Stability Analysis of Time-Varying Delay T–S Fuzzy Systems via Quadratic-Delay-Product Method

Yunfei Qiu, Ju H. Park, *Senior Member, IEEE*, Changchun Hua, *Senior Member, IEEE*, and Xijuan Wang

***Abstract*—The stability of Takagi–Sugeno (T–S) fuzzy systems with time-varying delay is investigated in this article. First of all, a novel Lyapunov–Krasovskii functional (LKF) is proposed by fully utilizing single integral polynomial-delay-product terms and membership-function-dependent matrices, where more delay information is considered. Second, by introducing negative integral estimation inequalities and polynomial inequality, the estimation gap of derivatives is further decreased. As consequence, the criterion with less conservatism is presented. Finally, the examples are utilized for verifying the validity of the stability approach.**

***Index Terms*—Linear matrix inequality, quadratic-delay-product method, T–S fuzzy systems, time-varying delay.**

I. Introduction

MAJORITY of industrial systems and physical processes are modeled as the complex forms, which drive us to develop more powerful nonlinear system control strategies. Since the T–S fuzzy systems combines the abundance of linear system theory with fuzzy logic theory into a weighed sum of linear subsystems to approximate the sophisticated nonlinear systems, amount of attempts have done with using Takagi–Sugeno (T–S) fuzzy model successfully [1]–[4]. However, time-delay is undoubtedly generated in the different practical systems owing to inherent network communication, chemical processes, etc., where oscillation, performance damage, or instability commonly outcome [5]–[7]. Since stability is a fundamental demand of the system, the stability analysis has theoretic sense and realistic sense simultaneously. The primary purpose of time-varying system stability is trying to decide the maximum allowable delay upper bound, under the condition of ensuring the stability of the system. Similarly, there exist time delays when nonlinear systems are represented in the form of a T–S fuzzy model. As a consequence, stability study about time delay T–S fuzzy systems turns out a crucial subject and it becomes a popular field in recent decades.

Lyapunov–Krasovskii functional (LKF) method as a standard and efficient method is used popularly to get criteria with time delay [8], [9]. However, how to obtain the maximum upper bound at the same time ensuring stability of the system is still an important issue. The permissible delay bound by the related criterion is commonly used to measure conservatism. Furthermore, the main roots of conservativeness reduction come from two sides: one is the structure of the LKF, another one is the estimation of the derivatives. In consequence, how to select LKF and how to approximate their derivatives are largely responsible for the conservativeness decrease.

On the one hand, constructing LKF is the critical matter to deal with time-delay T–S systems. Multiple integral terms method [10], delay-partitioning technology [11], and the augmented LKF approach [12], [13] were widely applied in the LKFs. In [14], negative quadratic terms were introduced to improve LKF, where the positive definiteness was ensured by combining the original terms and the additional negative term. In recent years, delay-product-type terms were introduced [14]–[16], where the delay and delay-related cross-terms have come out. In [15], the product of delayed-product-matrices and the augmented vectors became part of the LKF, where the integral states and delayed states were introduced to reduce conservatism. In [14], by bringing in delay-produced-type integral terms, more delay cross-terms and delay derivative cross-terms were generated as part of derivative of the LKF. Based on the previous research on the delay-product-type method, how to further reduce conservatism stimulates us to work more.

On the other hand, conservative estimation of LKF derivative have been studied for years. Jensen inequality [17], free-weighting matrices method [18], and free-matrix-based integral inequality [19] were widely used in estimation of single integral term to promote the result. In addition, the convex property plays a significant character in reducing conservatism. The simplest strategy is replacing time-varying delay with bounds [20].

Manuscript received 11 March 2022; revised 29 April 2022; accepted 7 June 2022. Date of publication 14 June 2022; date of current version 30 December 2022. This work was supported in part by the National Key R & D Program of China under Grant 2018YFB1308300, in part by the National Natural Science Foundation of China under Grant U20A20187 and Grant 618255304, in part by the Science Fund for Creative Research Groups of Hebei Province under Grant F2020203013, in part by the Science and Technology Development Grant of Hebei Province under Grant 20311803D and Grant 19011824Z, in part by the National Defence Fundamental Project under Grant 2020A130, in part by the Hebei Innovation Capability Improvement Plan Project under Grant 22567619H, and in part by the China Scholarship Council under Grant CSC202108130134. The work of J.H. Park was supported by the National Research Foundation of Korea (NRF) Grant funded by the Korea government (MSIT) under Grant 2020R1A2B5B02002002. *(Corresponding authors: Ju H. Park; Changchun Hua.)*

Yunfei Qiu is with the Institute of Electrical Engineering, Yanshan University, Qinhuangdao 066004, China, and also with the Department of Electrical Engineering, Yeungnam University, Gyongsan 38541, South Korea (e-mail: yunfeiiqiu@163.com).

Ju H. Park and Xijuan Wang are with the Department of Electrical Engineering, Yeungnam University, Gyongsan 38541, South Korea (e-mail: jessie@ynu.ac.kr; xijuanw02@163.com).

Changchun Hua is with the Institute of Electrical Engineering, Yanshan University, Qinhuangdao 066004, China (e-mail: cch@ysu.edu.cn).

Color versions of one or more figures in this article are available at https://doi.org/10.1109/TFUZZ.2022.3182786.

Digital Object Identifier 10.1109/TFUZZ.2022.3182786

In [21], the reciprocally convex combination lemma came up for the first time. In these years, plenty of works [22]–[25] were developed to make the improvement over the original reciprocally convex inequality. Sometimes, the derivative of LKF is expressed as a quadratic form like $f(x) = A_0 + A_1x + A_2x^2$, where $x \in [0, h]$. In [26], a sufficient condition was proposed for handling it from the convexity. Furthermore, several methods to deal with the negative definite determination of second-order polynomial with insightful analysis [27]–[29]. However, the abovementioned works still exist conservatism. Therefore, these methods should be further improved.

Inspired by the aforementioned materials, this article further studies the delay-dependent stability analysis of time delay T–S systems. By using integral quadratic delay-product-type terms and membership function based method, a novel LKF is constructed. This LKF together with Wirtinger inequality, extended reciprocally convex inequality and matrix-valued polynomial inequality, leads to the criterion with powerful performance. The major contributions are summarized as follows:

1) A new LKF is established, in which a single integral polynomial-delay-product method is devised. The derivatives of the LKF contain not only the delay-related cross terms, but also the delay-derivative-related terms and single integral quadratic-delay-related terms. Consequently, the delay-related information can be used more efficiently in the generated criterion.
2) Based on the inherent nature of membership function, a switched scheme is applied to construct the LKF. That is, the membership function is considered. Although some matrix constraints are added, a criterion with less conservatism can be obtained.
3) A new condition on a quadratic matrix inequality is used, which provides a powerful tool to delay with the quadratic inequality to formulate a novel criterion with less conservatism.

The rest of this article is organized as follows. The problem formulation of time delay T–S fuzzy systems is illustrated in Section II. The stability analysis is shown in Section III. Examples for demonstrating the validity of the presented method and the conclusion are separately shown in Sections IV and V.

Notations: Throughout this article, $\mathcal{R}$ represents the set of all real numbers; $\mathcal{S}_+^n$ means the set of all $n \times n$ symmetric positive definite matrices; $P > 0$ $(P \geq 0)$ stands P is a symmetric definite (semidefinite) matrix; diag$\{\cdot\}$ refers a block-diagonal matrix; col$\{\cdot\}$ stands a column vector or matrix.

