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Integrated Model-Based Control Allocation Strategies Oriented to Predictive Maintenance of Saturated Actuators

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***Abstract*— Predictive Maintenance approaches are gaining popularity in the new Industry 4.0 paradigm as they offer superior benefits in terms of time and money savings when it is required to assess the current working capabilities of operating equipment to carefully schedule maintenance operations. This work deals with a control allocation strategy inspired by Model Predictive Control ideas and able to address the loss of effectiveness of actuating equipment arising from their continuous usage. The scheme here presented comprises two modules: a prognostic unit for monitoring the reliability conditions of the actuators and a re-configurable control allocation block that operates according to the deterioration degree of the present actuators. The benefits of the proposed approach are testified by the numerical simulations carried out on both an unstable system and a tanks network. In particular, it can be observed that the proposed method is capable of suggesting a time-window for maintenance interventions that prevents either stability or feasibility issues.**

***Note to Practitioners*—Nowadays, in the viewpoint of financial and technical issues in the industries, the predictive maintenance (PM) plays the main and important subject. In the variety of industries, such as oil, gas, petrochemical, power plant, or transmission and distribution infrastructures, which are using the actuators and pipelines, predictive maintenance (PM) is a critical topic to prevent the unwanted shutdown and unrequired maintenance costs. However, most PM research and solutions have paid attention mainly to the estimation of the Remaining Useful Life (RUL) of critical devices (i.e., pipelines) only. On the contrary, in this work we focus on the possibility of influencing the RUL by properly acting on the actuators' effort. The proposed method consists in a dual-mode fault-tolerant control allocation technique developed for discrete-time systems subject to input saturation. The proposed predictive model-based control solution can be considered as a simple, structural and practical approach to minimize the maintenance cost and reliability by decreasing the number of maintenance interventions and optimizing the time for their repairing, by preventing or decreasing the probability of occurrence of unwanted failures events. The main limitation in this approach is that it assumes the RUL of the critical assets under investigation known. As a future research, this work will be extended to include a related RUL estimator hinging on online data.**

***Index Terms*— Predictive maintenance, fault-tolerant systems, control allocation methods, model predictive control, prognostics and health management, virtual actuators, industry 4.0.**

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I. Introduction

THE increasing penetration of new technologies into modern production systems turned the industry into a new era. Manufacturing, Power Grids, Healthy, Smart Buildings, Agriculture, Transportation, and Computer Networks are just a few of the fields involved in this process that gave rise to the so-called Industry 4.0 ([1], [2]). Adopting this new paradigm has resulted in novel challenges and solutions that can increase the production quality while decreasing system uncertainty and costs.

A. Literature Review

Over the past two decades, significant efforts have been focused on maintaining the integrity of industrial systems, which are becoming increasingly complex. One approach that has gained popularity in this context is the implementation of Predictive Maintenance (PM) [4].

PM exploits several tools and techniques based on either physical models [7] or data-driven approaches [5], [6] to monitor the condition of industrial equipment and anticipate imminent failures in order to take smart decisions regarding maintenance activities. Typically, these activities are performed at fixed intervals, which can result in either increased costs or serious component failures if repairs are conducted either prematurely or with delay. For these reasons, this paper addresses the concept of *just-in-time* PM, which refers to the capability to schedule asset repairs at the precise moment they are required.

The development of an efficient supervisory and fault diagnostic approach to improving the fault-tolerant and resilient performance of industrial closed-loop control systems is of special importance in this context.

To this end, several degrees of performance need to be taken into account in various failure conditions depending on the types and severity of defects that could impact the system. When moving from a safe to a faulty operating scenario, degraded performance can be usually accepted.

In view of this, to guarantee the tracking ability of input commands or reference model/trajectory by a closed-loop system in the cases of the undesired event, the system is usually provided with a redundant number of actuators that can be orchestrated in a smart way to maintain safety and adequate performance [13].

This aspect has motivated several research activities in prognostics and health monitoring (PHM) for actuating devices [8], [9]. Particular attention has been devoted to achieving an estimation procedure to assess the actuators' healthy status quantified by the so-called remaining useful lifetime (RUL) [10], [11]. Within maintenance plans, the RUL is usually considered as an indicator to establish timely interventions aimed at preventing complete breakdowns [15].

PHM involves three main operating axes: *observation, analysis, and action* [12]. In this context, PM is usually concerned only with the first two axes and it is aimed at predicting possible failures while the third axis is related to the actual maintenance operations. From a control perspective, PM involves also the *action* axis to address any degradation in actuators. This is achieved by utilizing information such as RUL or other PHM data to balance control efforts and compensate for degradation [8], [16]. In this way, some working conditions leading to precipitous failures are corrected in advance to increase as much as possible the RUL of the equipment and postpone repairing intervention. This *modus operandi* is referred to in the literature as Proactive Maintenance [17]. In this paper, this is achieved by re-configuring the control operations in order to extend actuators' availability until the next maintenance intervention while preserving the stability of the system.

In a more conventional setting, where no maintenance concepts are taken into account, two main approaches have been developed in the literature to provide closed-loop systems with some control reconfiguration capabilities in presence of redundancy of actuators. The first approach is the well-known Fault Tolerant Control (FTC) [18] and relies on the design of a specific control law capable of online redistributing the control effort by using the reachable redundant actuators. The second one is Control Allocation (CA) [19], [20] and foresees adding an *allocation unit* between the existing controller and a plant that is in charge of allocating the control signal to physical actuators. A graphic idea of such a method can be observed in Fig. 1. There, the controller has been implemented by considering the system fed by no-redundant virtual actuators with minimal numbers. Such a controller outputs a virtual control signal $v(t)$ that is allocated by the allocation block among the physical actuators $u(t)$ based on their currently healthy level.

B. Main Contribution

There are various contributions aimed at enabling the development of enhanced predictive maintenance plans by directly utilizing information from PHM [13], [14]. However, within the context of Control Allocation (CA), this concept has received limited attention and lacks in-depth investigation. Building upon these premises, this paper introduces an MPC (Model Predictive Control) method with CA capabilities to address the decline in actuator effectiveness caused by gradual degradation. Typically, MPC-based control methods optimize an open-loop control objective over a future time horizon at each control step to compute control inputs. As a result, MPC is widely acknowledged as an effective approach for reconfiguration and fault-tolerant control [21]. Numerous contributions in this domain have been proposed in the literature, as evident in [22], [23], [24], [25], and [26].

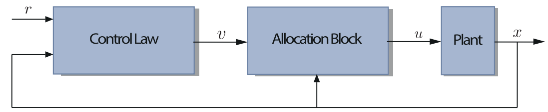

Fig. 1. Control system structure including control allocation remotely connected to maintenance services.

It is important to note that this study does not explicitly aim to introduce a new MPC approach. To be more precise, we employ MPC concepts to devise a novel CA method based on the health status of actuators. While a similar alternative use of MPC for a specific class of vehicles was presented in [27], this work generalizes and adapts it to address the principles of predictive maintenance within a broader context. Specifically, we assume that actuators can be affected by two potential conditions leading to a loss of effectiveness. In the first condition, the degradation effect directly correlates with the control effort, whereas the second scenario involves a deterioration factor influenced by exogenous phenomena affecting the actuator channels.

To design the proposed control allocation scheme based on the behavior of actuator degradation, a dual-mode approach is adopted. In the "nominal" mode, allocation operations are guided by the solution of an optimization problem, provided that the actuators demonstrate an acceptable level of efficiency. Conversely, in the "maintenance" mode, as some actuators approach the brink of failure, a strategy is employed to ensure system stability and the viability of the MPC allocation unit until maintenance can be performed. This involves guiding the system states toward a safety equilibrium area by leveraging the remaining capabilities of the actuators.

It is demonstrated that the proposed MPC allocation scheme, when applied under typical operating conditions, exhibits recursive feasibility, guaranteeing the availability of an acceptable physical input at each time instant and achieving asymptotic convergence of the system state toward a specified admissible set-point. Furthermore, to facilitate an effective just-in-time maintenance policy and reduce maintenance activities, the scheme is capable of triggering maintenance operations within a guaranteed minimum time-to-failure threshold.

