Regularized Deconvolution-Based Approaches for Estimating Room Occupancies

Afrooz Ebadat, Giulio Bottegal, Damiano Varagnolo, Bo Wahlberg, *Fellow, IEEE*, and Karl H. Johansson, *Fellow, IEEE*

***Abstract*—We address the problem of estimating the number of people in a room using information available in standard HVAC systems. We propose an estimation scheme based on two phases. In the first phase, we assume the availability of pilot data and identify a model for the dynamic relations occurring between occupancy levels, CO_2 concentration and room temperature. In the second phase, we make use of the identified model to formulate the occupancy estimation task as a deconvolution problem. In particular, we aim at obtaining an estimated occupancy pattern by trading off between adherence to the current measurements and regularity of the pattern. To achieve this goal, we employ a special instance of the so-called *fused lasso* estimator, which promotes piecewise constant estimates by including an ℓ_1 norm-dependent term in the associated cost function. We extend the proposed estimator to include different sources of information, such as actuation of the ventilation system and door opening/closing events. We also provide conditions under which the occupancy estimator provides correct estimates within a guaranteed probability. We test the estimator running experiments on a real testbed, in order to compare it with other occupancy estimation techniques and assess the value of having additional information sources.**

***Note to Practitioners*—Home automation systems benefit from automatic recognition of human presence in the built environment. Since dedicated hardware is costly, it may be preferable to detect occupancy with software-based systems which do not require the installation of additional devices. The object of this study is the reconstruction of occupancy patterns in a room using measurements of CO_2 concentration, temperature, fresh air inflow, and door opening/closing events. All these signals are information sources often available in HVAC systems of modern buildings and homes. We assess the value of such information sources in terms of their relevance in detecting occupancy in small and medium-sized rooms. The proposed estimation scheme is composed of two distinct phases. The first is a training phase where the goal is to derive a mathematical model relating the number of occupants with the CO_2 concentration. It is required to record the actual occupants in the room for a time period spanning few days, a task that can be performed either with manual logging or with temporary dedicated hardware counting systems. In a second phase, we use the derived model to design an online software which collects measurements of the environmental signals and provides the number of people currently in the room. The estimated occupancy levels can then be employed to enhance the efficiency of the HVAC system of the building. We notice that, in modern residential buildings composed by structurally equal flats, the training phase can be run in one flat only, since the obtained model will be reasonably valid for the other flats.**

***Index Terms*—Deconvolution, occupancy estimation, regularization, system identification.**

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A. Ebadat, G. Bottegal, B. Wahlberg, and K. H. Johansson are with the Department of Automatic Control, School of Electrical Engineering, KTH Royal Institute of Technology, Stockholm, 10044 Sweden (e-mail: ebadat@kth.se; bottegal@kth.se; bo@kth.se; kallej@kth.se).

D. Varagnolo is with the Division of Signals and Systems, Department of Computer Science, Electrical and Space Engineering, Luleå University of Innovation and Technology, Luleå 97187, Sweden (e-mail: damiano.varagnolo@ltu.se).

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I. Introduction

A. Motivations and Objective

Monitoring the number of occupants of rooms is important for home automation applications, e.g., to automate the control of lighting, thermostats, security locks, and home entertainment systems [1]–[3]. It is also a key enabling factor for improving energy efficiency in smart buildings: it has been shown in [4] the exact knowledge of the building occupancy may decrease the annual energy consumptions of about 10%–42% by optimizing the performance of Air Conditioning (HVAC) systems (see also [5] and [6]). Direct experience indicates that some standard off-the-shelf dedicated hardware for occupancy estimation (such as cameras and radio-frequency identification (RFID) tags) suffer from several problems. First, they may be insufficiently accurate for the employment in HVAC control systems. Second, they may induce large additional deployment and maintenance costs. Last, they may have installation feasibility problems in old buildings. Moreover, hardware-based occupancy detectors may trigger privacy concerns [7]. Consequently, it is interesting to study how and to what extent hardware-based people counters can be replaced by software-based occupancy estimators that only employ available information in standard HVAC systems (such as CO_2 concentration and temperature), which information sources have to be considered, and what type of statistical processing leads to efficient estimators.

The main objective of this paper is to address the above questions by proposing occupancy estimators that give information on the number of occupants using commonly available signals, namely, measurements of CO_2 concentration and temperature, HVAC actuation levels (i.e., the amount of fresh air injected in a room), and information on door opening/closing events.

B. Related Work

The currently available techniques for monitoring the occupancy in rooms and buildings can be categorized into *hardware-based* and *model-based* approaches.

The first category includes methods working with dedicated hardware. For instance, [5] and [8] deploy networks of cameras detecting people crossing a determined area under surveillance, whereas [9]–[11], utlilize RFIDs; see also [12] for a recent survey. As mentioned before, dedicated hardware may be expensive and applicable only to certain situations, due to potential drawbacks (privacy or others). For example, [13] uses inexpensive magnetic reed switches and passive infra red (PIR) sensors, but the method cannot provide the exact number of people, detecting only whether a room is occupied or not.

The second category exploits the fact that occupants affect the indoor environment by emitting CO_2, heat and humidity. Thus, occupancy is inferred indirectly using dynamical models that relate environmental signals with occupancy. These models may be obtained by employing data-driven techniques (i.e., *identification-based* methods) or by exploiting knowledge of the underlying physical laws (i.e., *physics-based* methods). The latter techniques comprise strategies based on mass balance equations or first principle considerations to derive dynamical models relating the number of occupants, CO_2 concentration, temperature and humidity [14]–[17]. Identification-based approaches aim at constructing input–output models from datasets of past measured data. Using the obtained model, the number of occupants is estimated by inverting the CO_2 dynamics [18]–[20]. The same idea is at the base of black-box machine learning techniques such as support vector machines (SVMs), neural networks (NNs), and hidden Markov models (HMMs). For instance, in [21], information regarding CO_2 concentration and data acquired by acoustic and passive infrared sensors are employed to estimate the number of occupants in an office using SVMs, NNs, and HMMs. Other proposed approaches estimate the occupancy using other CO_2 features (e.g., averages of the signals in time, first-/second-order temporal differences), see [22] and [23]. In [24], an autoregressive hidden Markov model (ARHMM) is developed to estimate occupancy levels based on environmental signal measurements also taking into account their correlation. The strategy is further developed in [25], where the technique is integrated with a wireless sensor network and tested in a research laboratory. A different solution is studied in [26], where occupancy is inferred from electricity consumption.

C. Statement of Contributions

This paper, extension of [27], describes and analyzes from theoretical perspectives a two-tier software-based occupancy estimation scheme. The first tier assumes the availability of both environmental signals and true occupancy levels (as pilot data) for a short and well defined period of time. The data regarding occupancy may come from manual logging, or from dedicated temporary people counting hardware. Black-box modeling is then used to model the room under consideration, i.e., no other *a priori* knowledge on the room properties is assumed. The second tier formulates the occupancy estimation problem as an inverse problem, i.e., it searches for the occupancy pattern that best explains the measured data given the identified model. In this tier, we exploit the fact that the occupancy signal is *piecewise constant* and integer, in order to formulate the estimation problem within a *fused-lasso* framework [28].

A contribution of the manuscript is to derive different estimators based on the availability of the various information sources. More specifically, we consider the case of adding knowledge of HVAC actuation signals (how much air is injected in the room). We also study the case of adding a boolean signal accounting for door opening/closing events and derive theoretical statistical properties of the strategy, which have not been considered in [27]. More precisely, we compute bounds on the probability of obtaining incorrect estimates, given the levels of measurement noise, the identified model and the design parameters of the estimators.

The proposed method is then employed to estimate the number of people in a medium-sized room instrumented as a university laboratory. Even if our tests are performed in a laboratory, we notice that there is no limitation in using the method in other rooms: the strategy is suitable for any kind of home or office environment as long as standard HVAC measurements are available. Moreover, in buildings with same-size flats one can perform the training phase in only one flat since the obtained model can be employed to design the occupancy estimator for the other flats. Therefore, the method is suitable for estimation of occupancy patterns in such buildings.

D. Structure of the Manuscript

Section II formulates the mathematical problem and the solution methodology. Sections III and IV describe, respectively, how to identify the model of the room from a training set, and how to exploit this model for estimation purposes. Section V characterizes the performance of the estimator from a statistical perspective. Section VI describes how to modify the original estimation strategy when considering also HVAC actuation levels and information on door opening and closing. Section VII introduces the considered estimation performance indexes, the experimental setup, the results of the estimation processes, and some comparisons with standard tools of Machine Learning. Section VIII then wraps some conclusions, remarks, and ideas for future directions. Proofs are collected in the Appendix.

II. Problem Definition and Methodology

We consider a schematic representation of the dynamics of the concentration of the CO_2 and temperature in a room under well-mixed air assumptions (i.e., these quantities are assumed to be spatially constant). In Fig. 1, $c(k)$ represents the concentration of CO_2, $t(k)$ the temperature, $v(k)$ the amount of injected fresh air, $o(k)$ the occupancy, all at time k. G represents an initially unknown dynamic system relating disturbances, events, ventilation and building occupancy levels with temperature and CO_2 concentration signals. In addition, we consider a variable $e(k)$ which is a Boolean measurement of door opening and closing events, defined as follows:

$$e(k) = \begin{cases} 1, & \text{if the door is open,} \\ 0, & \text{if the door is closed} \end{cases}. \tag{1}$$

The problem we consider in this paper is to find an effective algorithm that transforms measurements of $c(k), c(k-1), \ldots$

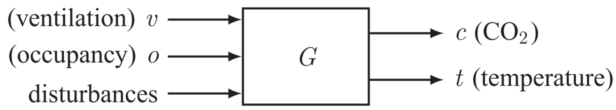

Fig. 1. Schematic representation of the signals and models under consideration.

and $t(k), t(k-1), \ldots$ into estimates of $o(k)$. Our proposal is the following two-tier estimator:

- **Tier 1, training phase:** identify a linear time invariant (LTI) system that captures the dynamics of G from pilot data of $c(k)$, $t(k)$, and $o(k)$, (Section III).
- **Tier 2, test phase:** estimate $o(k)$ from measurements of $c(k)$ and $t(k)$ and the estimated model of the room (Section IV).

The first phase addresses a *system identification* problem, while the second phase addresses a *deconvolution* problem.

