Radar Polarimetry: Classical Versus Quad-Pol Methodologies

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***Abstract*— In the synthetic aperture radar (SAR) context, "fully polarized" has two conflicting meanings: measurement of the scattering matrix, or an output image product that is a complete polarimetric characterization of an observed scene. Modern quadrature-polarimetric (quad-pol) SARs focus on the scattering matrix, while the typical user's objective is a fully polarimetric output product. Conventional quad-pol polarimetric retrieval processing relies on matrix decomposition, which leads to output products that are not fully polarimetric. The Stokes parameters from classical radar architectures are fully polarimetric, thus meeting users' objectives. A quad-pol SAR will produce fully polarimetric output products if and only if it is polarization-conserving, via the Mueller matrix for example. On this path, decomposition algorithms are not needed. Approximations and models are not needed. Classical full-polarimetry—which has no use for the scattering matrix—achieves the same goal by transmitting circular polarization and receiving two orthogonal polarizations, sufficient to evaluate the Stokes vector. Circular polarization (either *L* or *R*) coupled with a dual-polarized receiver are required. Both classical and user-oriented quad-pol approaches are founded on fundamental principles from classical optics, the Stokes parameters. These provide a full polarimetric portrait of the incoming electromagnetic field, including polarized and unpolarized constituents. Both the approaches share a requirement for circularly polarized transmissions, actually realized in the classical precedent, but virtually emulated in a quad-pol radar only when polarization-conserving methods are used.**

***Index Terms*— Full polarization, Mueller matrix, quad-pol, radar polarimeter, stokes, synthetic aperture radar (SAR).**

I. Introduction

CLASSICAL optics [1] includes the fundamental principle that a quasi-monochromatic partially polarized electromagnetic (*EM*) field may be completely characterized by any pair of orthogonally polarized constituents. It follows that four thoughtfully chosen real numbers—the Stokes parameters for example [2], [3], [4]—fully characterize the polarimetric properties of a partially polarized *EM* field. Consequently, a dual-polarized receiving system is sufficient to support fully polarized data collections [5].

These classical principles apply equally to a dual-pol microwave radiometer or to an active system [6], [7], [8], such as the Arecibo Radio Telescope [6] (Fig. 1).

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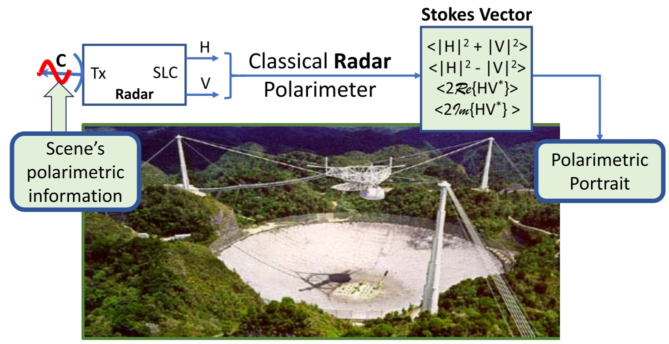

Fig. 1. Arecibo Radio Telescope (1963–2020) operated as a radiometer or a radar. It transmitted circular polarization in its radar mode.

The Stokes parameters are evaluated from data collected by the receiver through applicable formulas according to the orthogonal polarization pair used by the observing system [1], [7]. The measured values of the Stokes parameters do not depend on the specific polarimetric pair with which the *EM* field is observed [2]. Hence any pair will suffice. (In practice, that enables receiver polarization selection to be guided by implementation optimization [3], [4].) Radio and radar astronomy usually use *L* and *R* circularly polarized reception channels [10], but alternatives such as the linearly polarized *H* and *V* channels work just well [5].

The analytic Poincaré parameters may be calculated [4], [11] from the Stokes parameters. These consist of two powers (total and polarized), and two angles (that, respectively, describe the shape and orientation of the observed *EM* field's polarimetric ellipse) [3]. The Poincaré parameters have the advantage that they are canonical, unlike the Stokes parameters each of which reflect the value of two or three independent variables.

In the classic configuration, the polarimetric information collected by the receiver is preserved—through its collection by an antenna, passage through the receiver, and subsequent processing stages—then presented to the user, as a Stokes vector in traditional practice. End-to-end polarimetric conservation is a top-level requirement for most users.

