

# Guidelines for Stability Analysis of the DDSRF-PLL Using LTI and LTP Modelling in the Presence of Imbalance

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**ABSTRACT** In this article, the Linear Time Invariant (LTI) and Linear Time Periodic (LTP) models of two different implementations of the DDSRF-PLL in the presence of voltage imbalance are derived analytically. The accuracy of the models is investigated with time domain simulations, frequency scans, and stability analysis. On top of this, a guideline for properly choosing between LTI and LTP models for stability assessment of the DDSRF-PLL according to the degree of grid voltage imbalance is proposed. Furthermore, it is revealed that, depending on the DDSRF-PLL implementation, the positive-sequence voltage might also cause LTP dynamics, rendering the LTI model inaccurate even when the imbalance is low.

**INDEX TERMS** DDSRF-PLL, Linear Time Periodic, LVRT, small-signal stability, unbalanced system, voltage imbalance.

## I. INTRODUCTION

In many applications, power-electronic converters are required to provide Low Voltage Ride Through (LVRT) response. In order to achieve this, it is necessary that the converter keeps synchronism with the grid in the presence of all levels of voltage imbalance. This is especially challenging under weak grid conditions. Consequently, several advanced Phase-Locked Loop (PLL) methods have been developed in the literature [1], like for example, the Decoupled Double Synchronous Reference Frame PLL (DDSRF-PLL) [2]. In order to assess the different stability properties, stability margins, or the parameter design of the PLLs, small-signal models are a common tool for engineers.

Usually, small-signal models are either Linear Time Invariant (LTI) or Linear Time Periodic (LTP). In an LTI system, the state variables are constant (i.e. time invariant) in steady-state, and therefore, linearisation of the system equations can be undertaken around an operating point. Sometimes, systems are naturally LTI, but sometimes, the state variables are forced to be time invariant in steady state using different methods. For example, the system might have an oscillatory steady-state in one reference frame, but if the equations are taken

to another frame, then the state variables become constant in steady-state, which allows for LTI linearisation around an operating point. A well-known example is two-level converters in balanced situations with only current control: in the  $abc$  frame their steady-state is oscillatory, but in the  $dq$  frame the steady-state is constant, and therefore in the  $dq$  frame an LTI model can be derived. In this case, there is no loss of accuracy in the process since the process is simply a frame transformation [3]. Sometimes, for example with certain converter topologies, more than one  $dq$  frame is necessary to force the obtention of an LTI model [4]. Another example of forcing the time-invariance of the system is when the oscillatory behaviour is ignored or considered negligible. For example, in certain systems, grid voltage imbalance or harmonic distortion might induce an oscillatory behaviour in the system equations, so if these elements are ignored, it might be possible to obtain an LTI system. In this case, there is a risk of accuracy loss in the process, since part of the system dynamics are ignored.

In an LTP system, the state variables are oscillatory in steady-state. This means that the state variables are variant in a very specific way: if the variables are in steady-state, their Fourier decomposition needs to lead to harmonic magnitudes

with constant magnitudes and phase-angles. That is to say, there is a steady-state, but it is periodic. In this case, linearisation of the system equations has to be performed around an operating trajectory, instead of an operating point.

Numerous articles have been published about small-signal modelling of PLLs. In the particular case of single-phase PLLs or Frequency-Locked Loops (FLLs), the Linear Time Invariant (LTI) models have been recently challenged, as they are not capable of modelling the double-frequency oscillation typical in these structures. Several articles [5]–[7] have recently shown that the LTI models of different single-phase PLLs or FLLs are not able to predict the stability boundaries for different parameter variations, in comparison to the Linear Time Periodic (LTP) models which perform the task [8]. In order to model more accurately the dynamic effect of certain non-linearities in SOGI-based PLLs/FLLs, [9] proposes also the use of LTP theory. Further, [10] proposes the use of signal-flow graphs for easier understanding of the harmonic propagation in LTP systems, with the application example of single-phase PLLs.

With respect to three-phase PLLs, recent literature shows that LTP modelling might be necessary in the presence of a DC component in the input voltage [11] or in the presence of imbalance [12], [13]. Specifically, [12] focuses on an SRF-PLL and shows that the 100 Hz oscillations that are caused by voltage imbalance in this type of PLL, brings LTP dynamics to the system.<sup>1</sup> When the positive-sequence voltage phase-angle detected by the PLL ( $\theta_{PLL+}$ ) is fed back within the SRF-PLL, if the negative-sequence voltage ( $V_n$ ) is high enough, the 100 Hz component in the PLL  $dq$  signals will inter-modulate with any perturbation  $f_p^{dq+}$  that may be present in the  $\theta_{PLL+}$ . This means that, if the voltage has a perturbation at  $f_p^{dq+}$  (defined in the  $dq$  frame), the frequencies  $f_p^{dq+}$  and  $f_p^{dq+} \pm 2f_1$  will appear at  $\theta_{PLL+}$  (where  $f_1$  is the fundamental frequency). In turn, these frequencies are part of the signal ( $\theta_{PLL+}$ ) which is fed back, so inter-modulation happens again and, therefore, in the end,  $\theta_{PLL+}$  will have the frequencies  $f_p^{dq+} \pm 2f_1$ ,  $f_p^{dq+} \pm 4f_1$ , etc. The presence of (infinite) frequency couplings is typical in LTP systems [14].