A contribution of this paper is the characterization of the proposed estimator in terms of detection error, i.e., probability of obtaining wrong estimates as a function of the parameters of the estimator. We also study extensions of the estimator to include information on venting levels $v(k), v(k-1), \ldots$, and door opening/closing events $e(k), e(k-1), \ldots$. We shall see that, while including venting levels does not change the structure and main properties of the estimator, accounting for door opening and closing requires some modifications of the problem by adding suitable constraints.

III. Identification of the Room Model

In this section, we describe how to obtain a model for G starting from pilot data of $c(k)$, $t(k)$, and $o(k)$.

As in [29]–[33], we assume the environmental signals to be stochastic processes, the dynamics of the room to be discrete LTI, measurement devices to be synchronized and operating at the same sample time. We further assume that T_{tr} samples of the aforementioned signals have been collected during an experimental phase.

The dynamics of the room can be expressed as

$$\begin{bmatrix} c(k) \\ t(k) \end{bmatrix} = \begin{bmatrix} \boldsymbol{G}_c\left(q^{-1}\right) \\ \boldsymbol{G}_t\left(q^{-1}\right) \end{bmatrix} \begin{bmatrix} c(k-1) \\ t(k-1) \\ o(k-1) \end{bmatrix} + \begin{bmatrix} w_c(k) \\ w_t(k) \end{bmatrix} \tag{2}$$

where without loss of generality

$$\begin{aligned} \boldsymbol{G}_c\left(q^{-1}\right) &:= \left[\, G_c^c\left(q^{-1}\right) \quad G_c^t\left(q^{-1}\right) \quad G_c^o\left(q^{-1}\right) \,\right] \\ \boldsymbol{G}_t\left(q^{-1}\right) &:= \left[\, G_t^c\left(q^{-1}\right) \quad G_t^t\left(q^{-1}\right) \quad G_t^o\left(q^{-1}\right) \,\right] \end{aligned}$$

are matrix polynomials with all the entries having the same order. The processes $w_c(k)$ and $w_t(k)$ are white Gaussian noises, independent of each other, representing the innovation process, i.e., part of $c(k)$ and $t(k)$ that cannot be predicted from past measurements.

To estimate the polynomials $\boldsymbol{G}_c\left(q^{-1}\right)$ and $\boldsymbol{G}_t\left(q^{-1}\right)$ we consider a prediction error method (PEM) paradigm. We define the best linear one-step-ahead predictor of the outputs, namely

$$\begin{bmatrix} \widehat{c}(k|k-1) \\ \widehat{t}(k|k-1) \end{bmatrix} = \begin{bmatrix} \boldsymbol{G}_c\left(q^{-1}\right) \\ \boldsymbol{G}_t\left(q^{-1}\right) \end{bmatrix} \begin{bmatrix} c(k-1) \\ t(k-1) \\ o(k-1) \end{bmatrix} \tag{3}$$

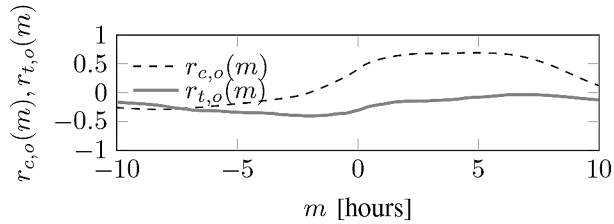

Fig. 2. Empirical cross-correlations between occupancy and either temperature ($r_{t,o}(m)$) or CO_2($r_{c,o}(m)$), computed using the dataset considered throughout the manuscript (sampling time $t_s = 5$ minutes). To highlight the features of the correlation signals, we use a time scale finer than the ones used in the subsequent figures.

obtained by simply neglecting the noise processes. Then, using PEM-based techniques we can obtain $\widehat{\boldsymbol{G}}_c\left(q^{-1}\right)$ and $\widehat{\boldsymbol{G}}_t\left(q^{-1}\right)$, such that the variance of the prediction errors $c(k) - \widehat{c}(k|k-1)$ and $t(k) - \widehat{t}(k|k-1)$ on the data collected during the training phase, is minimized. From (3), it follows that the predictors $\widehat{c}(k|k-1)$ and $\widehat{t}(k|k-1)$ exploit the same information of the past.

Fig. 2 plots the correlation functions defined in (4), and computed using the dataset considered throughout the manuscript. In (4), $\bar{c}(\cdot)$, $\bar{o}(\cdot)$ and $\bar{t}(\cdot)$ represent signals stripped of the mean, m is a time lag, and T_{tr} denotes the size of the dataset

$$\begin{aligned} r_{c,o}(m) &:= \frac{\sum_{k=0}^{T_{tr}} \bar{c}(k)\bar{o}(k-m)}{\sqrt{\left(\sum_{k=0}^{T_{Ts}} \bar{c}(k)^2\right)\left(\sum_{k=0}^{T_{Ts}} \bar{o}(k)^2\right)}} \\ r_{t,o}(m) &:= \frac{\sum_{k=0}^{T_{tr}} \bar{t}(k)\bar{o}(k-m)}{\sqrt{\left(\sum_{k=0}^{T_{Ts}} \bar{t}(k)^2\right)\left(\sum_{k=0}^{T_{Ts}} \bar{o}(k)^2\right)}}. \end{aligned} \tag{4}$$

The functions $r_{c,o}(m)$ and $r_{c,o}(m)$ indicate the dependency of the occupancy signal on the CO_2 concentration and temperature, respectively, as a function of the time lag m. It can be promptly seen that the signal mostly correlated with the occupancy is the CO_2 level. For this reason, in the rest of this paper, we shall consider only the predictor $\widehat{c}(k|k-1)$ and thus focus on the identification of $\widehat{\boldsymbol{G}}_c\left(q^{-1}\right)$.

A. Nonparametric Identification of the CO_2 Dynamics

In this paper, we adopt a nonparametric approach to the problem of identifying the CO_2 room dynamics. Instead of directly searching for the coefficients of the polynomials $G_c^o\left(q^{-1}\right)$, $G_c^t\left(q^{-1}\right)$ and $G_c^c\left(q^{-1}\right)$, we aim at estimating the system impulse responses, which are defined in the time domain and which are related to the frequency domain description of the system through the relations

$$\begin{aligned} G_c^o\left(q^{-1}\right) &= \sum_{k=0}^{+\infty} g_o(k) q^{-k} \\ G_c^t\left(q^{-1}\right) &= \sum_{k=0}^{+\infty} g_t(k) q^{-k} \\ G_c^c\left(q^{-1}\right) &= \sum_{k=0}^{+\infty} g_c(k) q^{-k} \end{aligned} \tag{5}$$

where $g_o(k)$, $g_t(k)$, $g_c(k)$ are the impulse responses having the occupancy, temperature and CO_2 as inputs, respectively. We can simplify the problem by truncating the impulse response to a fixed large index p and estimate the first p coefficients of each impulse response. The estimated coefficients can then be used to form polynomials[1] that well-approximate the transfer functions. To make the estimation problem well-posed, we define a suitable hypothesis space for the unknown impulse responses. Such a space is a reproducing kernel Hilbert space (RKHS) [34], and its associated kernel is the so-called *stable spline kernel* [35], [36], defined as

$$[K_\beta]_{i,j} = \beta^{\max\{i,j\}}, \quad 0 < \beta < 1 \tag{6}$$

where β is a hyperparameter tuning the decay rate. The choice of this kernel is motivated by the fact that the associated RKHS contains smooth and exponentially decaying functions. These are desirable properties in impulse responses modeling of physical systems such as those considered in this problem. We refer to [37] for a thorough description of kernel-based methods in system identification.

Let the training set be indexed by the time instances $0, 1, \ldots, T_{tr}$ and $\boldsymbol{g}_c$, $\boldsymbol{g}_t$, and $\boldsymbol{g}_o$ be column vectors containing the impulse responses related to $c(k)$, $t(k)$, and $o(k)$, respectively, and

$$\begin{aligned} \boldsymbol{g} &:= [\boldsymbol{g}_c^T \quad \boldsymbol{g}_t^T \quad \boldsymbol{g}_o^T]^T \\ \phi_c(k) &:= [\, c(k-1) \quad \ldots \quad c(k-p)\,] \\ \phi_t(k) &:= [\, t(k-1) \quad \ldots \quad t(k-p)\,] \\ \phi_o(k) &:= [\, o(k-1) \quad \ldots \quad o(k-p)\,] \end{aligned} \tag{7}$$

with $k > p$. Defining

$$\begin{aligned} \Phi &:= \begin{bmatrix} \phi_c(p+1) & \phi_t(p+1) & \phi_o(p+1) \\ \phi_c(2) & \phi_t(2) & \phi_o(2) \\ \vdots & \vdots & \vdots \\ \phi_c(T_{tr}) & \phi_t(T_{tr}) & \phi_o(T_{tr}) \end{bmatrix} \\ \boldsymbol{c}_{tr} &:= [\, c(1) \quad c(2) \quad \ldots \quad c(T_{tr})\,]^T \end{aligned}$$

we can formulate the system identification problem as

$$\widehat{\boldsymbol{g}} = \arg\min_{\boldsymbol{g}\in\mathbb{R}^{3p}} \|\boldsymbol{c}_{tr} - \Phi\boldsymbol{g}\|_2^2 + \gamma\left(\|\boldsymbol{g}_c\|_P^2 + \|\boldsymbol{g}_t\|_P^2 + \|\boldsymbol{g}_o\|_P^2\right) \tag{8}$$

i.e., as regularized least-squares (LS), where

- $\|\boldsymbol{g}\|_P^2 = \boldsymbol{g}^T P \boldsymbol{g}$ with P a positive definite weighting matrix penalizing candidate impulse responses which do not decay to zero for large values of the time index. In this way, P favors outcomes $\widehat{\boldsymbol{g}}$ that well represent impulse responses of stable systems. Here, we set the matrix P as $P = K_\beta^{-1}$; the choice of the hyperparameter β is discussed below.
- γ is a positive real number representing a tradeoff between variance and bias of the estimator, leading to the LS estimate of $\boldsymbol{g}$ for $\gamma = 0$.

The optimal values of γ and β can be computed using either cross validation [38] or empirical Bayes techniques [39], [35].

[1]In this paper, we set $p = 50$.

Once these values have been established, the solution can be computed in closed form [40] as

$$\widehat{\boldsymbol{g}} = \left(\Phi^T\Phi + \gamma D_P\right)^{-1} \Phi^T \boldsymbol{c}_{tr} \tag{9}$$

where D_P is block diagonal with four blocks all equal to P.

