Closed-Loop Frequency Analysis of Reset Control Systems

Ali Ahmadi Dastjerdi, *Member, IEEE*, Alessandro Astolfi, *Fellow, IEEE*, Niranjan Saikumar, *Member, IEEE*, Nima Karbasizadeh, *Member, IEEE*, Duarte Valério, *Member, IEEE*, and S. Hassan HosseinNia, *Senior Member, IEEE*

***Abstract*—This article introduces a closed-loop frequency analysis tool for reset control systems. To begin with sufficient conditions for the existence of the steady-state response for a closed-loop system with a reset element and driven by periodic references are provided. It is then shown that, under specific conditions, such a steady-state response for periodic inputs is periodic with the same period as the input. Furthermore, a framework to obtain the steady-state response and to define a notion of closed-loop frequency response, including high order harmonics, is presented. Finally, pseudosensitivities for reset control systems are defined. These simplify the analysis of this class of systems and allow a direct software implementation of the analysis tool. This methods gives deeper insight into the performance of the system than that achieved with the describing function method.**

***Index Terms*—Convergent, frequency-domain analysis, pseudosensitivities, reset controllers.**

I. Introduction

Proportional Integral Derivative (PID) controllers are used in more than 90% of industrial control applications [1], [2]. However, cutting-edge industrial applications have control requirements that cannot be fulfilled by PID controllers. To overcome this problem, linear controllers may be substituted by nonlinear ones. Reset controllers are one such type of nonlinear controllers, which have attracted attention due to their simple structure and their ability to improve closed-loop performance [3]–[18].

A traditional reset controller consists of a linear element the state of which is reset to zero when its input equals zero. The simplest reset element is the Clegg Integrator (CI), which is an integrator with a reset mechanism [3]. To provide design freedom and applicability, reset controllers such as first-order reset elements [19], [20] and second-order reset elements have been introduced [13]. These reset elements are utilized to construct new compensators to achieve significant performance enhancement [16], [21]–[24]. In order to further improve the performance of reset control systems several techniques, such as the considerations of nonzero reset values [8], [20], reset bands [25], [26], fixed reset instants, and $PI + CI$ configurations [27], [28] have been introduced.

Frequency-domain analysis is preferred in industry, since it allows ascertaining closed-loop performance measures in an intuitive way. In addition, frequency-domain analysis gives valuable information on the steady-state behaviour of the system. However, the lack of such methods for nonlinear controllers is one of the reasons why non-linear controllers are not widely popular in industry. The describing function (DF) method is one of the few methods for approximately studying nonlinear controllers in the frequency-domain and this has been widely used also in the literature of reset controllers [8], [16], [24], [29]. The DF method relies on an approximation of the steady-state output of a nonlinear system considering only the first harmonic of the Fourier series expansion of the input and output signals (assumed periodic). The general formulation of the DF method for reset controllers is presented in [29], which however does not provide any information on the closed-loop steady-state response.

In this article, first, sufficient conditions for the existence of the steady-state response for a closed-loop system with a reset element and driven by a periodic input are given. Then, a notion of closed-loop frequency response for reset control systems, including high-order harmonics, is introduced. Pseudosensitivities to combine harmonics and facilitate analyzing reset control systems in closed-loop configuration are then defined. All of these ideas are utilized to develop a toolbox, which is briefly discussed. Note finally that, contrary to the DF method, which provides only approximations for the periodic steady-state response of reset control systems, the proposed tools allow computing exact steady-state responses to periodic excitations.

This article is organized as follows. In Section II, sufficient conditions to define a notion of frequency response are presented. Then, a method to obtain closed-loop frequency responses for reset control systems, including high-order harmonics, is developed, and pseudosensitivities are defined. In Section III, the steady-state response of reset controllers to periodic inputs is studied. Finally, Section IV concludes this article.

II. Closed-Loop Frequency Response of Reset Control Systems

Consider the single-input single-output control architecture in the top diagram of Fig. 1. This includes as particular cases all schemes discussed in Section I. The closed-loop system consists of a linear plant with transfer function $G(s)$, two linear controllers with proper transfer function $C_{\mathfrak{L}_1}(s)$ and $C_{\mathfrak{L}_2}(s)$, and a reset controller with base transfer function $C_{\mathcal{R}}(s)$. Let $\mathcal{L}$ be the LTI part of the system and assume that $G(s)$ is strictly proper. The state-space realization of $\mathcal{L}$ is described by

Manuscript received 30 September 2021; accepted 21 January 2022. Date of publication 20 June 2022; date of current version 30 January 2023. This work was supported in part by the NWO through OTP TTW project under Grant #16335, in part by the EACEA, by the European Union's Horizon 2020 Research and Innovation Programme under Grant 739551 (KIOS CoE), and in part by the Italian Ministry for Research under Grant 2017YKXYXJ. Recommended by Associate Editor D. Dochain. *(Corresponding author: S. Hassan HosseinNia.)*

Ali Ahmadi Dastjerdi, Niranjan Saikumar, Nima Karbasizadeh, and S. Hassan HosseinNia are with the Department of Precision and Microsystems Engineering, Delft University of Technology, 2628 CD Delft, The Netherlands (e-mail: a.ahmadi-dastjerdi20@alumni.imperial.ac.uk; niranjans87@gmail.com; n.karbasizadehesfahani@tudelft.nl; s.h.hosseinniakani@tudelft.nl).

Alessandro Astolfi is with the Department of Electrical and Electronic Engineering, Imperial College London, SW7 2AZ London, U.K., and also with the Dipartimento di Ingegneria Civile e Ingegneria Informatica, Universita di Roma "Tor Vergata", 00133 Rome, Italy (e-mail: a.astolfi@imperial.ac.uk).

Duarte Valério is with the Faculty of IDMEC, Instituto Superior Técnico, Universidade de Lisboa, 649-004 Lisboa, Portugal (e-mail: duarte.valerio@tecnico.ulisboa.pt).

Color versions of one or more figures in this article are available at https://doi.org/10.1109/TAC.2022.3184039.

