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Theoretical Trade-Off Between Fairness and Efficiency in the Cooperative Driving Problem for CAVs at On-Ramps

ZIMIN HE [1], HUAXIN PEI [1], YUQING GUO [1], DANYA YAO [2] (Member, IEEE), AND LI LI [2] (Fellow, IEEE)

[1] Department of Automation, Tsinghua University, Beijing 100084, China

[2] Department of Automation, BNRist, Tsinghua University, Beijing 100084, China

CORRESPONDING AUTHOR: L. LI (e-mail: li-li@tsinghua.edu.cn)

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ABSTRACT Cooperative driving is crucial for improving traffic efficiency and safety for connected and automated vehicles (CAVs), especially in traffic bottlenecks. However, most of the state-of-the-art cooperative driving strategies neglect the issue of fairness. Fairness is essential to properly allocate road resources and improve the travel experience. In this paper, we focus on the fairness concerns in the on-ramp cooperative driving problem. First, we note that enhancing traffic efficiency usually leads to unfairness, but we propose solutions to balance both aspects. Using the fundamental relation in traffic flow theory, we illustrate the existence of the trade-off at congested on-ramps. We then make some modifications to the cooperative driving strategies to incorporate fairness considerations. Simulation results show that the modified strategies achieve trade-offs in agreement with the theoretical one, laying the foundation for implementing the trade-off in real-world scenarios. These findings are enlightening for the increasing research on fairness issues in cooperative driving, and contribute to the optimization of traffic management strategies.

INDEX TERMS Connected and automated vehicles (CAVs), cooperative driving, on-ramp scenario, fairness and efficiency.

I. INTRODUCTION

TRAFFIC bottlenecks, widely recognized by researchers as a major cause of congestion and accidents [1], [2], are particularly prevalent at on-ramps [3]. With advances in wireless communication technologies and connected and automated vehicles (CAVs), cooperative driving is emerging as an effective approach to improving traffic efficiency and safety. Researchers have shown great interest in cooperative driving, and have extensively studied the vehicle coordination problem in typical traffic scenarios [4], [5], [6], [7].

Cooperative driving leverages autonomous driving and intelligent network technologies to organize and schedule the movements of nearby vehicles [8], [9], thereby ensuring safe and efficient traffic management. As noted in [10], [11], the cooperative driving strategy usually consists of two parts: a scheduling problem to determine the passing order and a control problem to plan the vehicle movements. A well-planned passing order through the conflict zone can significantly reduce the vehicle travel time, which is the core issue of the cooperative driving problem [12], [13]. After the passing order is determined, a variety of motion planning methods are utilized to coordinate the movements of the vehicles and ensure that they reach the conflict zone at the required time.

However, most state-of-the-art cooperative driving strategies focus only on traffic efficiency and safety, without considering fairness measures. Fairness is also a critical issue that contributes to effective traffic management

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and promotes a balanced and harmonious society, which deserves further exploration and research. Unfair control instructions can cause queues at bottlenecks, resulting in longer travel times for some vehicles and a poor travel experience for passengers. This prevents individuals from having equal access to transportation benefits. In addition, certain areas may be plagued by persistent congestion that hinders residents' access to essential services, employment opportunities, and overall quality of life. On the other hand, a fair control strategy under congested traffic conditions may lead to inefficient traffic flows and low completion rates for some journeys. For example, the first-in-first-out (FIFO) rule schedules vehicles through the conflict zone based on their arrival time at the control zone, ensuring fairness by distributing the total travel time among vehicles equitably [14]. However, this approach is often ineffective in alleviating traffic congestion [13], [15]. Balancing fairness and efficiency can be a challenging task, as traffic controllers aim to minimize the total time spent in the network, which may result in the increased travel time for some road users. Therefore, research efforts are necessary to introduce appropriate fairness measures that strike a balance between fairness and efficiency, where both metrics reach a satisfactory level at the same time.

In our prior work [16], we utilized the standard deviation of vehicle travel time as a fairness metric, similar to [14], [17], [18]. Our findings showed that the cooperative driving strategies that reorder vehicles could provide benefits at congested on-ramps, but such improvement usually involves sacrificing fairness. By making some appropriate adjustments to the strategies, we achieved a better trade-off between fairness and traffic efficiency. However, we discovered this only through simulations and lacked a rigorous theoretical guarantee for the existence of such a trade-off. In addition, our research was limited to simple on-ramps with one lane in each direction.

In this study, we build on our previous work [16] and provide a theoretical analysis of the trade-off in the cooperative driving problem at on-ramps. We derive an analytical solution from the theory and compare it with the simulation results to confirm the existence of the trade-off. Furthermore, we extend the analysis from simple on-ramps to more complex ones, laying the foundation for further research on fairness issues.

First, we study the phenomenon that cooperative driving strategies can enhance traffic efficiency even under high traffic demand, but often lead to unfairness. Assuming that CAVs pass through the merging zone as a group, we use the fundamental relation in traffic flow theory to show that the trade-off between fairness and traffic efficiency exists for simple single-lane on-ramps. We then modify the rule-based strategy [19] and the dynamic programming (DP)-based strategy [20] to balance traffic efficiency and fairness. The simulation results validate the soundness of the assumption, verify the existence of the trade-off, and demonstrate the approach for adjusting the trade-offs in practical use. Finally, we explain how to extend the conclusions to more complex on-ramps with multiple lanes.

While the current study may not encapsulate all of the complexities, it effectively captures the core contradictions in the problem and derives a theoretical solution that matches the simulation results. The theoretical exploration of the trade-off between efficiency and fairness in the context of cooperative driving problems provides insights for more complex scenarios. By delving into the balance between efficiency and fairness, this study serves as a foundation for incorporating fairness considerations into practical applications, and underscores the importance of fairness solutions in the cooperative driving domain.

The main contributions of this paper include: (a) the inclusion of fairness concerns in the cooperative driving problem, improving the equitable distribution of transportation benefits and enhancing the travel experience; (b) a theoretical analysis on the existence of the trade-off between traffic efficiency and fairness, providing a critical foundation for integrating fairness into real-world applications; (c) the modification of cooperative driving strategies to achieve the trade-off between the two, introducing a new perspective on addressing the cooperative driving problem.

To present our work in a clear and organized manner, the remaining sections of this paper are structured as follows. Section II provides a brief review of cooperative driving strategies and the current literature on fairness issues. In Section III, we formulate the merging problem at on-ramps and compare the performance of different cooperative driving strategies in terms of fairness and traffic efficiency. The theoretical analysis of the trade-off in simple on-ramp scenarios is presented in Section IV. In Section V, we conduct some simulations to verify the theoretical trade-off. Section VI extends the conclusions from Section IV to complex on-ramps with multiple lanes. Finally, we summarize our findings and discuss potential directions for future research in Section VII.

II. LITERATURE REVIEW

In this section, we first introduce the cooperative driving strategies commonly utilized in the typical traffic bottlenecks, such as on-ramps and intersections. Then, we review the existing literature on how to distribute the benefits and losses in a fair and appropriate manner.

A. COOPERATIVE DRIVING STRATEGY

As summarized in [11], [21], the existing research on the cooperative driving problem can be classified into two approaches: optimization-based strategy and heuristics-based strategy. The optimization-based strategy seeks the global optimal passing order through the conflict zone. Li and Zhou [22] considered the environment full of CAVs, modeled the vehicle merging problem as a mixed integer linear programming (MILP) problem, and utilized the branch-and-bound method to obtain the optimal solution. Meanwhile, the cooperative merging problem was formulated as an optimization problem to minimize the travel time and maximize the number of merging vehicles, and an optimal merging sequence could be obtained by using a genetic algorithm [23]. However, optimization-based strategy usually entails a huge computation cost, rendering it impractical for real-world applications. To address this issue, Pei et al. [20] adopted the DP method in the on-ramp scenario and obtained the global optimal passing order with polynomial computational complexity. Moreover, Chen et al. [24] and Xue et al. [25] leveraged model predictive control method to construct a hierarchical merging control algorithm, where a tactical layer controller and an operational layer controller minimize the corresponding objective function to derive control commands.