Remark 1: The impulse response estimator (8) may be seen also as a maximum a posteriori (MAP) estimator under a Gaussian prior assumption of the unknown impulse responses. Then, the choice of such a prior is well motivated by the underlying theory of the RKHS induced the stable spline kernel (see [34] for further details).

IV. Deconvolution of the Occupancy Levels

In this section, we derive an estimator $\widehat{o}(k)$ of $o(k)$ as a function of the measurements $c(k)$ and $t(k)$ and the estimated room dynamics $\widehat{G}_c^c$, $\widehat{G}_c^t$, $\widehat{G}_c^o$. Let

$$\widehat{c}(k|k-1) = \widehat{\boldsymbol{G}}_c\left(q^{-1}\right) \begin{bmatrix} c(k-1) \\ t(k-1) \\ o(k-1) \end{bmatrix} \tag{10}$$

and consider the CO_2 levels prediction error

$$\varepsilon(k) := c(k) - \widehat{c}(k|k-1). \tag{11}$$

Under the stated assumptions $\varepsilon(k)$ is a zero-mean Gaussian white noise [41]. Substituting (10) into (11) and rearranging properly, we obtain

$$\begin{aligned} &\widehat{G}_c^o\left(q^{-1}\right) o(k-1) \\ &\quad = c(k) - \left[\widehat{G}_c^c\left(q^{-1}\right) \widehat{G}_c^t\left(q^{-1}\right)\right] \begin{bmatrix} c(k-1) \\ t(k-1) \end{bmatrix} - \varepsilon(k) \end{aligned} \tag{12}$$

where the unknowns are only $o(k-1)$ and $\varepsilon(k)$, since

$$\widetilde{c}(k) := c(k) - \left[\widehat{G}_c^c\left(q^{-1}\right) \widehat{G}_c^t\left(q^{-1}\right)\right] \begin{bmatrix} c(k-1) \\ t(k-1) \end{bmatrix}$$

can be computed given the available information. Thus (12) becomes

$$\widetilde{c}(k) = \widehat{G}_c^o\left(q^{-1}\right) o(k-1) + \varepsilon(k) \tag{13}$$

which shows that the problem of estimating the unknown occupancy pattern is a deconvolution problem, i.e., the signal $o(k)$ can be estimated as the signal best describing the observed output $\widetilde{c}(k)$, given the knowledge of the transfer function $\widehat{G}_c^o$. Since $\varepsilon(k)$ is assumed white and Gaussian, the natural approach to this problem would be to employ a LS estimator of $o(\cdot)$, because this would minimize the residual error [38, Ch. 7]. More specifically, let $\widehat{G}_c^o\left(q^{-1}\right) = g_1 q^{-1} + \cdots + g_p q^{-p}$; we consider two variants of the occupancy estimation problem:

1) online monitoring;
2) offline estimation.

We begin by dealing with the first case. At each time instant, we consider a window in the past of N samples of each signal, from $k-N+1$ to k, with $N \geq p$. Considering the auxiliary notation

$$\widehat{G} := \begin{bmatrix} g_1 & 0 & \cdots & & & 0 \\ g_2 & g_1 & & & & \\ \vdots & \ddots & \ddots & & & \vdots \\ g_p & \cdots & g_2 & g_1 & & \\ & \ddots & & \ddots & \ddots & \\ 0 & & g_p & \cdots & g_2 & g_1 \end{bmatrix}$$

$$\widetilde{\boldsymbol{o}} := \begin{bmatrix} o(k-N) \\ \vdots \\ o(k-1) \end{bmatrix}, \widetilde{\boldsymbol{c}} := \begin{bmatrix} \widetilde{c}(k-N+1) \\ \vdots \\ \widetilde{c}(k) \end{bmatrix} \tag{14}$$

a basic occupancy estimator can be formulated as the LS-type problem

$$\widehat{\boldsymbol{o}} = \arg \min_{\widetilde{\boldsymbol{o}} \in \mathbb{R}_+^N} \left\| \widetilde{\boldsymbol{c}} - \widehat{G}\widetilde{\boldsymbol{o}} \right\|_2^2. \tag{15}$$

The performance of this estimator is usually unsatisfactory, since the estimates are noisy, due to the high variance, and they do not reflect suitable room occupancy patterns. To overcome this issue we account for the prior information that $o(k)$ *is non-negative, integer, and piecewise constant* and we formulate the deconvolution problem as the problem of finding the least-changing positive piecewise constant input signal giving a prescribed mismatch between the estimated and measured outputs of the system. Let us define

$$\Delta o(i) := o(i) - o(i-1), \quad \Delta \boldsymbol{o} := \begin{bmatrix} \Delta o(k-N+1) \\ \vdots \\ \Delta o(k-1) \end{bmatrix}. \tag{16}$$

The estimation problem then becomes

$$\widehat{\boldsymbol{o}}(k-1) = \arg \min_{\widetilde{\boldsymbol{o}} \in \mathbb{N}_+^N} \quad \|\Delta \widetilde{\boldsymbol{o}}\|_0$$
$$\text{s.t.} \quad \left\| \widetilde{\boldsymbol{c}} - \widehat{G}\widetilde{\boldsymbol{o}} \right\|_2^2 \leq \rho \tag{17}$$

where

- $\widehat{\boldsymbol{o}}(k-1)$ is a N-dimensional vector with the estimated values of occupancy at the time instants $k-1, \ldots, k-N$ (for online estimation purposes and HVAC control one might consider to use just its first entry $\widehat{o}(k-1)$);
- the cost function $\|\cdot\|_0$, the ℓ_0 norm, counts the number of variations of the candidate inputs, thus penalizing signals with frequent variations;
- the LS-type term accounts for adherence to data and tries to match the estimated and measured outputs of the system, up to a precision given by the user-choice parameter ρ.

Problem (17) can be reformulated as follows [42]:

$$\widehat{\boldsymbol{o}}(k-1) = \arg \min_{\widetilde{\boldsymbol{o}} \in \mathbb{N}_+^N} \left\| \widetilde{\boldsymbol{c}} - \widehat{G}\widetilde{\boldsymbol{o}} \right\|_2^2 + \lambda \|\Delta \widetilde{\boldsymbol{o}}\|_0 \tag{18}$$

where λ is a regularization parameter (strictly related to ρ) that trades off the two previous terms; the choice of λ is discussed in details in Section IV-A. Unfortunately, Problem (18) is a nonconvex nonlinear integer program; to solve it directly one must search through all possible combinations of nonzero elements in $\Delta\widetilde{\boldsymbol{o}}$. Hence, the search space increases exponentially with the number of parameters and the problem cannot be solved efficiently [43]. To circumvent this computational drawback we adopt two relaxations. First, we substitute the ℓ_0 norm with the ℓ_1 norm [44, Ch. 3.4], which represents its best convex relaxation. Second, we extend the domain of the plausible inputs to $\mathbb{R}_+^N$ instead of $\mathbb{N}_+^N$, so that the estimation problem becomes

$$\widehat{\boldsymbol{o}}(k-1) = \left\lfloor \arg \min_{\widetilde{\boldsymbol{o}} \in \mathbb{R}_+^N} \left\| \widetilde{\boldsymbol{c}} - \widehat{G}\widetilde{\boldsymbol{o}} \right\|_2^2 + \lambda \|\Delta \widetilde{\boldsymbol{o}}\|_1 \right\rceil \tag{19}$$

where $\lfloor \cdot \rceil$ is the vector-wise rounding operator. Problem (19) is a particular case of *fused-lasso* estimator, where the solution is searched among *sparse* regressor vectors where less frequent jumps (i.e., nonzero impulses in the derivatives) are preferred, and the strength of this preference is dictated by the regularization parameter λ.

Remark 2: The estimator in (19) can also be seen as a MAP estimator of the occupancy signal (when the rounding operator is removed). In this case, the prior distribution on the unknown process is Laplacian with independent components (see, e.g., [45]), that is

$$p(\Delta \boldsymbol{o}) = \frac{1}{\alpha} \prod_{i=1}^{N-1} \exp\left(-\frac{|\Delta \widetilde{\boldsymbol{o}}(i)|}{\alpha}\right) \tag{20}$$

where α is a user parameter that tunes the sharpness (and so the sparsity) of the pdf. Since $\varepsilon(k)$ is white and Gaussian, then the distribution of the vector $\widetilde{\boldsymbol{c}}$ given $\Delta\widetilde{\boldsymbol{o}}$ is Gaussian, with mean $\widehat{G}\widetilde{\boldsymbol{o}}$ and variance $\sigma_\varepsilon^2 I$, where σ_ε^2 is the variance of the prediction error appearing in (13) and I is the identity matrix. The MAP estimation of $\widetilde{\boldsymbol{o}}$ can thus be formulated as

$$\widehat{\boldsymbol{o}}(k-1) = \arg \max_{\widetilde{\boldsymbol{o}} \in \mathbb{R}_+^N} \log\left(p\left(\widetilde{\boldsymbol{c}} | \Delta\widetilde{\boldsymbol{o}}\right) p\left(\Delta\widetilde{\boldsymbol{o}}\right)\right)$$
$$= \arg \min_{\widetilde{\boldsymbol{o}} \in \mathbb{R}_+^N} \frac{1}{\sigma_\varepsilon^2} \left\| \widetilde{\boldsymbol{c}} - \widehat{G}\widetilde{\boldsymbol{o}} \right\|_2^2 + \frac{2}{\alpha} \|\Delta\widetilde{\boldsymbol{o}}\|_1. \tag{21}$$

The above expression reveals that the regularization parameter $\lambda = 2\sigma_\varepsilon^2/\alpha$ is the ratio of the noise variance and the user parameter α regulating the (prior) sparseness of the derivative of the occupancy.

The parameter N plays an important role in (19), since it defines the amount of data employed for estimating $\widehat{\boldsymbol{o}}(k-1)$ (and in particular $\widehat{o}(k-1)$) at each time instant. Clearly, a large value of N yields more accurate estimates, since more information is used. However, a large value of N brings computational issues which could make the computation of (19) too slow for online operations. Thus, as will be discussed in Section VII-B, a good choice of N should consider both these aspects.

The derivation of the offline estimator is straightforward. Let the test set be indexed by the time instants $1, \ldots, T_{ts}$. Redefining the vectors introduced in (14) and (16), so that they include all the T_{ts} measurement of the test set, we can re-utilize the estimator defined in (19). In this case, its output will be a vector $\widehat{\boldsymbol{o}}$ containing the estimated occupancy pattern for the time instants $0, \ldots, T_{ts} - 1$.

