

# Command-Filter-Based Finite-Time Adaptive Control for Nonlinear Systems With Quantized Input

Jiali Ma , Ju H. Park , and Shengyuan Xu 

**Abstract**—This article considers the finite-time adaptive control problem of nonlinear systems with quantized input signal. Compared with existing results, the quantized parameters are unknown and the bound of the disturbance is not required. By utilizing the command filter backstepping method, an adaptive switching-type controller is designed and a novel switching mechanism is also proposed. By regulating the controller parameters online, practical finite-time stability can be guaranteed for the closed-loop system. Finally, a simulation example is given to illustrate the effectiveness of the proposed method.

**Index Terms**—Command filter, finite-time control, nonlinear systems, quantized input.

## I. INTRODUCTION

In recent years, nonlinear quantized systems have attracted great attention because of the application of digital control systems and networked systems [1], [2]. However, quantization may lead to undesired system performance or even instability. Due to this practical and theoretical significance, the control problem of nonlinear quantized systems has been a hot topic in the nonlinear control area and numerous results have been reported (see [3]–[7] and references therein). However, some challenging problems still exist.

On one hand, most results just ensured that the states of closed-loop systems were bounded or asymptotically converged to zero [3]–[7]. Specifically, backstepping-based adaptive stabilization was considered for strict-feedback nonlinear systems with the quantized input signal in [3]. By choosing suitable quantization and controller parameters, the ultimate boundedness was achieved. However, global Lipschitz conditions were required for the nonlinear functions. In [4], a simple adaptive backstepping method was proposed such that the Lipschitz conditions required in [3] were removed. Then, the output feedback control problems were solved for nonlinear systems and interconnected nonlinear

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systems in [6] and [7], respectively. Compared with asymptotical stability, finite-time stability has unique advantages, such as faster convergence, higher accuracy, and wonderful disturbance rejection [8]–[10]. Considering these factors, a finite-time control problem was solved for strict-feedback nonlinear systems with input quantization in [11]–[13]. By using the adding a power integrator technique, the adaptive finite-time controllers were designed in [11] and [12] to ensure the global stability of the quantized systems. By utilizing the neural networks and backstepping techniques, an adaptive neural network output feedback controller was also designed in [13] where semiglobal practical finite-time stability can be achieved. However, it should be mentioned that the aforementioned results [3]–[7], [11]–[13] were all obtained based on the assumption that the quantized parameters were known. When the quantized parameters are unknown, the control problem will become more difficult for the nonlinear systems. Recently, the adaptive control problem was investigated for nonlinear quantized systems in [14] where the quantized parameters were assumed to be unknown. However, only asymptotical stability could be ensured. Therefore, *it is a challenging and difficult problem to design an adaptive finite-time controller for nonlinear quantized systems whose quantized parameters are totally unknown*. To the author's knowledge, there is no result reported on this issue because of the challenging problem involved.

On the other hand, it should be pointed out that all aforementioned papers [3]–[7], [11]–[14] utilized the backstepping technique. As we all know, an inherent problem of backstepping is the “explosion of complexity,” which was caused by the repeated differentiation of the virtual controllers. To handle this problem, considerable efforts have been devoted [15]–[18]. In [15] and [16], dynamic surface control was proposed where first-order filters were introduced at each step of the backstepping design procedures. However, both papers ignored the compensation errors caused by the filter-order filters, which may degrade the system performance. Then, a modified technique, command filter backstepping, was first introduced in [17] where command filters were designed to approximate the derivative of the virtual controllers. Based on the technique in [17], fruitful results have been reported (see [18]–[20]). Besides, command-filter-based adaptive finite-time control has also been investigated in [21]–[25]. In [21], a neural network finite-time adaptive controller was designed for multiagent systems by constructing novel error compensation signals. Then, modified error compensation systems were designed in [22] and [23] where sign functions were utilized to eliminate the filter error in finite time. It should be mentioned that in the aforementioned references [21]–[23], Levant differentiator was utilized to approximate the derivative of the virtual controller in finite time. However, a drawback of this method is that the  $n$ th order derivative of the virtual controller should exist and be bounded. Obviously, it is hard to check this condition in advance. Consequently, *it is also a challenging problem to design a command-filter-based finite-time controller without the restriction in [21]–[23]*.

Motivated by the aforementioned discussions, command-filter-based adaptive finite-time control is considered for a class of nonlinear systems with the quantized input signal, unknown control direction, and

disturbance. Based on the command filter and the modified error compensation system, a novel adaptive finite-time controller is designed. By tuning the controller parameters via the switching mechanism, the system output can converge to a desired set in finite time. Compared with the existing results, the contributions of this article can be listed as follows.

