Received 23 December 2021; revised 17 April 2022; accepted 26 July 2022. Date of publication 8 August 2022; date of current version 2 September 2022.

Digital Object Identifier 10.1109/OJITS.2022.3196811

Event-Triggered Finite-Time Sliding Mode Control for Leader-Following Second-Order Nonlinear Multi-Agent Systems

QIANG HAN [1,2], YONG-SHUAI ZHOU [1], YU-XIN TANG [1], XIAN-GUO TUO [1], AND PING HE [1]

[1] School of Automation and Information Engineering, Sichuan University of Science and Engineering, Zigong 643000, Sichuan, China

[2] Robot Teclnology Used for Special Environment Key Laboratory of Sichuan Province, Southwest University of Science and Technology, Mianyang 621010, Sichuan, China

CORRESPONDING AUTHOR: P. HE (e-mail: pinghecn@qq.com)

This work was supported in part by the Science and Technology Project of Sichuan Province of Commerce under Grant 2021YFS0339; in part by the Fund of Robot Technology Used for Special Environment Key Laboratory of Sichuan Province of Commerce under Grant 17kftk05; in part by the Sichuan Science and Technology Program of China under Grant 2020YFH0124; and in part by the Zigong Key Science and Technology Project of China under Grant 2020YGJC01.

ABSTRACT Finite-time leader-following consensus problem of second-order multi-agent system (MAS) with nonlinear dynamics under directed communication topology is investigated in this paper. Based on the event triggered mechanism, a novel finite-time integral sliding mode control strategy is proposed to guarantee that the consensus stability of MAS can be reached within the upper limit of the time, which can be predicted according to the initial state of MAS. The event-triggered condition on the basis of the defined novel measurement error effectively reduces the energy dissipation of the system and the update frequency of the controller, and the deduced lower bound expression of the event-triggered time interval avoids the occurrence of zeno behavior. Numerical simulation results illustrate the effectiveness and feasibility of the proposed algorithm.

INDEX TERMS Nonlinear, multi-agent system, event-triggered, finite-time, sliding mode control.

I. INTRODUCTION

IN RECENT years, with the rapid development of high and novel technologies such as computers and artificial intelligence, the MAS with parallel computing capabilities and distributed collaborative control technology has been widely used in the fields of filters in sensor networks, collaborative control of drone formations, intelligent transportation and so on. The MAS interacts with information through the communication topology, thereby cooperating to complete tasks, so it has stronger fault tolerance, robustness and scalability [1]. And the information transmission and sharing capabilities of the system have become important factors affecting the consensus control of MAS.

In the traditional time-triggered mechanism, Agents exchange information and update control protocols after a fixed period of time. Its effectiveness and robustness have been relatively mature in the 1990s [2], [3], and have been widely used in first-order [4], [5], [6] and second-order [7], [8], [9] MAS. In an actual MAS, communication resources such as the bandwidth of the communication network and the energy of the agent itself are extremely limited. The traditional periodic sampling mechanism can reduce the communication frequency of the system to a certain extent. However, when the system is operating under relatively ideal situations or the states of the system tends to be consistent, still adopting a fixed time period for information interaction will inevitably cause the waste of limited communication resources. Therefore, it is necessary to design a reasonable communication trigger mechanism to economize limited communication resources, so as to realize the consensus control of MAS. Astrom K.J. proposed an event-triggered control strategy in the monograph [10], which is different from the time-triggered mechanism. The event-triggered mechanism is a communication trigger mechanism that only conducts information interaction when it is "needed", that is, when the behavior set by the mechanism occurs (such as the error value of the system state reaches a certain threshold),

The review of this article was arranged by Associate Editor Chi-Hua Chen.

the system conducts information interaction, control protocols update and other operations. Therefore, the introduction of the event-triggered mechanism in the consensus control of MAS can greatly reduce the communication frequency, economize communication resources and reduce the energy loss of the agent itself. The control strategy based on the event-triggered mechanism has become a hotspot of research by scholars due to its flexibility, intelligence and development prospects.

The event-triggered mechanism has been applied earlier in the single agent system. In [11], the injection engine compressor control protocol based on the event-triggered mechanism can effectively reduce the update frequency of the controller. In MAS, Dimarogonas proposed a distributed event-triggered control protocol in [12], in which each agent updates the control protocol at its own event-triggered time, but needs to consider the state of the neighbor agent at the latest trigger time.In order to further economize communication resources and reduce the event-triggered frequency, the dynamic event-triggered mechanism starts from another angle and introduces internal dynamic variables on the basis of static event-triggered control, such as [13].

Another key factor that affects the consensus control of MAS is the convergence time. The asymptotic consensus of the system means that the state of the agent tends to be consistent when time tends to infinity, so the convergence speed cannot be guaranteed, such as the asymptotic consensus controllers in [5], [6], [7]. However, the finite-time consensus can make the system state consistent in a predictable time according to the initial conditions. It has greater research significance due to its faster convergence speed and better anti-interference performance, for example [8], [9]. In order to reduce the number of updates of the system while ensuring the convergence speed of the MAS, the finite-time consensus problem based on event-triggered mechanism under the fixed topology and switching topology respectively was studied [14], and reference [15] discussed the finite-time consensus problem based on event-triggered mechanism under the directed communication topology. However, in practical MAS, there is often the influence of nonlinear dynamics. Literature [16] designed a distributed finite-time consensus controller for the second-order nonlinear MAS under undirected communication topology, and reference [17] proposed a finite-time sliding mode algorithm with faster convergence speed for the consensus problem of second-order nonlinear MAS under directed network topology, but the controllers designed in [16], [17] did not possess the event-triggered mechanism that could economize communication resources. For two situations of leader and no leader, two non-linear event-triggered control strategies were proposed for the finite-time consensus problem under the undirected network topology in [18]. Due to the robustness of the sliding mode control for model uncertainty and external interference, in [19], a finite-time integral sliding mode control method was proposed to solve the consensus problem of second-order MAS with disturbances. Compared with the reference [19], not only a finite-time integral sliding mode control algorithm is utilized to eliminate the influence of bounded disturbances for the consensus problem of first-order MAS, but also the event-triggered mechanism was introduced in the controller [20].

If the event is triggered infinitely many times within a finite time, the phenomenon is called zeno behavior [21]. In the research of event-triggered strategy, a key task is to exclude zeno behavior. At present, there are two main ways to eliminate the occurrence of zeno behavior. One is that a fixed positive lower bound proved existing in time interval between any two adjacent event-triggered moments. For example, the event-triggered strategy combined with periodic sampling proposed only detected events at the sampling point [22], so the event-triggered time only occurs at the sampling point, and reference [23] proposed a non-periodic intermittent detection event-triggered condition ensuring that the time interval of the event-triggered moment is a fixed constant number. Another way is that proving that zeno behavior does not exist is the method of proof by contradiction. First assume that there is zeno behavior, then there is a convergence point in the sequence of event-triggered moments (the limit value of the event-triggered moment is a fixed normal number), and then prove that this assumption contradicts the inherent nature of the system, excluding the existence of zeno behavior [24], [25].The application of event-triggered mechanism in the consensus control of MAS has achieved certain theoretical results, but most of the theoretical researches focus on the first-order MAS. In the second-order MAS, not only including the consensus of the position, but also the consensus of the speed, so it is more complicated and more difficult. In [26], a necessary condition for solving the consensus problem of second-order MAS is that the directed graph has a directed spanning tree. Finite-time event-triggered sliding mode control for one-sided Lipschitz nonlinear systems was investigated in [27], an observer-based sliding mode controller was utilized and LMIs was also used to get some sufficient conditions for finite time stability.

