

# Review of Resonator's Q-Factor Measurement With Focus on Design of Analog and Mixed Circuits for In-Situ Measurement

MING ZHANG<sup>1</sup> (Member, IEEE), AND NICOLAS LLASER<sup>2</sup>

<sup>1</sup>Faculty of Science Orsay, University of Paris Saclay, 91405 Orsay, France

<sup>2</sup>Department of Computer Science, College LPO Dorian, 75011 Paris, France

CORRESPONDING AUTHOR: M. ZHANG (e-mail: ming.zhang@u-psud.fr)

This work was supported in part by the European "Design for Micro and Nano Manufacture" Patent Network of Excellence and in part by the French Eiffel Doctoral Program.

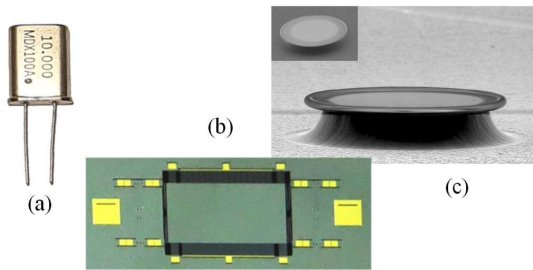
**ABSTRACT** A resonator is one of the important components in electronic systems. It is often used in an electronic system to give an accurate time reference. But it can also be used as a highly sensitive physical parameter sensor especially with the rapid development of fabrication technology for Micro Electro Mechanic System (MEMS) sensors. One of the parameters used to describe the performance of a resonator is its quality factor, also called Q-factor for short. Knowing the Q-factor of a resonator can make sure the performance of the system in which the resonator will be used. For applications in which a resonator is used as a sensing device, it is crucial for the Q-factor to be surveyed and measured in real time. In this paper, a review of a resonator's quality factor (Q-factor) measurement is given. Three major approaches can be identified from the literature: frequency domain, time-frequency domain and time domain. Both advantages and limitations of each approach are presented. Based on the published results, a comparison of the three approaches is conducted. As the time-domain measurement is the only way to present the potential for an in-situ Q-factor measurement but it is relatively less exploited compared with others, a special focus on the time-domain measurement is granted with two time-domain circuit designs.

**INDEX TERMS** CMOS integrated circuit, frequency domain, mechanical resonator, Q-factor measurement, time domain.

## I. INTRODUCTION

A RESONATOR is among the most often used components in an electronic system. It is a device that can produce a mechanical vibration movement. The two major parameters used to describe the performances of a resonator are the quality factor, also called Q-factor for short, and the resonance frequency. Having a high Q-factor for a resonator means that the resonator disposes of a highly stable resonance frequency and that it can be used as an accurate time reference in an electronic system. The well-known Quartz-based resonator disposes of a Q-factor reaching as high as  $10^6$ . This is the major reason that it is widely used as a time reference within electronic systems.

However, in spite of its high performance, a Quartz-based resonator can hardly be integrated on an integrated circuit chip due to its nature, a discrete component measured 13.5mm x 11mm x 4mm such as shown in Fig. 1 (a), which results in an irreducible system size. By comparison, a MEMS (Micro-Electro-Mechanical-System) based resonator, being naturally an integrated device, has the advantages of not only having a miniaturized size, 8mm x 4mm x 400 $\mu$ m such as shown in Fig. 1(b), but also being possible to be integrated on the same chip as an integrated circuit, reducing considerably a system size. Its quality factor can also reach as high as that of a Quartz-based resonator if it is packaged under vacuum condition. This is why MEMS resonators have found more and more applications [1].



**FIGURE 1.** Examples of a resonating device: (a) Quartz-based resonator; (b) MEMS resonator; (c) toroidal micro-resonator [9].

Being also an integrated resonator, an optical resonator with diameter  $120\mu\text{m}$ , as shown in Fig. 1(c), compared to a MEMS resonator, exhibits even more advantages such as ultra-high-Q ( $10^8$ - $10^{11}$ ), spectral efficiency, energy efficiency as well as accurate frequency spacing when used for comb of frequency [9]–[12].

Apart from being used as a time reference in an electronic system, a resonator can also be used as a sensor for its high sensitivity to physical parameters with either its Q-factor or its resonance frequency such as environmental parameters (temperature, humidity, pressure), gyroscope, viscosity, satellite positioning system, and electrical nose [2]–[12]. In the literature, there are also reports in which a micro cavity can be used as a resonating test structure to measure material's properties such as a material's rigidity, photo-conductivity, magnetic susceptibility and dielectric parameter sensing by way of Q-factor measurement [7], [13]–[15].

Different approaches have been reported in the literature to perform Q-factor measurement. Generally speaking, they can be distinguished as three major approaches: frequency domain, time-frequency domain and time domain. Whatever the approach is, they share the similar excitation to achieve a mechanical vibration and excited vibration detection techniques, such as electrical, magnetic even acoustical ways. But they do have fundamental differences among the three approaches.

For frequency-domain Q-factor measurement, a frequency sweeping is necessary. The steady response of the resonator to different frequencies is measured and with the help of expensive and cumbersome instruments [16]–[20]. Moreover, the measurement must be conducted with the resonator taken out of its application context, i.e., *ex-situ*. The measurement accuracy depends on that of measurement instruments as well as the frequency step taken. The smaller the step is, the better the accuracy will be, but the longer the measurement will last. Besides, more memory will be needed for more data storing, too. Usually, frequency domain can be used for a wide range Q-factor measurement. An accuracy in the order of  $10^{-4}$  can be expected with some modern instruments. Both Q-factor and resonance frequency can be extracted from frequency measurement.

To simplify the frequency-domain measurement, the mixed approach is proposed [7], [13]–[14]. A single frequency operation is used instead of a frequency sweeping. Then

**TABLE 1.** Comparison of Q-factor measurement techniques.

Parameter	Frequency	Mixed	Time
Accuracy	$10^{-4}$	$10^{-5}$	$10^{-4}$
Data storing	yes	yes	no
Frequency	GHz	MHz	MHz
Q-factor	$10^5$	$10^8$	$10^6$
Time consuming	minute	second	real time
Instrument	yes	yes	no
Measurement	<i>ex-situ</i>	<i>ex-situ</i>	<i>in-situ</i>

the vibration is left in free oscillation state, i.e., ring-down oscillation, and the ring-down signal is detected in time domain and in transient state. However, the Q-factor of the resonator is extracted in frequency domain, for which a Fourier transformation of the transient signal is necessary. Still a large memory is needed to store the digitalized transient signal along with the signal processing in frequency domain. Therefore, this approach benefits the same high Q-factor as the frequency domain measurement and measurement accuracy can even be better, i.e., in the order of  $10^{-5}$ .