- 1) Different from [3]–[7], [11]–[13], the quantization parameters are not required to be known. The bounds of the control coefficient and the disturbance are also not required.
- 2) Unlike the results in [21]–[23], a modified error compensation system is constructed in this article. By regulating the system's parameters online, finite-time stability of the error compensation system can be achieved.
- 3) Compared with the results in [14] where asymptotic stability was achieved, finite-time stability can be obtained for closed-loop system such that a better performance can be obtained.

The remainder of this article is organized as follows. In Section II, the problem formulation is shown and some useful lemmas are also provided. In Section III, command-filter-based controller is designed. In Section IV, we provide the stability analysis. In Section V, a numerical simulation example is given to illustrate the effectiveness of the proposed method.

## II. PROBLEM FORMULATION AND PRELIMINARIES

Consider the following nonlinear system with input quantization:

$$\begin{cases} \dot{x}_i = x_{i+1} + f_i(\bar{x}_i), i = 1, \dots, n-1 \\ \dot{x}_n = g(x)q(u) + f_n(\bar{x}_n) + d(x, t) \\ y = x_1 \end{cases} \quad (1)$$

where  $x = [x_1, \dots, x_n] \in \mathcal{R}^n$  denotes the system state;  $y$  is the system output;  $u$  is the system input and  $q(u)$  represent the quantizer;  $g(x)$  is an unknown function satisfying  $g(x) \neq 0$ ;  $f_i(\bar{x}_i)$  are assumed to be known where  $\bar{x}_i = [x_1, \dots, x_i]^T$ ; and  $d(x, t)$  denotes the external disturbance.

In this article, the quantizer is described as [12]

$$q(u) = \begin{cases} u_i \operatorname{sgn}(u), & \frac{u_i}{1+\delta} < |u| \leq u_i, \dot{u} < 0, \text{ or} \\ & u_i < |u| \leq \frac{u_i}{1-\delta}, \dot{u} > 0 \\ u_i(1+\delta) \operatorname{sgn}(u), & u_i < |u| \leq \frac{u_i}{1-\delta}, \dot{u} < 0, \text{ or} \\ & \frac{u_i}{1-\delta} < |u| \leq \frac{u_i(1+\delta)}{1-\delta}, \dot{u} > 0 \\ 0, & 0 \leq |u| < \frac{u_0}{1+\delta}, \dot{u} < 0, \text{ or} \\ & \frac{u_0}{1+\delta} \leq |u| \leq u_0, \dot{u} > 0 \\ q(u(t^-)), & \dot{u} = 0 \end{cases} \quad (2)$$

where  $u_i = \sigma^{1-i} u_{\min}$ ,  $i = 1, 2, \dots$ ,  $u_{\min} > 0$ ,  $0 < \sigma < 1$  and  $\delta = \frac{1-\sigma}{1+\sigma}$ . More details about the quantizer can be found in [12]. Quantizer (2) satisfies

$$q(u) = b_1(t)u + b_2(t) \quad (3)$$

where  $b_1 \geq 1 - \delta > 0$  and  $|b_2| \leq u_{\min}$ .

The control objective of this article is to design an adaptive controller such that all the states of the closed-loop system are bounded and the system output  $y$  can converge to a desired set in finite time. The following assumptions are made for the system (1).

*Assumption 1:* It is assumed that the sign of  $g(x)$  is known. There exist unknown positive constants  $\underline{g}$  and  $\bar{g}$  such that  $\underline{g} \leq |g(x)| \leq \bar{g}$ .

*Assumption 2:* The external disturbance  $d(x, t)$  satisfies  $|d(x, t)| \leq \bar{d}$ , where  $\bar{d}$  is an unknown positive constant.

*Assumption 3:* The parameters of the quantizer are unknown, which means  $\sigma$  and  $u_{\min}$  are unknown.

*Remark 1:* In the existing results about finite-time control [11]–[13], [21], *a priori* knowledge of the control coefficient and disturbance is always required. As shown in Assumptions 1 and 2, these restrictions are removed in this article. Therefore, a more general result can be achieved. Without loss of generality, it is assumed that  $g(x) > 0$ .

*Remark 2:* In the existing works [3]–[7], [14], only global boundedness or asymptotical stability could be obtained for quantized nonlinear systems. Although finite-time stability was achieved in [11]–[13], the quantized parameters were required to be known. In this article, the finite-time control problem is studied based on the assumption that the quantized parameters are unknown. Therefore, the proposed result is quite different from the existing results.

Besides, the following lemma is also introduced.

*Lemma 1 (see [10]):* Let  $V(x) : \mathcal{R}^n \rightarrow \mathcal{R}$  be a positive definite Lyapunov function. There exist positive constants  $c_1 > 0$ ,  $c_2 > 0$ , and  $0 < \gamma < 1$  such that  $\dot{V} + c_1 V + c_2 V^\gamma \leq 0$ . Then, there exists a positive constant  $\bar{t}$ , such that  $x(t) \equiv 0$  for  $t \geq \bar{t}$ , where  $\bar{t} = \frac{1}{c_1(1-\gamma)} \ln(1 + \frac{c_1}{c_2} V^{1-\gamma}(x(0)))$ .

