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Selfish Yet Optimal Routing by Adjusting Perceived Traffic Information of Road Networks

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ABSTRACT Traffic congestion in urban areas causes economic and time loss. Such traffic congestion is caused by selfish routing where users aim to minimize their own travel time. Even if a navigation system provides them with recommended routes, they may not follow the routes, due to dissatisfaction with the expected travel time. In this article, inspired by the concept of "Nudge," we propose "selfish yet optimal routing," where all users rationally aim to minimize their own travel time but social optimal routing emerges from such selfish routing by adjusting their perceived traffic information appropriately. We propose a scheme to derive nudging traffic information that fills the gap between selfish routing criterion and altruistic one by internalizing the marginal cost into the perceived traffic information. Users will conduct the selfish routing under their perceived traffic information, which unconsciously results in the optimal routing. Through numerical experiments using both the artificial road network and the real one, we show that the proposed scheme achieves almost the same performance compared with the optimal routing. In addition, the proposed scheme reduces the average travel time by 19.1% compared with notification of actual traffic information, in case of the central-area road network of Nagoya city, Japan.

INDEX TERMS Selfish yet optimal routing, selfish routing, social optimum, user equilibrium, nudging traffic information, road traffic congestion.

I. INTRODUCTION

TRAFFIC congestion in urban areas has been one of the serious problems all over the world because it causes both economic and time loss. It has been reported that 12 trillion yen of economic loss per year and 30 hours of time loss per person occur in Japan, due to traffic congestion [1]. In addition, it has been forecasted that traffic congestion will also increase total costs of the four advanced economies, i.e., U.K., France, Germany, and the USA, by 46% from 2013 to 2030 [2].

Such a traffic congestion problem can be modeled as a congestion game in game theory [3]. Route selection by a certain user corresponds to the usage of roads included in the selected route. When all users select their own routes, the degree of congestion of each road is determined, and thus we can estimate the travel time of both roads and routes. It is rational for each user to select a route that seems to have the minimum travel time. Such route selection is called *selfish routing* and results in a *Wardrop equilibrium* where each user cannot reduce its travel time by changing the route [4], [5]. In the Wardrop equilibrium, there is no incentive for any user to change its own route, which means the system reaches the steady state. However, in general, the average travel time among users in the Wardrop equilibrium may be far from *social optimum*, where the average travel time is minimized [6], [7].

In [6], Roughgarden and Tardos pointed out three kinds of ways to overcome selfish routing: (1) increasing the road capacity, (2) routing (part of) users in a central manner, i.e., Stackelberg routing, and (3) internalizing the externalities

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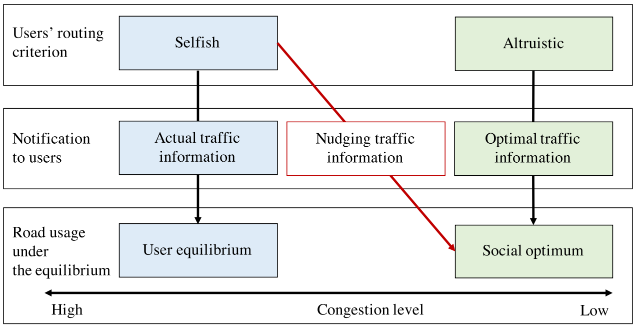

FIGURE 1. Relationship among user criteria, traffic information perceived by a user, and road usage.

by introducing taxes, i.e., congestion pricing. Cooperative routing [8], [9] and Stackelberg routing [10]–[14] cannot achieve social optimum under the users' rational decision making because these approaches fully or partly rely on the users' cooperation. Congestion pricing can alleviate traffic congestion by internalizing the externalities [15]–[22], however, it also has political and economic issues for the introduction [23], [24].

Since the selfish routing comes from users' rational decision making, it is difficult to prohibit the selfish routing itself. In this article, we aim at achieving the social optimum routing even under such users' rational (selfish) route selection by appropriately adjusting their perceived traffic information. Our approach is inspired by the Nudge theory [25]. In behavioral science, the concept of "Nudge" has been attracting many researchers to make decision making of individuals leading to desirable situations by means of indirect suggestions [26]. The Nudge concept is similar to the idea of internalizing the externalities in the congestion pricing but our approach uses the traffic information perceived by each user as the nudging information. Furthermore, the nudging traffic information is personalized per user, which is also different point compared with the standard congestion pricing. Since most of the current vehicle navigation systems and navigation software of smartphones (e.g., Google Maps [27]) have the function of notifying users about the actual traffic information, our approach can be easily introduced by replacing the advertised traffic information with the nudging traffic information. Compared with the conventional congestion pricing, our approach can be deployed anytime and anywhere.

Navigation services have become one of the most fundamental services of intelligent transportation systems (ITSs) [28]–[32]. Users can acquire not only the routes from their current locations to destinations but also the current traffic conditions from the navigation software. Since the navigation software is an agent for the corresponding user, its route selection also tends to be selfish routing. In what follows, the terms users and agents will be used interchangeably.

Even if all users rationally aim to minimize their own travel time, their behavior may change depending on their perceived traffic information. Fig. 1 illustrates the relationship among users' routing criterion, traffic information perceived by users, and road usage. The usage of each road will converge to user equilibrium (UE), which is a Wardrop equilibrium under the selfish routing criterion, when each user is selfish and receives the actual traffic information based on selfish route selection by others. On the other hand, when each user is altruistic and receives the optimal traffic information based on altruistic route selection by others, it will converge to social optimum (SO), which is also a Wardrop equilibrium under the altruistic routing criterion.

The proposed scheme, which leads selfish routing to social optimum, can be achieved by combining the following two functions. First one is the distributed route selection scheme [8], which can achieve either Wardrop equilibrium based on users' selfish criterion, i.e., user equilibrium, or that based on users' altruistic criterion, i.e., social optimum. Second one is the distribution of the nudging traffic information from the server to each user, which affects the users' perception of traffic congestion and leads their selfish routing to social optimum as shown in Fig. 1. We assume the current navigation software and/or traffic support systems (e.g., vehicle information and communication system (VICS) in Japan [28]) can provides users with the nudging traffic information instead of the actual traffic information.

Someone might think that users doubt whether the perceived traffic information is tweaked and defect the proposed system. Unfortunately, the proposed scheme cannot prohibit such (selfish) behavior. This might come from the dissatisfaction with the actual (experienced) travel time compared to the pre-expected travel time based on the nudging traffic information. There are several studies on dealing with such phenomena [33]–[37]. In [33], the information affecting travelers' route choices is categorized into the three types of information: experimental information (EI), descriptive information (DI), and prescriptive information (PI). Without any external information, it is assumed that a traveler will choose the route based on EI, which is the knowledge acquired from his/her own experience during the past choices. DI presents the information about the travel condition (e.g., expected travel time) notified by systems either before or during the travel. PI is the information for direct suggestion and guidance (e.g., a recommended route), which is provided by systems. In our case, the nudging traffic information and the resulting expected travel time can be regarded as DI.

The impact of the three kinds of information (i.e., EI, DI, and PI) and their combinations on the travelers' route choices have been studied from the aspects of both the short-term and long-term behavior [34]–[37]. In this article, we assume that all users follow the proposed scheme, in order to focus on the concept of the nudging traffic information itself. The precise modeling of users' satisfaction with their own travel time and the behavior modification based on the degree of satisfaction are possible future directions.

The main contributions of this article are given as follows:

1) Inspired by the Nudge theory, we propose "selfish yet optimal routing," which can achieve social optimum even under the rational (selfish) decision making of users by internalizing the marginal cost into the traffic information, i.e., perceived travel time. In contrast to the existing approaches (e.g., cooperative routing, centralized routing, and congestion pricing), the proposed scheme does not rely on the users' cooperative behavior and can be introduced anytime and anywhere.
2) In contrast to the standard cost pricing where the link level tolls are identical for all users, the provision of the link-level nudging traffic information in the proposed scheme can be viewed as the personalized pricing that considers heterogeneity in user behavior.
3) We demonstrate the fundamental characteristics of the proposed scheme through numerical experiments under a grid-like road network. Some of the main results are as follows: (1) the proposed scheme can achieve almost the same performance compared with the optimal routing as we expected and (2) the proposed scheme can decrease individual travel time of 82% users compared with the notification of actual (user-equilibrium) traffic information.
4) Furthermore, we also evaluate the practicality and scalability of the proposed scheme through numerical experiments under two kinds of real road networks (i.e., local-level and city-level road networks of Nagoya city, Japan). In particular, we find that the proposed scheme can improve the average travel time by 19.1% (resp. 7.4%), compared with the notification of the actual traffic information, in case of the local-level (resp. city-level) road network with 1,197 (resp. 10,004) users.

The remaining of the paper is given as follows. Section II gives related work. In Section III, we explain the existing distributed route selection scheme [8]. After describing the proposed scheme in Section IV, we demonstrate numerical results in Section V. Finally, Section VI provides conclusions.

II. RELATED WORK

Roughgarden first studied the traffic congestion problem from the viewpoint of selfish routing [6]. He revealed each user's selfish routing results in a Wardrop equilibrium and focused on the performance ratio of travel time based on selfish routing to that of optimal routing, i.e., Price of Anarchy (PoA) [7], in game theory. PoA can be equal to or greater than one and smaller PoA indicates that selfish routing can achieve shorter travel time, which is competitive with that of the optimal routing. For example, PoA becomes $4/3$ if the travel time of a road linearly increases with the flow over the road. Wang *et al.* revealed that selfish rerouting against unpredictable trouble, e.g., traffic accidents, also had a negative impact on traffic congestion [38]. In [6], Roughgarden pointed out three ways to overcome the selfish routing: (1) increasing the road capacity, (2) routing (part of) users in a centralized manner, i.e., cooperative routing and Stackelberg routing, and (3) internalizing the externalities, i.e., congestion pricing.

