# Collocated Adaptive Control of Underactuated Mechanical Systems

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***Abstract*—Collocated adaptive control of underactuated mechanical systems is still a concern for the control community. The main difficulty comes from the nonlinearity of the collocated inverse dynamics with respect to the base parameters, which forbids the direct application of classical adaptive control schemes. This paper extends and encompasses the Slotine's adaptive control, which was developed for fully actuated mechanical systems, to stabilize the collocated state space of an underactuated mechanical system. The key point is to define the sliding variable as the difference between the system's velocity and an exogenous state whose dynamics is considered as control input. We first revisit the Slotine's result in view of this definition and then show how to extend it to the underactuated case. Stability and convergence of time-varying reference trajectories for the collocated dynamics are shown to be in the sense of Lyapunov. Global well-posedness of the control laws is achieved by means of a new algebraic property of the mass matrix. Simulations, comparisons to existing control strategies, and experimental results on a two-link manipulator verify the soundness of the proposed approach.**

***Index Terms*—Adaptive control, collocated control, underactuated mechanical systems.**

## I. Introduction

Feedback control of underactuated mechanical systems is not new to the scientific community (see, e.g., [1]–[3] and the references therein). Aircraft, underwater vehicles, and humanoid robots are only a few examples where the number of control inputs is fewer than the system's degrees of freedom, which characterizes the nature of an underactuated system [4]. Clearly, the lack of actuation along with model uncertainties significantly complexify the control problem associated with these systems. Given an open-chain mechanical system, this study proposes control strategies for a subset of the system's degrees of freedom by using estimates of its dynamical model. In the language of automatic control, the laws presented in this paper fall into the category of *adaptive control schemes* [5].

Underactuated mechanical systems raise specific issues when attempting the control of the entire state space. For instance, assuming that the system's desired configuration is feasible, the nature of a stabilizing controller for this configuration is intimately related to the nature of the system itself. In particular, mechanical systems without potential terms in general forbid the existence of time-invariant feedback continuous stabilizers [6]. This claim, which follows from an application of Brockett's Theorem [7], has motivated the development of discontinuous and/or time-varying feedback stabilizers for specific classes of underactuated systems (see, e.g., [8]–[11]).

Complexity of the control problem associated with underactuated mechanical systems reduces when attempting to stabilize only a subset of the system's degrees of freedom. In the specialized literature, several methods have been proposed to achieve this objective. Inverse dynamics [4], [12], sliding mode [13], and energy-based techniques [14] are among the main tools exploited by these works. The common denominator of these approaches is to partition the set of degrees of freedom into two subsets, usually referred to as *collocated* and *noncollocated*. The former, whose cardinality equals the number of control inputs, contains the *actuated* degrees of freedom. The latter accounts for the remaining *nonactuated* degrees; see [4] for additional details. Then, the control objective is usually defined as the asymptotic stabilization of either set to desired values.

To cope with model uncertainties, the adaptive control of generic systems has received much attention from the control community. Most works in the specialized literature make specific assumptions on the relationship between the system's dynamics and the set of parameters that characterize it. More precisely, adaptive control of feedback linearizable systems is feasible [15]. This work, however, assumes that the dynamics can be expressed linearly with respect to the system's parameters, and this is not the case for the collocated dynamics of an underactuated mechanical system. An attempt to the adaptive control of nonlinearly parameterized systems can be found in [16]; but the assumption that there exists a parameter independent input ensuring global stability irrespective of the parameters complexifies the application of this theory to our case.

When considering the specific class of mechanical systems, adaptive stabilizations of the collocated and noncollocated dynamics can be achieved [17]. The main drawback of the approach is that the measurement of the system's acceleration is required by the feedback action. Leaving aside causality issues, this measurement may not be always available.

In the case of fully actuated mechanical systems, adaptive stabilization of time-varying reference trajectories can be achieved [18], [19]. The key assumption is that the system's inverse dynamics (see [20, p. 54]) can be expressed linearly with respect to a set of constant *base parameters*. The extension of these works to the underactuated case is not straightforward. As a matter of fact, the collocated inverse dynamics is no longer linear with respect to the base parameters when expressed independently from the noncollocated accelerations.

Assuming that the control objective is the asymptotic stabilization of the collocated state space, this paper basically extends the Slotine's adaptive controller [18] to the underactuated case.

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Inspired by the back-stepping method, the key point is to view the sliding variable as the difference between the system's velocity and an exogenous state, whose dynamics is considered as control input. We then show that this formulation can encompass the Slotine's result [18] for fully actuated mechanical systems. In contrast with [18] and inspired by the work by Spong *et al.* [19], we show that stability is in the sense of Lyapunov. The new definition of the sliding variable allows us to extend directly the controller [18] to the underactuated case when the objective is the stabilization of the collocated dynamics. No acceleration measurement is required by the proposed control laws. Simulation results, comparisons to a linear controller and to the approach [17], and experiments carried out on a two-link manipulator verify the soundness of the proposed control laws. The reader must be aware that it is beyond the scope of this work to address the classical, and well-known, drawbacks of adaptive control schemes (see, e.g., [21] and the references therein).

This paper is organized as follows. Section II presents the assumptions and the system's model for the mechanical system. Section III revisits the Slotine's adaptive control result [18]. Section IV presents the main theoretical results concerning the extension of [18] to the underactuated case. Validations of the approach are presented in Section V, first through simulations and then through experiments. Remarks and perspectives conclude the paper.

## II. Background

### A. Notation

The following notation is used throughout the paper.

- The set of real numbers is denoted by $\mathbb{R}$.
- Let $u$ and $v$ be two $n$-dimensional column vectors of real numbers, i.e., $u, v \in \mathbb{R}^n$, their inner product is denoted as $x^\top y$, with "$\top$" the transpose operator.
- Given a time function $f(t) \in \mathbb{R}^n$, its first- and second-order time derivative are denoted by $\dot{f}(t)$ and $\ddot{f}(t)$, respectively. Given a function $f(\cdot)$ of several variables, its gradient w.r.t. some of them, say $x$, is the row vector denoted as $\partial_x f$.
- The Euclidean norm of either a vector or a matrix of real numbers is denoted by $|\cdot|$.
- $I_n \in \mathbb{R}^{n\times n}$ denotes the identity matrix of dimension $n$; $0_n \in \mathbb{R}^n$ denotes the zero column vector of dimension $n$; $0_{n\times m} \in \mathbb{R}^{n\times m}$ is the zero matrix of dimension $n \times m$.

### B. System Modeling and Properties

We assume that the application of Lagrange formulation to the mechanical system yields a model of the following form [20]:

$$\begin{aligned} M(q,\pi)\ddot{q} + C(q,\dot{q},\pi)\dot{q} + g(q,\pi) \\ + F_v(\pi)\dot{q} + F(q,\dot{q},\pi) = \tau \end{aligned} \tag{1}$$

where $q \in \mathbb{R}^n$ denotes the generalized coordinates of the mechanical system; $M(\cdot) \in \mathbb{R}^{n\times n}$, $C(\cdot) \in \mathbb{R}^{n\times n}$, and $g(\cdot) \in \mathbb{R}^n$ denote the inertia matrix, the Coriolis matrix, and the gravity torques, respectively; $\pi \in \mathbb{R}^p$ is the vector of the (constant) system's base parameters [22]; $F_v \in \mathbb{R}^{n\times n}$ and $F(\cdot) \in \mathbb{R}^n$ model viscous and nonlinear friction torques (i.e., $F_v$ is a positive definite matrix); and $\tau$ is the vector of control inputs (i.e., desired actuators' torques) to be designed for achieving specific control objectives.