Motivated by the aforementioned works, event-triggered mechanism is utilized in the finite-time consensus control for second-order MAS. A novel event-triggered function is proposed for the designed finite-time event-triggered controller. The rest of this paper is organized as follows. Preliminaries including some algebraic knowledge and some usful lemmas is given in Section II, dynamics of the MAS is given in Section III, and the problem is proposed. Finite-time event-triggered controller is proposed in Section IV, also rigious proof of stablity and no zeno behavior is given. In Section V, a simulation example is given to illustrate the property of the proposed algorithm. And conclusion is given in Section VI.

II. PRELIMINARIES

For a system composed of n agents, it can be represented by a directed graph $G = (V, E, A)$, where $V = \{1, 2, \ldots, N\}$

is used to describe the collection of agents in system, $V = \{(i,j), i,j \in V, i \neq j\}$ represents the set of edges, and the weighted adjacency matrix $A = (a_{ij}) \in \mathbb{R}^{n\times n}$. The directed edge $(i,j) \in E$ means that the agent can receive information from the agent j, and the direction of information flow is irreversible. If $(i,j) \in E$ then $a_{ij} > 0$ otherwise $a_{ij} = 0$. In a directed graph G , if there is at least one node that has a directed path to other nodes, then the graph G contains a directed spanning tree. The directed path from node i to node j can be expressed as $(i, i_1), (i_1, i_2), \ldots, (i_l, j)$, where node $i_k \in V, k = 1, 2, \ldots, l$. The Laplacian matrix L of the directed graph G is defined as $L = D - A$, where $D = diag(d_1, \ldots, d_n), d_i = \sum_{i=1}^{n} a_{ij}$.

For the MAS composed of one leader agent and n follower agents, the leader agent is represented by node 0, and the follower agent is represented by node $i(i = 1, \ldots, n)$, and its communication topology is a directed graph $\bar{G}$. The communication topology of n follower agents is represented by a directed graph G is a subgraph of $\bar{G}$. The weight matrix of the graph $\bar{G}$ is $\bar{L} = L + B$. Where $B = diag\{b_1, \ldots, b_n\}$, if the follower agent i can receive the status information of leader agent, then $b_i > 0$, otherwise $b_i = 0$.

Lemma 1 [28]: For the system given by $\dot{x} = f(x)$, f(0)=0, $x \in \mathbb{R}^n$, If there exists a function $V(x) \in \mathbb{C}^1$ defined on a neighbourhood of the origin such that

1) $V(x)$ is positive definite;
2) $\dot{V}(x) + cV^{\eta}(x) \leq 0$, where $\eta \in (0, 1)$, $c > 0$.

Then the origin is locally finite-time stable, and the finite time T depending on the initial state $x(0)$ is:

$$T(x(0)) \leq \frac{V^{1-\eta}(x(0))}{c(1-\eta)} \tag{1}$$

Lemma 2 [29]: For the system given by $\dot{x} = f(x)$, $f(0) = 0$, $x \in \mathbb{R}^n$, If there exists a function $V(x) \in \mathbb{C}^1$ defined on a neighbourhood of the origin such that

1) $V(x)$ is positive definite;
2) $\dot{V}(x) + c_1 V^{\eta}(x) + c_2 V(x) \leq 0$, where $\eta \in (0, 1)$, $c_1, c_2 > 0$.

Then the origin is locally finite-time stable, and the finite time T depending on the initial state $x(0)$ is:

$$T(x(0)) \leq \frac{\ln\left[\left(c_2 V^{1-\eta}(x(0)) + c_1\right)/c_1\right]}{c_2(1-\eta)} \tag{2}$$

Lemma 3 [30]: If the directed graph $\bar{G}$ of MAS contains a directed spanning tree, then the matrix $\bar{L}$ is invertible, and all its eigenvalues have positive real parts.

Lemma 4 [31]: For $x_i \in \mathbb{R}$, $i = 1, \ldots, n$, $\alpha \in (0, 1]$, then

$$(\sum_{i=1}^{n} |x_i|)^{\alpha} \leq \sum_{i=1}^{n} |x_i|^{\alpha} \leq n^{1-\alpha}(\sum_{i=1}^{n} |x_i|)^{\alpha} \tag{3}$$

For $x \in \mathbb{R}^n$, $|\alpha| \in (0, 1)$, then

$$\left\|x^{\alpha}\right\| \leq n^{1-\alpha}\|x\|^{\alpha} \tag{4}$$

III. PROBLEM STATEMENT

A. SYSTEMS DESCRIPTIONS

Considering the second-order nonlinear MAS composed of one leader agent and n follower agents. The dynamics of the agent $j(i = 1, \ldots, n)$ are described as

$$\begin{aligned} \dot{x}_i &= v_i & x_i \in \mathbb{R}^m \\ \dot{v}_i &= u_i + f(x_i, v_i, t) & v_i \in \mathbb{R}^m \end{aligned} \tag{5}$$

where $x_i, v_i, f(x_i, v_i, t), u_i$ are the position, the velocity, the unknown nonlinear vector function, and the coupling input respectively.

The dynamics of the leader is given as below

$$\begin{aligned} \dot{x}_0 &= v_0 & x_i \in \mathbb{R}^m \\ \dot{v}_0 &= u_0 + f(x_0, v_0, t) & v_i \in \mathbb{R}^m \end{aligned} \tag{6}$$

where x_0, $v_0, f(x_0, v_0, t), u_0$ are the position, the velocity, the unknown nonlinear vector function, and the controller of the leader,respectively.

Assumption 1: The directed graph $\bar{G}$ contains a directed spanning tree.

Assumption 2: There exist non-negative constants μ_1 and μ_2, such that

$$\|f(x_i, v_i, t) - f(x_0, v_0, t)\|_2 \leq \rho_1\|x_i - x_0\|_2 + \rho_2\|v_i - v_0\|_2$$

B. NOTATION

For any vector $\psi = [\psi_1, \psi_2, \ldots, \psi_n]^T$,$\psi_i \in \mathbb{R}$, signum function $sign(\psi) = [sign(\psi_1), \ldots, sign(\psi_n)]^T$, and hyperbolic tangent function $tanh(\psi) = [tanh(\psi_1), \ldots, tanh(\psi_n)]^T$. For matrix $\Theta \in \mathbb{R}^{n\times n}$, then $\|\Theta\|_2 = \sqrt{\lambda_{max}(\Theta^T\Theta)}$, $\lambda_{max}(\bullet)$ and $\lambda_{min}(\bullet)$ represent the maximum and minimum eigenvalues of the matrix, respectively. Let I_m represents the m-dimensional unit matrix, and $\mathbf{1}_m$ represents the m-dimensional unit vector. In the following, let "$\otimes$" denotes the Kronecker product, and "$\circ$" denote the product of the corresponding elements of two vectors of the same dimension, and let "$\|\bullet\|$" instead of "$\|\bullet\|_2$" to denote the 2-norm.

