A Background Calibration for Joint Mismatch in the OFDM System With Time-Interleaved ADC

Haoyang Shen, *Graduate Student Member, IEEE*, Deepu John, *Senior Member, IEEE*, and Barry Cardiff, *Senior Member, IEEE*

***Abstract*—Time-interleaved analog-to-digital converters (TIADCs) are widely used in multi-Gigabit communication systems due to their high conversion rates. In practice, the offset, gain, and time-skew mismatch of TIADC significantly degrade the performance of an OFDM-TIADC system. It is also the case that the addition of channel coding can even degrade the system further. In this brief, we analyze the modulation error floor caused by mismatches and propose a two-stage background joint mismatch calibration using Least-square estimation to detect the offset and gain mismatches, and the time-skew mismatches are estimated by golden-section search. This method significantly reduces the bit error rate (BER) and improves the reliability of the OFDM-TIADC system.**

***Index Terms*—OFDM, time-interleaved analog-to-digital converter (TIADC), mismatches estimation, background calibration.**

I. Introduction

ORTHOGONAL frequency division multiplexing (OFDM) is widely used in wired and wireless broadband communication systems [1]. The authors in [2] state that OFDM is a promising technology for optical communications, and the optical OFDM system increases the data rate to 100Gb/s and beyond; thus, it requires a high-speed analog-to-digital converter (ADC) at the receiving end. Time-interleaved analog-to-digital Converter (TIADC) is widely used in the OFDM systems due to its high conversion rates, e.g., [3]–[5]. However, the TIADC is sensitive to offset, gain, and time-skew mismatches due to the process, voltage, and temperature (PVT) variations [6]. The performance of OFDM-TIADC system is usually limited by these mismatches. The effect of these mismatches on the bit error rate (BER) of the OFDM-TIADC system is analyzed in [5], [7]. Some mismatch calibration algorithm is proposed to relax the TIADC limitation in [8]–[12].

In communication modulation, the convolutional code is usually used to correct the bit error in a bit stream for the OFDM system [13], [14]. In theory, it can correct the bit error caused by the TIADC mismatches, but these convolution coding techniques are designed to tolerate the expected worst-case BER and fail to work at all or make the BER curve worse if the BER is above the threshold. Therefore, the existence of the TIADC mismatches may make the convolutional code fail to work or make the worse results.

In this brief, we analyze the error floor as a function of offset, gain and time-skew mismatch standard deviation separately allowing us to set correction targets for each of these imperfections. This tolerable mismatch level can serve as a guideline for circuit designers to do the digital signal processing and hardware design. We use the (2, 1, 7) convolution code as per the IEEE 802.11a standard [15] for the modulation and find the expected worst-case of BER is approximately 0.09. The mismatches may make the BER higher than the threshold, which makes the performance of the OFDM system worse due to the coded system.

We propose a two-stage digital background calibration to estimate the mismatches from TIADC. The estimation algorithm is divided into offset and gain estimation and time-skew estimation. We ignore the time-skew mismatch to estimate the offset and gain mismatches using the Least-square estimation, which implies that the decision feedback mechanism and channel estimation. After compensating offset and gain errors, the time-skew mismatch is explored by a gold-section search. In the correction block, the offset and gain mismatch is corrected by the addition and multiplication operations. The time-skew mismatch is corrected by variable delay lines (VDLs) [8], [9]. The digital background calibration significantly reduces the BER of the OFDM system, and it also ensures the BER is below the threshold of the coded system to improve the reliability of the communication system.

This brief is organized as follows: the OFDM-TIADC system math model is introduced in Section II. Section III proposes the digital estimation algorithm for offset, gain, and time-skew mismatches. The correction structure is mentioned in Section IV. The simulation result on the MATLAB is presented in Section V. Finally, the conclusion is shown in Section VI.

II. OFDM & TIADC Model

Fig. 1 shows a block diagram of an OFDM system using an M-channel TIADC at the receiving end [4].

Manuscript received 11 May 2022; accepted 6 June 2022. Date of publication 10 June 2022; date of current version 30 August 2022. This work was supported by the China Scholarship Council. This brief was recommended by Associate Editor E. Moisello. *(Corresponding author: Haoyang Shen.)*

The authors are with the School of Electrical and Electronic Engineering, University College Dublin, Dublin 4, D04 V1W8, Ireland (e-mail: haoyang.shen@ucdconnect.ie; deepu.john@ucd.ie; barry.cardiff@ucd.ie).

