whole M-A systems. Nevertheless, there is not a systematic design process for the control gain of this item. Referring to Remark 2 in [19], one can know that the gain is not easy to find for time-varying formation. Moreover, the additional assumption for the completion of leader's formation is required in the containment analysis of [18], [19], [20], and [21].

5) Different from the existing F-and-C control approach, the formation is achieved by the individual FSM TC of each leader in this research. Thus, the signal transmission problems between leaders, who are farthest from each other in the F-and-C problem, are efficiently avoided. The complexity of the stability analysis process is also reduced. Developing a simpler and more convenient controller design approach to ensure the performance of leaders, who are the most essential agents in the whole system, is very beneficial. Significantly, the time-varying formation and the assignment of whole system dynamics can be achieved at the same time by the FSM TC approach without an additional controller.

6) By virtue of the IT-2 T-SFM, the controller design problem for NMASs can be recast to a linear problem. It is worth noting that the analysis method for linear M-A systems in [16] can be utilized to solve the containment problem without the additional assumption. There are still hardly any papers discussing the control problem of NMASs with IT-2 T-SFM [22], [23]. However, the containment problem has only been considered in [22] and [23], which set leaders as open-loop systems. The stability of leaders is also necessary to be ensured in practical situations. By the IT-2 F-and-C FCs, stability and formation are both achieved for leaders.

The organization of this article is presented as follows. In Section II, the IT-2 T-SFM is established for NMASs. Then, the IT-2 F-and-C FCs are developed by the IPM and SMC. In Section III, the stability analyses for the F-and-C purposes are proposed. In Section IV, a simulation of the NMSS is provided. Finally, Section V concludes this article.

II. IT-2 T-SFM AND PROBLEM STATEMENTS

For NMASs with uncertainties and disturbances, the IT-2 T-SFM and the IPM IT-2 F-and-C FCs are developed in this section. Then, the F-and-C problems are also stated.

A. System Description

First, the following IT-2 T-SFM is presented for the NMASs with the problem of uncertainties and disturbances.

Model rule μ:

If $\vartheta_1^\beta(t)$ is $\tilde{\Phi}_{\mu 1}$ and $\vartheta_2^\beta(t)$ is $\tilde{\Phi}_{\mu 2}$ and …and $\vartheta_\ell^\beta(t)$ is $\tilde{\Phi}_{\mu\ell}$

$$\text{Then}\begin{cases}\dot{x}^\beta(t) = \mathbf{A}_\mu x^\beta(t) + \mathbf{B}_\mu u^\beta(t) + \mathbf{D}_\mu w^\beta(t)\\ y^\beta(t) = \mathbf{C}_\mu x^\beta(t) + \mathbf{E}_\mu w^\beta(t)\end{cases} \tag{1}$$

where $x^{\beta}(t) \in \mathbb{R}^{q}$, $u^{\beta}(t) \in \mathbb{R}^{p}$ and $y^{\beta}(t) \in \mathbb{R}^{m}$ are the state, input, and output vectors, $\mathbf{A}_{\mu}$, $\mathbf{B}_{\mu}$, $\mathbf{C}_{\mu}$, $\mathbf{D}_{\mu}$, and $\mathbf{E}_{\mu}$ are the constant matrices, $\vartheta_{\varepsilon}^{\beta}(t)$ is the premise variable, $\tilde{\Phi}_{\mu\varepsilon}$ is the IT-2 fuzzy set, $\beta = 1, 2, \ldots, \phi, \phi+1, \phi+2, \ldots, \phi+\tau$ is the agent's number, $\varepsilon = 1, 2, \ldots, \ell$ and $\mu = 1, 2, \ldots, \nu$ are the numbers of premise variable and fuzzy rule, and $w^{\beta}(t) \in \mathbb{R}^{h}$ denotes the disturbance, which is selected as the Gaussian white noise in this research. For the IT-2 T-SFM (1), the firing strength is obtained as follows by referring to [9] and [10]:

$$\mathbf{\Phi}_{\mu}\left(\vartheta^{\beta}(t)\right) = \left[\underline{\Phi}_{\mu}\left(\vartheta^{\beta}(t)\right), \bar{\Phi}_{\mu}\left(\vartheta^{\beta}(t)\right)\right] \tag{2}$$

where upper and lower bound grades of membership are denoted as

$$\underline{\Phi}_{\mu}\left(\vartheta^{\beta}(t)\right) = \prod_{\varepsilon=1}^{\ell} \underline{\Phi}_{\mu\varepsilon}\left(\vartheta_{\varepsilon}^{\beta}(t)\right) \geq 0 \tag{3}$$

$$\bar{\Phi}_{\mu}\left(\vartheta^{\beta}(t)\right) = \prod_{\varepsilon=1}^{\ell} \bar{\Phi}_{\mu\varepsilon}\left(\vartheta_{\varepsilon}^{\beta}(t)\right) \geq 0. \tag{4}$$

In (3) and (4), $\underline{\Phi}_{\mu\varepsilon}(\vartheta_{\varepsilon}^{\beta}(t))$ and $\bar{\Phi}_{\mu\varepsilon}(\vartheta_{\varepsilon}^{\beta}(t))$ are the lower and upper bound MFs satisfying the relationship $0 \leq \underline{\Phi}_{\mu\varepsilon}(\vartheta_{\varepsilon}^{\beta}(t)) \leq \bar{\Phi}_{\mu\varepsilon}(\vartheta_{\varepsilon}^{\beta}(t)) \leq 1$. $\underline{\Phi}_{\mu}(\vartheta^{\beta}(t))$ and $\bar{\Phi}_{\mu}(\vartheta^{\beta}(t))$ are lower and upper grades of membership with the relationship $0 \leq \underline{\Phi}_{\mu}(\vartheta^{\beta}(t)) \leq \bar{\Phi}_{\mu}(\vartheta^{\beta}(t)) \leq 1$. As a result, the IT-2 T-SFM (1) is inferred into the following form:

$$\dot{x}^{\beta}(t) = \sum_{\mu=1}^{\nu} \tilde{\Phi}_{\mu}\left(\vartheta^{\beta}(t)\right)\left\{\mathbf{A}_{\mu}x^{\beta}(t) + \mathbf{B}_{\mu}u^{\beta}(t) + \mathbf{D}_{\mu}w^{\beta}(t)\right\} \tag{5}$$

$$y^{\beta}(t) = \sum_{\mu=1}^{\nu} \tilde{\Phi}_{\mu}\left(\vartheta^{\beta}(t)\right)\left\{\mathbf{C}_{\mu}x^{\beta}(t) + \mathbf{E}_{\mu}w^{\beta}(t)\right\}. \tag{6}$$

IT-2 MF in the IT-2 T-SFM (5) and (6) is obtained with (3) and (4) as

$$\begin{aligned}\tilde{\Phi}_{\mu}\left(\vartheta^{\beta}(t)\right) = {} & \bar{\Omega}_{\mu}\left(\vartheta^{\beta}(t)\right)\bar{\Phi}_{\mu}\left(\vartheta^{\beta}(t)\right) \\ & + \underline{\Omega}_{\mu}\left(\vartheta^{\beta}(t)\right)\underline{\Phi}_{\mu}\left(\vartheta^{\beta}(t)\right).\end{aligned} \tag{7}$$

where $\sum_{\mu=1}^{\nu} \tilde{\Phi}_{\mu}(\vartheta^{\beta}(t)) = 1$, $\bar{\Omega}_{\mu}(\vartheta^{\beta}(t))$, and $\underline{\Omega}_{\mu}(\vartheta^{\beta}(t))$ are the nonlinear functions related to the uncertainties, which are not required to be known, $\underline{\Omega}_{\mu}(\vartheta^{\beta}(t)) + \bar{\Omega}_{\mu}(\vartheta^{\beta}(t)) = 1$, $0 \leq \bar{\Omega}_{\mu}(\vartheta^{\beta}(t)) \leq 1$, and $0 \leq \underline{\Omega}_{\mu}(\vartheta^{\beta}(t)) \leq 1$.

With the IT-2 T-SFM (5)–(7), the IT-2 FCs can be developed for NMASs to achieve the requirement of F-and-C. For followers, the interaction relationship in the IT-2 containment FC (CFC) is defined according to the graph theory in the following definition.

Definition 1: Considering an undirected graph $\Theta = (\aleph, \mathfrak{C}, \mathbb{A})$, the node set and edge set are respectively represented as $\aleph = \{\mathfrak{N}_1, \mathfrak{N}_2, \ldots, \mathfrak{N}_{\phi+\tau}\}$ and $\mathfrak{C} \subseteq \aleph \times \aleph$ so that $(\mathfrak{N}_{\beta}, \mathfrak{N}_{\alpha}) \in \mathfrak{C}$ denotes there is an edge between the nodes $\mathfrak{N}_{\beta}$ and $\mathfrak{N}_{\alpha}$. The node set $\mathfrak{A} = \{\mathfrak{N}_{\alpha} \in \aleph : (\mathfrak{N}_{\beta}, \mathfrak{N}_{\alpha}) \in \mathfrak{C}\}$ is defined for the neighbors of node $\mathfrak{N}_{\beta}$. Based on the definition of node and edge sets, the adjacency matrix $\mathbb{A} = [\mathfrak{a}_{\beta\alpha}] \in \mathbb{R}^{(\phi+\tau)\times(\phi+\tau)}$, whose element values $\mathfrak{a}_{\beta\alpha} = 1$ and $\mathfrak{a}_{\beta\alpha} = 0$, respectively, denote the situations $(\mathfrak{N}_{\beta}, \mathfrak{N}_{\alpha}) \in \mathfrak{C}$ and $(\mathfrak{N}_{\beta}, \mathfrak{N}_{\alpha}) \notin \mathfrak{C}$, is constructed to describe the relationship of all nodes. The degree matrix $\mathbb{D} = [d_{\beta\beta}] \in \mathbb{R}^{(\phi+\tau)\times(\phi+\tau)}$, in which $d_{\beta\beta} = \sum_{\alpha=1}^{\phi+\tau} \mathfrak{a}_{\beta\alpha}$ and $\alpha \neq \beta$, is also obtained. Therefore, the Laplacian matrix is obtained with $\mathbf{L} = \mathbb{D} - \mathbb{A}$.

Note that nodes in $\aleph$ and edges in $\mathfrak{C}$ of the graph Θ, respectively, indicate each agent and the interaction between agents. In the NMASs, the total number ϕ of leaders, which are labeled from 1 to ϕ, is considered. Otherwise, the followers are labeled from $\phi+1$ to $\phi+\tau$ with total number τ. Then, the Laplacian matrix is further constructed as

$$\mathbf{L} = \begin{bmatrix} \mathbf{0}_{\phi\times\phi} & \mathbf{0}_{\phi\times\tau} \\ \mathbf{L}_{FL} & \mathbf{L}_{FF} \end{bmatrix} \in \mathbb{R}^{(\phi+\tau)\times(\phi+\tau)} \tag{8}$$

where matrices $\mathbf{L}_{FF}$ and $\mathbf{L}_{FL}$ represent the interaction topology among all followers and from leaders to followers, $\mathbf{0}_{\phi\times\phi}$ and $\mathbf{0}_{\phi\times\tau}$ are the zero matrices.

Then, the Jordan canonical of matrix $\mathbf{L}_{FF}$ is defined with $\Im_{FF} = \mathbf{T}^{-1}\mathbf{L}_{FF}\mathbf{T}$, in which $\mathbf{T}$ is a nonsingular matrix. Besides, the eigenvalues in each diagonal position of $\Im_{FF}$ is defined as λ^{β} for $\beta = \phi+1, \phi+2, \ldots, \phi+\tau$.

In this research, the IT-2 T-SFM is also established as follows for the target trajectory of leaders in the NMASs:

$$\dot{x}_{d}^{\beta}(t) = \sum_{\mu=1}^{\nu} \tilde{\Phi}_{\mu}\left(\vartheta^{\beta}(t)\right)\left\{\mathbf{A}_{\mu}x_{d}^{\beta}(t)\right\} \tag{9}$$

$$y_{d}^{\beta}(t) = \sum_{\mu=1}^{\nu} \tilde{\Phi}_{\mu}\left(\vartheta^{\beta}(t)\right)\left\{\mathbf{C}_{\mu}x_{d}^{\beta}(t)\right\} \tag{10}$$

where $x_{\mathrm{d}}^{\beta}(t)$ and $y_{\mathrm{d}}^{\beta}(t)$ denote the target trajectory for the states and outputs.