Both in the case of single-phase or three-phase PLLs in the presence of imbalance, the chosen approach for LTP analysis is usually to develop a Harmonic Transfer Function (HTF) model in the frequency domain (e.g. [5]–[7], [11], [12]). Alternatively, [13] proposes a state-space in the time domain, with which stability can be assessed with the eigenvalues of the monodromy matrix.

One common message in the literature concerning LTP modelling of three-phase PLLs is that LTP models are necessary in the presence of significant imbalance [12], [13]; however, no guidance exists into what level of imbalance constitutes significant enough to justify the use of the LTP method. A comparison of the stability results obtained by the

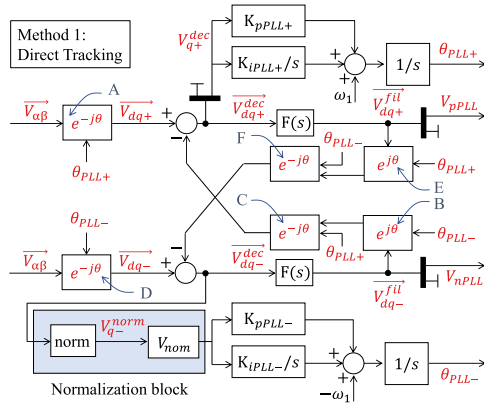
LTP and LTI methods at different imbalance levels (showing the need, or not, to use the LTP method depending on  $V_n$ ) is missing in the literature. Furthermore, the underlying assumption in previous literature is that, when  $V_n$  is low enough (and the voltage does not present other perturbations), the PLL necessarily behaves as an LTI system. This article shows that this is not always the case. In this article, two different implementations of the DDSRF-PLL are presented and modelled in the presence of voltage imbalance. One of them has significant periodic terms only when  $V_n$  is high; however, the other has significant LTP dynamics (also) due to the positive-sequence voltage ( $V_p$ ) and, therefore, has significant LTP dynamics even if  $V_n$  is low, which is a behaviour not predicted by previous literature.

Finally, previous literature assumes that the couplings created by the voltage imbalance must happen at  $2f_1$ ; however, it is shown here that, depending on the specific implementation, the LTP dynamics might appear at  $f_p^{dq+} \pm 2f_1$  (and beyond) or at  $f_p^{dq+} \pm 4f_1$  (and beyond). Thus, this article effectively contributes to the literature of small-signal models of three-phase PLLs.

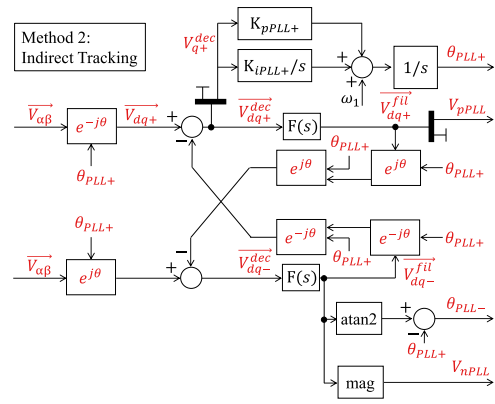
When designing the PLL, one critical design requirement is whether it is necessary to obtain the positive-sequence voltage characteristics only (i.e. magnitude and phase angle), or both the positive-sequence and negative-sequence voltage characteristics. If the intention is to only obtain the positive-sequence characteristics, a variety of solutions exist to eliminate the ripple that is produced by the negative-sequence voltage. One common solution is to place a filter inside the positive-sequence loop or before the PLL. A review of three-phase PLLs with enhanced filtering capabilities is shown in [1], although some examples are: using a notch filter [15], using a moving average filter [16], or using a Delayed Signal Cancellation (DSC) approach [17]. These options are very efficient in terms of computational burden. However, if the negative-sequence characteristics are desired as well, these filtering methods might not be sufficient. In this case, different solutions have been proposed, for example: the DDSRF-PLL [2], the Dual Second Order Generalized Integrator PLL (DSOGI-PLL) [18], the three-phase Enhanced PLL (3ph-EPLL) [19], the use of Adaptive Notch Filters (ANFs) combined with a symmetrical components calculator [20], or the use of methods derived from optimization problems [21]. Some of these structures are analysed and compared in [22], where it is shown that the DDSRF-PLL has very good performance under unbalanced conditions. Since the DDSRF-PLL is very popular phase-tracking scheme in three-phase applications [1], it was selected as an example in this article for showcasing the LTI vs. LTP modelling techniques, although other methods could have been an interesting case study as well.