Digital Object Identifier 10.1109/TAC.2022.3184039

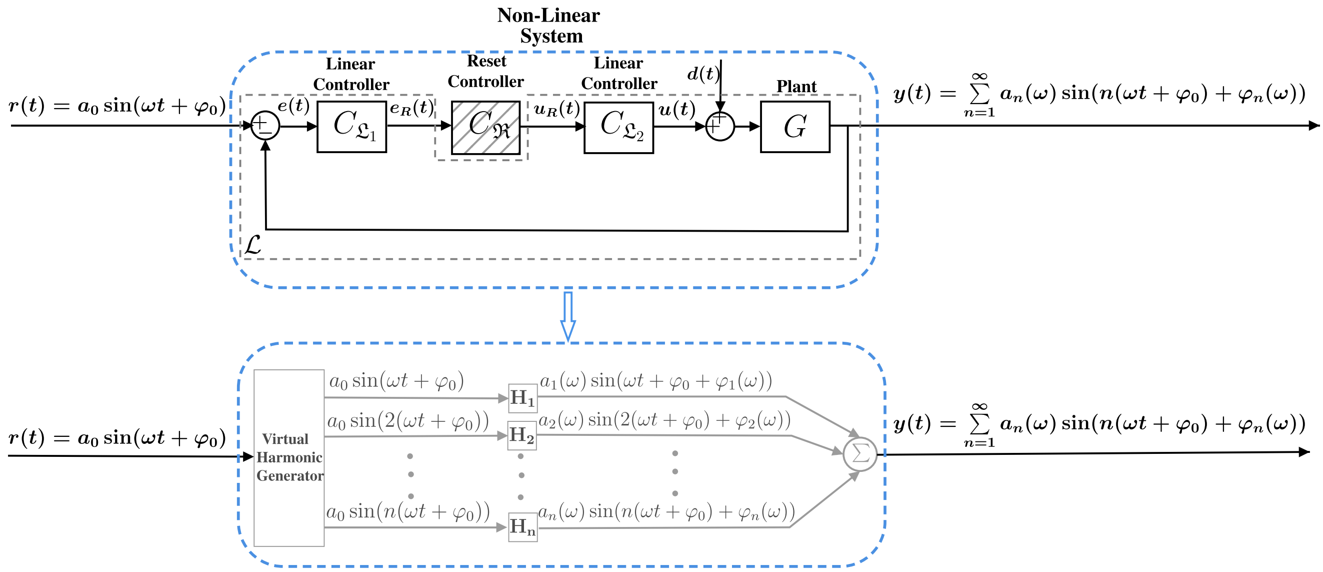

Fig. 1. Closed-loop architecture with reset controller (top). HOSIDF representation of the closed-loop configuration (bottom).

the equations

$$\mathcal{L}: \begin{cases} \dot{\zeta}(t) = A\zeta(t) + Bw(t) + B_u u_R(t) \\ u(t) = C_u\zeta(t) + D_u r(t) \\ e_R(t) = C_{e_R}\zeta(t) + D_{e_R} r(t) \\ y(t) = C\zeta(t) \end{cases} \tag{1}$$

where $\zeta(t) \in \mathbb{R}^{n_p}$ describes the states of the plant and of the linear controllers (n_p is the number of states of the linear part), A, B, C, B_u, C_{e_R}, C_u, D_u, and D_{e_R} are the corresponding dynamic matrices, $y(t) \in \mathbb{R}$ is the output of the plant and $w(t) = [r(t)\ d(t)]^{\mathrm{T}} \in \mathbb{R}^2$ is an external input. The state-space representation of the reset controller is given by the equations

$$\begin{cases} \dot{x}_r(t) = A_r x_r(t) + B_r e_R(t), & e_R(t) \neq 0 \\ x_r(t^+) = A_\rho x_r(t), & e_R(t) = 0 \\ u_R(t) = C_r x_r(t) + D_r e_R(t). \end{cases} \tag{2}$$

The closed-loop state-space representation of the overall system can, therefore, be written as

$$\begin{cases} \dot{x}(t) = \bar{A}x(t) + \bar{B}w(t), & e_R(t) \neq 0 \\ x(t^+) = \bar{A}_\rho x(t), & e_R(t) = 0 \\ u(t) = \bar{C}_u x(t) + \bar{D}_u r(t) \\ e_R(t) = \bar{C}_{e_R} x(t) + D_{e_R} r(t) \\ y(t) = \bar{C}x(t) \end{cases} \tag{3}$$

where $x(t) = [x_r(t)^{\mathrm{T}}\ \ \zeta(t)^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{R}^{n_p+n_r}$, and $\bar{A} = \begin{bmatrix} A_r & B_r C_{eR} \\ B_u C_r & A + B_u D_r C_{eR} \end{bmatrix}$, $\bar{C} = [0_{1\times n_r}\ \ C]$, $\bar{B} = \begin{bmatrix} 0_{n_r\times 2} \\ B \end{bmatrix} + \begin{bmatrix} B_r D_{eR} & 0_{n_r\times 1} \\ B_u D_r D_{eR} & 0_{n_p\times 1} \end{bmatrix}$, $\bar{A}_\rho = \begin{bmatrix} A_\rho & 0_{n_r\times n_p} \\ 0_{n_p\times n_r} & I_{n_p\times n_p} \end{bmatrix}$, $\bar{C}_u = [C_r D_{\mathfrak{L}_2}\ \ C_{e_R} D_r D_{\mathfrak{L}_2} + C_u]$, $\bar{C}_{e_R} = [0_{1\times n_r}\ \ C_{e_R}]$, and $\bar{D}_u = D_u D_{e_R} D_r$ with $D_{\mathfrak{L}_2}$ the feedthrough matrix of $C_{\mathfrak{L}_2}(s)$.

A. Stability and Convergence

In this section, sufficient conditions for the existence of a steady-state solution for the closed-loop reset control system (3) driven by periodic inputs are provided. This is based on the H_β condition [4], [30]–[32], which we recall in what follows. Let

$$C_0 = \begin{bmatrix} \rho & \beta C_{eR} \end{bmatrix}, \quad B_0 = \begin{bmatrix} I_{n_r\times n_r} \\ 0_{n_p\times n_r} \end{bmatrix}$$
$$\rho = \rho^{\mathrm{T}} > 0, \quad \rho \in \mathbb{R}^{n_r\times n_r}, \quad \beta \in \mathbb{R}^{n_r\times 1}. \tag{4}$$

The H_β condition states that the reset control system (3) with $w = 0$ is quadratically stable if and only if there exist $\rho = \rho^{\mathrm{T}} > 0$ and β such that the transfer function

$$H(s) = C_0(sI - \bar{A})^{-1}B_0 \tag{5}$$

is strictly positive real (SPR), $(\bar{A}, B_0)$ and $(\bar{A}, C_0)$ are controllable and observable, respectively, and

$$A_\rho^{\mathrm{T}}\rho A_\rho - \rho < 0. \tag{6}$$

Definition 1: A time $\bar{T} > 0$ is called a reset instant for the reset control system (3) if $e_R(\bar{T}) = 0$. For any given initial condition and input w, the resulting set of all reset instants defines the reset sequence $\{t_k\}$, with $t_k \leq t_{k+1}$, for all $k \in \mathbb{N}$. The reset instants t_k of the reset control system (3) have the well-posedness property if for any initial condition x_0 and any input w, all reset instants are distinct, and there exists a $\lambda > 0$ such that for all $k \in \mathbb{N}$, $\lambda \leq t_{k+1} - t_k$ [8], [33].