Dividing the IT-2 T-SFM (5)–(7), the IT-2 T-SFM of leaders and followers can be represented as follows. For leaders, the tracking model is obtained by subtracting the target models (9) and (10) from the IT-2 T-SFM (5) and (6).

$$\dot{\tilde{x}}^{\beta}(t) = \sum_{\mu=1}^{\nu} \tilde{\Phi}_{\mu}\left(\vartheta^{\beta}(t)\right)\left\{\mathbf{A}_{\mu}\tilde{x}^{\beta}(t) + \mathbf{B}_{\mu}u^{\beta}(t) + \mathbf{D}_{\mu}w^{\beta}(t)\right\} \tag{11}$$

$$\tilde{y}^{\beta}(t) = \sum_{\mu=1}^{\nu} \tilde{\Phi}_{\mu}\left(\vartheta^{\beta}(t)\right)\left\{\mathbf{C}_{\mu}\tilde{x}^{\beta}(t) + \mathbf{E}_{\mu}w^{\beta}(t)\right\} \tag{12}$$

where $\beta = 1, 2, \ldots, \phi$, $\tilde{x}^{\beta}(t) = x^{\beta}(t) - x_{\mathrm{d}}^{\beta}(t)$ and $\tilde{y}^{\beta}(t) = y^{\beta}(t) - y_{\mathrm{d}}^{\beta}(t)$ denote the tracking error of states and outputs. For followers, the IT-2 T-SFM is also presented as follows:

$$\dot{x}^{\beta}(t) = \sum_{\mu=1}^{\nu} \tilde{\Phi}_{\mu}\left(\vartheta^{\beta}(t)\right)\left\{\mathbf{A}_{\mu}x^{\beta}(t) + \mathbf{B}_{\mu}u^{\beta}(t) + \mathbf{D}_{\mu}w^{\beta}(t)\right\} \tag{13}$$

$$y^{\beta}(t) = \sum_{\mu=1}^{\nu} \tilde{\Phi}_{\mu}\left(\vartheta^{\beta}(t)\right)\left\{\mathbf{C}_{\mu} x^{\beta}(t) + \mathbf{E}_{\mu} w^{\beta}(t)\right\} \tag{14}$$

where $\beta = \phi+1, \phi+2, \ldots, \phi+\tau$.

Then, the IT-2 formation FC (FFC) and CFC can be proposed for leaders and followers of the NMASs by using the FSM TC approach. In the rest of the research, the premise variables in the MFs are omitted and $\tilde{\Phi}_{\mu}^{\beta}$ is applied to replace $\tilde{\Phi}_{\mu}(\vartheta^{\beta}(t))$. For all the other MFs, the expression is also changed in the same manner.

B. IT-2 F-and-C FCs

Using the IPM design concept [10], the different fuzzy rule numbers and forms of IT-2 MF can be selected for the IT-2 FC design. Because of this reason, the IT-2 F-and-C FCs for the leaders and followers are presented as follows.

Leader's FFC rule η:

If $\vartheta_1^{\beta}(t)$ is $\tilde{\Psi}_{\eta 1}$ and $\vartheta_2^{\beta}(t)$ is $\tilde{\Psi}_{\eta 2}$ and …and $\vartheta_{\ell}^{\beta}(t)$ is $\tilde{\Psi}_{\eta \ell}$

$$\text{Then } u^{\beta}(t) = u_{eq}^{\beta}(t) + u_{s}^{\beta}(t) \tag{15}$$

where $\beta = 1, 2, \ldots, \phi$, $u_{eq}^{\beta}(t)$ and $u_{s}^{\beta}(t)$, i.e., the controller of sliding motion and the controller to force system dynamic to the sliding surface, are designed as follows:

$$u_{eq}^{\beta}(t) = \mathbf{F}_{\eta} \tilde{x}^{\beta}(t) \tag{16}$$

$$u_{s}^{\beta}(t) = -\left(\sum_{\mu=1}^{\nu} \bar{\Phi}_{\mu}^{\beta} \mathbf{S} \mathbf{B}_{\mu}\right)^{-1} \cdot \delta \cdot \operatorname{sgn}\left(S^{\beta}(t)\right) \tag{17}$$

in which the sliding surface is defined as

$$S^{\beta}(t) = \mathbf{S}\left(\tilde{x}^{\beta}(t) - \int_0^t \mathfrak{D} \tilde{x}^{\beta}(\mathfrak{s})\, d\mathfrak{s}\right) \tag{18}$$

where $\mathfrak{D} = \sum_{\mu=1}^{\nu} \sum_{\eta=1}^{\varsigma} \tilde{\Phi}_{\mu}^{\beta} \tilde{\Psi}_{\eta}^{\beta} \{\mathbf{A}_{\mu} + \mathbf{B}_{\mu} \mathbf{F}_{\eta}\}$ and $\tilde{\Psi}_{\eta}^{\beta}$ will be defined in the latter context.

Follower's CFC rule η:

If $\vartheta_1^{\beta}(t)$ is $\tilde{\Psi}_{\eta 1}$ and $\vartheta_2^{\beta}(t)$ is $\tilde{\Psi}_{\eta 2}$ and …and $\vartheta_{\ell}^{\beta}(t)$ is $\tilde{\Psi}_{\eta \ell}$

$$\text{Then } u^{\beta}(t) = \mathbf{K}_{\eta} \sum_{\alpha \in \mathfrak{A}} \mathfrak{a}_{\beta\alpha}\left(x^{\beta}(t) - x^{\alpha}(t)\right) \tag{19}$$

where $\beta = \phi+1, \phi+2, \ldots, \phi+\tau$, $\mathbf{F}_{\eta}$ and $\mathbf{K}_{\eta}$ are the control gains, $\mathbf{S}$ is the constant matrix such that $\mathbf{SB}_{\mu}$ is ensured to be positive definite, δ is the sliding gain, and $\eta = 1, 2, \ldots, \varsigma$ is the rule number of FC. Thus, the firing strength for IT-2 F-and-C FCs (15)–(19) is defined as follows:

$$\boldsymbol{\Psi}_{\eta}^{\beta} = \left[\underline{\Psi}_{\eta}^{\beta}, \bar{\Psi}_{\eta}^{\beta}\right] \tag{20}$$

where lower and upper grades of membership are

$$\underline{\Psi}_{\eta}^{\beta} = \prod_{\varepsilon=1}^{\ell} \underline{\Psi}_{\eta\varepsilon}^{\beta} \geq 0 \text{ and } \bar{\Psi}_{\eta}^{\beta} = \prod_{\varepsilon=1}^{\ell} \bar{\Psi}_{\eta\varepsilon}^{\beta} \geq 0. \tag{21}$$

The MFs $\bar{\Psi}_{\eta\varepsilon}^{\beta}$ and $\underline{\Psi}_{\eta\varepsilon}^{\beta}$ also satisfy the condition $0 \leq \underline{\Psi}_{\eta\varepsilon}^{\beta} \leq \bar{\Psi}_{\eta\varepsilon}^{\beta} \leq 1$. Then, the inferred IT-2 F-and-C FCs are obtained for (15)–(19) by (21) as follows:

For $\beta = 1, 2, \ldots, \phi$, one has

$$u^{\beta}(t) = \sum_{\eta=1}^{\varsigma} \tilde{\Psi}_{\eta}^{\beta}\left\{u_{eq}^{\beta}(t) + u_{s}^{\beta}(t)\right\} \tag{22}$$

For $\beta = \phi+1, \phi+2, \ldots, \phi+\tau$, one has

$$u^{\beta}(t) = \sum_{\eta=1}^{\varsigma} \tilde{\Psi}_{\eta}^{\beta}\left\{\mathbf{K}_{\eta} \sum_{\alpha \in \mathfrak{A}} \mathfrak{a}_{\beta\alpha}\left(x^{\beta}(t) - x^{\alpha}(t)\right)\right\} \tag{23}$$

where $\tilde{\Psi}_{\eta}^{\beta} = \frac{\bar{\mho}_{\eta}^{\beta}\bar{\Psi}_{\eta}^{\beta} + \underline{\mho}_{\eta}^{\beta}\underline{\Psi}_{\eta}^{\beta}}{\sum_{\kappa=1}^{\varsigma}\bar{\mho}_{\kappa}^{\beta}\bar{\Psi}_{\kappa}^{\beta} + \underline{\mho}_{\kappa}^{\beta}\underline{\Psi}_{\kappa}^{\beta}}$ and $\sum_{\eta=1}^{\varsigma} \tilde{\Psi}_{\eta}^{\beta} = 1$. In (22) and (23), $\bar{\mho}_{\eta}^{\beta}$ and $\underline{\mho}_{\eta}^{\beta}$ are predefined functions with $0 \leq \bar{\mho}_{\eta}^{\beta} \leq 1$, $0 \leq \underline{\mho}_{\eta}^{\beta} \leq 1$, and $\underline{\mho}_{\eta}^{\beta} + \bar{\mho}_{\eta}^{\beta} = 1$.

By the IT-2 CFC (23), the containment purpose for followers is achieved with the interaction relationship, which satisfies the following assumption, among all agents.

Assumption 1: There is not less than one interaction from each leader to followers.

If Assumption 1 is satisfied for agents in the NMASs, a lemma is also given for the Laplacian matrix (8) as follows.

Lemma 1 (see [22] and [23]): All the eigenvalues of matrix $\mathbf{L}_{FF}$ consist of the positive real part. The sum of each row in $-\mathbf{L}_{FF}^{-1}\mathbf{L}_{FL}$, whose elements are nonnegative, is equal to one.

Therefore, the following closed-loop IT-2 T-SFMs are obtained by, respectively, substituting the control inputs (22) and (23) into IT-2 T-SFMs (11)–(14). For leaders, one can obtain

$$\begin{aligned}\dot{\tilde{x}}^{L}(t) = \sum_{\mu=1}^{\nu}\sum_{\eta=1}^{\varsigma} \tilde{\Lambda}_{\mu\eta}^{L}\{&(\mathbf{I}_L \otimes (\mathbf{A}_{\mu} + \mathbf{B}_{\mu}\mathbf{F}_{\eta}))\tilde{x}^{L}(t)\\ &+ (\mathbf{I}_L \otimes \mathbf{B}_{\mu}) u_s^{L}(t) + (\mathbf{I}_L \otimes \mathbf{D}_{\mu}) w^{L}(t)\}\end{aligned} \tag{24}$$

$$\tilde{y}^{L}(t) = \sum_{\mu=1}^{v} \tilde{\Phi}_{\mu}^{L}\{(\mathbf{I}_L \otimes \mathbf{C}_{\mu})\tilde{x}^{L}(t) + (\mathbf{I}_L \otimes \mathbf{E}_{\mu}) w^{L}(t)\} \tag{25}$$

where $\tilde{\Lambda}_{\mu\eta}^{L} = \tilde{\Phi}_{\mu}^{L}\tilde{\Psi}_{\eta}^{L}$ with $\sum_{\mu=1}^{v}\sum_{\eta=1}^{\varsigma}\tilde{\Lambda}_{\mu\eta}^{L} = 1$, $\mathbf{I}_L$ is the identity matrix whose dimension is related to leader's number, $\otimes$ is the Kronecker product, $\Bbbk^{L}(t) = [\Bbbk^{1}(t) \;\; \Bbbk^{2}(t) \;\; \cdots \;\; \Bbbk^{\phi}(t)]^{\mathrm{T}}$, and $\Bbbk^{L}(t)$ denotes the vectors $\tilde{x}^{L}(t)$, $u_s^{L}(t)$, $\tilde{y}^{L}(t)$, $w^{L}(t)$, $x^{L}(t)$. For followers, the following IT-2 T-SFM is obtained:

$$\begin{aligned}\dot{x}^{F}(t) = \sum_{\mu=1}^{\nu}\sum_{\eta=1}^{\varsigma} \tilde{\Lambda}_{\mu\eta}^{F}\{&(\mathbf{I}_F \otimes \mathbf{A}_{\mu} + \mathbf{L}_{FF} \otimes \mathbf{B}_{\mu}\mathbf{K}_{\eta}) x^{F}(t)\\ &+ (\mathbf{L}_{FL} \otimes \mathbf{B}_{\mu}\mathbf{K}_{\eta}) x^{L}(t) + (\mathbf{I}_F \otimes \mathbf{D}_{\mu}) w^{F}(t)\}\end{aligned} \tag{26}$$

$$y^{F}(t) = \sum_{\mu=1}^{\nu} \tilde{\Phi}_{\mu}^{F}\{(\mathbf{I}_F \otimes \mathbf{C}_{\mu}) x^{F}(t) + (\mathbf{I}_F \otimes \mathbf{E}_{\mu}) w^{F}(t)\} \tag{27}$$

where $\tilde{\Lambda}_{\mu\eta}^{F} = \tilde{\Phi}_{\mu}^{F}\tilde{\Psi}_{\eta}^{F}$, $\mathbf{I}_F$ is the identity matrix whose dimension is related to the follower's number, $\Bbbk^{F}(t) = [\Bbbk^{\phi+1}(t) \;\; \Bbbk^{\phi+2}(t) \;\; \cdots \;\; \Bbbk^{\phi+\tau}(t)\;]^{\mathrm{T}}$, and $\Bbbk^{F}(t)$ denotes the vectors $x^{F}(t)$, $y^{F}(t)$, $w^{F}(t)$.