*Lemma 2 (see [25]):* Consider the system  $\dot{x} = f(x)$ . Let  $V(x) : \mathcal{R}^n \rightarrow \mathcal{R}$  be a positive definite Lyapunov function. There exist positive constants  $c_1 > 0$ ,  $c_2 > 0$ ,  $0 < \gamma < 1$ , and  $0 < \delta < +\infty$  such that  $\dot{V} \leq -c_1 V - c_2 V^\gamma + \delta$ . Then, the trajectory of the system is practical finite-time stable, and the residual set of the solution of system is given by  $\{\lim_{t \rightarrow \bar{t}} |V^\gamma(x) \leq \frac{\delta}{(1-\lambda)c_2}\}$  where  $\bar{t} \leq \frac{1}{c_1(1-\gamma)} \ln(1 + \frac{c_1}{\lambda c_2} V^{1-\gamma}(x(0)))$ ,  $0 < \lambda < 1$ .

## III. CONTROLLER DESIGN

In this section, based on the command filter, an adaptive controller will be designed by using the backstepping scheme. First, introduce the following change of coordinates:

$$\begin{cases} \eta_1 = x_1 \\ \eta_i = x_i - \bar{\alpha}_{i-1}, i = 2, \dots, n \end{cases} \quad (4)$$

where  $\bar{\alpha}_{i-1}$  is the output of the command filter with  $\alpha_{i-1}$  as the input. Here, the command filter is designed as follows:

$$\begin{cases} \dot{\zeta}_{i1} = \beta \zeta_{i2} \\ \dot{\zeta}_{i2} = -2\gamma \beta \zeta_{i2} - \beta(\zeta_{i1} - \alpha_i), i = 1, \dots, n-1 \end{cases} \quad (5)$$

where  $\beta > 0$  and  $0 < \gamma \leq 1$ .  $\bar{\alpha}_i = \zeta_{i1}$  is the output of the command filter. The initial condition is  $\zeta_{i1}(0) = \alpha_i(0)$  and  $\zeta_{i2}(0) = 0$ .

The virtual controllers  $\alpha_i$ ,  $i = 1, \dots, n-1$  are designed as

$$\alpha_1 = -c_1 \eta_1 - f_1 - k_1 z_1^\mu \quad (6)$$

$$\alpha_i = -c_i \eta_i - f_i + \dot{\bar{\alpha}}_{i-1} - \eta_{i-1} - k_i z_i^\mu \quad (7)$$

where  $c_i$  and  $k_i > 0$  are designed parameters,  $\mu$  is a positive constant satisfying  $0 < \mu = \frac{\mu_1}{\mu_2} < 1$ , where  $\mu_1$  and  $\mu_2$  are odd integers.  $z$  is the compensation error signal defined as

$$z_i = \eta_i - \xi_i \quad (8)$$

and the error compensation signal  $\xi$  is defined as

$$\dot{\xi}_1 = -c_1 \xi_1 + \xi_2 + (\bar{\alpha}_1 - \alpha_1) - l \operatorname{sgn}(\xi_1) \quad (9)$$

$$\dot{\xi}_i = -c_i \xi_i - \xi_{i-1} + \xi_{i+1} + (\bar{\alpha}_i - \alpha_i) - l \operatorname{sgn}(\xi_i) \quad (10)$$

$$\dot{\xi}_n = -c_n \xi_n - \xi_{n-1} - l \operatorname{sgn}(\xi_n) \quad (11)$$

where  $l := l(s) + \frac{\sqrt{2}}{2} l_0$ .  $l_0$  is a positive constant and  $l(s)$  is a strictly increasing function with respect to  $s$ , which satisfies  $\lim_{s \rightarrow +\infty} l(s) = +\infty$ .

*Remark 3:* Different from the existing results [21]–[23], Levant differentiator is not utilized in this article to approximate the virtual controller. In a different way, a novel error compensation system with switching parameters is developed in this article. By tuning the parameter online, the finite-time stability of  $\xi$  can also be guaranteed.

### A. Controller Design

In this part, we will give the design procedures of the controller.