Remark 1: If the H_β condition holds, then the reset control system (3) has the uniform bounded-input bounded-state (UBIBS) property and the reset instants have the well-posedness property [34]. Therefore, the reset control system (3) has a unique well-defined solution for $t \geq t_0$ for any initial condition x_0 and input w which is a Bohl function [8], [33].

To develop a frequency-domain analysis method for the reset control system (3), the following assumption is required.

Assumption 1: The initial condition of the reset controller is zero. In addition, there are infinitely many reset instants and $\lim_{k\to\infty} t_k = \infty$.

The second term in Assumption 1 is introduced to rule out a trivial situation. In fact, if $\lim_{k\to\infty} t_k = T_K$, then for all $t \geq T_K$ the reset control system (3) is a stable linear system. Two important technical lemmas, which are used in the proof of the following theorem, are now formulated and proved.

Lemma 1: Let $\{t_k\}$ and $\{\tilde{t}_k\}$ be the reset sequences of the reset control system (3) for two different initial conditions ζ_0 and $\tilde{\zeta}_0$ of the linear part and for the same input. Suppose Assumption 1 and the H_β condition hold and w is a Bohl function. Then $\lim_{k\to\infty}(t_k - \tilde{t}_k) = 0$.

Proof: To begin with note that, for any initial condition $x_0 = \begin{bmatrix} 0^{\mathrm{T}} & \zeta_0^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}$, the signal e_R in (3) can be obtained through the equation

$$\begin{cases} \dot{x}_I(t) = \bar{A}x_I(t) + \bar{B}w(t) + \begin{bmatrix} B_r \\ 0_{n_p\times 2} \end{bmatrix} w_I(t), & e_R(t) \neq 0 \\ x_I(t^+) = \bar{A}_\rho x_I(t), & e_R(t) = 0 \\ e_R(t) = \bar{C}_{e_R} x_I(t) + D_{e_R} r(t) + [1\ 0] w_I(t) \end{cases} \tag{7}$$

with $x_I(0) = 0$ and

$$\begin{cases} \dot{Z}(t) = AZ(t), Z(0) = \zeta_0 \\ w_I(t) = \begin{bmatrix} C_{eR} \\ 0 \end{bmatrix} Z(t). \end{cases} \tag{8}$$

Since the linear part of the system contains the internal model (8) of w_I, and w is a Bohl function, based on [4], [31] e_R is asymptotically independent of w_I. This implies that $\lim_{k\to\infty}(t_k - \tilde{t}_k) = 0$. ■

Lemma 2: Consider the reset control system (3). Suppose Assumption 1 holds, w is a Bohl function, and the H_β condition is satisfied. Then, the reset control system (3) is uniformly exponentially convergent.

Proof: To begin with note that the property of uniformly exponentially convergence is as given in [35]. Since the H_β condition is satisfied, according to Remark 1, the reset control system (3) has a unique well-defined solution for any initial condition x_0 and any w which is a Bohl function. Let x and $\tilde{x}$ be two solutions of the reset control system (3) corresponding to the some input w and to two different initial conditions. Since the H_β condition is satisfied $x(t)$ and $\tilde{x}(t)$ are bounded for all t. Let $\Delta x := x - \tilde{x}$, and let $\{t_k\}$ and $\{\tilde{t}_k\}$ be the reset sequences of $x(t)$ and $\tilde{x}(t)$. Define $\mathcal{M} = \{t \in \mathbb{R}^+ |\ t \neq t_k \wedge t \neq \tilde{t}_k\}$. By Lemma 1

$$\forall\, \delta > 0,\ \exists\, \Pi > 0 \text{ such that } k > \Pi \Rightarrow |t_k - \tilde{t}_k| < \delta. \tag{9}$$

Moreover, by the well-posedness property, there exists a $\lambda > 0$ such that $\lambda \leq t_{k+1} - t_k$ and $\lambda \leq \tilde{t}_{k+1} - \tilde{t}_k$. Thus, selecting δ sufficiently small yields

$$x(t_k + \delta) = e^{\bar{A}\delta}\bar{A}_\rho x(t_k) + \int_{t_k}^{t_k+\delta} e^{\bar{A}(t_k+\delta-\tau)}\bar{B}w(\tau)d\tau \tag{10}$$

for all t_k sufficiently large. By (9), $\tilde{t}_k = t_k + \delta'$, with $0 \leq \delta' \leq \delta$. Thus

$$\begin{aligned} \tilde{x}(t_k + \delta) = e^{\bar{A}(\delta-\delta')}\bar{A}_\rho \Bigg(& e^{\bar{A}\delta'}\tilde{x}(t_k) \\ & + \int_{t_k}^{t_k+\delta'} e^{\bar{A}(t_k+\delta'-\tau)}\bar{B}w(\tau)d\tau \Bigg) \\ & + \int_{t_k+\delta'}^{t_k+\delta} e^{\bar{A}(t_k+\delta-\tau)}\bar{B}w(\tau)d\tau. \end{aligned}$$

Now, by (10) and (11)

$$\begin{aligned} \Delta x(t_k + \delta) &= \bar{A}_\rho \Delta x(t_k) + (e^{\bar{A}\delta}\bar{A}_\rho - e^{\bar{A}(\delta-\delta')}\bar{A}_\rho e^{\bar{A}\delta'})\tilde{x}(t_k) \\ &\quad - e^{\bar{A}(\delta-\delta')}\bar{A}_\rho \int_{t_k}^{t_k+\delta'} e^{\bar{A}(t_k+\delta'-\tau)}\bar{B}w(\tau)d\tau \\ &\quad + \int_{t_k}^{t_k+\delta'} e^{\bar{A}(t_k+\delta-\tau)}\bar{B}w(\tau)d\tau \\ &\quad + (e^{\bar{A}\delta} - I)\bar{A}_\rho \Delta x(t_k) \\ &= \bar{A}_\rho \Delta x(t_k) + O(\delta, \tilde{x}(t_k), x(t_k)) \end{aligned} \tag{11}$$

and, using (9)

$$\lim_{\delta\to 0} O(\delta, \tilde{x}(t_k), x(t_k)) = 0. \tag{12}$$

The same discussion applies to $\tilde{t}_k$. Hence, for t sufficiently large we have

$$\begin{cases} \Delta\dot{x}(t) = \bar{A}\Delta x(t), & t \in \mathcal{M} \\ \Delta x(t^+) = \bar{A}_\rho \Delta x(t), & t \notin \mathcal{M}. \end{cases} \tag{13}$$