Dealing with the disturbance in the IT-2 T-SFMs (24)–(27), the passive constraint in [4], [5] is extended as follows.

Lemma 2: The closed-loop IT-2 T-SFM (24) and (25) for leaders is strictly input passive if the following relationship for all $t_p \geq 0$ and $w^\beta(t) \neq 0$ is satisfied with the given positive scalar γ^L and matrix $\mathbf{H}^L$.

$$2\int_0^{t_p} \tilde{y}^{\beta^{\mathrm{T}}}(t)\,\mathbf{H}^L w^\beta(t)\,dt > \int_0^{t_p} \gamma^L w^{\beta^{\mathrm{T}}}(t)\,w^\beta(t)\,dt. \quad for\ \beta = 1, 2, \ldots, \phi \tag{28}$$

Lemma 3: The closed-loop IT-2 T-SFM (26) and (27) for followers is strictly input passive if the following relationship for all $t_p \geq 0$ and $w^F(t) \neq 0$ is satisfied with the given positive scalar γ^F and matrix $\mathbf{H}^F$:

$$2\int_0^{t_p} y_e^{F^{\mathrm{T}}}(t)\left(\mathbf{I}_F \otimes \mathbf{H}^F\right) w^F(t)\,dt > \int_0^{t_p} \gamma^F w^{F^{\mathrm{T}}}(t)\,w^F(t)\,dt \tag{29}$$

where $y_e^F(t)$ will be introduced in the later context. By satisfying the passive constraints (28) and (29), the disturbance energy can be dissipated to better complete the F-and-C tasks. Thus, the F-and-C problems are stated as follows.

Problem 1: If the stability of the tracking error $\tilde{x}^\beta(t)$ is guaranteed, then the formation task can be completed by assigning different target trajectories for each leader. Referring to [16], the containment task of followers is completed if there exists the gain $\mathbf{K}_\eta$ such that the upper bound for the peak-to-peak gain from $u^L(t)$ to the containment error, which is defined as $e^F(t)$, is minimized.

However, the conservative result is also caused due to the minimization and passive constraint even if the containment problem in Problem 1 is solved. By considering the IT-2 FC design approach in [10] and the following analysis approach for linear M-A systems in [17], the analysis process of IT-2 CFC design in this research can be efficiently relaxed.

Lemma 4 (see [17]): Rearranging all the eigenvalues λ^β in diagonal of the matrix $\Im_{FF}$ in accordance with the sequence $\mathrm{Re}\{\lambda^{\phi+1}\} < \mathrm{Re}\{\lambda^{\phi+2}\} < \ldots < \mathrm{Re}\{\lambda^{\phi+\tau}\}$, if the condition $\mathbf{O}_1 + \mathrm{Re}\{\tilde{\lambda}^\varpi\}\mathbf{O}_2 + \mathrm{Im}\{\tilde{\lambda}^\varpi\}\mathbf{O}_3 < 0$ for $\varpi = 1, 2, 3, 4$ is satisfied, where $\mathrm{Re}\{\cdot\}$ and $\mathrm{Im}\{\cdot\}$ are the real part and imaginary part of element $\cdot$, then the condition $\mathbf{O}_1 + \mathrm{Re}\{\lambda^\beta\}\mathbf{O}_2 + \mathrm{Im}\{\lambda^\beta\}\mathbf{O}_3 < 0$ for $\beta = \phi+1, \phi+2, \ldots, \phi+\tau$ is also ensured. Note that $\tilde{\lambda}^{1,2} = \mathrm{Re}\{\lambda^{\phi+1}\} \pm j\max\mathrm{Im}\{\lambda^\beta\}$, $\tilde{\lambda}^{3,4} = \mathrm{Re}\{\lambda^{\phi+\tau}\} \pm j\max\mathrm{Im}\{\lambda^\beta\}$ for $\beta = \phi+1, \phi+2, \ldots, \phi+\tau$, and $\mathbf{O}_1$, $\mathbf{O}_2$, and $\mathbf{O}_3$ are the real symmetric matrices.

III. IT-2 F-and-C FC Design and Stability Analysis

In accordance with the IT-2 T-SFMs (24)–(27) and passive constraints (28) and (29), an IT-2 F-and-C FC design approach is proposed for NMASs in this section. First, the IT-2 FFC design process is presented as follows.

Theorem 1: If the following condition is ensured, then the tracking errors of IT-2 T-SFM (24) and (25) can be forced to the sliding surface with the FSM TC (22):

$$\delta > \left\|\sum_{\mu=1}^{\nu} \tilde{\Phi}_\mu^\beta \mathbf{S}\mathbf{D}_\mu\right\| \cdot \left\|w^\beta(t)\right\|. \tag{30}$$

Referring to [5], the analysis process of Theorem 1 is provided in Appendix. By Theorem 1, the tracking errors can follow the dynamic of the sliding surface, which is controlled by the following theorem.

Theorem 2: If there exists the matrix $\mathbf{U}_\eta$, the positive definite matrices $\mathbf{G}_L$ and $\mathbf{Y}_{\mu\eta}^L$, the symmetric matrix $\mathbf{Z}^L$ such that the following sufficient conditions are satisfied with the given scalars γ^L, $\bar{\sigma}_{\mu\eta i_1 i_2\cdots i_\ell\rho}^L$, $\underline{\sigma}_{\mu\eta i_1 i_2\cdots i_\ell\rho}^L$, and the given matrix $\mathbf{H}^L$, the IT-2 T-SFM (24) and (25) is stable in the mean square and the formation task in Problem 1 is completed for leaders.

$$\mathbf{G}_L, \mathbf{Y}_{\mu\eta}^L > 0 \tag{31}$$

$$\mathbf{\Gamma}_{\mu\eta} - \mathbf{Y}_{\mu\eta}^L + \mathbf{Z}^L < 0 \qquad \text{for all } \mu, \eta \tag{32}$$

$$\sum_{\mu=1}^{v}\sum_{\eta=1}^{\varsigma}\left(\underline{\sigma}_{\mu\eta i_1 i_2\cdots i_\ell\rho}^L \mathbf{\Gamma}_{\mu\eta} - \left(\underline{\sigma}_{\mu\eta i_1 i_2\cdots i_\ell\rho}^L - \bar{\sigma}_{\mu\eta i_1 i_2\cdots i_\ell\rho}^L\right)\mathbf{Y}_{\mu\eta}^L + \underline{\sigma}_{\mu\eta i_1 i_2\cdots i_\ell\rho}^L \mathbf{Z}^L\right) - \mathbf{Z}^L < 0 \,\text{for all}\, i_1, i_2, \ldots, i_\ell, \rho \tag{33}$$

where $\underline{\sigma}_{\mu\eta i_1 i_2\cdots i_\ell\rho}^L$ and $\bar{\sigma}_{\mu\eta i_1 i_2\cdots i_\ell\rho}^L$ are the constant parameters depending on the upper and lower bound IT-2 MFs, which will be introduced in the derivation process of proof,

$$\mathbf{\Gamma}_{\mu\eta} = \begin{bmatrix} \mathbf{A}_\mu\mathbf{G}_L + \mathbf{G}_L\mathbf{A}_\mu^{\mathrm{T}} + \mathbf{B}_\mu\mathbf{U}_\eta + \mathbf{U}_\eta^{\mathrm{T}}\mathbf{B}_\mu^{\mathrm{T}} & * \\ \mathbf{D}_\mu^{\mathrm{T}} - \mathbf{H}^L\mathbf{C}_\mu\mathbf{G}_L & \gamma^L\mathbf{I}_h - \mathbf{E}_\mu^{\mathrm{T}}\mathbf{H}^L - \mathbf{H}^L\mathbf{E}_\mu \end{bmatrix},$$

$$\mathbf{U}_\eta = \mathbf{F}_\eta\mathbf{G}_L, \mathbf{G}_L = \mathbf{P}_L^{-1},$$

and $*$ is the transport item in the matrix.

Proof

It is worth noting that the stability analysis process only needs to be developed for one leader because of the homogeneity. Therefore, the candidate Lyapunov function for leader 1 is first selected as follows:

$$\mathrm{V}^1(t) = \tilde{x}^{1^{\mathrm{T}}}(t)\,\mathbf{P}_L\tilde{x}^1(t) \tag{34}$$

Then, the following differential of (34) is derived.

$$\dot{\mathrm{V}}^1(t) = \sum_{\mu=1}^{\nu}\sum_{\eta=1}^{\varsigma}\tilde{\Lambda}_{\mu\eta}^1 \times \left\{\tilde{\mathbf{X}}^{1^{\mathrm{T}}}(t) \begin{bmatrix} \mathbf{P}_L\mathbf{A}_\mu + \mathbf{A}_\mu^{\mathrm{T}}\mathbf{P}_L + \mathbf{P}_L\mathbf{B}_\mu\mathbf{F}_\eta + \mathbf{F}_\eta^{\mathrm{T}}\mathbf{B}_\mu^{\mathrm{T}}\mathbf{P}_L & * \\ \mathbf{D}_\mu^{\mathrm{T}}\mathbf{P}_L & 0 \end{bmatrix} \tilde{\mathbf{X}}^1(t)\right\} \tag{35}$$

where $\tilde{\mathbf{X}}^1(t) = \left[\tilde{x}^{1^{\mathrm{T}}}(t) \quad w^{1^{\mathrm{T}}}(t)\right]^{\mathrm{T}}$. Then, the passive constraint is considered for the design of IT-2 FFC. Referring to (28), the following cost function is defined for leader 1:

$$\int_0^{t_p}\left(\gamma^L w^{1^{\mathrm{T}}}(t)\,w^1(t) - 2y^{1^{\mathrm{T}}}(t)\,\mathbf{H}^L w^1(t)\right)dt. \tag{36}$$

From (35) and (36), the following relationship with passive performance can be obtained.

$$\begin{aligned}&\int_0^{t_p}\left(\gamma^L w^{1^{\mathrm{T}}}(t)\,w^1(t)-2y^{1^{\mathrm{T}}}(t)\,\mathbf{H}^L w^1(t)\right)dt\\&=\int_0^{t_p}\left(\gamma^L w^{1^{\mathrm{T}}}(t)\,w^1(t)-2y^{1^{\mathrm{T}}}(t)\,\mathbf{H}^L w^1(t)+\dot{\mathrm{V}}(t)\right)dt-\mathrm{V}(t)\\&\le\int_0^{t_p}\gamma^L w^{1^{\mathrm{T}}}(t)\,w^1(t)-2y^{1^{\mathrm{T}}}(t)\,\mathbf{H}^L w^1(t)+\dot{\mathrm{V}}(t)\,dt.\end{aligned}\tag{37}$$

Then, the integrand in the right-hand side of inequality (37) is derived as (38) shown at the bottom of this page.