II. Problem Formulation

Consider a time-delay T–S fuzzy system represented as follows:

Plant Rule i: If $z_1(t)$ is M_{i1} and … and $z_p(t)$ is M_{ip}, *then*

$$\begin{aligned}\dot{x}(t) &= A_i x(t) + A_{di} x(t-h(t))\\ x(t) &= \phi(t), \quad t \in [-h, 0]\end{aligned} \tag{1}$$

where $x(t) \in \mathcal{R}^n$ represents the state variable; $A_i \in \mathcal{R}^{n\times n}$, $A_{di} \in \mathcal{R}^{n\times n}$ and $B_i \in \mathcal{R}^{n\times m}$ are system matrices; $z_j(t)$ $(j = 1,\ldots,p)$ is the premise variable; M_{ij} $(i = 1,\ldots,r; j = 1,\ldots,p)$ is fuzzy set; r is the number of fuzzy rules; $h(t)$ is time delay with the conditions as follows:

$$0 \leq h(t) \leq h, \mu_1 \leq \dot{h}(t) \leq \mu_2, \mu_1 = -\mu_2. \tag{2}$$

By making use of the fuzzy inference approach, system (1) can be denoted as

$$\dot{x}(t) = \sum_{i=1}^{r} \lambda_i(z(t))(A_i x(t) + A_{di} x(t-h(t))) \tag{3}$$

and

$$\lambda_i(z(t)) = \frac{\omega_i(z(t))}{\Sigma_{i=1}^r \omega_i(z(t))}, \quad \omega_i(z(t)) = \prod_{j=1}^{p} M_{ij}(z_j(t))$$

with $\lambda_i(z(t))$ referring the normalized membership function, $M_{ij}(z(t))$ referring the grade of membership of $z_j(t)$ in M_{ij}. Suppose $\omega_i(z(t)) > 0$ for $t \geq 0$, then we obtain the nature of the membership function

$$\sum_{i=1}^{r} \lambda_i(z(t)) = 1, \quad \sum_{i=1}^{r} \dot{\lambda}_i(z(t)) = 0. \tag{4}$$

Before presenting the major result of this article, some useful lemmas are presented as follows:

Lemma 1 [30], [31]: For a differentiable vector $x(s) \in \mathcal{R}^n$, $a < b$ and a matrix $R \in \mathcal{S}_+^n$, we have the following inequalities:

$$\begin{cases} c\int_a^b x^T(s)Rx(s)ds \geq \hat{\psi}_1^T R\hat{\psi}_1 + 3\hat{\psi}_2^T R\hat{\psi}_2 \\ c\int_a^b \dot{x}^T(s)R\dot{x}(s)ds \geq \psi_1^T R\psi_1 + 3\psi_2^T R\psi_2 + 5\psi_3^T R\psi_3 \end{cases}$$

$$\begin{aligned}&\frac{c^2}{2}\int_a^b\int_\theta^b x^T(s)Rx(s)dsd\theta\\ &\quad\geq \int_a^b\int_\theta^b x^T(s)dsd\theta R\int_a^b\int_\theta^b x(s)dsd\theta + 2\psi_4^T R\psi_4\end{aligned}$$

where $c = b - a$ and

$$\begin{cases} \hat{\psi}_1 = \int_a^b x(s)ds \\ \hat{\psi}_2 = \int_a^b x(s)ds - \frac{2}{c}\int_a^b\int_\theta^b x(s)dsd\theta \\ \psi_1 = x(b) - x(a) \\ \psi_2 = x(b) + x(a) - \frac{2}{c}\int_a^b x(s)ds \\ \psi_3 = x(b) - x(a) + \frac{6}{c}\int_a^b x(s)ds - \frac{12}{c^2}\int_a^b\int_\theta^b x(s)dsd\theta \\ \psi_4 = -\int_a^b\int_\theta^b x(s)dsd\theta + \frac{3}{c}\int_a^b\int_\theta^b\int_s^b x(v)dvdsd\theta. \end{cases}$$

Lemma 2 [23]: Let $R_1, R_2 \in \mathcal{S}_+^n, \alpha \in (0,1)$, matrices $X_1, X_2 \in \mathcal{S}^n$ and real matrices $Y_1, Y_1 \in \mathcal{R}^{n\times n}$, if the following inequalities hold:

$$\begin{bmatrix} R_1 - X_1 & Y_1 \\ * & R_2 \end{bmatrix} \geq 0, \quad \begin{bmatrix} R_1 & Y_2 \\ * & R_2 - X_2 \end{bmatrix} \geq 0.$$

In this way, the following inequality holds:

$$\begin{bmatrix} \frac{1}{\alpha}R_1 & 0 \\ 0 & \frac{1}{1-\alpha}R_2 \end{bmatrix} \geq \begin{bmatrix} R_1 + (1-\alpha)X_1 & \alpha Y_1 + (1-\alpha)Y_2 \\ * & R_2 + \alpha X_2 \end{bmatrix}.$$

Lemma 3 [32]: For matrices $Z_h = \sum_{i=1}^{r} \lambda_i(z(t))Z_i$ and $X_h = \sum_{i=1}^{r} \lambda_i(z(t))X_i$ with $Z_i > 0$ and $X_i > 0$. We have $\dot{Z}_h \leq 0$ and $\dot{X}_h \leq 0$ if the following equalities (5) is satisfied:

$$\begin{cases} \dot{\lambda}_j(z(t)) < 0: & Z_j - Z_r > 0, X_j - X_r > 0 \\ \dot{\lambda}_j(z(t)) \geq 0: & Z_j - Z_r \leq 0, X_j - X_r \leq 0 \end{cases} \tag{5}$$

where $j = 1, \ldots, r-1$.

Lemma 4 [33]: The matrix polynomial inequality

$$A_0 + A_1 h(t) + A_2 h^2(t) < 0$$

with $A_0, A_1, A_2 \in \mathbb{R}^{n\times n}$ holds for arbitrary $h(t) \in [0, h]$ if and only if there are a skew-symmetric matrix $G \in \mathcal{R}^{n\times n}$ and $D \in \mathcal{S}_n^+$ such that

$$\begin{bmatrix} A_0 & \frac{1}{2}A_1 \\ \frac{1}{2}A_1 & A_2 \end{bmatrix} + \begin{bmatrix} C \\ J \end{bmatrix}^T \begin{bmatrix} D & G \\ G^T & -D \end{bmatrix} \begin{bmatrix} C \\ J \end{bmatrix} < 0$$

where $C = [hI \quad 0]$ and $J = [hI \quad -2I]$.

Remark 1: In [33], a necessary and sufficient condition about deciding negative definiteness of matrix polynomial inequality was established. A similar result was formulated in [34], where the decision variables are same as those in [33]. The condition of positive definiteness of matrix polynomial inequality can be easily obtained from Lemma 4, which will be used later.

III. Main Result

This article devotes to study the stability of the time delay T–S fuzzy systems with making use of membership function based matrices and single integral quadratic-delay-related terms. For simplicity, some expressions are listed as follows:

$$e_0 = 0_{n\times 10n}$$

$$e_i = \begin{bmatrix} 0_{n\times(i-1)n} & I_{n\times n} & 0_{n\times(10-i)n} \end{bmatrix}, i = 1, \ldots, 10$$

$$T_\varsigma = T_\varsigma(h(t), \dot{h}(t)), T_{\varsigma 0} = T_{\varsigma 0}(\dot{h}(t)), T_{\varsigma 1} = T_{\varsigma 1}(\dot{h}(t))$$

$$\hat{T}_\varsigma = \hat{T}_\varsigma(h(t), \dot{h}(t)), \hat{T}_{\varsigma 0} = \hat{T}_{\varsigma 0}(\dot{h}(t)), \hat{T}_{\varsigma 1} = \hat{T}_{\varsigma 1}(\dot{h}(t))$$

$$h_t = h - h(t), \varsigma = 1, 2, 3, 4$$

$$\eta_1(t) = col\{x(t), h(t)v_6(t), h_t v_7, h(t)v_8(t), h_t v_9\}$$

$$\eta_2(t,s) = col\{x(s), \dot{x}(s), x(t) \quad x(t-h(t)), x(t-h), \int_s^t x(u)du\}$$

$$\eta_3(t,s) = col\{x(s), \dot{x}(s), x(t) \quad x(t-h(t)), x(t-h), \int_s^{t-h(t)} x(u)du\}$$

$$v_6(t) = \int_{t-h(t)}^{t} \frac{x(s)}{h(t)} ds, v_7(t) = \int_{t-h}^{t-h(t)} \frac{x(s)}{h_t} ds$$

$$v_8(t) = \int_{t-h(t)}^{t} \int_s^t \frac{x(u)}{h^2(t)} du ds$$

$$v_9(t) = \int_{t-h}^{t-h(t)} \int_s^{t-h(t)} \frac{x(u)}{h_t^2} du ds$$

$$\xi(t) = col\{x(t), x(t-h(t)), x(t-h), \dot{x}(t-h(t)) \quad \dot{x}(t-h), v_6, v_7, v_8, v_9, \dot{x}(t)\}.$$