Due to the satisfaction of the H_β condition [4], [30], [31], there exist a matrix $P \in \mathbb{R}^{(n_p+n_r)\times(n_p+n_r)}$, $P = P^{\mathrm{T}} > 0$, and a scalar $\alpha > 0$ such that

$$P\bar{A} + \bar{A}^{\mathrm{T}}P \leq -2\alpha P \tag{14}$$

$$\bar{A}_\rho^{\mathrm{T}} P \bar{A}_\rho - P \leq 0. \tag{15}$$

Using the candidate Lyapunov function $V(\Delta x) = \frac{1}{2}(\Delta x)^{\mathrm{T}}P(\Delta x)$ yields

$$\begin{cases} \dot{V} \leq -\alpha V, & t \in \mathcal{M} \\ V(\Delta x(t^+)) = V(\Delta x(t)) + \Xi(t,\delta), & t \notin \mathcal{M}. \end{cases} \tag{16}$$

Thus, using (13) and (15) for t sufficiently large yields $\Xi(t,\delta) \leq 0$. Hence, since Δx is bounded, there exist $\alpha_m > 0$ and $\mathcal{K} > 0$ such that

$$||x_2(t) - x_1(t)||_P^2 \leq \mathcal{K}e^{-\alpha_m t} \tag{17}$$

for all $t \geq 0$ (see Lemma 3 in the Appendix). This implies that the reset control system (3) is uniformly exponentially convergent. ■

Theorem 1: Consider the reset control system (3). Suppose Assumption 1 holds, $w(t) = w_0 \sin(\omega t)$[1], and the H_β condition is satisfied. Then, the reset control system (3) has a periodic steady-state solution, which can be expressed as $\bar{x}(t) = \mathcal{S}(\sin(\omega t), \cos(\omega t), \omega)$ for some function $\mathcal{S} : \mathbb{R}^3 \to \mathbb{R}^{n_r+n_p}$.

Proof: Since the H_β condition holds and $w(t) = w_0 \sin(\omega t)$ is a Bohl function, by Remark 1 the reset control system (3) has a unique solution for any initial condition x_0. In addition, the reset control system (3) has the UBIBS property and, according to Lemma 2, it is uniformly exponentially convergent. Hence, the proof of the existence of the function $\mathcal{S}$ relies on the results in [35]. We only need to show that $\mathcal{S}$ is unique. To this end, similarly to [36], assume that the reset control system (3) has two steady-state solutions $\bar{x}_2(t) = \mathcal{S}_2(\sin(\omega t), \cos(\omega t), \omega)(t)$ and $\bar{x}_1(t) = \mathcal{S}_1(\sin(\omega t), \cos(\omega t), \omega)(t)$, for $w(t) = w_0 \sin(\omega t)$. Since the H_β condition holds, by Lemma 2 there exist $\alpha_m > 0$ and $\mathcal{K} > 0$ such that

$$||\bar{x}_2(t) - \bar{x}_1(t)||_P^2 \leq \mathcal{K}e^{-\alpha_m t} \tag{18}$$

hence, the claim. ■

Corollary 1: Consider the reset control system (3) with $r(t) = r_0 \sin(\omega t)$ and $d = 0$, for all $t \geq 0$. Then, the even harmonics and the subharmonics of the steady-state response have zero amplitude, and the sequence of reset instants is periodic with period $\frac{\pi}{\omega}$.

[1]For ease of notation, we consider $w(t) = w_0 \sin(\omega t)$. However, Theorem 1 is also applicable in the case in which $w(t) = [r_0 \sin(\omega_t + \phi_1),\ d_0 \sin(\omega_t + \phi_2)]^T$.

Proof: The response of (3) for $r = r_0 \sin(\omega t)$ and $d = 0$, for all $t \geq 0$, is given by

$$x(t) = r_0 \left(e^{\bar{A}(t-t_k)} (\xi_k + \psi(t_k)) - \psi(t) \right), \; t \in (t_k, t_{k+1}] \quad (19)$$

where

$$\psi(t) = (\omega I \cos(\omega t) + \bar{A} \sin(\omega t)) \mathcal{F}$$
$$\mathcal{F} = (\omega^2 I + \bar{A}^2)^{-1} \bar{B} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$
$$t_k = \{ t_k \in \mathbb{R}^+, \; k \in \mathbb{Z}^+ \mid e_R(t_k) = 0 \}$$
$$\xi_k = \frac{1}{r_0} x(t_k^+) = \frac{1}{r_0} \bar{A}_\rho x(t_k). \quad (20)$$

Thus

$$\bar{x}(t) = r_0 \left(e^{\bar{A}(t-t_s)} (\xi_s + \psi(t_s)) - \psi(t) \right), \; t \in (t_s, t_{s+1}] \quad (21)$$

with

$$\xi_s = \bar{A}_\rho e^{\bar{A}(t_s - t_{s-1})} \Bigg(\bar{A}_\rho e^{\bar{A}(t_{s-1} - t_{s-2})} \ldots \bar{A}_\rho e^{\bar{A}(t_1 - t_0)} (\xi_0 + \psi(t_0))$$
$$+ \bar{A}_\rho e^{\bar{A}(t_{s-1} - t_{s-2})} \ldots \bar{A}_\rho e^{\bar{A}(t_2 - t_1)} (I - \bar{A}_\rho) \psi(t_1)$$
$$+ \bar{A}_\rho e^{\bar{A}(t_{s-1} - t_{s-2})} \ldots \bar{A}_\rho e^{\bar{A}(t_2 - t_1)} (I - \bar{A}_\rho) \psi(t_2)$$
$$+ \cdots + (I - \bar{A}_\rho) \psi(t_{s-1}) \Bigg) - \bar{A}_\rho \psi(t_s). \quad (22)$$