To combine the information of IT-2 MF into the analysis process, the following definition is provided:

$$\tilde{\Lambda}^1_{\mu\eta}=\tilde{\Phi}^1_\mu\tilde{\Psi}^1_\eta=\underline{\psi}_{\mu\eta}\left(\vartheta^1(t)\right)\bar{\Lambda}^1_{\mu\eta}+\bar{\psi}_{\mu\eta}\left(\vartheta^1(t)\right)\underline{\Lambda}^1_{\mu\eta}\tag{39}$$

where $\underline{\psi}_{\mu\eta}(\vartheta^1(t))$ and $\bar{\psi}_{\mu\eta}(\vartheta^1(t))$ are also the functions not necessary to be known due to $\bar{\Omega}^1_\mu$ and $\underline{\Omega}^1_\mu$, and $0\le\underline{\Lambda}^1_{\mu\eta}\le\bar{\Lambda}^1_{\mu\eta}\le 1$. Then, the MFs in (39) can be further constructed with the following form:

$$\underline{\Lambda}^1_{\mu\eta}=\sum_{\rho=1}^{r}\sum_{i_1=1}^{2}\cdots\sum_{i_\ell=1}^{2}\prod_{\varepsilon=1}^{\ell}\mathfrak{X}_{\varepsilon i_\varepsilon\rho}\left(\vartheta^1_\varepsilon(t)\right)\underline{\sigma}^L_{\mu\eta i_1 i_2\cdots i_\ell\rho}\tag{40}$$

$$\bar{\Lambda}^1_{\mu\eta}=\sum_{\rho=1}^{r}\sum_{i_1=1}^{2}\cdots\sum_{i_\ell=1}^{2}\prod_{\varepsilon=1}^{\ell}\mathfrak{X}_{\varepsilon i_\varepsilon\rho}\left(\vartheta^1_\varepsilon(t)\right)\bar{\sigma}^L_{\mu\eta i_1 i_2\cdots i_\ell\rho}\tag{41}$$

which satisfy the situations $0\le\underline{\sigma}^L_{\mu\eta i_1 i_2\cdots i_\ell\rho}\le\bar{\sigma}^L_{\mu\eta i_1 i_2\cdots i_\ell\rho}\le 1$, $0\le\mathfrak{X}_{\varepsilon i_\varepsilon\rho}(\vartheta^1_\varepsilon(t))\le 1$, $\mathfrak{X}_{\varepsilon 1\rho}(\vartheta^1_\varepsilon(t))+\mathfrak{X}_{\varepsilon 2\rho}(\vartheta^1_\varepsilon(t))=1$for all $i_\varepsilon=1,2$ and $\varepsilon=1,2,\ldots,\ell$, where $\mathfrak{X}_{\varepsilon i_g\rho}(\vartheta^1_\varepsilon(t))$ is the so-called cross term, which is not related to fuzzy rules μ and η. The parameter ρ is designed according to the r connected substate space, which is defined as $\mathbb{S}_\rho$. The state space of interest is obtained with $\mathbb{S}=\cup^r_{\rho=1}\mathbb{S}_\rho$. The detailed information can be referred to [10]. Then, (38) is inferred as follows with (39) by multiplying diag$(\mathbf{G}_L,\mathbf{I}_h)$ on both sides, where diag$(\cdot)$ denotes the diagonal matrix with the elements $\cdot$.

$$\sum_{\mu=1}^{\nu}\sum_{\eta=1}^{\varsigma}\left(\underline{\psi}^1_{\mu\eta}\bar{\Lambda}^1_{\mu\eta}+\bar{\psi}^1_{\mu\eta}\underline{\Lambda}^1_{\mu\eta}\right)\left\{\tilde{\mathbf{X}}^{1^{\mathrm{T}}}(t)\,\mathbf{\Gamma}_{\mu\eta}\tilde{\mathbf{X}}^1(t)\right\}.\tag{42}$$

Slack matrices are also introduced with the following form as

$$\begin{aligned}&\left(\sum_{\mu=1}^{\nu}\sum_{\eta=1}^{\varsigma}\underline{\psi}^1_{\mu\eta}\bar{\Lambda}^1_{\mu\eta}+\bar{\psi}^1_{\mu\eta}\underline{\Lambda}^1_{\mu\eta}-1\right)\mathbf{Z}^L=0\\&-\sum_{\mu=1}^{\nu}\sum_{\eta=1}^{\varsigma}\left(1-\bar{\psi}_{\mu\eta}\right)\left(\underline{\Lambda}_{\mu\eta}-\bar{\Lambda}_{\mu\eta}\right)\mathbf{Y}^L_{\mu\eta}\ge 0.\end{aligned}\tag{43}$$

Thus, the relationship is obtained from (42) and (43) as

$$\begin{aligned}&\sum_{\mu=1}^{\nu}\sum_{\eta=1}^{\varsigma}\left(\underline{\psi}^1_{\mu\eta}\bar{\Lambda}^1_{\mu\eta}+\bar{\psi}^1_{\mu\eta}\underline{\Lambda}^1_{\mu\eta}\right)\left\{\tilde{\mathbf{X}}^{1^{\mathrm{T}}}(t)\,\mathbf{\Gamma}_{\mu\eta}\tilde{\mathbf{X}}^1(t)\right\}\le\tilde{\mathbf{X}}^{1\mathrm{T}}(t)\\&\left(\sum_{\mu=1}^{\nu}\sum_{\eta=1}^{\varsigma}\left(\underline{\Lambda}^1_{\mu\eta}\mathbf{\Gamma}_{\mu\eta}-\left(\underline{\Lambda}^1_{\mu\eta}-\bar{\Lambda}^1_{\mu\eta}\right)\mathbf{Y}^L_{\mu\eta}+\underline{\Lambda}^1_{\mu\eta}\mathbf{Z}^L\right)-\mathbf{Z}^L\right)\tilde{\mathbf{X}}^1(t)\\&-\sum_{\mu=1}^{\nu}\sum_{\eta=1}^{\varsigma}\bar{\psi}\left(\underline{\Lambda}^1_{\mu\eta}-\bar{\Lambda}^1_{\mu\eta}\right)\left\{\tilde{\mathbf{X}}^{1^{\mathrm{T}}}(t)\left(\mathbf{\Gamma}_{\mu\eta}-\mathbf{Y}^L_{\mu\eta}+\mathbf{Z}^L\right)\tilde{\mathbf{X}}^1(t)\right\}.\end{aligned}\tag{44}$$

Based on the representation of (40) and (41), the first item on the right-hand side of (44) is further expressed as follows by extracting the cross term:

$$\begin{aligned}&\sum_{\rho=1}^{r}\sum_{i_1=1}^{2}\cdots\sum_{i_\ell=1}^{2}\prod_{\varepsilon=1}^{\ell}\mathfrak{X}^1_{\varepsilon i_\varepsilon\rho}\left(\underline{\sigma}^L_{\mu\eta i_1 i_2\cdots i_\ell\rho}\mathbf{\Gamma}_{\mu\eta}\right.\\&\left.-\left(\underline{\sigma}^L_{\mu\eta i_1 i_2\cdots i_\ell\rho}-\bar{\sigma}^L_{\mu\eta i_1 i_2\cdots i_\ell\rho}\right)\mathbf{Y}^L_{\mu\eta}+\underline{\sigma}^L_{\mu\eta i_1 i_2\cdots i_\ell\rho}\mathbf{Z}^L\right)-\mathbf{Z}^L.\end{aligned}\tag{45}$$

According to the definition of (39)–(41) and the relationship (44) and (45), the negative definiteness of (42) can be ensured if the sufficient conditions (31)–(33) are satisfied by Theorem 2. The cost function (36) is also ensured to be negative because of the inequality (37). Consequently, the passive performance constraint (28) for leader 1 is achieved. Besides, the stability of leader 1 in the IT-2 T-SFM (24) and (25) can also be proved by ensuring (35) to be negative with the negative definiteness of (42) when the disturbance $w^1(t)=0$ is set. The analysis method is similar to the type-1 passive FC design approach in [4] and [5] and is not going to be provided in this article.

Therefore, better stability and resistance to the effect of uncertainties and disturbances can be ensured for the tracking error dynamic of leader 1 by Theorems 1 and 2. Notably, the stability of tracking error, passive performance, and the convergence to the sliding surface of all the other leaders can also be achieved by Theorems 1 and 2. If the leaders have the ability to track the target trajectories, the formation in Problem 1 is obviously achieved by designing the different target trajectories for each leader.

Remark 1: For the F-and-C control approach, the function of leaders is to construct a specific convex region such that all the followers can be contained. In other words, if better control performances can be ensured for leaders, the performance of followers, whose dynamic behaviors heavily depend on leaders, are certainly ensured. Thus, a simpler and more convenient controller design scheme for leaders can provide many benefits

$$\sum_{\mu=1}^{\nu}\sum_{\eta=1}^{\varsigma}\tilde{\Lambda}^1_{\mu\eta}\tilde{\mathbf{X}}^{1^{\mathrm{T}}}(t)\begin{bmatrix}\mathbf{P}_L\mathbf{A}_\mu+\mathbf{A}^{\mathrm{T}}_\mu\mathbf{P}_L+\mathbf{P}_L\mathbf{B}_\mu\mathbf{F}_\eta+\mathbf{F}^{\mathrm{T}}_\eta\mathbf{B}^{\mathrm{T}}_\mu\mathbf{P}_L & *\\ \mathbf{D}^{\mathrm{T}}_\mu\mathbf{P}_L-\mathbf{H}^L\mathbf{C}_\mu & \gamma^L\mathbf{I}-\mathbf{E}^{\mathrm{T}}_\mu\mathbf{H}^L-\mathbf{H}^L\mathbf{E}_\mu\end{bmatrix}\tilde{\mathbf{X}}^1(t).\tag{38}$$

in practical applications. Moreover, various performance constraints can also be easily combined with the proposed IT-2 FFC design approach.

Theorem 3: If there exist the matrix $\tilde{\mathbf{J}}_\eta$, the positive definite matrices $\tilde{\mathbf{G}}_F$ and $\mathbf{Y}^F_{\mu\eta}$, the symmetric matrix $\mathbf{Z}^F$, the scalar $\wp$ such that the following sufficient conditions are satisfied with the given scalars $\tilde{\gamma}^F$, $\bar{\sigma}^F_{\mu\eta i_1 i_2\cdots i_\ell g}$, $\underline{\sigma}^F_{\mu\eta i_1 i_2\cdots i_\ell g}$, $\Im$ and given matrix $\mathbf{H}^F$, the containment purpose in Problem 1 is achieved for followers.

$$\tilde{\mathbf{G}}_F, \mathbf{Y}^F_{\mu\eta} > 0 \tag{46}$$

$$\begin{gathered}\mathbf{\Xi}^{\varpi}_{\mu\eta} - \mathbf{Y}^F_{\mu\eta} + \mathbf{Z}^F < 0\\ \text{for all } \mu,\ \eta \text{ and } \varpi = 1,2,3,4\end{gathered} \tag{47}$$

$$\begin{gathered}\sum_{\mu=1}^{v}\sum_{\eta=1}^{\varsigma}\left(\underline{\sigma}^F_{\mu\eta i_1 i_2\cdots i_\ell g}\mathbf{\Xi}^{\varpi}_{\mu\eta} - \left(\underline{\sigma}^F_{\mu\eta i_1 i_2\cdots i_\ell g} - \bar{\sigma}^F_{\mu\eta i_1 i_2\cdots i_\ell g}\right)\mathbf{Y}^F_{\mu\eta}\right.\\ \left.+\underline{\sigma}^F_{\mu\eta i_1 i_2\cdots i_\ell g}\mathbf{Z}^F\right) - \mathbf{Z}^F < 0\\ \text{for all } i_1, i_2, \ldots, i_\ell, g \text{ and } \varpi = 1,2,3,4\end{gathered} \tag{48}$$

$$\begin{bmatrix}\tilde{\mathbf{G}}_F & \tilde{\mathbf{G}}_F\\ * & \wp^2\mathbf{I}_q\end{bmatrix} > 0 \tag{49}$$

$$\min\left\{\wp^2\right\} \tag{50}$$

where $\underline{\sigma}^F_{\mu\eta i_1 i_2\cdots i_\ell g}$ and $\bar{\sigma}^F_{\mu\eta i_1 i_2\cdots i_\ell g}$ are the IT-2 MF parameters, which is obtained with the similar process of (39)–(41), the matrices $\mathbf{\Xi}^{\varpi}_{\mu\eta}, \mathfrak{T}^{\varpi}$, and $\tilde{\mathbf{J}}_\eta$ are shown at the bottom of this page, $\tilde{\mathbb{Z}}_\mu = \operatorname{diag}(\mathbb{Z}_\mu, \mathbb{Z}_\mu)$ is defined for system matrices $\mathbb{Z}_\mu = \mathbf{A}_\mu, \mathbf{B}_\mu, \mathbf{C}_\mu, \mathbf{D}_\mu, \mathbf{E}_\mu, \mathbf{K}_\eta, \mathbf{G}_F$ and $\mathbf{G}_F = \mathbf{P}_F^{-1}$.