The Stokes parameters are an example of "full polarimetry" in the classic meaning [7]. In contrast, "full polarimetry" since 1985 [12] has taken on a different and narrower meaning in the context of quadrature-polarimetric (quad-pol or full-pol) synthetic aperture radar (SAR). The resulting dichotomy has led to considerable misunderstanding and controversy over

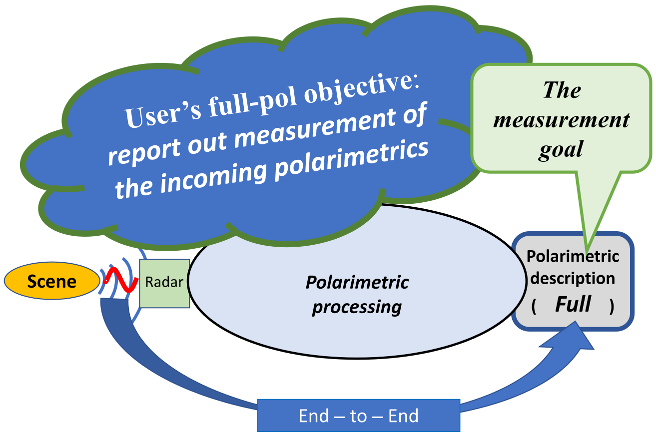

Fig. 2. Users, especially those in operational agencies responsible for large-scale information retrieval and classification tasks, expect that a polarimeter measures and then reports an accurate and complete characterization of the observed scene.

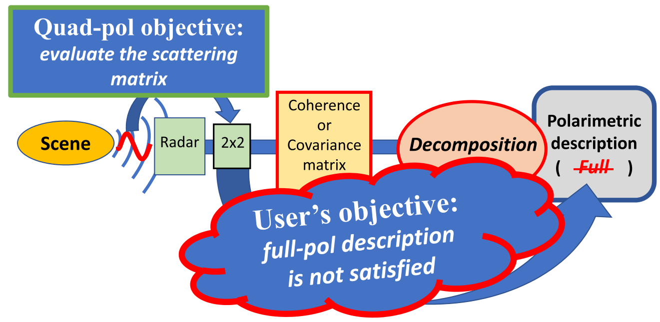

Fig. 3. Conventional (modern) quad-pol radars preserve a scene's complete (full) polarization properties in the 2 × 2 Sinclair scattering matrix, but the following decomposition algorithms do not conserve an accurate nor complete version of the scene's polarimetrics in the resulting data product.

the past several decades. Section II of this letter highlights the classical case, while Section III reviews the quad-pol SAR environment. Those two discussions set up the full-pol dichotomy. Section IV offers an explanation and resolution of the conflict. The letter closes with Section V, Conclusion.

II. Classical Fully Polarized Radar

The classical meaning of "full polarization" (Fig. 2) is that, as a *polarimeter*, the device must do what is promised by its name, to *measure* observed data, and then *report* to the user what had been measured. Users expect to see an accurate and complete characterization of the polarimetric *EM* field observed by the instrument.

For an active system, the observed field's polarization properties depend on the polarization of the transmitted field [12]. When the objective is to capture a scene's polarimetric portrait [13], the transmitted field must be *balanced* so that all the linearly polarized backscatter constituents have equal opportunity to shine. Circularly polarized transmissions from a radar (or a navigation satellite) satisfy that requirement with only one transmitted polarization.

Note that the classical approach does not depend on evaluation of the scattering matrix; dual-polarized data are sufficient to evaluate the Stokes parameters.

III. Modern Quad-Pol Radar

The quad-pol definition of full polarimetry is restricted to a specific class of radars, usually SARs, and hence, it has a much narrower meaning than the classical precedent. The objective of "full polarization" in the quad-pol context is to measure the four (complex) elements that comprise the Sinclair (scattering) matrix (Fig. 3) that characterizes an observed scene [14], [15]. Full-pol succeeds at that stage, but when followed by polarimetric matrix decomposition schemes [16], it falls short from a user's point of view. As stated above, users expect to see an accurate and complete characterization of the polarimetric *EM* field observed by the instrument. One major cause for that shortcoming is that popular decomposition methods do not include the uncorrelated (randomly polarized) constituents. Consequently, these methods cannot satisfy an SAR's simple basic conservation of energy principle. It is unlikely ever to be proven that the users' quad-pol product, following decomposition, is an accurate and complete characterization of the polarimetric properties of the observed *EM* field.