According to [37], uniformly convergent systems forget their initial conditions. By Lemmas 1 and 2, ξ_s and the reset instants are unique for any t_0 and ζ_0. Hence, the transient response of ξ_s converges to zero, which implies that

$$\xi_s = \bar{A}_\rho e^{\bar{A}(t_s - t_{s-1})} \Bigg((I - \bar{A}_\rho) \psi(t_{s-1})$$
$$+ \bar{A}_\rho e^{\bar{A}(t_{s-1} - t_{s-2})} (I - \bar{A}_\rho) \psi(t_{s-2})$$
$$+ \bar{A}_\rho e^{\bar{A}(t_{s-1} - t_{s-2})} \bar{A}_\rho e^{\bar{A}(t_{s-2} - t_{s-3})} (I - \bar{A}_\rho) \psi(t_{s-3})$$
$$+ \cdots + \bar{A}_\rho e^{\bar{A}(t_{s-1} - t_{s-2})} \ldots \bar{A}_\rho e^{\bar{A}(t_{s-m+1} - t_{s-m})}$$
$$\times (I - \bar{A}_\rho) \psi(t_{s-m}) \Bigg) - \bar{A}_\rho \psi(t_s). \quad (23)$$

Therefore, since reset occurs when

$$\bar{C}_{e_R} \bar{x}(t) + D_{e_R} r_0 \sin(\omega t) = 0 \quad (24)$$

if $\{t_s, t_{s-1}, \ldots, t_{s-m}\}$ are reset instants and satisfy (24), then $\{t_s, t_{s-1}, \ldots, t_{s-m}\} + \frac{\pi}{\omega}$ also satisfy (24), which implies that the sequence of reset instants is periodic with period $\frac{\pi}{\omega}$. Using this property in (21) shows that $\bar{x}(t) = -\bar{x}(t + \frac{\pi}{\omega})$ and $t_{s+q} - t_s = \frac{\pi}{\omega}$, hence $\xi_s = -\xi_{s+q}$. This means that the even harmonics of the steady-state response of the reset control system (3) have zero amplitude. In addition, $\bar{x}(t) = \bar{x}(t + \frac{2\pi}{\omega})$, which implies that the steady-state response of the reset control system (3) does not contain any subharmonic. ■

Remark 2: The reset sequence $\{t_k\}$ and the reset values ζ_k are independent of the input amplitude for $r(t) = r_0 \sin(\omega_t)$.

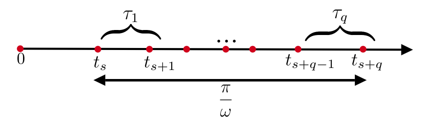

Fig. 2. Steady-state reset instants of the reset control system (3).

We now show that the function $\mathcal{S}$ is derived explicitly for $r(t) = r_0 \sin(\omega_t)$ and $d = 0$. Suppose that there are $q - 1$ reset instants between t_s and $t_s + \frac{\pi}{\omega}$ (see Fig. 2). Assume $\sin(\omega t_s) = \kappa$, then $\cos(\omega t_s) = \pm\sqrt{1 - \kappa^2}$ (without loss of generality, we consider the positive value here). Using trigonometry relations, one has that

$$\psi(t_s) = f_0(\kappa)$$
$$\psi(t_s + \tau_1) = f_1(\kappa, \tau_1)$$
$$\vdots$$
$$\psi(t_s + \tau_1 + \cdots + \tau_q) = f_q(\kappa, \tau_1, \tau_2, \ldots, \tau_q). \quad (25)$$

Moreover

$$\xi_{s+i} = \bar{A}_\rho \Bigg(e^{\bar{A}\tau_i} (g_{i-1}(\kappa, \xi_s, \tau_1, .., \tau_{i-1}) + f_{i-1}(\kappa, \tau_1, .., \tau_{i-1}))$$
$$- f_i(\kappa, \tau_1, \tau_2, \ldots, \tau_i) \Bigg) = g_i(\kappa, \xi_s, \tau_1, \tau_2, \ldots, \tau_i) \quad (26)$$

with $i = 1, 2, \ldots, q$ and $g_0(\kappa, \xi_s) = \xi_s$. Now, since $e_R(t)$ is zero at reset instants, one has that

$$\bar{C}_{e_R} \Bigg(e^{\bar{A}\tau_i} (g_{i-1}(\kappa, \xi_s, \tau_1, .., \tau_{i-1}) + f_{i-1}(\kappa, \tau_1, .., \tau_{i-1}))$$
$$- f_i(\kappa, \tau_1, \tau_2, \ldots, \tau_i) \Bigg) + D_{eR} \sin(\omega(t_s + \tau_1 + \cdots + \tau_i))$$
$$= E_i(\kappa, \xi_s, \tau_1, \ldots, \tau_i) = 0 \quad (27)$$

with $i = 1, 2, \ldots, q$. In addition

$$\tau_1 + \tau_2 + \cdots + \tau_q = \frac{\pi}{\omega}$$
$$\xi_s = -\xi_{s+q} \Rightarrow g_q(\kappa, \xi_s, \tau_1, \tau_2, \ldots, \tau_q) + \xi_s = 0. \quad (28)$$

Moreover, by the well-posedness property of reset instants (see Definition 1), reset instants are distinct. Hence, there are $q + 2$ independent equations and $q + 2$ parameters $(\kappa, \xi_s, q, \tau_1, \tau_2, \ldots, \tau_q)$, $q \in \mathbb{N}$. In addition, the well-posedness property implies that the reset intervals are lower bounded [8]. Hence

$$\exists \, \lambda \leq \tau_i \Rightarrow q \leq \frac{\pi}{\lambda\omega} - 1. \quad (29)$$

Furthermore, for $q = 1$, the equations have always a unique solution. Thus, there exists a bounded nonempty set $Q = \{Q_i \in \mathbb{N} | Q_i \leq q_{\max}\}$ such that for $q \in Q$, the equations have a solution. Hence, $\bar{x}$, the steady-state response of the reset control system (3) to $r(t) = r_0 \sin(\omega t)$, is the solution of (27)–(28) for $q = q_{\max}$. Since $\bar{x}$ is periodic with period $\frac{2\pi}{\omega}$, one has

$$\bar{x}(t) = \sum_{n=1}^{\infty} a_n \cos((2n+1)\omega t) + b_n \sin((2n+1)\omega t). \quad (30)$$

According to Theorem 1, $\bar{x}$ is unique and equal to the function $\mathcal{S}$. Thus

$$\bar{x}(t) = \sum_{n=1}^{\infty} a_n \cos((2n+1)\omega t) + b_n \sin((2n+1)\omega t) = \mathcal{S}(\sin(\omega t), \cos(\omega t), \omega). \tag{31}$$

Finally, one could also use De Moivre's formula to find a formal polynomial expansion for $\mathcal{S}$ in terms of $\sin(\omega t)$ and $\cos(\omega t)$.