*Step 1:* Consider the Lyapunov function candidate  $V_1 = \frac{1}{2} z_1^2$ . Based on (1), (4), and (9), we have

$$\begin{aligned} \dot{V}_1 &= z_1(x_2 + f_1 + c_1 \xi_1 - \xi_2 - (\bar{\alpha}_1 - \alpha_1) + l \operatorname{sgn}(\xi_1)) \\ &= z_1(z_2 + \alpha_1 + f_1 + c_1 \xi_1 + l \operatorname{sgn}(\xi_1)). \end{aligned} \quad (12)$$

Substituting (6) into (12) yields

$$\dot{V}_1 = -c_1 z_1^2 - k_1 z_1^{1+\mu} + l z_1 \operatorname{sgn}(\xi_1) + z_1 z_2. \quad (13)$$

*Step  $i = 2, \dots, n-1$ :* Consider the Lyapunov function candidate

$$V_i = V_{i-1} + \frac{1}{2} z_i^2. \quad (14)$$

Then, the derivative of  $V_i$  satisfies

$$\begin{aligned} \dot{V}_i &\leq \sum_{j=1}^{i-1} (-c_j z_j^2 - k_j z_j^{1+\mu} + l_j z_j \operatorname{sgn}(\xi_j)) \\ &\quad + z_i(z_{i+1} + \alpha_i + f_i - \dot{\alpha}_{i-1} + c_i \xi_i \\ &\quad + \eta_{i-1} + l \operatorname{sgn}(\xi_i)). \end{aligned} \quad (15)$$

Substituting (7) into (15) yields

$$\dot{V}_i \leq \sum_{j=1}^i (-c_j z_j^2 - k_j z_j^{1+\mu} + l z_j \operatorname{sgn}(\xi_j)) + z_i z_{i+1}. \quad (16)$$

*Step  $n$ :* Consider the Lyapunov function candidate

$$V_n = V_{n-1} + \frac{1}{2} z_n^2. \quad (17)$$

Then, we have

$$\begin{aligned} \dot{V}_n &\leq \sum_{j=1}^{n-1} (-c_j z_j^2 - k_j z_j^{1+\mu} + l z_j \operatorname{sgn}(\xi_j)) \\ &\quad + z_n(gq(u) + f_n + d - \dot{\alpha}_{n-1} + c_n \xi_n \\ &\quad + \eta_{n-1} + l \operatorname{sgn}(\xi_n)). \end{aligned} \quad (18)$$

Substituting (3) into (18) yields

$$\begin{aligned} \dot{V}_n &\leq \sum_{j=1}^{n-1} (-c_j z_j^2 - k_j z_j^{1+\mu} + l z_j \operatorname{sgn}(\xi_j)) \\ &\quad + z_n(g_0 u + f_0 + d_0 + l \operatorname{sgn}(\xi_n)) \end{aligned} \quad (19)$$

where  $g_0 = gb_1$ ,  $f_0 = f_n - \dot{\alpha}_{n-1} + c_n \xi_n + \eta_{n-1}$ , and  $d_0 = gb_2 + d$ . Then, design the controller as

$$u = -h(k)(c_n z_n + k_n z_n^\mu + \operatorname{sgn}(z_n f_0) f_0 + \operatorname{sgn}(z_n)) \quad (20)$$

where  $h(k)$  a strictly increasing function, which satisfies  $\lim_{k \rightarrow +\infty} h(k) = +\infty$ .

Under the control law (20), the inequality (19) becomes

$$\begin{aligned} \dot{V}_n &\leq \sum_{j=1}^n (-c_j z_j^2 - k_j z_j^{1+\mu} + l z_j \operatorname{sgn}(\xi_j)) \\ &\quad - (g_0 h(k) - 1) \phi - (g_0 h(k) - d_0) |z_n| \end{aligned} \quad (21)$$

where  $\phi = c_n z_n^2 + k_n z_n^{1+\mu} + |z_n f_0| \geq 0$ .

### B. Switching Mechanism

In this section, we will propose a switching mechanism to regulate the parameter  $k$ . Define the Lyapunov function

$$V_\xi = \frac{1}{2} \sum_{i=1}^n \xi_i^2. \quad (22)$$

Let  $k_0 = \min_{i=1, \dots, n} \{2^{\frac{1+\mu}{2}} k_i\}$ ;  $\gamma_v = \frac{1+\mu}{2}$ ;  $\gamma_\xi = \frac{1}{2}$ ;  $c = \min_{i=1, \dots, n} \{2c_i\}$ , and  $c_0$  is a positive constant. Then, design the switching mechanism as follows.

*Step 1: Initialization:* Select appropriate strictly increasing functions  $h(k)$  and  $l(s)$ . Choose a constant  $\delta > 0$ ,  $0 < \lambda < 1$ , and set  $k = s = 0$  and  $t_0 = 0$ .

*Step 2: Switching condition.*

1) For  $t > t_k$ , if  $V^{\gamma_v}(x) > \frac{\delta}{(1-\lambda)k_0}$ , and

$$\begin{aligned} V_n^{1-\gamma_v}(t) &> \max \left\{ \left( V_n^{1-\gamma_v}(t_k) \right. \right. \\ &\quad \left. \left. + \frac{\lambda k_0}{c_0} (1 - e^{(1-\gamma_v)c_0(t-t_k)}) \right) e^{-c_0(1-\gamma_v)(t-t_k)}, 0 \right\} \end{aligned} \quad (23)$$

then  $k \rightarrow k+1$ , and reset  $t_k = t$ . Otherwise,  $k$  and  $t_k$  remain unchanged.