Finally, time-domain measurement is performed with an acquisition of transient response of the resonator under test to a single frequency excitation but the extraction of Q-factor is carried out by using a specific designed circuit or a specific signal processing in time domain via a PC computer [21]–[27]. If the circuit is implemented on the same chip as the resonator, the Q-factor measurement can be made *in-situ* and even during the application of the resonator. The experimental results achieved in [25] show a Q-factor measurement accuracy in the order of  $10^{-4}$ , which is comparable to that obtained in frequency-domain measurement and for most applications, this accuracy is good enough. The measured Q-factor can reach as high as  $10^6$ . Both Q-factor and resonance frequency can be measured in time domain, as demonstrated in [21].

A general comparison of the three measurement techniques is given in Table 1. We can see from Table 1 that for resonators with ultra-high Q-factor and GHz frequency, the frequency-domain measurement is the only available technique. However, for resonators with relatively low Q-factor but high frequency, it is preferable to use mixed technique because by recording the ring-down oscillation with high precision, after FFT, the Q-factor can be identified with sufficient accuracy in frequency domain. Indeed, it can benefit the advantages from both frequency and time domain measurement. Finally for resonators with performances lying in-between, both time-domain and mixed can be exploited. But time-domain measurement is evidently the only approach that can be performed *in-situ* in real time with low cost and low power.

As a resonator in a large sense has been playing an important role in an electronic system, in material characterization as well as in physical parameters sensing, knowing the Q-factor along with its resonance frequency both accurately

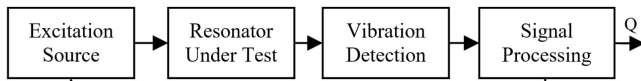


FIGURE 2. General diagram of a resonator's Q-factor measurement system.

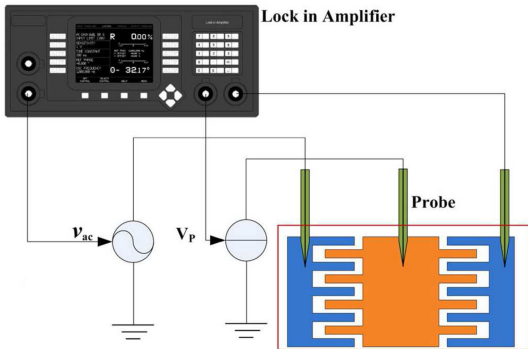


FIGURE 3. Excitation by an electrostatic force [24].

and quickly becomes crucial. Also, each approach has its own application domains, its advantages and its limitations. A review of different approaches is given in this paper, which can help users to choose an adequate approach according to different application requirements. For further details on implementing these particular methods, readers can refer to the cited references. A special focus of this paper will be granted on the design of analog and mixed circuits for time-domain Q-factor measurement because we believe that it is the only way to offer an *in-situ* Q-factor measurement but not sufficiently developed yet.

This paper is organized as follows. An overview of a general Q-factor measurement system is presented in Section II. Then three major measurement approaches are reviewed and compared in Section III. To better understand how the Q-factor can be extracted in time domain, *in-situ* time-domain Q-factor measurement circuit design is illustrated in Section IV, followed by conclusion in Section V.

## II. GENERAL Q-FACTOR MEASUREMENT DESCRIPTION

A conceptual diagram of a resonator's Q-factor measurement is shown in Fig. 2. Around the resonator under test (RUT), an external force excitation source, on one side, is used to put the resonator into mechanical vibration and the generated mechanical vibration, on the other side, is sensed by a detection circuit in form of electrical or optical signal without direct contact with the RUT. Once the sensed vibration is transformed into an electrical signal, adequate signal processing will be practiced to the electrical signal in order to extract both Q-factor and resonance frequency.

### A. EXCITATION

Several excitation ways have been reported in the literature: by an electrostatic force generated by an electrical field [24] (Fig. 3), with an acoustic wave generated with an acoustic

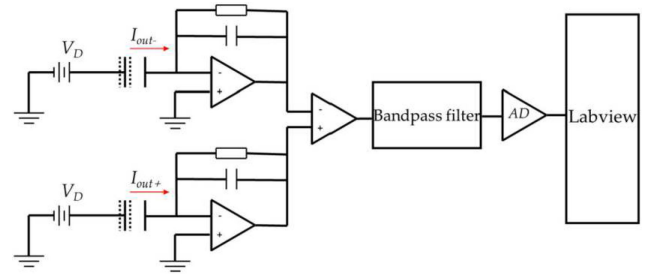


FIGURE 4. Capacitive detection [4].

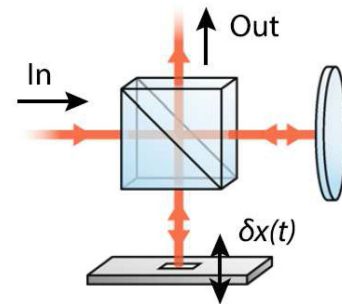


FIGURE 5. Principle of LASER-based Interferometer used for measuring the displacement of the resonator under test (RUT) [8].

source [6] and by electromagnetic force generated by an electromagnetic field [28].

### B. DETECTION

Different detections can be identified from the literature: capacitive detection [4], laser-based Doppler [4] or interferometer [6]–[7]. By capacitive detection, the capacitive variation due to the resonator's vibration is sensed and transformed into voltage variation based on using a specifically designed circuit (Fig. 4). The resonator is coupled to the detection circuit by a capacitor. So, there is no direct physical contact with the RUT.

An optical detection can also be used for detection of the vibration of a resonator. The displacement of the resonator is measured by means of frequency change in case of laser-based Doppler [4] or two optical beams' interference to get the image of the mechanical vibration indirectly with the principle diagram shown in Fig. 5. This image is then converted to an electrical signal.

### C. SIGNAL PROCESSING

Before signal processing, the electrical signal will be first sampled and converted from analog into digital signal. A computer, an oscilloscope, a network analyzer and even a circuit can be used to make the necessary signal processing depending on the measurement approach used. From the literature, three approaches can be distinguished: frequency domain, time-frequency mixed domain and time domain.

For frequency domain measurement (Fig. 6), the spectrum of the resonator can be obtained by frequency sweeping (Fig. 7), from which both Q-factor and resonance frequency

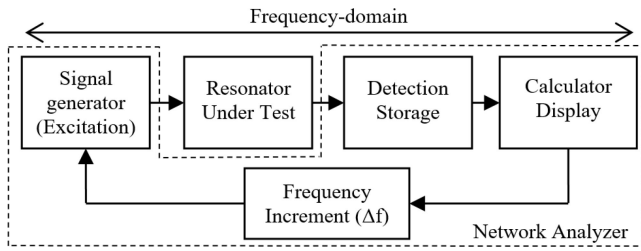


FIGURE 6. Principle diagram for frequency domain measurement.