IV. EVENT-TRIGGERED FINITE-TIME SLIDING MODE CONSENSUS ALGORITHM DESIGN

This section considers the finite-time consensus problem of the second-order nonlinear MAS, and its directed communication topology satisfies the Assumption

Define the consensus error of the agent i as

$$\begin{aligned} e_{1i} &= \sum_{j=1}^{n} a_{ij}\left(\bar{x}_i - \bar{x}_j\right) + b_i\bar{x}_i \\ e_{2i} &= \sum_{j=1}^{n} a_{ij}\left(\bar{v}_i - \bar{v}_j\right) + b_i\bar{v}_i \end{aligned} \tag{7}$$

where $\bar{x}_i$ and $\bar{v}_i$ respectively represent the position tracking errors and speed tracking errors of the agent i, and $\dot{\bar{x}}(t) = \bar{v}_i(t)$, which are defined as follows

$$\bar{x}_i(t) = x_i(t) - x_0(t) + d_i$$
$$\bar{v}_i(t) = v_i(t) - v_0(t) \tag{8}$$

where $d_i \in \mathbb{R}^m$ represents the distance expected to maintain between the follower agent and the leader agent, and $D = [d_1^T, \ldots, d_n^T]^T$. Let $\varepsilon_1 = [e_{11}^T, \ldots, e_{1n}^T]^T$, $\varepsilon_2 = [e_{21}^T, \ldots, e_{2n}^T]^T$, $\bar{X} = [\bar{x}_1^T, \ldots, \bar{x}_n^T]^T$, $\bar{V} = [\bar{v}_1^T, \ldots, \bar{v}_n^T]^T$, then

$$\varepsilon_1 = (L + B) \otimes I_m \cdot \bar{X}$$
$$\varepsilon_2 = (L + B) \otimes I_m \cdot \bar{V} \tag{9}$$

Taking the derivative of (8), we can obtain

$$\dot{\varepsilon}_1 = \varepsilon_2$$
$$\dot{\varepsilon}_2 = (L + B) \otimes I_m \cdot (\widetilde{U} + F - \mathbf{1}_n \otimes f_0) \tag{10}$$

where $\tilde{U} = [\tilde{u}_1^{\mathrm{T}}, \ldots, \tilde{u}_n^{\mathrm{T}}]^{\mathrm{T}}$, $\tilde{u}_i = u_i - u_0$, and $F = [f(x_1, v_1, t)^{\mathrm{T}}, \ldots, f(x_n, v_n, t)^{\mathrm{T}}]^{\mathrm{T}}$. The sliding surface $\bar{S}$ is defined as

$$\bar{S} = C_1 \otimes I_m \cdot \mathrm{sign}(\varepsilon_1) + C_2 \otimes I_m \cdot \varepsilon_2 + C_3 \otimes I_m \cdot \varepsilon_1 \tag{11}$$

where $C_1 = \mathrm{diag}(c_{11}, \ldots, c_{1n})$, $C_2 = \mathrm{diag}(c_{21}, \ldots, c_{2n})$, $C_3 = \mathrm{diag}(c_{31}, \ldots, c_{3n})$, $c_{ij} > 0$, $\forall i, j$. If $\beta \gg 1$, then $\mathrm{sign}(\varepsilon_1) \approx \tanh(\beta\varepsilon_1)$, and it can be obtained that

$$\bar{S} = C_1 \otimes I_m \cdot \tanh(\beta\varepsilon_1) + C_2 \otimes I_m \cdot \varepsilon_2 + C_3 \otimes I_m \cdot \varepsilon_1 \tag{12}$$

Taking the derivative of (11), we can obtain

$$\begin{aligned}\dot{\bar{S}} = {} & C_1 \otimes I_m \cdot \beta\Big[(\mathbf{1}_{mn} - \tanh^2(\beta\varepsilon_1)) \circ \varepsilon_2\Big] \\ & + [C_2 \cdot (L + B)] \otimes I_m \cdot \Big(\tilde{U} + F - \mathbf{1}_n \otimes f_0\Big) \\ & + C_3 \otimes I_m \cdot \varepsilon_2 \end{aligned} \tag{13}$$

The integral sliding mode surfaceis S defined as follows

$$S = \bar{S} - \int_0^t W^\eta(t)dt \tag{14}$$

where $W(t) = -(L + B) \otimes I_m \cdot \bar{S}$, and $\eta \in (0.5, 1)$ is strictly the ratio of positive odd numbers. The derivative of (13) is

$$\dot{S} = \dot{\bar{S}} - W^\eta \tag{15}$$

The event-triggered finite-time sliding mode consensus controller is designed as follows

$$\begin{aligned}\tilde{U}(t) = {} & [C_2 \cdot (L + B)]^{-1} \otimes I_m \cdot \Big\{-C_1 \otimes I_m \cdot \beta\Big[\Big(\mathbf{1}_{mn} \\ & - \tanh^2(\beta\varepsilon_1)\Big) \circ \varepsilon_2\Big] - C_3 \otimes I_m \cdot \varepsilon_2(t_k) \\ & + W^\eta(t_k) - K_1 \otimes I_m \cdot \mathrm{sign}(S(t_k)) \\ & - \mu \cdot \mathrm{sign}(S(t_k)) - K_2 \otimes I_m \cdot S(t_k)\Big\} \end{aligned} \tag{16}$$

where $K_1 = \mathrm{diag}(k_{11}, \ldots, k_{1n})$, $K_2 = \mathrm{diag}(k_{21}, \ldots, k_{2n})$, $k_{ij} > 0$, $\forall i, j$, and $\mu = \|C_2 \cdot (L+B)\|[\|(L+B)^{-1}\|(\rho_1\|\varepsilon_1\| + \rho_2\|\varepsilon_2\|) + \rho_1\|D\|]$. For $t \in [t_k, t_{k+1})$, t_k is the latest event-triggered time for all follower agents, and the follower agent only updates the control protocol at the event-triggered time. Since the leader agent moves independently and its trajectory is not affected by the follower agents, the leader agent has no trigger time. The measurement error of the event-triggered mechanism is defined as

$$\begin{aligned} e(t) = {} & C_1 \otimes I_m \cdot \beta\Big[\Big(\mathbf{1}_{mn} - \tanh^2(\beta\varepsilon_1)\Big) \circ \varepsilon_2 \\ & - \Big(\mathbf{1}_{mn} - \tanh^2(\beta\varepsilon_1(t_k))\Big) \circ \varepsilon_2(t_k)\Big] \\ & + C_3 \otimes I_m \cdot (\varepsilon_2 - \varepsilon_2(t_k)) - W^\eta + W^\eta(t_k) \\ & + K_1 \otimes I_m \cdot (\mathrm{sign}(S) - \mathrm{sign}(S(t_k))) \\ & + \mu \cdot (\mathrm{sign}(S) - \mathrm{sign}(S(t_k))) \\ & + K_2 \otimes I_m \cdot (S - S(t_k)) \end{aligned} \tag{17}$$

Theorem 1: Consider the leader-following second-order non-linear MAS (5) and (6), the event-trigger error controller (16) can make the tracking errors converge to zero in finite time, if the event-triggered function is chosen as

$$h(t) = \|e\| - \lambda_{\min}(K_1) - \lambda_{\min}(K_2)\|S\| + \delta \tag{18}$$

where $0 < \delta < \lambda_{\min}(K_1)$, and the convergence time T meets

$$\begin{aligned} T \leq {} & \frac{\sqrt{2V_1(0)}}{\delta} + \frac{(2V_2(0))^{\frac{1-\eta}{2}}}{(1-\eta)\lambda_{\min}(L+B)} \\ & + \frac{\ln\Big(\sqrt{2V_3(0)}\lambda_{\min}(C_2^{-1}C_3)/\lambda_{\min}(C_2^{-1}C_1) + 1\Big)}{\lambda_{\min}\Big(C_2^{-1}C_3\Big)} \end{aligned} \tag{19}$$

where V_1, V_2 and V_3 are the selected Lyapunov functions which are given in the proof.
When the event-triggered function $h(t) \geq 0$, whch means that $\|e\| - \lambda_{\min}(K_1) - \lambda_{\min}(K_2)\|S\| + \delta$, the event is triggered, then all followers update the control protocol at the same time.