A preliminary version of this work was presented in [32], where the aforementioned strategy was applied within the practical context of addressing constrained LFC (Load Frequency Control) control challenges in multi-area power grids. This paper adds significant value in the following ways:

1) It introduces the problem to be solved in a more formal and comprehensive manner and generalizes it to consider not only actuators that degrade due to control effort but also those influenced by uncontrollable exogenous factors.

2) It provides formal proofs for all theoretical results contained in Lemmas and Propositions, presenting them in a more comprehensive manner.
3) The simulation section is enriched with additional more realistic examples involving a tank system and a continuous stirred tank reactor, further enhancing the practical applicability of the proposed approach.

C. Paper Outline

The rest of the paper is organized as follows; the problem formulation is addressed in Section II. In Section III the proposed control allocation scheme is described while its main properties are illustrated in Section IV. In Section V, two challenging examples are presented for assessing the proposed method's pros. Eventually, some conclusions end the paper.

PRELIMINARIES

The notations 0_q and 1_q are entirely zero and unity vectors with the dimension q and q, respectively. The element-wise k-power of vector $x \in \mathbf{R}^n$, is defined as $x^k := [x_1^k, \ldots, x_n^k]^T$. The weighted 2-norm of vector $x \in \mathbf{R}^n$, is defined as $\|x\|_P^2 := x^T P x$ where $P = P^T \in \mathbf{R}^{n\times n}$ is a symmetric positive definite matrix.

II. PROBLEM FORMULATION

A. Plant Modeling and Control Law

The given discrete state-space realization of the dynamic system is as follows

$$x(t+1) = Ax(t) + B(\rho(t))u(t) \tag{1}$$

where the vector $x(t) \in \mathbf{R}^n$ represents the system state. The vector $u(t) \in \mathbf{R}^m$ the command input delivered via $|\mathcal{A}| = m$ physical actuators, where $\mathcal{A} := \{1, \ldots m\}$.

Moreover A is the dynamical matrix, while $B(\cdot)$ is the parameter-varying input matrix depending on the vector $\rho(t) = [\rho_1(t), \ldots, \rho_m(t)] \in \mathbf{R}^m$ whose components denote the degradation status of the i-th actuator in $\mathcal{A}$ and directly depend on the magnitude of the physical input command $u(t)$ (control effort).

Moreover, there could exist degradation/defects on the plant equipment not depending directly on $u(t)$, characterized by some exogenous variable $\theta(t)$ (e.g. the corrosion in the pipelines of a tank system) that however impose limitations on the actuators usage (e.g. pipeline failure pressure, etc). Such defects can be considered in our approach by assuming parameter-dependent (possibly time-varying) input saturation constraints on the control input. Therefore, it is hereafter assumed:

Assumption 1: The control input is subject to the following parameter-dependent (possibly time-varying) saturation constraints

$$\begin{aligned} &u(t) \in \mathcal{U}_\theta, \forall t \geq 0, \\ &\mathcal{U}_\theta := \{u \in \mathbf{R}^m \, -\theta\bar{u} \leq u \leq \theta\bar{u}\} \end{aligned} \tag{2}$$

where $\bar{u} \in \mathbf{R}^m$ is a vector of constant positive components and $\theta := \mathrm{diag}\{\theta_1, \theta_2, \ldots, \theta_m\}$ is composed by scalars $\theta_i \in [0, 1]$ that quantify the *input-independent* loss of actuator effectiveness not related to the input commands.

In particular, $\theta_i(t)$ is assumed to be a monotonic rate-bounded decreasing function, i.e. $\theta_i(0) = 1$, $\lim_{t\to\infty} \theta_i(t) = 0$ and $-\theta_0 \leq \theta_i(t+1) - \theta_i(t) \leq 0$ with $\theta_0 > 0$. □

The latter assumption is oriented to model the loss of actuator effectiveness not related to the input commands. For instance, this aspect often involves pipelines affected by corrosion phenomena (see [33]). Corrosion can restrict the flow of fluids through pipelines, leading to increased friction and pressure losses. This reduces the overall efficiency of the system, requiring higher energy inputs to maintain the desired flow rates. In the case of oil and gas pipelines, this can result in decreased throughput and revenue loss.

Please notice that θ belongs to the generic polytopic set $\Theta \subseteq \{\theta : 0_m \leq \theta \leq 1_m\}$.

Assumption 2: The matrix $B(\rho(t))$ is column-rank deficient where $Rank(B(\rho)) = l < m, \forall \rho \in \mathbf{R}^m$. □

Such an assumption refers to a property typically enjoyed by over-actuated plants and guarantees that, thanks to actuator redundancy, equation (4) has always a solution in $u(t)$. Examples of these systems are over-actuated vehicles, robot manipulators, industrial processes.

Given assumptions 1 and 2, plant (1) can be represented as follows

$$x(t+1) = Ax(t) + B_v v(t) \tag{3}$$
$$B_v v(t) = B(\rho(t))u(t) \tag{4}$$

where $v(t) \in \mathbf{R}^l$ and $B_v \in \mathbf{R}^{n\times l}$ are the *virtual control* input and full column-rank matrix, respectively. More in details, equation (3) will be considered as the representation of the *virtual plant* while (4) describes the formal link that relates the physical and virtual inputs and is here denoted as the *parity equation* of the system. More precisely, the virtual control input $v(t)$ stands for the required control effort, supposedly provided by a controller designed on the virtual plant, to be applied to the plant via the physical actuators by finding a suitable solution to the parity equation (4). In particular, it is assumed that $v(t)$ is provided by the following controller

$$v(t) = K(x(t) - x_r) + v_r \tag{5}$$

where K is a gain determined with the aim of making $(A + B_v K)$ a Schur matrix, r is a desired reference with (x_r, v_r) its steady-state pair, i.e.

$$\begin{bmatrix} A - I_n & B_v \\ I_n & 0_{n,l} \end{bmatrix} \begin{bmatrix} x_r \\ v_r \end{bmatrix} = \begin{bmatrix} 0_{n,1} \\ r \end{bmatrix} \tag{6}$$

B. Faulty Mode and Maintenance Policy

We assume that the $i-th$ actuator's degradation would be increased whenever non-zero control signals are applied in the way modeled by the following law ([25])

$$\rho_i(t+1) = \rho_i(t) + \alpha_i |u_i(t)|, \ i \in \mathcal{A} \tag{7}$$

that, in a vector form, becomes

$$\rho(t+1) = \rho(t) + diag(\alpha_1, \ldots, \alpha_m)|u(t)| \tag{8}$$

The impact of $\rho_i(t)$ on the actuator effectiveness is assumed to be governed by the following rule

$$B_i(\rho_i) = B_u^i \delta_i(\rho_i) \tag{9}$$

where B_u^i stands for the nominal input column while the continuous coefficient $\delta_i : \mathbf{R} \rightarrow [0, 1]$ represents the *input-dependent* loss of effectiveness scalar, which is directly related to the signal ρ_i. In particular, $\delta_i(\rho_i)$ belongs to the class of monotonic decreasing functions, i.e. $\delta_i(0) = 1$ and $\lim_{\rho\rightarrow\infty} \delta_i(\rho_i) = 0$

Moving from these considerations, system (1), can be recast in following form

$$x(t+1) = Ax(t) + B_u\Delta(\rho(t))u(t) \tag{10}$$

where $\Delta := diag(\delta_1, \ldots, \delta_m)$ and $B_u := [B_u^1|\ldots|B_u^m]$. The latter formulation allows to recast the parity equation of (4) as follows

$$B_v v(t) = B_u\Delta(\rho(t))u(t) \tag{11}$$

From (10) it is quite trivial to see that when actuators are not affected by degradation phenomena, it could be considered the $\delta = 1_m$ and $\Delta(\rho) = I_m$. Nevertheless, for more generality, the $\Delta(\rho) \neq I_m$ by the reason that $\delta := [\delta_1, \ldots, \delta_m]$ belongs to a generic polytopic set

$$\Gamma \subseteq \{\delta : 0_m \leq \delta_i \leq 1_m\} \tag{12}$$

such that the next assumption is satisfied.