From the viewpoint of system, it is desirable to achieve the social optimum routing that relies on cooperative route selection among all users. In [8], Lim and Rus proposed a distributed route selection scheme that can achieve either user equilibrium or social optimum according to users' routing criterion, i.e., selfish or altruistic, which will be introduced in Section III. In [9], Aslam *et al.* evaluated the performance of the distributed route selection scheme by using a congestion model learned from the actual traffic data of taxis. They found that the scheme could reduce the travel time by 15% compared with greedy optimal planning. However, it may be difficult to obtain cooperative support by all users, due to the potential selfishness of individuals. In particular, the selfish routing of the individual user tends to be prioritized in an emergency situation, i.e., evacuation, because evacuees want to move to a refuge as fast as possible.

If the system can obtain cooperative support by part of users, Stackelberg routing is one of the promising approaches to cope with the selfish routing problem [10]–[14]. Korilis *et al.* proposed Stackelberg strategies to improve the whole system performance, i.e., average travel time among users [11]. In [11], cooperative users first act as leaders by selecting routes that can lead the route selection of selfish users to good Nash (Wardrop) equilibria. Then, the selfish users, called followers, conduct selfish routing under the environment yielded by the leaders' decision, which will result in the expected Nash equilibria. Roughgarden showed that the derivation of the optimal Stackelberg strategy to minimize PoA was NP-hard and proposed three heuristic Stackelberg strategies [12]. Yang *et al.* also proposed a Stackelberg routing game and formulated a mixed behavior equilibrium model as variational inequalities which described players' routing behavior aiming at user equilibrium, social optimum, and Cournot-Nash equilibrium, respectively [13]. In [14], Groot *et al.* proposed a game-theoretic approach in order to maximize traffic throughput on a freeway network by introducing a reverse Stackelberg routing [39] with a monetary incentive [40]. Our selfish yet optimal routing is similar to the idea of Stackelberg routing, where a server acts as a leader by notifying the nudging traffic information to users, and then the users act as followers by conducting selfish routing with the perceived information.

To mitigate a negative impact of selfish routing, there are also several studies on indirect control by internalizing the externalities: congestion pricing [15]–[22] and gate control [41]. Congestion pricing imposes taxes on the road usage according to the congestion level. Cole *et al.* showed that the selfish routing could result in the optimal routing by appropriately introducing congestion pricing [16]. In actual, congestion pricing has been introduced to many cities and showed good results to mitigate urban congestion [17]–[19]. On the other hand, it was also pointed out that the congestion pricing had political and economic issues [23], [24]. Bazzan and Junges studied the route selection to achieve the social optimum by internalizing the route congestion into the congestion tolls [22]. In this work, a control center provides the users with the congestion tolls based on (imperfect) traffic information about the number of users selecting the corresponding route. The users select the corresponding route with the probability based on the congestion toll. Bu *et al.* showed that gate control could improve the crowd evacuation under emergent situations [41]. These indirect control schemes are also similar to the concept of "Nudge," which aims to lead individuals to desirable decision making through indirect suggestions [25], [26]. The main difference of the proposed scheme from the congestion pricing is the internalization of the marginal cost into traffic information, i.e., perceived travel time. In the proposed scheme, users' selfish routing unconsciously results in the optimal routing by leveraging the nudging traffic information without the user cooperation [8], [10]–[14] and the payment of congestion fees [15]–[21].

The difference of optimality between the individual users and the system, i.e., user equilibrium and social optimum, stems from the different goals among them. There are several studies to fill this gap [42]–[44]. Angelelli *et al.* proposed a proactive route guiding scheme to avoid traffic congestion, which considered not only the system performance to suppress traffic congestion but also the user performance to suppress the increase of individual travel time [44]. In [42], [43], they balance the individual user and the system objectives by deriving system optimal flows under the user constraints.

With the proliferation of vehicle navigation systems and smartphones, each individual can easily acquire the traffic information, which would affect the route selection [37], [45]–[48]. Essen *et al.* insisted that considering both the user behavior and the system performance was important to evaluate how the traffic information notified to users would affect the traffic congestion [45]. To alleviate congestion, Hassan *et al.* proposed a distributed traffic coordination scheme based on the travel information exchanged through the driver's social network [46]. In [48], Ramos *et al.* proposed a route selection scheme based on regret minimization [49], where each agent estimated the regret of route choice based on the local information obtained by its own experience and global information provided by a mobile navigation application, and then selected the route with the smallest estimated regret. The proposed scheme is compatible with the conventional navigation systems by replacing the advertising traffic information with the nudging one.

III. DISTRIBUTED ROUTE SELECTION SCHEME

In this section, we describe the details of the existing distributed route selection scheme [8], which will be used in part of the proposed scheme in Section IV.

TABLE 1. Notations.

Symbol	Description
G	Directed graph of the road network
$\mathcal{V}$	Set of vertices (road intersections)
$\mathcal{E}$	Set of edges (roads)
e	Road
$\mathcal{A}$	Set of all users in the road network
N	The number of all users, ($N = \|\mathcal{A}\|$)
$\boldsymbol{\pi}_i$	Set of route candidates for user i
$\boldsymbol{p}_i$	Vector of route choice probabilities for user i
$\mathcal{K}_i$	Set of route indices for user i
K_i	The number of route candidates, $\boldsymbol{\pi}_i$, ($K_i = \|\mathcal{K}_i\|$)
π_{ik}	User i's k-th route
p_{ik}	Probability that user i selects route π_{ik}
$\mathcal{C}_i$	Set of users j whose routes $\boldsymbol{\pi}_j$ (partly) conflict with $\boldsymbol{\pi}_i$
$\boldsymbol{p}_{\mathcal{C}_i}$	Vector of $\boldsymbol{p}_j$ for $j \in \mathcal{C}_i$
$f_e(\cdot)$	Flow of road e
$\mathbb{I}(\cdot)$	Indicator function
$t_e(\cdot)$	Travel time of road e
$c_e(\cdot)$	Cost of road e
$c_e^{(\mathrm{UE})}(\cdot)$	User-equilibrium based cost of road e
$c_e^{(\mathrm{SO})}(\cdot)$	Social-optimum based cost of road e
$c_{ik}(\cdot)$	Cost of route π_{ik}
d_i	Index of the route with minimum cost among $\boldsymbol{\pi}_i$
$V_i(\cdot)$	Local cost of user i
$V(\cdot)$	Global cost
m	Navigation server
G_i	Directed graph of the road network consisting of $\boldsymbol{\pi}_i$
$\mathcal{V}_i$	Set of vertices (road intersections) consisting of $\boldsymbol{\pi}_i$
$\mathcal{E}_i$	Set of edges (roads) consisting of $\boldsymbol{\pi}_i$
$\boldsymbol{p}_i^{(m)}$	Vector of social-optimum based route choice probability for user i
$\boldsymbol{p}_i^{(i)}$	Vector of selfish route choice probability for user i
$\boldsymbol{p}_{\mathcal{C}_i}^{(m)}$	Vector of $\boldsymbol{p}_j^{(m)}$ for $j \in \mathcal{C}_i$
$\boldsymbol{f}_i^{(m)}$	Nudging traffic information for user i
$f_e^{(m)}$	Nudging traffic information of road e, $f_e^{(m)} \in \boldsymbol{f}_i^{(m)}$
$c_e^{(\mathrm{UE})-1}(\cdot)$	Inverse function of $c_e^{(\mathrm{UE})}(\cdot)$
ϵ	Error tolerance
$T_i(\cdot)$	Travel time for user i

A. OVERVIEW

Table 1 presents the symbols and the notations used throughout the paper. $G = (\mathcal{V}, \mathcal{E})$ denotes a graph representing the internal structure of a road network, where $\mathcal{V}$ is a set of vertices, i.e., intersections, and $\mathcal{E}$ is a set of edges, i.e., roads, in the road network. There are $N > 0$ users, e.g., vehicles, in the road network and $\mathcal{A} = \{1, 2, \ldots, N\}$ denotes a set of users.

In the distributed route selection scheme, each user $i \in \mathcal{A}$ first calculates $K_i > 0$ route candidates $\boldsymbol{\pi}_i = \{\pi_{i1}, \pi_{i2}, \ldots, \pi_{iK_i}\}$, where π_{ik} is the k-th route candidate that is a set of edges in the corresponding route. Let $\mathcal{K}_i = \{1, 2, \ldots, K_i\}$ be a set of route indices for user i. Next, each user autonomously calculates route choice probabilities $\boldsymbol{p}_i = (p_{i1}, p_{i2}, \ldots, p_{iK_i})$ by using a gradient descent method [8]. Here, p_{ik} is the probability that user i selects k-th route, where p_{ik} ranges [0, 1] and $\sum_{k \in \mathcal{K}_i} p_{ik} = 1$. Note that $\boldsymbol{p}_i$ can be regarded as the mixed strategy in game theory [49]. In the route selection, each peer i considers $\boldsymbol{p}_i$ and $\boldsymbol{p}_j$ for all competitors $j \in \mathcal{C}_i$, where $\mathcal{C}_i$ denotes the set of users j whose route candidates $\boldsymbol{\pi}_j$ (partly) conflict with user i's route candidates $\boldsymbol{\pi}_i$. Please note that small increase/decrease of p_{ik} may change the congestion level of the roads in the route π_{ik}, which affects not only the travel time of user i but also that of i's competitors. The relationship between the route choice probabilities and resulting travel time will be described later. In addition, the route choice probabilities are controlled by each user in a distributed manner, with the help of the gradient descent method. Please see the detail mechanism in [8, Sec. 3.3].

