Event-Based HVAC Control—A Complexity-Based Approach

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***Abstract*— The optimal control of the heating, ventilation, and air-conditioning (HVAC) system in buildings has a significant energy saving potential and therefore is of a great practical interest. An event-based HVAC control adjusts control actions when certain events occur, which may be faster and more scalable than state-based or time-driven control methods. However, events may capture either local or global changes in the rooms. The choice of events is a tradeoff between the computational efficiency and the control performance. This challenging problem remains open. We consider this as an important problem in this paper and make three major contributions. First, we define local and global events for the HVAC control problem. The complexity of these event-based control policies is defined. Second, based on hypothesis testing, we develop a method to select events that capture a sufficient state information and with a relatively small event space. Third, we demonstrate the performance of this method on two groups of examples, including one group of small-scale problems for the proof of concept and the other group of large-scale problems in the HVAC control. It is shown that our method outperforms the Levin search, which is a traditional complexity-based search method and finds event-based HVAC control policies with a good performance.**

***Note to Practitioners*—When there are multiple rooms in a building, the HVAC control may achieve a significant energy saving and an indoor comfort satisfaction in the same time through exploring the coupling among the rooms. By appropriately defining the events, the size of the event space is usually much smaller than the state space. Therefore, an event-based control is more scalable and preferred in practice. Local events capture the state changes of rooms in a small neighborhood, which leads to a small event space but limited information. Global events capture the state changes of rooms in a large neighborhood, which leads to more information but a large event space. It remains open how to select events in large-scale HVAC control problems, especially when the computing budget is limited. In this paper, we define the complexity of an event-based control policy by the number of neighboring rooms considered. We develop a method based on hypothesis testing to select events with a proper complexity in order to achieve a good system performance. The performance of this method is demonstrated on an HVAC control problem.**

***Index Terms*— Discrete event dynamic systems, Markov decision processes (MDPs), event-based optimization, building energy saving, complexity.**

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I. Introduction

BUILDINGS consume more than 70% of the electricity in the United States [1], and 35% of the primary energy in China [2]. In buildings, about 40% of the energy is consumed by the heating, ventilation, and air-conditioning (HVAC) system [3]. It is of a great practical interest to optimize the control of HVAC for energy saving.

As people spend more than 90% of their time indoors on the average [4], the thermal comfort is an important factor in the HVAC control. The temperature and humidity of the indoor air are affected by the lights, the natural ventilation, the number of occupants, the plug-in load, the HVACs, and the outdoor environment, including the outdoor air temperature, the direct and diffuse radiation, and so on. The HVAC system needs to provide the required comfortable environment to the occupants with the minimal energy cost. There is usually a tradeoff between the energy cost of the HVAC and the thermal comfort of the indoor air.

When there are multiple rooms in a building, a room is coupled with the others in which the heat transfers through the walls, and the occupants may move from one room to another. So the optimal control of the HVAC system of an individual room should also consider the states of the neighboring rooms, such as the temperature of the air and the wall, the number of occupants, and the plug-in load in the room [3]. Based on this information, a control policy then determines whether to adjust the terminal devices of the HVACs [say a variable air volume (VAV)] in all the rooms of the building at each decision stage. This decision making problem may be formulated as a Markov decision process (MDP). (A detailed problem formulation in provided in Section III.) One challenge to solve this MDP is the large state space. The size of the state space increases exponentially fast when the number of rooms increases. Many efforts have been devoted to solve these large-scale MDPs. A brief review is presented in Section II.

Event-based optimization (EBO) [5]–[7] provides an alternative approach to solve large-scale MDPs. The basic idea is to define events as the sets of state transitions and then take actions only when certain events occur. When defined appropriately, the event space can stay at a small size though the state space may be extremely large. Therefore, on the one hand, event-based control is more scalable than state-based control methods. On the other hand, compared with time-driven control methods, which take actions periodically, the EBO can respond to the changes in the state faster. So event-based control methods have attracted a lot of attention and been applied to different fields, such as wireless sensor network [8], communication systems [9], HVAC control [10], and microgrids [11]–[18].

In the event-based HVAC control, events may capture either local or global changes in the rooms. Local events capture the state changes of rooms in a small neighborhood, which leads to a small event space but limited information. Global events capture state changes of rooms in a large neighborhood, which leads to more information but a large event space. The choice of events is a tradeoff between the computational efficiency and the control performance. Such a choice is challenging, because there usually does not exist any closed-form functions to quantify the relationship between the choice of events and the performance of the event-based control policies. In many cases, physical experiments or computer simulations are the only ways for performance evaluation. Due to the randomness in the system, multiple experiments or simulation replications are needed for accurate performance estimation. It remains open how to select events in large-scale HVAC control problems, especially when the computing budget is limited.

We consider this important problem in this paper and make three major contributions. First, we define local and global events for the HVAC control problem. The complexity of these event-based control policies is defined. Second, we propose a method to select events that capture a sufficient state information and with a relatively small event space. Third, we demonstrate the performance of this method on two groups of examples. One group is in small scale and for the proof of concept. The other group is an HVAC control problem with ten rooms in the building. We compare our method with the Levin search (LS) [19], which is a traditional complexity-based search method. Our method outperforms the LS and finds event-based HVAC control policies with a good performance.

The rest of this paper is organized as follows. We briefly review the related literature in Section II. We mathematically formulate the HVAC control problem in Section III. We provide the main results in Section IV, including the definition of local and global events in Section IV-A and an approach to select the complexity of the event-based control policies in Section IV-B. We provide the numerical results in Section V, including the example for the proof of concept in Section V-A and the HVAC control example in Section V-B. We briefly conclude this paper in Section VI.

II. Literature Review

There are a lot of existing studies on controlling the HVAC systems for energy saving. Most of the studies focus on individual rooms. For example, in [20], a computational tool was presented to optimize the proportional–integral–derivative (PID) (a PID controller is a control concept used in automation) coefficients for the HVAC system. A nonlinear controller that consists of a thermal load estimator and a disturbance rejection component was used for an HVAC system in [21]. A personalized-occupancy-profile-based HVAC schedule method was used in [22]. The start/stop of the VAV system was controlled by the occupancy profile instead of the predetermined HVAC schedules. But this was a rule-based control method, not necessarily the optimal control. The fuzzy logic approach is widely used in building the HVAC system control [23]–[25]. For example, in [24], a fuzzy adaptive controller is used to determine the PID parameters of the variable flow-rate HVAC system. Simulation showed that the performance of the proposed control algorithm was better than that of the classical PID and the fuzzy-PD controllers.