Heuristics-based strategy typically employs some explicit rules to tackle the cooperative driving problem and derive a near-optimal solution with low computational cost. The simplest rule is FIFO, which determines the vehicle passing order based on the time to reach the control zone. Autonomous intersection management [26] and reservation strategy [27] coordinate the vehicles to pass through the conflict zone in a rough FIFO order. However, studies have shown that the FIFO rule is not effective in alleviating traffic congestion [13], [20]. To improve upon this rule, Zhang and Cassandras [28] developed a strategy to dynamically adjust the passing order, resulting in reduced total travel time and energy consumption. Additionally, Ding et al. [19] proposed a rule-based adjustment strategy and summarized four cases of adjusting the passing order to facilitate the merging of vehicles in a near-optimal order. Grouping the vehicles in the same direction into platoons can reduce the search space size [15], [29] and has the potential to reduce the average vehicle delay even in congested traffic [30]. To enhance merging safety, the local gap-optimal rule creates gaps for vehicles, reducing the risk of merging in mixed traffic [31].

While most cooperative driving strategies significantly improve the efficiency and safety of traffic management, they tend to overlook one crucial aspect: fairness. Fairness, however, is essential to creating an effective and balanced transportation system. Fair allocation of roadway resources ensures that each traveler has access to efficient travel options, and promotes an even distribution of the challenges and benefits of travel, making daily mobility a more positive experience for all.

B. FAIRNESS STUDY

Fairness usually refers to the fair and appropriate distribution of benefits and costs among people [32]. In the field of transportation studies, the definition and measurement of fairness vary significantly based on specific application scenarios, such as the transport provision and demand [33], [34], [35], [36], environmental externalities of transport systems [37], road pricing, congestion charges, and travel costs [38], [39], [40], and opportunities for work and other matters [41], [42]. According to [43], three key components are crucial for analyzing transportation fairness: the benefits and burdens to be distributed, the populations or social groups affected, and the principle of fairness in distribution. Overall, fairness is a complex concept that varies depending on different perspectives, making it difficult to adopt a comprehensive measure that considers all involved traffic participants equitably.

In the vehicle coordination problem, the equitable distribution of vehicle travel delay or waiting time is typically used as a measure of fairness. Zhang and Levinson [44] proposed the weighted travel time as a new objective function to distribute delays equitably and balance efficiency and fairness in ramp metering. Meng and Khoo [45] introduced the minimum-to-maximum average travel delay ratio as an equity index, and explored the Pareto optimality between the total system delay and the equity index. Other fairness measures included the standard deviation of vehicle delay or travel time [14], [16], [17] and the maximum delay that an individual vehicle can experience [18]. Furthermore, fairness and equity have been evaluated in various studies using measures such as the Gini coefficient [46], [47], Jain's fairness index [48], [49], maximum queue length [50], [51], and regional speed [52].

In the studies mentioned above, fairness is not the only control objective, as efficiency, such as total travel time or total system delay, is also taken into consideration. Researchers have effectively improved traffic efficiency while maintaining fairness by implementing a reinforcement learning-based system [53], a multi-objective optimization framework [54], [55], or a dynamic bargaining game theory approach [56]. The numerical simulations suggest that a trade-off between fairness and efficiency can be attained with appropriate parameter settings. Nevertheless, a strong theoretical guarantee for the existence of such a trade-off remains absent.

III. PROBLEM PRESENTATION

We initially focus on the typical on-ramp scenario with a single lane in the same direction, as shown in FIGURE 1. In Section VI, we elaborate on how to extend this simple case to other scenarios. The merging zone, highlighted in red shading, is where rear-end or lateral collisions may occur. The control zone, outlined by a dotted line, is where the centralized controller coordinates the vehicles for a safe and efficient merging process. In general, the centralized controller determines the passing order of the vehicles and assigns each vehicle an access time to the merging zone. Based on the assigned access time, the vehicles then use distributed algorithms to plan their movement to the merging zone. In this section, we focus primarily on the passing order scheduling problem.

To simplify the problem, we consider each vehicle as a CAV whose driving state (including parameters such as position and speed) is instantly accessible to all other CAVs through vehicle-to-everything (V2X) communication. This assumption ensures real-time information sharing between

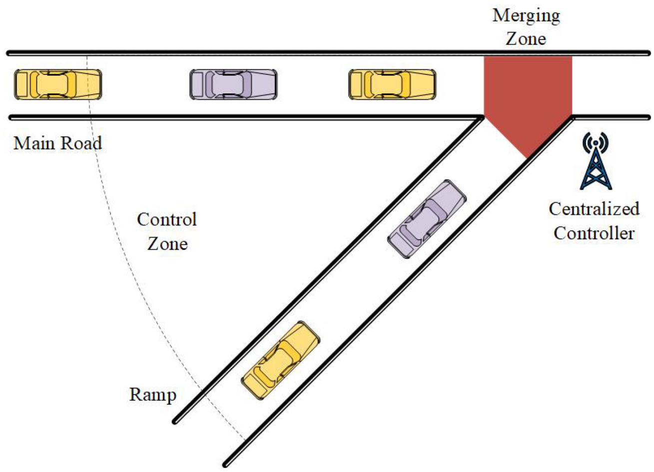

FIGURE 1. A typical merging scenario at the simple single-lane on-ramp.

the centralized controller and the vehicles, facilitating coordinated driving of all vehicles within the control zone to enhance the performance of the cooperative merging process [22], [57], [58].

A. PASSING ORDER SCHEDULING PROBLEM

Once a CAV arrives at the entrance of the control zone, we assign it a unique identity i, meaning the ith CAV to enter the control zone. For the sake of clarity, the main symbols used in this paper and their corresponding definitions are listed in TABLE 1.

To optimize the passing order of CAVs and improve traffic efficiency, we can formulate the merging problem as:

$$\min_{t_{assign},b} \omega_1 \max\left(t^i_{assign}\right) + \omega_2 \sum_{i=1}^{n}\left(t^i_{assign} - t^i_{\min}\right) \quad (1a)$$

$$s.t. \quad t^i_{assign} \geq t^i_{\min} \quad (1b)$$

$$t^i_{assign} - t^{i-1}_{assign} \geq \Delta t_1 \quad (1c)$$

$$t^k_{assign} - t^l_{assign} + M \cdot b_{k,l} \geq \Delta t_2 \quad (1d)$$

$$t^l_{assign} - t^k_{assign} + M \cdot \left(1 - b_{k,l}\right) \geq \Delta t_2 \quad (1e)$$

$$k \in N_1 = \{1, 2, .., n_1\} \quad (1f)$$

$$l \in N_2 = \{1, 2, .., n_2\} \quad (1g)$$

$$b_{k,l} \in \{0, 1\} \quad (1h)$$

where t^i_{assign} and $t^i_{\min}$ denote the assigned access time and the minimum access time to the merging zone of CAV i, n denotes the total number of vehicles in the control zone, M is a positive and sufficiently large constant, and $b_{k,l}$ denotes the passing order between CAV k and CAV l. Δt_1 and Δt_2 are the minimum safety gap for CAVs from the same and different lanes to enter the merging zone, respectively. N_1 and N_2 denote two sets containing all CAVs traveling on the lane, of size n_1 and n_2, respectively.

Hence, the two terms in Eq. (1a) can be interpreted as the total travel time and the total delay time, respectively. Constraint (1b) serves as a lower bound on the assigned access time, which is derived from the basic kinematics within the maximum speed and maximum acceleration limits, while constraint (1c) ensures the safety of CAVs from the same lane approaching the merging zone. If $b_{k,l} = 1$, CAV k reaches the merging zone earlier than CAV l. Then constraint (1d) must be met and constraint (1e) can prevent collisions between CAVs from different lanes when they arrive at the merging zone. Further details on the optimization problem formulation can be found in [15], [20].

Problem (1) defines a MILP problem that is NP-hard. Although the branch-and-bound method is commonly used to obtain the global optimal solution, it requires a computational

TABLE 1. The nomenclature lists.