A. Finding the Optimal Regularization Parameter λ

The regularization parameter λ establishes the typical variability of the room occupancy signal. Large values of λ penalize changes in the value of estimated occupancy, leading to estimates that are constant for long periods. Small values of λ, instead, yield occupancy signals with high-frequency components, thus behaving similarly to the outcomes of the LS estimator (which is obtained by setting $\lambda = 0$).

A reasonable choice of λ is obtained by finding the value of such a parameter giving the best estimation performance during the training phase. This optimal value can then be computed with the following procedure:

1) define a grid Λ of candidate values of λ;
2) for each $\lambda \in \Lambda$ solve Problem (19) using the $c(k)$ and $t(k)$ collected *during the training phase*, obtaining $\widehat{\boldsymbol{o}}(\lambda)$, i.e., an occupancy estimate as function of λ;
3) compute the optimal regularization parameter as

$$\widehat{\lambda} = \arg\min_{\lambda \in \Lambda} \|\widehat{\boldsymbol{o}}(\lambda) - \boldsymbol{o}\|_2^2 \tag{22}$$

with $\boldsymbol{o}$ being the occupancy signal collected during the training phase.

Remark 3: To find the set Λ one can start by first finding an opportune $\lambda_{\max}$ for which the problem (19) leads to constant occupancy estimates. Consider moreover the cardinality of Λ be given; then one can define the set Λ between 0 and the obtained $\lambda_{\max}$ exploiting a logarithmic grid. The main advantage of logarithmic gridding is that the grid will be finer for smaller values of λ, where the sensitivity is usually higher (see Fig. 5).

V. Characterization of the Occupancy Estimator

In this section, we derive relations between the probability of obtaining wrong occupancy estimates and the quantities parameterizing the estimator, namely, the identified linear models, the noise level of the measurements, and the regularization parameter λ.

Our first result regards the performance of the estimator when the occupancy is constant in a window of N past values.

Proposition 4: Let σ_ε be the variance of the noise in (13), N the window length in the estimator, and λ the regularization parameter. Assume that $o(k)$ is a constant signal. Define

$$\Delta := \begin{bmatrix} -1 & 1 & & \\ & \ddots & \ddots & \\ & & -1 & 1 \end{bmatrix} \in \mathbb{R}^{N-1 \times N} \tag{23}$$

and $V^T := \left(\Delta \widehat{G}^{-1}\right)^\dagger$, where $(X)^\dagger$ denotes the Moore–Penrose pseudoinverse of X. Then $\widehat{o}(k)$ is detected as constant with probability of at least α if

$$\lambda^2 > \sigma_\varepsilon^2 \chi_\alpha^{-1}(N) \|V_m\|^2 \tag{24}$$

where $\chi_\alpha^{-1}(N)$ is the inverse of the chi-square cumulative distribution function (CDF) with N degrees of freedom for the corresponding probability α and $\|V_m\|^2 := \max_i \|V_i\|^2$, with V_i the ith row of V.

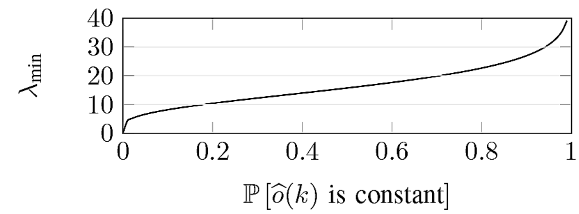

Fig. 3. Graphical representation of bound (24) as functions of the probability α for a given σ_ε^2.

The following result studies the case where $o(k)$ has a variation.

Proposition 5: Let σ_ε be the variance of the noise in (13), N the window length in the estimator, λ the regularization parameter. Define $\bar{\Delta} \in \mathbb{R}^{N-1 \times N-1}$, obtained removing the first column of Δ and $\bar{V}^T := (\bar{\Delta}\bar{H}^{-1})^{-1}$. Assume that the first value of the estimated occupancy is set to the true one, i.e., $\widehat{o}(k - N) = o(k - N)$, and that $o(k)$ has a unique discontinuity given by a variation of one unit. Then, $\widehat{o}(k)$ is detected as constant, i.e., there is a missed change with probability of at least α if

$$\lambda^2 > \sigma_\varepsilon^2 \chi_\alpha^{-1}(N) \|\bar{V}_1\|^2 + (1 + o(k-N)^2) \|\bar{V}_1\|^4 \tag{25}$$

where $\bar{V}_1$ is the first row of $\bar{V}$.

Fig. 3 shows the behavior of the bound derived in Proposition 4 as a function of the probability that the detected occupancy pattern is constant.

The previous results can easily be extended to the more general case where the true occupancy is piecewise constant with ρ discontinuities of +1 units. The sufficient condition to estimate a constant signal in this case is

$$\lambda^2 > \sigma_\varepsilon^2 \chi_\alpha^{-1}(N) \|V_1\|^2 + (\rho + o(k-N)^2) \|V_1\|^4.$$

VI. Accounting for Additional Information

In this section, we address the cases where the available information contains the additional signals $v(k)$ (venting levels) and $e(k)$ [door events, defined in (1)].

A. Accounting for Venting Levels

When the signal $v(k)$ is available, a straightforward generalization of (10) yields

$$\widehat{c}(k|k-1) = \widehat{\boldsymbol{G}}_c(q^{-1}) \begin{bmatrix} c(k-1) \\ t(k-1) \\ v(k-1) \\ o(k-1) \end{bmatrix}$$

$$\widehat{\boldsymbol{G}}_c(q^{-1}) = \left[\widehat{G}_c^c(q^{-1})\ \widehat{G}_c^t(q^{-1})\ \widehat{G}_c^v(q^{-1})\ \widehat{G}_c^o(q^{-1})\right]. \tag{26}$$

Consequently, we can extend the system identification procedure of Section III, with $\boldsymbol{g} := [\,\boldsymbol{g}_c^T \quad \boldsymbol{g}_t^T \quad \boldsymbol{g}_v^T \quad \boldsymbol{g}_o^T\,]^T$, and

$$\widehat{\boldsymbol{g}} = \arg\min_{\boldsymbol{g} \in \mathbb{R}^{4p}} \|\boldsymbol{c}_{tr} - \Phi \boldsymbol{g}\|_2^2 + \gamma(\|\boldsymbol{g}_c\|_P^2 + \|\boldsymbol{g}_t\|_P^2 + \|\boldsymbol{g}_v\|_P^2 + \|\boldsymbol{g}_o\|_P^2).$$

The same extension applies to the deconvolution step: the estimator (13) remains structurally the same as soon as $\widetilde{c}(k)$ is redefined as

$$\widetilde{c}(k) := c(k) - \left[\widehat{G}_c^c(q^{-1}) \; \widehat{G}_c^t(q^{-1}) \; \widehat{G}_c^v(q^{-1}) \right] \begin{bmatrix} c(k-1) \\ t(k-1) \\ v(k-1) \end{bmatrix}.$$

B. Accounting for Door Opening and Closing Events

Assume now the knowledge of $e(k)$, i.e., a Boolean signal measuring door opening and closing events. Using the definition in (1) we infer that $e(k) = 0$ implies $o(k) = o(k-1)$, while no deduction on the behavior of $o(k)$ can be made when $e(k) \neq 0$.

In the system identification problem of Section III, information on $e(k)$ is non-influential, i.e., it does not modify the derivations in Section III, since during the identification the occupancy levels are assumed known. In other words, $o(k)$ contains already the information in $e(k)$.

As for the deconvolution problem, knowing $e(k)$ changes the structure of the estimator, since $e(k)$ naturally constraints the estimand occupancy levels to be identical when $e(k) = 0$. More precisely, knowing $e(k)$ corresponds to knowing the sparsity pattern of the to-be-reconstructed signal. This imply that the regularization term $\|\Delta\widetilde{o}\|_0$ in (18) is a constant factor that does not depend on the decision variables; thus (18) is equivalent to the integer quadratic program (IQP)

$$\begin{aligned} \widehat{\boldsymbol{o}}(k-1) = \arg\min_{\widetilde{\boldsymbol{o}} \in \mathbb{N}_+^N} \quad & \left\| \widetilde{\boldsymbol{c}} - \widehat{G}\,\widetilde{\boldsymbol{o}} \right\|_2^2 \\ \text{s.t.} \quad & \Delta\widetilde{o}(k) = 0 \text{ for all } e(k) = 0. \end{aligned} \tag{27}$$

Following the motivations that brought from (18) to (19), (27) can be relaxed with

$$\begin{aligned} \widehat{\boldsymbol{o}} = \arg\min_{\widetilde{\boldsymbol{o}} \in \mathbb{R}_+^{T_{ts}}} \quad & \left[\left\| \widetilde{\boldsymbol{c}} - \widehat{G}\,\widetilde{\boldsymbol{o}} \right\|_2^2 \right] \\ \text{s.t.} \quad & \Delta\widetilde{o}(k) = 0 \text{ for all } e(k) = 0. \end{aligned} \tag{28}$$

Due to the lack of the regularization term, (28) does not require tunings of regularization parameters.

Estimator (28) is based on the hypothesis that the noise process $\varepsilon(k)$ is white and Gaussian. Such an assumption may be unrealistic, since the identification phase is likely to yield non-exact models (due to disturbances and unmodeled dynamics). One way to address this issue and robustify the estimator is to further modify (28) adding back the ℓ_1 regularization term $\lambda\|\Delta\widetilde{o}\|_1$ to obtain

$$\begin{aligned} \widehat{\boldsymbol{o}} = \arg\min_{\widetilde{\boldsymbol{o}} \in \mathbb{R}_+^{T_{ts}}} \quad & \left[\left\| \widetilde{\boldsymbol{c}} - \widehat{G}\,\widetilde{\boldsymbol{o}} \right\|_2^2 + \lambda \|\Delta\widetilde{\boldsymbol{o}}\|_1 \right] \\ \text{s.t.} \quad & \Delta\widetilde{o}(k) = 0 \text{ for all } e(k) = 0. \end{aligned} \tag{29}$$

As noticed before, this regularization term corresponds to promoting small changes in the occupancy signal, with the strength of this preference dictated by the regularization parameter λ. Obviously, implementing estimator (29) requires to find the optimal λ, as described in Section IV-A.