Proof

To solve Problem 1 for followers, the containment error is defined with the interaction relationship as follows:

$$e^F(t) = x^F(t) + \left(\mathbf{L}^{-1}_{FF}\mathbf{L}_{FL}\otimes\mathbf{I}_q\right)x^L(t). \tag{51}$$

With (51), the following error dynamic system is obtained by considering (24) and (26):

$$\begin{aligned}\dot{e}^F(t) &= \dot{x}^F(t) + \left(\mathbf{L}^{-1}_{FF}\mathbf{L}_{FL}\otimes\mathbf{I}_q\right)\dot{x}^L(t)\\ &= \sum_{\mu=1}^{\nu}\sum_{\eta=1}^{\varsigma}\tilde{\Lambda}^F_{\mu\eta}\left\{\left(\mathbf{I}_F\otimes\mathbf{A}_\mu + \mathbf{L}_{FF}\otimes\mathbf{B}_\mu\mathbf{K}_\eta\right)e^F(t)\right.\\ &\quad\left.+\left(\mathbf{L}^{-1}_{FF}\mathbf{L}_{FL}\otimes\mathbf{B}_\mu\right)u^L(t) + \left(\mathbf{I}_F\otimes\mathbf{D}_\mu\right)w^F(t)\right\}.\end{aligned} \tag{52}$$

It is worth noting that the disturbance effect is dissipated by the IT-2 FFC designed with Theorems 1 and 2 for leaders such that the disturbance in the IT-2 T-SFM (24) is neglected in the containment analysis. However, the IT-2 FFC also results in the containment analysis problem of the error dynamic system (52). Extending the result in [16], this analysis problem is also solved in this research as follows. First, the containment error (51) is transferred as follows by considering Lemma 1:

$$e^\beta(t) = x^\beta(t) + \sum_{\alpha=1}^{\phi}\theta_{\beta\alpha}x^\alpha(t) \tag{53}$$

where $\theta_{\beta\alpha}$ denotes the (β,α)th element of matrix $\mathbf{L}^{-1}_{FF}\mathbf{L}_{FL}$ which satisfies the conditions $-\theta_{\beta\alpha}\geq 0$ and $\sum_{\alpha=1}^{\phi}-\theta_{\beta\alpha} = 1$. Then, the leader's input signal is reconstructed as

$$u_c^\beta(t) \;:= -\sum_{\alpha=1}^{\phi}\theta_{\beta\alpha}u^\alpha(t) \text{ for } \beta = \phi+1, \phi+2, \ldots, \phi+\tau. \tag{54}$$

According to $2u^{\alpha^{\mathrm{T}}}(t)u^{\hbar}(t) \leq u^{\alpha^{\mathrm{T}}}(t)u^{\alpha}(t) + u^{\hbar^{\mathrm{T}}}(t)u^{\hbar}(t)$, the following relationship can be obtained:

$$\begin{aligned}u_c^{\beta^{\mathrm{T}}}(t)\,u_c^\beta(t) &= \sum_{\alpha=1}^{\phi}\sum_{\hbar=1}^{\phi}\theta_{\beta\alpha}\theta_{\beta\hbar}u^{\alpha^{\mathrm{T}}}(t)\,u^{\hbar}(t)\\ &\leq \frac{1}{2}\sum_{\alpha=1}^{\phi}\sum_{\hbar=1}^{\phi}\theta_{\beta\alpha}\theta_{\beta\hbar}\left(u^{\alpha^{\mathrm{T}}}(t)\,u^{\alpha}(t) + u^{\hbar^{\mathrm{T}}}(t)\,u^{\hbar}(t)\right)\\ &\leq \left(\sum_{\alpha=1}^{\phi}\theta_{\beta\alpha}\right)^2\bar{u}^2 = \bar{u}^2\end{aligned} \tag{55}$$

where $\bar{u}$ denotes the upper bound of the leader's input. In [16], an assumption has been given for ensuring the boundedness of the leader's input, which is also unnecessary in this research since the stability of leaders has already been ensured by Theorems 1 and 2. Then, the item related to the leader's input in (52) can be represented as follows:

$$\left(\mathbf{L}^{-1}_{FF}\mathbf{L}_{FL}\otimes\mathbf{B}_\mu\right)u^L(t) = -\left(\mathbf{I}_F\otimes\mathbf{B}_\mu\right)u_c^L(t) \tag{56}$$

where $u_c^L(t) = \left[u_c^{\phi+1}(t)\quad u_c^{\phi+2}(t)\quad\cdots\quad u_c^{\phi+\tau}(t)\right]^{\mathrm{T}}$ satisfies the condition $\left\|u_c^L(t)\right\|^2 \leq \tau\bar{u}^2$ with (55). Therefore, the error dynamic system (52) can be further represented in the following

$$\mathbf{\Xi}^{\varpi}_{\mu\eta} = \left[\begin{array}{c:c:c}\begin{gathered}\tilde{\mathbf{A}}_\mu\tilde{\mathbf{G}}_F + \mathfrak{T}^{\varpi}\tilde{\mathbf{B}}_\mu\tilde{\mathbf{J}}_\eta\\ +\tilde{\mathbf{G}}_F\tilde{\mathbf{A}}_\mu^{\mathrm{T}} + \tilde{\mathbf{J}}_\eta^{\mathrm{T}}\tilde{\mathbf{B}}_\mu^{\mathrm{T}}\mathfrak{T}^{\varpi^{\mathrm{T}}} + \Im\tilde{\mathbf{G}}_F\end{gathered} & * & *\\ \hdashline \tilde{\mathbf{D}}_\mu^{\mathrm{T}} - \mathbf{H}^F\tilde{\mathbf{C}}_\mu\mathbf{G}_L & \tilde{\gamma}^F\mathbf{I}_h - \tilde{\mathbf{E}}_\mu^{\mathrm{T}}\mathbf{H}^F - \mathbf{H}^F\tilde{\mathbf{E}}_\mu & *\\ \hdashline -\tilde{\mathbf{B}}_\mu^{\mathrm{T}} & 0 & -\Im\mathbf{I}_p\end{array}\right]$$

$$\mathfrak{T}^{\varpi} = \begin{bmatrix}\operatorname{Re}\left\{\tilde{\lambda}^{\varpi}\right\}\mathbf{I}_q & -\operatorname{Im}\left\{\tilde{\lambda}^{\varpi}\right\}\mathbf{I}_q\\ \operatorname{Im}\left\{\tilde{\lambda}^{\varpi}\right\}\mathbf{I}_q & \operatorname{Re}\left\{\tilde{\lambda}^{\varpi}\right\}\mathbf{I}_q\end{bmatrix},\ \tilde{\mathbf{J}}_\eta = \tilde{\mathbf{K}}_\eta\tilde{\mathbf{G}}_F$$

form with (56):

$$\dot{e}^F(t) = \sum_{\mu=1}^{\nu}\sum_{\eta=1}^{\varsigma}\tilde{\Lambda}_{\mu\eta}^F\left\{(\mathbf{I}_F\otimes\mathbf{A}_\mu+\mathbf{L}_{FF}\otimes\mathbf{B}_\mu\mathbf{K}_\eta)e^F(t) - (\mathbf{I}_F\otimes\mathbf{B}_\mu)u_c^L(t)+(\mathbf{I}_F\otimes\mathbf{D}_\mu)w^F(t)\right\}. \quad (57)$$

Then, the output of IT-2 T-SFM (27) is also extended for the containment error as follows:

$$y_e^F(t) = \sum_{\mu=1}^{\nu}\tilde{\Phi}_\mu^F\left\{(\mathbf{I}_F\otimes\mathbf{C}_\mu)e^F(t)+(\mathbf{I}_F\otimes\mathbf{E}_\mu)w^F(t)\right\}. \quad (58)$$

Applying the nonsingular matrix $\mathbf{T}$, the Jordan canonical form of (57) and (58) is obtained as follows:

$$\dot{\tilde{e}}^F(t) = \sum_{\mu=1}^{\nu}\sum_{\eta=1}^{\varsigma}\tilde{\Lambda}_{\mu\eta}^F\left\{(\mathbf{I}_F\otimes\mathbf{A}_\mu+\Im_{FF}\otimes\mathbf{B}_\mu\mathbf{K}_\eta)\tilde{e}^F(t) - (\mathbf{I}_F\otimes\mathbf{B}_\mu)\tilde{u}_c^L(t)+(\mathbf{I}_F\otimes\mathbf{D}_\mu)\tilde{w}^F(t)\right\} \quad (59)$$

$$\tilde{y}_e^F(t) = \sum_{\mu=1}^{\nu}\tilde{\Phi}_\mu^F\left\{(\mathbf{I}_F\otimes\mathbf{C}_\mu)\tilde{e}^F(t)+(\mathbf{I}_F\otimes\mathbf{E}_\mu)\tilde{w}^F(t)\right\} \quad (60)$$

where $\tilde{e}^F(t)=(\mathbf{T}^{-1}\otimes\mathbf{I}_q)e^F(t)$, $\tilde{w}^F(t)=(\mathbf{T}^{-1}\otimes\mathbf{I}_h)w^F(t)$, $\tilde{y}_e^F(t)=(\mathbf{T}^{-1}\otimes\mathbf{I}_m)y_e^F(t)$, and $\tilde{u}_c^L(t)=(\mathbf{T}^{-1}\otimes\mathbf{I}_p)u_c^L(t)$. Then, the IT-2 T-SFM (59) and (60) is further expressed as follows:

$$\dot{\tilde{\tilde{e}}}^\beta(t) = \sum_{\mu=1}^{\nu}\sum_{\eta=1}^{\varsigma}\tilde{\Lambda}_{\mu\eta}^\beta\left\{\left(\tilde{\mathbf{A}}_\mu+\mathfrak{T}^\beta\tilde{\mathbf{B}}_\mu\tilde{\mathbf{K}}_\eta\right)\tilde{\tilde{e}}^\beta(t) - \tilde{\mathbf{B}}_\mu\tilde{\tilde{u}}_c^\beta(t)+\tilde{\mathbf{D}}_\mu\tilde{\tilde{w}}^\beta(t)\right\} \quad (61)$$

$$\tilde{\tilde{y}}_e^\beta(t) = \sum_{\mu=1}^{\nu}\tilde{\Phi}_\mu^\beta\left\{\tilde{\mathbf{C}}_\mu\tilde{\tilde{e}}^\beta(t)+\tilde{\mathbf{E}}_\mu\tilde{\tilde{w}}^\beta(t)\right\} \quad (62)$$

where $\tilde{\tilde{\mathrm{fl}}}^\beta(t)=[\mathrm{Re}^{\mathrm{T}}\{\tilde{\mathrm{fl}}^\beta(t)\}\mathrm{Im}^{\mathrm{T}}\{\tilde{\mathrm{fl}}^\beta(t)\}]^{\mathrm{T}}$, $\tilde{\mathrm{fl}}^\beta(t)$ denotes the vectors $\tilde{e}^\beta(t)$, $\tilde{w}^\beta(t)$, $\tilde{y}_e^\beta(t)$, and $\tilde{u}_c^\beta(t)$ in (61) and (62) and $\mathfrak{T}^\beta=\begin{bmatrix}\mathrm{Re}\{\lambda^\beta\}\mathbf{I}_q & -\mathrm{Im}\{\lambda^\beta\}\mathbf{I}_q\\ \mathrm{Im}\{\lambda^\beta\}\mathbf{I}_q & \mathrm{Re}\{\lambda^\beta\}\mathbf{I}_q\end{bmatrix}$. To solve Problem 1, the following Lyapunov function is defined:

$$\mathrm{V}^F(t) = \tilde{\tilde{e}}^{\beta^{\mathrm{T}}}(t)\tilde{\mathbf{P}}_F\tilde{\tilde{e}}^\beta(t) \quad (63)$$

where $\tilde{\mathbf{P}}_F=\mathrm{diag}(\mathbf{P}_F,\mathbf{P}_F)$. Referring to [16], the following ellipsoid is also defined for the error dynamic:

$$\mathbb{U} := \left\{\tilde{\tilde{e}}^\beta(t)\,|\,\tilde{\tilde{e}}^{\beta^{\mathrm{T}}}(t)\tilde{\mathbf{P}}_F\tilde{\tilde{e}}^\beta(t)\le\bar{u}^2\right\}. \quad (64)$$

Then, if the following inequality is satisfied, the ellipsoid (64) is ensured to be an attractive invariant set

$$\dot{\tilde{\mathrm{V}}}^F(t) < 0 \quad (65)$$

where $\dot{\tilde{\mathrm{V}}}^F(t)=\dot{\mathrm{V}}^F(t)+\Im(\mathrm{V}^F(t)-\tilde{\tilde{u}}_c^{\beta^{\mathrm{T}}}(t)\tilde{\tilde{u}}_c^\beta(t))$, and $\Im$ is a positive scalar. However, the error dynamic system (61) and (62) is also affected by the disturbance, which makes the containment purpose difficult to achieve well. To deal with the disturbance effect by the passive constraint in Lemma 3, the following cost function is also selected:

$$\int_0^{t_p}\left(\tilde{\gamma}^F\tilde{\tilde{w}}^{\beta^{\mathrm{T}}}(t)\tilde{\tilde{w}}^\beta(t)-2\tilde{\tilde{y}}^{\beta^{\mathrm{T}}}(t)\mathbf{H}^F\tilde{\tilde{w}}^\beta(t)\right)dt. \quad (66)$$

From the condition (65) and cost function (66), the following inequality can be established:

$$\begin{aligned}&\int_0^{t_p}\left(\tilde{\gamma}^F\tilde{\tilde{w}}^{\beta^{\mathrm{T}}}(t)\tilde{\tilde{w}}^\beta(t)-2\tilde{\tilde{y}}^{\beta^{\mathrm{T}}}(t)\mathbf{H}^F\tilde{\tilde{w}}^\beta(t)\right)dt\\ &=\int_0^{t_p}\left(\tilde{\gamma}^F\tilde{\tilde{w}}^{\beta^{\mathrm{T}}}(t)\tilde{\tilde{w}}^\beta(t)-2\tilde{\tilde{y}}^{\beta^{\mathrm{T}}}(t)\mathbf{H}^F\tilde{\tilde{w}}^\beta(t)+\dot{\tilde{\mathrm{V}}}^F(t)\right)dt\\ &\quad-\tilde{\mathrm{V}}^F(t_p)\\ &\le\int_0^{t_p}\tilde{\gamma}^F\tilde{\tilde{w}}^{\beta^{\mathrm{T}}}(t)\tilde{\tilde{w}}^\beta(t)-2\tilde{\tilde{y}}^{\beta^{\mathrm{T}}}(t)\mathbf{H}^F\tilde{\tilde{w}}^\beta(t)+\dot{\tilde{\mathrm{V}}}^F(t)\,dt.\end{aligned} \quad (67)$$

Then, the integrand of the right-hand side of (67) is presented in the following form:

$$\sum_{\mu=1}^{\nu}\sum_{\eta=1}^{\varsigma}\left(\underline{\psi}_{\mu\eta}^F\bar{\Lambda}_{\mu\eta}^F+\bar{\psi}_{\mu\eta}^F\underline{\Lambda}_{\mu\eta}^F\right)\left\{\tilde{\tilde{\mathbf{X}}}^{\beta^{\mathrm{T}}}(t)\coprod_{\mu\eta}^{\beta}\tilde{\tilde{\mathbf{X}}}^\beta(t)\right\} \quad (68)$$

where $\tilde{\tilde{\mathbf{X}}}^{\beta^{\mathrm{T}}}(t)=[\tilde{\tilde{e}}^{\beta^{\mathrm{T}}}(t)\quad\tilde{\tilde{w}}^{\beta^{\mathrm{T}}}(t)\quad\tilde{\tilde{u}}_c^{\beta^{\mathrm{T}}}(t)]^{\mathrm{T}}$ and the matrix $\coprod_{\mu\eta}^\beta$ is shown at the bottom of next page.