Symbol	Definition
The symbols below are treated as constants.	
L	The length of the control zone
Δt_1	The minimum safety gap for CAVs in the same lane to enter the merging zone
Δt_2	The minimum safety gap for CAVs from different lanes to enter the merging zone
$v_{\max}, v_{\min}$	The maximum and minimum speeds of CAVs
$u_{\max}, u_{\min}$	The maximum and minimum accelerations of CAVs
ω_1, ω_2	The weights of the objective function in the passing order scheduling problem
N	The number of CAVs in each group
d	The safety distance for CAVs to depart from the point queue
The symbols below are treated as variables.	
t^i_{assign}	The access time to the merging zone assigned to CAV i
$t^i_{\min}$	The minimum access time to the merging zone of CAV i
t^i_{travel}	The travel time to the merging zone of CAV i
t^i_{depart}	The departure time from the control zone entrance of CAV i
$b_{k,l}$	The binary decision variable indicating the passing order between CAV k and CAV l
ρ, μ	The standard deviation and average of vehicle travel times
λ	The vehicle arrival rate to the control zone
v	The space mean speed of all CAVs in the control zone
q	The average number of CAVs passing through the merging zone in an hour
k	The average number of CAVs per kilometer of lane
T	The average time interval for CAVs to dequeue from the point queue
T_h	The average headway of CAVs traveling on the lane
α, β	The balancing factors to balance the performance of fairness and efficiency

Algorithm 1 An Iterative Algorithm to Calculate Assigned Access Time for All CAVs

Input: The passing order of CAVs P.

Output: The assigned access time t^i_{assign} for each CAV, and objective function J.

1: $t^{P_1}_{assign} \leftarrow t^{P_1}_{\min}$
2: **for** each $i \in [2, length(P)]$ **do**
3: **if** CAV P_{i-1} and CAV P_i are in the same lane **then**
4: $t^{P_i}_{assign} \leftarrow \max\left(t^{P_{i-1}}_{assign} + \Delta t_1,\ t^{P_i}_{\min}\right)$
5: **else**
6: $t^{P_i}_{assign} \leftarrow \max\left(t^{P_{i-1}}_{assign} + \Delta t_2,\ t^{P_i}_{\min}\right)$
7: **end if**
8: **end for**
9: $J \leftarrow \omega_1 \max\left(t^{P_i}_{assign}\right) + \omega_2 \sum_{i=1}^{n}\left(t^{P_i}_{assign} - t^{P_i}_{\min}\right)$

time of $O(b^d)$, where d is the search depth, approximately equal to the number of vehicles under consideration. The computational complexity increases exponentially with the number of vehicles, making the computational process exceedingly time-consuming and impractical for real-world implementation.

However, there is an alternative approach to addressing the problem. We first determine the vehicle passing order and then solve a linear programming problem [14], [59]. In this case, constraints (1d) and (1e) become linear without the binary variable $b_{k,l}$, making it possible to solve problem (1) using a iterative algorithm with a time complexity of $O(n)$, where n represents the number of vehicles within the control zone. Algorithm 1 outlines the details of this approach.

Building on this idea, the original problem can be transformed into determining the vehicle passing order to optimize traffic efficiency. In recent years, researchers have proposed some cutting-edge cooperative driving strategies to address this issue. Two notable strategies are the rule-based strategy [19] and the DP-based strategy [20]. Specifically, the rule-based strategy proposes four cases to adjust the vehicle passing order, and coordinates multiple CAVs in the same lane through the merging zone sequentially. The DP-based strategy defines the state space, state transition, and criterion function using relevant domain knowledge to derive the global optimal passing order under the polynomial time complexity of $O(n^2)$. For more information, readers can refer to [19], [20]. Numerical simulation results presented in [16] indicate that compared to the FIFO-based strategy, both the rule-based and the DP-based strategies produce better solutions in a short computation time.

After the passing order and access time to the merging zone are determined, the CAVs use methods to plan their movement to the merging zone, including optimal control method [59], [60], simple motion planning method [15], [20], and virtual vehicle technique [13], [61]. These motion planning methods ensure that the CAVs reach the merging zone at the required time under the vehicle control constraints.

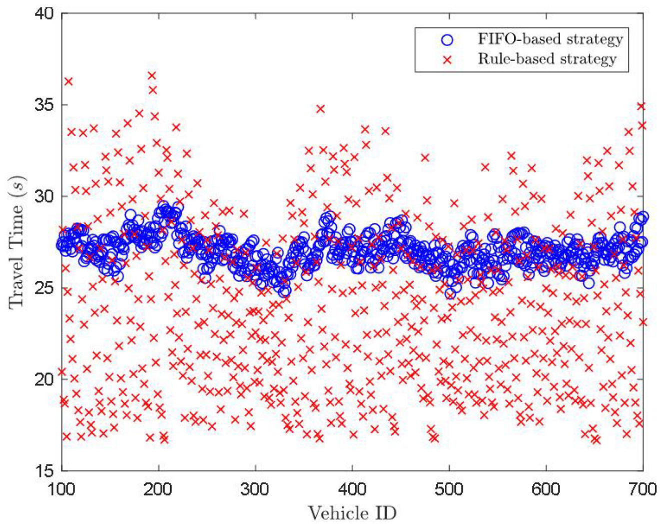

FIGURE 2. Scatter plot for vehicle travel times under two different strategies.

B. FAIRNESS AND EFFICIENCY DISCUSSION

In addition to traffic efficiency and computation time, fairness is also a critical concern that ensures equitable access and balanced distribution of transportation benefits among different individuals and communities. Under congested traffic conditions, the cooperative driving strategy of reordering vehicles has the potential to improve traffic efficiency but may come at the cost of fairness. To quantify fairness, we use the standard deviation of vehicle travel times as a metric [14], [16], as it can prevent an individual vehicle from being overburdened by excessive congestion, thereby promoting a balanced travel experience for all.

$$\rho = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(t^i_{travel} - \mu\right)^2} \tag{2a}$$

$$\mu = \frac{1}{n}\sum_{i=1}^{n} t^i_{travel} \tag{2b}$$

where t^i_{travel} represents the time CAV i takes to reach the merging zone, then μ is the average travel time, and ρ is the standard deviation, with a larger value indicating greater unfairness.

Taking the comparison between the FIFO-based strategy and the rule-based strategy as an example, we present the scatter plot for vehicle travel times under the two strategies in FIGURE 2, which is derived from our previous work [16]. Although the rule-based strategy outperforms the FIFO-based strategy in terms of the average vehicle travel time, it leads to an uneven distribution of travel times with a larger standard deviation. In other words, the rule-based strategy enhances traffic efficiency but sacrifices fairness. Therefore, neither the rule-based strategy nor the FIFO-based strategy can achieve a good balance between fairness and traffic efficiency performance.

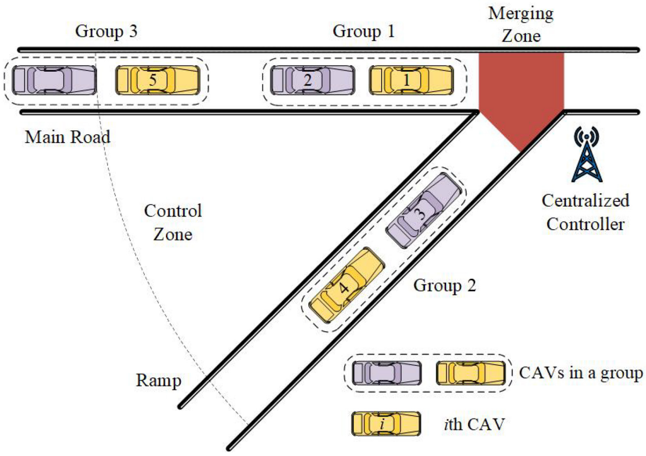

FIGURE 3. Presentation for the approach. Assume that $N = 2$ and that every two CAVs on the main road and the ramp form a group. The passing order through the merging zone is "Group 1 → Group 2 → Group 3", meaning that Group 1 passes first, followed by Group 2 and Group 3. We can alter the value of N and arrange different numbers of CAVs through the merging zone in sequence.