Hence, we assume that the mechanical system has $n$ constant degrees of freedom globally parameterized by the coordinates $q$. Consequently, we say that system (1) is "underactuated"[1] when the number of available torque inputs is smaller than $n$.

We assume the following properties of the model (1).

*Property 1:* The inertia matrix $M$ is bounded and symmetric positive definite for any $q$, i.e.,

$$\lambda_1(\pi) I_n \le M(q,\pi) \le \lambda_2(\pi) I_n \quad \forall q$$

with $\lambda_1$ and $\lambda_2$ two strictly positive constants.

*Property 2:* The matrix $\dot{M} - 2C$ is skew-symmetric, i.e.,

$$x^\top(\dot{M} - 2C)x = 0 \quad \forall x \in \mathbb{R}^n.$$

*Property 3:* The Coriolis matrix $C(q,\dot{q},\pi)$ satisfies

$$|C(q,\dot{q},\pi)| \le \lambda_0(\pi)|\dot{q}| \quad \forall q$$

for some bounded constant $\lambda_0$.

*Property 4:* The gravity vector $g(q,\pi)$ satisfies

$$|g(q,\pi)| \le \gamma_0(\pi) \quad \forall q$$

for some bounded constant $\gamma_0$.

*Property 5:* The model (1) can be expressed linearly with respect to the system's base parameters $\pi$. In addition, there exists a *regressor matrix* $Y(\cdot) \in \mathbb{R}^{n\times p}$ such that

$$\begin{aligned} M(q,\pi)\ddot{q} + C(q,\dot{q},\pi)\xi + g(q,\pi) \\ + F_v(\pi)\xi + F(q,\dot{q},\pi) = Y(q,\dot{q},\xi,\ddot{q})\pi \end{aligned}$$

for any vector $\xi \in \mathbb{R}^n$.

The matrix $Y(\cdot)$ is the so-called Slotine–Li regressor. Observe that in view of the algebraic Property 5, dynamics (1) can be compactly written by substituting $\xi$ with $\dot{q}$, i.e., $Y(q,\dot{q},\dot{q},\ddot{q})\pi = \tau$. As an example, all above properties hold in the case of rigid robot manipulators [20].

## III. Revisiting the Slotine's Adaptive Control

Let $r(t) \in \mathbb{R}^n$ denote a time-varying reference trajectory for the variables $q$. Throughout the paper, we assume the following.

*Assumption 1:* The reference trajectory $r(t)$ is bounded in norm on $\mathbb{R}^+$, and its first- and second-order time derivatives are well defined and bounded on this set.

We present below a revisited version of the scheme [18] that ensures the asymptotic stabilization of the tracking error

$$e := q - r \tag{2}$$

to zero without the knowledge of the system's parameters $\pi$. The benefits of the following slightly different formulation will

[1]For more rigorous definitions of "underactuated mechanical systems," see [23].

be clear in the next section. First, define

$$\tilde{\xi} := \xi - \dot{r} \tag{3a}$$

$$s := \dot{q} - \xi \tag{3b}$$

$$\tilde{\pi} := \hat{\pi} - \pi \tag{3c}$$

where $\tilde{\pi}$ is the base parameters estimation error. By considering $\dot{\hat{\pi}}$ and $\dot{\xi}$ as auxiliary control inputs, fusing and reformulating the results [18], [19] lead to the following lemma.

*Lemma 1:* Assume that Properties 1–5 and Assumption 1 hold. Apply the following control laws to system (1)

$$\tau = Y(q, \dot{q}, \xi, \dot{\xi})\hat{\pi} - Ks \tag{4a}$$

$$\dot{\xi} = \ddot{r} - \Lambda_1 \dot{e} - \Lambda_2 e \tag{4b}$$

$$\dot{\hat{\pi}} = -\Gamma Y^{\top}(q, \dot{q}, \xi, \dot{\xi})s \tag{4c}$$

with $K, \Lambda_1, \Lambda_2 \in \mathbb{R}^{n\times n}$ diagonal, constant positive-definite matrices, and $\Gamma \in \mathbb{R}^{p\times p}$ a constant positive definite matrix. Then, the following results hold.

1) The equilibrium point $(e, \tilde{\xi}, s, \tilde{\pi}) = (0_n, 0_n, 0_n, 0_p)$ of the closed-loop dynamics $(\dot{e}, \dot{\tilde{\xi}}, \dot{s}, \dot{\tilde{\pi}})$ is stable.
2) For any initial condition $(e, \tilde{\xi}, s, \tilde{\pi})(0)$, the trajectories of the closed-loop dynamics are bounded, and the tracking error $e(t)$ converges to zero.

The proof is given in Appendix. The main difference between the above formulation and that of [18] resides in the definition (3b), and in the fact that $\dot{\xi}$ is viewed as an auxiliary input. The above lemma shows that this standpoint does not affect stability, in terms of Lyapunov, and convergence. Observe also that boundedness and convergence are independent from the initial condition $\tilde{\xi}(0)$ thanks to the additional term $\Lambda_2 e$ in (4b). This term plays the role of an integral action in the expression of (3b) and compensates for the initial condition $\tilde{\xi}(0)$.

For Lemma 1 to hold, it is assumed that system (1) is fully actuated. The following section proposes an extension of Lemma 1 to the case where system (1) is underactuated.

## IV. Collocated Adaptive Control

Assume that system (1) possesses only $m < n$ torque control inputs so that the first $k := n - m$ rows on the right-hand side of (1) are identically equal to zero, i.e.,

$$Y(q, \dot{q}, \dot{q}, \ddot{q})\pi = \begin{pmatrix} 0_k \\ \bar{\tau} \end{pmatrix} \tag{5}$$

with $\bar{\tau} \in \mathbb{R}^m$ the control inputs. Now, partition the generalized coordinate vector $q$ as follows:

$$q := \begin{pmatrix} q_n \\ q_c \end{pmatrix} \tag{6}$$

where $q_n \in \mathbb{R}^k$, $q_c \in \mathbb{R}^m$, and the suffixes "$n$" and "$c$" stand for *noncollocated* and *collocated*, respectively. Assume that the control objective is the asymptotic stabilization of the collocated coordinates $q_c$ about reference trajectories $r(t) \in \mathbb{R}^m$, i.e., the stabilization of the tracking error

$$e := q_c - r \tag{7}$$

to zero. As before, we want to design control laws for this control objective without the knowledge of the parameters $\pi$.