Assumption 3: Given system (10), its admissible equilibrium set

$$\mathcal{X}_f := \{x \in \mathbf{R}^n : \forall(\delta,\theta) \in \Gamma\times\Theta, \exists u \in \mathcal{U}_\theta | (I-A)x = B_u\Delta u\} \tag{13}$$

is such that $\mathcal{X}_f \supset \{0_n\}$. □

The latter basically prescribes that the state of the plant (10) can be maintained on an admissible manifold even in the case of actuator faults. This is feasible if the set (13) is not trivial, meaning it encompasses more than just the origin. Such a set is a generalization of the admissible steady states ([30]) to a family of Linear Parameter Varying (LPV) systems. The latter are a class of dynamic systems that exhibit time-varying behavior, where the system dynamics change in response to variations in one or more parameters. These systems are widely used in control theory and engineering to model and control processes that depend on external factors, such as changing operating conditions or varying environmental conditions (e.g. degradation phenomena).

Moreover, the following assumption is in order

Assumption 4: There exist two vectors $\underline{\delta}$, $\underline{\theta}$ and an integer $T_{min} > 0$ such that for each possible subset $\tilde{\mathcal{A}} \subset \mathcal{A}$, $|\tilde{\mathcal{A}}| = l$, the following conditions hold true

$$\forall\delta_j \in [0,1], \delta_j > \underline{\delta}_j, j \in \mathcal{A}\setminus\tilde{\mathcal{A}}, \delta(t') = [\underline{\delta}_1, \ldots, \delta_j, \ldots, \underline{\delta}_m],$$
$$\text{one has that } \delta(t) \in \Gamma$$
$$\forall\theta_j \in [0,1], \theta_j > \underline{\theta}_j, j \in \mathcal{A}\setminus\tilde{\mathcal{A}}, \theta(t') = [\underline{\theta}_1, \ldots, \theta_j, \ldots, \underline{\theta}_m],$$
$$\text{one has that } \theta(t) \in \Theta$$

for all $t \in [t', t' + T_{min}]$ with $t' > 0$ being a generic time instant. □

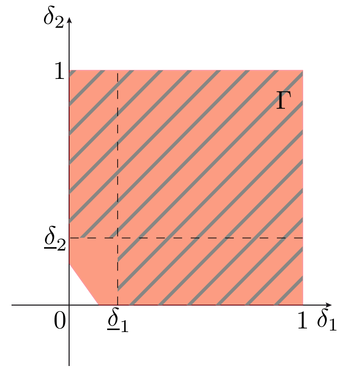

Fig. 2. Graphic Idea of Assumption 4 with $l = 1$ and $m = 2$. Each vector $\delta = [\delta_1, \delta_2]^T$ belonging to the textured area can be steered outside Γ in at least T_{min} time instant.

Such an assumption relates to the typical property enjoyed by industrial equipment of being characterized by rate-bounded degradation effects ([34]). In fact, for this kind of devices, it is possible to assume the existence of a minimum time interval T_{min} needed for a complete actuator failure starting from any possible faulty configuration. A graphic interpretation of it can be found in Fig. 2 where a special case with two actuators is considered ($\mathcal{A} = \{1, 2\}$). Let us account for the case where $\tilde{\mathcal{A}} = \{1\}$. Then, Assumption 4 guarantees that if the 2nd actuator is completely out of service ($\delta_2 = 0$) and $\delta_1 > \underline{\delta}_1$, then the complete breakdown of actuator 1 ($\delta_1 < \underline{\delta}_1$) can occur only after a finite time T_{min}.

In this respect, the following four scenarios (working modes) can be useful to describe the possible working operations of the system. To this end, the set $\bar{\mathcal{A}}(t) := \{i \in \mathcal{A} : \delta_i(t) \leq \underline{\delta}_i \text{ or } \theta_i(t) \leq \underline{\theta}_i\}$ is used to collect the damaged actuating channels at time t:

1) **Normal Mode**, $\bar{\mathcal{A}}(t) \equiv \emptyset$: both of the actuators and related actuating channels mostly work in nominal conditions. In this case, the redundancy of the actuators is sufficient to guarantee suitable control performance;
2) **Partial Failure Mode**, $0 < |\bar{\mathcal{A}}(t)| \leq m - l$: the maximum number of distinct actuators or actuating channels with serious damage is $m - l$. In this case, the control performance are significantly undermined but the system stability could be guaranteed through a proper control allocation policy;
3) **Maintenance Mode**: $|\bar{\mathcal{A}}(t)| > m - l$, for $t \geq t'$: the system is working with at least l distinct actuators and related actuating channels that are able to guarantee constraints fulfillment for next T_{min} time instants at least, where T_{min} will be rigorously defined later in the next section;
4) **Complete Failure Mode** $|\bar{\mathcal{A}}(t)| > m - l$ and $\delta(\rho(t)) \notin \Gamma$ or $\theta(t) \notin \Theta$: almost all actuators or actuating channels are under serious damage conditions. The *AC* task is not any longer guaranteed.

It is also assumed that a specific *Maintenance Service Provider* (MSP) is responsible for repairing damaged actuators or actuating channels. While the MSP can be requested for intervention at any time t', any repair or replacement is possible only after a specific interval, referred to as the *time-to-repair* $T_M < T_{min}$. This timeframe encompasses any

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logistical, administrative, and technical delays, along with the time required to actively intervene on the devices belonging to $\bar{A}(t)$, $t \geq t' + T_M$.

C. Problem Statement

The problem at hand is hereinafter defined:

Predictive Maintenance - Control Allocation Problem (PM-CAP): *Given a virtual input* $v(t) \in \mathbf{R}^l$ *and based on the present degradation value of the actuating channels and actuators, determine an actual control effort* $u(t) \in \mathbf{R}^m$ *satisfying (11) and such that: i) the constraints (2) are fulfilled, ii) guarantee that the closed-loop system is asymptotically stable, and iii)* ***Complete Failure*** *mode is avoided.* □

III. Control Allocation Scheme for Predictive Maintenance Operations (PM-CAS)

The solution of the above stated **PM-CAP** problem is presented in this Section and consists of an *Allocation Block* capable to map the control law (5) into the admissible actual control effort $u(t)$ for ever $t \geq 0$ while keeping $\delta(t) \in \Gamma$ and $\theta(t) \in \Theta$. Moreover, in order to implement a strategy of *punctual* maintenance, such a device should be instructed to convey a warning to the MSP within the *time-to-repair* T_M. In this way, a **Complete Failure** mode is always avoided. To this end, the *Allocation Block* will be designed according to the notion of the *minimum guaranteed time-to-failure estimate* T_{safe} ([15]), which will be proved to be greater than T_M.

The presented strategy is intended to effectively handle the allocation task in both **Normal** and **Partial Failure** modes by utilizing a model predictive control approach. Conversely, in **Maintenance** mode, the system state is kept around a safe equilibrium to stabilize the closed-loop system until MSP intervention is available.

A schematic of the proposed **CAS** is sketched in Fig. 3. It comprises two modules: a reconfigurable *Model Based Predictive Control Allocation unit* (MBPC Allocation Unit) and a *Prognostic module*. The former is related to the allocation of the virtual control effort $v(t)$ onto the actuators $u(t)$ regardless of the current effectiveness $(\delta(t), \theta(t))$ and *operating mode* $\sigma(t) \in \{ALLOCATION, MAINTENANCE\}$ imposed by the *Prognostic module*. In fact, the latter is endowed with a *Predictive Maintenance Algorithm* that defines the operating scenario $\sigma(\cdot)$ on the basis of the information given by the set of m distinct systems which are responsible to measure the degraded value of the i-th actuator.