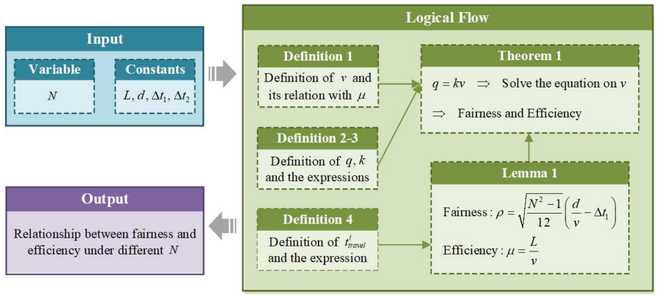

FIGURE 4. Structure of the entire proof.

Nevertheless, the studies on ramp metering have successfully achieved the trade-off between fairness and traffic efficiency [44], [56], [62]. Similarly, a trade-off should exist in the cooperative driving problem at on-ramps. We can pursue sub-optimal solutions for both traffic efficiency and fairness simultaneously to obtain better performance of the two [14], [16]. Alternatively, we can construct a multi-objective optimization framework that accounts for both traffic efficiency and fairness to achieve the balance [55], [63]. Before implementing the trade-off, we will provide a theoretical analysis of its existence in the next section.

IV. THEORETICAL ANALYSIS ON THE TRADE-OFF

This section proposes a theoretical analysis of the trade-off between fairness and efficiency in the cooperative driving problem at simple single-lane on-ramps. When vehicle arrival rates are lower, the cooperative driving strategies can improve traffic efficiency by effectively coordinating and optimizing the use of road resources. This improves overall traffic efficiency while still maintaining a fair playing field for all road users. However, under heavy traffic conditions, the cooperative driving strategies aim to improve traffic performance by rearranging CAVs, which may result in longer travel times for some CAVs. Therefore, our research focuses primarily on investigating scenarios with high vehicle arrival rates in two lanes, with the goal of determining how to balance fairness and efficiency under such circumstances.

Our approach is based on organizing the CAVs into groups and coordinating the CAV groups on the main road and the ramp to alternate through the merging zone, where each group contains N CAVs, as shown in FIGURE 3. Although this approach may not be optimal, it provides a feasible solution to Problem (1), which is particularly relevant in heavy traffic situations. Consider a scenario where an unbalanced allocation of time to different groups results in a larger number of CAVs from one lane entering the merging zone in sequence. This could create a bottleneck in the other lane and impede the flow of vehicles, resulting in increased congestion and reduced overall traffic efficiency. Conversely, a uniform group size and balanced time allocation can mitigate the risk of such bottlenecks and promote a smoother merging of traffic. This approach aims to strike a balance between promoting smooth traffic flow and maintaining equitable road use for all vehicles by modifying the group size N.

The overall proof structure is depicted in FIGURE 4, and Theorem 1 shows the primary conclusion of the study. To prove this theorem, we first introduce the definitions of the space mean speed v, flow rate q, and density k, along with their corresponding expressions in Definition 1-3. Using the fundamental relation in traffic flow theory, we derive an equation involving v and then solve it. Lemma 1 outlines how to obtain the fairness and efficiency metrics with a given v. By varying the value of group size N, we can analyze the trend of both metrics and identify the trade-off.

Definition 1: The space mean speed v is defined based on the average time taken to cross a given distance L, the length of the control zone.

$$v = \frac{L}{\frac{1}{N}\sum_{i=1} t_{travel}^{i}} = \frac{L}{\mu} \tag{3}$$

where t_{travel}^{i} is the travel time of CAV i to the merging zone, and μ is the average travel time of all CAVs.

In traffic flow theory, there is a fundamental relation that establishes a close link between the three parameters: flow rate, density, and speed.

$$q = kv \tag{4}$$

where q is the flow rate and k is the density. v is the space mean speed, as defined in Eq. (3). Knowing two of these variables immediately leads to the remaining third variable.

Based on Definition 1, the main conclusion of this study is stated as follows.

Theorem 1: Using the fundamental relation from traffic flow theory, we can formulate an equation for the space mean speed v. This equation can be solved by setting certain constants. As a result, we derive the standard deviation and the average of the vehicle travel times. These two quantifiable metrics serve as indicators to evaluate the fairness and efficiency of the system, respectively.

To prove Theorem 1, we need to take two steps. First, we introduce the definitions and expressions for the flow rate and density in Definition 2 and Definition 3, and derive an equation for the space mean speed. Second, we demonstrate how to compute the fairness and efficiency metrics with a known space mean speed in Lemma 1.

A. THE EQUATION ABOUT V

The following are the definitions of flow rate and density in traffic flow theory, along with their expressions. By substituting these expressions into Eq. (4), we can obtain the equation for the space mean speed v.

Definition 2: The flow rate q is usually defined as the number of vehicles passing through a given cross-section per unit of time, and is estimated by dividing the number of vehicles by the elapsed time. If we take the sum of the headways as the elapsed time, then there exists a reciprocal relationship between the flow rate and the average headway.

$$q = \frac{N_q}{T_q} = \frac{N_q}{\Sigma_{i=1} h_i} = \frac{1}{\frac{1}{N_q}\Sigma_{i=1} h_i} = \frac{1}{T_h} \tag{5}$$

where N_q is the number of vehicles counted, T_q is the total elapsed study time, h_i is the headway recorded for each CAV, and T_h is the average headway.

We adopt the time iteration equation in Algorithm 1 to compute T_h. Due to the congestion at the on-ramp, the assigned access time is always greater than the minimum access time, so the time iteration equation can be simplified as:

$$\begin{aligned} t^i_{assign} &= \max\left(t^{i-1}_{assign} + \Delta,\ t^i_{\min}\right) = t^{i-1}_{assign} + \Delta \\ &= \begin{cases} t^{i-1}_{assign} + \Delta t_1, & \text{if on the same lane} \\ t^{i-1}_{assign} + \Delta t_2, & \text{otherwise} \end{cases} \end{aligned} \tag{6}$$

In other words, the headway is Δt_1 for CAVs within the group, and Δt_2 for CAVs between the groups. Thus, the average headway for CAVs traveling on the lane is:

$$T_h = \frac{(N-1)\Delta t_1 + \Delta t_2}{N} \tag{7}$$

By Eq. (5) and Eq. (7), we can express the flow rate as:

$$q = \frac{N}{(N-1)\Delta t_1 + \Delta t_2} \tag{8}$$

Definition 3: Density is a standard concept in physics that is adopted by traffic flow theory. It disregards the effects of traffic composition or vehicle lengths and considers only the abstract number of vehicles. The density k is measured in a given spatial region and reflects the number of vehicles per kilometer of lane.

$$k = \frac{N_k}{L} \tag{9}$$

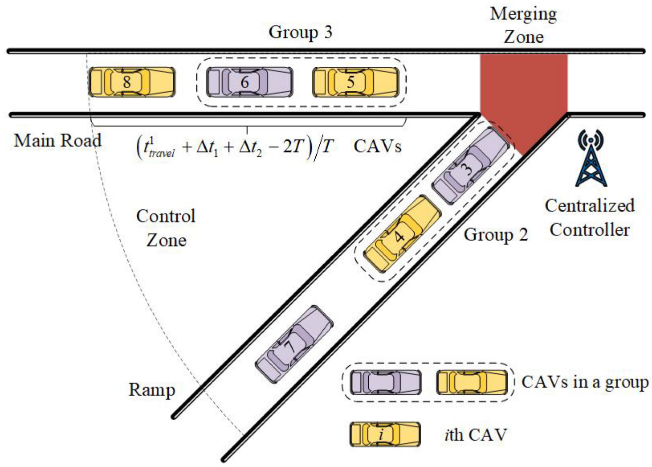

FIGURE 5. Estimation of the number of CAVs on the main road at time $t = t_0 + \Delta t_1 + \Delta t_2$.

where N_k indicates the number of vehicles at a given moment, and L is the length of the on-ramp control zone. We try to estimate N_k and get the density of the on-ramp.