2) For  $t > t_s$ , if

$$\begin{aligned} V_\xi^{1-\gamma_\xi}(t) &> \max \left\{ \left( V_\xi^{1-\gamma_\xi}(t_s) \right. \right. \\ &\quad \left. \left. + \frac{l_0}{c} (1 - e^{(1-\gamma_\xi)c(t-t_s)}) \right) e^{-c(1-\gamma_\xi)(t-t_s)}, 0 \right\} \end{aligned} \quad (24)$$

then  $s \rightarrow s+1$ , and reset  $t_s = t$ . Otherwise,  $s$  and  $t_s$  remain unchanged.

*Step 3:* Repeat Step 2.

## IV. STABILITY ANALYSIS

By the work in Section III, we can obtain the following results, as shown in Theorem 1.

*Theorem 1:* Consider the nonlinear system (1), which satisfies Assumptions 1–3. Under the adaptive control law (20) and the switching mechanism, all the signals of the closed-loop system are bounded and finite-time stability can be achieved.

*Proof:* As shown in [8] and [9], for any fixed  $k$  and  $s$ , the solution of the closed-loop system exists until the state escapes to the infinity or time  $t$  tends to infinity. Define the maximum internal of the solution as  $[0, t_m)$ . In the following part, we will first prove that the closed-loop system states are bounded and  $t_m = +\infty$ . The details are as follows.

During every switch of  $k$ , for  $t \in [t_k, t_{k+1})$ , we have  $V^{\gamma_v}(x) \leq \frac{\delta}{(1-\lambda)k_0}$  or

$$\begin{aligned} V_n^{1-\gamma_v}(t) &\leq \max \left\{ \left( V_n^{1-\gamma_v}(t_k) \right. \right. \\ &\quad \left. \left. + \frac{\lambda k_0}{c_0} (1 - e^{(1-\gamma_v)c_0(t-t_k)}) \right) e^{-c_0(1-\gamma_v)(t-t_k)}, 0 \right\} \\ &\leq V_n^{1-\gamma_v}(t_k) + \frac{k_0}{c_0}. \end{aligned} \quad (25)$$

According to (25), it is obvious that  $z$  is uniformly bounded on  $[t_k, t_{k+1})$ . Then, we assume that the last switching occurs at time  $t_{k^*}$ . For  $t \geq t_{k^*}$ , we also have  $V^{\gamma v}(x) \leq \frac{\delta}{(1-\lambda)k_0}$  or

$$\begin{aligned} V_n^{1-\gamma v}(t) &\leq \max \left\{ \left( V_n^{1-\gamma v}(t_{k^*}) \right. \right. \\ &\quad \left. \left. + \frac{\lambda k_0}{c_0} (1 - e^{-(1-\gamma v)c_0(t-t_{k^*})}) \right) e^{-c_0(1-\gamma v)(t-t_{k^*})}, 0 \right\} \\ &\leq V_n^{1-\gamma v}(t_{k^*}) + \frac{k_0}{c_0} \end{aligned} \quad (26)$$

which also implies the boundedness of  $z$ . Based on these facts, we have that closed-loop system states are bounded on  $[0, t_m)$ . Since no finite-time escape phenomenon occurs, we have  $t_m = +\infty$ .

Next, we can prove that the switching times are finite. The reason is: define  $\underline{g}_0 = \inf_{t \geq 0} g_0$  and  $\bar{d}_0 = \sup_{t \geq 0} |d_0|$ . According to Assumptions 1 and 2, it is easy to prove that  $\underline{g}_0 > 0$  and  $\bar{d}_0$  is bounded. Since  $h(k)$  is a strictly increasing function with respect to  $k$ , there exists a switching number  $\bar{k}$  such that

$$\underline{g}_0 h(\bar{k}) - 1 \geq 0, \quad \underline{g}_0 h(\bar{k}) - \bar{d}_0 \geq 0. \quad (27)$$

Substituting (27) into (21) yields

$$\begin{aligned} \dot{V}_n &\leq \sum_{j=1}^n (-c_j z_j^2 - k_j z_j^{1+\mu} + l z_j \operatorname{sgn}(\xi_j)) \\ &\leq \sum_{j=1}^n \left( - \left( c_j - \frac{l^2}{4\omega} \right) z_j^2 - k_j z_j^{1+\mu} \right) + n\omega \end{aligned} \quad (28)$$

where  $\omega > 0$  is a design parameter satisfying  $n\omega \leq \delta$ .