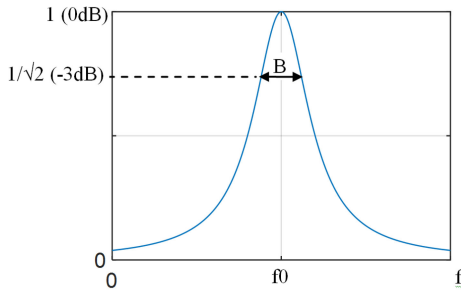


FIGURE 7. Frequency domain measurement.

can be directly deduced. While for time domain measurement, after being put into vibration, the resonator is then left to free ring-down vibration (Fig. 9). As the ring-down time constant depends on the resonator’s Q-factor, the Q-factor can be deduced from the transient response in time domain. With mixed measurement, the ring-down transient response is transformed back into frequency domain by Fourier transformation, from which the Q-factor is then extracted just like frequency-domain measurement. More details will be given in the following.

III. REVIEW OF Q-FACTOR MEASUREMENT

A. FREQUENCY Q-FACTOR MEASUREMENT

The principle diagram for frequency domain measurement is shown in Fig. 6. As indicated previously, by sweeping the excitation source’s frequency, the frequency response of the resonator is measured in steady state. The frequency sweeping can be achieved either by continuous sweeping [17] or stepped frequencies as in network analyzer. The vibration under different frequencies can be sensed either by capacitive detection or optical detection as mentioned previously. In both cases, the mechanical vibration is converted into an electrical signal. Both Q-factor and resonance frequency can be extracted.

Based on frequency response (Fig. 7), different extractions are proposed in the literature [16]. -3dB bandwidth measurement is one of the commonly used as given by (1)

$$Q = \frac{f_0}{B} \tag{1}$$

where  $f_0$  is the resonance frequency and B the -3dB bandwidth of the resonator, which will be extracted from the measured frequency response.

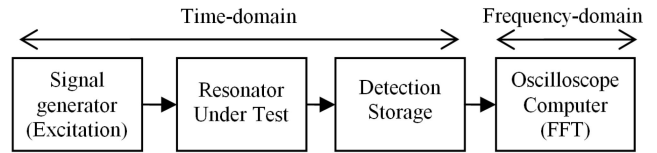


FIGURE 8. Principle diagram for mixed time-frequency domain measurement.

Lorentzian fitting is also one of the commonly used methods. By Lorentzian fitting, we mean the magnitude fitting by a mathematic formula. Then by means of this formula, the Q-factor can be identified. Under the condition that SNR is high enough, phase fitting can give the best accuracy for both Q-factor and resonance frequency [16]. Based on the transmission parameter  $S_{21}$  measured with Network analyzer, three other methods are available [16]. As in all instrumentation, Q-factor measurement is also quite sensible to noise. The higher the Q-factor is, the more sensible the measurement will be. Therefore, an average is usually conducted among several measurements in order to filter out the impact from high frequency noise on frequency measurement.

Frequency domain measurement can be used to measure both high Q-factor and high resonance frequency, as illustrated later in comparison. But for a high Q-factor, B will be very small. More frequency sweeping is needed in order to achieve a good accuracy. Therefore, frequency domain measurement is rather time consuming. Also, expensive and cumbersome instruments are necessary to make the measurement.

As those measurement instruments are cumbersome, the measurement cannot be carried out *in-situ*. It can only be conducted in the laboratory where the measurement instrument is installed.

B. MIXED TIME-FREQUENCY DOMAIN Q-FACTOR MEASUREMENT

By mixed time-frequency domain, we mean that the Q-factor measurement is conducted in both time-domain and frequency domain. The principle diagram for mixed time-frequency domain measurement is shown in Fig. 8. As illustrated, both excitation of vibration and the acquisition of ring-down vibration are performed in time-domain. They can be done by way of optical or electrical measurement as mentioned above.

The mixed time-frequency domain Q-factor measurement is introduced to simplify the frequency-domain measurement. A single frequency excitation is used to put the resonator into vibration, which can be done either by an external pulsed source in an open loop [21] (Fig. 9) or by a feedback system in a closed loop [14] (Fig. 10) by using a signal amplifier to ensure certain phase shift requirement. The advantage of the feedback system is that no external signal generator is necessary. Once the vibration reaches its steady state, the excitation source is removed leaving the resonator in its free ring-down vibration whose displacement amplitude denoted by A, follows an exponential law given by (2). The ring-down

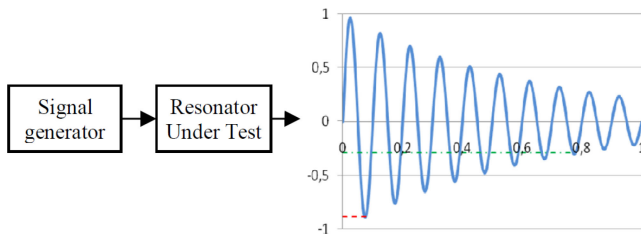


FIGURE 9. Excitation for time-frequency measurement.

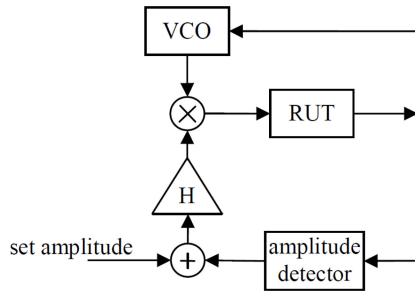


FIGURE 10. Simplified feedback system for resonator's excitation [22].

constant is directly linked to the Q-factor (3).

$$A(t) = A_0 \cdot e^{-t/\tau} \tag{2}$$

$$Q = \pi \cdot f_0 \cdot \tau \tag{3}$$

where  $A_0$  is the initial displacement of the resonator,  $\tau$  ring-down constant and  $f_0$  the resonator's resonance frequency.

Compared with frequency-domain measurement, there is no frequency sweeping. Therefore, mixed measurement enhances the measurement efficiency, decreases the equipment cost and reduces the measurement time considerably. This is why mixed Q-factor measurement is gaining more utilities.

Besides, with mixed measurement, high Q-factor measurement can be expected. However, the conversion from time domain to frequency domain still needs one additional step. The ADC, before FFT, degrades the measurement accuracy inevitably. In addition, with the current measurement system proposed in the literature, a large memory is also needed to store the transient response as long as the measurement takes in order to make the FFT more accurate. The need of instrument to make FFT operation prevents it from an *in-situ* Q-factor measurement.