To provide the reader with a better comprehension of the genesis of this paper, let us show the difficulties in attempting to apply Lemma 1 for controlling the collocated state space $q_c$. This lemma assumes that Property 5 holds, i.e., the inverse dynamics of the controlled variables is linear with respect to the parameters $\pi$. In view of the system dynamics (5), this linearity still holds for the collocated state space $q_c$. Then, the stabilization of the tracking error $e$ to zero may be achieved by applying the laws (4) as follows:

$$\bar{\tau} = Y_c\left(q, \dot{q}, \begin{pmatrix} \dot{q}_n \\ \xi \end{pmatrix}, \begin{pmatrix} \ddot{q}_n \\ \dot{\xi} \end{pmatrix}\right)\hat{\pi} - Ks_c \tag{8a}$$

$$\dot{\hat{\pi}} = -\Gamma Y_c^{\top}\left(q, \dot{q}, \begin{pmatrix} \dot{q}_n \\ \xi \end{pmatrix}, \begin{pmatrix} \ddot{q}_n \\ \dot{\xi} \end{pmatrix}\right)s_c \tag{8b}$$

where

$$Y(q, \dot{q}, \xi, \ddot{q}) = \begin{pmatrix} Y_n(q, \dot{q}, \xi, \ddot{q}) \\ Y_c(q, \dot{q}, \xi, \ddot{q}) \end{pmatrix}, \quad s := \begin{pmatrix} s_n \\ s_c \end{pmatrix} \tag{9}$$

with $s_n \in \mathbb{R}^k$, $s_c \in \mathbb{R}^m$, $Y_n \in \mathbb{R}^{k\times p}$, $Y_c \in \mathbb{R}^{m\times p}$, and $\dot{\xi} \in \mathbb{R}^m$ still governed by (4b). Note that $Y_c(\cdot)$ in (8) depends upon the accelerations of the noncollocated state space $\ddot{q}_n$. Therefore, if the measurement of this acceleration were available, one might apply the laws (4) for the adaptive control of $e$ to zero.

The above approach, which is basically that of [17], does pose causality issues. In fact, the acceleration $\ddot{q}_n$ in (8a) depends upon the control input $\bar{\tau}$ via the dynamic equation (1), i.e., $\ddot{q}_n = \ddot{q}_n(\bar{\tau})$. To avoid these causality concerns, one may think of substituting the acceleration $\ddot{q}_n$ in (5) with its expression deduced by the dynamical model (1). However, in this case, it is simple to show that the obtained inverse dynamics

$$\bar{\tau} = \bar{\tau}(\pi, q, \dot{q}, \ddot{q}_c)$$

is no longer linear with respect to the base parameters $\pi$, which destroys the stability and convergence arguments of Lemma 1.

The next theorem presents control laws that ensure the adaptive asymptotic stabilization of the collocated variables without acceleration feedback, thus avoiding causality concerns and circumventing the nonlinearity of the collocated inverse dynamics with respect to the base parameters.

*Theorem 1:* Assume that Properties 1–5 and Assumption 1 hold. Partition the variables $\xi$ as follows:

$$\xi := \begin{pmatrix} \xi_n \\ \xi_c \end{pmatrix} \tag{10}$$

where $\xi_n \in \mathbb{R}^k$ and $\xi_c \in \mathbb{R}^m$.

Apply the following control laws to system (5):

$$\bar{\tau} = Y_c(q,\dot{q},\xi,\dot{\xi})\hat{\pi} - Ks_c, \tag{11a}$$

$$\dot{\xi} = \begin{pmatrix} \dot{\xi}_n \\ \dot{\xi}_c \end{pmatrix} = \begin{pmatrix} \widehat{M}_n^{-1}\left[K_n s_n - Y_n\left(q,\dot{q},\xi,\begin{pmatrix} 0_k \\ \dot{\xi}_c \end{pmatrix}\right)\hat{\pi}\right] \\ \ddot{r} - \Lambda_1\dot{e} - \Lambda_2 e \end{pmatrix} \tag{11b}$$

$$\dot{\hat{\pi}} = -\Gamma Y^\top(q,\dot{q},\xi,\dot{\xi})s \tag{12}$$

with $K, \Lambda_1, \Lambda_2 \in \mathbb{R}^{m\times m}$ and $K_n \in \mathbb{R}^{k\times k}$ diagonal, constant positive-definite matrices, and the matrix $\widehat{M}_n$ defined as the $k$th-order leading principal minor of the mass matrix $M$ evaluated with estimated base parameters, i.e.,

$$\widehat{M}_n := S_k M(q,\hat{\pi})S_k^\top \tag{13}$$

where the selector $S_k$ is given by

$$S_k := \begin{pmatrix} I_k & 0_{k\times(n-k)} \end{pmatrix}. \tag{14}$$

Then, the following results hold.

1) The equilibrium point $(e, \tilde{\xi}_c, s, \tilde{\pi}) = (0_m, 0_m, 0_n, 0_p)$ of the closed-loop dynamics $(\dot{e}, \dot{\tilde{\xi}}_c, \dot{s}, \dot{\tilde{\pi}})$ is stable.
2) Assume that the noncollocated velocities remain bounded, i.e., $\exists\delta > 0$ such that $|\dot{q}_n| < \delta \quad \forall t$. There exists a neighborhood $\mathcal{I}$ of the origin $(0_m, 0_m, 0_n, 0_p)$ such that if the initial condition $(e, \tilde{\xi}_c, s, \tilde{\pi})(0)$ belongs to $\mathcal{I}$, then the tracking error $e(t)$ converges to zero.

The proof is given in Appendix. The appeal of the invoked reformulation of the Slotine's adaptive control presented in Lemma 1 lies in the similarity between the control laws (4) and (11) and (12). More precisely, in both cases, the evolution of the variable $\xi$ can be obtained by numerical integration of its dynamics $\dot{\xi}$. When the system possesses $k$ unactuated degrees of freedom, it suffices to modify the first $k$ elements of this dynamics —see (11b)—to still ensure stability and convergence of the collocated coordinates. Note that the dynamics $\dot{\xi}_n$ in (11b) plays the role of an estimator for the noncollocated acceleration $\ddot{q}_n$ when the system's trajectories belong to a neighborhood of the equilibrium point.

Convergence of the tracking error $e$ to zero is guaranteed, however, when the noncollocated velocities $|\dot{q}_n|$ remain bounded. This requirement, which follows from the application of Barbalat's lemma, is reminiscent of the condition on the stability of the zero dynamics in [15]. Let us remark that stability is here guaranteed independently from the boundedness of $\dot{q}_n$, which cannot be in general satisfied by an appropriate choice of the control input $\bar{\tau}$. In fact, the influence of this input on the noncollocated dynamics cannot be guaranteed to have general properties. Clearly, friction effects may play a role in guaranteeing the boundedness of $|\dot{q}_n|$ and, consequently, the convergence of the tracking error $e(t)$ to zero.

### A. Desingularization for a Globally Defined Controller

The local nature of the controls (11) is due to the fact that matrix (13) may not be invertible far from the point $\tilde{\pi}=0_p$. Observe that the invertibility of (13) in a neighborhood of this point is guaranteed by Property 1, which implies that each leading principal minor of the mass matrix $M(q,\pi)$ is positive definite and, therefore, invertible.