Proof: First select the Lyapunov function as

$$V_1 = \frac{1}{2}S^{\mathrm{T}}S \tag{20}$$

Taking the derivative of (19), substituting (13) and (15) into it, we can obtain

$$\begin{aligned}\dot{V}_1 = {} & S^{\mathrm{T}}[C_2 \cdot (L + B)] \otimes I_m \cdot (F - \mathbf{1}_n \otimes f_0) \\ & + S^{\mathrm{T}}\Big\{C_1 \otimes I_m \cdot \beta\Big[\Big(\mathbf{1}_{mn} - \tanh^2(\beta\varepsilon_1)\Big) \circ \varepsilon_2\Big] \\ & + [C_2 \cdot (L + B)] \otimes I_m \cdot \tilde{U} \\ & + C_3 \otimes I_m \cdot \varepsilon_2 - W^\eta\Big\} \end{aligned} \tag{21}$$

Based (15 and (16)

$$\begin{aligned}\dot{V}_1 = {} & S^{\mathrm{T}}[C_2 \cdot (L + B)] \otimes I_m \cdot (F - \mathbf{1}_n \otimes f_0) \\ & + S^{\mathrm{T}}(e - K_1 \otimes I_m \cdot \mathrm{sign}(S) \\ & - \mu \cdot \mathrm{sign}(S) - K_2 \otimes I_m \cdot S \\ \leq {} & \|S\|\Big[\|C_2 \cdot (L + B)\|\|F - \mathbf{1}_n \otimes f_0\| + \|e\| \\ & - (\lambda_{\min}(K_1) + \mu) - \lambda_{\min}(K_2)\|S\|\Big] \end{aligned} \tag{22}$$

Based on Assumption 2 and the properties of the norm, we can find that

$$
\begin{aligned}
&\|F-\mathbf{1}_n\otimes f_0\| \\
&= \left\|(f_1-f_0)^{\mathrm T},\ldots,(f_n-f_0)^{\mathrm T}]^{\mathrm T}\right\| \\
&\le \left\|(\|f_1-f_0\|,\ldots,\|f_n-f_0\|)^{\mathrm T}\right\| \\
&\le \|(\rho_1\|x_1-x_0\|+\rho_2\|v_1-v_0\|,\ldots, \\
&\qquad \rho_1\|x_n-x_0\|+\rho_2\|v_n-v_0\|)^{\mathrm T}\| \\
&\le \|(\rho_1\|\bar x_1-d_1\|+\rho_2\|\bar v_1\|,\ldots, \\
&\qquad \rho_1\|\bar x_n-d_n\|+\rho_2\|\bar v_n\|)^{\mathrm T}\| \\
&\le \|[\rho_1(\|\bar x_1\|+\|d_1\|)+\rho_2\|\bar v_1\|,\ldots, \\
&\qquad \rho_1(\|\bar x_n\|+\|d_n\|)+\rho_2\|\bar v_n\|]^{\mathrm T}\| \\
&\le \rho_1\left\|(\|\bar x_1\|,\ldots,\|\bar x_n\|)^{\mathrm T}\right\|+\rho_1\left\|(\|d_1\|,\ldots,\|d_n\|)^{\mathrm T}\right\| \\
&+\ \rho_2\left\|(\|\bar v_1\|,\ldots,\|\bar v_n\|)^{\mathrm T}\right\| \\
&\le \rho_1\left\|\bar X\right\|+\rho_2\left\|\bar V\right\|+\rho_1\|D\| \\
&\le \left\|(L+B)^{-1}\right\|(\rho_1\|\varepsilon_1\|+\rho_2\|\varepsilon_2\|)+\rho_1\|D\| \qquad (23)
\end{aligned}
$$

Hence using (22) and $\mu=\|C_2\cdot(L+B)\|[\|(L+B)^{-1}\|(\rho_1\|\varepsilon_1\|+\rho_2\|\varepsilon_2\|)+\rho_1\|D\|]$, we have

$$V_1 \le \|S\|(\|e\|-\lambda_{\min}(K_1)-\lambda_{\min}(K_2)\|S\|) \qquad (24)$$

When the event-triggered function $h(t)\le 0$, $\|e\|\le \lambda_{\min}(K_1)+\lambda_{\min}(K_2)\|S\|-\delta$, then

$$V_1 \le -\delta\|S\| \le -\sqrt{2}\delta V_1^{\frac{1}{2}} \qquad (25)$$

According to Lemma 1, it can be seen that under the action of the controller (16), the sliding mode surface S can realize $S=0$ and $\dot S=0$ in finite time, and the convergence time T_1 satisfies

$$T_1 \le \frac{\sqrt{2V_1(0)}}{\delta} \qquad (26)$$

Then select the Lyapunov function as

$$V_2=\frac{1}{2}\bar S^{\mathrm T}[(L+B)\otimes I_m]^{\mathrm T}[(L+B)\otimes I_m]\bar S \qquad (27)$$

Let $p_i=\sum_{j\in n_i}a_{ij}(\bar s_i-\bar s_j)+b_i\bar s_i$, $P=[p_1^{\mathrm T},\ldots,p_n^{\mathrm T}]^{\mathrm T}=(L+B)\otimes I_m\cdot\bar S$, then

$$V_2=\frac{1}{2}P^{\mathrm T}P \qquad (28)$$

When $\dot S=0$, then $\dot{\bar S}=W^\eta=-[(L+B)\otimes I_m\cdot\bar S]^\eta$, and taking the derivative of (27), we can obtain

$$
\begin{aligned}
\dot V_2 &= -P^{\mathrm T}[(L+B)\otimes I_m][(L+B)\otimes I_m\cdot\bar S]^\eta \\
&= -P^{\mathrm T}[(L+B)\otimes I_m]P^\eta \\
&\le -\lambda_{\min}(L+B)P^{\mathrm T}P^\eta \qquad (29)
\end{aligned}
$$

Since the positive odd ratio parameter $\eta\in(0.5,1)$, and combined with Lemma 4, we know

$$
\begin{aligned}
\dot V_2 &\le -\lambda_{\min}(L+B)(\sum_{i=1}^{mn}|p_i|^{1+\eta}) \\
&\le -\lambda_{\min}(L+B)(\|P\|^2)^{\frac{1+\eta}{2}} \\
&\le -2^{\frac{1+\eta}{2}}\lambda_{\min}(L+B)V_2^{\frac{1+\eta}{2}} \qquad (30)
\end{aligned}
$$

According to Lemma 1, under the action of the controller (16), the state of the system can reach and remain on the sliding mode surface $\bar S=0$ within finite time, and the convergence time T_2 satisfies

$$T_2 \le \frac{(2V_2(0))^{\frac{1-\eta}{2}}}{(1-\eta)\lambda_{\min}(L+B)} \qquad (31)$$

Based on (12) and the sliding mode surface $\bar S=0$, we can obtain

$$\varepsilon_2=-\left(C_2^{-1}C_1\right)\otimes I_m\cdot\mathrm{sign}(\varepsilon_1)-\left(C_2^{-1}C_3\right)\otimes I_m\cdot\varepsilon_1 \qquad (32)$$

Selecting the Lyapunov function as

$$V_3=\frac{1}{2}\varepsilon_1^{\mathrm T}\varepsilon_1 \qquad (33)$$

Taking the derivative of (33) and substituting (32) into the derivative, we can obtain

$$
\begin{aligned}
\dot V_3 &= \varepsilon_1^{\mathrm T}\Big[-\left(C_2^{-1}C_1\right)\otimes I_m\cdot\mathrm{sign}(\varepsilon_1) \\
&\qquad -\left(C_2^{-1}C_3\right)\otimes I_m\cdot\varepsilon_1\Big] \\
&\le -\lambda_{\min}\left(C_2^{-1}C_1\right)\|\varepsilon_1\|-\lambda_{\min}\left(C_2^{-1}C_3\right)\|\varepsilon_1\|^2 \\
&\le -\sqrt{2}\lambda_{\min}\left(C_2^{-1}C_1\right)V_3^{\frac{1}{2}}-2\lambda_{\min}\left(C_2^{-1}C_3\right)V_3 \qquad (34)
\end{aligned}
$$