This scheme assumes that the number of users in the road network is large enough such that each user's route choice probability can be regarded as a fractional flow [50]–[53]. As a result, the flow of a road can also be regarded as the probabilistic occupation by users. In addition, this scheme also assumes that the user's probabilistic occupation of a road is static during the whole time horizon of the user's travel as in [52]–[55]. With these assumptions, flow $f_e(\boldsymbol{p}_i, \boldsymbol{p}_{\mathcal{C}_i})$ of road $e \in \mathcal{E}$ can be expressed as the sum of the probabilities that user i and competitors $\mathcal{C}_i$ use road e, where $\boldsymbol{p}_{\mathcal{C}_i}$ denotes the vector of $\boldsymbol{p}_j$ for $j \in \mathcal{C}_i$:

$$f_e(\boldsymbol{p}_i, \boldsymbol{p}_{\mathcal{C}_i}) = \sum_{j \in \{i\} \cup \mathcal{C}_i} \sum_{k \in \mathcal{K}_j} \mathbb{I}(e \in \pi_{jk}) \cdot p_{jk}, \tag{1}$$

where $\mathbb{I}(\cdot)$ denotes an indicator function. The cost of route π_{ik} of user i is a routing criterion and can be expressed as the sum of the cost of each road along the route:

$$c_{ik}\big(\boldsymbol{p}_i, \boldsymbol{p}_{\mathcal{C}_i}\big) = \sum_{\forall e \in \pi_{ik}} c_e\big(f_e(\boldsymbol{p}_i, \boldsymbol{p}_{\mathcal{C}_i})\big),$$

where $c_e(f_e(\cdot))$ is the cost of road e under flow $f_e(\cdot)$, which is a differentiable non-decreasing function. One possible definition of $c_e(f_e(\cdot))$ is travel time $t_e(f_e(\cdot))$ of road e under flow $f_e(\cdot)$. The cost function represents the user's sense of value and will be described in Section III-B.

Each user i defines local cost $V_i(\boldsymbol{p}_i, \boldsymbol{p}_{\mathcal{C}_i})$ as the difference between the expected cost among all route candidates and the minimum route cost:

$$V_i(\boldsymbol{p}_i, \boldsymbol{p}_{\mathcal{C}_i}) = \sum_{k \in \mathcal{K}_i} p_{ik} c_{ik}(\boldsymbol{p}_i, \boldsymbol{p}_{\mathcal{C}_i}) - c_{d_i}(\boldsymbol{p}_i, \boldsymbol{p}_{\mathcal{C}_i}), \tag{2}$$

where d_i is the index of the route with the minimum cost among route candidates, i.e., $d_i = \arg\min_{k \in \mathcal{K}_i} c_{ik}$. Each user i controls route choice probabilities $\boldsymbol{p}_i$ such that V_i approaches 0. Since (2) can be rewritten as

$$V_i(\boldsymbol{p}_i, \boldsymbol{p}_{\mathcal{C}_i}) = \sum_{k \in \mathcal{K}_i} p_{ik}\big(c_{ik}(\boldsymbol{p}_i, \boldsymbol{p}_{\mathcal{C}_i}) - c_{d_i}(\boldsymbol{p}_i, \boldsymbol{p}_{\mathcal{C}_i})\big), \tag{3}$$

$V_i = 0$ results in the two conditions of Wardrop equilibrium:

$$\begin{cases} c_{ik}(\boldsymbol{p}_i, \boldsymbol{p}_{\mathcal{C}_i}) = c_{d_i}(\boldsymbol{p}_i, \boldsymbol{p}_{\mathcal{C}_i}), & \text{if } p_{ik} > 0, \\ c_{ik}(\boldsymbol{p}_i, \boldsymbol{p}_{\mathcal{C}_i}) \geq c_{d_i}(\boldsymbol{p}_i, \boldsymbol{p}_{\mathcal{C}_i}), & \text{otherwise.} \end{cases}$$

As a result, each user i will select the minimum cost when $V_i = 0$. In addition, the global cost $V(\{\boldsymbol{p}_i\}_{i \in \mathcal{A}}, \{\mathcal{C}_i\}_{i \in \mathcal{A}})$ is defined as the sum of local cost $V_i(\cdot)$ among all users:

$$V(\{\boldsymbol{p}_i\}_{i \in \mathcal{A}}, \{\mathcal{C}_i\}_{i \in \mathcal{A}}) = \sum_{i \in \mathcal{A}} V_i\big(\boldsymbol{p}_i, \boldsymbol{p}_{\mathcal{C}_i}\big).$$

When each user i control $\boldsymbol{p}_i$ to achieve $V_i = 0$, global cost V can also converge to 0.

In [8], a distributed gradient controller is developed, in which each user i can control $\boldsymbol{p}_i$ to achieve $V_i = 0$ in a distributed manner. The distributed controller governs the time derivative of the route choice probabilities using the competitors' current route choice probabilities. Please refer to [8, Sec. 3.3] for the detail of the mechanism.

We should note here that equilibrium $\boldsymbol{p}_i^*$ of $\boldsymbol{p}_i$ will change depending on the shape of local cost function $c_e(\cdot)$, i.e., user equilibrium or social optimum, and the resulting $(\boldsymbol{p}_1^*, \ldots, \boldsymbol{p}_N^*)$ is the global goal with the corresponding local cost function. The detail of local cost function $c_e(\cdot)$ will be given in the next subsection.

In addition, equilibrium $\boldsymbol{p}_i^*$ can be regarded as the *stochastic user equilibrium (SUE)* [56], which is a special case of the *generalized stochastic user equilibrium (GSUE)* [57]. In [57], the author also indicated that the achievement of SUE is guaranteed under the large sample approximation theorem, which assumes the absolute demand (i.e., the product of the demand rate and time period) is sufficiently large. As for this point, we will discuss in the evaluation part (Section V-A).

The uniqueness of SUE with heterogeneous users is guaranteed under some simplified settings [58], [59]. However, in our case, the SUE may not be unique because there is heterogeneity in individuals' route selection from their candidates, which differ among users even for the same origin and destination pair. The random nature of route updating order among users in the distributed gradient controller would also result in multiple SUEs.

B. ROUTING CRITERIA

We assume that the routing criteria depend on the user's selfishness and its cooperativeness. From the viewpoint of user's selfish decision making, cost of each road e, $c_e^{(\mathrm{UE})}(f_e(\cdot))$, can be directly expressed as the travel time $t_e(f_e(\cdot))$ under flow $f_e(\cdot)$:

$$c_e^{(\mathrm{UE})}(f_e(\cdot)) = t_e(f_e(\cdot)). \tag{4}$$

On the contrary, from the viewpoint of user's social-optimum decision making, cost of each road e, $c_e^{(\mathrm{SO})}(f_e)$, can be defined as follows:

$$c_e^{(\mathrm{SO})}(f_e) = t_e(f_e(\cdot)) + f_e(\cdot) \left.\frac{\partial t_e(f)}{\partial f}\right|_{f = f_e(\cdot)}, \tag{5}$$

which is the marginal cost of road e, i.e., the total cost increase of all the users using road e due to a small increase of the flow on road e. The existing scheme [8] converges to Wardrop equilibrium, i.e., user equilibrium (UE), (resp. social optimum (SO)) when all users select routes based on $c_e^{(\mathrm{UE})}(\cdot)$ (resp. $c_e^{(\mathrm{SO})}(\cdot)$).

IV. PROPOSED SCHEME

In this section, we propose selfish yet optimal routing by adjusting the perceived traffic information. After introducing the system overview, we describe the detail of the proposed scheme.

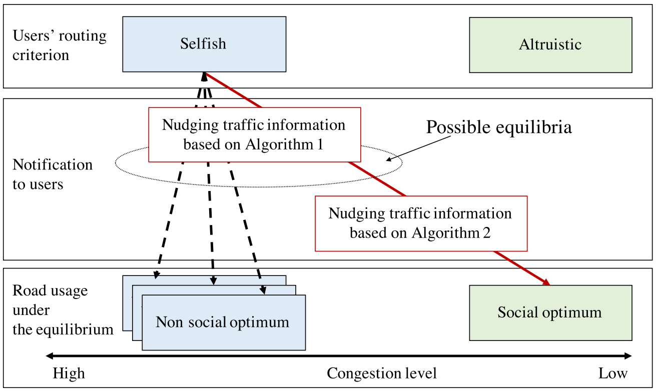

FIGURE 2. Relationship among user criteria, nudging traffic information, and road usage.

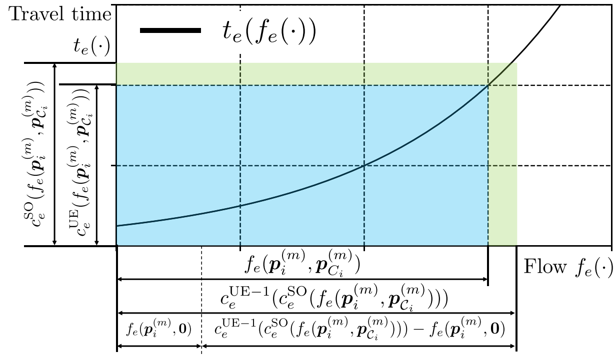

FIGURE 3. Gap between the UE-based routing criterion and SO-based routing criterion.