Theorem 1: Given h, μ_k, system (3) is asymptotically stable if there are symmetric matrices with appropriate dimension $P_i > 0, Q_{1i} > 0, Q_{2i} > 0, R_\varsigma > 0, H > 0, Z_{11}, Z_1, Z_2, Z_{22}, Z_3, Z_4, M_{11}, M_1, M_2, M_{22}, M_3, M_4, X_1, X_2, D_{1\varsigma}, D_{2i}$, any matrices Y_1, Y_2, K_ι, and skew-symmetric matrices $G_{1\varsigma}, G_{2i}$, for $k = 1, 2$, $i = 1, \ldots, r$, $\varsigma = 1, 2, 3, 4$, $\iota = 1, 2, 3$, $\nu = 0, 1$ *or none*, satisfying the following equalities:

$$\dot{P}_h \leq 0, \dot{Q}_{1\,h} \leq 0, \dot{Q}_{2\,h} \leq 0 \tag{6}$$

$$ZM_\varsigma + \begin{bmatrix} C_1 \\ J_1 \end{bmatrix}^T \begin{bmatrix} D_{1\varsigma} & G_{1\varsigma} \\ G_{1\varsigma}^T & -D_{1\varsigma} \end{bmatrix} \begin{bmatrix} C_1 \\ J_1 \end{bmatrix} < 0 \tag{7}$$

$$T_\varsigma(h(t), u_k) > 0 \tag{8}$$

$$\begin{bmatrix} \hat{T}_1 - X_1 & Y_1 \\ * & \hat{T}_2 \end{bmatrix} \geq 0 \quad \begin{bmatrix} \hat{T}_1 & Y_2 \\ * & \hat{T}_2 - X_2 \end{bmatrix} \geq 0 \tag{9}$$

$$\begin{bmatrix} A_i^0(u_k) & \frac{1}{2}A_i^1(u_k) \\ * & A_i^2(u_k) \end{bmatrix} + \begin{bmatrix} C_2 \\ J_2 \end{bmatrix}^T \begin{bmatrix} D_{2i} & G_{2i} \\ G_{2i}^T & -D_{2i} \end{bmatrix} \begin{bmatrix} C_2 \\ J_2 \end{bmatrix} < 0 \tag{10}$$

where other equations are listed in Appendix.

Proof: Considering the following LKF:

$$V(t) = \sum_{i=1}^{5} V_i(t) \tag{11}$$

where

$$V_1(t) = \eta_1^T(t) P_h \eta_1(t)$$

$$V_2(t) = \int_{t-h(t)}^{t} \eta_2^T(t,s) Q_{1\,h} \eta_2(t,s) ds + \int_{t-h}^{t-h(t)} \eta_3^T(t,s) Q_{2\,h} \eta_3(t,s) ds$$

$$V_3(t) = \int_{t-h(t)}^{t} \dot{x}^T(s)(h^2(t)Z_{11} + h(t)Z_1 + Z_2)\dot{x}(s)ds + \int_{t-h}^{t-h(t)} \dot{x}^T(s)(h_t^2 Z_{22} + h_t Z_3 + Z_4)\dot{x}(s)ds + \int_{-h(t)}^{0} \int_{t+\theta}^{t} \dot{x}^T(s) R_1 \dot{x}(s) ds d\theta + \int_{-h}^{-h(t)} \int_{t+\theta}^{t} \dot{x}^T(s) R_2 \dot{x}(s) ds d\theta,$$

$$V_4(t) = \int_{t-h(t)}^{t} x^T(s)(h^2(t)M_{11} + h(t)M_1 + M_2)x(s)ds + \int_{t-h}^{t-h(t)} x^T(s)(h_t^2 M_{22} + h_t M_3 + M_4)x(s)ds + \int_{-h(t)}^{0} \int_{t+\theta}^{t} x^T(s) R_3 x(s) ds d\theta$$

$$+\int_{-h}^{-h(t)}\int_{t+\theta}^{t} x^T(s)R_4x(s)dsd\theta$$

$$V_5(t)=\int_{t-h}^{t}\int_{\theta}^{t}\int_{u}^{t}\dot{x}^T(s)H\dot{x}(s)dsdud\theta$$

with $P_h=\sum_{i=1}^{r}\lambda_i(z(t))P_i$, $Q_{1\,h}=\sum_{i=1}^{r}\lambda_i(z(t))Q_{1i}$ and $Q_{2\,h}=\sum_{i=1}^{r}\lambda_i(z(t))Q_{2i}$.

The positive definiteness of $V_1(t)$, $V_2(t)$ can be proved by $P_i>0, Q_{1i}>0$ and $Q_{2i}>0$. The positive definiteness of $V_3(t)$, $V_4(t)$ and $V_5(t)$ can be guaranteed by (7), $R_\varsigma>0$ ($\varsigma=1,2,3,4$) and $H>0$.