Color versions of one or more figures in this article are available at https://doi.org/10.1109/TCSII.2022.3182235.

Digital Object Identifier 10.1109/TCSII.2022.3182235

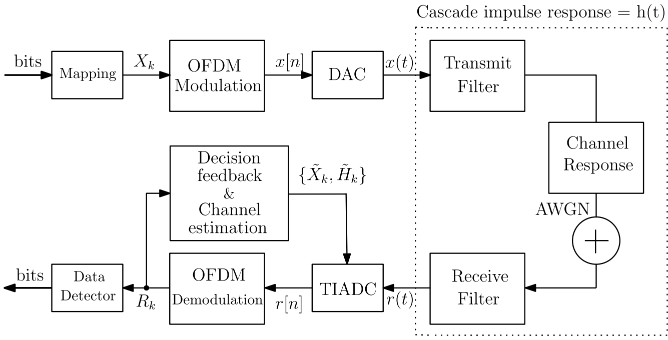

Fig. 1. Block diagram of an OFDM-TIADC system with decision feedback mechanism and channel estimation techniques [4].

The transmitting signal, $x[n]$, can be expressed as following:

$$x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X_k e^{\frac{j2\pi kn}{N}} \tag{1}$$

where the X_k is the bit stream with quadrature modulation, e.g., PSK and QAM. The N is the number of subcarriers.

In Fig. 1, assuming the impulse response of cascading the transmit filter, channel, and receiver filter is $h(t)$ in this system. We define the sampling rate of TIADC is T, and $\eta(t)$ is independently and identically distributed (i.i.d.) AWGN noise with noise spectral density N_0 caused by the communication channel. When the receiver gets the signal, the signal, $r(t)$, can be written as:

$$\begin{aligned} r(t) &= \sum_{n=0}^{N-1} x(t)h(t-nT) + \eta(t) \\ &= \frac{1}{N}\sum_{k=0}^{N-1} X_k \sum_{n=0}^{N-1} h(t-nT)e^{\frac{j2\pi nk}{N}} + \eta(t) \\ &= \frac{1}{N}\sum_{k=0}^{N-1} X_k e^{\frac{j2\pi kt}{NT}} \sum_{n=0}^{N-1} h(t-nT)e^{-\frac{j2\pi k(t-nT)}{NT}} + \eta(t) \\ &= \frac{1}{N}\sum_{k=0}^{N-1} X_k H_k e^{\frac{j2\pi kt}{NT}} + \eta(t) \end{aligned} \tag{2}$$

Then the signal goes into an M-channel TIADC with offset, gain, and time-skew mismatches, and the output of the TIADC can be expressed as:

$$r[n] = \frac{1+g_n}{N}\sum_{k=0}^{N-1} X_k H_k e^{\frac{j2\pi k(n+\tau_n)}{N}} + o_n + \eta[n] \tag{3}$$

where

$$\begin{aligned} o_n &\triangleq o_{m(\mathrm{mod}M)} \\ g_n &\triangleq g_{m(\mathrm{mod}M)} \\ \tau_n &\triangleq \tau_{m(\mathrm{mod}M)} \end{aligned}$$

Observing (3), the interference caused by offset mismatch is independent of $r[n]$, which is easy to estimate using the Least-square estimation and corrected by the addition operation. The gain and time-skew mismatches cause signal-dependent interference that complicates the receiver signal and jointly affect the signal result. We propose a digital background calibration by addressing the gain and time-skew mismatches separately in Section III.

III. Mismatch Estimation

In this section, we propose a two-stage digital mismatch estimation algorithm. The first stage is offset and gain estimation using the Least-square estimation and the second stage is using golden-section search to detect the time-skew mismatch.

A. Offset and Gain Mismatches

We define $s[n] = \frac{1}{N}\sum_{k=0}^{N-1} X_k H_k e^{\frac{j2\pi k(n+\tau_n)}{N}}$. If the $s[n]$ were know we could compute the following set of values:

$$\zeta[n] \triangleq r[n]/s[n] \tag{4}$$

These relate to the gain and offset errors as follows:

$$\zeta[n] = 1 + g_n + o_n/s[n] + \eta'[n] \tag{5}$$

where we redefined the noise term as $\eta'[n] \triangleq \eta[n]/s[n]$ which can be considered to be another i.i.d. additive noise contribution.