A. SYSTEM OVERVIEW

In the road network $G = (\mathcal{V}, \mathcal{E})$, each user $i \in \mathcal{A}$ first requests a route from navigation server m via its user agent. After receiving a designated route (i.e., social optimal route) from server m, altruistic user i will follow the designated route. On the other hand, selfish user i may not follow the designated route and then requests other route (i.e., selfish route) to its user agent. The user i's agent asks navigation server m for traffic information of each road in its $K_i > 0$ route candidates $\boldsymbol{\pi}_i$. Server m first calculates a vector of social optimum route choice probabilities, $\boldsymbol{p}_i^{(m)} = (p_{i1}^{(m)}, p_{i2}^{(m)}, \ldots, p_{iK_i}^{(m)})$, for each user $i \in \mathcal{A}$, with the help of the existing scheme in Section III. Then, it derives nudging traffic information $\boldsymbol{f}_i^{(m)} = \{f_e^{(m)}\}_{e \in \mathcal{E}_i}$, which is a vector of nudging traffic for each road included in $\mathcal{E}_i$ and required to lead the user's selfish routing to the optimal routing, where $\mathcal{E}_i$ denotes a set of roads included in $\boldsymbol{\pi}_i$, i.e., $\mathcal{E}_i = \{e \in \cup_{k=1}^{K_i} \pi_{ik}\}$. We also define $\mathcal{V}_i$ and G_i as a set of nodes consisting of $\mathcal{E}_i$ and graph $(\mathcal{V}_i, \mathcal{E}_i)$, respectively. After retrieving $\boldsymbol{f}_i^{(m)}$ from server m, each user agent i calculates selfish route choice probability $\boldsymbol{p}_i^{(i)}$ under $\boldsymbol{f}_i^{(m)}$ with help of the existing scheme [8]. Finally, user agent i selects route π_{ik^*} from $\boldsymbol{\pi}_i$ according to $\boldsymbol{p}_i^{(i)}$. Hereafter, the user and the corresponding agent will be used interchangeably.

B. NUDGING TRAFFIC INFORMATION ACHIEVING SELFISH YET OPTIMAL ROUTING

In this section, we explain how server m derives nudging traffic information for each user i, which affects the user's perception of traffic congestion and leads the user's selfish routing to the optimal routing. Recall that the routing criterion is different between selfish routing and optimal routing, i.e., UE-based routing criterion (4) and SO-based routing criterion (5). If all users follow the SO-based routing criterion, the optimal routing can be achieved as a Wardrop equilibrium. However, rational decisions of users tend to follow the UE-based routing criterion where they only consider their own travel time. In what follows, we aim to lead the selfish routing to the optimal routing by appropriately modifying the users' perception of traffic congestion through the nudging traffic information.

As mentioned above, server m calculates nudging traffic information $\boldsymbol{f}_i^{(m)}$ for each user $i \in \mathcal{A}$. To achieve selfish yet optimal routing, $\boldsymbol{f}_i^{(m)}$ should satisfies the following conditions:

1) The social optimum assignment for each user is equivalent to one of the Wardrop equilibria under the UE-based routing criterion, which will be satisfied by Algorithm 1.
2) Any Wardrop equilibrium for each user can be equivalent to the social optimum assignment under the UE-based criterion, which will be satisfied by Algorithm 2.

Fig. 2 shows the graphical implication of these conditions and the details will be given in the following.

We first focus on the first condition. Even if all competitors $j \in \mathcal{C}_i$ of user i follow social optimum route choice probability $\boldsymbol{p}_{\mathcal{C}_i}^{(m)}$, the selfish route choice probability for user i may not be equivalent to social optimum one $\boldsymbol{p}_i^{(m)}$. This situation will come from the user i's underestimation of the traffic congestion, which is caused by the UE-based routing criterion under social optimum route assignment $(\boldsymbol{p}_i^{(m)}, \boldsymbol{p}_{\mathcal{C}_i}^{(m)})$. Recall that the usage of each road for user i is only affected by i's competitors $j \in \mathcal{C}_i$ rather than all the others.

Fig. 3 depicts the relationship between flow $f_e(\boldsymbol{p}_i^{(m)}, \boldsymbol{p}_{\mathcal{C}_i}^{(m)})$ on road e, travel time $t_e(f_e(\boldsymbol{p}_i^{(m)}, \boldsymbol{p}_{\mathcal{C}_i}^{(m)}))$, UE-based routing criterion $c_e^{(\mathrm{UE})}(f_e(\boldsymbol{p}_i^{(m)}, \boldsymbol{p}_{\mathcal{C}_i}^{(m)}))$, and SO-based routing criterion $c_e^{(\mathrm{SO})}(f_e(\boldsymbol{p}_i^{(m)}, \boldsymbol{p}_{\mathcal{C}_i}^{(m)}))$. Note that $t_e(f_e(\boldsymbol{p}_i^{(m)}, \boldsymbol{p}_{\mathcal{C}_i}^{(m)})) = c_e^{(\mathrm{UE})}(f_e(\boldsymbol{p}_i^{(m)}, \boldsymbol{p}_{\mathcal{C}_i}^{(m)}))$. We can confirm that the UE-based cost $c_e^{(\mathrm{UE})}(\cdot)$ underestimates the cost of road e as $t_e(f_e(\boldsymbol{p}_i^{(m)}, \boldsymbol{p}_{\mathcal{C}_i}^{(m)}))$, which is smaller than $c_e^{(\mathrm{SO})}(f_e(\boldsymbol{p}_i^{(m)}, \boldsymbol{p}_{\mathcal{C}_i}^{(m)}))$ by the corresponding marginal cost as in (5). This SO-based cost $c_e^{(\mathrm{SO})}(f_e(\boldsymbol{p}_i^{(m)}, \boldsymbol{p}_{\mathcal{C}_i}^{(m)}))$ can be transformed into the corresponding flow $c_e^{(\mathrm{UE})-1}(c_e^{(\mathrm{SO})}(f_e(\boldsymbol{p}_i^{(m)}, \boldsymbol{p}_{\mathcal{C}_i}^{(m)})))$ under the UE-based cost. Here, $c_e^{(\mathrm{UE})-1}(\cdot)$ is the inverse function of $c_e^{(\mathrm{UE})}(\cdot)$. Due to the linearity in (1),

$$f_e\left(\boldsymbol{p}_i^{(m)}, \boldsymbol{p}_{\mathcal{C}_i}^{(m)}\right) = f_e\left(\boldsymbol{p}_i^{(m)}, \boldsymbol{0}\right) + f_e\left(\boldsymbol{0}, \boldsymbol{p}_{\mathcal{C}_i}^{(m)}\right) \tag{6}$$

Algorithm 1 Nudging Traffic Information $\boldsymbol{f}_i^{(m)}$ for User i, Which Leads the Social Optimum Assignment for User i to a Wardrop Equilibrium Under UE-Based Route Criterion

Input: $G_i = (\mathcal{V}_i, \mathcal{E}_i)$, $\boldsymbol{p}_i^{(m)}$, $\boldsymbol{p}_{\mathcal{C}_i}^{(m)}$

Output: $\boldsymbol{f}_i^{(m)}$

1: **for** $\forall e \in \mathcal{E}_i$ **do**

2: $f_e(\mathbf{0}, \boldsymbol{p}_{\mathcal{C}_i}^{(m)}) \leftarrow \sum_{j\in\mathcal{C}_i} \sum_{k\in\mathcal{K}_j} I(e \in \pi_{jk}) \cdot p_{jk}^{(m)}$ ▷ Calculate social optimal flow of road e among competitors $\mathcal{C}_i$

3: $f_e(\boldsymbol{p}_i^{(m)}, \boldsymbol{p}_{\mathcal{C}_i}^{(m)}) \leftarrow \sum_{k\in\mathcal{K}_i} I(e \in \pi_{ik}) \cdot p_{ik}^{(m)} + f_e(\mathbf{0}, \boldsymbol{p}_{\mathcal{C}_i}^{(m)})$ ▷ Calculate social optimal flow of road e among user i and competitors $\mathcal{C}_i$

4: $c_e^{(\mathrm{SO})}(f_e(\boldsymbol{p}_i^{(m)}, \boldsymbol{p}_{\mathcal{C}_i}^{(m)})) \leftarrow t_e(f_e(\boldsymbol{p}_i^{(m)}, \boldsymbol{p}_{\mathcal{C}_i}^{(m)})) + f_e(\boldsymbol{p}_i^{(m)}, \boldsymbol{p}_{\mathcal{C}_i}^{(m)}) \frac{\partial t_e(f)}{\partial f}\Big|_{f=f_e(\boldsymbol{p}_i^{(m)}, \boldsymbol{p}_{\mathcal{C}_i}^{(m)})}$ ▷ Calculate SO-based cost of road e

5: $f_e^{(m)} \leftarrow c_e^{(\mathrm{UE})-1}(c_e^{(\mathrm{SO})}(f_e(\boldsymbol{p}_i^{(m)}, \boldsymbol{p}_{\mathcal{C}_i}^{(m)}))) - f_e(\boldsymbol{p}_i^{(m)}, \mathbf{0})$ ▷ Derive nudging traffic information $\boldsymbol{f}_i^{(m)}$

6: **return** $\boldsymbol{f}_i^{(m)}$

is satisfied. To make $\boldsymbol{p}_i^{(m)}$ to be the optimal usage of each road even under the UE-based routing criterion, user i should perceive traffic information $f_e^{(m)}$ $(e \in \mathcal{E}_i)$, which satisfies the following:

$$f_e^{(m)} = c_e^{(\mathrm{UE})-1}\Big(c_e^{(\mathrm{SO})}\Big(f_e\Big(\boldsymbol{p}_i^{(m)}, \boldsymbol{p}_{\mathcal{C}_i}^{(m)}\Big)\Big)\Big) - f_e\Big(\boldsymbol{p}_i^{(m)}, \mathbf{0}\Big). \quad (7)$$

Algorithm 1 presents the calculation of nudging traffic information $\boldsymbol{f}_i^{(m)}$ for user i, which satisfies the first condition. Given road network $G_i = (\mathcal{V}_i, \mathcal{E}_i)$, social optimum route choice probability of user i, $\boldsymbol{p}_i^{(m)}$, and that of user i's competitors, $\boldsymbol{p}_{\mathcal{C}_i}^{(m)}$, system m first calculates social optimal flow of road e among competitors $\mathcal{C}_i$, i.e., $f_e(\mathbf{0}, \boldsymbol{p}_{\mathcal{C}_i}^{(m)})$, and that among user i and competitors $\mathcal{C}_i$, i.e., $f_e(\boldsymbol{p}_i^{(m)}, \boldsymbol{p}_{\mathcal{C}_i}^{(m)})$, (lines 2–3). Next, it also calculates the SO-based cost of road e, i.e., sum of the travel time and marginal cost, from (5) (line 4). Finally, it derives the nudging traffic information of road e using (7) (line 5).