Next, calculating the derivatives yields

$$\dot{V}(t)=\sum_{i=1}^{5}\dot{V}_1(t) \tag{12}$$

where

$$\begin{aligned}
\dot{V}_1(t)&=\text{sym}\{\eta_1^T(t)P_h\dot{\eta}_1(t)\}+\eta_1^T(t)\dot{P}_h\eta_1(t)\\
&\le\sum_{i=1}^{r}\lambda_i(z(t))\xi^T(t)(\Phi_{10}+h(t)\Phi_{11})\xi(t)\\
\dot{V}_2(t)&=\eta_2^T(t,t)Q_{1\,h}\eta_2(t,t)-(1-\dot{h}(t))\eta_2^T(t,h_t)Q_{1\,h}\eta_2(t,h_t)\\
&\quad+\text{sym}\left\{\int_{t-h(t)}^{t}\eta_2^T(t,s)Q_{1\,h}\frac{\partial\eta_2(t,s)}{\partial t}ds\right\}\\
&\quad+\int_{t-h(t)}^{t}\eta_2^T(t,s)\dot{Q}_{1\,h}\eta_2(t,s)ds\\
&\quad+(1-\dot{h}(t))\eta_3^T(t,h_t)Q_{2\,h}\eta_3(t,h_t)\\
&\quad-\eta_3^T(t,t-h)Q_{2\,h}\eta_3(t,t-h)\\
&\quad+\text{sym}\left\{\int_{t-h}^{t-h(t)}\eta_3^T(t,s)Q_{2\,h}\frac{\partial\eta_3(t,s)}{\partial t}ds\right\}\\
&\quad+\int_{t-h}^{t-h(t)}\eta_3^T(t,s)\dot{Q}_{2\,h}\eta_3(t,s)ds\\
&\le\sum_{i=1}^{r}\lambda_i(z(t))\xi^T(t)(\Phi_{20}(\dot{h}(t))+h(t)\Phi_{21}(\dot{h}(t))\\
&\qquad+h^2(t)\Phi_{22}(\dot{h}(t)))\xi(t)\\
\dot{V}_3(t)&=\dot{x}^T(t)[h^2(t)Z_{11}+h(t)Z_1+Z_2+h(t)R_1+h_tR_2]\\
&\quad\times\dot{x}(t)+(1-\dot{h}(t))\dot{x}^T(t-h(t))[-h^2(t)Z_{11}\\
&\quad-h(t)Z_1-Z_2+h_t^2Z_{22}+h_tZ_3+Z_4]\dot{x}(t-h(t))\\
&\quad+\dot{x}^T(t-h)[-h_t^2Z_{22}-h_tZ_3-Z_4]\dot{x}(t-h)\\
&\quad-\int_{t-h(t)}^{t}\dot{x}^T(s)\Big((1-\dot{h}(t))R_1+\dot{h}(t)R_2\\
&\quad-2h(t)\dot{h}(t)Z_{11}-\dot{h}(t)Z_1\Big)\dot{x}(s)ds\\
&\quad-\int_{t-h}^{t-h(t)}\dot{x}^T(s)T_2\dot{x}(s)ds\\
&=\xi^T(t)(\Phi_{30}(\dot{h}(t))+h(t)\Phi_{31}(\dot{h}(t))+h^2(t)\Phi_{32}(\dot{h}(t)))\\
&\quad\times\xi(t)-\int_{t-h(t)}^{t}\dot{x}^T(s)\Big((1-\dot{h}(t))R_1+\dot{h}(t)R_2\\
&\quad-2h(t)\dot{h}(t)Z_{11}-\dot{h}(t)Z_1\Big)\dot{x}(s)ds\\
&\quad-\int_{t-h}^{t-h(t)}\dot{x}^T(s)T_2\dot{x}(s)ds\\
\dot{V}_4(t)&=x^T(t)[h^2(t)M_{11}+h(t)M_1+M_2+h(t)R_3\\
&\quad+h_tR_4]x(t)+(1-\dot{h}(t))x^T(t-h(t))\\
&\quad\times[-h^2(t)M_{11}-h(t)M_1-M_2+h_t^2M_{22}\\
&\quad+h_tM_3+M_4]x(t-h(t))\\
&\quad+x^T(t-h)[-h_t^2M_{22}-h_tM_3\\
&\quad-M_4]x(t-h)-\int_{t-h(t)}^{t}x^T(s)T_3x(s)ds\\
&\quad-\int_{t-h}^{t-h(t)}x^T(s)T_4x(s)ds\\
&=\xi^T(t)(\Phi_{40}(\dot{h}(t))+h(t)\Phi_{41}(\dot{h}(t))+h^2(t)\Phi_{42}(\dot{h}(t)))\\
&\quad\times\xi(t)-\int_{t-h(t)}^{t}x^T(s)T_3x(s)ds.\\
&\quad-\int_{t-h}^{t-h(t)}x^T(s)T_4x(s)ds\\
\dot{V}_5(t)&=\frac{h^2}{2}\dot{x}^T(t)H\dot{x}(t)-\int_{t-h(t)}^{t}\int_{\theta}^{t}\dot{x}^T(s)H\dot{x}(s)dsd\theta\\
&\quad-\int_{t-h}^{t-h(t)}\int_{\theta}^{t-h(t)}\dot{x}^T(s)H\dot{x}(s)dsd\theta\\
&\quad-h_t\int_{t-h(t)}^{t}\dot{x}^T(s)H\dot{x}(s)ds.
\end{aligned}$$

Using Lemma 1 to calculate the above double integral terms, then the following holds:

$$\dot{V}_5(t)\le\xi^T(t)\Phi_{50}\xi(t)-h_t\int_{t-h(t)}^{t}\dot{x}^T(s)H\dot{x}(s)ds.$$

Based on convex combination technique, inequalities (8) can ensure the positiveness of T_ς. Next, utilizing Lemma 1 to estimate T_1, T_2 related integral terms yields

$$\begin{aligned}
&-\int_{t-h(t)}^{t}\dot{x}^T(s)T_1\dot{x}(s)ds-\int_{t-h}^{t-h(t)}\dot{x}^T(s)T_2\dot{x}(s)ds\\
&\le-\xi^T(t)\begin{bmatrix}E_1\\E_2\end{bmatrix}^T\begin{bmatrix}\frac{1}{h(t)}\hat{T}_1 & 0\\ 0 & \frac{1}{h_t}\hat{T}_2\end{bmatrix}\begin{bmatrix}E_1\\E_2\end{bmatrix}\xi(t).
\end{aligned}\tag{13}$$

Applying Lemma 2, if there exist suitable dimensional symmetrical matrices X_1, X_2, any matrices Y_1, Y_2 satisfying condition (9), then the following holds:

$$-\begin{bmatrix}\frac{1}{h(t)}\hat{T}_1 & 0\\ 0 & \frac{1}{h_t}\hat{T}_2\end{bmatrix} \leq -\frac{1}{h}\begin{bmatrix}\hat{T}_1+(1-\frac{h(t)}{h})X_1 & \frac{h(t)}{h}Y_1+(1-\frac{h(t)}{h})Y_2\\ * & \hat{T}_2+\frac{h(t)}{h}X_2\end{bmatrix}. \quad (14)$$

Applying the Lemma 1 to estimate T_3 and T_4 yields

$$-\int_{t-h(t)}^{t} x^T(s)T_3x(s)ds - \int_{t-h}^{t-h(t)} x^T(s)T_4x(s)ds \leq -\xi^T(t)\begin{bmatrix}E_3\\E_4\end{bmatrix}^T\begin{bmatrix}h(t)\hat{T}_3 & 0\\ 0 & h_t\hat{T}_4\end{bmatrix}\begin{bmatrix}E_3\\E_4\end{bmatrix}\xi(t). \quad (15)$$

For any proper dimensional matrices K_1, K_2, and K_3, we can easily obtain the below constant zero term (16)

$$2\sum_{i=1}^{r}\lambda_i(z(t))\Gamma(t)[-\dot{x}(t)+A_ix(t)+A_{di}x(t-h(t)]=0 \quad (16)$$

in which $\Gamma(t)=\xi^T(t)(e_1^TK_1+e_2^TK_2+e_{10}^TK_3)$.

By combing the above formulations (11)–(16), we can get the following inequalities:

$$\dot{V}(t) \leq \sum_{i=1}^{r}\lambda_i(z(t))\xi^T(t)\mathcal{A}_i\xi(t) \quad (17)$$

where $\mathcal{A}_i = A_i^1 + h(t)A_i^1 + h^2(t)A_i^2$.

Based on Lemma 4, inequality $\mathcal{A}_i<0$ holds if there are skew-symmetric matrices G_{2i}, symmetric matrices D_{2i} and fixed matrices C_2 and J_2 such that inequalities (10) hold. This completes the proof. ■

Remark 2: As previously stated in [15], [16], and [35], delay and delay derivative related terms may effectively reveal the delay information, which can help reduce the conservatism. In [14], delay-product-type terms were employed. In our work, we consider the single integral quadratic-delay-related terms in $V_3(t)$ and $V_4(t)$. As a result, there are delay and delay derivative dependent terms that can be produced, as well as a number of single integral quadratic-delay-related terms such as $\dot{h}(t)\int_{t-h(t)}^{t}\dot{x}^T(s)Z_1\dot{x}(s)ds$, $2\dot{h}(t)h(t)\int_{t-h(t)}^{t}\dot{x}^T(s)Z_{11}\dot{x}(s)ds$.

Remark 3: In order to make LKF (11) keep positive, the positiveness of $h^2(t)Z_{11}+h(t)Z_1+Z_2$, $h_t^2Z_{22}+h_tZ_3+Z_4$, $h^2(t)M_{11}+h(t)M_1+M_2$ and $h_t^2M_{22}+h_tM_3+M_4$ can be guaranteed by condition (7), which can be easily obtained by Lemma 4.

Remark 4: A few publications used membership function information to address stability issues about T–S fuzzy systems [16]. Different from the LKFs employed in the previous studies, the developed LKF in this article employs a membership function-dependent matrix approach, resulting in a superior stability criterion with a larger time delay upper bound when compared to the existing literature. In spite of some constraints on matrices increasing, the criterion also reduces the conservatism.