By the same process of (39)–(41), the IT-2 MF dependent parameter $\underline{\sigma}_{\mu\eta i_1i_2\cdots i_{\ell g}}^F$ and $\bar{\sigma}_{\mu\eta i_1i_2\cdots i_{\ell g}}^F$ can be obtained and the substate space g can be selected differently from the leaders.

Then, $\mathbf{Y}_{\mu\eta}^F$ and $\mathbf{Z}^F$ are combined into the analysis process with the same form of (43). It is not difficult to find that if the conditions (46)–(48) are satisfied by Theorem 3, the cost function (66) can be ensured to be negative definite by referring to the analysis process (36)–(45) and Lemma 4. Considering the situation $\tilde{\tilde{w}}^\beta(t)=0$, the condition (65) also can be satisfied with (46)–(48) such that the stability of the error dynamic is also guaranteed to be in the attractive invariant set (64). Consequently, the conditions (49) and (50) can be derived as follows to solve the minimization problem in Problem 1 for containment purposes. Multiplying $\tilde{\mathbf{G}}_F$on both sides and applying Schur complement to (49), the following inequality is obtained with $\tilde{\mathbf{P}}_F-\wp^{-2}\mathbf{I}_q>0$ and (64).

$$\bar{u}^2\ge\tilde{\tilde{e}}^{\beta^{\mathrm{T}}}(t)\tilde{\mathbf{P}}_F\tilde{\tilde{e}}^\beta(t)>\tilde{\tilde{e}}^{\beta^{\mathrm{T}}}(t)\left(\wp^{-2}\mathbf{I}_q\right)\tilde{\tilde{e}}^\beta(t) \quad (69)$$

where $\|\tilde{\tilde{e}}^\beta(t)\|^2<\wp^2\bar{u}^2$. If $\wp^2$ is minimized with the condition (50) in Theorem 3, the dynamic of containment error also can be minimized based on (69). Therefore, the containment problem in Problem 1 is solved for followers.

Finally, the passive constraint (29) is required to be proved with the cost function (66). Using the definition of signals in (61) and (62) and defining the maximum and minimization of eigenvalues for $\mathbf{T}^{-\#}\mathbf{T}^{-1}$ as $\bar{\lambda}_{\mathrm{T}^{-1}}$ and $\underline{\lambda}_{\mathrm{T}^{-1}}$, where $\#$ denotes the conjugate transpose, the following inequality can be obtained from (66):

$$\bar{\lambda}_{\mathrm{T}^{-1}} \int_0^{t_p} 2y^{F\mathrm{T}}(t)\left(\mathbf{I}_F \otimes \mathbf{H}^F\right) w^F(t)\,dt$$

$$\underline{\lambda}_{\mathrm{T}^{-1}} \tilde{\gamma}^F \int_0^{t_p} w^{F\mathrm{T}}(t)\, w^F(t)\,dt. \tag{70}$$

From the relationships from (70), it is obvious that the passive constraint (29) in Lemma 3 can also be satisfied by ensuring the negative definiteness of (66) if the parameter $\tilde{\gamma}^F = (\bar{\lambda}_{\mathrm{T}^{-1}}/\underline{\lambda}_{\mathrm{T}^{-1}})\gamma^F$ in Theorem 3 is set. Accordingly, the proof is accomplished for Theorem 3.

Remark 2: In the existing papers [18], [19], [20], [21], the containment analysis is required to be implemented with the additional assumption for the completion of a leader's formation, which is unnecessary in Theorem 3. Although this assumption can fulfill the IT-2 CFC design, the collision of followers may be caused in practical applications because the containment is unable to be ensured until the formation is completed.

By Theorems 1–3, a simulation of NMSS is given to illustrate the advantage of the proposed IT-2 F-and-C FC design approach in the next section.

IV. Simulation of NMSS

Referring to the parameters in [24], the nonlinear ship dynamic systems can be extended to the following NMSS with uncertainties and disturbances.

$$\dot{x}_1^\beta(t) = \left(\cos\left(x_3^\beta(t)\right) + \Delta(t)\right) x_4^\beta(t) - \left(\sin\left(x_3^\beta(t)\right) + \Delta(t)\right) x_5^\beta(t) \tag{71}$$

$$\dot{x}_2^\beta(t) = \left(\sin\left(x_3^\beta(t)\right) + \Delta(t)\right) x_4^\beta(t) + \left(\cos\left(x_3^\beta(t)\right) + \Delta(t)\right) x_5^\beta(t) \tag{72}$$

$$\dot{x}_3^\beta(t) = (1 + \Delta(t))\, x_6^\beta(t) \tag{73}$$

$$\dot{x}_4^\beta(t) = -0.0318 x_4^\beta(t) + 0.8870 u_1^\beta(t) + 0.1 w^\beta(t) \tag{74}$$

$$\dot{x}_5^\beta(t) = -0.0628 x_5^\beta(t) - 0.0030 x_6^\beta(t) + 0.5415 u_2^\beta(t) + 0.3152 u_3^\beta(t) + 0.1 w^\beta(t) \tag{75}$$

$$\dot{x}_6^\beta(t) = -0.0045 x_5^\beta(t) - 0.2427 x_6^\beta(t) + 0.3152 u_2^\beta(t) + 8.0082 u_3^\beta(t) + 0.1 w^\beta(t) \tag{76}$$

$$y_1^\beta(t) = x_3^\beta(t) + w^\beta(t) \tag{77}$$

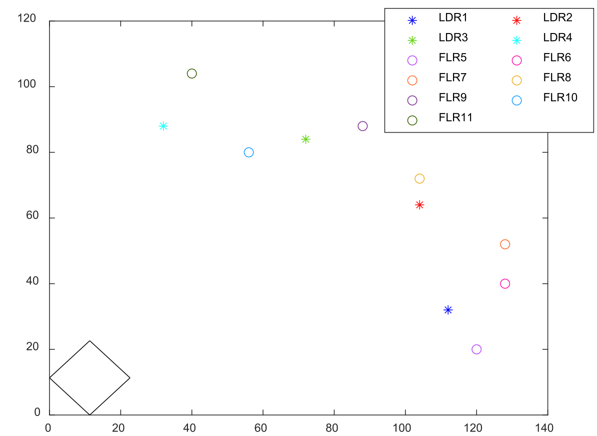

Fig. 1. F-and-C control problems of NMSS.

where $x_1^\beta(t)$, $x_2^\beta(t)$, and $x_3^\beta(t)$ are the earth-fixed X position, Y position, and yaw angle, and $x_4^\beta(t)$, $x_5^\beta(t)$, and $x_6^\beta(t)$ are the ship's body-fixed surge, sway, and yaw angular velocity, respectively, and $u_1^\beta(t)$, $u_2^\beta(t)$, and $u_3^\beta(t)$ are the force and moment produced by the thrusters. Note that the uncertainties and disturbances are selected as $\Delta(t) = 0.1\sin(t)$ and $w^\beta(t)$ with the density $\mathbf{W} = 1$. Then, the following IT-2 T-SFM is constructed:

$$\dot{x}^\beta(t) = \sum_{\mu=1}^{3} \tilde{\Phi}_\mu^\beta \left\{\mathbf{A}_\mu x^\beta(t) + \mathbf{B}_\mu u^\beta(t) + \mathbf{D}_\mu w^\beta(t)\right\} \tag{78}$$

$$y^\beta(t) = \sum_{\mu=1}^{3} \tilde{\Phi}_\mu^\beta \left\{\mathbf{C}_\mu x^\beta(t) + \mathbf{E}_\mu w^\beta(t)\right\} \tag{79}$$

where model matrices and IT-2 MF of IT-2 T-SFM (78) and (79) are given in the Appendix.

Dividing the NMSS into four leaders and seven followers, the F-and-C control problems can be stated in Fig. 1. Note that LDR denotes the leader ship and FLR denotes the follower ship. In this simulation, the four LDRs are required to be placed at vertices such that the square region can be formed. Then, all the FLRs are necessary to be contained in the region.

With (9) and (10), the IT-2 T-SFM of the target trajectory is also constructed by the same IT-2 MFs of (78) and (79). According to the ship positions in Fig. 1, the Laplacian matrix is obtained for the containment purpose in the Appendix. Then, the following IT-2 F-and-C FCs are designed for LDRs and FLRs:

$$u^\beta(t) = \sum_{\eta=1}^{2} \tilde{\Psi}_\eta^\beta \left\{u_{eq}^\beta(t) + u_s^\beta(t)\right\} \text{ for } \quad \beta = 1,2,3,4 \tag{80}$$

$$\coprod_{\mu\eta}^{\beta} = \left[\begin{array}{c:c:c} \left(\tilde{\mathbf{A}}_\mu + \mathfrak{T}^\beta \tilde{\mathbf{B}}_\mu \tilde{\mathbf{K}}_\eta\right)^{\mathrm{T}} \tilde{\mathbf{P}}_F + \tilde{\mathbf{P}}_F \left(\tilde{\mathbf{A}}_\mu + \mathfrak{T}^\beta \tilde{\mathbf{B}}_\mu \tilde{\mathbf{K}}_\eta\right) + \Im \tilde{\mathbf{P}}_F & * & * \\ \hdashline \tilde{\mathbf{D}}_\mu^{\mathrm{T}} \tilde{\mathbf{P}}_F - \tilde{\mathbf{H}}^F \tilde{\mathbf{C}}_\mu & \tilde{\gamma}^F \mathbf{I}_h - \tilde{\mathbf{E}}_\mu^{\mathrm{T}} \tilde{\mathbf{H}}^F - \tilde{\mathbf{H}}^F \tilde{\mathbf{E}}_\mu & * \\ \hdashline -\tilde{\mathbf{B}}_\mu^{\mathrm{T}} \tilde{\mathbf{P}}_F & 0 & -\Im \mathbf{I}_p \end{array}\right].$$

$$u^{\beta}(t) = \sum_{\eta=1}^{2} \tilde{\Psi}_{\eta}^{\beta} \left\{ \mathbf{K}_{\eta} \sum_{\alpha \in \mathfrak{A}} \mathfrak{a}_{\beta\alpha} \left(x^{\beta}(t) - x^{\alpha}(t) \right) \right\}$$
$$\text{for } \beta = 5, 6, \ldots, 11 \tag{81}$$

where the design of IT-2 MF for F-and-C FCs is also presented in the Appendix.

In this simulation, the norm of disturbance is assumed within 11. Thus, the sliding gain $\delta = 10$ and the matrix $\mathbf{S} = [\text{diag}\{0,0,0\} \quad \text{diag}\{5,5,5\}]$ are selected such that Theorem 1 is satisfied and $\mathbf{SB}_{\mu}$ is the positive definite matrix.

Then, dividing the state space with $\rho = 30$ and solving Theorem 2 with MATLAB according to $\gamma^L = 0.1$, $\mathbf{H}^L = 1$, and the IT-2 MFs in the Appendix, the gains are obtained as (82) and (83) shown at the bottom of this page for LDRs.