B. HOSIDF of the Closed-Loop Reset Control Systems

In Section II-A, sufficient conditions for the existence of the steady-state solution for the reset control system (3) driven by periodic inputs have been presented. Moreover, the steady-state solution has been explicitly calculated. In this section, the HOSIDF technique [38] is applied to the steady-state response of the system to derive a notion of frequency response for the reset control system (3), which allows analyzing tracking and disturbance rejection performance (see the bottom diagram of Fig. 1).

1) Tracking Performance: Consider the reset control system (3) with $r(t) = r_0 \sin(\omega t)$ and $d(t) = 0$, for all $t \geq 0$. We now derive relations between the input r and the steady-state response of the output y, of the error e, and of the control input u. To this end, consider the steady-state reset instants $t_s, t_{s+1}, \ldots, t_{s+q}$ and their associated reset values $\xi_s, \xi_{s+1}, \ldots, \xi_{s+q}$, which are calculated through (27) and (28).

Theorem 2: Consider the reset control system (3) with $r(t) = r_0 \sin(\omega t)$ and $d(t) = 0$, for all $t \geq 0$. Let $T_n(j\omega)$ be the ratio of the nth harmonic component of the output signal y to the first harmonic component of r. Then

$$T_n(j\omega) = \begin{cases} \mathfrak{T}(1,\omega) - \bar{C}(j\omega I + \bar{A})\mathcal{F}, & n = 1 \\ \mathfrak{T}(n,\omega), & n > 1 \text{ odd} \\ 0, & n \text{ even} \end{cases} \tag{32}$$

in which

$$\mathfrak{T}(n,\omega) = \frac{2j\omega\bar{C}}{\pi}(\bar{A} - jn\omega I)^{-1}\left(\sum_{i=1}^{q} \mathcal{R}(i,n,\omega)\right) \tag{33}$$

$$\mathcal{R}(i,n,\omega) = \left(\frac{e^{\bar{A}(t_{s+i}-t_{s+i-1})}}{e^{jn\omega t_{s+i}}} - \frac{I}{e^{jn\omega t_{s+i-1}}}\right) \times \left(\xi_{s+i-1} + \psi(t_{s+i-1})\right). \tag{34}$$

Proof: The proof requires a straightforward calculation, hence, it is omitted (see also [39]). Note that $\psi(t)$ is defined in (20). ∎

Definition 2: The family of complex valued functions $T_n(j\omega), n = 1, 2, \ldots$ is the complementary sensitivity of the reset control system (3).

Corollary 2: Consider the reset control system (3) with $r(t) = r_0 \sin(\omega t)$ and $d(t) = 0$, for all $t \geq 0$. Let $S_n(j\omega)$ be the ratio of the nth harmonic component of the error signal e to the first harmonic component of r. Then

$$S_n(j\omega) + T_n(j\omega) = \begin{cases} 1, & n = 1 \\ 0, & n > 1. \end{cases} \tag{35}$$

Corollary 3: Consider the reset control system (3) with $r(t) = r_0 \sin(\omega t)$ and $d(t) = 0$, for all $t \geq 0$. Let $CS_n(j\omega)$ be the ratio of the nth harmonic component of the control input signal u to the first harmonic component of r. If the plant is stable, then

$$CS_n(j\omega) = \frac{T_n(j\omega)}{G(nj\omega)}. \tag{36}$$

Definition 3: The families of complex valued functions $S_n(j\omega)$ and $CS_n(j\omega)$, $n = 1, 2, \ldots$, are the sensitivity and the control sensitivity of the reset control system (3), respectively.

2) Disturbance Rejection: In this section, relations between $d(t) = \sin(\omega t)$ and the error e and the control input u are found in the case in which $r(t) = 0$ for the reset control system (3) using the same procedure provided in Section II-B1. To this end, the matrix $\psi(t)$ has to be replaced by

$$\psi_{\mathcal{D}}(t) = (\omega I \cos(\omega t) + \bar{A}\sin(\omega t))\mathcal{F}_{\mathcal{D}}$$

$$\mathcal{F}_{\mathcal{D}} = (\omega^2 I + \bar{A}^2)^{-1}\bar{B}\begin{bmatrix} 0 \\ 1 \end{bmatrix}. \tag{37}$$

Let $t'_s, t'_{s+1}, \ldots, t'_{s+q'}$ and $\xi'_s, \xi'_{s+1}, \ldots, \xi'_{s+q'}$ be the steady-state reset instants and their associated reset values for the reset control system (3) with $d(t) = d_0 \sin(\omega t)$ and $r(t) = 0$, respectively. In addition, since $r(t) = 0$, (27) is changed to

$$\bar{C}_{e_R}\left(e^{\bar{A}\tau'_i}(g_{i-1}(\kappa', \xi'_s, \tau'_1, .., \tau'_{i-1}) + f_{i-1}(\kappa', \tau'_1, .., \tau'_{i-1})) - f_i(\kappa', \tau'_1, \tau'_2, \ldots, \tau'_i)\right) = E_i(\kappa', \xi'_s, \tau'_1, \ldots, \tau'_i) = 0 \tag{38}$$

with $i = 1, 2, \ldots, q'$. Now, substituting $\psi(t)$ with $\psi_{\mathcal{D}}(t)$ in the relations (25) and (26), and considering (38) instead of (27), the steady-state response of the reset control system (3) for $d(t) = d_0 \sin(\omega t)$ and $r(t) = 0$ is found using the same procedure provided in Section II-A.

Corollary 4: Consider the reset control system (3) with $d(t) = d_0 \sin(\omega t)$ and $r(t) = 0$, for all $t \geq 0$. Let $PS_n(j\omega)$ be the ratio of the nth harmonic component of the error signal e to the first harmonic component of d. Then

$$PS_n(j\omega) = \begin{cases} \mathfrak{P}(1,\omega) + \bar{C}(j\omega I + \bar{A})\mathcal{F}_{\mathcal{D}}, & n = 1 \\ \mathfrak{P}(n,\omega), & n > 1 \text{ odd} \\ 0, & n \text{ even} \end{cases} \tag{39}$$

in which

$$\mathfrak{P}(n,\omega) = \frac{2j\omega\bar{C}}{\pi}(jn\omega I - \bar{A})^{-1}\left(\sum_{i=1}^{q'} \mathcal{R}_{\mathcal{D}}(i,n,\omega)\right) \tag{40}$$

$$\mathcal{R}_{\mathcal{D}}(i,n,\omega) = \left(\frac{e^{\bar{A}(t'_{s+i}-t'_{s+i-1})}}{e^{jn\omega t'_{s+i}}} - \frac{I}{e^{jn\omega t'_{s+i-1}}}\right) \times \left(\xi'_{s+i-1} + \psi_{\mathcal{D}}(t'_{s+i-1})\right).$$