A. MBPC Allocation Unit

In the following, the execution procedure of the MBPC Allocation Unit will be illustrated. The main part of such a procedure relies on the following optimal control problem that, by exploiting the information conveyed by the *Prognostic module* $(\sigma(t), \delta(t), \theta(t))$, determines an admissible allocation $u(t)$ from the control law $v(t)$

$$
\begin{aligned}
(\mathbf{x}^\star(t), \mathbf{u}^\star(t), \mathbf{p}^\star(t)) = {} & \arg\min_{\mathbf{u}} J(\mathbf{x}, \mathbf{u}, \mathbf{p}) \\
subject\ to \quad \mathbf{x}(k+1) &= A\mathbf{x}(k) + B_u \Delta(k)\mathbf{u}(k) \\
\mathbf{x}(0) &= x(t) \\
\mathbf{p}(k+1) &= \mathbf{p}(k) + \mathrm{diag}(\alpha)|\mathbf{u}(k)| \\
\mathbf{p}(0) &= \rho(t) \\
\theta(0) &= \theta(t) \\
\Delta(k) &= \mathrm{diag}(\delta(\mathbf{p}(k))) \\
B_v v_{\mathbf{x}(N)} &= (I_n - A)\mathbf{x}(N) \\
\mathbf{u}(k) &\in \mathcal{U}_{\theta(k)},\ \mathbf{x}(N) \in \mathcal{X}_f
\end{aligned} \tag{14}
$$

where the cost function $J(\mathbf{x}, \mathbf{u}, \mathbf{p})$ is defined as follows:

$$
\begin{aligned}
J(\mathbf{x}, \mathbf{u}, \mathbf{p}) := \sum_{k=0}^{N-1} \Big(& \| B_v K(\mathbf{x}(k) - \mathbf{x}(N)) \|_Q^2 \\
& + \| B_v v_{\mathbf{x}(N)} - B_u \Delta(k)\mathbf{u}(k)) \|_R^2 \Big) \\
& + \| B_v K(\mathbf{x}(N) - x_r) \|_P^2.
\end{aligned} \tag{15}
$$

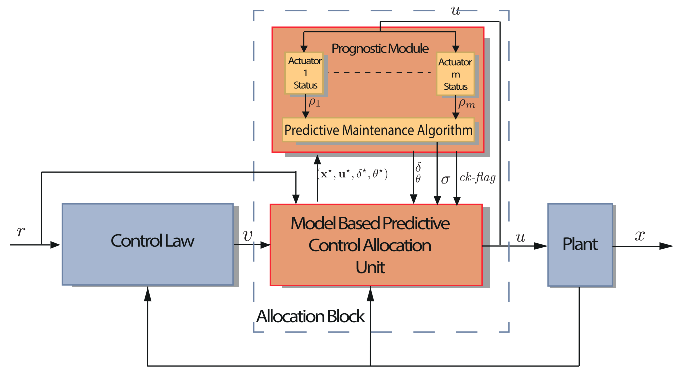

Fig. 3. The PM-CAS architecture.

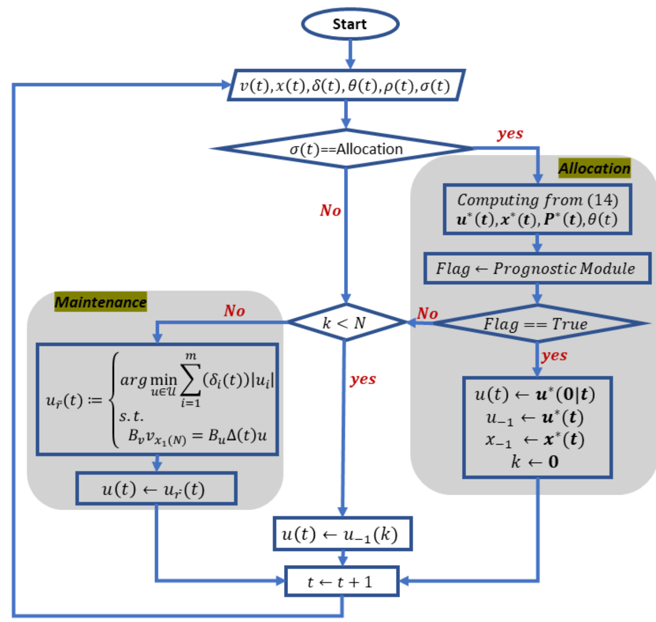

Fig. 4. Flowchart of proposed PM-CAS procedure.

Please note that explicit constraints on the effectiveness of actuators are intentionally omitted to avoid potential issues with feasibility and sub-optimal control performance. Instead, these constraints are indirectly handled by the *Prognostic module*, as explained in the following section. The ensuing algorithm is shown in Fig. 4.

B. Prognostic Module

In the MBPC Allocation Unit of PM, degradation signals received from the physical actuators' monitoring bank is exploited to establish the appropriate *active mode* $\sigma(t)$ and to execute the MSP operations. The rationale for the operations performed by the PM is established on next Proposition 1.

Proposition 1: Let us consider the following constrained optimal control problem to compute the required control effort for maintaining the system state vector $x(t)$ of (10) in the position of a generic equilibrium $x_f \in \mathcal{X}_f$:

$$J_W(\bar{\mathcal{A}}, \delta, \theta, x_f, u) := \min_{u \in \mathcal{U}_\theta} \sum_{i \in \mathcal{A} \setminus \bar{\mathcal{A}}} \delta_i |u_i|$$
$$s.t. \quad B_v v_{x_f} = \sum_{i \in \mathcal{A} \setminus \bar{\mathcal{A}}} B_u^i \delta_i u_i \tag{16}$$

Then, the input with the worst control effort when using only actuators in $\mathcal{A} \setminus \bar{\mathcal{A}}$ is represented by

$$u^s := \arg\min_u J_W(\bar{\mathcal{A}}, \underline{\delta}, \theta, x_f, u),$$

such that $|u_i^s| \geq |u_i^p|, \forall i \in \mathcal{A} \setminus \bar{\mathcal{A}}$, where

$$u^p := \arg\min_u J_W(\bar{\mathcal{A}}, \delta', \theta, x_f, u)$$

is an arbitrary solution with $\delta' \geq \underline{\delta}$.

Proof - Please observe that thanks to Assumption 3 u_s always exists. Then, u^s would be an admissible solution for (16) with cost $J_W(\bar{\mathcal{A}}, \delta', \theta, x_f, u)$, then

$$J_W(\bar{\mathcal{A}}, \delta', \theta, x_f, u^p) \leq J_W(\bar{\mathcal{A}}, \delta', \theta, x_f, u^s)$$
$$\Downarrow$$
$$\sum_{i \in \mathcal{A} \setminus \bar{\mathcal{A}}} \delta_i' |u_i^p| \leq \sum_{i \in \mathcal{A} \setminus \bar{\mathcal{A}}} \underline{\delta_i} |u_i^s|$$
$$\Downarrow$$
$$|u_i^s| \geq |u_i^p|, \forall i \in \mathcal{A} \setminus \bar{\mathcal{A}}$$

where the latter inequality follows by the fact that $\delta' \geq \underline{\delta}$. □

The resulting *predictive maintenance procedure* is described by the *state automata* shown in Fig. 5 and satisfies the following simplified conditions:

- the transition from the *allocation mode* to the *maintenance mode* is activated at time t if 1) at least $m - l$ actuators or the related actuating channels are damaged, 2) at least one component $\delta_i^\star(N|t)$ and/or its channel $\theta_i^\star(N|t)$ with their estimated degraded vectors $\delta^\star(t)$ and $\theta^\star(t)$ lower than $\underline{\delta}_i$ and/or $\underline{\theta}_i$ respectively for some $i \in \mathcal{A} \setminus \bar{\mathcal{A}}(t)$. Both mentioned conditions can be distinctly formulated as follows

$$|\bar{\mathcal{A}}(t)| \geq m - l \tag{17}$$
$$\delta_i^\star(N|t) \leq \underline{\delta}_i, \ i \in \mathcal{A} \setminus \bar{\mathcal{A}}(t) \tag{18}$$
$$\theta_i^\star(N|t) \leq \underline{\theta}_i, \ i \in \mathcal{A} \setminus \bar{\mathcal{A}}(t) \tag{19}$$

  In this case, the *check-flag=false* is sent to the *MBPC Allocation Unit* so that the solution $\mathbf{u}_i^\star(t)$ is discarded. Furthermore, the system state is forced to stay close last computed admissible steady-state equilibrium $\mathbf{x}_{-1}(N)$ by applying the minimum-energy allocated input u^s resulting from (16) with cost $J_W(\bar{\mathcal{A}}(t), \underline{\delta}, \underline{\theta}, \mathbf{x}_{-1}(N), u^s)$. Then, the MSP intervention is represented by

$$T_{safe} = T_{min} + \min_{i \in \mathcal{A} \setminus \bar{\mathcal{A}}(t)} (T_{lim,i}) \tag{20}$$

  where

$$T_{lim,i}$$
$$:= \max\Big(0, \min\big(\big(t_{\underline{\delta}_i} - (t+N)\big), \big(t_{\underline{\theta}_i} - (t+N)\big)\big)\Big) \tag{21}$$

  while $t_{\underline{\delta}_i}$ and $t_{\underline{\theta}_i}$ are the time instants where $\delta_i(t) = \underline{\delta}_i$ and $\theta_i(t) = \underline{\theta}_i$ respectively. Observe also that $T_{lim,i}$ denotes the minimum of time needed to $\theta_i(t)$ and/or $\delta_i(t)$, $i \in \mathcal{A} \setminus \bar{\mathcal{A}}$, to prevail their safety limits $\underline{\delta}_i$ and $\underline{\theta}_i$ respectively.
- the *Maintenance Mode* will evolve to the *Allocation Mode* at time t if the repair operation occurs allowing the system to operate with at least $m - l - 1$ healthy actuators,

$$|\bar{\mathcal{A}}(t)| < m - l \tag{22}$$

  Therefore, the *MBPC Allocation Unit* is always able to propose an admissible solution for (14). This aspect will be discussed in the next session.