This can be written as the following matrix equation:

$$\mathbf{Z} = \left[\boldsymbol{A}\,\middle|\,\mathrm{diag}\left\{\frac{1}{s[n]}\right\}\boldsymbol{A}\right]\left[\frac{\boldsymbol{g}+\mathbf{1}}{\boldsymbol{o}}\right] + \boldsymbol{\eta}' \tag{6}$$

where $\boldsymbol{g}$ and $\boldsymbol{o}$ are vectorized[1] versions of g_n, and o_n respectively and $\boldsymbol{A}$ is the $N \times M$ matrix defined as:

$$\boldsymbol{A} \triangleq \begin{bmatrix} \boldsymbol{I_M} \\ \hline \boldsymbol{I_M} \\ \hline \vdots \\ \hline \boldsymbol{I_M} \end{bmatrix}$$

Initial estimates $\tilde{\boldsymbol{g}}$ and $\tilde{\boldsymbol{o}}$ for $\boldsymbol{g}$ and $\boldsymbol{o}$ respectfully can be obtained via a pseudo inverse of (6) provided the $s[n]$ are known. Specifically we compute the estimates as follows:

$$\left[\frac{\tilde{\boldsymbol{g}}+\mathbf{1}}{\tilde{\boldsymbol{o}}}\right] = \left[\boldsymbol{A}\,\middle|\,\mathrm{diag}\left\{\frac{1}{\tilde{s}[n]}\right\}\boldsymbol{A}\right]^{\dagger}\tilde{\boldsymbol{Z}} \tag{7}$$

where the $\tilde{s}[n]$ are the $s[n]$ computed using prior knowledge of the X_k and the H_k and by letting the $\tau_n = 0$. This implies the use of a decision feedback mechanism and a channel estimation block. The $\tilde{Z}$ is $\tilde{Z}$ computed with $\tilde{s}[n]$ rather than $s[n]$ as previously defined. Final estimates of g_n, o_n and τ_n will be computed in a later processing step.

B. Time-Skew Mismatch

For time-skew mismatch estimation, we compensate the offset and gain mismatch at first; thus, the (3) for m^{th} sub-ADC can be rewritten as:

$$\begin{aligned} r^{(m)}[n] &\triangleq \frac{r[nM+m] - \tilde{o}_m}{1+\tilde{g}_m} \\ &= \frac{1}{N}\sum_{k=0}^{N-1} X_k H_k e^{\frac{j2\pi k((nM+m)+\tau_m)}{N}} + \eta^{(m)}[n] \end{aligned}$$

[1] Unless otherwise stated, all vectors are column vectors.

Algorithm 1: The Golden-Section Search to Find the $\tilde{\tau}_m$ That Minimizes a Real Value Function $f(\tilde{\tau}_m)$

```
Let: L and U be Lower and Upper possible values of τ_m
  resp., and let x_1 = L + (1 − δ)(U − L), and
  x_2 = L + δ(U − L) where δ = (√5 − 1)/2.
do
  if f(x_1) < f(x_2) then
    U = x_2
    x_2 = x_1
    x_1 = L + (1 − δ)(U − L)
  else
    L = x_1
    x_1 = x_2
    x_2 = L + δ(U − L)
  end
while U − L > ϵ & k < max iterations;
if f(x_1) < f(x_2) then
  τ̃_m = x_1
else
  τ̃_m = x_2
end
```

$$= \sum_{k=0}^{N-1} \alpha_{(n,k,m)} e^{\frac{j2\pi k \tau_m}{N}} + \eta^{(m)}[n] \tag{8}$$

where $\alpha_{(n,k,m)} \triangleq \frac{1}{N} X_k H_k e^{\frac{j2\pi k((nM+m))}{N}}$.

We define $s^{(m)}[n] \triangleq \sum_{k=0}^{N-1} \alpha_{(n,k,m)} e^{\frac{j2\pi k \tilde{\tau}_m}{N}}$. The $\tilde{\tau}_m$ can be obtained by a minimum squared error (MSE) approach as follows: [4]

$$\tilde{\tau}_m = \underset{\tilde{\tau}_m}{\operatorname{argmin}} \sum_{n=0}^{N-1} \left| r^{(m)}[n] - s^{(m)}[n] \right|^2 \tag{9}$$

We propose the use of a golden-section search to efficiently obtain the solution of this MSE problem statement (9). The golden-section search algorithm is described in Algorithm 1 where the parameters L and U are initial lower and upper bounds for each τ_m. For example, if the time-skew has a standard deviation of 10% then we would set $L = -0.3$ and $U = 0.3$ to obtain a 3-sigma solution. ϵ is the termination condition when the interval is less than it. Fig. 2 demonstrates the convergence of the error to zero where 1000's of randomly generated time skews errors were simulated. For a value of $\epsilon = 10^{-8}$ we found that typically the algorithm requires approximately 36 iterations. For this simulation we used an OFDM system with 256 sub-carriers each containing 16-QAM and the SNR is 25 dB.