Solving Theorem 3 with the setting of $\gamma^F = 0.1$, $\mathbf{H}^F = 1$, $g = 20$, the parameter $\Im = 0.01$, and the IT-2 MFs in the Appendix, the gains are obtained as (84) and (85) shown at the bottom of this page for FLRs.

Based on the initial condition of all LDRs and FLRs in the Appendix, the responses of the NMSS (71)–(77) are obtained by the IT-2 F-and-C FCs (80) and (81) with the gains of (82)–(85) in Figs. 2–8.

Obviously, the desired dynamics of target trajectories are successfully tracked by the states of LDRs in Figs. 2 and 3. Even under the effect of disturbances, the state responses of all FLRs still can be forced into the interval between LDRs.

According to Fig. 8, one can see that the time-varying target trajectories are all successfully tracked by LDRs with the IT-2 FSM tracking controller (80). The formation is also achieved by the design of these trajectories. Significantly, it is witnessed that the problem of time-varying formation and the assignment for the whole system dynamics can be solved by the IT-2 FSM TC approach without the additional item.

By the setting of target trajectories in Fig. 8, the F-and-C tasks with the situation that exists obstacles between the starting point and the destination can be efficiently completed. Moreover, the FLRs are all forced into the formation of LDRs almost in the first black dashed-square because of the IT-2 CFC designed with Theorem 3. Then, all the FLRs are continuously maintained in the leader's formation until the destination. In the results of Figs. 2–7, the effect of uncertainties and disturbances is greatly reduced by the combination of SMC and passive constraint.

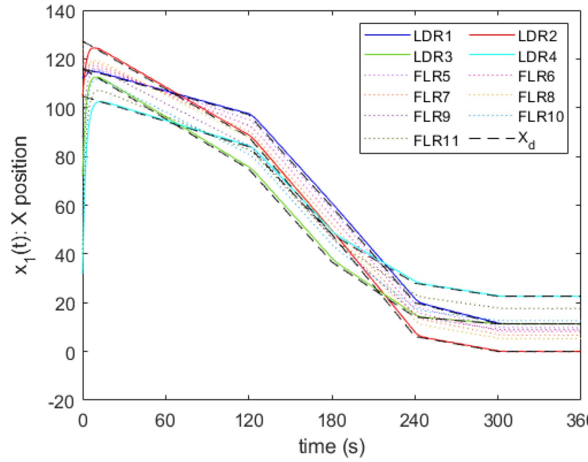

Fig. 2. Responses of the *X* position of NMSS.

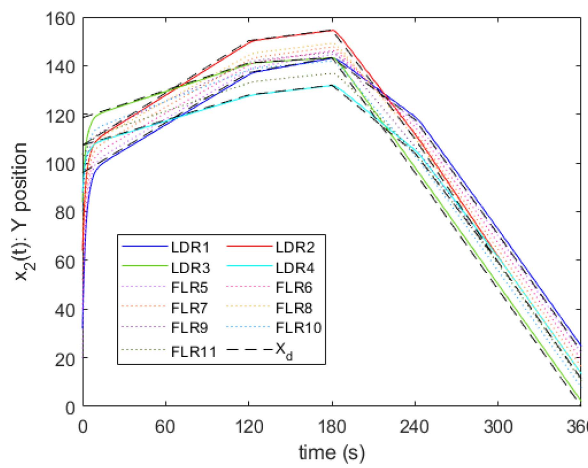

Fig. 3. Responses of the *Y* position of NMSS.

$$\mathbf{F}_1 = \begin{bmatrix} -30.8975 & 5.7778 & -1.5009 & -77.1708 & 14.9090 & -0.4183 \\ -7.3556 & -60.3299 & 1.7714 & -18.0812 & -160.0127 & 1.6802 \\ 0.2902 & 2.4320 & -6.1071 & 0.7079 & 6.4465 & -2.2626 \end{bmatrix} \tag{82}$$

$$\mathbf{F}_2 = \begin{bmatrix} -25.8501 & -6.0774 & -1.3988 & -65.8357 & -15.7362 & -0.2274 \\ 7.3022 & -51.4029 & 2.1355 & 17.9898 & -136.3137 & 1.5656 \\ -0.2451 & 2.0857 & -5.8603 & -0.6067 & 5.5284 & -2.1521 \end{bmatrix}. \tag{83}$$

$$\mathbf{K}_1 = 10^3 \times \begin{bmatrix} -2.8920 & 0.6187 & 0.0105 & -4.3293 & 0.9178 & 0.0022 \\ 0.7593 & -6.7497 & 0.2040 & 1.1251 & -10.0611 & 0.0165 \\ -0.0276 & 0.2537 & -1.2718 & -0.0409 & 0.3782 & -0.1888 \end{bmatrix} \tag{84}$$

$$\mathbf{K}_2 = 10^3 \times \begin{bmatrix} -2.7946 & 0.3501 & 0.0173 & -4.1840 & 0.5184 & 0.0027 \\ 0.8628 & -6.6115 & 0.1229 & 1.2790 & -9.8550 & 0.0047 \\ -0.0336 & 0.2533 & -1.2675 & -0.0499 & 0.3775 & -0.1882 \end{bmatrix}. \tag{85}$$

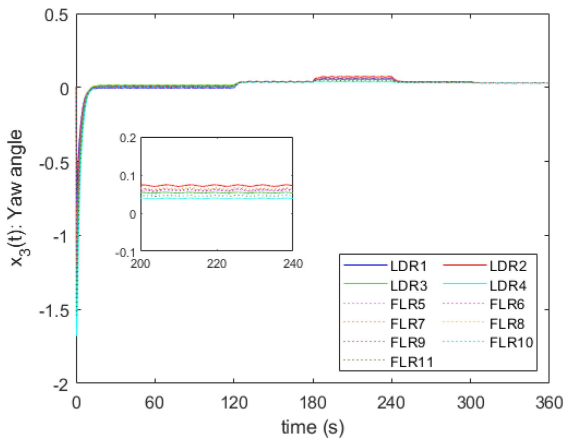

Fig. 4. Responses of yaw angle of NMSS.

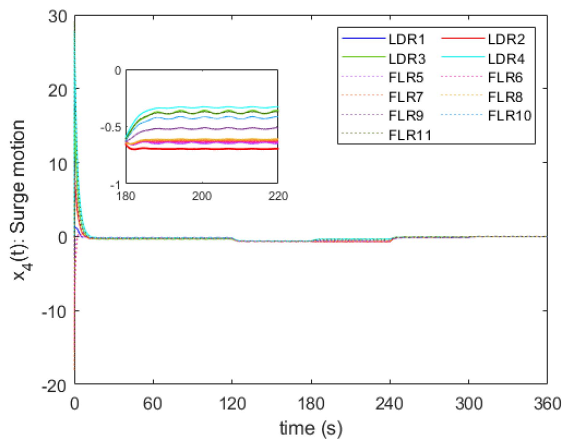

Fig. 5. Responses of surge motion of NMSS.

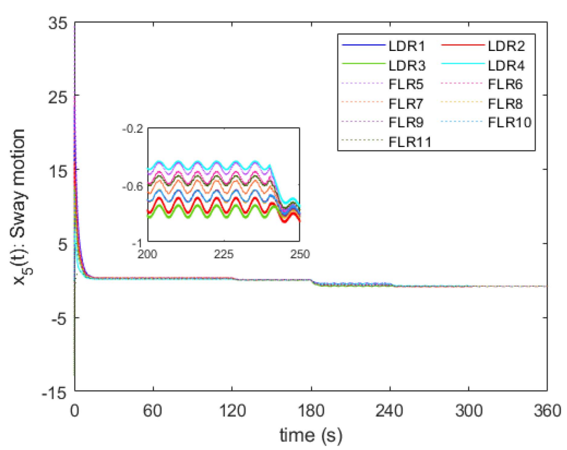

Fig. 6. Responses of sway motion of NMSS.

Thus, the tracking performances are enhanced for better formation by the simpler IT-2 FFC design process of leaders.

Based on Remark 2, the simulation results of the comparison are provided as follows. Note that the containment control problem based on (52) can be reduced to a typical problem for the following error dynamic system with the assumption.

$$\dot{e}^F(t) = \sum_{\mu=1}^{\nu}\sum_{\eta=1}^{\varsigma}\tilde{\Lambda}^F_{\mu\eta}\left\{(\mathbf{I}_F\otimes\mathbf{A}_\mu + \mathbf{L}_{FF}\otimes\mathbf{B}_\mu\mathbf{K}_\eta)\,e^F(t) + (\mathbf{I}_F\otimes\mathbf{D}_\mu)\,w^F(t)\right\}. \tag{86}$$

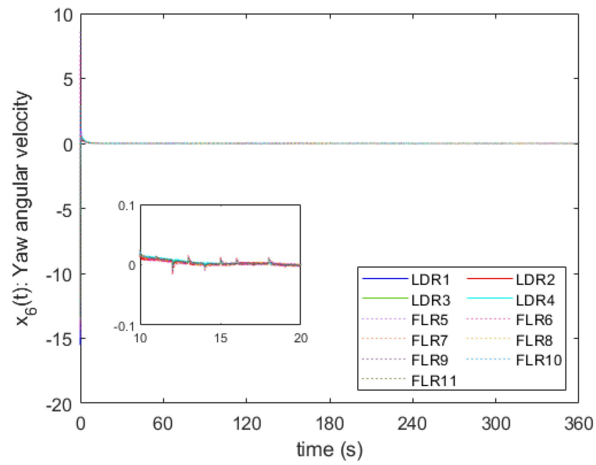

Fig. 7. Responses of yaw velocity of NMSS.

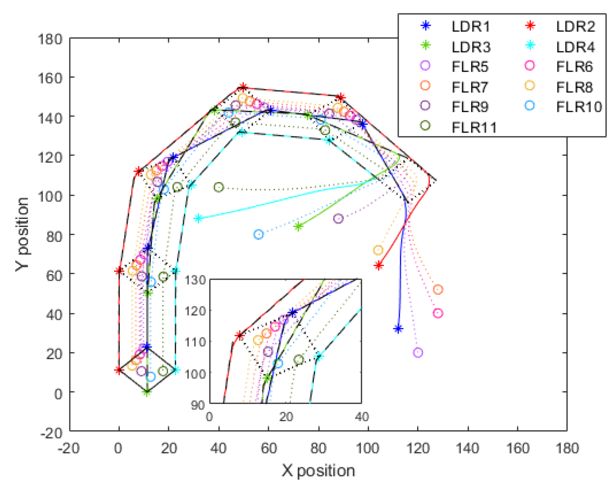

Fig. 8. Trajectory of all ships in NMSS.

Using the typical containment analysis method for (86), which can be referred to in many existing papers, with the IT-2 FC design approach and the passive constraint (29), the control gains are obtained as (87) and (88) shown at the bottom of next page with the same settings of (84) and (85).

To make the comparison clearer, the point tracking problem is considered for Fig. 1. With the same initial and destination, the different trajectories can be obtained in Figs. 9 and 10 by the same IT-2 FFC (80) with gains (82) and (83) and the IT-2 CFC (81) respectively with (84) and (85) and (87) and (88). Note that the figure of the ship's trajectories is only presented to save space.

Comparing Figs. 9 and 10, the containment task of followers is completed almost at the end in Fig. 9 since the additional assumption. This reason will possibly lead to the collision between the obstacles, FLR 11 and FLR 5. By the IT-2 CFC designed with Theorem 3, the followers are forced into the region between leaders faster. It is also verified that the assumption is not proper for the containment analysis.

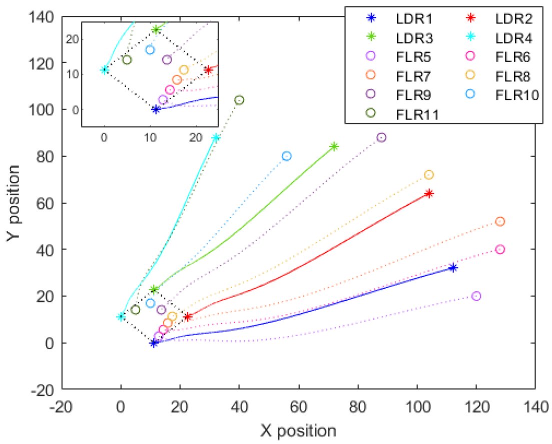

Fig. 9. Point tracking of all ships with assumption.