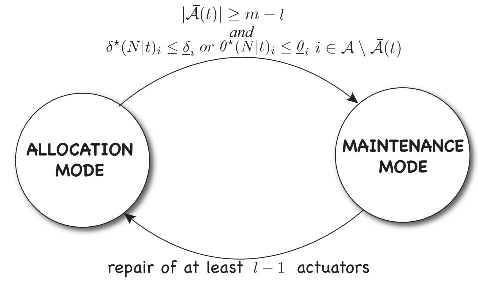

Fig. 5. The State Automata of *Predictive Maintenance Procedure* in *Prognostic Module*.

IV. Properties

Important properties of the **PM-CAS** procedure are summarized as follows.

Proposition 2: Given the over-actuated system (1) and the related virtual plant (3), let the proposed **PM-CAS** procedure described in Fig. 4 and Fig. 5 be performed with Prediction Horizon $N \leq T_{min}$ in the presence of faults models which are illustrated in Section II-B. Then:

1) the effectiveness vectors $\delta(t)$ and/or $\theta(t)$ are maintained respectively into the sets Γ and/or Θ along the evolutions of system (10) for all $t \geq 0$. Any MSP recovery intervention lasts at most T_{safe} time instants.

2) if a solution $\mathbf{u}(0)$ exists for problem (14) at time $t = 0$, then a solution will exist for all $t > 0$ and the signal $v(t)$ is always allocated into an admissible input $u(t)$ via Algorithm 1.

3) Closed-loop asymptotic stability of (1) is ensured. More in detail, for any constant reference $r(t) \equiv r$, the state $x(t)$ is steered either to its best approximation $\hat{r} \in \mathcal{X}_f$ until the *Allocation Mode* is active or to a generic admissible equilibrium $\bar{r} \in \mathcal{X}_f$ if the *Maintenance Mode* was activated.

Proof - 1) When problem (14) is solved in *Allocation Mode*, the condition $|\bar{\mathcal{A}}| < m - l$ holds true. Then, because $N \leq T_{min}$, every admissible solution $\mathbf{u}(t)$ for (14) can be applied for N steps without steering $\delta(t)$ and $\theta(t)$ outside Γ and Θ. During the *Maintenance Mode*, it is sufficient to observe that $\delta(t) \in \Gamma$ and $\theta(t) \in \Theta$ for at least $T_{safe} > T_M$ time instants, which are sufficient for MSP to recover the damaged actuators.

2) Let $\mathbf{u}^\star(t)$ be a solution to (14) at time t. Then, a solution at time $t+1$ can be constructed as $\mathbf{u}(t+1) = [\mathbf{u}^\star(1|t), \ldots, \mathbf{u}^\star(N-1|t), u_{\bar{r}(t)}]$ with $u_{\bar{r}(t)}$ computed as in (16). This sequence is always feasible thanks to Assumption 3. This approach is still valid in *Maintenance Mode* where it can be repeated until $\delta(t) \in \Gamma$. For this reason, if a recovery occurs, the existence of a feasible solution is always ensured for all t.

3) Let assume that the scheme is working in *Allocation Mode* and $\mathbf{u}^\star(t)$ be an optimal solution of (14) at time t. Correspondingly, let $J^\star(t) := J(\mathbf{x}^\star(t), \mathbf{u}^\star(t), \delta(t), \theta(t))$ be the value of the optimal cost. By applying the same suboptimal feasible control sequence introduced in the previous item, one achieves the suboptimal cost

$$J(t+1) = J^\star(t) - \|B_v K(\mathbf{x}^\star(0|t) - \mathbf{x}^\star(N|t))\|_Q^2 - \|B_v v_{\mathbf{x}^\star(N|t)} - B_u \delta(t)\mathbf{u}^\star(0|t))\|_R^2$$

which implies

$$\begin{aligned} J^\star(t+1) &\leq J(t+1) \\ &\leq J^\star(t) - \|B_v K(\mathbf{x}^\star(0|t) - \mathbf{x}^\star(N|t))\|_Q^2 \\ &\quad - \|B_v v_{\mathbf{x}^\star(N|t)} - B_u \Delta(t) u(t)\|_R^2 \end{aligned}$$

Roughly speaking, $J^\star(t)$ is strictly decreasing as long as $x(t) \neq \mathbf{x}^\star(N|t)$ and $B_u \Delta(t) u(t) \neq B_v v_{\mathbf{x}^\star(N|t)}$. Hence $x(t) \to \mathbf{x}^\star(N|t)$, $B_u \Delta(t) u(t) \to B_v v_{\mathbf{x}^\star(N|t)}$ and $J^\star(t) \to \|B_v K(\mathbf{x}^\star(N|t) - x_r)\|_P^2$. Then, by exploiting arguments presented in [30], it is possible to prove that if the *Allocation Mode* persists, then $x(t)$ converges to the best approximation $x_{\bar{r}}$ of x_r in $\mathcal{X}_f$. Otherwise, if a transition to the *Allocation Mode* would be activated at t', the state is steered to the latter computed terminal state $\mathbf{x}(N|t')$. □

V. Simulations

A. Unstable System

This section considers an unstable discrete-time system having the same structure of (1) with matrices $A = 1.1$, $B(\rho(t)) = B_u \Delta(\rho(t))$ where $B_u = [1, 1, 1]$. The parameter $\rho(t)$ denotes the actuator's degradation law (7) and $\alpha_1 = 0.48, \alpha_2 = 0.6$ and $\alpha_3 = 0.08$. In particular, the degradation evolution affects the actuators effectiveness δ_i, $i = 1, 2, 3$ according to the following expression $\delta_i = 1 - \frac{1}{1+10e^{-(\rho_i - 5)}}$, $i = 1, 2, 3$. Furthermore, the parameters $\underline{\delta}_i = 0.24$,, $T_{min} = 5$ and the polytope $\Gamma = \delta \in [0_3, 1_3] : \delta_1 + \delta_2 + \delta_3 \geq \delta_{min}, \delta_{min} = 0.06$ are selected that satisfy Assumptions 3-4 for $i = 1, 2, 3$. No *input-independent* loss of actuator effectiveness has been considered here. It has been assumed that the related virtual system has an input matrix $B_v = 1$. As a consequence, the virtual control gain $K = -0.6$ is suitable for guaranteeing asymptotic stability.

The control objective is the tracking of the squared signal set-points $r(t)$, shown in Fig. 6 (top), starting with from the initial condition $x(0) = 0$ and $\rho(0) = [8.2, 7.2, 7.2]$, while ensuring that the control input remains within the bounds $|u(t)| \leq 2$. In order to accomplish this task, the **PM-CAS** strategy is employed and it is initialized according to the instructions given in Section III. Specifically, it is assumed that the MSP can guarantee a *time-to-repair* $T_M = 11$, while the *MBPC Allocation Unit* employs a prediction horizon $N = 5$.