IV. Mismatch Correction

For the mismatch correction, the addition and multiplication operations easily correct the offset and gain mismatches. In some TIADC calibration methods, e.g., [8], [9], variable delay lines (VDLs) can introduce small timing delay to the clock path to compensate the time-skew mismatch. Adjusting the configurations of these VDLs based on time-skew estimation values to keep all sub-ADCs have the same timing delay. The complete TIADC correction structure is depicted in Fig. 3.

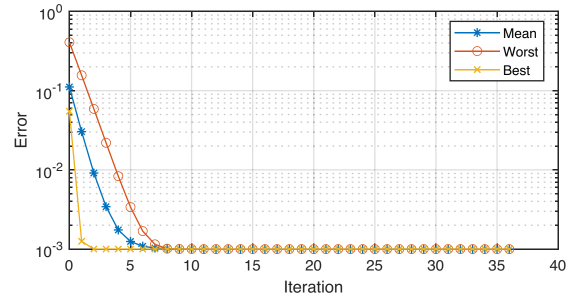

Fig. 2. The evaluated error during the iterations for 8-channel TIADC with 10% time-skew mismatch under 25 dB SNR AWGN.

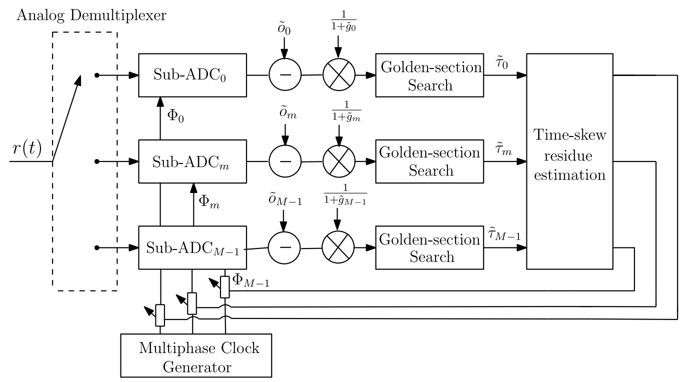

Fig. 3. The M-channel TIADC system with adjusting VDL to compensate the time-skew mismatch.

Note that the VDLs are only capable to correct the time-skew mismatch in their dynamic correction range. If the τ_m lies the outside of the correction range, the VDLs' performance will degrade. The authors in [16] propose the selection of time-reference affects the time-skew correction range. Usually, many applications use the one of the sub-ADC as time reference, e.g., [11], [12], and the authors in [16] proposes a time reference, which can minimize the time-skew correction range for every sub-ADC, and it is expressed as:

$$r \triangleq \frac{\operatorname{Max}_m(\{\tau_m\}) - \operatorname{Min}_m(\{\tau_m\})}{2} \tag{10}$$

We define the dynamic correction range of VDL is Δ and the target yield is η, which can be written as:

$$\eta \triangleq \mathrm{P}(|\tau_m - r| \leq \Delta \quad \forall 0 \leq m < M) \tag{11}$$

Fig. 4 depicts the statistic model of the two time reference selections. The detail of analysis statistic is introduced in [16]. Assuming the correction range of VDL is $0.02T_s$, using the proposed time reference in (10) making the target yield are more than 95%; however, using the any one sub-ADC as time reference can only reach 60%.

V. Simulation Result

The simulation results are carried out using an OFDM-TIADC system with each sub-carrier having independent channel gain drawn from a Rayleigh fading random process

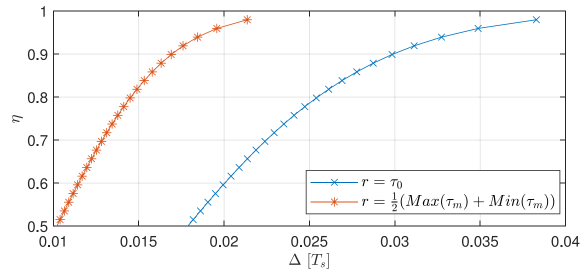

Fig. 4. The target yield, η, versus the dynamic correction range, Δ, for an 8-channel TIADC.