In recent years, it is demonstrated that a significant energy saving is possible by exploring the correlation among multiple rooms in the HVAC system [3]. The joint control of the HVAC system in a multiroom building therefore attracted a much attention both in theoretical studies as well as in practice.

The MDP provides a general framework for many control, decision-making, and optimization problems [26], [27], but it is usually challenging to solve large-scale MDPs in practice, because the state space and/or the action space increase exponentially fast when the scale of the problem increases. Attempts to address this challenge can be roughly classified into two categories. One category explores the structural property of the system to reduce the state space and the policy space. Examples include state aggregation [28], time aggregation [29], and action elimination [30], [31]. The other category considers approximate solutions. Examples include neurodynamic programming [32], approximate dynamic programming [33], reinforcement learning [34], the rollout method [35], and the recent successful stories of AlphaGo [36].

The event-based optimization provides an alternative to solve MDPs in discrete-event dynamic systems [37], [38]. Both exact and approximate solution methods have been developed for finite-stage and infinite-stage EBO [39], [39]. The EBO has been applied to many problems. For example, in a dispatching problem of the material handling system, the EBO reduces the computation time by one order of magnitude [40]. In the wireless sensor network, an event-triggered broadcasting policy can maintain a small state estimation error but substantially reduce the communication power consumption [8]. The EBO has also been applied to HVAC control problems. In a multiroom joint HVAC control problem, local-event-based methods achieve near-optimal control policies using a small event space. In an integrated building control

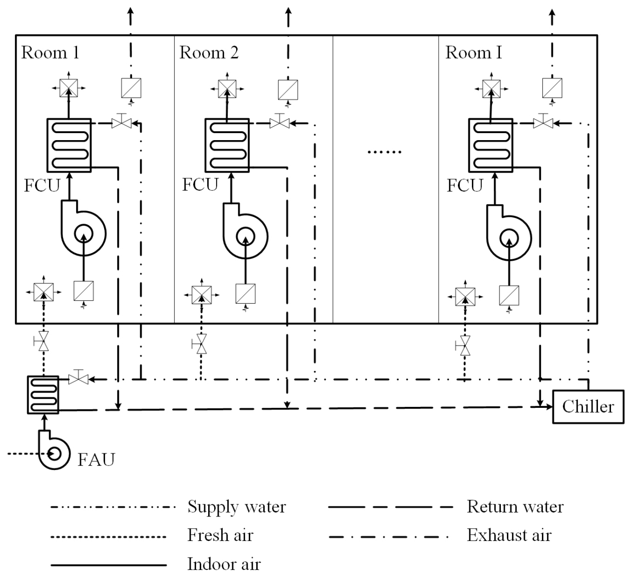

Fig. 1. Structure of the room and HVAC system.

problem, the EBO is combined with the Lagrangian relaxation to reduce the overall energy cost while maintaining similar levels of human comfort compared with time-based control methods [10].

Despite the aforementioned successful applications, the definition of events in these works is usually predetermined by heuristics or experience. It remains open how to select events in large-scale HVAC control problems, especially when the computing budget is limited. In the rest of this paper, we develop a method to select events with a proper complexity in order to achieve a good system performance.

III. Problem Formulation

The multiroom HVAC control problem is mathematically formulated in this section. Part of the model is from the previous work [41]. Beyond that, we also model the correlation among neighboring rooms, including the indoor air temperature dynamics and the changes of the number of occupants. We also provide a systematic definition of events in Section IV. The system model includes the room and device model in Section III-A, the thermal comfort model in Section III-B, and the system dynamics in Section III-C.

A. Room and Device Model

Consider a K-stage discrete-time HVAC control problem serving I rooms, as shown in Fig. 1. Each decision stage is with equal duration Δt. The rooms are in a row. Each two adjacent rooms are separated by a glass wall. There is a glass curtain wall with blinds for shading in each room. Consider a cooling season. The indoor air is supplied to the fan coil unit (FCU) in the room by a fan, cooled and dehumidified by chilled water from the chiller, and then supplied back to the room. We focus on the control of the FCU in each room and do not consider the control of the fresh air unit in this paper.

In the FCU, the power is mainly consumed by two parts. The first part is by the fan for the air flow. Let $P_{\text{fan,FCU},i,k}$ denote the power of the fan-in room i at stage k, which is a nonlinear function of the air flow rate as follows [42]:

$$P_{\text{fan,FCU},i,k} = P_{\text{fan,FCU,Rated},i,k} \cdot (G_{\text{FCU},i,a,k}/G_{\text{FCU,Rated},i,a,k})^3 \tag{1}$$

where $P_{\text{fan,FCU,Rated},i,k}$ and $G_{\text{FCU,Rated},i,a,k}$ are the rated FCU fan power and the rated FCU air flow rate in room i at stage k, respectively; $G_{\text{FCU},i,a,k}$ is the air flow rate in room i at stage k.

The second part is for cooling and dehumidification. Let $P_{\text{FCU},i,k}$ denote this second part of energy consumption by the FCU in room i at stage k. Note that $P_{\text{FCU},i,k}$ equals to the difference between the inlet air enthalpy $Q_{\text{FCU,inlet},i,k}$ and the outlet air enthalpy $Q_{\text{FCU,outlet},i,k}$. The enthalpy quantifies the energy that is contained in the air and the water vapor in the air. We have [42]

$$\begin{aligned} &P_{\text{FCU},i,k} \\ &\quad = Q_{\text{FCU,inlet},i,k} - Q_{\text{FCU,outlet},i,k} \\ &\quad = G_{\text{FCU},i,a,k}[C_p T_{i,a,k} + H_{i,k}(2500 + 1.84T_{i,a,k})] \\ &\qquad - G_{\text{FCU},i,a,k}[C_p T_{\text{FCU},i,k} + H_{\text{FCU},i,k}(2500 + 1.84T_{\text{FCU},i,k})] \end{aligned} \tag{2}$$

where C_p is the air specific heat; $T_{i,a,k}$ and $H_{i,k}$ are the air temperature and humidity in room i at stage k, respectively; $T_{\text{FCU},i,k}$ and $H_{\text{FCU},i,k}$ are the outlet air temperature and humidity of the FCU in room i, respectively.