Select  $c_j = \frac{c_0}{2} + \frac{l^2}{4\omega}$ . Then, we have

$$\begin{aligned} \dot{V}_n &\leq \sum_{j=1}^n \left( -\frac{c_0}{2} z_j^2 - k_j z_j^{1+\mu} \right) + n\omega \\ &\leq -c_0 V_n - k_0 V^{\gamma v} + n\omega. \end{aligned} \quad (29)$$

If  $V^{\gamma v}(x) \leq \frac{\delta}{(1-\lambda)k_0}$ , it is obvious that switching condition is not satisfied. If  $V^{\gamma v}(x) > \frac{\delta}{(1-\lambda)k_0}$ , based on (29), we have

$$\dot{V}_n \leq -c_0 V_n - \lambda k_0 V^{\gamma v}. \quad (30)$$

Based on (30) and Lemma 2, it is easy to prove

$$\begin{aligned} V_n^{1-\gamma v}(t) &\leq \max \left\{ \left( V_n^{1-\gamma v}(t_{\bar{k}}) \right. \right. \\ &\quad \left. \left. + \frac{\lambda k_0}{c_0} (1 - e^{-(1-\gamma v)c_0(t-t_{\bar{k}})}) \right) e^{-c_0(1-\gamma v)(t-t_{\bar{k}})}, 0 \right\} \end{aligned} \quad (31)$$

which also implies that switching condition (23) will also never be satisfied. Therefore, the switching times are finite. Besides, we also have  $k^* \leq \bar{k}$ . Finally, finite-time stability can be obtained for  $z$ . The reason is: if  $V^{\gamma v}(x) > \frac{\delta}{(1-\lambda)k_0}$ ,  $\exists t \geq t_{k^*} + \frac{1}{c_0(1-\gamma v)} \ln(1 + \frac{c_0}{\lambda k_0} V^{1-\gamma v}(t^*))$ , then we have

$$\begin{aligned} 0 \leq V_n^{1-\gamma v}(t) &\leq \max \left\{ \left( V_n^{1-\gamma v}(t_{t^*}) + \frac{\lambda k_0}{c_0} \right. \right. \\ &\quad \left. \left. \times (1 - e^{-(1-\gamma v)c_0(t-t_{t^*})}) \right) e^{-c_0(1-\gamma v)(t-t_{t^*})}, 0 \right\} = 0 \end{aligned} \quad (32)$$

which results in a contradiction. Thus, the finite-time stability is proved.

Since (32) hold for  $t \geq t_{k^*}$ , based on the results in [18], [19], [22], and [25], we have  $|\bar{\alpha}_i - \alpha_i| \leq \varsigma_i$ , where  $\varsigma_i$  is unknown but bounded.

Then, consider the Lyapunov function  $V_\xi$ . According to (9)–(11), we have

$$\begin{aligned} \dot{V}_\xi &= - \sum_{i=1}^n c_i \xi_i^2 - \sum_{i=1}^n l |\xi_i| - \sum_{i=1}^{n-1} \xi_i (\bar{\alpha}_i - \alpha_i) \\ &\leq - \sum_{i=1}^n c_i \xi_i^2 - \sum_{i=1}^n \frac{\sqrt{2}}{2} l_0 |\xi_i| - \sum_{i=1}^{n-1} (l(s) - \varsigma_i) |\xi_i|. \end{aligned} \quad (33)$$

Now, we will prove the finite-time stability of  $\xi$ . First of all, we can prove that  $s$  can just be switched a finite number of times. Then, we can prove that  $\xi$  can converge to zero in finite time. Then, reasons are shown as follows.

1) It is claimed that the switching number of  $s$  is finite. The reason is: since  $l(s)$  is a strictly increasing function, there exists a switching number  $s_1$  such that  $l(s_1) - \bar{\varsigma} \geq 0$ , where  $\bar{\varsigma} = \max_{j=1, \dots, n} \{\varsigma_j\}$ . Hence, for  $t \geq t_{s_1}$ , we have

$$\dot{V}_\xi \leq - \sum_{i=1}^n c_i \xi_i^2 - \sum_{i=1}^n \frac{\sqrt{2}}{2} l_0 |\xi_i| \leq -c V_\xi - l_0 V_\xi^{\frac{1}{2}}. \quad (34)$$

By using Lemma 1 and the results in [10], we have

$$\begin{cases} V_\xi(t) \leq \left( V_\xi^{1-\gamma_\xi}(t_{s_1}) + \frac{l_0}{c} \right. \\ \quad \left. (1 - e^{-(1-\gamma_\xi)c(t-t_{s_1})}) \right)^{\frac{1}{1-\gamma_\xi}} e^{-c(t-t_{s_1})}, t \in [t_{s_1}, \bar{T}) \\ V_\xi(t) = 0, t \in [\bar{T}, +\infty) \end{cases} \quad (35)$$

where  $\bar{T} = \frac{1}{c(1-\gamma_\xi)} \ln(1 + \frac{c}{l_0} V^{1-\gamma_\xi}(t_{s_1}))$ .