Remark 5: To fully utilize the membership function information, we employ the method proposed in [32], which is on account of the essence of the membership function. In this way, a new time-varying matrices are constructed by using the membership function, and the matrices are negative definite after derivation. There exist 2^{r-1} situations, the finally largest upper bound is obtained by $h=\min_{1\leq q\leq 2^{r-1}}\{h_q\}$. For example, we can get two different upper bounds under two circumstances in Example 1. When $\dot{\lambda}_1(t)<0$, the sampling interval is h_1; When $\dot{\lambda}_1(t)\geq 0$, the sampling interval is h_2. The ultimate largest upper bound is $h=\min\{h_1,h_2\}$.

TABLE I
MAXIMUM UPPER BOUNDS

Methods	[36]	[16]	[37]	[38]	Theorem 1
u=0	1.9011	2.3269	2.5932	3.6167	3.6446
u=0.1	1.7411	2.1190	2.3268	-	3.2464

IV. Simulation Examples

Two examples are shown for verifying the validity of the suggested strategy. The maximal upper bounds are given in Tables, which embodies the superiority.

Example 1: The widely studied time-delay fuzzy system is considered [32], [38]

$$\dot{x}(t)=\sum_{i=1}^{2}\lambda_i(z(t))[A_ix(t)+A_{di}x(t-h(t))]$$

where

$$A_1=\begin{bmatrix}-2 & 0\\ 0 & -0.9\end{bmatrix},\quad A_2=\begin{bmatrix}-1 & 0.5\\ 0 & -1\end{bmatrix}$$

$$A_{d1}=\begin{bmatrix}-1 & 0\\ -1 & -1\end{bmatrix},\quad A_{d2}=\begin{bmatrix}-1 & 0\\ 0.1 & -1\end{bmatrix}$$

and the membership functions are

$$\lambda_1(t)=\frac{1}{1+e^{-2x_1(t)}},\quad \lambda_2(t)=1-\lambda_1(t).$$

The upper bound of maximum delay h is computed via Theorem 1. First, making $u_1=0$ means the system is studied as constant delay system. When $\{P_1>P_2, Q_{11}>Q_{12}, Q_{21}>Q_{22}\}$, $h_1=3.7439$; when $\{P_1\leq P_2, Q_{11}\leq Q_{12}, Q_{21}\leq Q_{22}\}$, $h_2=3.6446$, so the final maximum sample interval $h=\min\{h_1,h_2\}=3.6446$. Second, making $u_1=0.1$ means the system is studied as the time-varying delay system. When $\{P_1>P_2, Q_{11}>Q_{12}, Q_{21}>Q_{22}\}$, $h_1=3.3210$; when $\{P_1\leq P_2, Q_{11}\leq Q_{12}, Q_{21}\leq Q_{22}\}$, $h_2=3.2464$, so the final maximum sample interval $h=\min\{h_1,h_2\}=3.2464$. By making $u_1=\{0,0.1\}$, Table I displays the different values and shows Theorem 1 is less conservative.

TABLE II
MAXIMUM UPPER BOUNDS

Methods	[39]	[40]	[37]	[41]	Theorem 1
u=0	3.82	4.37	5.5826	5.5973	5.6897
u=0.1	3.09	3.41	4.2044	4.2928	4.4982
u=0.5	1.95	1.95	2.0685	2.2571	2.9666

The responses of the system is demonstrated in Fig. 1. The curves shows the system stays stable with $h(t) = \frac{3.2464}{2}\sin(\frac{0.2}{3.2464}t) + \frac{3.2464}{2}$ and $x_0(t) = [-1; 1.5]$.

Example 2: The widely studied time-delay fuzzy system is considered [39]–[41]

$$\dot{x}(t) = \sum_{i=1}^{2} \lambda_i(z(t))[A_i x(t) + A_{di} x(t - h(t))]$$

where

$$A_1 = \begin{bmatrix} -2.1 & 0.1 \\ -0.2 & -0.9 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -1.9 & 0 \\ -0.2 & -1.1 \end{bmatrix}$$

$$A_{d1} = \begin{bmatrix} -1.1 & 0.1 \\ -0.8 & -0.9 \end{bmatrix}, \quad A_{d2} = \begin{bmatrix} -1.9 & 0 \\ -1.1 & -1.2 \end{bmatrix}$$

and the membership functions are

$$\lambda_1(t) = \frac{1}{1 + e^{-2x_1(t)}}, \quad \lambda_2(t) = 1 - \lambda_1(t).$$

The maximum delay bounds for different derivatives are given in Table II based on the method in Theorem 1. In this way, this novel method can offer less conservative results compared with the results in Example 2.

Remark 6: There are some papers that have shown the effectiveness of the result may be greatly affected by the type of the antecedent set [42]–[44]. The CNO type antecedents and SNNN type antecedents are used to test the generality of the method. The obtained CNO related results provide better performance than the T–S related results. In the future, we may focus on how to select fuzzy antecedents to gain better results.

V. CONCLUSION

In this article, stability analysis of time delay T–S fuzzy systems is investigated. A new LKF makes full use of single integral polynomial-delay-product terms and membership-function-dependent matrices. To cope with the derivatives of the LKF, Wirtinger inequality, auxiliary function-based integral inequality, and matrix-valued polynomial inequalities are utilized. Then, the novel stability criterion with less conservatism is constructed using the Lyapunov stability theory. Ultimately, two examples are utilized for verifying the advantage of the criterion. However, how to further minimize conservatism of the stability and reduce the computation still matters. Based on the existing works, designing a more powerful method with using membership function information to deal with the T–S fuzzy system is our future research plan

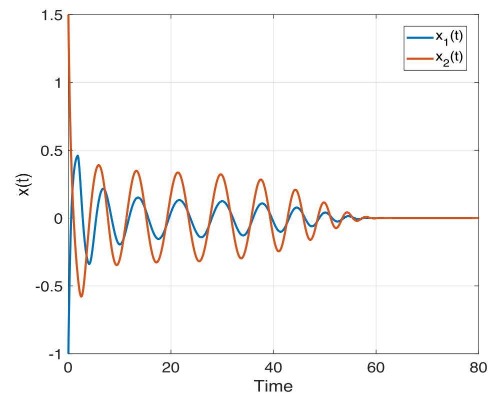

Fig. 1. Responses with h(t) $= \frac{3:2464}{2}\sin(\frac{0:2}{3:2464}t) + \frac{3:2464}{2}$.

APPENDIX

$$A_i^0 = \Phi_{10} + \Phi_{20} + \Phi_{30} + \Phi_{40} + \Phi_{50} + \Phi_{60} + \Phi_{70} + \Phi_{80}$$

$$A_i^1 = \Phi_{11} + \Phi_{21} + \Phi_{31} + \Phi_{41} + \Phi_{61} + \Phi_{71}$$

$$A_i^2 = \Phi_{22} + \Phi_{32} + \Phi_{42} + \Phi_{72}$$

$$\Phi_{10} = \mathrm{sym}\{f_{10}^T P_i f_2\}, \Phi_{11} = \mathrm{sym}\{f_{11}^T P_i f_2\}$$

$$\Phi_{20}(\dot{h}(t)) = F_1^T Q_{1i} F_1 - (1 - \dot{h}(t)) F_{20}^T Q_{1i} F_{20} - F_{40}^T Q_{2i} F_{40} + (1 - \dot{h}(t)) F_3^T Q_{2i} F_3 + \mathrm{sym}\{F_{50}^T Q_{1i} F_6 + F_{70}^T Q_{2i} F_8\}$$

$$\Phi_{21}(\dot{h}(t)) = \mathrm{sym}\{-(1 - \dot{h}(t)) F_{21}^T Q_{1i} F_{20} - F_{41}^T Q_{2i} F_{40} + F_{51}^T Q_{1i} F_6 + F_{71}^T Q_{2i} F_8\}$$

$$\Phi_{22}(\dot{h}(t)) = -(1 - \dot{h}(t)) F_{21}^T Q_{1i} F_{21} - F_{41}^T Q_{2i} F_{41} + \mathrm{sym}\{F_{52}^T Q_{1i} F_6 + F_{72}^T Q_{2i} F_8\}$$