VII. Experiments

We have tested the proposed estimator on one of the rooms of the KTH ACL-HVAC testbed, see http://hvac.ee.kth.se/ for more information. The collected information, available at http://hvac.ee.kth.se/datasets.html, comprises two weeks of measurements of CO_2 and temperature levels from HDH sensors, and of venting, cooling, and heating actuation levels from the central HVAC system. Occupancy levels were manually registered for the whole period. To uniform the sampling times of the various signals (5 min), or in case of missing measurements, the information was resampled using linear interpolation schemes. The first week was used as a training set, while the second week was used as a test set.

Definition of the Performance Indexes

We consider four performance indexes: (i) the *mean squared error (MSE)* (30), characterizing the relative estimation errors; (ii) the *accuracy* (32), reporting how many times the estimator returns the correct value; and (iii) the *false positive/false negative occupancy detection rates* (35), describing the ability of discriminating the presence/absence of occupants in terms of false positives (when the room is estimated to be occupied while it is not) and false negatives (when the room is estimated to be empty while it is not).

The MSE associated with $\boldsymbol{o}$ and $\widehat{\boldsymbol{o}}$ is

$$\text{MSE}(\widehat{\boldsymbol{o}}) := \frac{\|\widehat{\boldsymbol{o}} - \boldsymbol{o}\|_2^2}{\|\boldsymbol{o}\|_2^2}. \tag{30}$$

To define the other performance indexes we then transform the signals $\boldsymbol{o}, \widehat{\boldsymbol{o}}$ with codomain $\mathbb{N}_+$ (number of occupants) to signals with codomain $\{0, 1\}$ (corresponding to the states "room is not occupied" and "room is occupied," respectively) through the indicator function

$$\mathbb{1}(o(k)) := \begin{cases} 1, & \text{if } o(k) > 0 \\ 0, & \text{otherwise} \end{cases}, \quad \mathbb{1}(\boldsymbol{o}) := \begin{bmatrix} \mathbb{1}(o(1)) \\ \vdots \\ \mathbb{1}(o(N)) \end{bmatrix}. \tag{31}$$

Given (31), the accuracy of the estimate $\widehat{\boldsymbol{o}}$ is

$$\text{Acc}(\widehat{\boldsymbol{o}}) := \frac{N - \sum_{k=1}^{N} \mathbb{1}(o(k) - \widehat{o}(k))}{N}. \tag{32}$$

To define the false positive/negative rates we introduce

$$\mathcal{N}_\theta := \{k \text{ s.t. } \mathbb{1}(o(k)) = \theta\} \tag{33}$$

dividing the time indexes in two sets: $\mathcal{N}_0$, for the time indexes k for which the room was not occupied, and $\mathcal{N}_1$, for the k's for which the room was occupied. Using this definition we may capture wrong matches of the type "the room is estimated to be occupied while it is empty" and "the room is considered empty while it is occupied," We define

$$\widehat{\beta}(\theta) := \frac{1}{|\mathcal{N}_\theta|} \sum_{k \in \mathcal{N}_\theta} \mathbb{1}(\widehat{o}(k)) \tag{34}$$

TABLE I
COMPARISON OF THE PERFORMANCE OF ESTIMATORS (19), (19) WITH KNOWLEDGE OF VENTILATION LEVELS $v(k)$, (28) AND (29)

Estimator	MSE	Accuracy	FP	FN
(19)	0.208	0.822	0.039	0.028
(19)+v	0.124	0.888	0.007	0.018
(28)	0.217	0.884	0.001	0.028
(29)	0.109	0.886	0.006	0.008

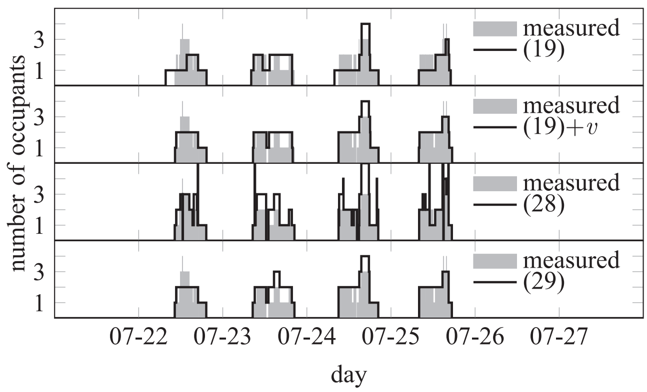

Fig. 4. Realizations of the estimates for the test set considered in our experiments for the various estimators proposed in this manuscript.

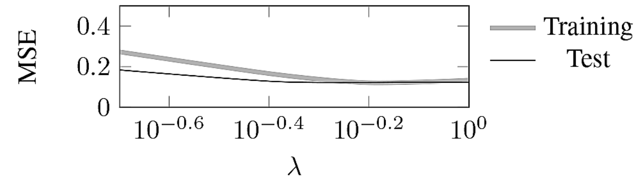

Fig. 5. Sensitivity of the performance of estimator (19) $+v$ w.r.t. the choice of λ.

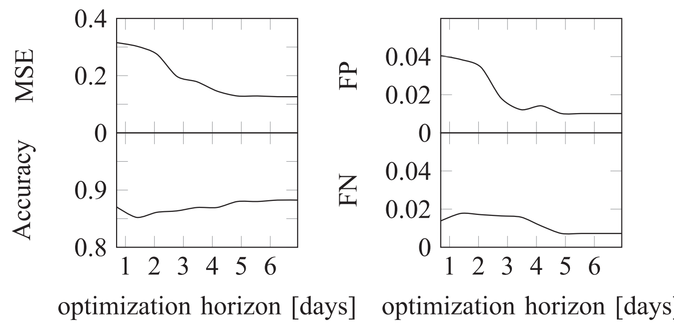

Fig. 6. Dependency of the performance of estimator (29) w.r.t. the choice of N.

where we remark that the summation is performed over the set $\mathcal{N}_\theta$. With (34) the false positive and false negative rates become

$$\mathrm{FP}(\widehat{\boldsymbol{o}}) := \widehat{\beta}(0), \qquad \mathrm{FN}(\widehat{\boldsymbol{o}}) := 1 - \widehat{\beta}(1). \tag{35}$$

A. Summary of the Results

1) Evaluation of the Importance of Additional Information: We assume that the parameters λ and N are optimally scaled (we discuss tuning of these parameters in the following sections). Table I numerically assesses the value of knowing the ventilation levels $v(k)$ and the door openings/closing events $e(k)$, while Fig. 4 depicts graphically the realizations of the results. From Table I, it can be seen that adding the information regarding ventilation levels can improve the accuracy of the estimator. Moreover, the estimator (29) has the best performance in terms of MSE. It is worth mentioning that due to the constraint on $\Delta\widetilde{o}$ in (29), the ℓ_1 regularization term does not impose (further) sparsity, however, it shrinks the estimates of the occupancy. It is well known that shrinking may improve the MSE [44]; we can get similar results with other shrinkage methods such as the ridge regression [44].

2) Evaluation of the Sensitivity to the Regularization Parameter λ: We evaluate the effectiveness of selection strategy for the parameter λ described in Section IV-A. Since the best value of such a parameter for the test set may be different from its best value in the training set, it is important to evaluate the effects of this unavoidable mismatch.

Fig. 5 plots the MSE for different values of λ for estimator (19) $+v$ for both the training and test sets. The dependency on λ appears relatively weak in the test set, and the MSEs of the training and test sets attain their minima at approximately the same point. This suggests that the proposed estimation strategy for λ is reliable and effective.

3) Evaluation of the Sensitivity to the Optimization Horizon N: The parameter N trades-off computational requirements with information: the larger the optimization horizon, the more information the estimators have about the dynamics of the system. Intuition suggests that, beyond a certain horizon, adding more information does not improve the estimation performance, i.e., beyond this horizon the room dynamics do not influence the current estimates. The results shown in Fig. 6 indicate that in our experiments the horizon is of about five days.

B. Alternative Occupancy Estimation Methods

We hereby consider two classical Machine Learning strategies and compare them against estimator (19) with knowledge of ventilation levels $v(k)$.

1) Estimation Using SVM: In their basic form, SVMs perform classification tasks as follows: given a dataset $\mathcal{D}$ of samples $(\boldsymbol{x}_k, y_k)$ for $k = 0, \ldots, N$ with $\boldsymbol{x}_k \in \mathbb{R}^n$ and $y_k \in \{-1, +1\}$, try to find a separating hyperplane in $\mathbb{R}^{n+1}$) that: (i) separates the points of the form $(\mathbf{x}_k, +1)$ from those of the form $(\boldsymbol{x}_k, -1)$ and (ii) maximizes its minimum distance from the $\boldsymbol{x}_i$'s. This concept can then be extended to cope with nonlinear and imperfect separation rules, and with multiclasses classification tasks [46, Part II].

SVMs have already been exploited for building occupancy estimation tasks, e.g., in [22] and [23]. The most common approach is to let $\boldsymbol{x}_k$ contain functions of the current and past CO_2, temperature and ventilation levels (e.g., the average of $c(k), \ldots, c(k-n)$). y_k instead usually represents the building occupancy level $o(k)$. With these definitions it is possible to train a general multiclass SVM on the couples $(\boldsymbol{x}_k, y_k)$ that form the training set. After this step one can then estimate the unknown building occupancy by applying the trained SVM on the $\boldsymbol{x}_k$ that form the test set.

The SVM implemented in our tests that led to the best estimation error performance is a C-SVM exploiting a polynomial kernel of order 3. As features, it considers current and past values of the temperature, CO_2, and ventilation levels up to 1 hour in the past, and their first and second derivatives in time.

2) Estimation Using NN: The (NNs) maps considered in this manuscript are of the form [47, Sec. 44]

$$y_k = \Psi'' \left(\sum_i \omega_i'' h_i(\boldsymbol{x}_k) + \theta'' \right)$$
$$h_i(\boldsymbol{x}_k) = \Psi_i' \left(\sum_j \omega_j' x_{k,j} + \theta_i' \right)$$

with y_k and $\boldsymbol{x}_k$ having the same meanings of Section VII-C1. The structures of the functions Ψ'', Ψ_i', h_i are design parameters, that usually remind how biological neurons electrically react to external stimuli. Once the design parameters have been chosen, training the network corresponds to search for a set of weights $\boldsymbol{w}$ for which the corresponding NN best fits the training examples. Once this function has been learned, it can be used for prediction purposes analogously to the SVM case.

The NN implemented in our tests that led to the best estimation error performance is a complete feed-forward network with Sigmoid activation rules and one hidden layer composed by 8 neurons. It considers the same features exploited to train the SVM based estimator.