It can be seen from Lemma 2 that the position consensus error of the MAS can achieve in finite time. And on the sliding surface , it can be seen from (33) that the speed consensus error can also achieve in finite time. The convergence time satisfies

$$T_3 \le \frac{\ln\left(\sqrt{2V_3(0)}\lambda_{\min}\left(C_2^{-1}C_3\right)/\lambda_{\min}\left(C_2^{-1}C_1\right)+1\right)}{\lambda_{\min}\left(C_2^{-1}C_3\right)} \qquad (35)$$

Next, we need to analyze whether the system has a minimum event-triggered time interval strictly greater than zero, which means that there is no zeno behavior. When the event-triggered function (18) satisfies $h(t)\ge 0$, the event is triggered. Combining (17), we can see that between any two adjacent event-triggered moments, $\|e\|$ increases from zero to $\lambda_{\min}(K_1)+\lambda_{\min}(K_2)\|S\|-\delta$. Therefore, when the growth rate is the fastest, the event-triggered time interval is the smallest. In this case, when the minimum time interval is a value greater than zero, it can be guaranteed that there is no zeno behavior. ■

Theorem 2: Based on Assumptions 1 and 2, the MAS which is given in (5) and (6). The event-triggered errror controller is designed in (16) and event-triggered function is given in (18), the system does not have zeno behavior under any initial conditions, and the event-triggered time interval T_m satisfies

$$T_{\mathrm m} \ge \frac{\lambda_{\min}(K_1)-\delta}{\sigma} \qquad (36)$$

where σ represents the maximum growth rate of $\|e\|$ which is given as below

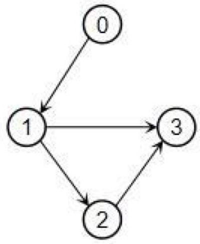


FIGURE 1. Communication topology.

$$\sigma = 2mn\beta^2\|C_1\|\|\varepsilon_2^2\|_{\max}\left(\beta\sqrt{mn}\|C_1\| + \|C_3\|\right)$$
$$\|\dot{\varepsilon}_2\|_{\max} + \eta(mn)^{3-2\eta}\|L+B\|(2V_2(0))^{\frac{2\eta-1}{2}} \tag{37}$$

Proof: When the sliding mode surface $S = 0$ and $\dot{S} = 0$, combining (16) to derive $\|e\|$ as follows

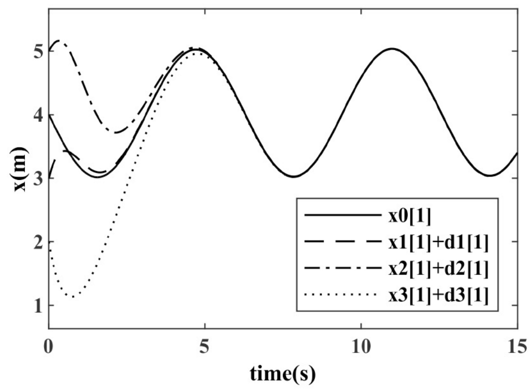
$$\begin{aligned}
&\frac{d\|e\|}{dt} \\
&\le \left\|\frac{de}{dt}\right\| \\
&\le \left\|\frac{d}{dt}\left\{C_1 \otimes I_m \cdot \beta\left[\left(\mathbf{1}_{mn} - \tanh^2(\beta\varepsilon_1)\right) \circ \varepsilon_2\right]\right.\right. \\
&\qquad \left.\left. + C_3 \otimes I_m \cdot \varepsilon_2 - W^\eta\right\}\right\| \\
&\le \beta\|C_1\|\| - 2\beta\tanh(\beta\varepsilon_1) \circ \left(\mathbf{1}_{mn} - \tanh^2(\beta\varepsilon_1)\right) \circ \varepsilon_2^2 \\
&\quad + \left(\mathbf{1}_{mn} - \tanh^2(\beta\varepsilon_1)\right) \circ \dot{\varepsilon}_2\| + \|C_3\|\|\dot{\varepsilon}_2\| \\
&\quad + \left\|\eta W^{\eta-1} \circ \dot{W}\right\| \\
&\le 2mn\beta^2\|C_1\|\left\|\varepsilon_2^2\right\| + \left(\beta\sqrt{mn}\|C_1\| + \|C_3\|\right)\|\dot{\varepsilon}_2\| \\
&\quad + \eta\left\|W^{\eta-1}\right\|\left\|-(L+B) \otimes I_m \cdot \dot{\dot{S}}\right\| \\
&\le 2mn\beta^2\|C_1\|\left\|\varepsilon_2^2\right\| + \left(\beta\sqrt{mn}\|C_1\| + \|C_3\|\right)\|\dot{\varepsilon}_2\| \\
&\quad + \eta\|L+B\|\left\|W^{\eta-1}\right\|\left\|W^\eta\right\|
\end{aligned} \tag{38}$$

By Lemma 4, we can obtain

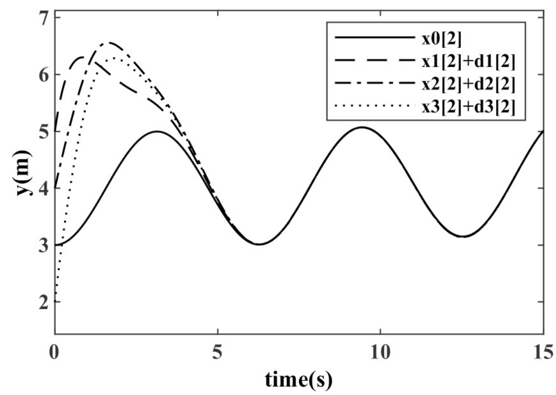
$$\begin{aligned}
&\eta\|L+B\|\left\|W^{\eta-1}\right\|\left\|W^\eta\right\| \\
&\le \eta(mn)^{3-2\eta}\|L+B\|\|W\|^{2\eta-1} \\
&\le \eta(mn)^{3-2\eta}\|L+B\|\|P\|^{2\eta-1} \\
&\le \eta(mn)^{3-2\eta}\|L+B\|(2V_2(0))^{\frac{2\eta-1}{2}}
\end{aligned} \tag{39}$$

Since the MAS (5) can achieve finite-time consensus under the action of the controller (16) and the event-triggered function (18), both $\|\varepsilon_2^2\|$ and $\|\dot{\varepsilon}_2\|$ have upper bounds, which are taken as $\|\varepsilon_2^2\|_{\max}$ and $\|\dot{\varepsilon}_2\|_{\max}$. And combining (39), we can obtain

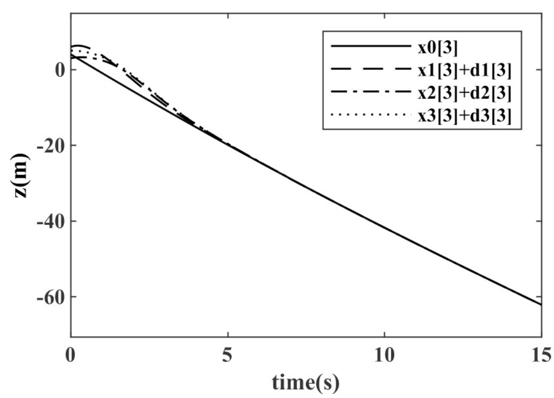
$$\begin{aligned}
\frac{d\|e\|}{dt} &\le 2mn\beta^2\|C_1\|\|\varepsilon_2^2\|_{\max} + \left(\beta\sqrt{mn}\|C_1\| + \|C_3\|\right) \\
&\quad \|\dot{\varepsilon}_2\|_{\max} + \eta(mn)^{3-2\eta}\|L+B\|(2V_2(0))^{\frac{2\eta-1}{2}} \\
&\le \sigma
\end{aligned} \tag{40}$$



(a) x-axis



(b) y-axis



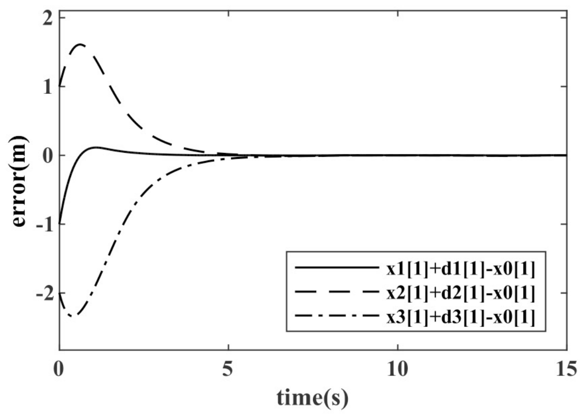
(c) z-axis

FIGURE 2. Position tracking trajectory.