The noninvertibility of the matrix (13) is related to the *standard inertial parameters*[2] associated with the estimated base parameters $\hat{\pi}$. For instance, when an estimate $\hat{\pi}$ induces a negative mass of a rigid body composing the underlying mechanical system, the inertia matrix $M(q,\hat{\pi})$ may not be positive definite [24], thus eventually resulting in an ill-conditioned controller. Now, let us remark that if

$$\det\left(M_n(q(t),\hat{\pi}(t))\right) > 0 \quad \forall t \tag{15}$$

independently of the initial conditions, laws (11) ensure global convergence of the tracking error and global boundedness. This may not be always the case, however. To avoid a possible ill-conditioning of the laws (11), a desingularization policy must be defined when the above determinant gets close to zero. The desingularization policy used in this paper exploits the following result on the inertia matrix $M(q,\pi)$.

*Lemma 2:* Properties 1 and 5 imply that

$$[\partial_\pi \det\left(S_i M(q,\pi)S_i^\top\right)]\pi = i\det(S_i M(q,\pi)S_i^\top) \tag{16}$$

$\forall i \in \{1,\ldots,n\}$ and with $S_i$ given by (14). Then, the gradient with respect to $\pi$ of the determinant of each leading principal minor of the mass matrix $M(q,\pi)$ has a norm different from zero, i.e., there exists $\gamma > 0$ such that

$$|\partial_\pi \det\left(S_i M(q,\pi)S_i^\top\right)| > \gamma. \tag{17}$$

The proof is in Appendix. The above lemma in turn implies that there exists a choice for $\dot{\hat{\pi}}$ such that the time derivative of (15) can be imposed at will. Then it is theoretically possible to modify the law (12) to ensure that the determinant of $\widehat{M}_n$ never decreases below a certain threshold. The next proposition presents such a modification of the adaptation law $\dot{\hat{\pi}}$.

*Proposition 1.* Consider the laws (11) with the adaptation law redefined as follows:

$$\dot{\hat{\pi}} = -\Gamma[Y^\top(q,\dot{q},\xi,\dot{\xi})s - \eta\delta] \tag{18}$$

with $\eta \in \mathbb{R}$ and $\delta \in \mathbb{R}^p$ given by

$$\eta := \begin{cases} 0, & \text{if } \operatorname{tr}(\widehat{M}_n^{-1}\Upsilon)\geq 0 \text{ or } \det(\widehat{M}_n)>\varepsilon \\ -\dfrac{\operatorname{tr}(\widehat{M}_n^{-1}\Upsilon)}{\delta^T\Gamma\delta}, & \text{otherwise} \end{cases} \tag{19a}$$

$$\delta := \sum_{i=1}^{k} Y_{M_n}^\top(q,e_i)\widehat{M}_n^{-1}e_i \tag{19b}$$

[2]The standard inertial parameters of a rigid body consist in a 10-D vector composed of the mass, the three first moments of mass, and the six elements of the inertia matrix [22].

where $\varepsilon \in \mathbb{R}^+$,

$$Y_{M_n} := S_k \left[ Y\left(q, 0_n, 0_n, \begin{pmatrix} e_i \\ 0_m \end{pmatrix}\right) - Y(q, 0_n, 0_n, 0_n) \right] \quad (20a)$$

$$\Upsilon := (\upsilon_1, \ldots, \upsilon_i, \ldots, \upsilon_k) \quad (20b)$$

$$\upsilon_i := \left[\frac{\partial}{\partial q}(Y_{M_n}\hat{\pi})\right]\dot{q} - Y_{M_n}\Gamma Y^\top(q, \dot{q}, \xi, \dot{\xi})s \quad (20c)$$

and $e_i \in \mathbb{R}^k$ denotes a vector of $k$ zeros except for the $i$th coordinate, which is equal to 1.

Then, the following results hold.

1) If $\det\left(\widehat{M}_n\right) > 0$, then

$$|\delta| > 0 \quad \text{and} \quad |\hat{\pi}| > 0.$$

2) Assume that $\det\left(\widehat{M}_n\right)(0) > \varepsilon$. Then

$$\det\left(\widehat{M}_n\right)(t) \geq \varepsilon \quad \forall t.$$

The proof is in Appendix. This proposition states that it is always possible to maintain the determinant of $\widehat{M}_n$ above a certain threshold $\varepsilon$. In fact, the desingularizing term $\eta$ in (19) would be ill-conditioned only at $|\delta| = 0$, but this never occurs provided that $\det(\widehat{M}_n) > 0$ —see result 1. When compared with existing desingularization procedures, the main advantage of the above policy is that it does not affect the instantaneous value of the control torque $\bar{\tau}$, but only its derivative with respect to time. This characteristic is of a pivotal role in practice since it helps minimize the additional effort that the actuators must withstand close to the ill-conditioning point of the control laws.

In light of the above, the always-defined control laws are given by (11)–(18). Clearly, the larger the threshold $\varepsilon$, the larger the influence of the desingularizing term $\eta\delta$ on the results of Theorem 1. Consequently, this threshold must be tuned depending on the specific system. Assuming that one is given with best guesses $\bar{\pi}$ of the system's base parameters $\pi$, we suggest to set $\varepsilon = \det(M(\bar{q}, \bar{\pi}))$, with $\bar{q}$ a tunable parameter. For instance, if $r(t)$ converges to some desired values $r_d$, one may set $\bar{q} = (q_n^\top(0), r_d^\top)^\top$. Simulations and experimental results presented next show that the influence of this desingularizing term $\eta\delta$ does not significantly affect the practical stability and boundedness of the tracking error $e$.

*Remark 1:* Dynamics (1) assumes that no external wrench acts on the system. The effects of an external measurable wrench $w_e$ can be modeled as a disturbance $d := J^\top(q)w_e \in \mathbb{R}^n$, with $J(q)$ the Jacobian of the frame associated with the application point of the wrench $w_e$. The term $d$ must be then added on the right-hand side of (1). Now, partition $d = \begin{pmatrix} d_n^\top & d_c^\top \end{pmatrix}^\top$, where $d_n \in \mathbb{R}^k$ and $d_c \in \mathbb{R}^m$. To retain stability and convergence of the collocated variables, it suffices to apply (11) and (18) with

$$\bar{\tau} = Y_c(q, \dot{q}, \xi, \dot{\xi})\hat{\pi} - K s_c - d_c$$

$$\dot{\xi}_n = \widehat{M}_n^{-1}\left[K_n s_n + d_n - Y_n\left(q, \dot{q}, \xi, \begin{pmatrix} 0_k \\ \dot{\xi}_c \end{pmatrix}\right)\hat{\pi}\right].$$

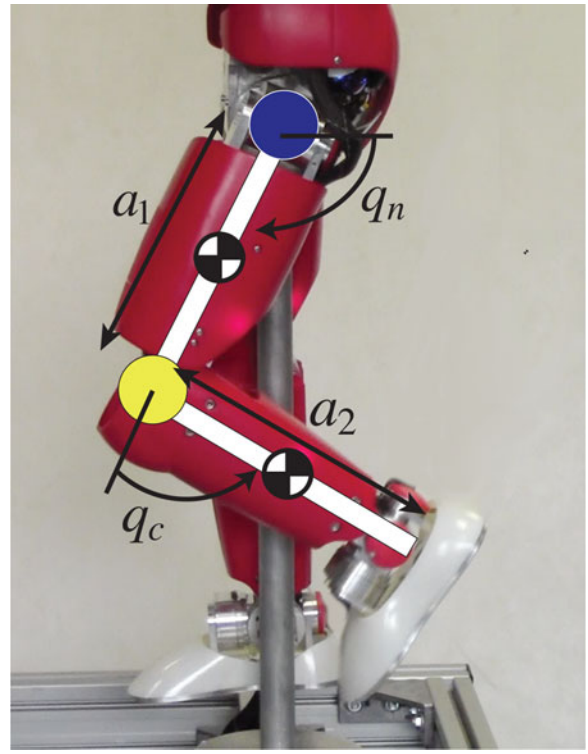


Fig. 1. Two-link manipulator obtained from the iCub's leg.