B. Thermal Comfort Model

We follow Fanger's model [43] to quantify the human thermal comfort. In this model, a predicted mean vote (PMV) index is defined as the predicted mean thermal sensation vote on a standard scale for a large group of indoor occupants with different combinations of thermal environmental variables, activity, and clothing levels. To be specific, the PMV value F is given by

$$F = [0.303\exp(-0.036Y) + 0.028]P \tag{3}$$

where Y is the metabolic activity level; P is the thermal load on the body, which is defined as the difference between the internal heat production and the heat loss to the actual environment for an occupant. Though the PMV model is well adopted both in the literature and in practice, there are other models [44]–[48] to quantify thermal comfort, especially for individual occupants. When other thermal comfort models are used, the method developed in this paper may still be applied.

C. System Dynamics

The multiroom HVAC joint control problem can be modeled as a discrete-time MDP. The state of room i at stage k is defined as

$$s_{i,k} = \{T_{i,a,k}, T_{i,w,k}, H_{i,k}, x_{i,l,k}, n_{i,k}, L_{i,d,k}, T_{o,k}, L_{i,o,k}\} \tag{4}$$

where

$T_{i,a,k}$	air temperature of room i;
$T_{i,w,k}$	wall temperature of room i;
$H_{i,k}$	humidity of room i;
$x_{i,l,k}$	ON/OFF state of the lights in room i;
$n_{i,k}$	number of the occupants in room i;
$L_{i,d,k}$	power of the plug-in device in room i;
$T_{o,k}$	outdoor temperature;
$L_{i,o,k}$	outdoor radiant heat gains of room i.

Therefore, $s_{i,k} \in \mathbb{R}^6 \times \mathbb{B}^2$, where $\mathbb{R}$ is the set of real numbers and $\mathbb{B} = \{0, 1\}$. Define the system state at stage k as

$$s_k = (s_{1,k}, s_{2,k}, \ldots, s_{I,k}).$$

Let $\mathscr{S}$ denote the state space, i.e., $s_k \in \mathscr{S}$ for all k.

The state transition is described by dynamic equations, which are given in detail as follows.

First is the indoor air temperature. For each of the rooms from 2 to $I-1$, there are two adjacent rooms. The indoor air temperature at room i at stage $k+1$ is affected by: 1) the heat that is generated by the lights, $x_{i,l,k}L_{i,l,k}$, the occupants $n_{i,k}L_g$, where L_g is the heat generation rate per occupant, and the other plug-in devices, $L_{i,d,k}$; 2) the heat that is transferred from the interior wall, $h_w A_{i,w}(T_{i,w,k} - T_{i,a,k})$, where h_w is the heat convection coefficient between the interior walls and the indoor air, and $A_{i,w}$ is the area of the interior walls; 3) the heat that is transferred through the glass curtain wall from outside, $h_{\text{gs}}A_{i,\text{gs}}(T_{o,k} - T_{i,a,k})$, where h_{gs} is the heat transfer coefficient between outdoor and indoor air through the glass curtain wall, and $A_{i,\text{gs}}$ is the area of the glass curtain; 4) the heat that is transferred through the interior wall from the two adjacent rooms $i+1$ and $i-1$, $h_{\text{gw}}A_{\text{gw}}(T_{i+1,a,k} + T_{i-1,a,k} - 2T_{i,a,k})$, where h_{gw} is the heat transfer coefficient between the adjacent rooms through the interior glass wall, and A_{gw} is the area of the interior glass wall; 5) the heat that is provided by the HVAC system, $\Delta G_{\text{FCU},i,a,k}(T_{\text{FCU},i,k} - T_{i,a,k})$; and 6) the heat that is contained in the remaining indoor air, $m_{i,a}T_{i,a,k}$, where $m_{i,a}$ is the mass of the air in room i. So we have

$$\begin{aligned} m_{i,a}&T_{i,a,k+1} \\ &= m_{i,a}T_{i,a,k} + \Delta t[n_{i,k}L_g + x_{i,l,k}L_{i,l,k} \\ &\qquad + L_{i,d,k} + h_{\text{gs}}A_{i,g,s}(T_{o,k} - T_{i,a,k}) \\ &\qquad + h_{\text{gw}}A_{\text{gw}}(T_{i+1,a,k} + T_{i-1,a,k} - 2T_{i,a,k}) \\ &\qquad + h_w A_{i,w}(T_{i,w,k} - T_{i,a,k})]/C_p \\ &\qquad + \Delta t G_{\text{FCU},i,a,k}(T_{\text{FCU},i,k} - T_{i,a,k}). \end{aligned} \tag{5}$$

For rooms 1 and I, there is only one adjacent room. So we have

$$\begin{aligned} m_{1,a}&T_{1,a,k+1} \\ &= m_{1,a}T_{1,a,k} + \Delta t[n_{1,k}L_g + x_{1,l,k}L_{1,l,k} \\ &\qquad + L_{1,d,k} + h_{\text{gs}}A_{1,\text{gs}}(T_{o,k} - T_{1,a,k}) \\ &\qquad + h_{\text{gw}}A_{\text{gw}}(T_{2,a,k} - T_{1,a,k}) \\ &\qquad + h_w A_{1,w}(T_{1,w,k} - T_{1,a,k})]/C_p \\ &\qquad + \Delta t G_{\text{FCU},1,a,k}(T_{\text{FCU},1,k} - T_{1,a,k}) \end{aligned} \tag{6}$$

and

$$\begin{aligned} m_{I,a}&T_{I,a,k+1} \\ &= m_{I,a}T_{I,a,k} + \Delta t[n_{I,k}L_g + x_{I,l,k}L_{I,l,k} \\ &\qquad + L_{I,d,k} + h_{\text{gs}}A_{I,\text{gs}}(T_{o,k} - T_{I,a,k}) \\ &\qquad + h_{\text{gw}}A_{\text{gw}}(T_{I-1,a,k} - T_{I,a,k}) \\ &\qquad + h_w A_{I,w}(T_{I,w,k} - T_{I,a,k})]/C_p \\ &\qquad + \Delta t G_{\text{FCU},I,a,k}(T_{\text{FCU},I,k} - T_{I,a,k}). \end{aligned} \tag{7}$$

Second is the interior wall temperature. The interior wall temperature is affected by: 1) the heat convection between the wall and the indoor air $h_w A_{i,w}(T_{i,a,k} - T_{i,w,k})$ and 2) the radiant heat gains $L_{i,o,k}$ which is related to the orientation of the window. So we have

$$\begin{aligned} \frac{m_{i,w}}{2}C_w T_{i,w,k+1} = \frac{m_{i,w}}{2}&C_w T_{i,w,k} \\ &+ \Delta t[h_w A_{i,w}(T_{i,a,k} - T_{i,w,k}) + L_{i,o,k}] \end{aligned} \tag{8}$$

where C_w is the wall capacitance, $m_{i,w}$ is the mass of the wall, and $L_{i,o,k}$ is related to the orientations of the window. Note that the wall is divided into two halves each of which is shared by one of the two adjacent rooms. The temperature of the two halves may be different and affected by the two rooms, respectively.