### C. TIME DOMAIN Q-FACTOR MEASUREMENT

By time-domain Q-factor measurement, we mean not only the acquisition of a resonator's ring-down vibration but also the extraction of Q-factor is performed in time domain, as illustrated in Fig. 11. To do so, a specific electronic circuit should be designed to implement the Q-factor measurement in time-domain [21]–[27]. After the detection of the mechanic vibration, the corresponding ring-down voltage

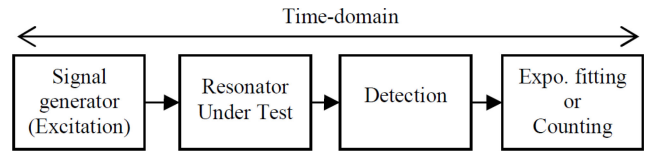


FIGURE 11. Principle diagram for time-domain measurement.

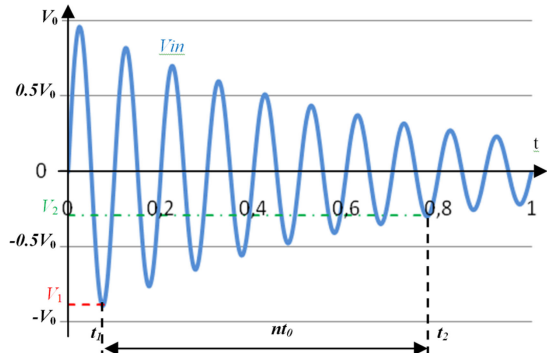


FIGURE 12. Resonator's free ring-down vibration.

can be expressed by (4), as shown in Fig. 12.

$$V(t) = V_0 e^{-\frac{\omega_0}{2Q}t} \left[ \cos\left(\omega_0 t \sqrt{1 - \frac{1}{4Q^2}}\right) + \frac{1}{\sqrt{4Q^2 - 1}} \sin\left(\omega_0 t \sqrt{1 - \frac{1}{4Q^2}}\right) \right] \tag{4}$$

where  $V_0$  is the initial voltage corresponding to the resonator's initial displacement,  $\omega_0 = 2\pi f_0$  and  $Q$  the Q-factor of the resonator.

Even though the ring-down signal acquisition is sensed in the same way as mixed Q-factor measurement, different techniques have been proposed in the literature to extract Q-factor directly in time domain. A straight exponential fitting in time domain is proposed in [24]. A better accuracy in time domain is observed compared with frequency domain measurement.

As the ring-down amplitude is linked to Q-factor through the ring-down time constant  $\tau$ , the time constant can be determined by measuring the time interval corresponding to the amplitude decrease within the predefined interval  $[V_2, V_1]$ . Furthermore, the time interval can be measured by counting the pseudo-periods number ( $n$ ) of the ring-down signal within the interval  $[V_2, V_1]$ . As a result, the Q-factor can be calculated by (5) [19].

$$Q = \sqrt{\frac{1}{4} + \left[\frac{n \cdot \pi}{\ln(k)}\right]^2} \approx \frac{n \cdot \pi}{\ln(k)} \tag{5}$$

where  $k = V_1/V_2$ ,  $V_1$  and  $V_2$  are two predefined voltage references with  $V_0 > V_1 > V_2$  and  $n$  is the counting number of signal periods.

For pseudo-periods counting, a simple comparator is used in [21]. After comparison with two thresholds successively,

**TABLE 2.** Comparison (simu/mes) of Q-factor measurement results.

Ref	Q	relative error	frequency	tech
[16]	$10^2 - 10^5$	$10^{-4}$	9.6GHz	freq
[17]	$2.5 \times 10^4$	$10^{-3}$	10GHz	freq
[18]	5564	$10^{-2}$	24.58GHz	freq
[14]	$10^4$	$6 \times 10^{-5}$	20-50MHZ	mixed
[7]	$3 \times 10^8$	$2 \times 10^{-1}$	100KHz	mixed
[22]	$7.5 \times 10^5$	$8 \times 10^{-2}$	106.1Hz	time
[24]	$2.36 \times 10^5$	$4.9 \times 10^{-3}$	8KHz	time
[21]	$2.49 \times 10^5$	$1.04 \times 10^{-1}$	8KHz	freq
[21]	661	$10^{-2}$	2MHZ	time
[4]	$7.3 \times 10^5$	$6.9 \times 10^{-3}$	7KHz	time
[25]	$10^6$	$2.5 \times 10^{-4}$	10KHz	time
[27]	$2 \times 10^3$	$2 \times 10^{-3}$ *	0.1-1.5MHz	time

\* simulation results

the difference of the two counting numbers is used for Q calculation. An accuracy of 1% is observed for a Q-factor of around 661. In this design, apart from the Q-factor measurement, the ring-down signal is also used for the resonance frequency measurement by using a separate counter. Another way to count the pseudo-periods is proposed in [25]–[27], in which a peak detector is used and each detected peak is kept on a memory during the comparison with voltage reference. Only the periods whose amplitudes are lying between the two predefined references are counted for Q-factor calculation. The experimental results show an accuracy of  $2.5 \times 10^{-4}$  for a Q-factor up to  $10^6$ , which is comparable with frequency domain measurement accuracy.

In order to benefit a good SNR in time domain, instead of using the ring-down signal, a constant amplitude oscillation is proposed in [22] (Fig. 10). The constant amplitude is kept by supplying the necessary energy to the resonator by an external circuit to compensate its loss. The experimental results showed an accuracy of 8% for a Q-factor of  $7.5 \times 10^5$ .

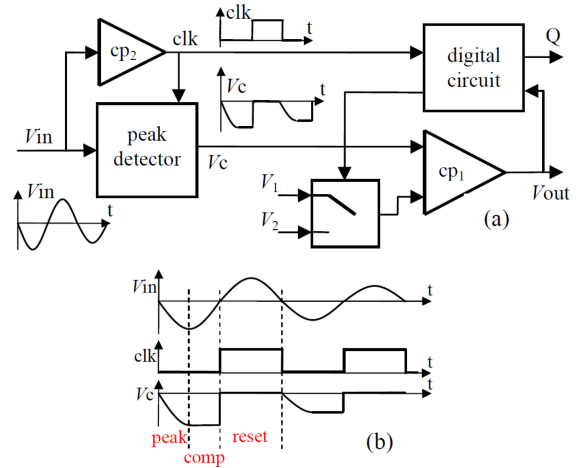
As the measurement is carried out by a designed circuit, it offers the possibility to be integrated on the same chip as the resonator or to be packaged together with the resonator to make an in-situ measurement.

In order to illustrate how to extract Q-factor in time domain, two specific circuit designs aiming at in-situ accurate pseudo-periods counting are given in the following section.