$$\Phi_{30}(\dot{h}(t)) = e_{10}^T (Z_2 + hR_2) e_{10} + (1 - \dot{h}(t)) e_4^T (-Z_2 + Z_4 + h^2 Z_{22} + hZ_3) e_4 - e_5^T (h^2 Z_{22} + hZ_3 + Z_4) e_5$$

$$\Phi_{31}(\dot{h}(t)) = e_{10}^T (Z_1 + R_1 - R_2) e_{10} + (1 - \dot{h}(t)) e_4^T (-Z_1 - 2hZ_{22} - Z_3) e_4 + e_5^T (2hZ_{22} + Z_3) e_5$$

$$\Phi_{32}(\dot{h}(t)) = e_{10}^T Z_{11} e_{10} + (1 - \dot{h}(t)) e_4^T (-Z_{11} + Z_{22}) e_4 - e_5^T Z_{22} e_5$$

$$\Phi_{40}(\dot{h}(t)) = e_1^T (M_2 + hR_4) e_1 + (1 - \dot{h}(t)) e_2^T (-M_2 + M_4 + h^2 M_{22} + hM_3) e_2 - e_3^T (h^2 M_{22} + hM_3 + M_4) e_3$$

$$\Phi_{41}(\dot{h}(t)) = e_1^T (M_1 + R_3 - R_4) e_1 + (1 - \dot{h}(t)) e_2^T (-M_1 - 2hM_{22} - M_3) e_2 + e_3^T (2hM_{22} + M_3) e_3$$

$$\Phi_{42}(\dot{h}(t)) = e_1^T M_{11} e_1 + (1 - \dot{h}(t)) e_2^T (-M_{11} + M_{22}) e_2 - e_3^T M_{22} e_3$$

$$\Phi_{50} = \frac{h^2}{2} e_{10}^T H e_{10} - 2f_3^T H f_3 - f_4^T H f_4 - 2f_5^T H f_5 - f_6^T H f_6$$

$$\Phi_{60} = -\frac{1}{h}\begin{bmatrix} E_1 \\ E_2 \end{bmatrix}^T \begin{bmatrix} \hat{T}_{10} + X_1 & Y_2 \\ * & \hat{T}_{20} \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \end{bmatrix}$$

$$\Phi_{61} = -\frac{1}{h}\begin{bmatrix} E_1 \\ E_2 \end{bmatrix}^T \begin{bmatrix} \hat{T}_{11} - \frac{1}{h}X_1 & \frac{1}{h}Y_1 - \frac{1}{h}Y_2 \\ * & \hat{T}_{21} + \frac{1}{h}X_2 \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \end{bmatrix}$$

$$\Phi_{70} = -hE_4^T \hat{T}_{40} E_4$$

$$\Phi_{71} = -E_3^T \hat{T}_{30} E_3 - hE_4^T \hat{T}_{41} E_4 + E_4^T \hat{T}_{40} E_4$$

$$\Phi_{72} = -E_3^T \hat{T}_{31} E_3 + E_4^T \hat{T}_{41} E_4$$

$$\Phi_{80} = \mathrm{sym}\{\Gamma_0 \times (e_{10} - A_i e_1 - A_{di} e_2)\}$$

$$\Gamma_0 = e_1^T K_1 + e_2^T K_2 + e_{10}^T K_3$$

$$T_1 = (1-\dot{h}(t))R_1 + \dot{h}(t)R_2 - 2h(t)\dot{h}(t)Z_{11} - \dot{h}(t)Z_1 + h_t H$$

$$T_2 = R_2 + 2h_t\dot{h}(t)Z_{22} + \dot{h}(t)Z_3$$

$$T_3 = (1-\dot{h}(t))R_3 + \dot{h}(t)R_4 - 2h(t)\dot{h}(t)M_{11} - \dot{h}(t)M_1$$

$$T_4 = R_4 + 2h_t\dot{h}(t)M_{22} + \dot{h}(t)M_3$$

$$T_{10} = (1-\dot{h}(t))R_1 + \dot{h}(t)R_2 - \dot{h}(t)Z_1 + hH$$

$$T_{11} = -2\dot{h}(t)Z_{11} - H$$

$$T_{20} = R_2 + 2h\dot{h}(t)Z_{22} + \dot{h}(t)Z_3$$

$$T_{21} = -2\dot{h}(t)Z_{22}$$

$$T_{30} = (1-\dot{h}(t))R_3 + \dot{h}(t)R_4 - \dot{h}(t)M_1$$

$$T_{31} = -2\dot{h}(t)M_{11}$$

$$T_{40} = R_4 + 2h\dot{h}(t)M_{22} + \dot{h}(t)M_3$$

$$T_{41} = -2\dot{h}(t)M_{22}$$

$$\hat{T}_{1\nu} = \begin{bmatrix} T_{1\nu} & 0 & 0 \\ * & 3T_{1\nu} & 0 \\ * & * & 5T_{1\nu} \end{bmatrix}, \quad \hat{T}_{3\nu} = \begin{bmatrix} T_{3\nu} & 0 \\ * & 3T_{3\nu} \end{bmatrix}$$

$$\hat{T}_{2\nu} = \begin{bmatrix} T_{2\nu} & 0 & 0 \\ * & 3T_{2\nu} & 0 \\ * & * & 5T_{2\nu} \end{bmatrix}, \quad \hat{T}_{4\nu} = \begin{bmatrix} T_{4\nu} & 0 \\ * & 3T_{4\nu} \end{bmatrix}$$

$$C_1 = [hI_{n\times n}\ 0_{n\times n}]; C_2 = [hI_{10n\times 10n}\ 0_{10n\times 10n}]$$

$$J_1 = [hI_{n\times n}\ -2I_{n\times n}]; J_2 = [hI_{10n\times 10n}\ -2I_{10n\times 10n}]$$

$$F_1 = col\{e_1, e_{10}, e_1, e_2, e_3, e_0\}$$

$$F_{20} = \mathrm{col}\{e_2, e_4, e_1, e_2, e_3, e_0\}$$

$$F_{21} = \mathrm{col}\{e_0, e_0, e_0, e_0, e_0, e_6\}$$

$$F_3 = \mathrm{col}\{e_2, e_4, e_1, e_2, e_3, e_0\}$$

$$F_{40} = \mathrm{col}\{e_3, e_5, e_1, e_2, e_3, he_7\}$$

$$F_{41} = \mathrm{col}\{e_0, e_0, e_0, e_0, e_0, -e_7\}$$

$$F_{50} = \mathrm{col}\{e_0, e_1 - e_2, e_0, e_0, e_0, e_0\}$$

$$F_{51} = \mathrm{col}\{e_6, e_0, e_1, e_2, e_3, e_0\}$$

$$F_{52} = \mathrm{col}\{e_0, e_0, e_0, e_0, e_0, e_8\}$$

$$F_6 = \mathrm{col}\{e_0, e_0, e_{10}, (1-\dot{h}(t))e_4, e_5, e_1\}$$

$$F_{70} = \mathrm{col}\{he_7, e_2 - e_3, he_1, he_2, he_3, h^2 e_9\}$$

$$F_{71} = \mathrm{col}\{-e_7, e_0, -e_1, -e_2, -e_3, -2he_9\}$$

$$F_{72} = \mathrm{col}\{e_0, e_0, e_0, e_0, e_0, e_9\}$$

$$F_8 = \mathrm{col}\{e_0, e_0, e_{10}, (1-\dot{h}(t))e_4, e_5, (1-\dot{h}(t))e_2\}$$

$$f_{10} = \mathrm{col}\{e_1, e_0, he_7, e_0, he_9\}$$

$$f_{11} = \mathrm{col}\{e_0, e_6, -e_7, e_8, -e_9\}$$

$$f_2 = \mathrm{col}\{e_{10}, e_1 - (1-\dot{h}(t))e_2, (1-\dot{h}(t))e_2 - e_3, e_1 - (1-\dot{h}(t))e_6 - \dot{h}(t)e_8, (1-\dot{h}(t))e_2 - e_7 + \dot{h}(t)e_9\}$$