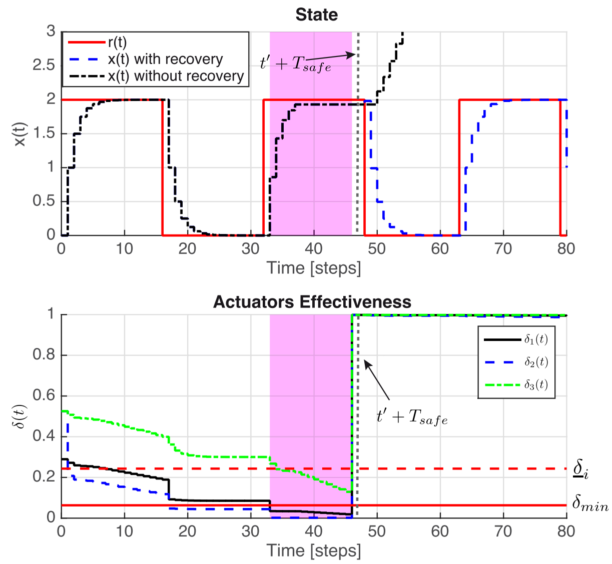

Fig. 6. System state (top), effectiveness signal (down). The shady area denotes the time interval where the *Maintenance mode* is active.

Simulations have been performed in Matlab by using the IPOPT solver[1] and numerical outcomes are presented in the subsequent figures (Fig. 6-7). At the beginning, the **PM-CAS** scheme works under the *Allocation Mode* because the set $\bar{\mathcal{A}}(0)$ of damaged actuators is empty at time $t = 0$. Within 9 time steps the second actuator first ($t = 2$) and the first actuator later reach $\underline{\delta}$ (Fig. 6 (bottom)). As a consequence, $\bar{\mathcal{A}}(9) = \{1, 2\}$ and, in turn, $|\bar{\mathcal{A}}(9)| \geq m - l = 2$. In this situation, a *Partial Failure* scenario arises, that is actuator 3 only can convey the whole control effort to the plant (Fig. 7 (top) - Fig. 6 (top)). Such a scenario ends at instant time $t' = 34$ when conditions (17) and (18) become active for $i \in 3$. Therefore, the MSP is asked to complete a maintenance operation within $T_{safe} = 14$ time instants on the basis of (20). Hence, the *Maintenance Mode* is activated so that the *Allocation Unit* can apply the remaining part of the MPC sequence $\mathbf{u}_{-1}$ (see step 15 of Algorithm 1) until the feasible equilibrium $x(t) = 1.93$ is reached at time $t = 38$. Then, the system state is kept at this equilibrium by applying for $t > 38$ the input $u(t)$ defined in (16) (see Fig. 7 (bottom)). Finally, at time $t = 46$ the damaged actuators are repaired and the degradation signals are updated to their default value as depicted in Fig. 7 (bottom). As a result, a transition to the *Allocation mode* is triggered and the control goals can be achieved with a significantly reduced individual actuators energy. It is worth noticing that the use of the presented scheme enforces a *just-in-time* maintenance policy. As a matter of fact, in Fig. 6 (bottom) where the system state $x(t)$ without any recovery operations is depicted, is shown that a *Complete Failure* could arise within only two-time instants after $t' + T_{safe}$.

[1] https://projects.coin-or.org/Ipopt

This article has been accepted for inclusion in a future issue of this journal. Content is final as presented, with the exception of pagination.

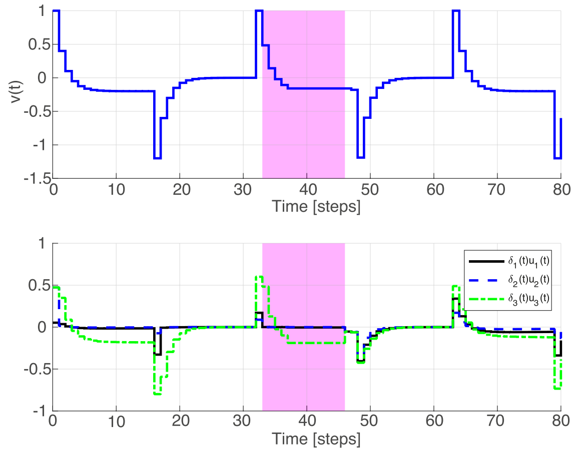

Fig. 7. Virtual input (top), physical input (down). The shady area denotes the time interval where the *Maintenance mode* is active.

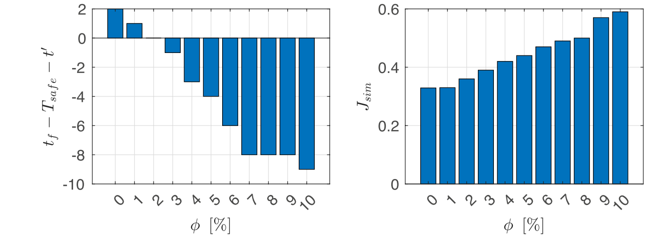

Fig. 8. Robustness analysis: (left) $t_f - (t' + T_{safe})$ on the basis of error percentage ϕ, (right) average simulation cost J_{sim} on the basis of error percentage ϕ.

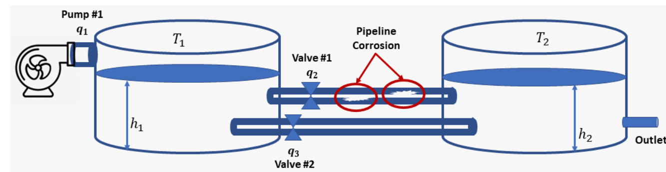

Fig. 9. System schema.

1) Robustness Analysis: A further simulation scenario has been considered to investigate the robustness of **PM-CAS** strategy in the case where the value of δ_i, provided by the *Prognostic Module*, are not accurate. Specifically, we consider a worst-case scenario where the *MBPC Control Allocation Unit* receives the following signal $\tilde{\delta}_i := \delta_i + \phi\delta_i$, $\phi \in [0, 1]$ so it has a more *optimistic* assessment of the current actuator healthy status. To asses the robustness, we examine the quantity $t_f - T_{safe} - t'$ where t_f is the time instant where the actuator configuration of the system enters in **Complete Failure Mode**. If this value is positive, then the MSP intervention can occur while the system is still in **Maintenance Mode**; otherwise, the repair operations start when a failure is already ongoing. Moreover, to evaluate the control performance, the following indicator $J_{sim} := \sum_{t=0}^{t_f} |x(t) - r(t)|$ has been also computed throughout the simulation. The results are presented in Figure 8, which include several simulation with parameter ϕ ranging from 0 to 0.1. It is not surprising to observe an increasing loss of performance on the right side of the figure as ϕ increases. However, the left part of the figure clearly shows that, in the current configuration, the PM-CAS can tolerate up to 2% of inaccuracy in the measurement of δ. On the other hand, a possible way to increase such a tolerance percentage is to reduce T_{safe} by increasing δ_{min} in the definition of set Γ. However, it is worth remarking that such a design modification does not come without any loss of control performance and, most importantly, presents an inherent limitation in the form of requirement that $T_{safe} \geq T_M$. As a consequence, in this case, T_{safe} can be lowered by at most of 3 units. As shown in Figure 8 (left), this adjustment allows for a deviation of up to 4% in the measurement of the parameter δ to be tolerated.

B. Stable System

This simulation example involves the system of connected tanks considered in [31]. As shown in Fig. 9, the level of liquid in tanks T_1 and T_2 are h_1 and h_2, respectively. Also, both tanks are interconnected by actuators (valves) u_2 and u_3 that are installed in the actuating channel (pipeline connections) #2 and #3 respectively. Also, tank T_1 is filled via pump u_1 that is installed in the actuating channel (pipeline connection) #1. By considering the state vector $x = [h_1, h_2]^T$, the input vector $u = [u_1, u_2, u_3]^T = [0.48, 0.75, 0.2]^T$ and operating point $\hat{x} = [0.4, 0.06]^T$, the plant parameters of the discretized model (1) are given by

$$A = \begin{bmatrix} 0.9830 & 0.0085 \\ 0.0167 & 0.9563 \end{bmatrix}, \quad B_u = \begin{bmatrix} 0.0201 & -0.0079 & -0.0084 \\ 0.000 & 0.0078 & 0.0082 \end{bmatrix}$$

where $\rho(t)$ follows the degradation law (7) affecting the three actuators with $\alpha_1 = 0.008, \alpha_2 = 0.7$ and $\alpha_3 = 0.7$. For all actuators, the loss of effectiveness, δ_i is related to the degradation by

$$\delta_i = 1 - \frac{1}{1 + 10e^{-(\rho_i - 5)}}, \; i = 1, 2, 3$$

The input-independent effectiveness of pipeline connections #1, #2, and #3 are $\theta_1(t) = 1$, $\theta_2(t) = e^{-\gamma t}$, and $\theta_3(t) = 1$, respectively. Moreover, parameters $\underline{\delta} = [0.4, 0.2322, 0.2322]^T$, $\underline{\theta} = [0, 0.2322, 0]^T, i = 1, 2, 3$, $T_{min} = 60$ and polytopes $\Gamma := \{\delta \in [0_3, 1_3] : \delta_1 + \delta_2 + \delta_3 \geq \delta_{min}\}$, $\delta_{min} = 0.1$, $\Theta = \{\theta \in [0_3, 1_3] : \theta_1 + \theta_2 + \theta_3 \geq \theta_{min}\}$, $\theta_{min} = 0.1$, $\gamma = 0.0005$, are chosen such that satisfy Assumptions 3-4. Regarding the system (3) - (5), the matrices B_v and K are given by

$$B_v = \begin{bmatrix} 0.20 & 0.01 \\ 0.00 & 0.10 \end{bmatrix}, \quad K = \begin{bmatrix} 3.4066 & -0.3356 \\ 0.1670 & 7.5630 \end{bmatrix}$$