Third is the indoor air humidity. The indoor air humidity is affected by: 1) the humidity that is provided by the HVAC system $\Delta t G_{\text{FCU},i,a,k}(H_{\text{FCU},i,k} - H_{i,k})$; 2) the humidity that is generated by the occupants $\Delta t n_{i,k}H_g$, where H_g is the humidity generation rate per occupant; and 3) the humidity that is contained in the remaining indoor air $m_{i,a}H_{i,k}$. So we have

$$\begin{aligned} m_{i,a}H_{i,k+1} = m_{i,a}&H_{i,k} + \Delta t n_{i,k}H_g \\ &+ \Delta t G_{\text{FCU},i,a,k}(H_{\text{FCU},i,k} - H_{i,k}). \end{aligned} \tag{9}$$

Fourth is the number of occupants. Assume that the dynamics of the distribution of the occupants among the rooms follow the Markov chain. The one-step transition matrix is [49]

$$P\{n_{i,k+1} = b | n_{i,k} = C\} = \pi_{bc,i}, \quad i = 1, \ldots, I. \tag{10}$$

Fifth is the plug-in load. The plug-in load in room i is modeled as a constant $C_{i,d}$ plus a Gaussian disturbance $w_{i,d,k}$ [49], i.e.,

$$L_{i,d,k} = C_{i,d} + w_{i,d,k}.$$

The mean and variance of $w_{i,d,k}$ can be obtained from historical data.

Sixth is the light. Assume that the lights are ON during office hours and OFF otherwise. So we have

$$x_{i,l,k} = \begin{cases} 1, & k \in K_w \\ 0, & k \notin k_w \end{cases} \tag{11}$$

where K_w is the office hour of a day.

The dynamics of the outdoor air temperature $T_{o,k}$ and the outdoor radiant heat gain of room i, $L_{i,o,k}$, is well formulated in EnergyPlus. We refer to [50] and [51] for more details.

Consider an ON/OFF control of the FCU at each room. Let $a_{i,k} \in \mathbb{B}$ denote the action in room i at stage k. Let $a_k = (a_{1,k}, \ldots, a_{I,k})$ denote the action vector for the whole system. Then, the action space $\mathscr{A} = \mathbb{B}^I$.

We consider both the energy consumption and the comfort satisfaction in the one-step cost. Let $E_{i,k}$ denote the energy consumption in room i at stage k. We have

$$E_{i,k}(a_{i,k}) = (a_{i,k} P_{\text{fan,FCU},i,k} + x_{i,l,k} L_{i,l,k} + L_{i,d,k} + a_{i,k} P_{\text{FCU},i,k}/\rho)\Delta t \quad (12)$$

where ρ is the coefficient of performance. Let $F_{i,k}$ be the PMV value in room i at stage k. Then, the one-step cost is

$$f_k(s_k, a_k) = \sum_{i=1}^{I} (\alpha E_{i,k}(a_{i,k}) + (1-\alpha)|F_{i,k}|)$$

where $\alpha \in (0, 1)$ is a weighting factor.

A state-based policy d_s is defined as a mapping from $\mathscr{S}$ to $\mathscr{A}$, which specifies an action to take $d_s(s) \in \mathscr{A}$ when the system state is s. Denote $\mathscr{D}_s$ as the state-based policy space. We are interested in the long-run average cost of a policy, i.e.,

$$\eta^{d_s} = \lim_{K\to\infty} \frac{1}{K} E\left(\sum_{k=1}^{K} f_k(s_k, d_s(s_k))\right) \quad (13)$$

where the expectation is taken over different outside temperatures, the number of occupants, and the plug-in loads. The optimization problem is to find

$$\eta^{d_s^*} = \min_{d_s \in \mathscr{D}_s} \eta^{d_s} \quad (14)$$

where d_s^* is the optimal state-based policy and $\eta^{d_s^*}$ is the corresponding optimal performance. This problem is challenging due to the large state space $\mathscr{S}$. We want to aggregate the states into aggregated states. When the system enters one such aggregated state, an event $e \subset \mathscr{S}$ is triggered. Let $\mathscr{E}$ denote the event space. An event-based policy d_e is defined as a mapping from $\mathscr{E}$ to $\mathscr{A}$, which specifies an action to take $d_e(e) \in \mathscr{A}$ when the system enters an aggregated state $e(s_k)$. Denote $\mathscr{D}_e$ as the event-based policy space. The average cost of such a policy is

$$\eta^{d_e} = \lim_{K\to\infty} \frac{1}{K} E\left(\sum_{k=1}^{K} f_k(s_k, d_e(e(s_k)))\right). \quad (15)$$

The event-based optimization problem is to find

$$\eta^{d_e^*} = \min_{d_e \in \mathscr{D}_e} \eta^{d_e}. \quad (16)$$

The question is how to define the events, i.e., how to aggregate the states in this paper.

IV. Main Results

In this section, we first define local and global events and event-based control policies with different complexities in Section IV-A and then provide the approach to select the complexity of the event-based control policies in Section IV-B.