Knowing the average time interval T from the entrance of control zone and the travel time T_0 of the first CAV, we can calculate the number of CAVs in the lane as $T_0/.T$. In our proposed approach, CAV groups on the main road and the ramp alternate through the merging zone in cycles. This allows us to estimate the overall density by counting CAVs in a single cycle. Given the similarity between the main road and ramp traffic flows, we only need to count the main road CAVs and multiply the total by two. The calculation is illustrated using the scenario in FIGURE 3 as an example.

Suppose that at time $t = t_0$, CAV 1 travels on the main road for time t^1_{travel} and arrives at the merging zone. Therefore, there are approximately $t^1_{travel}/.T$ CAVs on the main road at time $t = t_0$. According to the time iteration relations in Eq. (6), CAV 2 reaches the merging zone at time $t = t_0 + \Delta t_1$, and its travel time is $t^1_{travel} + \Delta t_1 - T$. For more details, see Definition 4 in the next subsection. Furthermore, at time $t = t_0 + \Delta t_1 + \Delta t_2$, CAV 3 reaches the merging zone and CAV 5 has been traveling in the lane for time $t^1_{travel} + \Delta t_1 + \Delta t_2 - 2T$. So, the number of CAVs on the main road at this time is $(t^1_{travel} + \Delta t_1 + \Delta t_2 - 2T)/.T$, as shown in FIGURE 5. We perform the same calculation for time $t = t_0 + 2\Delta t_1 + \Delta t_2$ when CAV 4 arrives at the merging zone. Finally, we average the values for the four time points to estimate the number of CAVs on the main road.

For the general case with a group size of N, we count $2N$ moments, representing when the CAV groups from the main road and the ramp reach the merging zone. With the same calculation, we can estimate the average number of CAVs on the main road at these $2N$ moments. Then, by doubling it, we can obtain the number of CAVs in the entire on-ramp.

We treat the main road and the ramp that share an exit as one long straight road, so the density k is expressed as:

$$k = 2\left(\frac{t^1_{travel}}{T} + \frac{(N-1)\Delta t_1}{T} + \frac{\Delta t_2}{2T} - \frac{3N-1}{4}\right) \Big/ L \quad (10)$$

Combining Eq. (4), Eq. (8), and Eq. (10), we can establish an equation for the space mean speed v. Although there are two unknown variables T and t^1_{travel}, they are related to v and the expressions are shown in Eq. (14) and Eq. (17). With these equations, the simplified equation can finally be obtained:

$$av^2 + bv + c = 0 \quad (11a)$$
$$a = (4\Delta t_1 - 4\Delta t_2 - 4N\Delta t_1)m + 2\Delta t_1\Delta t_2 - 2\Delta t_2^2 - 2N\Delta t_1\Delta t_2 \quad (11b)$$
$$b = 3d\Delta t_2 - 3d\Delta t_1 + 4Nd\Delta t_1 - Nd\Delta t_2 - N^2 d\Delta t_1 \quad (11c)$$
$$c = 2LNd \quad (11d)$$
$$m = \log\left(\exp\left(\frac{L}{v} - \frac{N-1}{2}\left(\frac{d}{v} - \Delta t_1\right)\right) + \exp\left(\frac{L}{v_{\max}}\right)\right) \quad (11e)$$

where a, b and c are three coefficient expressions, for the convenience of presentation. m is a LogSumExp function, which will be described in detail later in Eq. (16). $v_{\max}$ is the maximum speed for CAVs traveling in the lane.

So far, we have established an equation involving the space mean speed v, which can be solved by setting some constants. This completes the first part in the proof of Theorem 1.

B. THE CALCULATION ON FAIRNESS AND EFFICIENCY

Below we demonstrate how to calculate the fairness and efficiency metrics at a given space mean speed. Before presenting Lemma 1, we introduce the definition and expression of vehicle travel time.

Definition 4: The vehicle travel time is the time spent in the journey, and calculated by subtracting the departure time from the assigned access time to the merging zone.

$$t^i_{travel} = t^i_{assign} - t^i_{depart}, \quad i = 1, 2, .., N \quad (12)$$

where t^i_{assign} denotes the assigned access time to the merging zone of CAV i, and t^i_{depart} is the departure time from the point queue. t^i_{travel} is the travel time, and its calculation is shown in FIGURE 6. For simplicity, we assume that CAVs travel at a constant speed to the merging zone.

For t^i_{depart}, it is assumed that CAVs depart from the entrance of the control zone at time interval T and the iterative relation for the departure time can be stated as:

$$t^i_{depart} = t^{i-1}_{depart} + T, \quad i = 1, 2, .., N \quad (13)$$

At high arrival rates, regardless of the vehicle arrival pattern, the CAVs entering the control zone entrance will surpass the on-ramp capacity and are therefore stored in a point queue for safety purposes [13], [64]. A CAV is released from the point queue only when sufficient space is available. We can calculate the average time interval T for departure:

$$T = \frac{d}{v} \quad (14)$$

where d denotes the safety distance for CAVs to depart from the point queue. In general, once the leading CAV in the point queue has traveled for a duration of T and left sufficient space d, the second CAV can depart from the point queue and proceed on the lane.

For t^i_{assign}, the time iterative relation is shown in Eq. (6). Thus, by Eq. (6) and Eqs. (12)-(14), we have the travel time for each CAV in the group.

$$t^i_{travel} = t^1_{travel} - (i-1)\left(\frac{d}{v} - \Delta t_1\right), \quad i = 1, 2, .., N \quad (15)$$

The travel time of CAVs in the group follows an arithmetic progression, gradually decreasing from one term to the next, thus ensuring the efficiency of the traffic flow. Note that CAVs are subject to a maximum speed constraint at the on-ramp, which imposes a lower bound on the travel time. To facilitate the computational process, we adopt the smooth LogSumExp function as an approximation technique.

$$t^N_{travel} = \max\left(t^N_{travel}, \frac{L}{v_{\max}}\right) \approx \log\left(\exp\left(t^N_{travel}\right) + \exp\left(\frac{L}{v_{\max}}\right)\right) \quad (16)$$

By inserting the travel times from Eq. (15) into Eq. (3) and then replacing t^N_{travel} on the right side of Eq. (16), we arrive at Eq. (17b). As a result, Eq. (17) establishes the correlation between t^1_{travel} and v.

$$t^1_{travel} = t^N_{travel} + (N-1)\left(\frac{d}{v} - \Delta t_1\right) \quad (17a)$$
$$t^N_{travel} = \log\left(\exp\left(\frac{L}{v} - \frac{N-1}{2}\left(\frac{d}{v} - \Delta t_1\right)\right) + \exp\left(\frac{L}{v_{\max}}\right)\right) \quad (17b)$$

Note that T and t^1_{travel} are related to v, so Eq. (11) can ultimately be simplified.

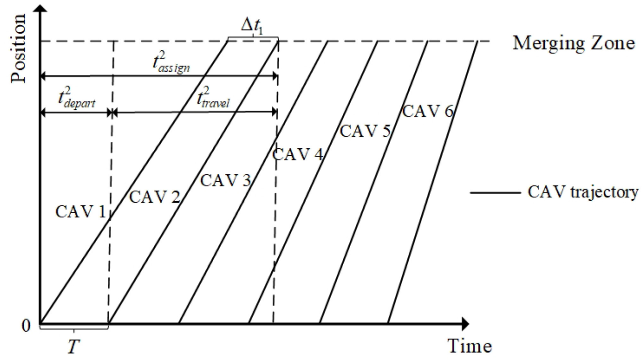

FIGURE 6. Illustration for the calculation of vehicle travel time.

TABLE 2. Parameter value settings.