Hence, the following inequality can be obtained:

$$\begin{aligned} V_\xi^{1-\gamma_\xi}(t) &\leq \max \left\{ \left( + \frac{l_0}{c} (1 - e^{-(1-\gamma_\xi)c(t-t_{s_1})}) \right) V_\xi^{1-\gamma_\xi}(t_{s_1}) \right. \\ &\quad \left. + \frac{l_0}{c} (1 - e^{-(1-\gamma_\xi)c(t-t_{s_1})}) \right) e^{-c(1-\gamma_\xi)(t-t_{s_1})}, 0 \right\} \end{aligned} \quad (36)$$

which means the switching condition (24) can never be satisfied for  $t \geq t_{s_1}$ . Therefore, the switching times for  $s$  are finite.

2) Let  $\bar{s}$  be the last switching number of  $s$ . Then, we have  $\bar{s} \leq s_1$ . For  $t \geq t_{\bar{s}}$ , we have

$$\begin{aligned} V_\xi^{1-\gamma_\xi}(t) &\leq \max \left\{ \left( V_\xi^{1-\gamma_\xi}(t_{\bar{s}}) \right. \right. \\ &\quad \left. \left. + \frac{l_0}{c} (1 - e^{-(1-\gamma_\xi)c(t-t_{\bar{s}})}) \right) e^{-c(1-\gamma_\xi)(t-t_{\bar{s}})}, 0 \right\}. \end{aligned} \quad (37)$$

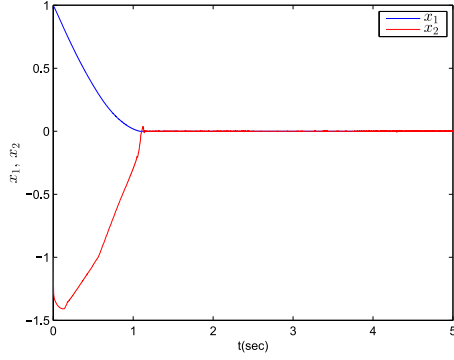
From (37), we can conclude that  $\xi$  can converge to zero in finite time. The reason is: if  $\xi \neq 0$ ,  $\exists t \geq t_{\bar{s}} + \frac{1}{c(1-\gamma_\xi)} \ln(1 + \frac{c}{l_0} V(t_{\bar{s}})^{1-\gamma_\xi})$ , we have

$$\begin{aligned} 0 \neq V_\xi^{1-\mu}(x(t)) &\leq \max \left\{ \left( V_\xi^{1-\mu}(t_{\bar{s}}) \right. \right. \\ &\quad \left. \left. + \frac{l_0}{c} (1 - e^{-(1-\gamma_\xi)c(t-t_{\bar{s}})}) \right) e^{-c(1-\gamma_\xi)(t-t_{\bar{s}})}, 0 \right\} = 0 \end{aligned} \quad (38)$$

which results in a contradiction. Therefore,  $\xi = 0$  for  $t \geq t_{\bar{s}} + \frac{1}{c(1-\gamma_\xi)} \ln(1 + \frac{c}{l_0} V(t_{\bar{s}})^{1-\gamma_\xi})$ .

In conclusion, finite-time stability of  $z$  and  $\xi$  is ensured. According to the definition of  $z$  and  $\eta$ , we can easily prove the finite-time stability of  $x$ . ■

*Remark 4:* In [14], parameter updating laws were developed to handle the unknown quantized parameters where two estimates were introduced in the adaptive controller. However, once introducing the


 Fig. 1. Response of the system state  $x$ .

adaptive estimates, the finite-time stability cannot be achieved. Different from the result in [14], switching parameters are introduced in the adaptive controller and error compensation systems. By tuning the switching parameters in a finite number of times, the effect of the unknown quantized parameters can be eliminated such that the finite-time convergence can be achieved.

*Remark 5:* Our proposed method can also be utilized to solve the finite-time control problem of nonlinear systems with actuator fault

$$\begin{cases} \dot{x}_i = x_{i+1} + f_i(\bar{x}_i), i = 1, \dots, n-1 \\ \dot{x}_n = g(x)f(u) + f_n(\bar{x}_n) + d(x, t) \end{cases} \quad (39)$$

where  $f(u)$  denotes the actuator fault, which can be modeled as [12]

$$f(u) = b(t)u + u_d(t) \quad (40)$$

where  $b(t) > 0$  and  $u_d(t)$  is a bounded signal. Obviously, system (39) has similar form with system (1). Therefore, our proposed method can be applied to system (39).

*Remark 6:* The finite-time fault-tolerant control has been investigated in [25] where the error signal can converge to a residual set in finite time. Compared with Li [25], the advantages of all papers are: first, the unknown system parameters are compensated by regulating the controller parameters; and second, the bound knowledge of  $|\bar{\alpha}_i - \alpha_i|$  is not required in this article.