A. Definition of Events

Without loss of generality, we assume that the FCU is ON when the room temperature is higher than an upper bound T_h, and that the FCU is OFF when the room temperature is below a lower bound T_l. The question is how to control the FCU when the room temperature is within $[T_l, T_h]$. A basic idea is to divide $[T_l, T_h]$ into a finite number of mutually disjoint intervals. For the proof of concept, suppose that $[T_l, T_h]$ is divided into two intervals $[T_l, T_m)$ and $[T_m, T_h]$, where $T_m \in (T_l, T_h]$ may be determined by experience or heuristics. In other words, we aggregate the state space $\mathscr{S}$ into 2^I aggregated states

$$\begin{aligned}
e_{\text{global}}(1) &= \{s_k \in \mathscr{S} | T_{1,a,k} < T_m, T_{2,a,k} < T_m, \ldots \\
&\qquad T_{I,a,k} < T_m\} \\
e_{\text{global}}(2) &= \{s_k \in \mathscr{S} | T_{1,a,k} \geq T_m, T_{2,a,k} < T_m, \ldots \\
&\qquad T_{I,a,k} < T_m\} \\
e_{\text{global}}(3) &= \{s_k \in \mathscr{S} | T_{1,a,k} < T_m, T_{2,a,k} \geq T_m, \ldots \\
&\qquad T_{I,a,k} < T_m\} \\
&\vdots \\
e_{\text{global}}(2^I) &= \{s_k \in \mathscr{S} | T_{1,a,k} \geq T_m, T_{2,a,k} \geq T_m, \ldots \\
&\qquad T_{I,a,k} \geq T_m\}.
\end{aligned}$$

Because these aggregated states contain information of the global state transitions, we call them global events. Define the global event space

$$\mathscr{E}_{\text{global}} = \{e_{\text{global}}(j), j = 1, \ldots, 2^I\}. \quad (17)$$

Note that the total number of global events is 2^I, which increases exponentially fast when there are more rooms in the system. One may want to further aggregate the states so that only local information is considered during the decision making. One extreme case is to control the FCU in room i based on the information in room i only. In other words, we define two aggregated states (or called local events) for the decision making in each room i, $i = 1, \ldots, I$

$$\begin{aligned}
e^i_{\text{local}}(1) &= \{s_k \in \mathscr{S} | T_{i,a,k} < T_m\} \\
e^i_{\text{local}}(2) &= \{s_k \in \mathscr{S} | T_{i,a,k} \geq T_m\}.
\end{aligned}$$

There are $2I$ local events in this case, which is much smaller than 2^I.

More generally, we may control the FCU in room i based on the information within the nearest r rooms, $r = 1, \ldots, I$. Define

$$\begin{aligned}
l(i, r) &= \max\{i - \lfloor r/2 \rfloor, 1\} \\
u(i, r) &= \min\{i + \lfloor r/2 \rfloor, I\}
\end{aligned}$$

where $\lfloor\cdot\rfloor$ is the floor function. Then, more specifically, we use the information in room $l(i, r), \ldots, u(i, r)$ to control the FCU in room i. Define the following aggregated states:

$$\begin{aligned}
e^i_r(1) = \{s_k \in \mathscr{S} | T_{l(i,r),a,k} &< T_m \\
T_{l(i,r)+1,a,k} &< T_m, \ldots \\
T_{u(i,r),a,k} &< T_m\}
\end{aligned}$$

$$
\begin{aligned}
e_r^i(2) = \{s_k \in \mathscr{S} | T_{l(i,r),a,k} &\geq T_m \\
T_{l(i,r)+1,a,k} &< T_m, \dots \\
T_{u(i,r),a,k} &< T_m\} \\
&\vdots \\
e_r^i(2^{u(i,r)-l(i,r)+1}) = \{s_k \in \mathscr{S} | T_{l(i,r),a,k} &\geq T_m \\
T_{l(i,r)+1,a,k} &\geq T_m, \dots \\
T_{u(i,r),a,k} &\geq T_m\}.
\end{aligned}
$$

These aggregated states are called events. There are $2^{u(i,r)-l(i,r)+1}$ events in total. Define

$$\mathscr{E}_r^i = \{e_r^i(j), j = 1, \dots, 2^{u(i,r)-l(i,r)+1}\}$$

which is the event space. Note that when $r = 1$

$$\mathscr{E}_r^i = \{e_{\text{local}}^i(1), e_{\text{local}}^i(2)\}$$

which is the set of the aforementioned local events. When $r = I$

$$\mathscr{E}_I^i = \{e_{\text{global}}^i(1), \dots, e_{\text{global}}^i(2^I)\} \tag{18}$$

which is the set of the global events.

Now, we have defined local and global events and the more general event space $\mathscr{E}_r^i$. When the control of the HVAC in room i only uses the events in $\mathscr{E}_r^i$, this defines an event-based policy $d_e^{i,r} : \mathscr{E}_r^i \mapsto \mathbb{B}$. Let $d_e^r = \{d_e^{i,r}, i = 1, \dots, I\}$. Then, d_e^r is the set of such event-based policies, each of which is for a room. Let $C(\cdot)$ denote the complexity of an event-based policy. Define

$$C(d_e^r) = r. \tag{19}$$

So an event-based policy with complexity r uses the state changes in r neighboring rooms to control the HVAC in a room and does so for all the rooms. Note that there are 2^r event-based control policies with complexity r for a room. Because there are I rooms in total, there are 2^{rI} event-based control policies with complexity r. This number increases exponentially fast when the complexity r increases. This means that the search space increases sharply, though more complicated policies utilize more information and may achieve a better control performance. Therefore, an interesting question is how to find a complexity r to achieve a good tradeoff between the search effort and the control performance. We provide a method to address this question in Section IV-B.

B. Selection Approach

In this section, we present a method to select event-based control policies within certain complexities. We will apply this method to the HVAC control problem in Section V. To provide a general method, let us introduce some variables first.