Parameter	Value	Parameter	Value
L	$250\ m$	Δt_1	$1.5\ s$
d	$30\ m$	Δt_2	$2\ s$
$u_{\max}$	$3\ m/s^2$	$v_{\max}$	$15\ m/s$
$u_{\min}$	$-5\ m/s^2$	$v_{\min}$	$0\ m/s$
ω_1, ω_2	0.5	λ	$0.5\ veh/(lane \cdot s)$

Lemma 1: Given a space mean speed v, we can get the fairness and efficiency metrics, denoted as ρ and μ, respectively.

$$\rho = \sqrt{\frac{N^2-1}{12}}\left(\frac{d}{v} - \Delta t_1\right) \tag{18a}$$

$$\mu = \frac{L}{v} \tag{18b}$$

Proof: As shown in Eq. (2), the standard deviation and the average of the vehicle travel time are used to measure fairness and efficiency. Since the travel time follows an arithmetic progression shown in Eq. (15), we can calculate the standard deviation as:

$$\begin{aligned}\rho &= \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(t^i_{travel} - mean\left(t^i_{travel}\right)\right)^2} \\ &= \left(\frac{d}{v} - \Delta t_1\right)\sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(\frac{N-1}{2} - i + 1\right)^2} \\ &= \sqrt{\frac{N^2-1}{12}}\left(\frac{d}{v} - \Delta t_1\right)\end{aligned} \tag{19}$$

In addition, based on Definition 1, we have the average of vehicle travel time shown in Eq. (18b). Therefore, we can obtain the fairness and efficiency metrics if given a v, which completes the second part in the proof of Theorem 1. ∎

V. SIMULATION ANALYSIS

In this section, we first present the results from traffic flow theory and analyze the relationship between fairness and traffic efficiency. Then, we apply the concept of improving efficiency while maintaining fairness to some state-of-the-art cooperative driving strategies, including the rule-based strategy [19] and the DP-based strategy [20]. Specifically, we introduce balancing factors to modify the strategies and balance the performance of fairness and efficiency. The theoretical analysis and the strategies introduce different perspectives to find the trade-off and obtain the consistent results, thus laying the foundation for implementing the trade-off in real traffic conditions.

The simulations are performed on the MATLAB platform of a desktop computer equipped with an Intel i9 CPU and 32GB RAM, using the parameter values listed in TABLE 2.

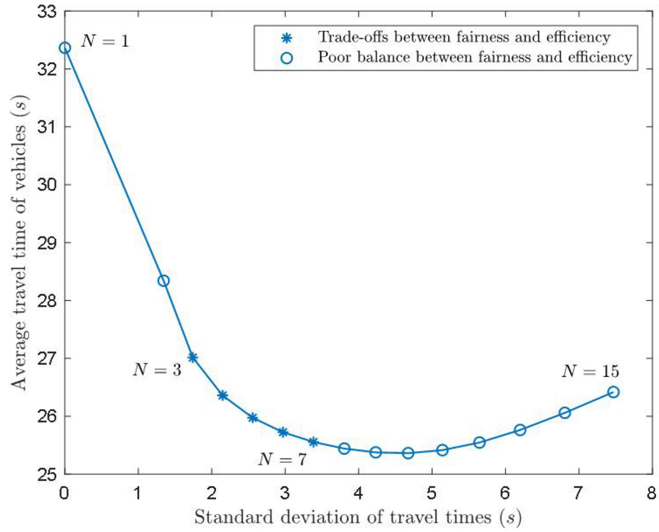

FIGURE 7. Relationship between fairness and efficiency in theory.

A. THEORETICAL RESULT

As previously mentioned, by applying some parameter settings, we can calculate the space mean speed v with Eq. (11), and further the standard deviation and the average of the travel times with Eq. (18). We vary the number of vehicles in each group N from 1 to 15 and analyze the relationship between the two, as shown in FIGURE 7. The standard deviation of travel times represents fairness, while the average travel time reflects traffic efficiency.

It can be observed that as N increases, the standard deviation of the travel times increases, while the average travel time initially decreases and then increases. According to Eq. (18a), the travel time for CAVs in a group becomes more evenly distributed as N decreases, suggesting a lower standard deviation and improved fairness among all CAVs. Note that $N = 1$ corresponds to each CAV forming a group, and since all groups are assumed to have the same travel time distribution, there is no standard deviation in travel time.

We also examine the dynamics of the average travel time concerning N. For small values of N ($N = 1, 2$), a handful of CAVs are grouped, resulting in frequent right-of-way changes and sluggish traffic flow due to merging delays from different lanes. As N increases, the flow rate also increases according to Eq. (8). More CAVs are coordinated through the merging zone without interruption, while CAVs in the other lane will slow down and yield, leading to increased congestion at the on-ramp. However, if N is not too large ($N = 3 \sim 9$), the right-of-way exchanges occur quickly and the congestion can dissipate at short intervals, allowing for faster travel and a reduction in the average travel time. Once N exceeds a certain threshold ($N = 10 \sim 15$), excessive CAVs leads to persistent congestion and escalating density, compromising traffic efficiency.

By selecting an appropriate group size N, the right-of-way can be exchanged between the two lanes at intervals, making full use of the road resources and improving traffic efficiency without causing congestion. At the same time, fairness among CAVs is ensured with a small N. Therefore, we can choose an appropriate N to strike a balance between fairness and traffic efficiency.

As shown in FIGURE 7, both the fairness and efficiency metrics reach an excellent level when $N = 3 \sim 7$. Thus, we conclude that $N = 3 \sim 7$ CAVs in each group can achieve a balance between fairness and efficiency. To enhance traffic efficiency, we set N to 7, while N can be 4 or 5 for a better trade-off. By adjusting the values of N, we can customize the trade-off according to our preferences.

B. MODIFICATION TO THE RULE-BASED STRATEGY

The rule-based strategy presents four cases for adjusting the vehicle passing order to improve traffic efficiency. When the requirement for adjustment is satisfied, the CAVs are rearranged to the position with better traffic efficiency. According to [19], the simulation results indicate that the strategy performs similarly to the global optimal solution. However, FIGURE 2 shows that the improvement in traffic efficiency usually comes at the cost of fairness, causing some CAVs to experience longer travel times before reaching the merging zone.

To strike a balance fairness and efficiency, we introduce a balancing factor α to consider fairness while pursuing traffic efficiency. The passing order adjustment is implemented only when both the requirement described in case 1 to 4 [19] and the following condition are satisfied.

$$J_{new} < J_{now} - \alpha J_{new} \cdot D_1 \tag{20}$$

where J_{now} refers to the objective function value under the current passing order, which can be calculated by Eq. (1a). And a smaller value indicates better traffic efficiency. The current passing order assigns the newly arrived CAV to the last position. J_{new} is the objective function value after adjusting the current passing order. D_1 is an integer that indicates how many orders differ between the current and new passing orders, and can prevent a large number of order changes with little improvement in traffic efficiency.

With the inclusion of α, the rule-based strategy adjusts the passing order only when it results in a significant increase in traffic efficiency. As α increases, it becomes more challenging to satisfy Eq. (20), so the rule-based strategy tends to maintain the current passing order instead of adjusting it. Ultimately, the rule-based strategy adopts a FIFO order, achieving the best fairness but low traffic efficiency. Therefore, we seek to find a trade-off between fairness and traffic efficiency by increasing the value of α.

To evaluate fairness and efficiency using Eq. (2), we simulate the movement of CAVs in the traffic flow scenario and collect all the travel times. For large vehicle arrival rates, CAVs are first stored in the point queue, and are only released to the lane when sufficient space is available. We use the rule-based strategy to determine the passing order and assign the access time for CAVs in the control zone, and then employ a simple motion planning method [15], [20] to guide the CAVs to reach the merging zone at the required time. Once a CAV arrives at the merging zone, we remove it from the passing order and record its travel time.