3) Results of Comparisons: We compare in Table II the performance of estimator (19) with knowledge of ventilation levels against the performance of the NN and support vector classification (SVC) strategies described above. All these estimators are comparable in the sense that they use the same amount of information. It can then be noticed that the estimation strategy proposed in this manuscript outperforms the aforementioned machine learning strategies on the considered dataset.

VIII. Conclusion

We have proposed methods for estimating occupancy levels in closed environments that exploit different sources of information. We have aimed at understanding which of such sources are mostly meaningful in addressing the task of estimating how occupancy levels change in time. The main standing assumption in our methodology is that it is possible to access to direct measurements of the true occupancy levels for a limited period.

The proposed estimation scheme first obtains a dynamic model by a suitable identification method using pilot data. Then, it formulates the occupancy estimation problem as a regularized deconvolution problem (where the regularization exploits prior information on the features of the searched signal). The obtained results show that adding information on ventilation and door opening/closing events can significantly improve the performance of the estimator.

We have also analyzed the statistical performance of the estimation scheme, showing that the probability of obtaining wrong estimates can be suitably bounded when we know specific design parameters and the measurement noise variance.

TABLE II
Comparison of the Performance of Estimator (19) With Knowledge of Ventilation Levels Against the Performance of Equivalent NN and SVC Strategies

Estimator	MSE	Accuracy	FP	FN
(19)+v	0.124	0.888	0.007	0.018
SVM	0.342	0.826	0.018	0.277
NN	0.268	0.811	0.067	0.095

The idea considered in this paper can be extended towards the construction of occupancy estimators for whole buildings, and towards the identification of building occupancy pattern models. Moreover it may be possible to adapt the models identified in a single room to other rooms of the same building, by an opportune rescaling of the identified impulse responses accounting variations in the structural properties of rooms.

Appendix

Proof of Proposition 4:

The proof is divided in three main parts: *i)* Rewrite (19), derive the dual of the rewritten problem and the structure of its solution. *ii)* Find some analytical relations between the estimated and the true occupancy levels. *iii)* Exploit these relations to derive bounds that characterize the statistical performance of the estimator.

i): Introduce the variable $\boldsymbol{z} := \Delta\widetilde{\boldsymbol{o}}$ and rewrite (19) as

$$\begin{aligned} \arg \min_{\substack{\widetilde{\boldsymbol{o}} \in \mathbb{R}_+^N \\ \boldsymbol{z} \in \mathbb{R}^{N-1}}} \quad & \frac{1}{2} \left\| \widetilde{\boldsymbol{c}} - \widehat{G}\widetilde{\boldsymbol{o}} \right\|_2^2 + \lambda \left\| \boldsymbol{z} \right\|_1 \\ \text{s.t.} \quad & \boldsymbol{z} = \Delta\widetilde{\boldsymbol{o}} \end{aligned} \tag{36}$$

where for the purposes of the proof, the function $\lfloor \cdot \rceil$ (the vector-wise rounding operator) is omitted. The Lagrangian of (36) is then

$$\mathcal{L}(\widetilde{\boldsymbol{o}}, \boldsymbol{z}, \boldsymbol{u}) = \frac{1}{2} \|\widetilde{\boldsymbol{c}} - \widehat{G}\, \widetilde{\boldsymbol{o}}\|_2^2 + \lambda \|\boldsymbol{z}\|_1 + \boldsymbol{u}^T (\Delta\widetilde{\boldsymbol{o}} - \boldsymbol{z}) \tag{37}$$

where $\boldsymbol{u}$ is the Lagrange multiplier. The dual problem, obtained minimizing $\mathcal{L}$ w.r.t. $\widetilde{\boldsymbol{o}}$ and $\boldsymbol{z}$, is [48]

$$\begin{aligned} \arg \min_{\boldsymbol{u} \in \mathbb{R}^N} \quad & \frac{1}{2} \left\| \widetilde{\boldsymbol{c}} - \left(\Delta \widehat{G}^{-1} \right)^T \mathbf{u} \right\|_2^2 \\ \text{s.t.} \quad & |\boldsymbol{u}|_\infty \leq \lambda. \end{aligned} \tag{38}$$

We notice that, since $\widehat{G}$ is a lower triangular matrix, it admits an inverse as long as $g_1 \neq 0$. This is satisfied as soon as there is (only) one delay in the effects of the occupancy on the CO_2 levels of the room.

To obtain the structure of the dual solution, consider again the derivative of the Lagrangian with respect to $\boldsymbol{z}$

$$\begin{aligned} \min_{\boldsymbol{z}} \mathcal{L}(\widetilde{\boldsymbol{o}}, \boldsymbol{z}, \boldsymbol{u}) &= \min_{\boldsymbol{z}} (\lambda \|\boldsymbol{z}\|_1 - \boldsymbol{u}^T \boldsymbol{z}) \\ &= \begin{cases} 0, & \text{if } \|\boldsymbol{u}\|_\infty \leq \lambda \\ -\infty, & \text{otherwise} \end{cases}. \end{aligned} \tag{39}$$

Let then $\widehat{\boldsymbol{u}}_\lambda$ be the dual solution and $\widehat{z} = \Delta\widehat{\boldsymbol{o}}_\lambda$ be the primal solution of (36) for a specific λ. Given the computations above, it satisfies

$$\widehat{u}_{\lambda,i} \in \begin{cases} \{+\lambda\} & \text{if } (\Delta\widehat{\boldsymbol{o}})_i > 0 \\ \{-\lambda\} & \text{if } (\Delta\widehat{\boldsymbol{o}})_i < 0 \\ [-\lambda, \lambda] & \text{if } (\Delta\widehat{\boldsymbol{o}})_i = 0. \end{cases} \tag{40}$$

In other words, to maximize (39), the i^{th} element of the dual solution, i.e., $\widehat{u}_{\lambda,i}$, should be equal to either $+\lambda$ if the corresponding element in the primal solution is positive or $-\lambda$ if the corresponding element in the primal solution is negative, see [48]. For those elements of the primal solution with zero values, we can only say that the dual problem must satisfy the condition $|\boldsymbol{u}|_\infty \leq \lambda$.

From (40), one can conclude that $|\widehat{u}_{\lambda,i}| \neq \lambda$, only if $(\Delta\widehat{\boldsymbol{o}})_i = 0$.

ii): Relax problem (38) by removing the ∞-norm constraint. The resulting problem is a unconstrained LS problem, with solution

$$\boldsymbol{u}_{\text{LS}} = \left(\Delta\widehat{G}^{-1}\right)^{T^\dagger} \widetilde{\boldsymbol{c}}. \tag{41}$$

If $\|\boldsymbol{u}_{\text{LS}}\|_\infty < \lambda$ holds, then two facts hold:
1) $\boldsymbol{u}_{\text{LS}}$ is also the solution of problem (38);
2) due to the last implication described in *i)*, $\Delta\widehat{\boldsymbol{o}} = 0$, i.e., the estimated occupancy is a constant signal.

These two facts connect variations in the estimate $\Delta\widehat{\boldsymbol{o}}$ with the measured signal $\widetilde{c}$, considering $V^T := \left(\Delta\widehat{G}^{-1}\right)^\dagger$, since they read as

$$\|V\widetilde{\boldsymbol{c}}\|_\infty < \lambda \quad \Rightarrow \quad \Delta\widehat{\boldsymbol{o}} = 0. \tag{42}$$

To explicit $\widetilde{\boldsymbol{c}}$, consider that the vectorized version of (13), namely

$$\widetilde{\boldsymbol{c}} = \widehat{G}\boldsymbol{o} + \varepsilon \tag{43}$$

with $\varepsilon \in \mathbb{R}^{T_{ts}}$ white and Gaussian innovation, and $\boldsymbol{o}$ the true occupancy signal. Rewriting V as

$$V = \left(\Delta\widehat{G}^{-1}\widehat{G}^{-T}\Delta^T\right)^{-1}\Delta\widehat{G}^{-1} \tag{44}$$

and substituting (44) into (42), we rewrite the latter as

$$\left\|\left(\Delta\widehat{G}^{-1}\widehat{G}^{-T}\Delta^T\right)^{-1}\Delta\widehat{G}^{-1}\left(\widehat{G}\boldsymbol{o} + \varepsilon\right)\right\|_\infty < \lambda, \tag{45}$$

which in turn implies $\Delta\widehat{\boldsymbol{o}} = 0$. As can be seen, (45) relates conditions on the true occupancy $\boldsymbol{o}$ and the innovation process ε with conditions on the final estimate $\widehat{\boldsymbol{o}}$.

iii): We now analyze the case when the true occupancy is constant ($\Delta\boldsymbol{o} = \boldsymbol{0}$). In this case, condition (45) reads as

$$\|V\varepsilon\|_\infty < \lambda \quad \Rightarrow \quad \Delta\widehat{\boldsymbol{o}} = 0 \tag{46}$$

that is equivalent to

$$\{|\langle V_i, \varepsilon\rangle|^2 < \lambda^2\}_{i=1,\ldots,N} \quad \Rightarrow \quad \Delta\widehat{\boldsymbol{o}} = 0. \tag{47}$$

The Cauchy–Schwarz inequality yields $\langle V_i, \varepsilon\rangle|^2 \leq \|V_i\|^2\|\varepsilon\|^2$. Letting $\|V_m\|^2 := \max_i \|V_i\|^2$, the sufficient condition for (46) becomes

$$\|V_m\|^2\|\varepsilon\|^2 < \lambda^2 \quad \Rightarrow \quad \Delta\widehat{\boldsymbol{o}} = 0. \tag{48}$$

In (48), V_m is known, while ε is white Gaussian noise: thanks to the PEM paradigm, $\varepsilon_i \sim \mathcal{N}(0, \sigma_\varepsilon^2)$, with σ_ε^2 estimated during the system identification phase. It thus follows that:

$$\left\|\frac{\varepsilon}{\sigma_\varepsilon}\right\|^2 = \sum_{k=1}^{T_{ts}}\left(\frac{\varepsilon(k)}{\sigma_\varepsilon}\right)^2 \sim \chi^2(N) \tag{49}$$

where $\chi^2(N)$ is a Chi-squared distribution with N degrees of freedom. Thus, with the probability of at least α, $\|\varepsilon\|^2$ will have the following upper bound

$$\|\varepsilon\|^2 \leq (\sigma_\varepsilon)^2\chi_\alpha^{-1}(N) \tag{50}$$

where $\chi_\alpha^{-1}(N)$ is the inverse of the Chi-square cdf with N degrees of freedom for the corresponding probability α. Substituting (50) into (48), we get the statement of the proposition.