If the first condition is satisfied by Algorithm 1, the social optimum can be one of the Wardrop equilibria under the UE-based routing criterion. (Remind that the Wardrop equilibrium (SUE) may not be unique, as mentioned in Section III.) However, it cannot guarantee that any Wardrop equilibrium under the UE-based routing criterion is equivalent to the social optimum as shown in Fig. 2, and thus the second condition will be required. To achieve the second condition, server m iteratively searches for selfish route choice probability $p_i^{(i)}$ of user i under nudging traffic $\boldsymbol{f}_i^{(m)}$ given by (7), and updates $\boldsymbol{f}_i^{(m)}$ such that the selfish flow will reach the social optimum flow. The details are given in Algorithm 2.

Given road network $G_i = (\mathcal{V}_i, \mathcal{E}_i)$, social optimum route choice probability of user i, $\boldsymbol{p}_i^{(m)}$, that of user i's competitors, $\boldsymbol{p}_{\mathcal{C}_i}^{(m)}$, nudging traffic information obtained from Algorithm 1, $\boldsymbol{f}_i^{(m)}$, and error tolerance $\epsilon \geq 0$, server m first obtains user i's selfish route choice probability $\boldsymbol{p}_i^{(i)}$ under the fictitio traffic information $\boldsymbol{f}_i^{(m)}$ using `calc_selfish_route_prob`$(\cdot)$ function (line 2). Here, `calc_selfish_route_prob`$(\cdot)$ function can be achieved by the existing scheme with UE-based routing criterion in Section III. Next, in lines 3–6, it calculates the flow of road e, which is caused by both selfish route choice probability $\boldsymbol{p}_i^{(i)}$ and latest nudging traffic information $\boldsymbol{f}_i^{(m)}$, and then updates nudging traffic information of each road $e \in \mathcal{E}_i$ as follows:

$$f_e^{(m)} \leftarrow f_e^{(m)} + f_e(\boldsymbol{p}_i^{(i)}, \mathbf{0}) - f_e\Big(\boldsymbol{p}_i^{(m)}, \mathbf{0}\Big). \quad (8)$$

Algorithm 2 Nudging Traffic Information $\boldsymbol{f}_i^{(m)}$ for User i, Which Leads Any Wardrop Equilibrium for User i to the Social Optimum Route Assignment Under UE-Based Route Criterion

Input: $G_i = (\mathcal{V}_i, \mathcal{E}_i)$, $\boldsymbol{p}_i^{(m)}$, $\boldsymbol{p}_{\mathcal{C}_i}^{(m)}$, $\boldsymbol{f}_i^{(m)}$, ϵ

Output: $\boldsymbol{f}_i^{(m)}$

1: **do**

2: $\boldsymbol{p}_i^{(i)} \leftarrow$ `calc_selfish_route_prob`$(\boldsymbol{f}_i^{(m)})$ ▷ Obtain the selfish route choice probability for user i

3: **for** $\forall e \in \mathcal{E}_i$ **do**

4: $f_e(\boldsymbol{p}_i^{(i)}, \mathbf{0}) \leftarrow \sum_{k\in\mathcal{K}_i} I(e \in \pi_{ik}) \cdot p_{ik}^{(i)}$

5: $f_e(\boldsymbol{p}_i^{(m)}, \mathbf{0}) \leftarrow \sum_{k\in\mathcal{K}_i} I(e \in \pi_{ik}) \cdot p_{ik}^{(m)}$

6: $f_e^{(m)} \leftarrow f_e^{(m)} + f_e(\boldsymbol{p}_i^{(i)}, \mathbf{0}) - f_e(\boldsymbol{p}_i^{(m)}, \mathbf{0})$ ▷ Update the nudging traffic information

7: $RMSE \leftarrow \sqrt{K_i^{-1} \sum_{i\in\mathcal{K}_i} (p_{ik}^{(i)} - p_{ik}^{(m)})^2}$

8: **while** $\epsilon < RMSE$

9: **return** $\boldsymbol{f}_i^{(m)}$

The second and third terms of the right-hand side indicate the flow difference between user i's selfish flow and social optimum flow. If $f_e(\boldsymbol{p}_i^{(i)}, \mathbf{0}) - f_e(\boldsymbol{p}_i^{(m)}, \mathbf{0}) > 0$, the usage of road e under the selfish route choice probability is higher than that under the social optimum route choice probability. Therefore, server m increases nudging traffic $f_e^{(m)}$ such that user i reduces the usage probability of road e. Otherwise, it decreases $f_e^{(m)}$ to increase the usage of road e by user i. Next, it calculates root mean square error (RMSE) between $\boldsymbol{p}_i^{(i)}$ and $\boldsymbol{p}_i^{(m)}$ (line 7). These processes (line 2–7) are repeated until the selfish yet social optimum routing is almost achieved, i.e., $RMSE \leq \epsilon$. Since the update rule of $f_e^{(m)}$ given by (8) aims to satisfy $f_e(\boldsymbol{p}_i^{(i)}, \mathbf{0}) = f_e(\boldsymbol{p}_i^{(m)}, \mathbf{0})$, we can expect $\boldsymbol{p}_i^{(i)}$ eventually approaches $\boldsymbol{p}_i^{(m)}$.

After conducting Algorithm 2, server m sends $\boldsymbol{p}_i^{(m)}$ and $\boldsymbol{f}_i^{(m)}$ to user i. Then, user i also calculates selfish route assignment $\boldsymbol{p}_i^{(i)}$ using these information and the gradient descent method [8] (please refer to [8, Sec. 3.3] for the detail of the gradient descent method). Since calc_selfish_route_prob(·) function is deterministic [8], $\boldsymbol{p}_i^{(i)}$ will be the almost same as $\boldsymbol{p}_i^{(m)}$, which is the selfish yet optimal routing as shown in Fig. 2. Note that the proposed scheme can achieve the selfish yet optimal routing under the situation where the altruistic users even exist because the nudging traffic information for each user is adjusted based on the social optimum assignment for each user.

V. NUMERICAL RESULTS

In this section, we first demonstrate fundamental characteristics of the proposed scheme through numerical experiments using a grid-like network: (1) convergence property and (2) degree of optimality in terms of average travel time. Next, we also evaluate the practicality of the proposed scheme through numerical experiments using the local/city level real road networks of Nagoya city in Japan.

A. EVALUATION MODEL

To evaluate the fundamental characteristics of the proposed scheme, we first use a grid road network consisting of 50×50 nodes (intersections). There are 50 users ($\mathcal{A} = \{1, \ldots, 50\}$). Each user $i \in \mathcal{A}$ travels from $(i, 1)$-st node to $(i, 50)$-th node. We assume the travel time of each road $e \in \mathcal{E}$ follows Bureau of Public Roads (BPR) function $t_e(f_e) = \underline{t}_e(1 + \alpha(f_e/c_e)^{\beta})$ [60]. $\underline{t}_e$ denotes the travel time without road congestion, which is proportional to the ratio of length to speed limit of road e. c_e denotes the capacity of road e, which is proportional to the ratio of road e's size, i.e., road width, to the size of a user. α and β represent the degree of road congestion. We set these four parameters by considering those used in [60]: $\underline{t}_e = [1, 5]$, $c_e = [3, 5]$, $\alpha = 0.15$, and $\beta = 4$.

As we will see later, the time period considered in the following evaluations may not be large enough to satisfy the large sample approximation. Since the BPR function with $\beta = 4$ is 4 times differentiable, the GSUE of order 4, i.e., GSUE(4), gives the most accurate mean flow at the expense of computing higher order moments. In [57, Sec. 6], the author illustrates the relationship between the time period τ and mean flow of GSUE(n) under a simple two-road network where the demand rate is 20 [vehicles/hour], the cost function of one road e_1 is 4 times differentiable function of $(f_{e_1}/10)^4$, and that of another road e_2 is 10. This result shows that the approximation error of the mean flow between the GSUE(4) and the GSUE(1) (i.e., SUE) actually exists in case of finite τ but it quickly decreases with increase of τ. In particular, the error is about 14% ($\tau = 0.1$ [hour]) and 7% ($\tau = 0.2$ [hour]). In this article, considering the tradeoff between accuracy and complexity, we adopt SUE.