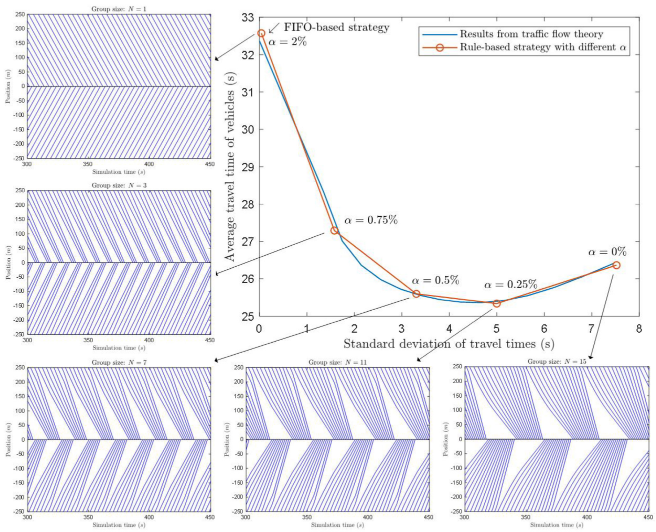

FIGURE 8. Relationship between fairness and efficiency in the modified rule-based strategy, compared to the theoretical result. The vehicle trajectories under different α are also drawn.

We vary the value of α from 0% to 2% and plot the relationship between fairness and efficiency in FIGURE 8. For each value, we run a twenty-minute simulation and collect all vehicle travel times to calculate fairness and efficiency using Eq. (2). As α increases, the average travel time initially decreases before increasing, and the standard deviation decreases continuously. Due to the presence of α, the rule-based strategy does not aim to obtain the optimal solution in each static scenario. Instead, the sub-optimal solution, which maintains the current passing order and reduces the number of sequence adjustments triggered by small efficiency gains, may have the potential to improve the overall traffic efficiency. For example, the strategy with $\alpha = 0.25\%$ and $\alpha = 0.5\%$ outperforms the pure rule-based strategy in terms of traffic efficiency. In addition, as α increases, the rule-based strategy tends to preserve the FIFO order resulting in fairness but low traffic efficiency. To balance the performance between the two, we can select the points with $\alpha = 0.5\%$ and $\alpha = 0.75\%$.

The simulation result closely approximates the theoretical result, as shown in FIGURE 8. For example, the simulation point $\alpha = 0\%$ almost coincides with the theoretical point $N = 15$. The vehicle trajectory shows the coordination of 15 CAVs moving sequentially through the merging zone. This proves that the theoretical analysis and the modified strategy are essentially the same, coordinating CAVs through the merging zone in groups with a fixed group size. The trajectory also reveals an excessive accumulation of CAVs at the on-ramp, reducing the overall efficiency to some extent. For other α values, we can identify the corresponding N in the theoretical result. More importantly, the trade-offs between fairness and efficiency observed in the simulation are consistent with the theoretical ones. Thus, by modifying the strategy properly, we can fairly distribute the benefits of transportation among road users while maintaining efficient traffic management, paving the way for balancing the performance of the two in practical use.

C. MODIFICATION TO THE DP-BASED STRATEGY

The DP-based strategy utilizes an objective function that considers traffic efficiency as the criterion. When dealing with multiple predecessor states, it selects the optimal predecessor state according to the criterion function and terminates the search for other states, ensuring optimality and computational efficiency. Although the objective function used in this study differs from that in [20], the simulation results in [16] indicate that the DP-based strategy also performs well in terms of traffic efficiency under the objective function in Eq. (1a). Similarly, we can modify the search process for the DP-based strategy to incorporate fairness by introducing a balancing factor β into the objective function:

$$\min \ \omega_1 \max\left(t^i_{assign}\right) + \omega_2 \sum_{i=1}^{n} \left(t^i_{assign} - t^i_{\min}\right) + \beta \cdot D_2 \quad (21)$$

where D_2 represents the difference between the searched and current passing orders. To perform the calculation, we subtract the order of each CAV in two passing orders and then sum the absolute values to get D_2. For instance, suppose the current passing order is CAV 1 $\rightarrow$ CAV 2, and the searched passing order is CAV 2 $\rightarrow$ CAV 1. In this case, the order difference is calculated as -1 for CAV 1 and 1 for CAV 2, so that D_2 is the sum of the absolute values as 2.

The modified objective function aims to prevent large changes in the passing order that result in small benefits. As β increases, the DP-based strategy explores only those passing orders that can significantly improve traffic efficiency. Otherwise, the strategy maintains the current passing order and stops searching. At a large β, the DP-based strategy also derives a FIFO order. By increasing the value of β from 0 to 1, we can observe the trends of fairness and efficiency in the traffic flow scenarios. The simulation results, along with the theoretical results, are shown in FIGURE 9.

As β increases, the standard deviation of travel times keeps decreasing, and the average travel time first decreases and then increases, for the same reason mentioned earlier. To strike a balance between fairness and efficiency, there are three points, i.e., $\beta = 0.15$, $\beta = 0.25$, and $\beta = 0.5$. Similar to FIGURE 8, the DP-based strategy coordinates a specific number of CAVs through the merging zone across varying β, closely approximating the theoretical results. The minor deviations may be attributed to some simulation-related random factors affecting the vehicle departure intervals. Nevertheless, the modified DP-based strategy offers fairness and efficiency trade-offs that are consistent with the theoretical ones. This agreement underscores its practical applicability in achieving effective traffic management.

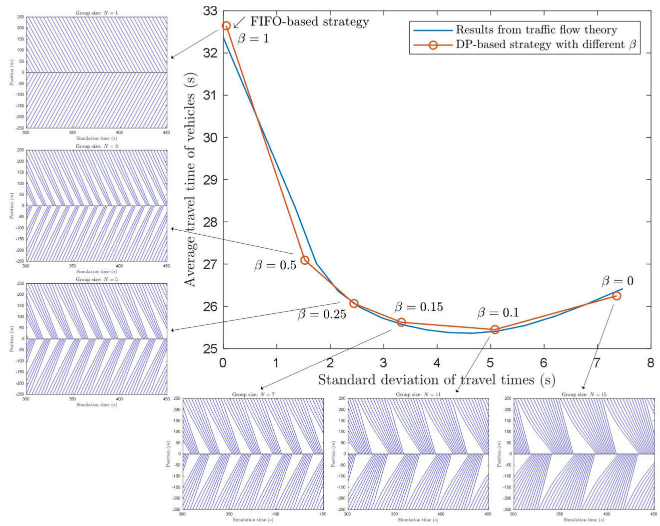

FIGURE 9. Relationship between fairness and efficiency in the modified DP-based strategy, compared to the theoretical result. The vehicle trajectories under different β are also drawn.

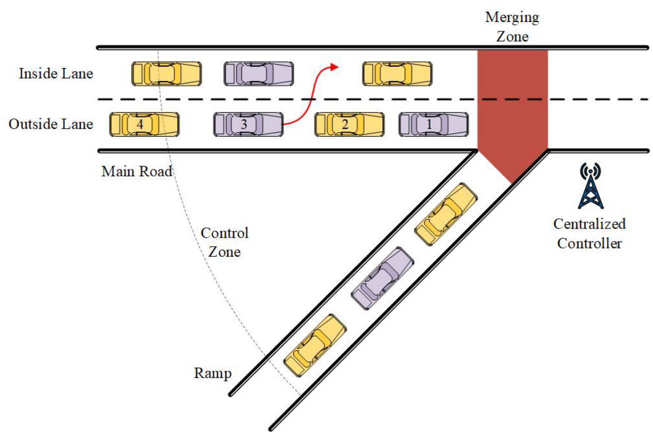

FIGURE 10. A multi-lane merging scenario at the on-ramp.

VI. DISCUSSION ON COMPLEX ON-RAMPS

In this section, we present a more complex scenario by adding lanes to the simple on-ramps. The above findings can be readily extended to the new situations.

Specifically, we focus on the multi-lane scenario depicted in FIGURE 10, where the main road consists of two lanes: an outside lane and an inside lane. In the congested on-ramps investigated in this study, a traffic bottleneck often occurs between the outside lane and the ramp, prompting outside lane CAVs to switch to the inside lane to quickly pass through the merging zone. To simply the analysis, we limit our attention to the lane changes from the outside lane to the inside lane. We present Modified Definition 4 to account for vehicle lane changes and propose a new formula for computing the travel time.