Proof of Proposition 5:

In this case, we impose another constraint on the optimization problem (19) by setting the first element in the occupancy signal to its true value. Using the same approach as in the proof of the Proposition 4, we will have (36) subject to $\widetilde{\boldsymbol{o}}(1) = o(k-N)$, where $o(k-N)$ is the true value of the occupancy signal at time $k-N$. Substituting the new constraint $\widetilde{\boldsymbol{o}}(1) = o(k-N)$ into the cost function, one can rewrite (36) as

$$\begin{aligned} \arg \min_{\substack{\bar{\boldsymbol{o}} \in \mathbb{R}_+^N \\ \boldsymbol{z} \in \mathbb{R}^{N-1}}} \quad & \frac{1}{2}\|\bar{\boldsymbol{c}} - \bar{H}\bar{\boldsymbol{o}}\|_2^2 + \lambda\|\boldsymbol{z} - \bar{\boldsymbol{o}}^*\|_1 \\ \text{s.t.} \quad & \boldsymbol{z} = \bar{\Delta}\bar{\boldsymbol{o}} \end{aligned} \tag{51}$$

where

$$\bar{\boldsymbol{o}}^* := [o(k-N)\ 0\ \cdots\ 0]^T \in \mathbb{R}^{N-1\times N-1}.$$

Using the same approach as the previous proof the dual problem for (51) can be written as

$$\begin{aligned} \arg \min_{\boldsymbol{u} \in \mathbb{R}^N} \quad & \frac{1}{2}\left\|\bar{\boldsymbol{c}} - (\bar{\Delta}\bar{H}^{-1})^T\boldsymbol{u}\right\|_2^2 \\ \text{s.t.} \quad & |\boldsymbol{u}|_\infty \leq \lambda \end{aligned} \tag{52}$$

where the Lagrange multipliers satisfy

$$\widehat{u}_{\lambda,i} \in \begin{cases} \{+\lambda\} & \text{if } (\bar{\Delta}\bar{\boldsymbol{o}} - \bar{\boldsymbol{o}}^*)_i > 0 \\ \{-\lambda\} & \text{if } (\bar{\Delta}\bar{\boldsymbol{o}} - \bar{\boldsymbol{o}}^*)_i < 0 \\ [-\lambda, \lambda] & \text{if } (\bar{\Delta}\bar{\boldsymbol{o}} - \bar{\boldsymbol{o}}^*)_i = 0. \end{cases} \tag{53}$$

Notice that $\bar{\Delta}$ is invertible and thus the condition (45) for this case reads as

$$\left\|(\bar{V}\bar{V}^T)^1 o^* + (\bar{V}\bar{V}^T)^k \pm \bar{V}\varepsilon\right\|_\infty < \lambda \Rightarrow \bar{\Delta}\bar{\boldsymbol{o}} - \bar{\boldsymbol{o}}^* = 0 \tag{54}$$

where $(\bar{V}\bar{V}^T)^k$ is the k-th column of $\bar{V}\bar{V}^T$ and $\bar{V} = (\bar{H}\bar{\Delta}^{-1})^T$. Notice that this is a upper triangular Toeplitz matrix, satisfying (letting $\bar{V}_j$ be the jth row of $\bar{V}$)

$$\|\bar{V}_1\|^2 \geq \|\bar{V}_2\|^2 \geq \cdots \geq \|\bar{V}_N\|^2. \tag{55}$$

This implication refers to the case where the estimator commits the error of missing the change in the occupancy signal at time k.

The ∞-norm above can be expanded, as before, to obtain the component-wise equivalent condition

$$\{|\langle \bar{V}_i, \bar{V}_1\rangle o^* + \langle \bar{V}_i, \bar{V}_k\rangle \pm \langle \bar{V}_i, \varepsilon\rangle| < \lambda\}_{i=1,\ldots,N} \Rightarrow \bar{\Delta}\bar{\boldsymbol{o}} = \bar{\boldsymbol{o}}^* \tag{56}$$

or using the bilinearity of inner products

$$\{|\langle \bar{V}_i, \bar{V}_1 o(k-N) + \bar{V}_k \pm \varepsilon\rangle| < \lambda\}_{i=1,\ldots,N} \quad \Rightarrow \quad \bar{\Delta}\bar{\boldsymbol{o}} = \bar{\boldsymbol{o}}^*. \tag{57}$$

Cascading now Cauchy–Schwarz and triangular inequalities with (55) and (57) it is possible to derive the sufficient condition

$$\|\bar{V}_1\|^2(\|\bar{V}_1\|^2 o(k-N)^2 + \|\bar{V}_1\|^2 + \|\varepsilon\|^2) < \lambda^2 \Rightarrow \bar{\Delta}\bar{\boldsymbol{o}} = \bar{\boldsymbol{o}}^* \tag{58}$$

or equivalently

$$\left\|\frac{\varepsilon}{\sigma_\varepsilon}\right\|^2 < \frac{\lambda^2 - (1 + o(k-N)^2)\|\bar{V}_1\|^4}{\sigma_\varepsilon^2\|\bar{V}_1\|^2} \quad \Rightarrow \quad \bar{\Delta}\bar{\boldsymbol{o}} = \bar{\boldsymbol{o}}^*. \tag{59}$$

Same considerations as in the previous case thus follow and (59) can be rewritten as

$$\lambda^2 > \sigma_\varepsilon^2 \chi_\alpha^{-1}(N)\|\bar{V}_1\|^2 + (1 + o(k-N)^2)\|\bar{V}_1\|^4. \tag{60}$$

REFERENCES

[1] F. Oldewurtel, D. Sturzenegger, and M. Morari, "Importance of occupancy information for building climate control," *Appl. Energy*, vol. 101, pp. 521–532, 2013.

[2] O. Guerra Santin, L. Itard, and H. Visscher, "The effect of occupancy and building characteristics on energy use for space and water heating in Dutch residential stock," *Energy Build.*, vol. 41, no. 11, pp. 1223–1232, Nov. 2009.

[3] T. A. Nguyen and M. Aiello, "Energy intelligent buildings based on user activity: A survey," *Energy Build.*, vol. 56, pp. 244–257, 2013.

[4] V. L. Erickson, M. Á. Carreira-Perpiñán, and A. E. Cerpa, "OBSERVE: Occupancy-based system for efficient reduction of HVAC energy," in *Proc. 10th Int. Conf. Inform. Process. Sens. Netw. (IPSN)*, 2011, pp. 258–269.

[5] V. L. Erickson, S. Achleitner, and A. E. Cerpa, "POEM: Power-efficient occupancy-based energy management system," in *Proc. ACM/IEEE Int. Conf. Inform. Process. Sens. Netw. (IPSN)* , 2013, pp. 203–216.

[6] J. Brooks, S. Goyal, R. Subramany, Y. Lin, T. Middelkoop, L. Arpan, L. Carloni, and P. Barooah, "An experimental investigation of occupancy-based control of commercial building climate," in *Proc. IEEE Conf. Decision Control*, 2014, pp. 5680–5685.

[7] Z. Han, R. Gao, and Z. Fan, "Occupancy and indoor environment quality sensing for smart buildings," in *Proc. Instrument. Meas. Technol. Conf.*, 2012, pp. 882–887.

[8] A. Kamthe, L. Jiang, M. Dudys, and A. Cerpa, "Scopes: Smart cameras object position estimation system," in *Wireless Sensor Networks*. New York, NY, USA: Springer, 2009, pp. 279–295.

[9] N. Li, G. Calis, and B. Becerik-Gerber, "Measuring and monitoring occupancy with an RFID based system for demand-driven HVAC operations," *Autom. Construction*, vol. 24, pp. 89–99, 2012.

[10] H. Wang, Q.-S. Jia, Y. Lei, Q. Zhao, and X. Guan, "Estimation of occupant distribution by detecting the entrance and leaving events of zones in building," in *Proc. IEEE Int. Conf. Multisen. Fusion Integr. Intell. Syst.*, 2012, pp. 27–32.

[11] S. Meyn, A. Surana, Y. Lin, S. M. Oggianu, S. Narayanan, and T. A. Frewen, "A sensor-utility-network method for estimation of occupancy in buildings," in *Proc. IEEE Conf. Decision Control*, 2009, pp. 1494–1500.

[12] T. Labeodan, W. Zeiler, G. Boxem, and Y. Zhao, "Occupancy measurement in commercial office buildings for demand-driven control applications—a survey and detection system evaluation," *Energy Build.*, vol. 93, pp. 303–3014, 2015.

[13] Y. Agarwal, B. Balaji, S. Dutta, R. K. Gupta, and T. Weng, "Duty-cycling buildings aggressively: The next frontier," in *Proc. IPSN HVAC Control*, 2011, pp. 246–257.

[14] T. Leephakpreeda, R. Thitipatanapong, T. Grittiyachot, and V. Yungchareon, "Occupancy-based control of indoor air ventilation: A theoretical and experimental study," *Sci. Asia*, vol. 27, no. 4, pp. 279–284, 2001.

[15] F. Scotton, L. Huang, S. A. Ahmadi, and B. Wahlberg, "Physics-based modeling and identification for HVAC systems," in *Proc. Eur. Control Conf.*, 2013, pp. 7404–7409.

[16] S. Wang, J. Burnett, and H. Chong, "Experimental validation of CO2-based occupancy detection for demand-controlled ventilation," *Indoor Built Environ.*, vol. 8, no. 6, pp. 377–391, Nov. 1999.

[17] S. Kar and P. K. Varshney, "Accurate estimation of indoor occupancy using gas sensors," in *Proc. Int. Conf. Intell. Sens., Sens. Netw. Inform. Process.*, 2009, pp. 355–360.

[18] D. Calì, P. Matthes, K. Huchtemann, R. Streblow, and D. Müller, "CO2 based occupancy detection algorithm: Experimental analysis and validation for office and residential buildings," *Build. Environ.*, vol. 86, pp. 39–49, 2015.

[19] M. Gruber, A. Trüschel, and J.-O. Dalenbäck, "CO2 sensors for occupancy estimations: Potential in building automation applications," *Energy Build.*, vol. 84, pp. 548–556, 2014.

[20] G. Ansanay-Alex, "Estimating occupancy using indoor carbon dioxide concentrations only in an office building: A method and qualitative assessment," in *Proc. REHVA World Cong. Energy Efficient, Smart and Healthy Build. (CLIMA)*, 2013, pp. 1–8.