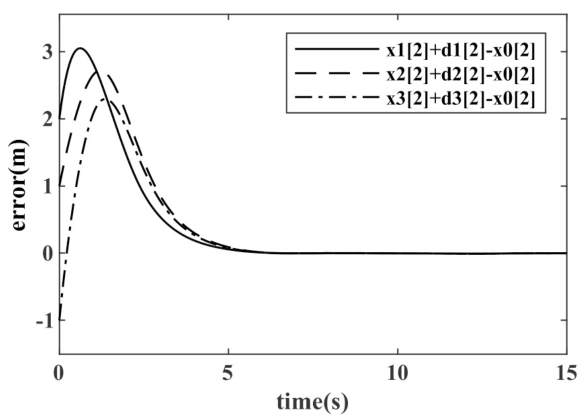
For any $t \in [t_k, t_{k+1})$, t_k is the latest event-triggered moment for all follower agents, the time interval $T_m = t_{k+1} - t_k$, and $\|e(t_k)\| = 0$ at the event-triggered moment, we can obtain

$$\|e(t)\| - \|e(t_k)\| = \|e(t)\| \le (t - t_k)\sigma \le T_m\sigma \tag{41}$$

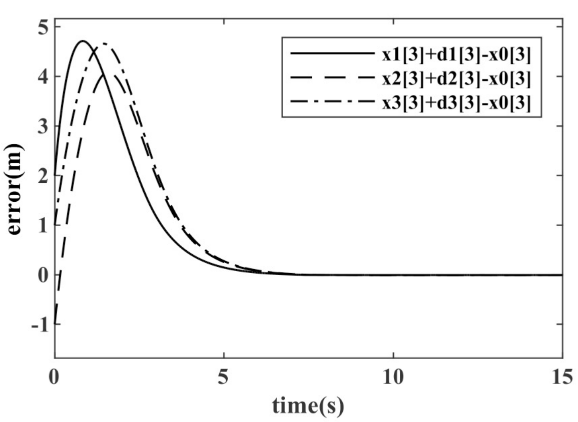
When the event-triggered function (18) satisfies $h(t) \ge 0$, the event is triggered, we have



(a) x-axis



(b) y-axis

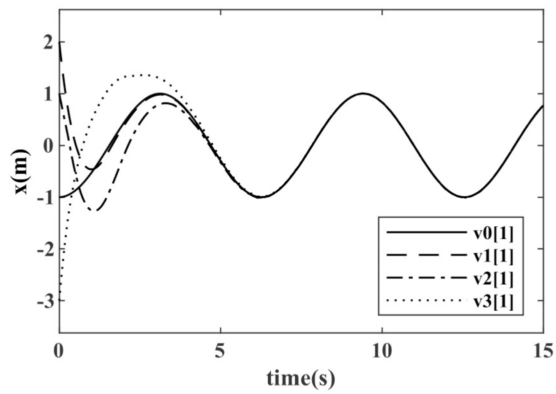


(c) z-axis

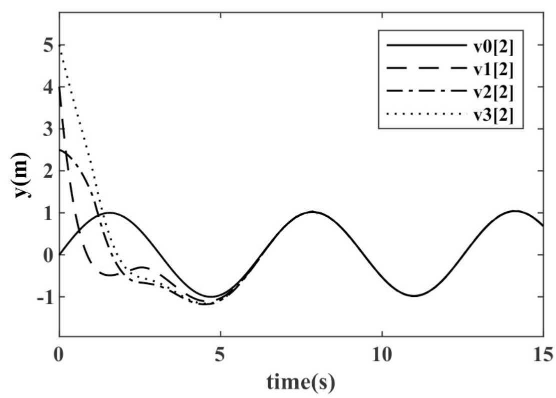
FIGURE 3. **Position tracking error.**

$$\begin{aligned}\|e(t)\| &\geq \lambda_{\min}(K_1) + \lambda_{\min}(K_2)\|S\| - \delta \\ &\geq \lambda_{\min}(K_1) - \delta \end{aligned} \tag{42}$$

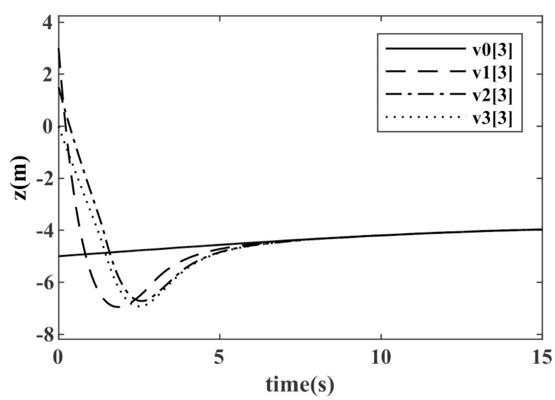
Combining (41) and (42), we can obtain (36). It can be seen from (36) that the event-triggered time interval is strictly greater than zero, so there is no zeno behavior. ■



(a) x-axis



(b) y-axis



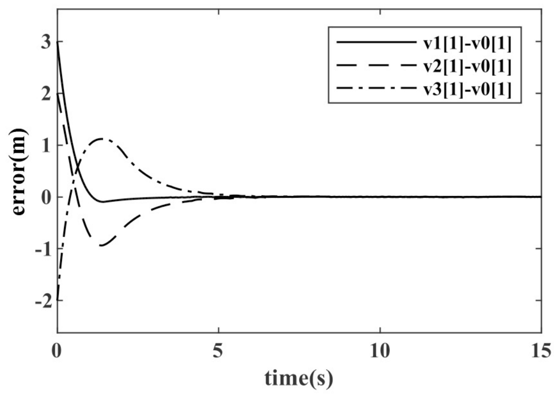
(c) z-axis

FIGURE 4. **Velocity tracking trajectory.**

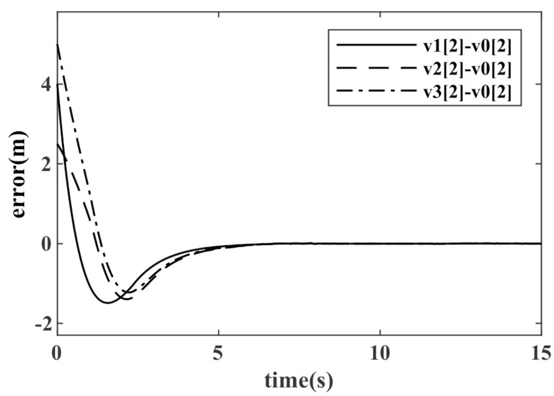
V. EXAMPLE SIMULATION

Considering the MAS composed of four agents, including three follower agents and one leader agent, and the leader node is 0. The directed communication topology is shown in Fig. 1. Hence we have

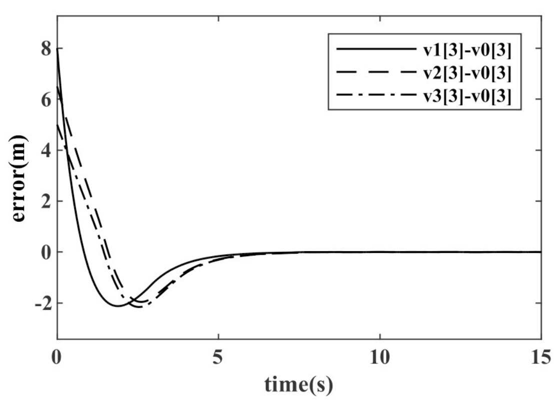
$$L = \begin{bmatrix} 0 & 0 & 0 \\ -1 & 1 & 0 \\ -1 & -1 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$



(a) x-axis



(b) y-axis



(c) z-axis

FIGURE 5. Velocity tracking error.