Let $\mathscr{D}_e^1, \mathscr{D}_e^2, \dots, \mathscr{D}_e^r, \dots, \mathscr{D}_e^M$ denote the sets of event-based control policies with complexities $1, 2, \dots, r, \dots, M$, respectively. M is the number of event-based policies. In the example of HVAC system, when the number of joint rooms is I, then the complexity $r = 1, 2, \dots, I$, which means $M = I$. We have explained it in Section IV-A. We have $\mathscr{D}_e = \cup_{r=1}^M \mathscr{D}_e^r$, which is the set of all the event-based control policies. For $\theta \in \mathscr{D}_e$, denote the true performance as $J(\theta)$, which is usually unknown, and may be estimated by simulation. Let $\hat{J}(\theta)$ be the estimation. We have

$$\hat{J}(\theta) = J(\theta) + W \tag{20}$$

where W is the observation noise. In general, W may depend on θ. To simplify the discussion, we assume that noise W is independent and identically distributed, which is a common practice. Suppose that we know the true performance of each policy $\theta \in \Theta_l, l = 1, \dots, M$. Then, we may rank the policies according to this true performance, from small to large, and obtain a curve, which is called the ordered performance curve (OPC), as shown in Fig. 2, where

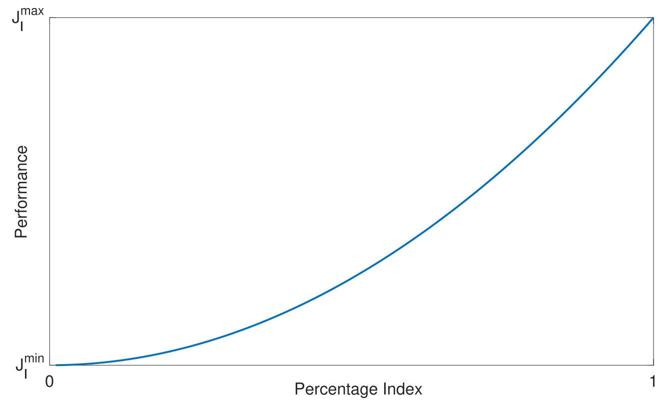

Fig. 2. OPC.

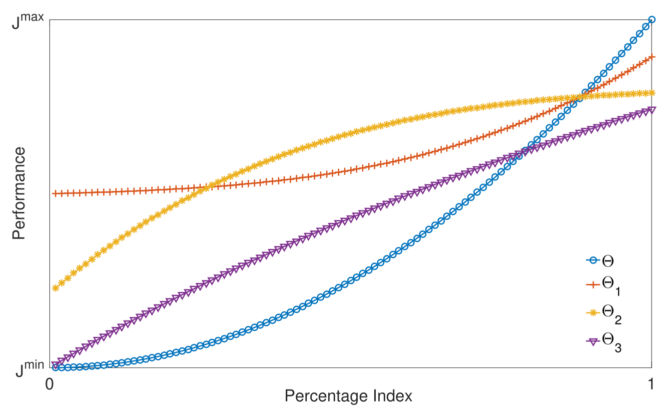

Fig. 3. Example of OPCs of different policy spaces.

$$
\begin{aligned}
J_l^{\min} &= \min_{\theta \in \Theta_l} J(\theta) \\
J_l^{\max} &= \max_{\theta \in \Theta_l} J(\theta).
\end{aligned}
$$

We may obtain the OPC for each of $\Theta_l, l = 1, \dots, M$, and also for the entire policy space Θ. One such example is shown in Fig. 3, where

$$
\begin{aligned}
J^{\min} &= \min_{l=1,\dots,M} J_l^{\min} \\
J^{\max} &= \max_{l=1,\dots,M} J_l^{\max}.
\end{aligned}
$$

Let the user specify a value $g\%$. We may compare the $g\%$ quantiles of Θ_l. The one that achieves the minimal $g\%$ quantile is the set that we should pick. Note that in practice, we do not know the exact OPCs. Otherwise, we should have already solved the problem. An alternative is to estimate the OPC by a function

$$\mathrm{OPC}_l\big(y|\alpha_0^l,\beta_0^l\big) = \big(J_l^{\max} - J_l^{\min}\big)\Lambda\big(y|\alpha_0^l,\beta_0^l\big) + J_l^{\min}$$
$$\Lambda\big(y|\alpha_0^l,\beta_0^l\big) = \int_0^y \frac{\Gamma\big(1/\alpha_0^l + 1/\beta_0^l\big)}{\Gamma\big(1/\alpha_0^l\big)\Gamma\big(1/\beta_0^l\big)} \cdot z^{1/\alpha_0^l - 1}(1-z)^{1/\beta_0^l - 1}dz$$

where $y \in [0,1]$ and $\Lambda(\cdot)$ is known as the incomplete Beta function [52]. We may also estimate the observation noise by $W_l \sim U[-\lambda_l, \lambda_l]$. In order to find Θ_l with the minimal $g\%$ quantile, we use the $g\%$ quantile of the entire policy space Θ as a ruler. Let us take N uniform samples from [0,1], denoted as $y_1, y_2, \ldots, y_N$. Their observed performance may be estimated by

$$\hat{J}(y_i) = \mathrm{OPC}(y_i|\alpha_0, \beta_0) + W. \tag{21}$$

We rank $\hat{J}(y_i)$ from small to large, i.e.,

$$\hat{J}(y_{[1]}) \le \hat{J}(y_{[2]}) \le \cdots \le \hat{J}(y_{[N]}). \tag{22}$$

Let $\hat{J}(y_{[\lceil Ng\% \rceil]})$ be the estimated $g\%$ quantile for set Θ, where $\lceil \cdot \rceil$ is the ceiling function. Let us take N_l uniform samples from [0, 1], denoted as $y_{l,1}, y_{l,2}, \ldots, y_{l,N_l}$. Their observed performance may be estimated by

$$\hat{J}(y_{l,i}) = \mathrm{OPC}_l\big(y_{l,i}|\alpha_0^l,\beta_0^l\big) + W_l. \tag{23}$$

We rank $\hat{J}(y_{l,i})$ from small to large, i.e.,

$$\hat{J}(y_{l,[1]}) \le \hat{J}(y_{l,[2]}) \le \cdots \le \hat{J}(y_{l,[N_l]}). \tag{24}$$

Let $\hat{J}(y_{l,[\lceil N_l g\% \rceil]})$ be the estimated $g\%$ quantile for set Θ_l. Define

$$P_l = \Pr\{\hat{J}(y_{l,[\lceil N_l g\% \rceil]}) \le \hat{J}(y_{[\lceil N g\% \rceil]})\}$$

which may be estimated by repeating the previous sampling procedure by multiple times. We are interested in

$$l^* = \arg\max_{l=1,\ldots,M} P_l.$$

The set Θ_{l^*} contains a better $g\%$ quantile than Θ with the largest probability. We summarize this approach in the following algorithm, call it the complexity-based approach (CBA), and demonstrate the performance in Section V.