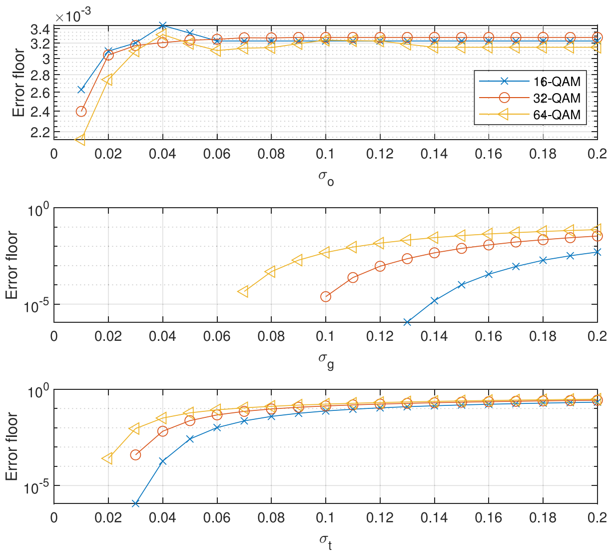

Fig. 5. The error floor versus the standard deviation of offset, gain, and time-skew mismatch.

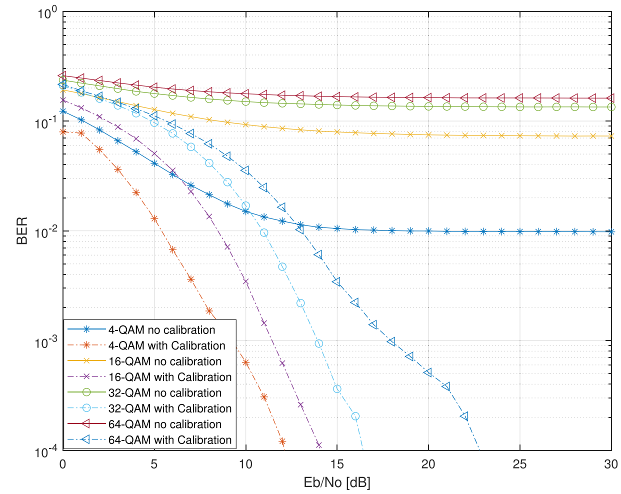

Fig. 6. The BER with/without calibration in 8-channel TIADC for OFDM system with FFT size is 256 employing 4-QAM, 16-QAM, 32-QAM, and 64-QAM constellation.

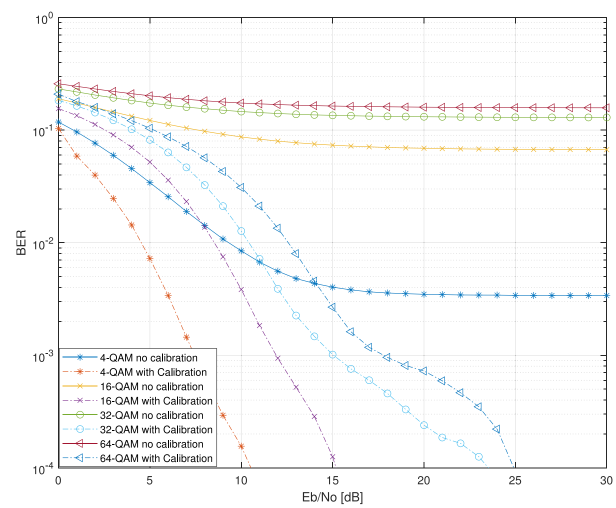

Fig. 7. The BER with/without calibration in 8-channel TIADC for OFDM system with FFT size is 1024 employing 4-QAM, 16-QAM, 32-QAM, and 64-QAM.

having unity average gain. This independent sub-carrier gain assumption is not realistic but, as we are also assuming perfect channel-state-information (CSI) in the receiver, it allows us to perform comprehensive simulations relatively quickly rather that having to perform each simulation over the very long time windows that would be required to average out the effects of correlated fading. In the golden-section search algorithm, we set the $\epsilon = 10^{-8}$ and the maximum number of iterations is 36.