Each user $i \in \mathcal{A}$ obtains K_i ($K_i \geq 1$) route candidates $\boldsymbol{\pi}_i$ as exclusively as possible so as to alleviate the route competition with others. Since it is hard to obtain the comprehensive route candidates due to the computational complexity [61], we obtain the route candidates according to the following heuristic approach, which is used in [8]. Each user $i \in \mathcal{A}$ first calculates the shortest route from the source, i.e., $(i, 1)$-st node to the destination, i.e., $(i, 50)$-th node, when the flow of user i only exists, i.e., $t_e(1) = \underline{t}_e(1 + \alpha(1/c)^{\beta})$. Next, it obtains the next route candidate by calculating the shortest route from the source to destination under the assumption that a predefined number of road segments, i.e., 30, in the first shortest route are randomly chosen and set to be unavailable. By repeating this procedure, each user $i \in \mathcal{A}$ obtains K_i route candidates, $\boldsymbol{\pi}_i$, which are exclusive to each other as much as possible. Note that the route candidates of all users for a given origin-destination pair are not necessarily identical because the heuristic approach includes the randomness. We also evaluate the impact of K_i on the system performance in Section V-B. Fig. 4 illustrates an example of route candidates for all users and those for user 25 are highlighted by blue color.

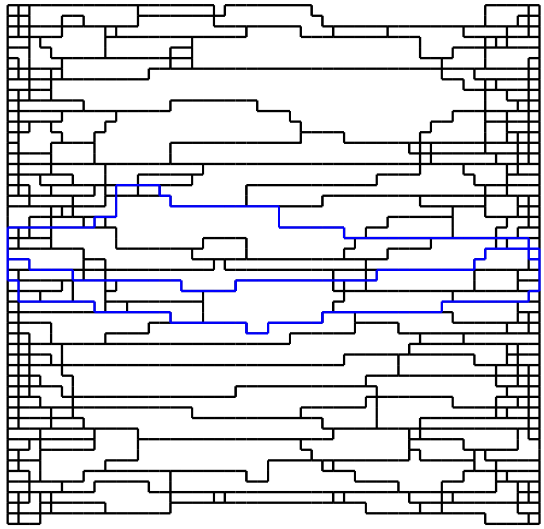

FIGURE 4. An example of route candidates for a certain user $i = 25$, π_j, (blue lines) and those for all users, $\{\pi_j\}_{\forall j \in \mathcal{A}}$ (black lines).

We evaluate the system performance in terms of the average travel time, T_{avg}, which is the mean of expected travel time among all users under route assignment $(\boldsymbol{p}_i^{(i)}, \boldsymbol{p}_{\mathcal{A}\setminus\{i\}}^{(i)})$:

$$\begin{aligned} T_{\text{avg}} &= N^{-1} \sum_{i \in \mathcal{A}} T_i\left(\boldsymbol{p}_i^{(i)}, \boldsymbol{p}_{\mathcal{A}\setminus\{i\}}^{(i)}\right) \\ &= N^{-1} \sum_{i \in \mathcal{A}} T_i\left(\boldsymbol{p}_i^{(i)}, \boldsymbol{p}_{\mathcal{C}_i}^{(i)}\right), \end{aligned} \tag{9}$$

where $T_i(\cdot)$ is given as follows:

$$T_i\left(\boldsymbol{p}_i^{(i)}, \boldsymbol{p}_{\mathcal{C}_i}^{(i)}\right) = \sum_{k \in \mathcal{K}_i} p_{ik}^{(i)} \sum_{e \in \pi_{ik}} t_e\left(f_e\left(\boldsymbol{p}_i^{(i)}, \boldsymbol{p}_{\mathcal{C}_i}^{(i)}\right)\right), \tag{10}$$

where $f_e(\boldsymbol{p}_i, \boldsymbol{f}_i)$ is given by

$$f_e\left(\boldsymbol{p}_i^{(i)}, \boldsymbol{f}_i\right) = \sum_{j \in \{i\} \cup \mathcal{C}_i} \sum_{k \in \mathcal{K}_j} I(e \in \pi_{jk}) \cdot p_{jk}^{(j)}.$$

The proposed scheme is composed of the notification of the nudging traffic information and the route selection based

TABLE 2. Schemes for evaluation.

Scheme	Notification of traffic information	User i's criterion
Notification of $\boldsymbol{p}_{\mathcal{C}_i}^{(i)}$	user equilibrium $\boldsymbol{p}_{\mathcal{C}_i}^{(i)}$	Selfish
Notification of $\boldsymbol{p}_{\mathcal{C}_i}^{(m)}$	social optimum $\boldsymbol{p}_{\mathcal{C}_i}^{(m)}$	Selfish
Proposed scheme	nudging traffic information $\boldsymbol{f}_i^{(m)}$	Selfish
Optimal routing	social optimum $\boldsymbol{p}_{\mathcal{C}_i}^{(m)}$	Altruistic

on the user's selfish criterion. For comparison purpose, we also use the following three schemes depending on the combination of traffic notification and user's criterion, as shown in Table 2.

- *Notification of user equilibrium route choice probabilities $\boldsymbol{p}_{\mathcal{C}_i}^{(i)}$:* Server m notifies each user $i \in \mathcal{A}$ of traffic information under the assumption that all user i's competitors $\mathcal{C}_i$ follow user equilibrium route choice probabilities $\boldsymbol{p}_{\mathcal{C}_i}^{(i)}$. Each user agent $i \in \mathcal{A}$ calculates selfish route choice probability $\boldsymbol{p}_i^{(i)}$ under $\boldsymbol{p}_{\mathcal{C}_i}^{(i)}$. As a result, this scheme results in the user equilibrium but may not be social optimum.
- *Notification of social optimum route choice probabilities $\boldsymbol{p}_{\mathcal{C}_i}^{(m)}$:* Server m first notifies each user $i \in \mathcal{A}$ of traffic information under the assumption that all user i's competitors $\mathcal{C}_i$ follow social optimum route choice probabilities $\boldsymbol{p}_{\mathcal{C}_i}^{(m)}$. Next, each user agent $i \in \mathcal{A}$ calculates route choice probability $\boldsymbol{p}_i^{(i)}$ under the selfish criterion with the social optimum traffic information $\boldsymbol{p}_{\mathcal{C}_i}^{(m)}$. This scheme may increase traffic congestion because each user $i \in \mathcal{A}$ tends to underestimate the congestion level under the assumption that others behave cooperatively.
- *Optimal routing:* The optimal routing is achieved when all users select the route with the probability $\boldsymbol{p}_i^{(m)}$ under the altruistic criterion with the social optimum traffic information $\boldsymbol{p}_{\mathcal{C}_i}^{(m)}$ obtained from the server.

We use the server with Intel Xeon E7-8895v3 (18 cores and 2.60 GHz) and 2 TB memory to obtain the following results.

B. EVALUATION UNDER A GRID-LIKE NETWORK

1) IMPACT OF NUMBER OF ROUTE CANDIDATES ON TRAVEL TIME

We first investigate the impact of the number K_i of route candidates on the average travel time. Fig. 5 depicts how the average travel time of two schemes (i.e., notification of $\boldsymbol{p}_{\mathcal{C}_i}^{(i)}$ and optimal routing) changes according to the number of route candidates, K_i. We observe that T_{avg} of both schemes decreases with K_i and almost converges in case of $K_i \geq 5$, due to lack of exclusive route candidates. In what follows, we use $K_i = 5$. There are also several studies on generating the route candidates [62]–[64].

2) CONVERGENCE PROPERTY

We first focus on the convergence property of the proposed scheme consisting of the distributed route selection scheme [8], Algorithms 1, and 2. The convergence property of the distributed route selection scheme was already discussed in [8]. Since Algorithm 1 calculates nudging traffic

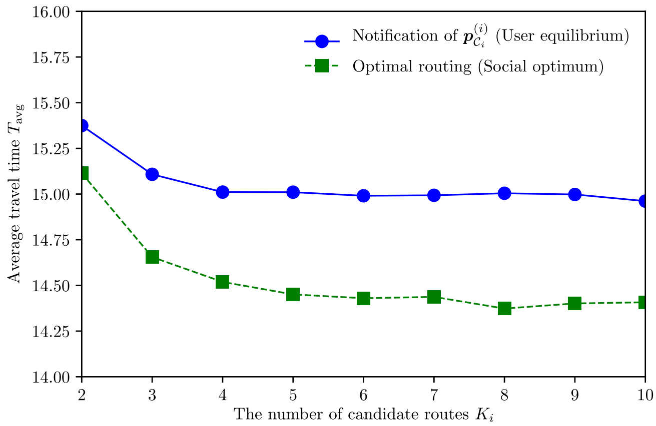

FIGURE 5. Impact of the number of route candidates K_i on the average travel time T_{avg} (grid-like network case).

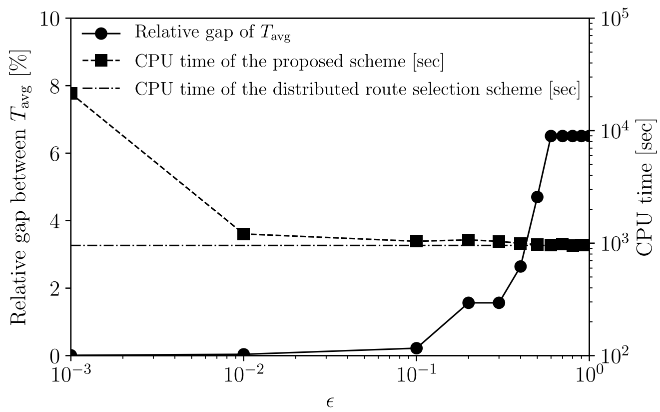

FIGURE 6. Impact of ϵ on convergence time and achievement level of selfish yet optimal routing (grid-like network case).