Algorithm 1 CBA

```
set g according to accuracy requirements
set n = 0, n_l = 0, l = 1, 2, ..., M
initialize A, N, N_l
while n < A do
  take N uniform samples from Θ
  count the number of samples that fall in subset Θ_l, denote
  as N'_l
  rank the observed Ĵ(y_i) as in Eq.(22)
  get the estimated OPC and Ĵ(y_[⌈Ng%⌉])
  for l = 0; l < M; l++ do
    take N_l − N'_l uniform samples from Θ_l
    rank the observed Ĵ(y_{l,i}) as in Eq.(24)
    get the estimated OPC_l and Ĵ(y_{l,[⌈N_l g%⌉]})
    if Ĵ(y_{l,[⌈N_l g%⌉]}) ≤ Ĵ(y_[⌈Ng%⌉]) then
      n_l = n_l + 1
    end if
  end for
  n = n + 1
end while
estimate P_l by calculating n_l/A
select set Θ_{l*}, which satisfies l* = arg max_{l=1,...,M} P_l
```

V. Numerical Results

We provide the two sets of numerical results in this section. In Section V-A, we compare our method with the LS, which is a traditional search algorithm that considers the complexity preference. In Section V-B, we demonstrate the performance of our method in the HVAC control problem that is proposed in Section IV-B.

A. Toy Examples

Traditionally, when there is preference on simple solutions during an optimization, a search method may prefer simple solution candidates when the performance is close. One such benchmark method is the LS [19], [53], which starts from the set of solution candidates that are with the smallest complexity. When all the solution candidates in the set are enumerated, the LS moves on to the next set with the smallest complexity. This process is repeated until all the computing budgets are consumed. In the end, the LS outputs the best solution candidate that has been explored. Note that the correlation between the complexity and the performance of the solution candidates affects the performance of the LS. In order to make a fair comparison between the LS and our method, we consider two types of problems, namely the first problem in which the complexity and the performance of the solution candidates are positively correlated (i.e., simple solutions are with a better performance) and the second problem in which the complexity and the performance are negatively correlated (i.e., simple solutions are with worse performance).

Problem 1: Consider the following optimization problem:

$$\min_{\theta\in\Theta} J(\theta) = \min_{\theta\in\Theta} \mathbf{E}_W(0.1\theta + 1 + W)$$

where $\Theta = \{1, 2, \ldots, 500\}$, and $W \sim U(-10, 10)$ follows a uniform distribution. Here, we assume that W is uniformly distributed noise for simplicity. Divide Θ into two subsets $\Theta_1 = \{1, 2, \ldots, 200\}$ and $\Theta_2 = \{201, 202, \ldots, 500\}$. Let $C(\Theta_1) = 1$ and $C(\Theta_2) = 2$. In other words, solution candidates in Θ_1 are simpler than those in Θ_2. Suppose that our computing budget is 200, which means that we can explore

TABLE I
PCS OF LS AND CBA ON SOLVING J_1

	# of selecting Θ_1	PCS
LS	1000	100%
CBA	1000	100%

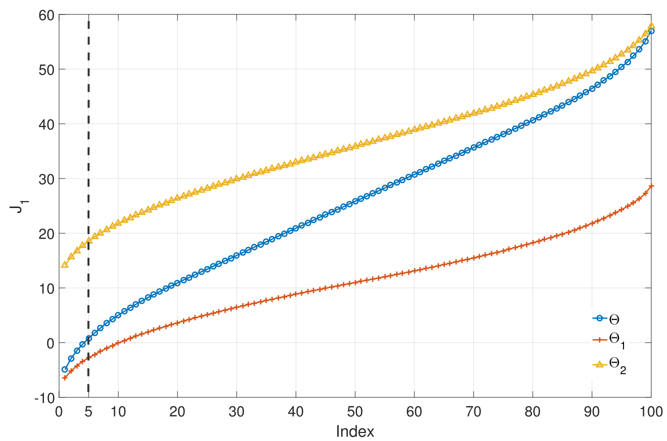

Fig. 4. Example of estimated OPC of Problem 1.

TABLE II
PCS OF LS AND CBA ON SOLVING J_2

	# of selecting Θ_2	PCS
LS	0	0%
CBA	1000	100%

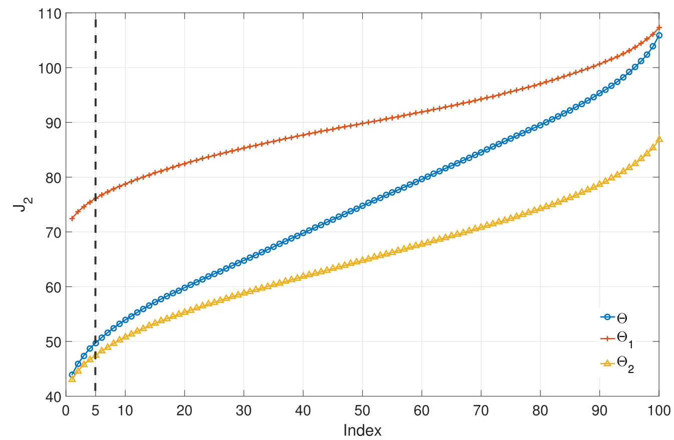

Fig. 5. Example of estimated OPC of Problem 2.

at most 200 solution candidates. We are satisfied by a top 25 solution, i.e., $g\% = 25/500 = 5\%$.

In this problem, Θ_1 contains better solution candidates than Θ_2. We are interested in the probability of correctly selecting (PCS) Θ_1. In the LS, the algorithm spends all the computing budgets in Θ_1, taking one observation for each solution candidate. Based on the observed performance, the LS outputs a set that contains the solution candidate that is observed as the best. We repeat this process by 1000 times and show the PCS of the LS in Table I. Then, we apply the CBA method. We uniformly sample $N = 100$ solution candidates from Θ. Suppose that there are N_1' and N_2' of these samples fall in Θ_1 and Θ_2, respectively. We have $N_1' + N_2' = N$. Then, we take $N_1'' = 100 - N_2'$ samples from Θ_2. Note that we have in total N_1 and N_2 uniform samples from Θ_1 and Θ_2, respectively, where

$$N_1 = N_1' + N_1'' = 100, \quad N_2 = N_2' + N_2'' = 100.$$

Based on these samples, we estimate the OPC of Θ_1, Θ_2, and Θ. An example is shown in Fig. 4. Using these estimated OPCs, we compare the $g\%$ quantile of Θ_1 (and Θ_2) with Θ and estimate P_l by 1000 replications. Then, the CBA method outputs the set with the minimal P_l, as shown in (25). We repeat the above process by 1000 times and show the PCS of CBA in Table I.