Our experiment includes three parts; firstly, we plot the error floor as a function of offset, gain and time-skew mismatch standard deviation separately allowing us to set correction targets for each of these imperfections. Next we test the performance of the calibration algorithm for different-order QAM constellation sizes (on each sub-carrier) using a typical values of offset, gain and time-skew mismatch standard deviation. Finally, we analyze the impact of the proposed calibration algorithm on convolutional coded system.

A. Error Floor

Fig. 5 depicts the effect of offset, gain, and time-skew mismatch on the BER of the OFDM system. The error floor versus mismatches standard deviation (std) is based on an 8-channel TIADC. From Fig. 5, we argue that the system can tolerate the std of offset mismatch is below 0.01, the std of gain mismatch is below 0.07, and the std of time-skew mismatch is below 0.02. Thus, we set the std of offset, gain and time-skew mismatches are 0.05, 0.15, and 0.1, respectively, for the following simulations.

B. Calibration Test

Fig. 6 depicts the performance of the proposed algorithm in an 8-channel TIADC for the OFDM system with the FFT size is 256 employing 4-QAM, 16-QAM, 32-QAM, and 64-QAM constellation. The digital calibration algorithm significantly reduces the BER for the system and removes the error floor caused by mismatches. Fig. 7 repeats the same experiment with Fig. 6 but different FFT size. It is clear that the digital calibration still has a good result that reduces the BER for the OFDM-TIADC system suffering from mismatches.

C. Coded Systems

To mitigate various channel and/or implementation errors communication systems employ channel coding prior to the

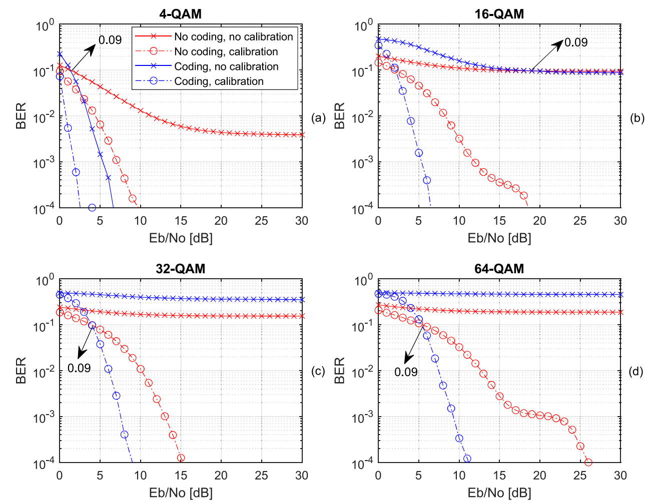

Fig. 8. The BER with/without calibration in 8-channel TIADC for OFDM system with/without convolutional coding.

modulation. To access the impact of this in the presence of gain, offset and timing errors in the receiver we perform the BER simulations using a convolutional code at the transmitter. The code used was a (2, 1, 7) convolution code as per the IEEE 802.11a standard [15].

Fig. 8 illustrates the BER for an 8-channel TIADC and the FFT size is 1024 under different-order QAM constellation. It is clear that the OFDM system with mismatch calibration and convolutional decoding techniques has the best performance in different-order QAM. From Fig. 8(a) and (b), we argue that the (2, 1, 7) convolution code can tolerate the expected worst-case bit error rate is approximately 0.09. The convolutional decoding works well in the 4-QAM constellation; however, for the higher-order QAM constellation, if we do not correct the mismatches, the BER will be above the threshold, the convolutional decoding fails to work or even worse in Fig. 8(c) and (d).

VI. Conclusion

This brief proposed a background joint mismatch calibration algorithm in an OFDM-TIADC system. A two-stage digital estimation method was introduced based on a decision feedback mechanism and channel estimation. We used Least-square estimation to estimate the offset and gain mismatch in the first stage. The time-skew mismatch is then estimated using a golden-section search in the second stage. We preseneted an error floor analysis for the OFDM-TIADC system to help us set the mismatch correction targets and to guide the circuit design. This calibration technique is verified by an OFDM-TIADC system with each sub-carrier having independent channel gain drawn from a Rayleigh fading random process. It significantly improves the OFDM system's performance and reduces the BER to ensure the reliability of the convolution coded system.

References

[1] S. D. Dissanayake and J. Armstrong, "Comparison of ACO-OFDM, DCO-OFDM and ADO-OFDM in IM/DD systems," *J. Lightw. Technol.*, vol. 31, no. 7, pp. 1063–1072, Apr. 1, 2013.