Corollary 5: Consider the reset control system (3) with $d(t) = d_0 \sin(\omega t)$ and $r(t) = 0$, for all $t \geq 0$. Let $CS_{d_n}(j\omega)$ be the ratio of the nth harmonic component of the control input signal u to the first harmonic component of d. If the plant is stable, then

$$CS_{d_n}(j\omega) = \begin{cases} \dfrac{-PS_1(j\omega)}{G(j\omega)} - 1, & n = 1 \\ \dfrac{-PS_n(j\omega)}{G(nj\omega)}, & n > 1. \end{cases} \tag{41}$$

Definition 4: The families of complex valued functions $PS_n(j\omega)$ and $CS_{d_n}(j\omega), n = 1, 2, \ldots$, are the process-sensitivity and the control

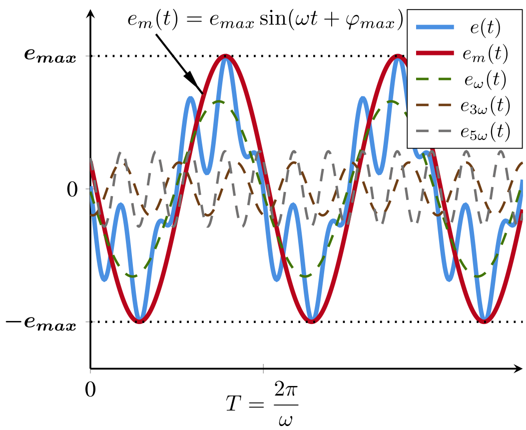

Fig. 3. Error signal $e(t)$ with its 1st, 3rd, and 5th harmonics. $e_m(t)$ is fitted to $e(t)$ and it is an indicator of the maximum error of the system.

sensitivity due to the presence of the disturbance of the reset control system (3), respectively.

C. Pseudosensitivities for Reset Control Systems

The analysis of the error signal e and of the control input u is one of the main factors while designing a controller. In linear systems this analysis is performed using the closed-loop transfer functions [40]. As discussed in Section I, although reset control systems may be analyzed using the DF of the reset controller in the closed-loop sensitivity equations, this yields an approximation, which is not precise due to the existence of high order harmonics. On the other hand, it is not trivial to analyze reset controllers considering all harmonics. In order to perform the analysis of reset control systems straightforwardly, we combine all harmonics into one frequency function for each closed-loop frequency response. In the literature, there are several definitions of Bode plot for nonlinear systems [41], [42]. However, all of these focus only on the gain of the system. In the following, pseudosensitivities, which have both gain and phase components, are defined.

It has been proven that the error and the control input signals of the reset control system (3) are periodic with period $\frac{2\pi}{\omega}$ (see Fig. 3). We define the pseudosensitivity as the ratio of the maximum error of the reset control system (3), for $r(t) = r_0\sin(\omega t)$ and $d(t) = 0$, for all $t \geq 0$, to the amplitude of the reference at each frequency.

Definition 5: The Pseudosensitivity S_∞ is, for all $\omega \in \mathbb{R}^+$

$$S_\infty(j\omega) = e_{\max}(\omega)e^{j\varphi_{\max}(\omega)},\ \varphi_{\max} = \frac{\pi}{2} - \omega t_{\max}$$

$$e_{\max}(\omega) = \frac{\max\limits_{t_s \leq t \leq t_{s+2q}} (r(t) - y(t))}{r_0} = \sin(\omega t_{\max}) - \frac{1}{r_0}\bar{C}\bar{x}(t_{\max})$$

with

$$t_{\max} \in \{t_{ext} \mid \dot{e}(t_{ext}) = 0,\ t_s \leq t_{ext} \leq t_{s+2q}\} \cup\{t_{s+i} \mid i \in \mathbb{Z},\ 0 \leq i \leq 2q\}.$$

Using (3) and (21), t_{ext} is obtained from

$$\begin{aligned}\dot{e}(t_{\text{ext}}) = 0 &\Rightarrow \omega\cos(\omega t_{\text{ext}}) - \bar{C}\bar{B}\begin{bmatrix}1\\0\end{bmatrix}\sin(\omega t_{\text{ext}})\\ &= \bar{C}\bar{A}\big(e^{\bar{A}(t_{\text{ext}}-t_{s+i})}(\xi_{s+i}\\ &\quad + \psi(t_{s+i})) - \psi(t_{\text{ext}})\big)\\ t_{\text{ext}} &\in (t_{s+i}, t_{s+i+1}],\ i = \{i \in \mathbb{Z}^+ \mid i < 2q\}.\end{aligned} \tag{42}$$

Similarly, the pseudoprocess sensitivity, the pseudocomplementary sensitivity, the pseudocontrol sensitivity, and the pseudocontrol sensitivity of the disturbance can be defined (for more detail see [39]). We conclude this section, with the following statement.

Corollary 6: Consider the reset control system (3). The pseudosensitivities and the closed-loop HOSIDFs are independent of the amplitude of the harmonic excitation input.

Remark 3: The presented results have been integrated into an open source toolbox, which has been developed using MATLAB, see [43]. This toolbox facilitates the analysis and design for reset control systems.

III. Periodic Inputs

In Section II, a notion of frequency response and pseudosensitivities for reset control systems have been defined. These serve as graphical tools for performance analysis of reset controllers. The pseudosensitivities determine how a system amplifies harmonic inputs at various frequencies, information which is essential for control designers. However, this information is obtained for a single harmonic excitation and since the superposition principle does not hold, it provides only an approximation in the case of multiharmonics excitation. In this section the steady-state performance in the presence of multiharmonics excitation and periodic inputs is investigated. This is reasonable since most references and disturbances are periodic [11]. For ease of notation let $\text{lcm}(\frac{a_1}{b_1}, \frac{a_2}{b_2}, \ldots, \frac{a_i}{b_i})$ denote the least common multiple of $\frac{a_1}{b_1}, \frac{a_2}{b_2}, \ldots,$ and $\frac{a_i}{b_i}$ and $\gcd(\frac{a_1}{b_1}, \frac{a_2}{b_2}, \ldots, \frac{a_i}{b_i})$ denote the greatest common divisor of $\frac{a_1}{b_1}, \frac{a_2}{b_2}, \ldots,$ and $\frac{a_i}{b_i}$ in which $a_i \in \mathbb{N}$ and $b_i \in \mathbb{N}$.