#### D. COMPARISON AND DISCUSSIONS

A comparison of different approaches of Q-factor measurement is made and shown in Table 2. In this table, all the results, either simulated ones or experimental ones, are taken into account. It aims to give an insight of the measurement potential in terms of Q-factor value, measurement accuracy and resonance frequency, as well as to guide the future users to choose the adequate approach according to their applications.

The comparison reveals that whatever the measurement approach is, the Q-factor measurement accuracy is comparable. However, we can see that frequency domain and mixed


**FIGURE 13.** Time-domain Q-factor measurement principle diagram [25].

measurements can cover both low and high Q-factors and the resonance frequency can be up to GHz when a microwave cavity measurement is used. However, time domain measurement is more suitable for intermediate Q-factors with resonance frequency in the order of MHz up to now.

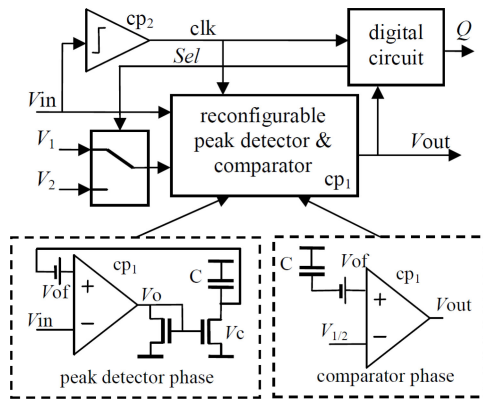
#### IV. IN-SITU TIME-DOMAIN Q-FACTOR MEASUREMENT CIRCUIT

The purpose of this section is to give a tutorial illustration of how to design a circuit for time-domain Q-factor measurement by highlighting the key design features and giving complementary explanations to already published contents.

To have a comprehensive description, based on the general principle illustrated in Fig. 12, first a straight implementation principle diagram for Q-factor measurement is given in Fig. 13. It contains two major functional blocks: peak detector and comparator and it has three-step operations: peak detection, comparison and reset. However, in practice, each functional block is imperfect, for instance offset voltage exists, and the non-null offset can degrade the measurement accuracy with such a design. After analyzing the defaults of Fig. 13 [26], an improved implementation diagram is then proposed and illustrated in Fig. 14. In the improved implementation, a reconfigurable circuit based on an *unique* OTA is used to implement both peak detection and comparison functions successively. As a result, the offset of OTA is cancelled automatically after the two operations greatly improving the Q-factor measurement accuracy. Finally, two previous designs are presented: one is designed with a PCB circuit for an operation frequency of 10kHz and the other with CMOS technology for a band of frequencies ranging from 0.1 to 1.5MHz.

##### A. CIRCUIT DESIGN PRINCIPLE

Based on the idea illustrated in Fig. 12, a simple implementation diagram for Q-factor measurement is given in Fig. 13(a) [25]. It consists of two major circuit blocks: peak



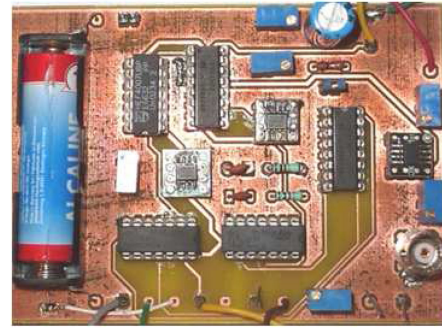
**FIGURE 14.** Configurable implementation structure with a unique OTA for both peak detector and comparator [25].

detector and comparator ( $cp_1$ ). The digital circuit is used to generate the necessary digital control signals and the counting of pseudo-periods. The clock signal used by the digital circuit is restored from the ring-down input signal and provided by the zero-crossing circuit ( $cp_2$ ). The circuit aims to measure the number of pseudo-periods whose amplitudes are lying between the two predefined voltage references [ $V_2$ ,  $V_1$ ]. The measurement circuit works with positive amplitude as well as negative amplitude. We have chosen the negative amplitude in the following design.

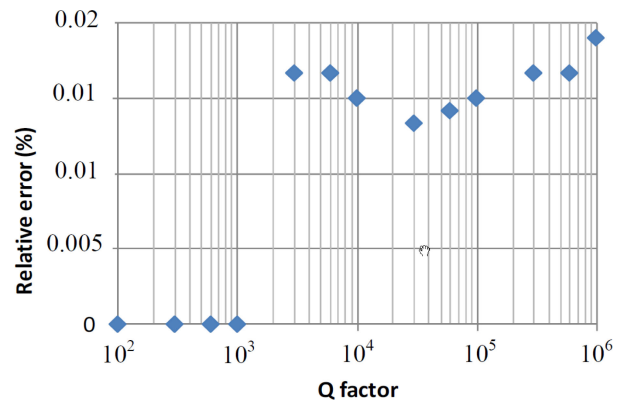
The measurement circuit operates in three steps during each signal cycle, as shown in Fig. 13 (b). During the first quarter of a period, the signal's amplitude, i.e., peak value is sensed by the peak detector. Once a peak is detected, it is kept on an analog memory ( $V_c$ ) and compared with the voltage reference  $V_1$  ( $cp_1$ ) during the second quarter of a period. If  $V_c < V_1$ , the memorized peak voltage will be reset to zero during the remaining half period, i.e., the third step. The same operation cycle will then be repeated. However, if  $V_c > V_1$ , the following digital circuit enables the periods counting, replaces  $V_1$  by  $V_2$  and resets the  $V_c$ . The same operation cycle restarts as before. Every time a new peak is sensed, the pseudo-periods counter increments, followed by a new peak detection until  $V_c > V_2$ .

However, offset exists in both peak detector and comparator. It degrades the Q-factor measurement accuracy [26]. To prevent this, a single OTA-based reconfigurable circuit is proposed to implement both peak detector and comparator ( $cp_1$ ) successively in time (Fig. 14), which is based on the fact that both peak detection and comparison are performed one after the other. Therefore, there is no timely conflict.

Suppose that the offset of OTA is  $V_{of}$ . During the peak detection (Fig. 14 in left dotted square), the detected peak voltage will be stored together with  $V_{of}$  across the capacitor  $C$  ( $V_c$ ), i.e.,  $V_c = V_{peak} - V_{of}$ . During the comparison, the spoiled peak voltage is compared with the reference voltage (Fig. 13 in right dotted square). As the comparator disposes of the same offset as that stored on  $C$ , the voltage appearing at the non-inverting input ( $V_+$ ) of the comparator becomes  $V_+ = V_c + V_{of} = V_{peak}$ , which is offset free. As



**FIGURE 15.** PCB realization of the time-domain Q-factor measurement [25].



**FIGURE 16.** The relative error of measurement results was traced with the Q-factor up to  $10^6$  [25].

a result, an offset free peak voltage is used to generate the counting signal even though the undesired imperfection of the designed circuit does exist.