The conventional linearly polarized quad-pol architecture takes advantage of the classical two-orthogonal-polarization principle as it uses a sequence of two interleaved orthogonal polarizations (usually implemented with H and V) [17] transmitted toward the scene. The quad-pol approach achieves polarimetric balance of the illuminating *EM* field, but at the cost of doubling the rate of transmission pulse repetition frequency (prf) with its attendant disadvantages [14], [15]. The prf must be doubled for two reasons, to respect the Nyquist lower sampling bound for each channel and to assure coherence between the respective cross-polarized backscatter constituents. Quad-pol's principal user disincentives include major restrictions on potential area coverage, the impossibility to implement ScanSAR, limited span of radar incident angles, and doubled data burden per pixel [18], [19].

One advantage of quad-pol is that its complete scattering matrix supports generation of polarimetric signatures [17], based on a transfer function relationship in the Stokes (averaged power) domain. The governing expression ([16, eq. (2)] for example) states that a scene's Mueller matrix [M] postmultiplied by the Stokes vector of a transmitted *EM* field yields the Stokes vector of the backscattered field in response to that particular transmission. Polarimetric signatures of a given object (or scene) are generated by cycling through all the polarization states in the transmission Stokes vector and displaying the corresponding like- and cross-polarized output Stokes vectors. (This feature when demonstrated with a video presentation during the 1985 IGARSS at the University of Massachusetts caused somewhat of a sensation.) Like its scattering matrix heritage, the Stokes operator is fully polarimetric.

IV. Back to Basics

In the special case of a circularly polarized transmission Stokes vector[1] driving the Mueller matrix, the output Stokes

[1]The (power-normalized) Stokes vector for circular polarization is $(\mathbf{S_C}) = (1,0,0,\pm 1)^T$ where the sign of the fourth parameter depends on the L- or R-handedness of the circularity.

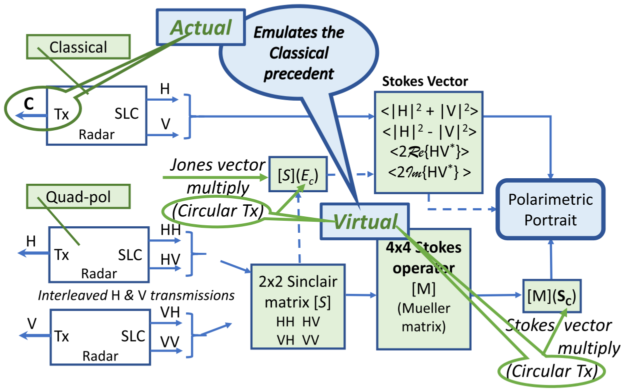

Fig. 4. Classical radar polarimeter (upper signal flow) produces the scene's polarimetric portrait in response to circularly polarized transmissions. A quad-pol radar (lower signal flow) satisfies conservation from the complete scattering polarization matrix. The scene's polarimetric portrait follows from virtual representation of circular polarization.

vector is the polarimetric portrait [13] of the observed scene, exactly the result desired by users. This result is reached by a well-defined objective closed-form methodology founded on well-established classical physical optics principles. When this method is adopted, there is no need for matrix decomposition strategies.

Appendix A (page 124) of Jakob van Zyl's PhD thesis [20] expands on the Stokes operator—variously known as the closely-related Mueller or Kennaugh matrices [16], [21]. As portrayed in Fig. 4, [M] maps the four (complex amplitude domain) scattering matrix elements into 16 real numbers (power domain), [20, eqs. (2)–(17)]. Being fully polarimetric, the Stokes operator has the great advantage—in favorable contrast to the usual quad-pol decomposition methods that rely on coherence or covariance matrices [16]—of preserving the unpolarized along with the polarized constituents of the observed *EM* field [7], [8], [20]. That property is important, because the backscatter may include as much as 50% or more of unpolarized constituents [22] that are evident in polarization signatures as the pedestal base above which the polarized constituents appear [17].

Use of this polarization-conserving matrix is a natural means of achieving users' full polarimetry from quad-pol SAR data, through exploitation of quad-pol's polarimetric signature feature [17], [20] for the particular case of circularly polarized transmissions. This method is appropriate for a wide variety of applications, including especially scenes containing different classes of backscatterers, as object-specific models or approximations are not needed.