In the simulations, the main goal is to exploit the proposed **PM-CAS** strategy to control the liquid levels (h_1, h_2) such that the storage tank levels track the reference signal $(h_{r_1}(t), h_{r_2}(t))$ depicted in Fig. 11 (red line). In the simulations, the degrading actuators and plant constraints are both considered. The considered initial conditions are $x(0) = [0.1, 0.3]^T$, $\rho(0) = [7, 8, 6]^T$, $\theta(0) = diag(1, 1, 1)$.

The assumed input constraints are $|u_i(t)| \leq 2\theta_i(t)$ and $\theta(t) = diag(1, e^{-\gamma t}, 1)$. The **PM-CAS** scheme has been implemented with parameters $T_M = 50$ for MSP with a prediction horizon $N = 20$ for the MBPC Allocation Unit.

In order to evaluate the proposed **PM-CAS** strategy performance, a comparison with the adaptive control allocation method presented in [19], here referred to as **FTCAP-CM**,

TABLE I
SIMULATION SCENARIOS

Control strategy	Repairing	Number of failure	Average tracking error per step (%) (x_1, x_2)
PM-CAS	Automata Fig. 5	0	(0.36, 0.69)
FTCAP-CM	Condition (23)	2	(0.58, 1.39)

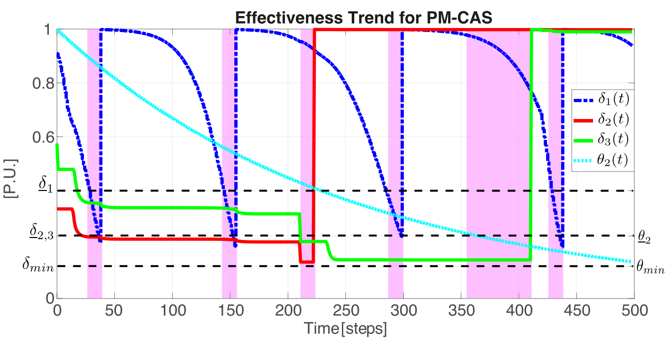

Fig. 10. Effectiveness trend for PM-CAS.

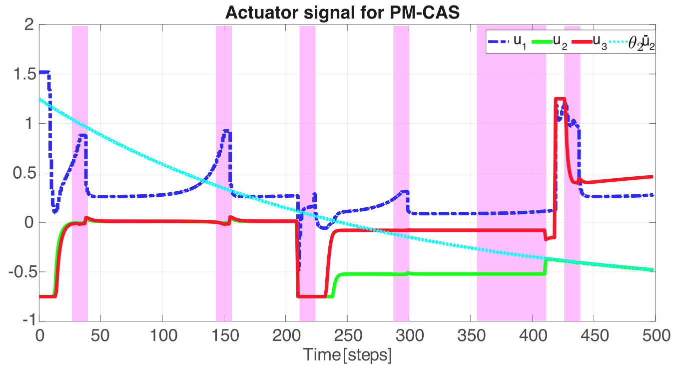

Fig. 11. Control effort of all actuators and permissible value of control signal for actuating channel #2 in case of using PM-CAS.

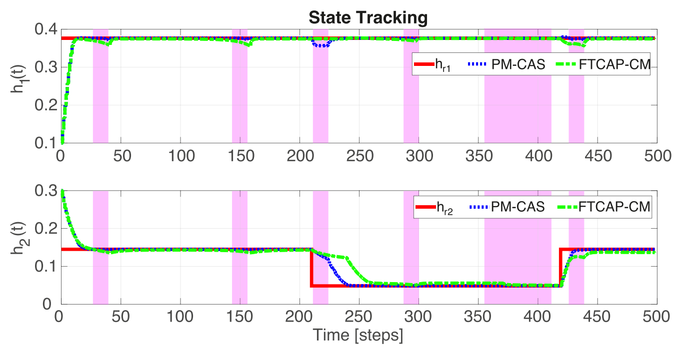

Fig. 12. State tracking.

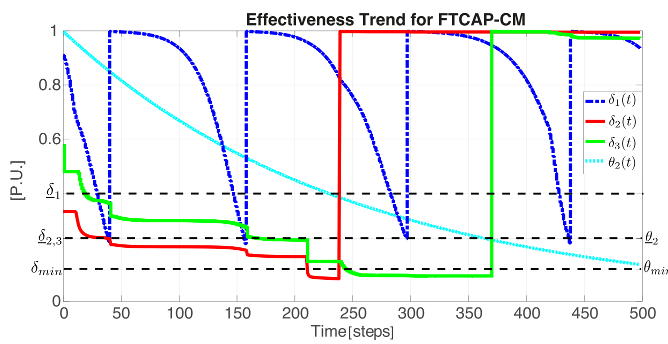

Fig. 13. Effectiveness trend for FTCAP-CM.

is presented. In particular, the counterpart algorithm, not in principle oriented to predictive maintenance, has been endowed with an adaptive repairing policy that foresees a notification to the MSP station when the following condition becomes true

$$(\delta_i \leq \underline{\delta_i} \text{ or } \delta_i \leq \underline{\theta}_i) \quad and \quad (\delta_i \leq \delta_{min,j} \text{ or } \delta_i \leq \theta_{min,j}),\ i \neq j \tag{23}$$

The simulation results are depicted in Figs. 10-13. As shown in Fig. 12, when the plant is equipped with **PM-CAS**, the reference signal is well tracked in both *Allocation mode* and *Maintenance mode* (see shady areas in Figs. 10-12). From the same figures, it results that the control performance by using the proposed strategy is better than **FTCAP-CM**. To quantify this aspect, the average tracking error per step for both states have been calculated in Table I. Moreover, by looking at Figs. 10 and 13, it can be observed that using **PM-CAS** to manage the control effort of valves, the RUL of the actuators is increased. In other words, the effectiveness of the actuators is lately entered in the maintenance mode range (between $\underline{\delta}_{2,3}$ and $\delta_{min_{2,3}}$) with respect to the FTCAP-CM algorithm (time interval [210-375] steps in Fig. 13).

Another important point can be observed in Figs. 12 and 11 within the time range [437-500]. Although the *input-dependent* effectiveness δ_i is not affected in both strategies, the actuating capability of the system is limited by the *input-independent* loss of actuators on the actuating channel #2 that is under the $\underline{\theta}_2$ threshold. Differently from **FTCAP-CM**, which is not able to allow the perfect tracking of x_{r2} (green dashed line in Fig. 12), the presented **PM-CAS** strategy clearly attains an exact tracking of the reference signal. Such a result is achieved by increasing the effort of actuator #2 (see solid red line in Fig. 11) to compensate for the limitation imposed on actuator #3 (see solid green line in Fig. 11).

C. CSTR Plant

This example aims at assessing the benefits achievable in terms of actuator degradation reduction in realistic plants of industrial interest during their normal operations from the use of the **PM-CAS** method in contrast to conventional industrial controllers. A continuous stirred tank reactors (CSTR), a schematic of which is depicted in Fig. 14, is here considered. This plant has been extensively used in the literature for evaluating the effectiveness of multi-scale process monitoring techniques [35], [36], [37].