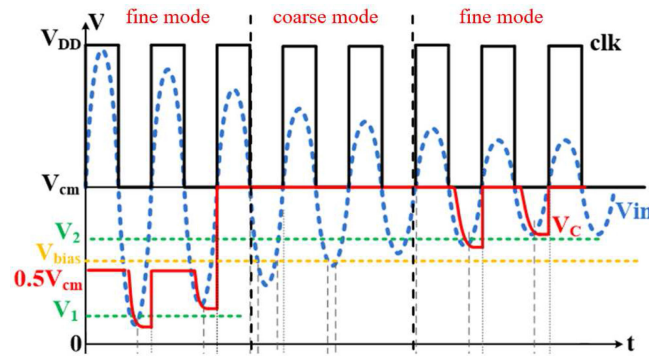
For the third step as well as the remaining operation, the improved circuit follows the same operation cycles as described previously until  $V_2$  is crossed, which ends the Q-factor measurement.

Two circuits have been designed: one with discrete components in PCB [25] and the other in CMOS  $0.35\mu\text{m}$  technology [27]. Please refer to the references for more details.

## B. PCB AND CMOS CIRCUIT DESIGNS

The first design is made with discrete components under a voltage supply of 5V. For the PCB realized circuit (Fig. 15), even though the circuit is designed for an operation frequency around 10 kHz, the so designed circuit has been tested up to 30 kHz with the desired functionality, such as offset cancellation. The experimental results, given in Fig. 16, have shown a measurement accuracy of 0.02% for a Q-factor up to  $10^6$ .

The limitation of operation frequency mainly comes from the discrete components used. For instance, the transition frequency of the operational amplifier is only 38MHz. Also, by using available functional blocks, we have are limited to exploit the available performance of each sub-circuit rather than to optimize the design globally. But it's easier, quicker



**FIGURE 17.** Chronograms of signals for design 2. The three-phase operation with 2 modes as indicated by signal  $V_c$  [27].

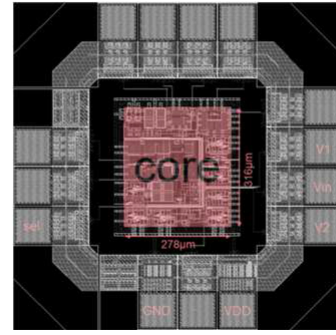
and lower expensive to make the circuit design, which can also achieve a good performance. To further increase the operation frequency and miniaturize the circuit, it is mandatory to make a full custom design with an integrated circuit technology.

In order to increase the circuit operation frequency, we have designed the Q-factor measurement circuit in CMOS  $0.35\mu\text{m}$  [27]. The circuit is designed under these conditions: a voltage supply of 5V, a Q-factor of almost 2000, and the two reference voltages chosen as 0.3V for  $V_1$  and 2.2V for  $V_2$ .

One of the difficulties to deal with the operation frequency increase is to guarantee the speed and the stability of the measurement circuit at the same time. This difficulty is further accentuated by the fact that the amplitude of the input signal is not constant but decreases with time. On one hand, with the operation frequency increase, the Slew Rate (SR)  $dV_c/dt$  of the designed circuit should be high enough to be able to follow the input signal. On the other hand, with the decrease of the input signal's amplitude in time, a smaller SR will be needed to guarantee the stability of the circuit. The significant difference in required SR combined with frequency increase makes the circuit design quite challenging. Otherwise, it might cause instability of the peak detection circuit. Also, along with operation frequency increase, the circuit power consumption will increase naturally. So, it might become a concern. To deal with these concerns, several techniques have been proposed.

To balance the different requirements in SR during the measurement, two operation modes have been designed for the circuit. The corresponding chronograms are illustrated in Fig. 17 [27]. For more details, please refer to [27].

Near both reference voltages  $V_2$  and  $V_1$ , a fine measurement mode with a reconfigurable structure presented previously will be employed to guarantee good measurement accuracy because it benefits all the discussed advantages from the reconfigurable structure. However, for the measurement in-between the two reference voltages, a coarse measurement mode with only a comparator can be used. Besides, in the fine measurement mode, to scale down the difference in SR, which is caused by the difference in signal



**FIGURE 18.** Layout of the implementation of Q-factor measurement in  $0.35\mu\text{m}$  CMOS technology with a core dimension of  $278\mu\text{m} \times 316\mu\text{m}$ .

amplitudes, a new reset voltage level  $V_{cm}/2$  is defined instead of the common mode voltage ( $V_{cm} = V_{dd}/2$ ) before  $V_1$  crossing. As a result, artificially the excursion for peak detection is reduced, as illustrated by  $V_c$ , until the reference voltage  $V_1$  crossing. Once  $V_1$  is crossed, the reset voltage level is switched back to  $V_{cm}$ . As a result, the difference in SR is greatly reduced, whatever the signal amplitude is.

Once  $V_1$  is crossed, the circuit operation passes to the coarse mode, in which the peak detection is no longer needed. Instead, only the comparator configuration is kept with the reference voltage  $V_{bias}$  to monitor the signal amplitude decrease. Meanwhile the pseudo-periods counting continues.

However, as soon as the intermediate reference  $V_{bias}$  is crossed, the circuit is switched back to the fine measurement mode but with  $V_2$  set as the reference voltage and  $V_{cm}$  as the reset voltage level. The same three-step operation as describe previously restarts until  $V_2$  crossing. As the signal amplitude decreases, the demand for SR remains almost the same as previous one and so the same designed SR will do.

As far as the coarse measurement mode is concerned, as we only need to know if the input signal amplitude crosses  $V_2$  or not, only a comparison operation is necessary. Having no need of peak detection, its operation is stopped during this mode and the reconfigurable circuit operates merely as a comparator. Unlike peak detection, comparison works mostly in nonlinear regime and less power is consumed, which not only reduces power consumption but also facilitates frequency increase. In fact, as there is no Slew Rate limitation for comparator, it's easier to increase operation frequency. The longer the coarse mode lasts, the more efficient the power reduction will be.