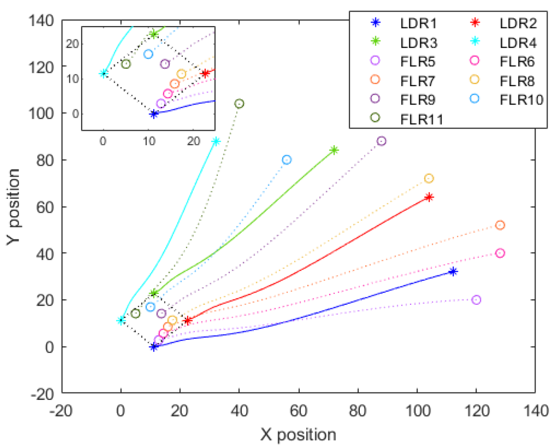

Fig. 10. Point tracking of all ships with Theorem 3.

V. Conclusion

An IT-2 FSM tracking approach is proposed for NMASs to solve the F-and-C control problem with uncertainties and disturbances in this research. Different from the existing papers, the formation is completed without communication between leaders such that signal transmission problems are efficiently avoided. Applying the IPM method and extending the results of linear M-A systems, a flexible design process can be provided for the IT-2 F-and-C FCs. Because a simpler analysis IT-2 FFC design process is provided, the SMC approach is conveniently combined to further enhance the formation performance. Moreover, the passive constraints are also combined to dissipate the disturbance energy for both leaders and followers. In the simulation, the proposed IT-2 FSM controller achieves good tracking performance and smooth trajectories for the requirement of F-and-C. Notably, the leaders play a critical role in the F-and-C problems. Because of this reason, how to apply a less-complicated or more convenient controller design process to achieve better control performances for leaders is also an important issue. In the future, many requirements such as actuator fault and saturation can be considered for NMASs by using the IT-2 F-and-C FC design scheme in this research.

Appendix

A. Stability Analysis of Theorem 1

First, the following Lyapunov function is selected based on the sliding surface (18):

$$\mathrm{V}_s^1(t) = (1/2)\, S^{1^{\mathrm{T}}}(t)\, S^1(t)\,. \tag{89}$$

Then, the derivative of (89) is obtained as follows:

$$\dot{\mathrm{V}}_s^1(t) = S^{1^{\mathrm{T}}}(t)\, \dot{S}^1(t) \tag{90}$$

where

$$\dot{S}^1(t) = \mathbf{S}\left(\sum_{\mu=1}^{\nu} \tilde{\Phi}_\mu^1 \left\{\mathbf{A}_\mu \tilde{x}^1(t) + \mathbf{B}_\mu u^1(t) + \mathbf{D}_\mu w^1(t)\right\} - \sum_{\mu=1}^{\nu}\sum_{\eta=1}^{\varsigma} \tilde{\Phi}_\mu^1 \tilde{\Psi}_\eta^1 \left\{\mathbf{A}_\mu + \mathbf{B}_\mu \mathbf{F}_\eta\right\} \tilde{x}^1(t)\right). \tag{91}$$

Substituting the IT-2 FFC (22) into (91), one can obtain

$$\dot{S}^1(t) = -\delta \cdot \operatorname{sgn}\left(S^1(t)\right) + \sum_{\mu=1}^{\nu} \tilde{\Phi}_\mu^1 \mathbf{S}\mathbf{D}_\mu w^1(t)\,. \tag{92}$$

Based on (92), (90) can be derived as follows:

$$\begin{aligned}\dot{\mathrm{V}}_s^1(t) &= S^{1^{\mathrm{T}}}(t)\left(-\delta \cdot \operatorname{sgn}\left(S^1(t)\right) + \sum_{\mu=1}^{\nu} \tilde{\Phi}_\mu^1 \mathbf{S}\mathbf{D}_\mu w^1(t)\right)\\ &= -\delta\left|S^1(t)\right| + S^{1^{\mathrm{T}}}(t)\left(\sum_{\mu=1}^{\nu} \tilde{\Phi}_\mu^1 \mathbf{S}\mathbf{D}_\mu\right) w^1(t)\\ &= \left(-\delta + \left\|\left(\sum_{\mu=1}^{\nu} \tilde{\Phi}_\mu^1 \mathbf{S}\mathbf{D}_\mu\right)\right\| \cdot \left\|w^1(t)\right\|\right) \cdot \left\|S^1(t)\right\|.\end{aligned} \tag{93}$$

Obviously, if the condition (30) is satisfied by Theorem 1, the negative definite of (90) can be ensured from (93). Therefore, the convergence of tracking error dynamic to the sliding surface is also achieved.

$$\mathbf{K}_1 = \begin{bmatrix} -70.9705 & 2.0875 & 2.3714 & -292.9315 & 8.7239 & 0.8099 \\ -1.6494 & -74.3279 & 8.6324 & -6.6814 & -318.2871 & 6.1594 \\ 0.0436 & 2.8196 & -9.7772 & 0.1751 & 12.0744 & -7.4309 \end{bmatrix} \tag{87}$$

$$\mathbf{K}_2 = \begin{bmatrix} -70.8147 & -2.5066 & 2.6904 & -292.2947 & -10.6723 & 1.0334 \\ 0.9486 & -74.7384 & 8.6047 & 3.5593 & -320.0415 & 6.1711 \\ -0.1657 & 2.8501 & -9.7526 & -0.6702 & 12.2047 & -7.4147 \end{bmatrix}. \tag{88}$$

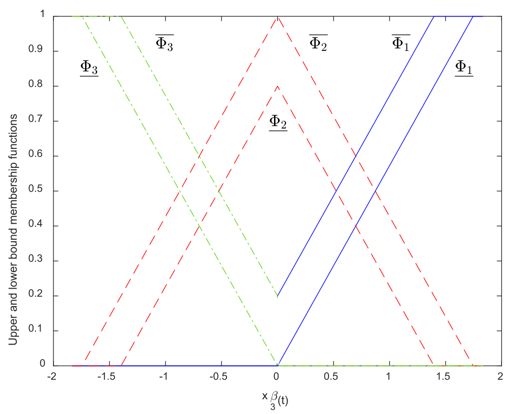

Fig. 11. IT-2 MF of T-SFM.

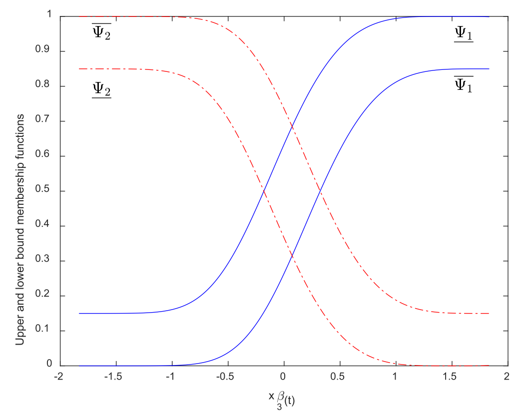

Fig. 12. IT-2 MF of F-and-C FCs.

B. Model Matrices of IT-2 T-SFM for NMSS

Selecting three operating points $x^{\beta}_{3(op1)}(t) = 90^{\circ}$, $x^{\beta}_{3(op2)}(t) = 0^{\circ}$, and $x^{\beta}_{3(op3)}(t) = -90^{\circ}$, the corresponding model matrices of IT-2 T-SFM (78) and (79) can be obtained for the NMSS (71)–(77) by the fuzzy modeling method as follows:

$$\mathbf{A}_1 = \begin{bmatrix} 0 & 0 & 0 & \theta_1 & -1 & 0 \\ 0 & 0 & 0 & 1 & \theta_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -0.0318 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.0628 & -0.0030 \\ 0 & 0 & 0 & 0 & -0.0045 & -0.2427 \end{bmatrix}$$

$$\mathbf{A}_2 = \begin{bmatrix} 0 & 0 & 0 & 1 & -\theta_2 & 0 \\ 0 & 0 & 0 & \theta_2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -0.0318 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.0628 & -0.0030 \\ 0 & 0 & 0 & 0 & -0.0045 & -0.2427 \end{bmatrix}$$

$$\mathbf{A}_3 = \begin{bmatrix} 0 & 0 & 0 & \theta_3 & 1 & 0 \\ 0 & 0 & 0 & -1 & \theta_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -0.0318 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.0628 & -0.0030 \\ 0 & 0 & 0 & 0 & -0.0045 & -0.2427 \end{bmatrix}$$

$$\mathbf{B}_1 = \mathbf{B}_2 = \mathbf{B}_3 = \begin{bmatrix} 0 & 0 & 0 & 0.8870 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.5415 & 0.3152 \\ 0 & 0 & 0 & 0 & 0.3152 & 8.0082 \end{bmatrix}^{\mathrm{T}}$$

$$\mathbf{C}_1 = \mathbf{C}_2 = \mathbf{C}_3 = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix},,$$

$$\mathbf{D}_1 = \mathbf{D}_2 = \mathbf{D}_3 = \begin{bmatrix} 0 & 0 & 0 & 0.1 & 0.1 & 0.1 \end{bmatrix}^{\mathrm{T}},$$

$$\mathbf{E}_1 = \mathbf{E}_2 = \mathbf{E}_3 = 1$$

where $\theta_1 = \cos(88^{\circ})$, $\theta_2 = \sin(2^{\circ})$, and $\theta_3 = \cos(-88^{\circ})$.

C. IT-2 MF of T-SFM and FC for NMSS

To cover the effect of uncertainties in the NMSS (71)–(77), the IT-2 MF is designed in the Fig. 11 by extending the type-1 MF in [15].

Then, the IT-2 MF of F-and-C FCs (80) and (81) is also designed to deal with the effect of uncertainties in Fig. 12.

Based on the design of IT-2 MFs in Figs. 11 and 12, the control gains of (82)–(85) and (87) and (88) can be obtained with the mentioned setting.

D. Interaction Relationship of NMSS

Considering the position of all LDRs and FLRs in the F-and-C problem of Fig. 1, the interaction relationship among all ships can be presented with the Laplacian matrix as follows:

$$\mathbf{L}_{FF} = \begin{bmatrix} 2 & -1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 3 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 3 & -1 \\ 0 & 0 & 0 & 0 & 0 & -1 & 2 \end{bmatrix}$$

$$\mathbf{L}_{FL} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}. \tag{94}$$

E. Initial Condition and Destination of LDRs and FLRs

For the F-and-C control problem in Fig. 1, the initial conditions of all ships are presented. Besides, the destination for four LDRs is also provided as follows:

$$x^1(0) = \begin{bmatrix} 112 & 32 & 0 & 0 & 0 & 0 \end{bmatrix}^{\mathrm{T}}$$
$$x^2(0) = \begin{bmatrix} 104 & 64 & 0 & 0 & 0 & 0 \end{bmatrix}^{\mathrm{T}}$$
$$x^3(0) = \begin{bmatrix} 72 & 84 & 0 & 0 & 0 & 0 \end{bmatrix}^{\mathrm{T}}$$
$$x^4(0) = \begin{bmatrix} 32 & 88 & 0 & 0 & 0 & 0 \end{bmatrix}^{\mathrm{T}}$$
$$x^5(0) = \begin{bmatrix} 120 & 20 & 0 & 0 & 0 & 0 \end{bmatrix}^{\mathrm{T}}$$
$$x^6(0) = \begin{bmatrix} 128 & 40 & 0 & 0 & 0 & 0 \end{bmatrix}^{\mathrm{T}}$$

$$x^{7}(0) = \begin{bmatrix}128 & 52 & 0 & 0 & 0 & 0\end{bmatrix}^{\mathrm{T}}$$
$$x^{8}(0) = \begin{bmatrix}104 & 72 & 0 & 0 & 0 & 0\end{bmatrix}^{\mathrm{T}}$$
$$x^{9}(0) = \begin{bmatrix}88 & 88 & 0 & 0 & 0 & 0\end{bmatrix}^{\mathrm{T}}$$
$$x^{10}(0) = \begin{bmatrix}56 & 80 & 0 & 0 & 0 & 0\end{bmatrix}^{\mathrm{T}}$$
$$x^{11}(0) = \begin{bmatrix}40 & 104 & 0 & 0 & 0 & 0\end{bmatrix}^{\mathrm{T}}. \quad (95)$$

The final positions for the destination of LDRs are set as

$$x_{\mathrm{des}}^{1} = \begin{bmatrix}16/\sqrt{2} & 0 & 0 & 0 & 0 & 0\end{bmatrix}^{\mathrm{T}}$$
$$x_{\mathrm{des}}^{2} = \begin{bmatrix}32/\sqrt{2} & 16/\sqrt{2} & 0 & 0 & 0 & 0\end{bmatrix}^{\mathrm{T}}$$
$$x_{\mathrm{des}}^{3} = \begin{bmatrix}16/\sqrt{2} & 32/\sqrt{2} & 0 & 0 & 0 & 0\end{bmatrix}^{\mathrm{T}}$$
$$x_{\mathrm{des}}^{4} = \begin{bmatrix}0 & 16/\sqrt{2} & 0 & 0 & 0 & 0\end{bmatrix}^{\mathrm{T}}. \quad (96)$$

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