The simulation model of MAS as follows

$$\dot{x}_i = v_i \qquad x_i \in \mathbb{R}^m$$
$$\dot{v}_i = u_i + f(x_i, v_i, t) \qquad v_i \in \mathbb{R}^m$$

The initial positions of the system are selected as $x_0(0) = [4, 3, 4]^{\mathrm{T}}$, $x_1(0) = [1, 3, 4]^{\mathrm{T}}$, $x_2(0) = [0, -1, -2]^{\mathrm{T}}$, $x_3(0) = [-6, -6, -3]^{\mathrm{T}}$. The initial states of speed are selected as $v_0(0) = [-1, 0, -5]^{\mathrm{T}}$, $v_1(0) = [2, 4, 3]^{\mathrm{T}}$,

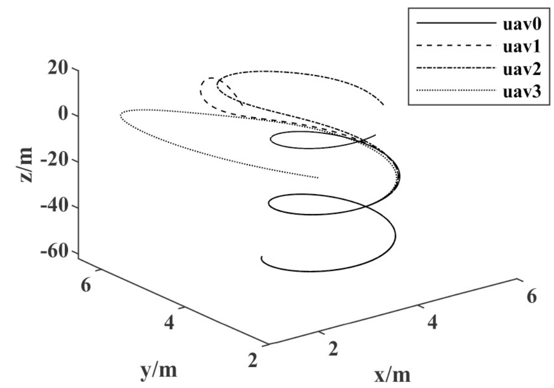


FIGURE 6. Three-dimensional trajectory of MAS.

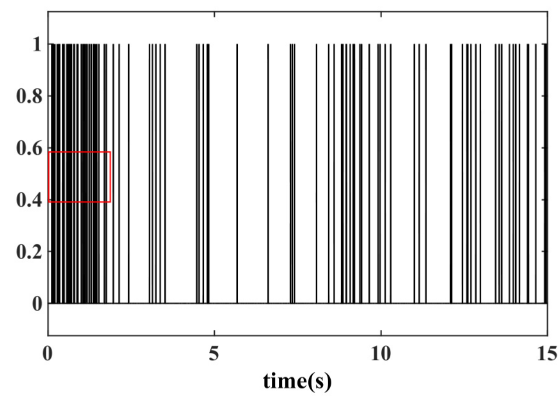


FIGURE 7. Event-triggered time.

$v_2(0) = [1, 2.5, 1.5]^{\mathrm{T}}$, $v_3(0) = [-3, 5, 0]^{\mathrm{T}}$. Select the distances expected to maintain between the follower agent and the leader agent as $d_1 = [2, 2, 2]^{\mathrm{T}}$, $d_2 = [5, 5, 5]^{\mathrm{T}}$, $d_3 = [8, 8, 8]^{\mathrm{T}}$. Take the nonlinear function as $f(x_i, v_i, t) = -0.01(\cos(t)\cos(x_i) + \sin(\dot{x}_i))$. Adopting controller (15) and event-triggered function (17), where $\beta = 100$, $\rho_1 = \rho_2 = 0.0025$, $\eta = 0.6$, $\delta = 0.001$, $K_1 = \mathrm{diag}(0.01, 0.01, 0.01)$, $K_2 = \mathrm{diag}(0.01, 0.01, 0.01)$, $C_1 = \mathrm{diag}(0.001, 0.001, 0.001)$, $C_2 = \mathrm{diag}(0.2, 0.2, 0.2)$, $C_3 = \mathrm{diag}(0.2, 0.2, 0.2)$. The controller of the leader agent is designed as $u_0 = [\sin(t), \cos(t), 0.1\cos(0.08t)]^{\mathrm{T}}$.

Under the action of the controller (16) and the event-triggered function (18), the position tracking trajectory, position tracking errors, velocity tracking trajectory, velocity tracking error and three-dimensional trajectory of MAS are shown in Fig. 2, Fig. 3, Fig. 4, Fig. 5, and Fig. 6 respectively.

From Fig. 2 to Fig. 6, it can be seen that the position and speed of all follower agents can accurately track the leader agent in finite time, and the upper bound of finte time of reaching consensus can be predicted by (19).

The event-triggered time interval of MAS for the 15s is shown Fig. 7. In order to show the superior performance of the proposed event-triggered control strategy in reducing the energy dissipation of the system and the update frequency of the controller, the denser part marked by the rectangular

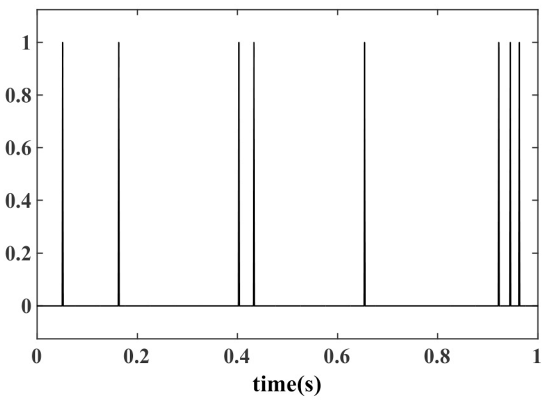


FIGURE 8. Magnification of the marked area.

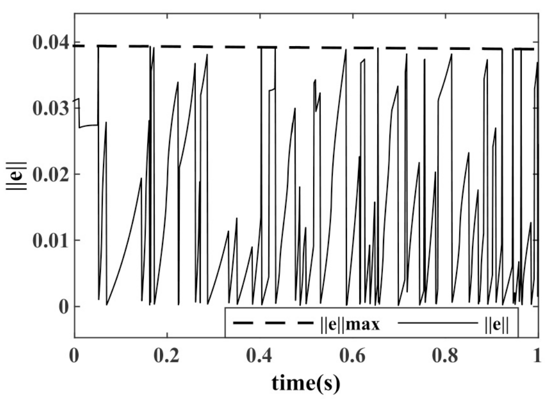


FIGURE 9. Variation trend of measurement error norm $\|e\|$ and threshold $\|e\|_{max}$.

box is enlarged, as shown in Fig. 8. Though the triggered times is a lot, there is also interval exists between these event-triggering instants for the 1s, we can know that there is no zeno behavior.

Fig. 9 shows the evolution process of the measurement error norm $\|e\|$ of the system in the time period marked by the rectangular box, where threshold $\|e\|_{\max}= \lambda_{\min}(K_1) + \lambda_{\min}(K_2)\|S\| - \delta$. When the value of $\|e\|$ increases from zero to $\|e\|_{\max}$, the event is triggered. At this time, all follower agents interact with information and update the controller at the same time.