The layout of the designed circuit with the core occupying surface of  $278\mu\text{m} \times 316\mu\text{m}$  is illustrated in Fig. 18. The post-layout simulations are illustrated in Fig. 19. First, we can see that the circuit operation frequency is extended to cover the frequency band from 0.1 up to 1.5MHz thanks to the proposed coarse and fine mode operation. Second, within the covered wide operation frequency range, Q-factor measurement accuracy still remains under 0.2% for a Q-factor of  $\sim 2000$ . This is the direct result of the offset compensation by using the reconfigurable structure combined with the



TABLE 3. Comparison of time-domain Q-factor extraction.

parameter	[22]	[24]	[21]	[4]	[25]	[27]
Q-factor	$7.5 \cdot 10^5$	$2.36 \cdot 10^5$	661	$7.3 \cdot 10^5$	$10^6$	$2 \cdot 10^3$
Frequency	106.1Hz	8KHz	2MHz	7KHz	10KHz	0.1 -1.5MHz
Relative error	$8 \cdot 10^{-2}$	$4.9 \cdot 10^{-3}$	$10^{-2}$	$6.9 \cdot 10^{-3}$	$2.5 \cdot 10^{-4}$	$2 \cdot 10^{-3}$
Date storing	yes	yes	no	yes	no	no
Equipment	yes	yes	no	yes	no	no
Measurement	ex-situ	ex-situ	in-situ	ex-situ	in-situ	in-situ

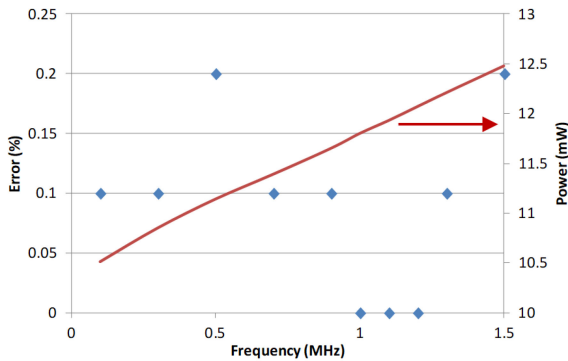


FIGURE 19. Simulation results on power consumption and relative error versus circuit operation frequency from 0.1 to 1.5MHz with a Q-factor of ~2000.

lead / lag circuit design technique. Third, as expected, the simulation results have confirmed that the power consumption increases with frequency. Still thanks to the proposed design scheme (coarse and fine mode), a 7.5% reduction of the total power consumption is achieved, i.e., 12.5mW with power saving against 13.5mW without power saving scheme.

### C. COMPARISON OF TIME-DOMAIN Q-FACTOR MEASUREMENT

A comparison of the time-domain Q-factor measurements is given in Table 3. According to the works reported in the literature, three approaches can be identified: time fitting, only comparison and peak/comparison.

For time fitting approach reported in [4], [22], [24], the ring-down signal should first recorded. Once the acquisition is finished, the signal ring-down envelop is then fitted by a mathematic function to measure the time constant  $\tau$ . As indicated by (2) and (3), the damping amplitude is linked to the Q-factor, the latter can therefore be identified from the fitting formula. It needs memory for the recording, a tool for the mathematical fitting and one more step to identify the Q-factor, which is done *ex-situ*. A better accuracy can be achieved but at the price of a bigger memory with more processing time but the measurement should be done *ex-situ*.

As far as the only comparison approach proposed in [21] is concerned, the circuit is relatively simple and the measurement can be done *in-situ* because it uses a comparator to identify the damping amplitudes used for pseudo-periods counting. This approach can facilitate the operation frequency increase but the measurement accuracy is relatively limited. As the comparison is made in a dynamic

way, the comparator should be designed with quite a timing performance.

As for the last approach peak/comparison reported in [25], [27], a peak detector is used to precede the comparison. By designing a reconfigurable circuit, the offset of the main functional blocks can be cancelled automatically making the comparison offset-free, which is the key element for achieving high accuracy measurement. Moreover, the comparison is carried out somehow in a static way, which not only relaxes the comparator's design considerably but also contributes the high accuracy measurement. Furthermore, the only comparison is used as the coarse mode, which allows both frequency increasing and power reduction. More important, it offers quite a potential to make *in-situ* Q-factor measurement.

### V. CONCLUSION

In this paper, we have given an overview of a Q-factor measurement system. We have also reviewed different Q-factor measurement approaches reported in the literature with their advantages as well as limitations. Generally speaking, frequency-domain and mixed measurements are suitable for all Q-factors measurement while the time-domain measurement is more adapted for intermediate Q-factors measurement with frequency limitation.

However expensive and cumbersome instruments are needed for frequency-domain-related measurement. Especially in case of optical measurement, the optical system, interferometer for instance, is fragile. Special attentions must be paid during measurement. Therefore, frequency domain measurement is rather reserved for those in which there is no need for either in-situ measurement or real time measurement, for instance, to characterize different materials.

As far as the time domain measurement is concerned, the measurement can be done with a specifically designed circuit and it can be performed in real time. Since the measurement circuit is usually small in size, it can be embedded together with the resonator used as a sensing device for an in-situ measurement.

As the measurement circuit is totally compatible with CMOS technology, another advantage of time-domain measurement is its potential to be integrated on the same chip as the resonator, making the measurement system more compact and facilitating the survey of the Q-factor before or even during application. Benefiting from this

prospective, time-domain Q-factor measurement will find more applications in the near future. Further increasing operation frequencies and reduction of power consumption can make it more attractive.

## ACKNOWLEDGMENT

The authors would like to express their gratitude to the reviewers for their valuable and constructive suggestions, which have helped greatly to the improvement of this paper.