When a polarization-conserving bridge such as the Stokes operator is used, there is only one functional difference between classical and quad-pol radar architectures: *actual* versus *virtual* means of providing circularly polarized transmissions. That difference has no impact on the polarimetric portrait [13]. When reduced to basics, the key requirement on both the approaches is to transmit circular polarization, either *actual* (classical) or *virtual* (quad-pol).

A. Actual

The classical method actually transmits circular polarization—*H* and *V* for example, simultaneously, 90° out of phase—from which backscatter is generated that includes all the information required to evaluate the scene's Stokes vector. (Actual transmission of circular polarization may be visualized as two orthogonal linear polarizations transmitted simultaneously in parallel.) Measurement of the four scattering matrix elements is not needed. The classical architecture affords an efficient approach to realizing users' fully polarimetric objective because it literally circumvents scattering matrix evaluation, thus avoiding its inherent disadvantages [18].

B. Virtual

The modern full-pol approach, which transmits sequentially interleaved linear polarizations—*H* then *V* for example—collects sufficient data to evaluate the four scattering matrix elements. (Virtual transmission of circular polarization may be visualized as two interleaved orthogonal linear polarizations transmitted in series. The like-and cross-polarized transmitted constituents must be closely spaced in time to preserve their relative coherence, as noted by van Zyl [20].) From those data, a mathematical representation of circular transmit polarization may be invoked to derive the radar's corresponding response. In effect, this emulates the classical precedent. The approach works in the complex amplitude domain (*via* Jones vectors) [13] or in the power (Stokes operator) domain as recommended by van Zyl [20]. When using polarization-conserving methods such as these in the quad-pol environment, there is no need for matrix decomposition.

V. Conclusion

It is known from classical optics that a partially polarized quasi-monochromatic *EM* field may be fully characterized by four real numbers, such as the Stokes parameters or any two orthogonal polarizations. Classical fully polarimetric radiometers are implemented with a dual-polarized receiver followed by transformation of the observed complex two-channel data into a Stokes vector that contains the complete (full) polarimetric characterization of the incoming *EM* field. A classical active system uses the same receiver arrangement, but requires that the transmitted *EM* field to be circularly polarized, hence providing polarimetrically balanced illumination of the scene. Illumination circularity may be from either a monostatic transmitter–receiver radar combination or an external source such as a navigation satellite. For either the active or passive classical arrangement, the output polarimetric portrait completely and accurately conveys the observed polarimetric information, exactly as required by users. Polarimetric portraits satisfy the end-to-end conservation of energy principle that is applicable to imaging radars.

In contrast, full polarimetry for quad-pol enthusiasts requires that the four (complex) Sinclair scattering matrix elements must be fully evaluated, which the conventional quad-pol radar architecture is designed to do. The scattering matrix once populated is fully polarimetric. Starting with a complete scattering matrix, subsequent retrievals may follow one of two different paths, *decomposition* or *polarimetric conservation*. Users expect that their end product should be a faithful measurement of the incoming *EM* field's polarimetric properties. Popular decomposition schemes fail to meet that expectation. Decomposition methodologies have never been proven to achieve the results required by users. In particular, energy conservation from input signal to output polarimetric information is not satisfied. Hence, the central user requirement that the polarimeter should report as its output product an accurate and complete characterization of the observed *EM* field is not met.

Quad-pol radars are capable of meeting users' expectations for a radar polarimeter, but if and only if a polarimetrically conservative processing path is followed, rather than matrix decomposition. The Mueller (Stokes operator) matrix guarantees energy and polarimetric conservation. The key step in that algorithm is postmultiplication of the Mueller matrix by a circularly polarized Stokes vector of the transmitted *EM* field, a singularly important case of quad-pol's polarization signature. That methodology is objective and reliable, having closed form, and requires no models or approximations. It renders decomposition schemes to be irrelevant. It is end-to-end energy conservative.

Quad-pol polarization-conserving paths lead to the same Stokes vector as from classical dual-pol architecture, a faithful polarimetric portrait of the observed scene. The key to that result is that both the approaches rely on circularly polarized transmissions, *actual* for the classical precedent, or *virtual* for the quad-pol radar configuration.

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