VI. CONCLUSION

In this paper, we deal with the finite-time leader-following consensus problem of the second-order MAS. The proposed finite-time integral sliding mode algorithm effectively eliminates the influence of nonlinear dynamics in the system due to its inherited robustness, and forces the states of MAS to reach the designed terminal sliding mode surface in finite time and maintain on it. Compared with the traditional time-triggered control strategy, the proposed algorithm combined with the event-triggered mechanism economizes the communication bandwidth and computing resources of the system. For the purpose of verifying the theoretical analysis and the effectiveness of the proposed algorithm, a simulation example is conducted for the MAS. Also, we can find that the event-triggered time interval has a lower limit, so zeno behavior can be avoided.

REFERENCES

[1] R. Olfati-Saber, J. A. Fax, and R. M. Murray, “Consensus and cooperation in networked multi-agent systems,” *Proc. IEEE*, vol. 95, no. 1, pp. 215–233, Jan. 2007.

[2] K. J. Astrom and B. Wittenmark, *Linear Networks and Systems*. Hoboken, NJ, USA: Prentice-Hall, 1990.

[3] T. Chen and B. Francis, “Optimal sampled-data control systems,” *Proc. IEEE*, vol. 86, no. 4, pp. 741–742, Apr. 1995.

[4] R. Olfati-Saber and R. M. Murray, “Consensus problems in networks of agents with switching topology and time-delays,” *IEEE Trans. Autom. Control*, vol. 49, no. 9, pp. 1520–1533, Sep. 2004.

[5] J. Xu, L. Xie, T. Li, and K. Y. Lum, “Consensus of multi-agent systems with general linear dynamics via dynamic output feedback control,” *IET Control Theory Appl.*, vol. 7, no. 1, pp. 108–115, 2013.

[6] D. Zhao, T. Zou, S. Li, and Q. Zhu, “Adaptive backstepping sliding mode control for leader-follower multi-agent systems,” *IET Control Theory Appl.*, vol. 6, no. 8, pp. 1109–1117, 2012.

[7] W. Yu, G. Chen, M. Cao, and J. Kurths, “Second-order consensus for multiagent systems with directed topologies and nonlinear dynamics,” *IEEE Trans. Syst. Man Cybern. B, Cybern.*, vol. 40, no. 3, pp. 881–891, Jun. 2010.

[8] X. Lu, S. Chen, and J. Lu, “Finite-time tracking for double-integrator multi-agent systems with bounded control input,” *IET Control Theory Appl.*, vol. 7, no. 11, pp. 1562–1573, 2013.

[9] X. Lu, R. Lu, S. Chen, and J. Lu, “Finite-time distributed tracking control for multi-agent systems with a virtual leader,” *IEEE Trans. Circuits Syst. I, Reg. Papers*, vol. 60, no. 2, pp. 352–362, Feb. 2013.

[10] K. J. Aström, “Event based control,” in *Analysis and Design of Nonlinear Control Systems*. Heidelberg, Germany: Springer, 2008, pp. 127–147.

[11] A. Anta and P. Tabuada, “To sample or not to sample: Self-triggered control for nonlinear systems,” *IEEE Trans. Autom. Control*, vol. 55, no. 9, pp. 2030–2042, Sep. 2010.

[12] D. V. Dimarogonas, E. Frazzoli, and K. H. Johansson, “Distributed event-triggered control for multi-agent systems,” *IEEE Trans. Autom. Control*, vol. 57, no. 5, pp. 1291–1297, May 2012.

[13] A. Girard, “Dynamic triggering mechanisms for event-triggered control,” *IEEE Trans. Autom. Control*, vol. 60, no. 7, pp. 1992–1997, Jul. 2015.

[14] H. Zhang, D. Yue, X. Yin, S. Hu, and C. Dou, “Finite-time distributed event-triggered consensus control for multi-agent systems,” *Inf. Sci.*, vol. 339, pp. 132–142, Apr. 2016.

[15] Y. Dong and J. G. Xian, “Finite-time event-triggered consensus for nonlinear multi-agent networks under directed network topology,” *IET Control Theory Appl.*, vol. 11, no. 15, pp. 2458–2464, 2017.

[16] A.-M. Zou, K. D. Kumar, and Z.-G. Hou, “Distributed consensus control for multi-agent systems using terminal sliding mode and Chebyshev neural networks,” *Int. J. Robust Nonlinear Control*, vol. 23, no. 3, pp. 334–357, 2013.

[17] H. Li, X. Liao, and G. Chen, “Leader-following finite-time consensus in second-order multi-agent networks with nonlinear dynamics,” *Int. J. Control Autom. Syst.*, vol. 11, no. 2, pp. 422–426, 2013.

[18] Y. Zhu, X. Guan, X. Luo, and S. Li, “Finite-time consensus of multi-agent system via nonlinear event-triggered control strategy,” *IET Control Theory Appl.*, vol. 9, no. 17, pp. 2548–2552, 2015.

[19] S. Yu and X. Long, “Finite-time consensus for second-order multi-agent systems with disturbances by integral sliding mode,” *Automatica*, vol. 54, no. 4, pp. 158–165, 2015.

[20] R. R. Nair, L. Behera, and S. Kumar, “Event-triggered finite-time integral sliding mode controller for consensus-based formation of multirobot systems with disturbances,” *IEEE Trans. Control Syst. Technol.*, vol. 27, no. 1, pp. 39–47, Jan. 2019.

[21] A. Selivanov and E. Fridman, “Event-triggered H∞ control: A switching approach,” *IEEE Trans. Autom. Control*, vol. 61, no. 10, pp. 3221–3226, Oct. 2016.

[22] X. G. Meng and T. W. Chen, "Event based agreement protocols for multi-agent networks," *Automatica*, vol. 49, no. 7, pp. 2125–2132, 2013.

[23] Y. Fan, L. Liu, and G. Feng, "Self-triggered consensus for multi-agent systems with Zeno-free triggers," *IEEE Trans. Autom. Control*, vol. 60, no. 10, pp. 2779–2784, Oct. 2015.

[24] W. Zhu and Z. P. Jiang, "Event-based leader-following consensus of multi-agent systems with input time delay," *IEEE Trans. Autom. Control*, vol. 60, no. 5, pp. 1362–1367, May 2015.

[25] X. L. Yi, K. Liu, D. V. Dimarogonas, and K. H. Johansson, "Distributed dynamic event-triggered control for multi-agent systems," in *Proc. IEEE 56th Annu. Conf. Decis. Control*, 2017, pp. 6683–6698.

[26] W. Ren and E. Atkins, "Distributed multi-vehicle coordinated control via local information exchange," *Int. J. Robust Nonlinear Control*, vol. 17, nos. 10–11, pp. 1002–1033, 2007.

[27] J. C. Ren, J. Sun, and J. Fu, "Finite-time event-triggered sliding mode control for one-sided Lipschitz nonlinear systems with uncertainties," *Nonlinear Dyn.*, vol. 103, no. 3, pp. 865–882, 2021.

[28] S. P. Bhat and D. S. Bernstein, "Finite-time stability of continuous autonomous systems," *SIAM J. Control Optim.*, vol. 38, no. 3, pp. 751–766, 2000.

[29] W. Gao and J. C. Hung, "Variable structure control of nonlinear systems: A new approach," *IEEE Trans. Ind. Electron.*, vol. 40, no. 1, pp. 45–55, Feb. 1993.

[30] S. Khoo, L. Xie, and Z. Man, "Robust finite-time consensus tracking algorithm for multirobot systems," *IEEE/ASME Trans. Mechatronics*, vol. 14, no. 2, pp. 219–228, Apr. 2009.

[31] S. Li, H. Du, and X. Lin, "Finite-time consensus algorithm for multi-agent systems with double-integrator dynamics," *Automatica*, vol. 47, no. 8, pp. 1706–1712, 2011.