Modified Definition 4: For the group in the outside lane, the travel time iterative relation for all CAVs is shown as:

$$t^i_{travel} = t^1_{travel} - (i-1)\left(\frac{4d}{3v} - \Delta t_1\right), \ \ i = 1, 2, .., N \quad (22)$$

Typically, the minimum safety gap for CAVs in the same lane to reach the merging zone Δt_1 is less than the time interval to leave the point queue T. Therefore, for the inside lane, more CAVs arrive at the merging zone than enter the control zone. Some CAVs in the outside lane tend to switch to the inside lane to quickly reach the merging zone.

For example, in FIGURE 10, CAV 3 switches lanes from the outside lane to the inside lane, and then CAV 2 and CAV 4 reach the merging zone in sequence. We simply ignore CAV 3 because the space can be quickly filled by CAV 4, but the departure interval from the point queue changes. Specifically, the departure interval between CAV 2 and CAV 4 becomes $2T$, one T longer than the original. Based on Definition 4, CAV 4 can reduce its travel time to the merging zone by T. About $\frac{d/v_{\max}-\Delta t_1}{\Delta t_1}$ of the CAVs in the outside lane will switch to the inside lane and this reduction T needs to be split. Thus, the iterative travel time takes the form:

$$t^i_{travel} = t^1_{travel} - (i-1)\left(T - \Delta t_1 + \frac{d/v_{\max} - \Delta t_1}{\Delta t_1} T\right)$$
$$= t^1_{travel} - (i-1)\left(\frac{4d}{3v} - \Delta t_1\right), \ i = 1, 2, .., N \quad (23)$$

Similar to the previous proof, we can establish the relationship between t^1_{travel} and v. Note that the expression for t^1_{travel} has changed, and thus the equation in Eq. (11) needs to be updated. However, since the probability of vehicle lane changes is unknown, we do not take into account the reduction in the number of vehicles in the outside lane resulting from lane changes.

The additional lane on the main road introduces differences in traffic flow between the outside lane and the ramp. Therefore, it is necessary to calculate the space mean speed for both lanes using the original and modified frameworks, respectively. By combining the data from two lanes, we can illustrate the overall fairness and traffic efficiency.

We vary the value of N from 1 to 15, and depict the relationship between fairness and traffic efficiency, as shown in FIGURE 11. A comparison with the curve in FIGURE 7 reveals that the additional lane yields traffic efficiency advantages for the on-ramp. However, these benefits primarily favor the outside lane, resulting in uneven traffic flow between the outside lane and the ramp. As a result, this curve shows a shorter average travel time but a higher standard deviation of travel time. Nevertheless, it confirms the balance between fairness and efficiency when there are $N = 3 \sim 7$ CAVs in each group.

In addition, we can incorporate more lanes to the on-ramp scenario using a similar approach. While the additional lanes can enhance traffic efficiency, they may also exacerbate the difference in the traffic flow between the outside lane and the ramp. However, there is always a trade-off between fairness and efficiency, and we can strike a balance by selecting a suitable value of N. Although the proposed framework seems to be relatively simple, it succeeds in obtaining theoretical solutions that are consistent with the simulation results,

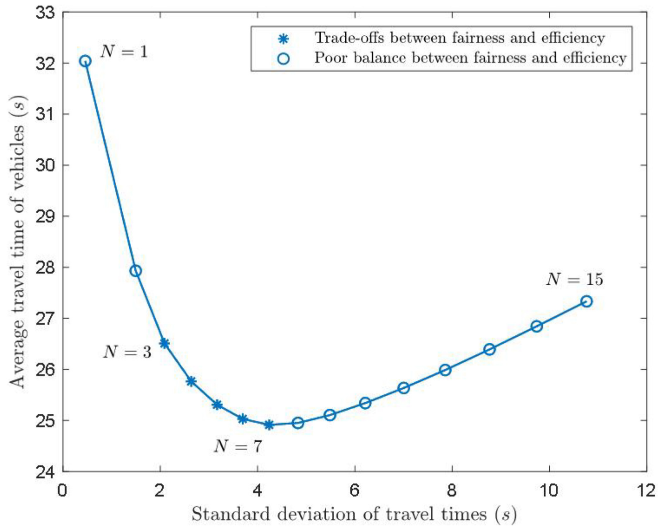

FIGURE 11. Theoretical relationship between fairness and efficiency in multi-lane merging scenarios.

allowing for a more comprehensive exploration of fairness in future research.

VII. CONCLUSION

In this paper, we focus on the fairness issues in the cooperative driving problem at on-ramps. Fairness is crucial for optimizing the equitable distribution of road resources and creating a positive travel experience for all road users. Although fairness and traffic efficiency are often conflicting goals, it is possible to strike a balance between the two. We utilize the fundamental relation in traffic flow theory to examine the trend of fairness and traffic efficiency, confirming the existence of a trade-off. Our findings show that as the group size N increases, the standard deviation of travel times increases, while the average travel time first decreases and then increases, indicating the theoretical trade-off between fairness and traffic efficiency for a reasonable group size. We also extend the analysis to complex multi-lane on-ramp scenarios and arrive at consistent conclusions.

Moreover, we achieve this trade-off in the cooperative driving strategies. In both the rule-based and DP-based strategies, we introduce a balancing factor to account for fairness. Our theoretical analysis is supported by simulation results, which show the adaptability of the trade-off in practical implementation through appropriate selecting of the balancing factor.

Although our study has primarily focused on the on-ramp scenario, the findings hold relevance for broader scenarios, including urban intersections and mixed traffic flows at on-ramps. By considering the limited capacity of the intersection and the on-ramp, and ensuring a fair distribution of travel time among vehicles, we can always find an approach to balance fairness and efficiency. Furthermore, there are several critical and interesting issues that require further investigation, such as relaxing the assumption of uniform departure intervals from the point queue, and improving the estimation of the number of vehicles in the outside lane. Given the constraints of space, we defer these topics for future research.

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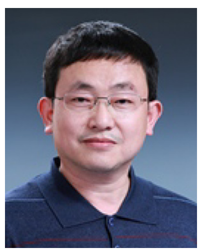
ZIMIN HE received the B.S. degree from the Department of Automation, Tsinghua University, Beijing, China, in 2022, where he is currently pursuing the master’s degree in control science and engineering. His current research interests include autonomous driving, cooperative driving, and intelligent vehicles.

HUAXIN PEI received the B.S. degree from the University of Electronic Science and Technology of China, China, in 2017, and the Ph.D. degree in control science and engineering with the Department of Automation, Tsinghua University, China, in 2023. His current research interests include intelligence testing, autonomous driving, cooperative driving, and intelligent vehicle-infrastructure cooperative systems.

YUQING GUO received the B.S. degree from the Harbin Institute of Technology, China, in 2018. She is currently pursuing the Ph.D. degree with the Department of Automation, Tsinghua University, China. Her current research interests include autonomous driving, cooperative driving, and trajectory planning.



DANYA YAO (Member, IEEE) received the B.S., M.S., and Ph.D. degrees from Tsinghua University, Beijing, China, in 1988, 1990, and 1994, respectively, where he is currently a Full Professor with the Department of Automation. His research interests include intelligent detection technology, system engineering, mixed traffic flow theory, and intelligent transportation systems.

LI LI (Fellow, IEEE) is currently a Full Professor with the Department of Automation, Tsinghua University, Beijing, China, where he was involved in artificial intelligence, intelligent control and sensing, intelligent transportation systems, and intelligent vehicles. He has authored over 110 SCI-indexed international journal papers and over 60 international conference papers.

Dr. Li was a member of the Editorial Advisory Board for *Transportation Research Part C: Emerging Technologies*, and a member of the Editorial Board of *Transport Reviews* and *Acta Automatica Sinica*. He serves as an Associate Editor for the IEEE TRANSACTIONS ON INTELLIGENT TRANSPORTATION SYSTEMS and IEEE TRANSACTIONS ON INTELLIGENT VEHICLES.