## V. SIMULATION EXAMPLE

In this section, to show the effectiveness of the proposed approach, the pendulum system in [26] is considered

$$ml\ddot{\theta} = -mg\sin\theta - kl\dot{\theta} + \frac{T}{l} + d. \quad (41)$$

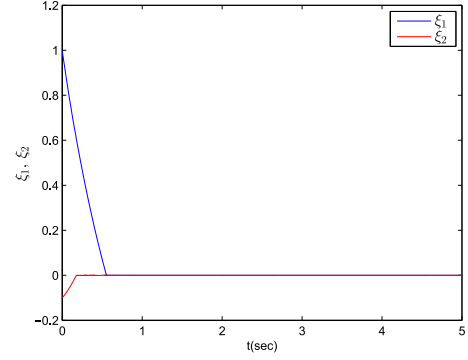
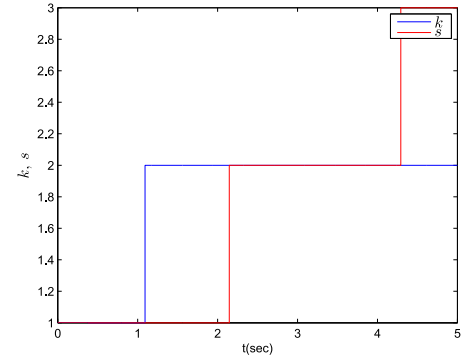
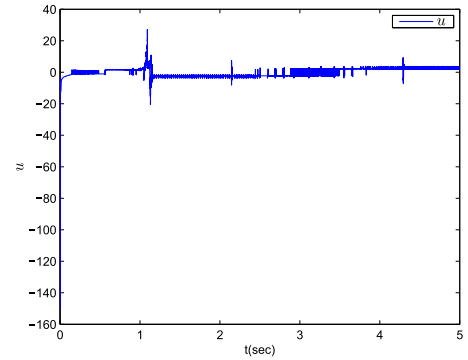
The detailed physical descriptions about the parameters  $m$ ,  $l$ ,  $g$ , and  $k$  can be found in [26].  $d$  is the unknown disturbance. Let  $x_1 = \theta$  and  $x_2 = \dot{\theta}$ , then we have

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \frac{1}{ml^2}q(u) - \frac{g}{l}\sin x_1 - \frac{k}{m}x_2 + d. \end{aligned} \quad (42)$$

According to the results in Section III, we can design the command filter and the error compensation system first. Then, the system controller can be designed as follows:

$$u = -h(k)(c_2z_2 + k_2z_2^H + \text{sgn}(z_2f_0)f_0 + \text{sgn}(z_2)).$$

The design parameters are chosen as:  $c_0 = 2$ ,  $k = 1$ ,  $l_0 = \sqrt{2}$ ,  $\beta = 1000$ ,  $\gamma = 0.75$ ,  $h(k) = 0.8k$ , and  $l(s) = 0.3s + 1$ . In the simulation, the initial conditions are chosen as:  $[x_1(0) \ x_2(0)]^T = [1 \ -0.5]^T$ ,


 Fig. 2. Response of error compensation system  $\xi$ .

 Fig. 3. Switching number  $k$  and  $s$ .

 Fig. 4. Response of system input  $u$ .

$[\zeta_1(0) \ \zeta_2(0)]^T = [-3 \ 0]^T$ , and  $[\xi_1(0) \ \xi_2(0)]^T = [1 \ -1]^T$ . The system parameters are chosen as  $m = 0.02$  kg,  $l = 9.8$  m, and  $k = 0.02$ . The unknown disturbance is chosen as  $d = \sin(5t)$ . The parameters for the quantizer is chosen as  $u_{\min} = 0.02$  and  $\sigma = 0.2$ . Then, the simulation results are shown in Figs. 1–4. Fig. 1 provides the response of the system state  $x$ . The response of the compensation system signal  $\xi$  is shown in Fig. 2. Figs. 3 and 4 provide the response of switching parameters  $k$ ,  $s$  and system input  $u$ . From Fig. 1, it can be seen that the finite-time stability can be obtained for the considered system, and we can also find that the compensation system  $\xi$  is finite-time stable from Fig. 2.

## VI. CONCLUSION

In this article, the command-filter-based finite-time adaptive control problem has been investigated for a class of nonlinear systems with

quantized input signal and disturbance. Unlike the existing results, the quantized parameters and the bound of the disturbance are not required in this article. By introducing the novel command filter and the error compensation system, the “explosion of complexity” problem can be solved. By tuning the control parameters online, the designed controller can ensure that the finite-time stability can be guaranteed for the closed-loop system. Moreover, in the future, we will focus on the command-filter-based finite-time control problem for nonlinear systems whose functions are unknown.

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