## V. Simulations and Experimental Results

In this section, we test the control laws (11)–(18) first through simulations and then through experiments carried out on a two-link manipulator with rotational joints.

The 2R manipulator is obtained from the *hip*—nonactuated joint—and the *knee*—actuated joint—of the iCub humanoid robot (see Fig. 1). The robot's ankle is kept fixed with a position controller. When applied to this case study, the laws (11)–(18) require the regressor $Y(\cdot)$ of a two-link manipulator [25, p. 268]. This regressor is computed with only viscous friction terms, which play a role when the controller is launched on the real robot. More precisely, the iCub platform is equipped with a low-level torque control loop that is in charge of stabilizing *any desired joint torque* [26], [27]. This loop is supposed to compensate for friction effects, but this compensation is never perfect. Therefore, the friction terms $F_v\dot{q}$ left in the regressor can account for imperfect viscous friction compensations by the low-level torque control.

Simulations are performed with the following parameters: $m_{l_1} = m_{l_2} = 2$ [kg], $I_{l_1} = I_{l_2} = 0.2528$ [kg m$^2$], $l_1 = l_2 = 0.75$ [m], $a_1 = a_2 = 1.5$ [m], $F_v = I_2$ [N m s/rad], where $(m_{l_1}, m_{l_2})$, $(I_{l_1}, I_{l_2})$, $(l_1, l_2)$, $(a_1, a_2)$ stand for the masses, inertias, center-of-mass positions, and lengths of the two links, respectively (see Fig. 1). The associated base parameters are $\pi = [2, -1.5, 1.3778, 2, -1.5, 1.3778, 1, 1]$ (see [25, p. 268] with zero motor masses), where the last two elements of $\pi$ are the diagonal of the matrix $F_v$. The control input $\bar{\tau}$ is saturated at 73 [N m]. The initial conditions are $\xi(0)=q(0)=\dot{q}(0)=0_2$.

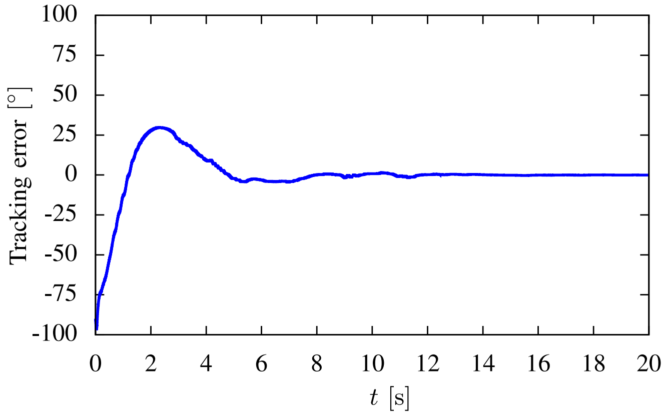


Fig. 2. Performances of the control laws (11)–(18).

Fig. 2 depicts the simulation result for the reference trajectory

$$r(t) = \frac{\pi}{2}\big(1 + \sin(\pi t)\big)$$

when the laws (11)–(18) are evaluated with $\Lambda_1{=}\Lambda_2{=}K{=}I_2$, $\Gamma{=}I_8$, $\varepsilon{=}1.5$, and $\hat{\pi}(0){=}\,(1.8, -1, 0.8, 2.6, -2, 1.7, 0.9, 1.1)$. Convergence of the tracking error is achieved with an overshoot of 25°.

As for elements of comparison with existing control techniques, Fig. 4 shows simulation results when applying the law (8) and the laws (11)–(18) with no feedforward term (i.e., $Y_c(q, \dot{q}, \xi, \dot{\xi})\hat{\pi} \equiv 0$), which result in a PID controller for all intents and purposes. The acceleration $\ddot{q}_n$ in (8) was estimated by using a filter of the form

$$\frac{s'}{(2\pi f s' + 1)}$$

where $s'$ is the Laplace variable, and $f$ is the cutoff frequency of the filter set at 10 [Hz]. By doing so, we basically compare our control strategy with that of [17]. Initial conditions, gains, and reference trajectory were kept equal to those associated with the simulation of Fig. 2.

Interestingly, Fig. 4 shows that the lack of the feedforward term $Y_c(q, \dot{q}, \xi, \dot{\xi})\hat{\pi}$ significantly worsens not only the steady state, but also the transient response. Increasing the PID's gains would reduce the error in this case, but would result in amplifying eventual measurement errors and noises.

Fig. 4 also shows that the law (8) evaluated with an estimated acceleration $\ddot{q}_n$ renders the variable $q_c$ unstable. This instability is the combined effect of the torque saturation and the cutoff frequency of the filter used to estimate the acceleration $\ddot{q}_n$. Simulations we have performed tend to show that increasing the cutoff frequency reduce the likelihood of rendering the actuated variable unstable, but this may be problematic in practice because of well-known issues such as high-frequency noise. Analogously, we verified that increasing the torque saturation would solve the instability problem, but this threshold may not be exceeded in practice.

We then went one step further and applied the laws (11)–(18) to the aforementioned robot obtained from the iCub's leg. The