[2] J. Armstrong, "OFDM for optical communications," *J. Lightw. Technol.*, vol. 27, no. 3, pp. 189–204, Feb. 1, 2009.

[3] Y. Oh and B. Murmann, "System embedded ADC calibration for OFDM receivers," *IEEE Trans. Circuits Syst. I, Reg. Papers*, vol. 53, no. 8, pp. 1693–1703, Aug. 2006.

[4] S. Ponnuru, M. Seo, U. Madhow, and M. Rodwell, "Joint mismatch and channel compensation for high-speed OFDM receivers with time-interleaved ADCs," *IEEE Trans. Commun.*, vol. 58, no. 8, pp. 2391–2401, Aug. 2010.

[5] V.-T.-D. Huynh, N. Noels, and H. Steendam, "Effect of offset mismatch in time-interleaved ADC circuits on OFDM-BER performance," *IEEE Trans. Circuits Syst. I, Reg. Papers*, vol. 64, no. 8, pp. 2195–2206, Aug. 2017.

[6] M. El-Chammas and B. Murmann, "General analysis on the impact of phase-skew in time-interleaved ADCs," in *Proc. IEEE Int. Symp. Circuits Syst.*, 2008, pp. 17–20.

[7] V.-T.-D. Huynh, N. Noels, and H. Steendam, "Closed-form BER expression for OFDM with the effect of TI-ADC's timing mismatch," in *Proc. 25th Int. Conf. Telecommun. (ICT)*, 2018, pp. 132–137.

[8] A. Salib, M. F. Flanagan, and B. Cardiff, "A high-precision time skew estimation and correction technique for time-interleaved ADCs," *IEEE Trans. Circuits Syst. I, Reg. Papers*, vol. 66, no. 10, pp. 3747–3760, Oct. 2019.

[9] J. Song, K. Ragab, X. Tang, and N. Sun, "A 10-b 800-ms/s time-interleaved SAR ADC with fast variance-based timing-skew calibration," *IEEE J. Solid-State Circuits*, vol. 52, no. 10, pp. 2563–2575, Oct. 2017.

[10] J. Li, S. Wu, Y. Liu, N. Ning, and Q. Yu, "A digital timing mismatch calibration technique in time-interleaved ADCs," *IEEE Trans. Circuits Syst. II, Exp. Briefs*, vol. 61, no. 7, pp. 486–490, Jul. 2014.

[11] H. Wei, P. Zhang, B. D. Sahoo, and B. Razavi, "An 8 bit 4 gs/s 120 mw CMOS ADC," *IEEE J. Solid-State Circuits*, vol. 49, no. 8, pp. 1751–1761, Aug. 2014.

[12] S. Chen, L. Wang, H. Zhang, R. Murugesu, D. Dunwell, and A. C. Carusone, "All-digital calibration of timing mismatch error in time-interleaved analog-to-digital converters," *IEEE Trans. Very Large Scale Integr. (VLSI) Syst.*, vol. 25, no. 9, pp. 2552–2560, Sep. 2017.

[13] E. Akay and E. Ayanoglu, "Achieving full frequency and space diversity in wireless systems via BICM, OFDM, STBC, and Viterbi decoding," *IEEE Trans. Commun.*, vol. 54, no. 12, pp. 2164–2172, Dec. 2006.

[14] A. Mishra, A. K. Jagannatham, and L. Hanzo, "Sparse Bayesian learning-aided joint sparse channel estimation and ML sequence detection in space-time trellis coded MIMO-OFDM systems," *IEEE Trans. Commun.*, vol. 68, no. 2, pp. 1132–1145, Feb. 2020.

[15] *ISO/IEC 8802-11: 1999 [IEEE std 802.11-1999(r2003)] Information Technology–Telecommunications and Information Exchange Between Systems–Local and Metropolitan Area Networks–Specific Requirements–Part 11: Wireless LAN Medium Access Control (MAC) and Physical Layer (PHY) Specifications*, ISO/IEC Standard 8802-11, Sep. 2005.

[16] A. Salib, M. F. Flanagan, and B. Cardiff, "A generic foreground calibration algorithm for ADCs with nonlinear impairments," *IEEE Trans. Circuits Syst. I, Reg. Papers*, vol. 66, no. 5, pp. 1874–1885, May 2019.