Theorem 3: Consider the reset control system (3). Suppose the H_β condition and Assumption 1 hold. Then for any periodic excitation of the form

$$w(t) = w_0\sin\left(\frac{2\pi}{T_0}t\right) + w_1\sin\left(\frac{2\pi}{T_1}t\right) + \cdots + w_N\sin\left(\frac{2\pi}{T_N}t\right) \tag{43}$$

with $w_i = [r_i,\ d_i]^{\text{T}}$, the reset control system (3) has a periodic steady-state solution of the form

$$\bar{x}(t) = a_0 + \sum_{n=1}^{\infty} a_n\cos(n\omega_M t) + b_n\sin(n\omega_M t)$$

$$\omega_M = 2\pi \times \gcd(\frac{1}{T_0}, \frac{1}{T_1}, \ldots, \frac{1}{T_N}).$$

Proof: Let $t_{s_{M+i}}$ be the steady-state reset instants of the reset control system (3) for w is given in (43). By (3), the steady-state solution for w as in (43) is given by

$$\bar{x}(t) = e^{\bar{A}(t-t_{s_M})}(\xi_{s_M} + \psi_M(t_{s_M})) - \psi_M(t),\ t \in (t_{s_M}, t_{s_M+1}] \tag{44}$$

where

$$\begin{aligned}\psi_M(t) &= \psi_0(t) + \psi_1(t) + \cdots + \psi_N(t)\\ \psi_i(t) &= (\omega_i I\cos(\omega_i t) + \bar{A}\sin(\omega_i t))\mathcal{F}_i\\ \mathcal{F}_i &= (\omega_i^2 I + \bar{A}^2)^{-1}\bar{B}w_i.\end{aligned}$$

By Lemma 2, the reset control system (3) forgets the initial condition; thus, using a procedure similar to the one in Section II-A, yields

$$\bar{x}(t) = e^{\bar{A}(t-t_{s_M})}\Big((I - \bar{A}_\rho)\psi_M(t_{s_M}) + e^{\bar{A}(t_{s_M}-t_{s_{M-1}})}$$
$$\Big((I - \bar{A}_\rho)\psi_M(t_{s_{M-1}}) + \cdots + \bar{A}_\rho e^{\bar{A}(t_{s_{M-1}}-t_{s_{M-2}})}\ldots$$

$$\bar{A}_\rho e^{\bar{A}(t_{s_{M-m+1}}-t_{s_{M-m}})}(I-\bar{A}_\rho)\psi_M(t_{s_{M-m}})\Big)\Big)$$
$$-\psi_M(t),\ t\in(t_{s_M},t_{s_M+1}].$$

Since the reseting condition is

$$\bar{C}_{e_R}\bar{x}(t)+D_{e_R}[1\ 0]w_M(t)=0 \tag{45}$$

if $\{t_{s_M},t_{s_M-1},\ldots,t_{s_M-m}\}$ are reset instants and satisfy (45), then $t\in\{t_{s_M},t_{s_M-1},\ldots,t_{s_M-m}\}+\frac{2\pi}{\omega_M}$ are such that (45) holds, which implies that the sequence of reset instants is periodic with period $\frac{2\pi}{\omega_M}$; hence, $\bar{x}(t)=\bar{x}(t+\frac{2\pi}{\omega_M})$, and using the Fourier series representation yields

$$\bar{x}(t)=a_0+\sum_{n=1}^{\infty}a_n\cos(n\omega_M t)+b_n\sin(n\omega_M t). \tag{46}$$

■

We conclude this section with the following statement the proof of which can be found in [39].

Corollary 7: Consider the reset control system (3). Suppose the H_β condition and Assumption 1 hold. Then for any periodic input $w_P(t)=w_P(t+T_P)$ the reset control system (3) has a steady-state periodic solution with the same period T_P[2].

IV. Conclusion

This article has proposed an analytical approach to obtain closed-loop frequency responses for reset control systems, including high-order harmonics. To this end, sufficient conditions for the existence of the steady-state solution of the closed-loop reset control systems driven by periodic inputs have been presented. Moreover, pseudosensitivities, which serve as a graphical tool for performance analysis of reset controllers, have been defined: these relate the error and control input of the system to the reference and the disturbance. All calculations are performed in a user-friendly toolbox to make this approach easy of use. The proposed method predicts the closed-loop performance of reset control systems more accurately than the DF method.

Appendix A

Lemma 3: Consider a positive and bounded function $V(t)$. Suppose that there exists $\alpha>0$ such that

$$\begin{cases}\dot{V}\le-\alpha V & t\in\mathcal{M}\\ V(\Delta x(t^+))=V(\Delta x(t))+\Xi(t,\delta), & t\notin\mathcal{M}.\end{cases} \tag{47}$$

If for t sufficiently large

$$\Xi(t,\delta)\le 0 \tag{48}$$

then there exist $\alpha_m>0$ and $\mathcal{K}>0$ such that

$$V(t)\le\mathcal{K}e^{-\alpha_m t},\ \text{for all } t\ge 0. \tag{49}$$

Proof: Since V is bounded, by (47) and (48), V achieves its maximum value at some time $t_{v_m}<\infty$. In other words, there exists a time $0\le t_{v_m}<\infty$ such that

$$\begin{cases}V(t_{v_m})\ge V(t), & t\le t_{v_m}\\ V(t_{v_m})>V(t). & t>t_{v_m}\end{cases} \tag{50}$$

[2]Due to lack of space, it is not possible to have illustrative examples in this article. In [39], there are several practical example which show the effectiveness of the proposed approach.

Therefore, by (48) and well-posedness property, there exists a bounded set $\mathcal{T}=\{t_i>t_{v_m}|\ t_i\notin\mathcal{M}\wedge\Xi(t_i,\delta)>0,\ i\in\mathbb{N}\}$. Thus, using (50) there exists a bounded set $\mathcal{A}=\{\alpha_i>0|\ \ V(t_i)=e^{-\alpha_i(t_i-t_{v_m})}V(t_{v_m}),\ t_i\in\mathcal{T}\}$. Since the set $\mathcal{A}$ is bounded, there exists a $\alpha'>0$ such that for all $\alpha_i\in\mathcal{A}$ one has that $\alpha'\le\alpha_i$. Now considering $\alpha_m=\min(\alpha,\alpha')$, based on (47) and (48), yields

$$V(t)\le e^{-\alpha_m(t-t_{v_m})}V(t_{v_m})=\mathcal{K}e^{-\alpha_m t},\ \text{for all } t\ge 0. \tag{51}$$

Finally, if $\mathcal{T}$ and $\mathcal{A}$ are empty sets, then selecting $\alpha_m=\alpha$ the claim yields.

Acknowledgment

The authors would like to thank Prof. Alfonso Baños from University of Murcia, Spain, for his kind guidance.

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