$f_e^{(m)}$ for each road $e \in \mathcal{E}_i$ at once, it obviously converges. On the other hand, Algorithm 2 repeatedly updates $f_e^{(m)}$ ($e \in \mathcal{E}_i$) until the selfish yet optimal routing is almost achieved, i.e., $RMSE \leq \epsilon$. ϵ controls the balance between convergence time and achievement level of selfish yet optimal routing. Fig. 6 illustrates how ϵ affects the convergence time, i.e., CPU time, and the achievement level, i.e., the relative gap between T_{avg} of the proposed scheme and that of optimal routing. We first observe that the relative gap of average travel time can be suppressed by appropriately adjusting ϵ. For example, $\epsilon = 0.01$ results in only 0.04% relative gap, which achieves almost the selfish yet optimal routing. On the contrary, the CPU time of the proposed scheme increases with a decrease of ϵ. To clarify which part of the proposed scheme contributes to the CPU time, we also show the CPU time of the distributed route selection scheme. Since the CPU time of Algorithm 1 is negligible, the difference between the CPU time of the proposed scheme and that of the distributed route selection scheme can be regarded as that of Algorithm 2. We can observe that the overhead of Algorithm 2 is much smaller than that of the distributed route selection scheme when $\epsilon \geq 0.01$. As a result, $\epsilon = 0.01$ can achieve selfish yet optimal routing while keeping the CPU time competitive

TABLE 3. Average travel time T_{avg} and maximum travel time $T_{\max}$ (grid-like network case).

Scheme	T_{avg} [min]	$T_{\max}$ [min]
Notification of $\boldsymbol{p}_{\mathcal{C}_i}^{(i)}$ (User equilibrium)	15.0	16.3
Notification of $\boldsymbol{p}_{\mathcal{C}_i}^{(m)}$	18.6	27.4
Proposed scheme	14.5	16.5
Optimal routing (Social optimum)	14.5	16.5

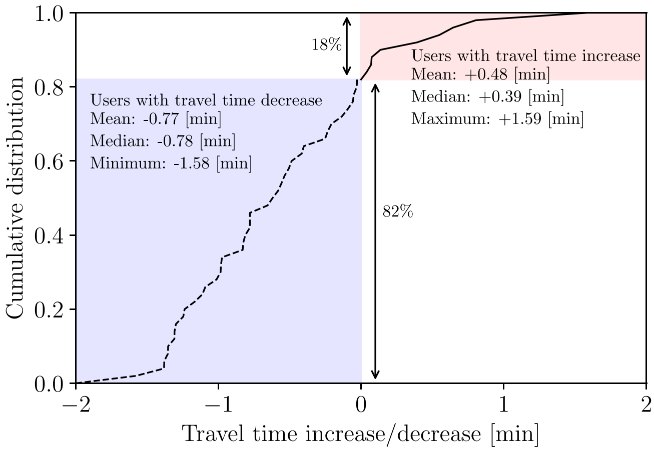

FIGURE 7. Cumulative distribution of users with travel time increase/decrease (grid-like network case).

with the distributed route selection scheme. In what follows, we use $\epsilon = 0.01$.

3) AVERAGE AND MAXIMUM TRAVEL TIME AMONG USERS

Table 3 presents average travel time T_{avg} and maximum travel time $T_{\max}$ of the four schemes. We first focus on the results of comparison schemes. We observe that T_{avg} (resp. $T_{\max}$) of the notification of $\boldsymbol{p}_{\mathcal{C}_i}^{(m)}$ increases by 3.6 [min] (24%) (resp. 11.1 [min] (68%)) compared with that of the notification of $\boldsymbol{p}_{\mathcal{C}_i}^{(i)}$. This is because the notification of $\boldsymbol{p}_{\mathcal{C}_i}^{(m)}$ results in underestimating traffic congestion, due to lack of considering other users' selfish route selection. In addition, we also observe that the notification of $\boldsymbol{p}_{\mathcal{C}_i}^{(i)}$ increases T_{avg}, i.e., 0.5 [min] (3.4%), compared with the optimal routing.

Next, we focus on the result of the proposed scheme. We confirm that the proposed scheme achieves almost the same T_{avg} compared with the optimal routing. As a result, the proposed scheme can reduce T_{avg} by 3.6 [min] and 0.5 [min] compared with the notification of $\boldsymbol{p}_{\mathcal{C}_i}^{(m)}$ and the notification of $\boldsymbol{p}_{\mathcal{C}_i}^{(i)}$, respectively. In addition, the PoA becomes 1.03, 1.28, and 1.00, in case of notification of $\boldsymbol{p}_{\mathcal{C}_i}^{(i)}$, notification of $\boldsymbol{p}_{\mathcal{C}_i}^{(m)}$, and proposed scheme, respectively. In Section V-C, we will show this improvement can become larger in the real road network.

4) INDIVIDUAL TRAVEL TIME INCREASE/DECREASE

The proposed scheme can improve the average travel time compared with the notification of $\boldsymbol{p}_{\mathcal{C}_i}^{(i)}$, however, it may also increase the travel time for some users. Fig. 7 illustrates the cumulative distribution of users with travel time increase/decrease, i.e., the difference between $T_i(\cdot)$ of the proposed scheme and that of the notification of $\boldsymbol{p}_{\mathcal{C}_i}^{(i)}$. We observe that most of the users, i.e., 82% users, experiences 0.77 [min] (5.1%) travel time decrease in average, with help of the proposed scheme. In addition, the remaining 18% users can also suppress the average travel time increase by 0.48 [min] (3.2%).

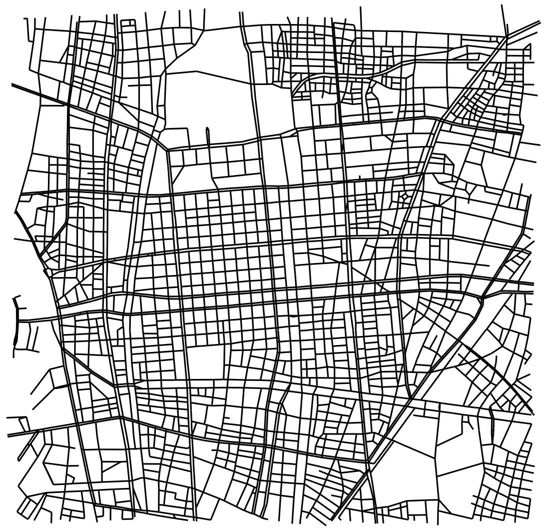

FIGURE 8. 4.7[km] × 4.5[km] east area of Nagoya station in Japan [65].

C. EVALUATION UNDER A LOCAL-LEVEL REAL ROAD NETWORK

In this section, we present the performance of the proposed scheme under the local-level real road network, i.e., the central part of Nagoya city, Japan.

1) EVALUATION MODEL

Fig. 8 illustrates the target area of 4.7 [km] × 4.5 [km] east area of Nagoya station in Japan. We use the digital road map provided by Japan Digital Road Map Association [65], in which the internal graph structure is composed of 3,173 vertices and 5,013 edges. This map also has important attribute information of each road, i.e., road length, the number of lanes, and speed limit, which can be used for parameters ($\underline{t}_e$ and c_e) of the BPR function $t_e(f_e) = \underline{t}_e(1 + \alpha(f_e/c_e)^{\beta})$. We set $\underline{t}_e$ by considering the road length and the speed limit, and c_e based on the number of lanes. As for α and β, we use the same settings, i.e., $\alpha = 0.15$ and $\beta = 4$.

To make the evaluation scenario more realistic, we also use the ordinary flow of people in the target area. In case of Nagoya city, Japan, we can also obtain such data called people flow data [66], [67], which describe the flow of people in a certain area during one day. The people flow data presents the number of people in the target road network and each person's origin and destination with its transportation method at a certain interval, e.g., an hour. In what follows,

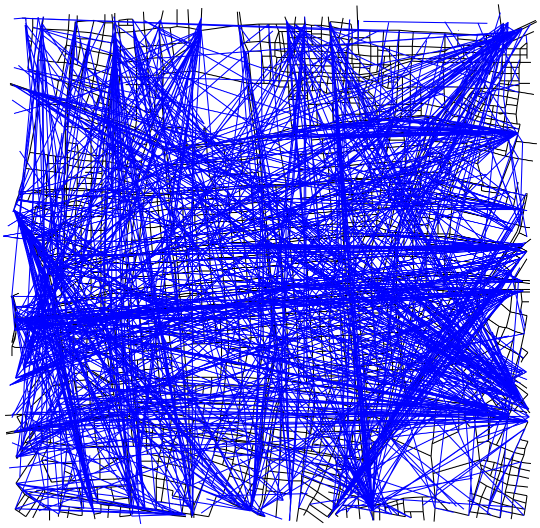

FIGURE 9. The pairs of origin and destination for each user based on the people flow data [66], [67].

TABLE 4. T_{avg} and T_{max} (real road network case).

Scheme	T_{avg} [min]	$T_{\max}$ [min]
Notification of $\boldsymbol{p}_{\mathcal{C}_i}^{(i)}$ (User equilibrium)	6.13	70.4
Notification of $\boldsymbol{p}_{\mathcal{C}_i}^{(m)}$	12.5	174
Proposed scheme	4.96	68.5
Optimal routing (Social optimum)	4.93	68.5

we focus on the start of office hours, i.e., 8:00–8:59, where 1,197 vehicle users exist in the road network. Fig. 9 illustrates the corresponding origin and destination pairs as blue lines.