## REFERENCES

- [1] N. Lobontiu, *Dynamics of Microelectromechanical Systems*. New York, NY, USA: Springer 2014.
- [2] V. Cimalla *et al.*, "Nanoelectromechanical devices for sensing applications," *Sens. Actuat. B*, vol. 126, no. 1, pp. 24–34, Sep. 2007.
- [3] I. Lee, K. Park, and J. Lee, "Note: Precision viscosity measurement using suspended microchannel resonators," *Rev. Sci. Instrum.*, vol. 83, Oct. 2012, Art. no. 116106.
- [4] Z. Qiu *et al.*, "Optical and electrical method characterizing the dynamic behavior of the fused silica cylindrical resonator," *Sensor*, vol. 19, p. 2928, Jul. 2019.
- [5] V. Petrescu, J. Pettine, D. M. Karabacak, M. Vandecasteele, M. C. Calama, and C. Van Hoof, "Power-efficient readout circuit for miniaturized electronic nose," in *Proc. IEEE Int. Solid-State Circuits Conf.*, San Francisco, CA, USA, 2012, pp. 318–320.
- [6] Y. Pan *et al.*, "Observation and analysis of the quality factor variation behavior in a monolithic fused silica cylindrical resonator," *Sens. Actuat. A, Phys.*, vol. 260, pp. 81–89, Jun. 2017.
- [7] R. Nawrodt *et al.*, "High mechanical Q-factor measurements on calcium fluoride at cryogenic temperatures," *Eur. Phys. J. Appl. Phys.*, vol. 38, pp. 53–59, Feb. 2007.
- [8] A. Barg, Y. Tsaturyan, E. Belhage, W. H. P. Nielsen, C. B. Møller, and A. Schliesser, "Measuring and imaging nanomechanical motion with laser light," *Appl. Phys. B*, vol. 123, no. 1, p. 8, 2017.
- [9] D. K. Armani, T. J. Kippenberg, S. M. Spillane, and K. J. Vahala, "Ultra-high-Q toroid microcavity on a chip," *Nature*, vol. 421, no. 6926, pp. 925–928, 2003.
- [10] G. Brunetti, F. D. Olio, D. Conteduca, M. N. Armenise, and C. Ciminelli, "Comprehensive mathematical modelling of ultra-high Q grating-assisted ring resonators," *J. Opt.*, vol. 22, no. 3, 2020, Art. no. 035802.
- [11] D. T. Spencer, J. F. Bauters, M. J. R. Heck, and J. E. Bowers, "Integrated waveguide coupled Si<sub>3</sub>N<sub>4</sub> resonators in the ultrahigh-Q regime," *Optica*, vol. 1, no. 3, pp. 153–157, 2014.
- [12] M. L. Gorodetsky, A. A. Savchenkov, and V. S. Ilchenko, "Ultimate Q of optical microsphere resonators," *Opt. Lett.*, vol. 21, no. 7, pp. 453–455, 1996.
- [13] B. Gyüre, B. G. M'arkus, B. Bernath, F. Mur'anyi, and F. Simon, "A time domain based method for the accurate measurement of Q-factor and resonance frequency of microwave resonators," *Rev. Sci. Instrum.*, vol. 86, Sep. 2015, Art. no. 094702.
- [14] B. Gyüre-Garami, O. Sagi, B. G. Márjys, and F. Simon, "A highly accurate measurement of resonator Q-factor and resonance frequency," *Rev. Sci. Instrum.*, vol. 89, no. 11, 2018, Art. no. 113903.
- [15] I. Gresits *et al.*, "A highly accurate determination of absorbed power during magnetic hyperthermia," *J. Phys. D, Appl. Phys.*, vol. 52, no. 37, 2019, Art. no. 375401.
- [16] P. J. Petersan and S. M. Anlage, "Measurement of resonant frequency and quality factor of microwave resonators: Comparison of methods," *J. Appl. Phys.*, vol. 84, p. 3392, Jun. 1998.
- [17] B. Nebendahl, D.-N. Peligrad, M. Požek, A. Dulčić, and M. Mehring, "An AC method for the precise measurement of Q-factor and resonance frequency of a microwave cavity," *Rev. Sci. Instrum.*, vol. 72, no. 3, p. 1876, 2001.
- [18] K. Leong and J. Mazierska, "Precise measurements of the Q factor of dielectric resonators in the transmission mode-accounting for noise, crosstalk, delay of uncalibrated lines, coupling loss, and coupling reactance," *IEEE Trans. Microw. Theory Techn.*, vol. 50, no. 9, pp. 2115–2127, Sep. 2002.
- [19] A. Luiten, "Q-factor measurements," *Encyclopaedia RF Microw. Eng.*, vol. 5, pp. 3948–3964, Apr. 2005.
- [20] D. Kajfez, *Q Factor Measurements Using MATLAB*. Boston, MA, USA: Artech House, 2011.
- [21] Z. Zeng, M. A. P. Pertijs, and D. M. Karabacak, "An energy-efficient readout circuit for resonant sensors based on ring-down measurement," *Rev. Sci. Instrum.*, vol. 84, Feb. 2013, Art. no. 025005.
- [22] N. D. Smith, "A technique for continuous measurement of the quality factor of mechanical oscillators," *Rev. Sci. Instrum.*, vol. 86, Apr. 2015, Art. no. 053907.
- [23] E. Kulikov, "Experimental measurement of quality factor measurement," *Meas. Techn.*, vol. 2, no. 6, pp. 462–465, 1959.
- [24] Y. Wang, Y. Xie, T. Zhang, G. Wu, G. Wang, and C. Yu, "Quality factor measurement for MEMS resonator using time-domain amplitude decaying method," *Microsyst. Technol.*, vol. 21, pp. 825–829, Apr. 2015.
- [25] M. Zhang and N. Llaser, "Exploiting time-domain approach for extremely high Q-factor measurement," *IEEE Trans. Instrum. Meas.*, vol. 64, no. 10, pp. 2730–2737, Oct. 2015.
- [26] M. Zhang, N. Llaser, and H. Mathias, "Improvement of the architecture for MEMS resonator quality factor measurement," in *Proc. 15th IEEE Int. Conf. Electron. Circuits Syst.*, Saint Julian's, Malta, Aug./Sep. 2008, pp. 255–258.
- [27] X. Ren, M. Zhang, N. Llaser, and Y. Wang, "CMOS implementation of wide frequency bandwidth resonator's Q-factor measurement circuit," in *Proc. IEEE Int. Symp. Circuits Syst. (ISCAS)*, Sapporo, Japan, May 2019, pp. 1700–1703.
- [28] C. A. Grimes *et al.*, "Wireless magnetoelastic resonance sensors: A critical review," *Sensors*, vol. 2, pp. 294–313, Jul. 2002.



**MING ZHANG** (Member, IEEE) received the M.Sc. degree in telecommunications and signal processing from Xidian University, Xi'an, China, in 1985, and the Ph.D. degree in electrical engineering from University of Paris-Sud, France, in 1994.

She was an Associate Professor with the Beijing Information Engineering Institute, Beijing, China, from 1985 to 1989, and the École Nationale Supérieure d'Électronique et de Radioélectricité de Bordeaux, France, from 1994 to 2001. Since 2002, she has been an Associate Professor with University of Paris-Sud (becoming University of Paris-Saclay since 2020), France. She holds a U.S. patent on voltage-to-voltage converters. Her current research interests include the study of analogue nonvolatile memory, integrated voltage-to-voltage converter, micro-electromechanical systems sensors, and integrated micro system for medical applications. She was a recipient of the IBM France Prize in 2003 for her contribution in voltage-to-voltage converters.



**NICOLAS LLASER** received the competitive examination for posts on the teaching staff in electrical engineering from the École Nationale Supérieure de CACHAN, Paris, France, in 1992, and the M.Sc. degree and the Ph.D. degree in electronic engineering from University of Paris-Sud, France in 1993 and 1999, respectively. Since 1997, he has been a Professor of Computer Science with College DORIAN, Paris, France, and has been collaborating with University of Paris-Sud (becoming University of Paris-Saclay since 2020). He holds a U.S. patent on voltage-to-voltage converters. His research interests include fast and sigma-delta ADC, voltage-to-voltage converter as well as low-voltage systems and micro-electromechanical systems sensors. He was a recipient of the IBM France Prize in 2003 for his contribution in voltage-to-voltage converters.