$$f_3 = e_1 - e_6$$

$$f_4 = e_1 + 2e_6 - 6e_8$$

$$f_5 = e_2 - e_7$$

$$f_6 = e_2 + 2e_7 - 6e_9$$

$$E_1 = \begin{bmatrix} e_1 - e_2 \\ e_1 + e_2 - 2e_6 \\ e_1 - e_2 + 6e_6 - 12e_8 \end{bmatrix}, E_3 = \begin{bmatrix} e_6 \\ e_6 - 2e_8 \end{bmatrix}$$

$$E_2 = \begin{bmatrix} e_2 - e_3 \\ e_2 + e_3 - 2e_7 \\ e_2 - e_3 + 6e_7 - 12e_9 \end{bmatrix}, E_4 = \begin{bmatrix} e_7 \\ e_7 - 2e_9 \end{bmatrix}$$

$$ZM_1 = \begin{bmatrix} -Z_2 & -\frac{1}{2}Z_1 \\ * & -Z_{11} \end{bmatrix}, \quad ZM_2 = \begin{bmatrix} -Z_4 & -\frac{1}{2}Z_3 \\ * & -Z_{22} \end{bmatrix}$$

$$ZM_3 = \begin{bmatrix} -M_2 & -\frac{1}{2}M_1 \\ * & -M_{11} \end{bmatrix}, \quad ZM_4 = \begin{bmatrix} -M_4 & -\frac{1}{2}M_3 \\ * & -M_{22} \end{bmatrix}$$

REFERENCES

[1] Y. Dong, Y. Song, and G. Wei, "Efficient model-predictive control for networked interval type-2 T-s fuzzy system with stochastic communication protocol," *IEEE Trans. Fuzzy Syst.*, vol. 29, no. 2, pp. 286–297, Feb. 2021.

[2] E. Ahmadi, J. Zarei, and R. Razavi-Far, "Robust ℓ_1-controller design for discrete-time positive T-S fuzzy systems using dual approach," *IEEE Trans. Syst., Man, Cybern. Syst.*, vol. 52, no. 2, pp. 706–715, Feb. 2022.

[3] X. Guo, X. Fan, and C. K. Ahn, "Adaptive event-triggered fault detection for interval type-2 T-S fuzzy systems with sensor saturation," *IEEE Trans. Fuzzy Syst.*, vol. 29, no. 8, pp. 2310–2321, Aug. 2021.

[4] X. Xie, C. Wei, Z. Gu, and K. Shi, "Relaxed resilient fuzzy stabilization of discrete-time takagi-sugeno systems via a higher order time-variant balanced matrix method," *IEEE Trans. Fuzzy Syst.*, early access, Jan. 31, 2022, doi: 10.1109/TFUZZ.2022.3145809.

[5] K. Li, C. Hua, X. You, and X.-P. Guan, "Output feedback-based consensus control for nonlinear time delay multiagent systems," *Automatica*, vol. 111, 2020, Art. no. 108669.

[6] C. Hua, Y. Wang, and S. Wu, "Stability analysis of neural networks with time-varying delay using a new augmented Lyapunov-Krasovskii functional," *Neurocomputing*, vol. 332, pp. 1–9, 2019.

[7] C. Hua and Y. Wang, "Delay-dependent stability for load frequency control system via linear operator inequality," *IEEE Trans. Cybern.*, early access, Dec. 11, 2020, doi: 10.1109/TCYB.2020.3037113.

[8] J. Kim, "Note on stability of linear systems with time-varying delay," *Automatica*, vol. 47, no. 9, pp. 2118–2121, 2011.

[9] C. C. Hua, S. S. Wu, and X. P. Guan, "Stabilization of T-S fuzzy system with time delay under sampled-data control using a new looped-functional," *IEEE Trans. Fuzzy Syst.*, vol. 28, no. 2, pp. 400–407, Feb. 2020.

[10] H. Zhang, S. Sun, C. Liu, and K. Zhang, "A novel approach to observer-based fault estimation and fault-tolerant controller design for T-S fuzzy systems with multiple time delays," *IEEE Trans. Fuzzy Syst.*, vol. 28, no. 8, pp. 1679–1693, Aug. 2020.

[11] S. Wang, W. Ji, Y. Jiang, and D. Liu, "Relaxed stability criteria for neural networks with time-varying delay using extended secondary delay partitioning and equivalent reciprocal convex combination techniques," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 31, no. 10, pp. 4157–4169, Oct. 2020.

[12] J. Chen, J. H. Park, and S. Xu, "Stability analysis of continuous-time systems with time-varying delay using new Lyapunov-Krasovskii functionals," *J. Franklin Inst.*, vol. 355, no. 13, pp. 5957–5967, 2018.

[13] R. Samidurai, R. Manivannan, C. K. Ahn, and H. R. Karimi, "New criteria for stability of generalized neural networks including Markov jump parameters and additive time delays," *IEEE Trans. Syst., Man, Cybern. Syst.*, vol. 48, no. 4, pp. 485–499, Apr. 2018.

[14] F. Long, C. K. Zhang, L. Jiang, Y. He, and M. Wu, "Stability analysis of systems with time-varying delay via improved Lyapxunov-Krasovskii functionals," *IEEE Trans. Syst., Man, Cybern. Syst.*, vol. 51, no. 4, pp. 2457–2466, Apr. 2021.

[15] H. Lian, S. Xiao, H. Yan, F. Yang, and H. Zeng, "Dissipativity analysis for neural networks with time-varying delays via a delay-product-type Lyapunov functional approach," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 32, no. 3, pp. 975–984, Mar. 2021.

[16] Z. Lian, Y. He, C. K. Zhang, P. Shi, and M. Wu, "Robust H_∞ control for T-S fuzzy systems with state and input time-varying delays via delay-product-type functional method," *IEEE Trans. Fuzzy Syst.*, vol. 27, no. 10, pp. 1917–1930, Oct. 2019.

[17] K. Gu, V. L. Kharitonov, and J. Chen, *Stability of Time-Delay Systems*. Boston, MA, USA: Birkhauser, 2003.

[18] Y. He, Q. Wang, and M. Wu, "Lmi-based stability criteria for neural networks with multiple time-varying delays," *Physica D: Nonlinear Phenomena*, vol. 212, no. 1, pp. 126–136, 2005.

[19] H. Zeng, X. Liu, and W. Wang, "A generalized free-matrix-based integral inequality for stability analysis of time-varying delay systems," *Appl. Math. Comput.*, vol. 354, pp. 1–8, 2019.

[20] Y. He, Q. Wang, C. Lin, and M. Wu, "Delay-range-dependent stability for systems with time-varying delay," *Automatica*, vol. 43, no. 2, pp. 371–376, 2007.

[21] P. Park, J. W. Ko, and C. Jeong, "Reciprocally convex approach to stability of systems with time-varying delays," *Automatica*, vol. 47, no. 1, pp. 235–238, 2011.

[22] A. Seuret, K. Liu, and F. Gouaisbaut, "Generalized reciprocally convex combination lemmas and its application to time-delay systems," *Automatica*, vol. 95, pp. 488–493, 2018.

[23] X. M. Zhang and Q. L. Han, "State estimation for static neural networks with time-varying delays based on an improved reciprocally convex inequality," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 29, no. 4, pp. 1376–1381, Apr. 2018.

[24] C. K. Zhang, Y. He, L. Jiang, M. Wu, and Q. G. Wang, "An extended reciprocally convex matrix inequality for stability analysis of systems with time-varying delay," *Automatica*, vol. 85, pp. 481–485, 2017.

[25] H. Lin, H. Zeng, X. Zhang, and W. Wang, "Stability analysis for delayed neural networks via a generalized reciprocally convex inequality," *IEEE Trans. Neural Netw. Learn. Syst.*, early access, Feb. 2, 2022, doi: 10.1109/TNNLS.2022.3144032.