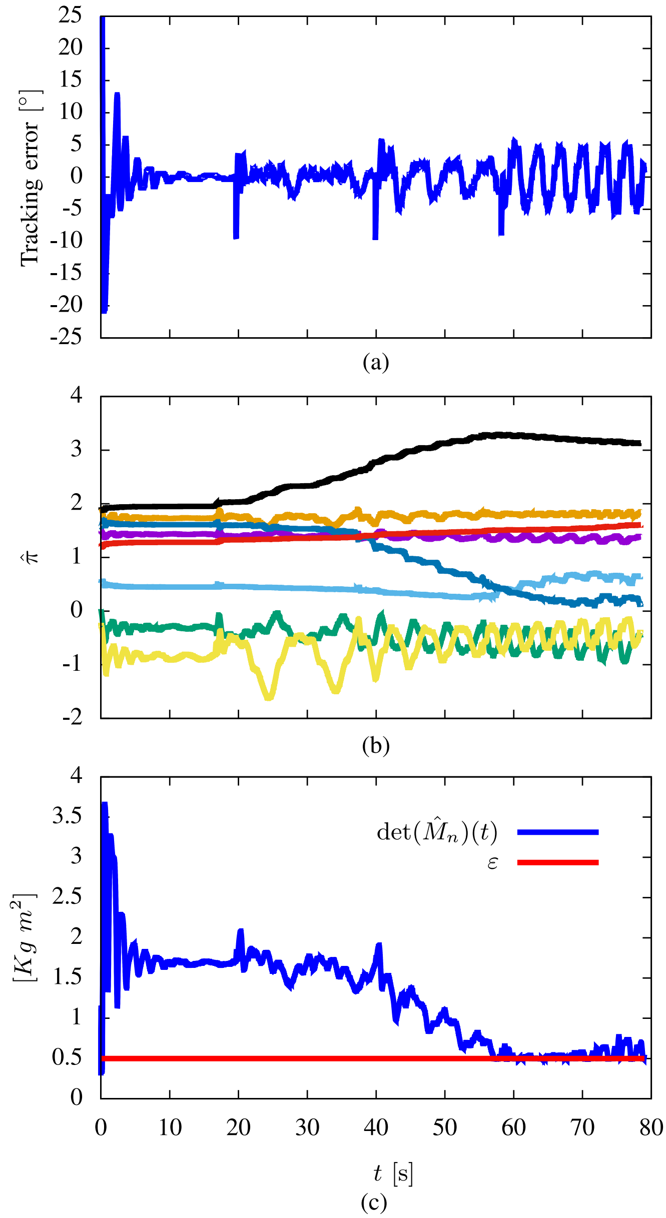


Fig. 3. Plots associated with the experimental result. (b) shows the evolution of $\hat{\pi}$. Parameters are given with the following color order: purple, green, light blue, orange, yellow, dark blue, red, black, which correspond to $\hat{\pi}_1, \ldots, \hat{\pi}_p$.

reference trajectory was chosen as

$$r(t) = \frac{\pi}{4}\sin(2\pi f_r(t)t) - \frac{\pi}{3}$$

with a piecewise constant frequency $f_r(t)$ given by

$$f_r(t) = \begin{cases} 0\text{Hz}, & 0\,\text{s} \le t < 20\,\text{s} \\ 0.1\text{Hz}, & 20\,\text{s} \le t < 40\,\text{s} \\ 0.2\text{Hz}, & 40\,\text{s} \le t < 58\,\text{s} \\ 0.3\text{Hz}, & 58\,\text{s} \le t < 80\,\text{s}. \end{cases} \tag{4}$$

The control parameters are $\Lambda_1 = 6,\ \Lambda_2 = 15,\ K = 1,\ K_n = 1,\ \Gamma = 0.2I_8,\ \varepsilon = 0.5,\quad e(0) = 40°,\ \hat{\pi}(0) = (1.5,\ -0.1,\ 0.01,\ 2,\ -0.24,\ 0.08,\ 0.05,\ 0.05)$.

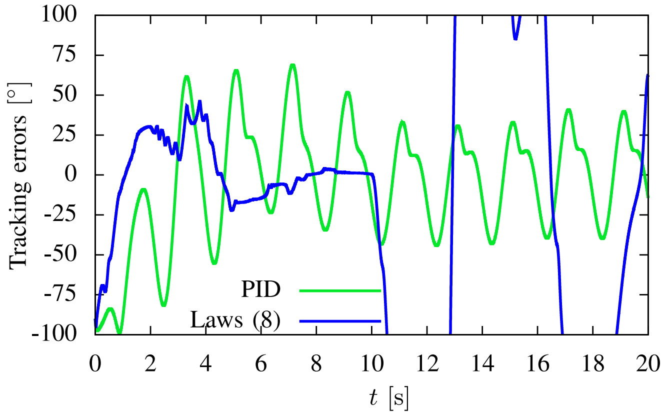


Fig. 4. Performances of a PID and the control law (8).

From top to bottom, Figs. 3 and 5 depict the tracking error $e$, the estimated base parameters $\hat{\pi}$, the determinant of the matrix $M_n(q, \hat{\pi})$, the sliding variable $s_c$, the torque control input $\bar{\tau}$, and the velocity $\dot{q}_n$ of the noncollocated variable. Observe that the tracking error converges to zero for a constant reference $r$. Sharp variations of the tracking error and of the torque at the time instants $t = 20$ [s], $t = 40$ [s], and $t = 58$ [s] are due to the discontinuities of the reference trajectory $r(t)$. Note also that unmodeled friction effects and imperfect tracking of the low-level torque control loop reflect in $\dot{q}_n = 0$ close to $t = 20$ [s] [see Fig. 5(c)]. As the frequency $f_r$ increases, the tracking error is kept relatively small despite a significant increase of the hip velocity (peak hip velocity at 40 [°/s]). The fact that the tracking error does not converge to zero is mainly due to the imperfect tracking of the low-level torque control loop implemented on the iCub platform.

Fig. 3(c) shows the effects of the desingularization policy defined in Proposition 1. Once above the threshold $\varepsilon$, the determinant of $M_n(q(t), \hat{\pi}(t))$ never goes below this threshold. Observe that this desingularization action is of particular importance for high-frequency reference trajectories, where the coupling effects between the collocated and noncollocated joints are no longer negligible. Although stability and convergence are not guaranteed when the desingularization action is active, the estimated parameters remain bounded on $t \in (60, 80)$ [s] [see Fig. 3(b)].

## VI. Conclusion and Perspectives

We have presented an extension of the Slotine's adaptive control [18], which was developed for fully actuated manipulators, to stabilize the collocated space of an underactuated mechanical system. Stability and convergence of the collocated variables were shown by using Lyapunov and Barbalat arguments. Compared with existing results, our approach does not make use of any acceleration measurement, thus avoiding altogether causality concerns. The control results were validated with simulations, comparisons, and with an implementation on a two-link manipulator obtained from the iCub humanoid robot. It was beyond the scope of this study to address the classical, and well-known, drawbacks of adaptive control schemes [21].