In this example, we consider a specific plant and a proportional output feedback controller, referred to hereafter as the conventional control, with detailed descriptions available in [36]. Additionally, for evaluating the proposed **PM-CAS** strategy, we have enhanced the model by introducing a redundant actuator dedicated to valve V_1. This valve is now replaced by two independent and redundant valves, namely V_{11} and V_{12} (see Fig. 14). The model, sufficiently complex for the intended assessment, exhibits rich dynamics, comprising 15 states, 5 outputs, all of which are measured for feedback, and 6 commanded inputs.

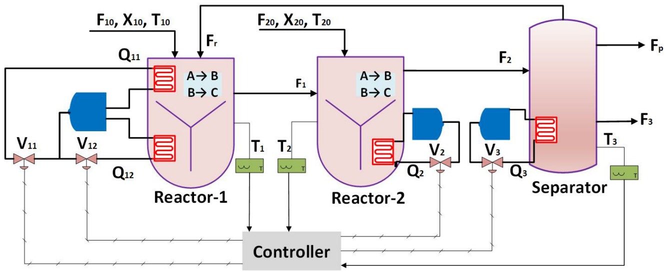

Fig. 14. CSTR plant schematic with controller architecture.

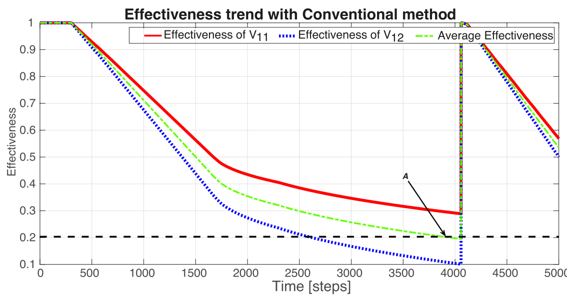

Fig. 15. Effectiveness trend of V_{11}/V_{12} with conventional method.

The rationale for linearizing the nonlinear system around a designated working point is justified in [37]. The resulting structural formulation of the system and control dynamics adhere to the framework established in (1) and (5), respectively.

The paramount control objective is the minimization of the temperature tracking errors with respect to the nominal set-points in order to optimize the quality of the final output. Consequently, the simulation scenario is delineated in alignment with reference set-points, $T_{1,s} = 510^oC$, $T_{2,s} = 500^oC$, and $T_{3,s} = 505^oC$, which are tracked by T_1, T_2, and T_3 respectively.

In actual industrial facilities, deviations exceeding $\pm 1^oC$ from nominal set-points under steady-state conditions are typically attributed to potential degradation of certain plant subsystems, necessitating repairs. Additionally, mirroring physical constraints in practical applications, the maximum allowable value of flow ($Q_{i_{\max}}$) and its rate of the variation ($dq_{i_{\max}}$) have been constrained, see [36].

For the sake of clarity, in the current simulation it is assumed that the degradation of the actuators is only considered for the redundant valves V_{11} and V_{12} and that the degradation effect is that of decreasing the value of the maximum allowable $Q_{i_{max}}$. The considered degradation laws are as follows

$$\rho_{11}(t+1) = \rho_{11}(t) + \alpha_{11} Q_{11}(t), \alpha_{11} = 14 \times 10^{-4} \quad (24)$$

$$\rho_{12}(t+1) = \rho_{12}(t) + \alpha_{12} Q_{12}(t), \alpha_{12} = 18 \times 10^{-4} \quad (25)$$

where Q_{11}, Q_{12} and α_{11}, α_{12} are the scaled-flow rates and the degradation coefficients for valve V_{11} V_{12} respectively. A fixed-time repairing policy is adopted in this simulation with repairing time instants at 5750 steps and 7600 steps. The remaining system simulation parameters are defined based on [36].

We further assume that actuators V_{11} and V_{12} are declared failed when their average effectiveness is below 0.2, so a

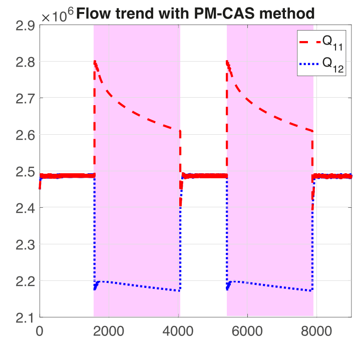

Fig. 16. Scaled flow trend of Q_{11}/Q_{12} with PM-CAS method.

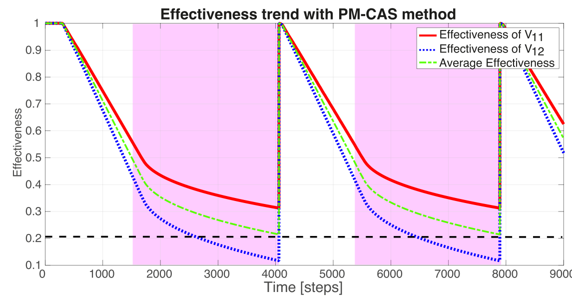

Fig. 17. Effectiveness trend of V_{11}/V_{12} with PM-CAS method.

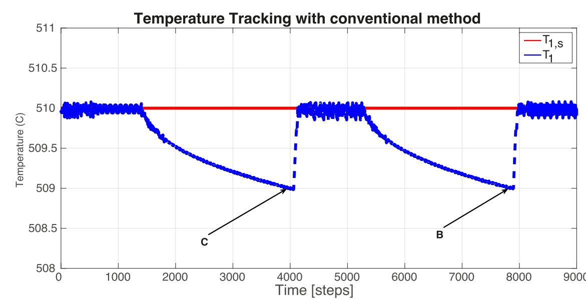

Fig. 18. Reference temperature tracking trend of T_1 with conventional method.

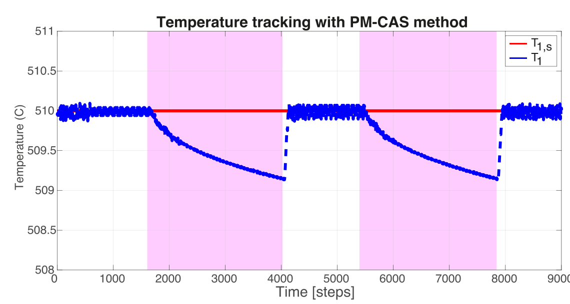

Fig. 19. Reference temperature tracking trend of T_1 with PM-CAS method.

Complete Failure mode occurs. In this respect, the conventional approach is not able to prevent the failure before the maintenance intervention (see Point A in Fig. 15). In this case, it would be possible to prevent the failure by making repairing earlier. However, it would cause an increasing of the number of repairing services with respect to usage of the **PM-CAS** method. In the latter case the *Maintenance* mode is promptly activated at 1521 and 5430 steps so that a safe input is applied (please refer to the shady area in Fig. 16). As a consequence, as depicted in Fig 17, a reduction of the degradation rate of the redundant valves has been produced and has prevented the occurrence of failure events.

The above discussed results have relevant effects on the control performance. In fact, as depicted in Fig. 19, the use of **PM-CAS** allows one to maintain the tracking error in an acceptable region when the system is reconfigured in the *Maintenance* mode. On the contrary, in the case of using the conventional method, it is concluded that the tracking error on T_1 exceeded (Points B & C) the permissible range ($\pm 1^o C$) before the repairing time, see Fig. 18.

VI. Conclusion

This paper has presented a receding horizon control allocation strategy to address tracking control problems in the presence of actuator effectiveness incipient degradation. To this end, actuators have been assumed to be affected by two possible degrading factors: the former is directly related to their usage, while the second depends on exogenous non-manipulable phenomena (e.g. corrosion effects in pipelines).

A novel method for fault-tolerant control allocation in discrete-time systems subject to input saturation has been developed. The primary aim of this approach was to focus on creating a framework that exploits predictive maintenance principles for an efficient handling of actuator failures and prompting corrective action in due course. Specifically, the proposed scheme is able to optimally allocate the control effort when all actuators present a proper healthy status. On the contrary, when some actuator failure is imminent, the control action is allocated on the remaining healthy actuators. Moreover, under severe fault situations, the residual actuators' effectiveness is used to steer the state of the system towards a safe equilibrium to keep the closed-loop system stable. The numerical outcomes obtained from simulations show that the proposed approach is effective in preventing critical scenarios and highlights its potential to improve system reliability.

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