[21] B. Dong, B. Andrews, K. P. Lam, M. Höynck, R. Zhang, Y.-S. Chiou, and D. Benitez, "An information technology enabled sustainability test-bed (itest) for occupancy detection through an environmental sensing network," *Energy Build.*, vol. 42, no. 7, pp. 1038–1046, 2010.

[22] K. P. Lam, M. Höynck, B. Dong, B. Andrews, Y.-s. Chiou, D. Benitez, and J. Choi, "Occupancy detection through an extensive environmental sensor network in an open-plan office building," in *Proc. IBPSA Conf.*, 2009, pp. 1452–1459.

[23] B. Dong, B. Andrews, K. P. Lam, M. Höynck, R. Zhang, Y.-S. Chiou, and D. Benitez, "An information technology enabled sustainability test-bed (ITEST) for occupancy detection through an environmental sensing network," *Energy Build.*, vol. 42, no. 7, pp. 1038–1046, 2010.

[24] Z. Han, R. X. Gao, and Z. Fan, "Occupancy and indoor environment quality sensing for smart buildings," in *Proc. IEEE Int. Instrument. Meas. Technol. Conf. (MTC'12)*, 2012, pp. 882–887.

[25] B. Ai, Z. Fan, and R. X. Gao, "Occupancy estimation for smart buildings by an auto-regressive hidden Markov model," in *Proc. Amer. Control Conf. (ACC'14)*, 2014, pp. 2234–2239.

[26] W. Kleiminger, C. Beckel, T. Staake, and S. Santini, "Occupancy detection from electricity consumption data," in *Proc. 5th ACM Workshop Embedded Syst. For Energy-Efficient Buildings (BuildSys'13)*, 2013, pp. 1–8.

[27] A. Ebadat, G. Bottegal, D. Varagnolo, B. Wahlberg, and K. H. Johansson, "Estimation of building occupancy levels through environmental signals deconvolution," in *Proc. ACM Workshop Embedded Syst. Energy-Efficient Build.*, 2013.

[28] R. Tibshirani and M. Saunders, "Sparsity and smoothness via the fused lasso," *J. Royal Stat. Soc.: Series B (Statistical Methodology)*, vol. 67, no. 1, pp. 91–108, 2005.

[29] G. Mustafaraj, J. Chen, and G. Lowry, "Development of room temperature and relative humidity linear parametric models for an open office using BMS data," *Energy Build.*, vol. 42, no. 3, pp. 348–356, 2010.

[30] D. Loveday and C. Craggs, "Stochastic modelling of temperatures for a full-scale occupied building zone subject to natural random influences," *Appl. Energy*, vol. 45, no. 4, pp. 295–312, 1993.

[31] G. Lowry and M.-W. Lee, "Modelling the passive thermal response of a building using sparse BMS data," *Appl. Energy*, vol. 78, no. 1, pp. 53–62, 2004.

[32] J. C.-M. Yiu and S. Wang, "Multiple ARMAX modeling scheme for forecasting air conditioning system performance," *Energy Conversion Manage.*, vol. 48, pp. 2276–2285, 2007.
[33] S. Wu and J.-Q. Sun, "A physics-based linear parametric model of room temperature in office buildings," *Build. Environ.*, vol. 50, pp. 1–9, 2012.
[34] G. Wahba, *Spline Models for Observational Data*. Philadelphia,, PA, USA: SIAM, 1990, vol. 59.
[35] G. Pillonetto and G. De Nicolao, "A new kernel-based approach for linear system identification," *Automatica*, vol. 46, no. 1, pp. 81–93, 2010.
[36] G. Bottegal and G. Pillonetto, "Regularized spectrum estimation using stable spline kernels," *Automatica*, vol. 49, no. 11, pp. 3199–3209, 2013.
[37] G. Pillonetto, F. Dinuzzo, T. Chen, G. De Nicolao, and L. Ljung, "Kernel methods in system identification, machine learning and function estimation: A survey," *Automatica*, vol. 50, no. 3, pp. 657–682, 2014.
[38] L. Ljung, *System Identification—Theory for the User*, 2nd ed. Englewood Cliffs, NJ, USA: Prentice-Hall, 1999.
[39] G. Pillonetto, A. Chiuso, and G. De Nicolao, "Prediction error identification of linear systems: A nonparametric Gaussian regression approach," *Automatica*, vol. 47, no. 2, pp. 291–305, 2011.
[40] T. Chen, H. Ohlsson, and L. Ljung, "On the estimation of transfer functions, regularizations and Gaussian processes—revisited," *Automatica*, vol. 48, no. 8, pp. 1525–1535, 2012.
[41] B. D. Anderson and J. B. Moore, *Optimal Filtering*. Englewood Cliffs, NJ, USA: Prentice-Hall, 1979.
[42] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge, U.K.: Cambridge Univ. Press, 2004.
[43] G. Davis, S. Mallat, and M. Avellaneda, "Adaptive greedy approximations," *Constructive Approx.*, vol. 13, pp. 57–98, 1997.
[44] T. Hastie, R. Tibshirani, and J. Friedman, *The Elements of Statistical Learning: Data Mining, Inference, and Prediction*, 2nd ed. New York, NY, USA: Springer, 2001.
[45] T. Park and G. Casella, "The Bayesian lasso," *J. Amer. Statist. Assoc.*, vol. 103, no. 482, pp. 681–686, 2008.
[46] B. Schölkopf and A. J. Smola, *Learning With Kernels: Support Vector Machines, Regularization, Optimization, and Beyond*. Cambridge, MA, USA: MIT Press, 2002.
[47] S. M. Kay, *Fundamentals of Statistical Signal Processing: Estimation Theory*. Englewood Cliffs, NJ, USA: Prentice-Hall, 1993.
[48] R. J. Tibshirani and J. Taylor, "The solution path of the generalized lasso," *Ann. Statist.*, vol. 39, no. 3, pp. 1335–1371, 2011.

Afrooz Ebadat received the B.Sc. and M.Sc. degrees in electrical engineering from Shiraz University, Shiraz, Iran, in 2008 and 2011, respectively. She is currently working towards the Ph.D. degree at the Department of Automatic Control, KTH Royal Institute of Technology, Stockholm, Sweden.

Her research interests include identification and control of HVAC systems, experiment design for system identification, identification for control and model predictive control.

Giulio Bottegal received the M.S. (*cum laude*) degree in automation engineering and the Ph.D. degree in information engineering from the University of Padova, Padova, Italy, in 2009 and 2013, respectively.

Currently, he is a Postdoctoral Research Fellow with the School of Electrical Engineering, KTH Royal Institute of Technology, Stockholm, Sweden. His research interests are in the area of system identification.

Damiano Varagnolo received the M.S. degree in automation engineering and the Ph.D. degree in information engineering from the University of Padova, Padova, Italy, in 2005 and 2011, respectively.

He was a Research Engineer at Tecnogamma S.p.A., Treviso, Italy, during 2006–2007. He visited UC Berkeley as a Researcher Scholar in 2010. From 2012 to 2013, he was a Postdoctoral Scholar at KTH, Royal Institute of Technology, Stockholm. Currently, he is Associate Senior Lecturer at LTU, Luleå University of Technology. His interests include distributed optimization, distributed estimation, identification and control of HVAC systems.

Bo Wahlberg (S'85–M'87–SM'02–F'07) was born in Norrköpingg, Sweden, in 1959. He received the M.Sc. degree in electrical engineering and the Ph.D. degree from Linköping University, Linköping, Sweden, in 1983 and 1987, respectively.

In 1991, he was a Professor of the Chair of Automatic Control at KTH Royal Institute of Technology, Stockholm, Sweden. He was a Visiting Professor at the Department of Electrical Engineering, Stanford University, Stanford, CA, USA, from August 1997 to July 1998 and August 2009 to June 2010, and Vice President of KTH from 1999 to 2001. He is a Co-Founder of the Centre of Autonomous Systems and the Linnaeus Center ACCESS on Networked Systems at KTH. He is the KTH Director and co-PI for the Wallenberg Autonomous Systems Program. He is a Fellow of the IEEE for his contributions to system identification using orthonormal basis functions. His research interests include modeling, system identification, estimation and control of industrial processes.

Karl H. Johansson (S'92–M'98–SM'08–F'13) received the M.Sc. and Ph.D. degrees in electrical engineering from Lund University, Lund, Sweden.

He is Director of the ACCESS Linnaeus Centre and Professor at the School of Electrical Engineering, KTH Royal Institute of Technology, Stockholm, Sweden. He is a Wallenberg Scholar and has held a Senior Researcher Position with the Swedish Research Council. He also heads the Stockholm Strategic Research Area ICT The Next Generation. He has held visiting positions at UC Berkeley, the California Institute of Technology, Nanyang Technological University, and the Institute of Advanced Studies, Hong Kong University of Science and Technology. His research interests are in networked control systems, cyber-physical systems, and applications in transportation, energy, and automation systems.

Prof. Johansson has been a member of the IEEE Control Systems Society Board of Governors and the Chair of the IFAC Technical Committee on Networked Systems. He has been on the Editorial Boards of several journals, including *Automatica*, the IEEE TRANSACTIONS ON AUTOMATIC CONTROL, and *IET Control Theory and Applications*. He is currently a Senior Editor of the IEEE TRANSACTIONS ON CONTROL OF NETWORK SYSTEMS and Associate Editor of the *European Journal of Control*. He has been Guest Editor for a Special Issue of the IEEE TRANSACTIONS ON AUTOMATIC CONTROL on cyber-physical systems and the *IEEE Control Systems Magazine* on cyber-physical security. He was the General Chair of the ACM/IEEE Cyber-Physical Systems Week 2010 in Stockholm and IPC Chair of many conferences. He has served on the Executive Committees of several European research projects in the area of networked embedded systems. He received the Best Paper Award at the IEEE International Conference on Mobile Ad-Hoc and Sensor Systems in 2009 and the Best Theory Paper Award at the World Congress on Intelligent Control and Automation in 2014. In 2009, he was awarded Wallenberg Scholar, as one of the first ten scholars from all sciences, by the Knut and Alice Wallenberg Foundation. He was awarded Future Research Leader from the Swedish Foundation for Strategic Research in 2005. He received the triennial Young Author Prize from IFAC in 1996 and the Peccei Award from the International Institute of System Analysis, Austria, in 1993. He received Young Researcher Awards from Scania in 1996 and from Ericsson in 1998 and 1999. He is a Fellow of the IEEE.