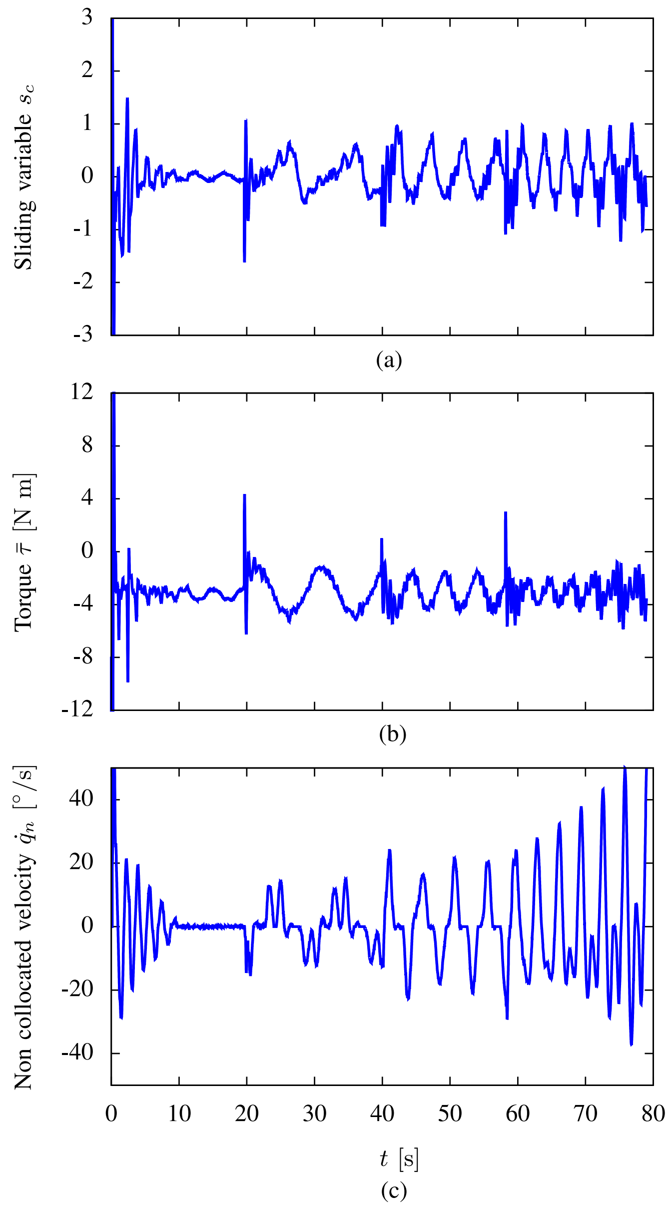


Fig. 5. Plots associated with the experimental result.

Applications of the presented approach include mechanical systems when the control task depends on the collocated variables only. Then, one can obtain the references for the collocated variables associated with the control task and stabilize them by means of the presented approach. The stabilization of the end effector of a manipulator attached to a space ship exemplifies this kind of applications [28]. More specifically, applications of the presented approach include the "floating base systems" [29], such as humanoid robots with the objective of stabilizing the joint space only. In this case, however, the theory presented in this paper must be extended to take into account external wrenches acting on the system. As mentioned in the remark at the end of Section IV, if these wrenches are measured by proper force/torque sensors, as in the case of iCub, this extension is straightforward. Hence, future work also consists in using the presented approach to make iCub walk, where joint trajectories are provided by independent planning algorithms.

The laws presented in this paper render the associated equilibrium point stable, but convergence of the tracking errors is shown when the initial conditions belong to a neighborhood of the equilibrium point. This local nature is due to the fact that the control laws make use of the invertibility of the system's inertia matrix along the estimated system's model. This matrix may not be invertible for physical inconsistent *base parameters* [24]. Then, our goal is to design an estimation dynamics such that the associated base parameters are always physical consistent. In this case, the control laws presented in this paper would guarantee global boundedness and convergence.

## Appendix
## Proof of Lemma 1

Consider the following candidate Lyapunov function:

$$\begin{aligned} V := \; & \frac{1}{2}[s^\top M s + \tilde{\pi}^\top \Gamma^{-1}\tilde{\pi} + 2e^\top K\Lambda_1 e \\ & + 2(\tilde{\xi} + \Lambda_1 e)^\top \Lambda_1 K \Lambda_2^{-1}(\tilde{\xi} + \Lambda_1 e)]. \end{aligned} \tag{22}$$

Observe that $V = 0 \iff (e, \tilde{\xi}, s, \tilde{\pi}) = (0_n, 0_n, 0_n, 0_p)$. Note also that since $K, \Lambda_1$, and $\Lambda_2$ are diagonal matrices, then the products $K\Lambda_1$ and $\Lambda_1 K \Lambda_2^{-1}$ are diagonal and positive-definite matrices. In view of Properties 2 and 5, the controls (4a)–(4c), and $\tilde{\xi} = \xi - \dot{r}$, one verifies that the time derivative of (22) yields

$$\begin{aligned} \dot{V} = \; & -s^\top K s - s^\top F_v s + 2e^\top K \Lambda_1 \dot{e} \\ & + 2(\dot{r} - \Lambda_1 e - \xi)^\top \Lambda_1 K e. \end{aligned} \tag{23}$$

Recall that $F_v$ is a positive-definite matrix. From (4b), observe that the variable $\xi$ can be obtained by integration, i.e.,

$$\xi(t) = \dot{r} - \Lambda_1 e - \Lambda_2 \zeta$$

with

$$\zeta := \int_0^t e(z)dz + \Lambda_2^{-1}\alpha$$

and $\alpha = \dot{r}(0) - \xi(0) - \Lambda_1 e(0) \in \mathbb{R}^n$, i.e., all initial conditions. By substituting

$$s = \dot{q} - \xi = \dot{e} + \Lambda_1 e + \Lambda_2 \zeta$$

in the first term on the right-hand side of (23), one has

$$\dot{V} = -(\dot{e} + \Lambda_2\zeta)^\top K(\dot{e} + \Lambda_2\zeta) - s^\top F_v s - e^\top \Lambda_1 K \Lambda_1 e. \tag{24}$$

As a consequence, $\dot{V} \leq 0$. Then, the stability of

$$(e, \tilde{\xi}, s, \tilde{\pi}) = (0_n, 0_n, 0_n, 0_p)$$

and the boundedness of the closed-loop trajectories for any initial condition follow [30, Th. 4.8, p. 151].

To show that the tracking error $e$ converges to zero, let us first prove that $\dot{V}$ converges to zero. By direct calculations, one verifies that

$$\ddot{V} = -2(\dot{e} + \Lambda_2\zeta)^\top K(\ddot{e} + \Lambda_2 e) - 2s^\top F_v \dot{s} - 2e^\top \Lambda_1 K \Lambda_1 \dot{e}.$$

By using Assumption 1, Properties 1, 3, 4, and the fact that the variables $e, \xi, s, \tilde{\pi}$ are bounded, one deduces that $\ddot{V}$ is bounded. This implies that $\dot{V}$ is uniformly continuous, and the application of Barbalat's Lemma ensures that $\dot{V}$ tends to zero. In view of (24), this implies that the error $e(t)$ converges to zero.

## Proof of Theorem 1

Thanks to the formulation of the control result of Lemma 1, this proof is similar to that above. In particular, reconsider the candidate Lyapunov function (22) with $\tilde{\xi}_c := \xi_c - \dot{r}$ instead of $\tilde{\xi}$. In view of Properties 2 and 5 and the partitioning (10), the application of the controls (11), (12) renders the time derivative of $V$ as follows:

$$\begin{aligned} \dot{V} = \; & -s^T \begin{pmatrix} Y_n(q, \dot{q}, \xi, \dot{\xi})\hat{\pi} \\ K s_c \end{pmatrix} - s^\top F_v s \\ & + 2e^\top K \Lambda_1 \dot{e} + 2(\dot{r} - \Lambda_1 e - \xi_c)^\top \Lambda_1 K e. \end{aligned} \tag{25}$$

Given Property 1, note that the auxiliary control input $\dot{\xi}_n$ is well defined in a neighborhood of $\tilde{\pi} = 0_p$ since each leading principal minor of the mass matrix $M(q, \pi)$ is invertible when $\hat{\pi}$ belongs to a neighborhood of $\pi$. As a consequence, the choice of the auxiliary control input $\dot{\xi}_n$ in (11b) implies that

$$\widehat{M}_n \dot{\xi}_n + Y_n\left(q, \dot{q}, \xi, \begin{pmatrix} 0_k \\ \dot{\xi}_c \end{pmatrix}\right)\hat{\pi} = Y_n(q, \dot{q}, \xi, \dot{\xi})\hat{\pi} = K_n s_n.$$