Note that in Problem 1, the LS only explores Θ_1 and therefore always outputs Θ_1 as the promising set. The PCS of the LS is therefore 100%. CBA samples from both Θ_1 and Θ_2 and also achieves a high PCS of 100%. The PCS of the LS and the CBA are comparable in this problem.

Problem 2: Consider the following optimization problem:

$$\min_{\theta \in \Theta} J(\theta) = \min_{\theta \in \Theta} \mathbf{E}_W(-0.1\theta + 100 + W).$$

The parameter settings of $\Theta, \Theta_1, \Theta_2$, and W are the same as in Problem 1. The computing budget is still 200. We are interested in a top 25 solution candidate, i.e., $g\% = 25/500 = 5\%$. In this problem, Θ_2 contains better solution candidates than Θ_1. We are interested in the PCS Θ_2. We still denote this probability by PCS.

In the LS, the algorithm starts from Θ_1, taking one observation for each solution candidate, and outputs a set that contains the solution candidate that is observed as the best. We repeat this process by 1000 times and show the PCS of LS in Table II. Then, we apply the CBA method in Problem 2 and follow the same parameter setting for N_1, N_2, and N as that in Problem 1. Based on these samples, we estimate the OPC of Θ_1, Θ_2, and Θ. An example is shown in Fig. 5. We repeat the process by 1000 times and show the PCS of the CBA in Table II.

Note that in problem 2, LS only explores Θ_1 and therefore always outputs Θ_1 as the promising set. The PCS of LS is therefore 0%. CBA samples from both Θ_1 and Θ_2 and achieves a high PCS of 100%.

B. HVAC Control Problem

In this section, we apply the CBA to the HVAC control problem that is formulated in Section III.

Here, the number of rooms is $I = 10$. For the smallest complexity $r = 1$, the size of the policy space for room i is $N(\mathscr{D}_i^1) = 4$, so the size of the policy space of the 10 rooms is 4^{10}, which is extremely large. If the complexity r becomes larger, the size of the policy space will increase exponentially. It takes unbearable computing budget by using the LS to find which complexity should be selected. So the method that is proposed in Section IV will be used here.

Here, the number of samples that are selected from $\mathscr{D}^r$ is 1000, where $r = 1, 2, \ldots, 10$. The numerical results are shown in Table III.

TABLE III
NUMERICAL RESULTS UNDER DIFFERENT COMPLEXITIES

Complexity	OPC type	Noise	P_l
1	Bell	0.0090	0.6243
2	U-Shaped	0.4000	0.7030
3	U-Shaped	0.1130	0.9917
4	Neutral	0.1500	0.6965
5	Neutral	0.0900	0.8174
6	Bell	0.0600	0.1946
7	Bell	0.0800	0.1148
8	Bell	0.0800	0.0199
9	Bell	0.0700	0.0001
10	Bell	0.0450	0.0551

TABLE IV
NUMERICAL RESULTS OF THE HVAC CONTROL PROBLEM UNDER DIFFERENT COMPLEXITIES

Index	$r = 1$		$r = 3$	
	$\bar{\eta}$	STD	$\bar{\eta}$	STD
1	44,759.98	74.49	42,423.72	30.79
2	44,933.23	51.55	42,380.25	33.46
3	44,960.50	55.16	42,406.23	39.02
4	44,746.93	93.53	42,372.61	37.85
5	44,813.31	59.26	42,391.10	39.99

From Table III, we can see that the biggest probability of P_l is 0.9917, and the corresponding complexity is 3. So in the HVAC control problem, that is proposed in Section III, the best complexity of the events is $r = 3$, which means that if the room index $i = 1$, the events of room 1 consider the state transitions of rooms 1–3; if the room index $i = 10$, the events of room 10 consider the state transitions of rooms 8–10; if the room index $i = 2, 3, \ldots, 9$, the events of room i consider the state transitions of room $i-1$, i, and $i+1$. To demonstrate the performance of the system under different complexities, we compare the solutions of the HVAC control problem under two cases, which are one event that captures the information of only one room [54] and one event that captures the information of three rooms. The solution method has been shown in [54], so we provide the numerical results of the two cases directly in Table IV, where STD stands for standard deviation, and $\bar{\eta}$ is the mean value of $\bar{\eta}$ over 30 simple paths.

In Table IV, we can see that when the complexity of the events $r = 3$, the performance of the HVAC control problem is better than the performance when $r = 1$. It is because the events with complexity $r = 3$ capture more information of the system than the events with complexity $r = 1$. Intuitively, more information is captured with a higher complexity of the events and thus the policy may have a better performance. However, the solution space becomes larger as the complexity of the events increased. Therefore, the probability of achieving a near-optimal solution may decrease. The complexity of the events $r = 3$ is a tradeoff between the computation budget and the performance of the event-based policy. Moreover, $r = 3$ means that a room would control the HVAC system based on the events of its nearest two neighbors. For room index $i = 2, 3, \ldots, I-1$, the room should consider the events of its left and right neighbors. For room index $i = 1$, the room should consider the events of room 2 and room 3. For room index $i = I$, the room should consider the events of room $I-1$ and room $I-2$. The major influence on the temperature of a room comes from its nearest two neighbors.

VI. CONCLUSION

In the EBO problem, the performance of the event-based policy highly depends on the complexity of the events. In this paper, we proposed a method called CBA for choosing a proper complexity of the events in a certain problem. We applied the CBA in solving the HVAC control problem. Numerical results showed that the method works well on choosing the complexity of the events in a large-scale HVAC problem.

In the future work, some extensions will be considered.

1) *Multitemperature Threshold:* In the HVAC system in this paper, we consider the case with only one predefined temperature threshold between the upper bound and the lower bound. We may apply the method to the case where there are multitemperature thresholds.
2) *Variant Obeservation Noise W:* We will consider the case where W is dependent on θ. In addition, W may follow other distributions, such as the normal distribution, the Cauchy distribution, and so on.
3) *Other Factors of Thermal Comfort:* As mentioned in this paper, thermal comfort is related to temperature, humidity, and many other factors.

We will take all these factors into account in the future work.

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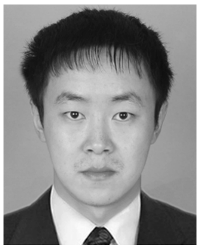

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