2) AVERAGE AND MAXIMUM TRAVEL TIME AMONG USERS

Table 4 presents average (resp. maximum) travel time among all users, T_{avg} (resp. T_{max}), among four schemes, i.e., the proposed scheme, Notification of $\boldsymbol{p}_{C_i}^{(i)}$, Notification of $\boldsymbol{p}_{C_i}^{(m)}$, and optimal routing. We first observe that the proposed scheme can improve T_{avg} and T_{max} by 1.17 [min] (19.1%) and 1.9 [min] (2.7%), compared with the notification of $\boldsymbol{p}_{C_i}^{(i)}$, respectively. We also observe that the PoA of each scheme becomes 1.24, 2.54, and 1.01, in case of notification of $\boldsymbol{p}_{C_i}^{(i)}$, notification of $\boldsymbol{p}_{C_i}^{(m)}$, and proposed scheme, respectively. The real road network case does not have a more complex graph structure but also have biased traffic demand, i.e., the origin and destination of users, compared with the grid-like network case. The proposed scheme can effectively distribute the traffic load as in the optimal routing, and thus the improvement ratio is larger than that of the grid-like network.

3) INDIVIDUAL TRAVEL TIME INCREASE/DECREASE

Next, we focus on the degree of optimality in terms of the travel time for each user in case of the proposed scheme. Fig. 10 illustrates the cumulative distribution of users with travel time increase/decrease, i.e., the difference between

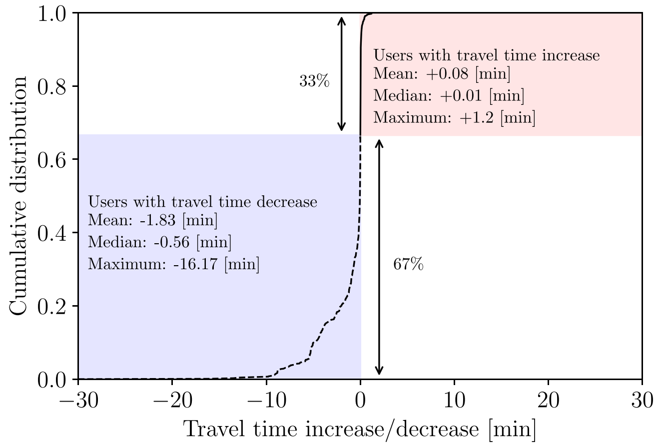

FIGURE 10. Cumulative distribution of users with travel time increase/decrease (real road network case).

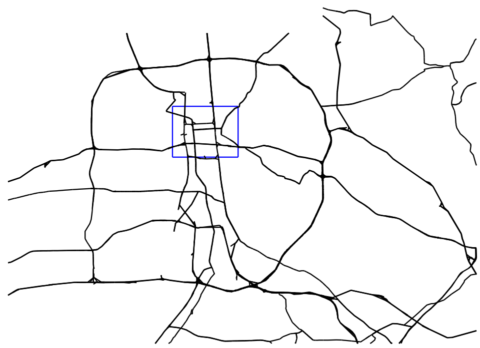

FIGURE 11. 33.9[km] × 29.7[km] area of Nagoya city in Japan [65].

$T_i(\cdot)$ of the proposed scheme and that of the notification of $\boldsymbol{p}_{C_i}^{(i)}$. We observe that the proposed scheme can reduce the travel time among 67% users with average (resp. maximum) travel time decrease of 1.83 [min] (23.1%) (resp. 16.17 [min]), compared with the notification of $\boldsymbol{p}_{C_i}^{(i)}$. On the other hand, as for the remaining 33% users, we observe that the average (resp. maximum) travel time increase is limited, 0.08 [min] (2.9%) (resp. 1.2 [min]). The above-mentioned results show that the proposed scheme can improve the travel time for most users with a slight increase of that for the remaining users.

D. EVALUATION UNDER A CITY-LEVEL REAL ROAD NETWORK

In this section, we present the performance of the proposed scheme under the city-level real road network, i.e., the whole area of Nagoya city, Japan.

1) EVALUATION MODEL

Fig. 11 depicts the target area of 33.9[km] × 29.7[km] area of Nagoya city in Japan, which is provided by Japan Digital Road Map Association [65]. The blue rectangle in Fig. 11 corresponds to the east area of Nagoya station in Fig. 8. The internal graph with 5,070 vertices and 6,332 edges consists

TABLE 5. T_{avg} **and** T_{max} **(real road network case).**

Scheme	T_{avg} [min]	$T_{\max}$ [min]
Notification of $\boldsymbol{p}_{C_i}^{(i)}$ (User equilibrium)	9.98	88.5
Notification of $\boldsymbol{p}_{C_i}^{(m)}$	11.6	124.9
Proposed scheme	9.24	70.7
Optimal routing (Social optimum)	9.21	70.1

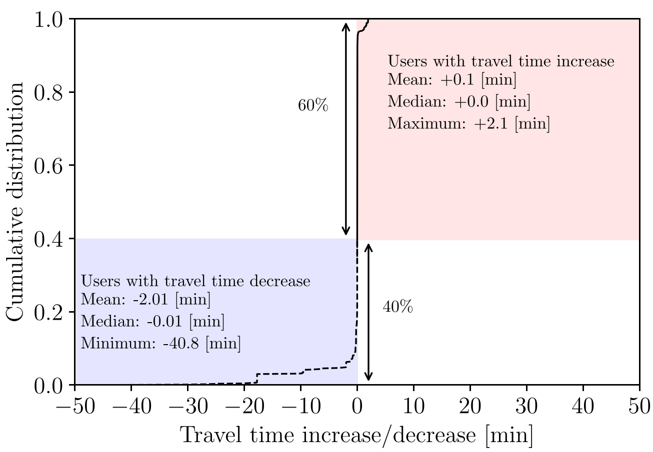

FIGURE 12. **Cumulative distribution of users with travel time increase/decrease (real road network case).**

of the major arterial roads in Nagoya city. As with the evaluation scenario at the local level, we also use the people flow data [66], [67] in the target area. In what follows, we also focus on the relatively crowded time period, i.e., start of office hours (8:00–8:59), where 10,004 vehicle users exist in the road network.

2) AVERAGE AND MAXIMUM TRAVEL TIME AMONG USERS

Table 5 presents average (resp. maximum) travel time among all users, T_{avg} (resp. T_{max}), among four schemes, i.e., the proposed scheme, Notification of $\boldsymbol{p}_{C_i}^{(i)}$, Notification of $\boldsymbol{p}_{C_i}^{(m)}$, and optimal routing. We observe that the proposed scheme improves T_{avg} and T_{max} by 0.74 [min] (7.4%) and 17.8 [min] (20.1%), compared with the notification of $\boldsymbol{p}_{C_i}^{(i)}$, respectively. We also observe that the proposed scheme has almost the same performance as the optimal routing. We confirm that the PoA of each scheme becomes 1.08, 1.26, and 1.00, in case of notification of $\boldsymbol{p}_{C_i}^{(i)}$, notification of $\boldsymbol{p}_{C_i}^{(m)}$, and proposed scheme, respectively.

3) INDIVIDUAL TRAVEL TIME INCREASE/DECREASE

Fig. 12 depicts the cumulative distribution of users with travel time increase/decrease, i.e., the difference between $T_i(\cdot)$ of the proposed scheme and that of the notification of $\boldsymbol{p}_{C_i}^{(i)}$. We observe that the proposed scheme reduces the travel time among 40% users with the average (resp. maximum) travel time decrease of 2.01 [min] (14.7%) (resp. 40.8 [min]), compared with the notification of $\boldsymbol{p}_{C_i}^{(m)}$. On the other hand, the average (resp. maximum) travel time increase is limited, 0.1 [min] (1.3%) (resp. 2.1 [min]).

VI. CONCLUSION

Traffic congestion in urban areas is mainly caused by selfish routing of users and results in considerable economic and time loss. In this article, we have proposed a scheme to achieve selfish yet optimal routing by adjusting the perceived traffic information, which is inspired by the concept of "Nudge." Selfish yet optimal routing internalizes the marginal cost into the perceived traffic information. In the proposed scheme, the server first calculates the social optimum route choice probability for each user. Then, it derives the nudging traffic information, which leads the selfish routing to the social optimum routing, and notifies it to each user. After retrieving the nudging traffic information, each user finds the selfish route choice probability under the perceived traffic information.

Through the numerical experiments under the grid-like road network, we have evaluated the following fundamental characteristics of the proposed scheme: (1) the proposed scheme achieves almost the same performance as the optimal routing and (2) it can improve the travel time of 82% users with the average travel time decrease of 0.77 [min], compared with the notification of $\boldsymbol{p}_{C_i}^{(i)}$.

Furthermore, we have also evaluated the practicality and scalability of the proposed scheme under the local-level and city-level road networks of Nagoya city. We have observed that the proposed scheme can improve the average travel time by 19.1% (resp. 7.4%), compared with the notification of $\boldsymbol{p}_{C_i}^{(i)}$, in case of the local-level (resp. city-level) road network with 1,197 (resp. 10,004) users. From the viewpoint of individual travel time under the local-level (resp. city-level) road network, the proposed scheme can reduce the travel time among 67% (resp. 40%) users with the average travel time decrease of 1.83 [min] (resp. 2.01 [min]) while suppressing the average travel time increase among the remaining users by 0.08 [min] (resp. 0.1 [min]).

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