[26] J. H. Kim, "Further improvement of Jensen inequality and application to stability of time-delayed systems," *Automatica*, vol. 64, pp. 121–125, 2016.

[27] C. K. Zhang, F. Long, Y. He, W. Yao, L. Jiang, and M. Wu, "A relaxed quadratic function negative-determination lemma and its application to time-delay systems," *Automatica*, vol. 113, 2020, Art. no. 108764.

[28] H. B. Zeng, H. C. Lin, Y. He, C. K. Zhang, and K. L. Teo, "Improved negativity condition for a quadratic function and its application to systems with time-varying delay," *IET Control Theory Appl.*, vol. 14, no. 18, pp. 2989–2993, 2020.

[29] H. B. Zeng, H. C. Lin, Y. He, K. L. Teo, and W. Wang, "Hierarchical stability conditions for time-varying delay systems via an extended reciprocally convex quadratic inequality," *J. Franklin Inst.*, vol. 357, no. 14, pp. 9930–9941, 2020.

[30] M. Park, O. Kwon, J. H. Park, S. Lee, and E. Cha, "Stability of time-delay systems via Wirtinger-based double integral inequality," *Automatica*, vol. 55, pp. 204–208, 2015.

[31] P. Park, W. I. Lee, and S. Y. Lee, "Auxiliary function-based integral inequalities for quadratic functions and their applications to time-delay systems," *J. Franklin Inst.*, vol. 352, no. 4, pp. 1378–1396, 2015.

[32] L. Wang and H. Lam, "New stability criterion for continuous-time Takagi-Sugeno fuzzy systems with time-varying delay," *IEEE Trans. Cybern.*, vol. 49, no. 4, pp. 1551–1556, Apr. 2019.

[33] X. Zhang, Q. Han, and X. Ge, "Novel stability criteria for linear time-delay systems using Lyapunov-Krasovskii functionals with a cubic polynomial on time-varying delay," *IEEE/CAA J. Automatica Sinica*, vol. 8, no. 1, pp. 77–85, Jan. 2021.

[34] J. M. Park and P. Park, "Finite-interval quadratic polynomial inequalities and their application to time-delay systems," *J. Franklin Inst.*, vol. 357, pp. 4316–4327, 2020.

[35] T. H. Lee, H. M. Trinh, and J. H. Park, "Stability analysis of neural networks with time-varying delay by constructing novel Lyapunov functionals," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 29, no. 9, pp. 4238–4247, Sep. 2018.

[36] M. Li, F. Shu, D. Liu, and S. Zhong, "Robust H_∞ control of T-S fuzzy systems with input time-varying delays: A delay partitioning method," *Appl. Math. Comput.*, vol. 321, pp. 209–222, 2018.

[37] O. Kwon, M. Park, J. H. Park, and S. Lee, "Stability and stabilization of T-S fuzzy systems with time-varying delays via augmented Lyapunov-Krasovskii functionals," *Inf. Sci.*, vol. 372, pp. 1–15, 2016.

[38] Z. Sheng, C. Lin, B. Chen, and Q. Wang, "An asymmetric Lyapunov-Krasovskii functional method on stability and stabilization for T-S fuzzy systems with time delay," *IEEE Trans. Fuzzy Syst.*, vol. 30, no. 6, pp. 2135–2140, Jun. 2021.

[39] Z. Zhang, D. Wang, X. Gao, and K. Cao, "Improved delay-dependent stability analysis for uncertain T-S fuzzy systems with time-varying delay," in *Proc. 12th Int. Conf. Fuzzy Syst. Knowl. Discov.*, 2015, pp. 73–77.

[40] H. B. Zeng, J. H. Park, J. W. Xia, and S. P. Xiao, "Improved delay-dependent stability criteria for T-S fuzzy systems with time-varying delay," *Appl. Math. Comput.*, vol. 235, pp. 492–501, 2014.

[41] Z. Lian, Y. He, and M. Wu, "Stability and stabilization for delayed fuzzy systems via reciprocally convex matrix inequality," *Fuzzy Sets Syst.*, vol. 402, pp. 124–141, 2021.

[42] P. Baranyi, *TP-Model Transformation-Based-Control Design Frameworks*. Berlin, Germany: Springer, 2016.

[43] P. Baranyi, "The generalized tp model transformation for T-S fuzzy model manipulation and generalized stability verification," *IEEE Trans. Fuzzy Syst.*, vol. 22, no. 4, pp. 934–948, Aug. 2014.

[44] P. Baranyi, "How to vary the input space of a T-S fuzzy model: A. TP model transformation-based approach," *IEEE Trans. Fuzzy Syst.*, vol. 30, no. 2, pp. 345–356, Feb. 2022.

Yunfei Qiu received the B.S. degree in building electricity an intelligence from Shenyang Jianzhu University, Shenyang, China, in 2013. She is currently working toward the Ph.D. degree in control science and engineering with Yanshan University, Qinhuangdao, China.

Her current research interests include model free adaptive control, iterative learning control, fuzzy systems, and time-delay systems.

Ju H. Park (Senior Member, IEEE) received the Ph.D. degree in electronics and electrical engineering from the Pohang University of Science and Technology (POSTECH), Pohang, South Korea, in 1997.

From May 1997 to February 2000, he was a Research Associate with Engineering Research Center-Automation Research Center, POSTECH. He joined Yeungnam University, Kyongsan, South Korea, in March 2000, where he is currently the Chuma Chair Professor. He is a co-author of the monographs *Recent Advances in Control and Filtering of Dynamic Systems with Constrained Signals* (New York, NY, USA: Springer-Nature, 2018) and *Dynamic Systems With Time Delays: Stability and Control* (New York, NY, USA: Springer-Nature, 2019), and is an Editor of an edited volume *Recent Advances in Control Problems of Dynamical Systems and Networks* (New York: Springer-Nature, 2020). His research interests include robust control and filtering, neural/complex networks, fuzzy systems, multiagent systems, and chaotic systems. He has published a number of articles in these areas.

Prof. Park is a fellow of the Korean Academy of Science and Technology (KAST). Since 2015, he has been a recipient of the Highly Cited Researchers Award by Clarivate Analytics (formerly, Thomson Reuters) and listed in three fields, Engineering, Computer Sciences, and Mathematics, in 2019, 2020, and 2021. He also serves as an Editor of the *International Journal of Control, Automation and Systems*. He is also a Subject Editor/Advisory Editor/Associate Editor/Editorial Board Member of several international journals, including *IET Control Theory & Applications, Applied Mathematics and Computation, Journal of The Franklin Institute, Nonlinear Dynamics, Engineering Reports, Cogent Engineering*, IEEE TRANSACTION ON FUZZY SYSTEMS, IEEE TRANSACTION ON NEURAL NETWORKS AND LEARNING SYSTEMS, and IEEE TRANSACTION ON CYBERNETICS.

Changchun Hua (Senior Member, IEEE) received the Ph.D degree in electrical engineering from Yanshan University, Qinhuangdao, China, in 2005.

He was a Research Fellow with the National University of Singapore from 2006 to 2007. From 2007 to 2009, he worked with Carleton University, Canada, funded by Province of Ontario Ministry of Research and Innovation Program. From 2009 to 2010, he worked with the University of Duisburg-Essen, Germany, funded by Alexander von Humboldt Foundation. Now, he is a full Professor in Yanshan University, China. He is the author or coauthor of more than 120 papers in mathematical, technical journals, and conferences. He has been involved in more than 15 projects supported by the National Natural Science Foundation of China, the National Education Committee Foundation of China, and other important foundations. He is Cheung Kong Scholars Programme Special appointment Professor. His research interests are in nonlinear control systems, multiagent systems, control systems design over network, teleoperation systems, and intelligent control.

Xijuan Wang received the B.S. degree in automation from the Hebei University of Science and Technology, Shijiazhuang, China, in 2016. She is currently working toward the Ph.D. degree in control science and engineering with Yanshan University, Qinhuangdao, China.

Her current research interests include multi-agent systems, cyber-physical systems attacks, and networked control systems.