In view of (10) and of the above equation, the expression of $\dot{V}$ in (25) becomes

$$\begin{aligned} \dot{V} = \; & -s_n^T K_n s_n - s_c K s_c - s^\top F_v s \\ & + 2e^\top K \Lambda_1 \dot{e} + 2(\dot{r} - \Lambda_1 e - \xi_c)^\top \Lambda_1 K e. \end{aligned}$$

Analogously to the proof of Lemma 1, the variable $\xi_c(t)$ can be obtained by integration, i.e.,

$$\xi_c(t) = \dot{r} - \Lambda_1 e - \Lambda_2 \int_0^t e(z)dz - \alpha = \dot{r} - \Lambda_1 e - \Lambda_2 \zeta$$

with the collocated error $e$ given by (7). Now, by substituting

$$s_c = \dot{q}_c - \xi_c = \dot{e} + \Lambda_1 e + \Lambda_2 \zeta$$

in the second term on the right-hand side of $\dot{V}$, one obtains

$$\begin{aligned} \dot{V} = \; & -s_n^T K_n s_n - (\dot{e} + \Lambda_2\zeta)^\top K(\dot{e} + \Lambda_2\zeta) \\ & - s^\top F_v s - e^\top \Lambda_1 K \Lambda_1 e \leq 0. \end{aligned} \tag{26}$$

Then, the stability of the equilibrium point

$$(e, \tilde{\xi}_c, s, \tilde{\pi}) = (0_m, 0_m, 0_n, 0_p)$$

follows, which clearly implies the boundedness of the system's trajectories when the initial conditions belong to a neighborhood of the equilibrium point.

Given Assumption 1, Properties 1, 3, 4, and the assumption that the noncollocated velocity $\dot{q}_n$ remains bounded, it is possible to verify that $\ddot{V}$ is bounded when the initial conditions belong to a neighborhood of the equilibrium point. Then, $\dot{V}$ is uniformly continuous, and analogously to the proof of Lemma 1, one shows that $e(t)$ converges to zero.

## Proof of Lemma 2

First, define

$$M_i(q, \pi) := S_i M(q, \pi) S_i^\top \in \mathbb{R}^{i \times i}$$

with $S_i$ given by (14), as the symmetric positive-definite leading minor of order $i$ of the mass matrix $M(q, \pi)$. Then, observe that

$$\begin{aligned} M(q, \pi)\ddot{q} := \; & Y_M(q, \ddot{q})\pi \\ = \; & [Y(q, 0_n, 0_n, \ddot{q}) - Y(q, 0_n, 0_n, 0_n)]\pi. \end{aligned} \tag{27}$$

Let $e_j \in \mathbb{R}^i$ denote the vector of $i$ zeros except for the $j$th coordinate, which is equal to 1. By using Jacobi's formula,[3] one has

$$\begin{aligned}
\partial_\pi \det(M_i) &= \det(M_i)\mathrm{tr}(M_i^{-1}\partial_\pi M_i)\\
&= \det(M_i)\mathrm{tr}(M_i^{-1}S_i\partial_\pi M S_i^\top)\\
&= \det(M_i)\sum_{j=1}^{i} e_j^\top M_i^{-1}S_i\partial_\pi M S_i^\top e_j\\
&= \det(M_i)\sum_{j=1}^{i} e_j^\top M_i^{-1}S_i\ \partial_\pi M \begin{pmatrix} e_j \\ 0_{n-i}\end{pmatrix}.
\end{aligned}$$

Then, in view of (27), one obtains

$$\partial_\pi \det(M_i) = \det(M_i)\sum_{j=1}^{i} e_j^\top M_i^{-1}S_i Y_M\left(q, \begin{pmatrix} e_j \\ 0_{n-i}\end{pmatrix}\right).$$

Consequently

$$\begin{aligned}
\partial_\pi \det(M_i)\,\pi &= \det(M_i)\sum_{j=1}^{i} e_j^\top M_i^{-1}S_i Y_M\left(q, \begin{pmatrix} e_j \\ 0_{n-i}\end{pmatrix}\right)\pi\\
&= \det(M_i)\sum_{j=1}^{i} e_j^\top M_i^{-1}S_i M \begin{pmatrix} e_j \\ 0_{n-i}\end{pmatrix}\\
&= \det(M_i)\sum_{j=1}^{i} e_j^\top M_i^{-1}S_i M S_i^\top e_j\\
&= \det(M_i)\sum_{j=1}^{i} e_j^\top M_i^{-1}M_i e_j\\
&= i\det(M_i).
\end{aligned}$$

Since the inertia matrix $M$ is positive definite, then

$$\det(M_i) > 0 \quad \forall i = \{1,\ldots,n\}.$$

This, in turn, implies that

$$\exists \gamma > 0 \text{ such that } |\partial_\pi \det(S_i M(q,\pi)S_i^\top)| > \gamma.$$

## Proof of Proposition 1

*Proof of 1)*: If

$$\det(\widehat{M}_n) > 0$$

then the matrix $\widehat{M}_n^{-1}$ exists. Note that $\widehat{M}_n$ is symmetric by construction. Then, analogously to the proof of the Lemma 2, multiplying (19b) times $\hat{\pi}^\top$ yields

$$\hat{\pi}^\top\delta = \sum_{i=1}^{k}\hat{\pi}^\top Y_{M_n}^\top(q,e_i)\widehat{M}_n^{-1}e_i = \sum_{i=1}^{k} e_i^\top \widehat{M}_n^\top \widehat{M}_n^{-1}e_i = k.$$

Since the system is underactuated, then $k \geq 1$. Consequently, $|\delta| > 0$ and $|\hat{\pi}| > 0$.

*Proof of 2):* Consider the following storage function:

$$V_d := \frac{1}{2}\det{}^2(\widehat{M}_n).$$

[3] Although the Jacobi's formula is usually applied to a single-parameter dependent matrix, it is possible to verify that the above application to a multi-variable dependent matrix is correct.

It is possible to verify that the time derivative of $V_d$ is

$$\begin{aligned}
\dot{V}_d &= \det{}^2(\widehat{M}_n)\mathrm{tr}(\widehat{M}_n^{-1}\dot{\widehat{M}}_n)\\
&= \det{}^2(\widehat{M}_n)\mathrm{tr}(\widehat{M}_n^{-1}[\cdots,\partial_q(Y_{M_n}\hat{\pi})\dot{q} + Y_{M_n}\dot{\hat{\pi}},\cdots])\\
&= \det{}^2(\widehat{M}_n)[\mathrm{tr}(\widehat{M}_n^{-1}\Upsilon) + \eta\delta^\top\Gamma\delta]
\end{aligned}$$

where $\eta$ is given by (19a), $\delta$ by (19b), $\Upsilon$ by (20b), and the adaptation $\dot{\hat{\pi}}$ by (18). If

$$\det(\widehat{M}_n) \leq \varepsilon$$

then one obtains that $\dot{V}_d \geq 0$ in view of the definition of $\eta$. As a consequence

$$\det(\widehat{M}_n) \geq \varepsilon \ \ \forall t \quad \text{if} \quad \det(\widehat{M}_n)(0) > \varepsilon.$$

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