In particular, two DDSRF-PLL implementations (from now on, called methods for brevity) are shown, which are focused on tracking both the positive and negative-sequence phase-angle. In a broad context, the negative-sequence properties might be useful for monitoring purposes, fault analysis and

<sup>1</sup>It is worth to observe that, from a mathematical perspective, a single-phase voltage is equivalent to a three-phase voltage with considerable imbalance.



**FIGURE 1. Method 1 (M1) for DDSRF-PLL implementation: Direct tracking of negative-sequence voltage phase-angle.**



**FIGURE 2. Method 2 (M2) for DDSRF-PLL implementation: Indirect tracking of negative-sequence voltage phase-angle.**

classification, or other applications [21]. According to [20], extracting the characteristics of all the symmetrical components in the grid voltage is important for many applications, including power quality and protection. Another usage can be the calculation of synchrophasors [23]. In the energy conversion context, the negative-sequence characteristics can be useful, for example, for generating a negative-sequence SRF for controlling the negative-sequence current (a strategy common in wind energy generation [24], [25] and other applications [26], [27]).

The DDSRF-PLL methods are presented in Section II. The LTI and LTP models are derived in Section III – Section VI, and are validated in Section VII. An explanation of the LTI and LTP model differences is shown in Section VIII. The comparison of the LTI and LTP models for stability studies is shown in Section IX. Conclusions are in Section XII.

## II. DESCRIPTION OF THE DDSRF-PLL METHODS

In this article, the phase-angle of the positive-sequence voltage is  $\theta_{1+} = \omega_1 t + \phi_{vp}$ , whereas for the negative-sequence is  $\theta_{1-} = -\omega_1 t - \phi_{vn}$ . The outputs of the PLL are  $\theta_{PLL+}$  and  $\theta_{PLL-}$ , which might not be exactly accurate, having some error as:  $\theta_{PLL+} = \theta_{1+} + \Delta\theta_{1+}$  and  $\theta_{PLL-} = \theta_{1-} + \Delta\theta_{1-}$ .

### A. METHOD 1: DIRECT TRACKING OF $\theta_{1-}$

The schematic of method 1 (M1) is shown in Fig. 1. This method incorporates two separate SRF-PLLs, one for tracking the positive sequence and one for the negative sequence. The logic behind this design is to try to make the PLL as symmetrical as possible. This method is called direct tracking since it uses an SRF-PLL to directly track the negative sequence. The phase angle obtained by this SRF-PLL,  $\theta_{PLL-}$ , is used within the decoupling network of the DDSRF-PLL, which means that, in steady state, the negative-sequence frame in the decoupling network rotates perfectly with  $\theta_{1-} = -\omega_1 t - \phi_{vn}$ . Thus,  $\vec{V}_{dq-}^{fil}$  has, in steady state,  $V_n$  in the  $d$  channel and 0 in the  $q$  channel.

Usually, the nominal voltage ( $V_{nom}$ ) is used for designing the parameters for the positive-sequence SRF-PLL (details in Appendix A). The input to the negative-sequence SRF-PLL, however, varies a lot, since  $V_n$  ranges from being very low in normal operating conditions to being very high during certain faults. For minimizing this effect, normalization can be performed as:

$$V_{q-}^{norm} = \frac{V_{q-}^{dec}}{\sqrt{(V_{d-}^{dec})^2 + (V_{q-}^{dec})^2}}. \quad (1)$$

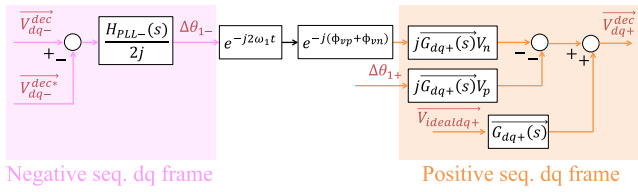
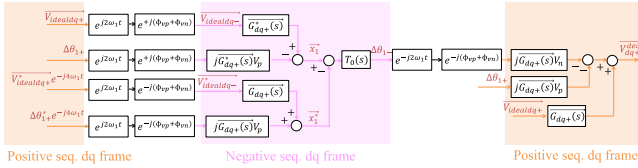
Thus, if  $V_{q-}^{norm}$  is multiplied by  $V_{nom}$ , the same PLL constants can be used for the positive- and negative-sequence SRF-PLL.

### B. METHOD 2: INDIRECT TRACKING OF $\theta_{1-}$

The schematic of method 2 (M2) is shown in Fig. 2. This method uses only a positive-sequence SRF-PLL, and uses the phase-angle it creates,  $\theta_{PLL+}$  in all the decoupling network transformations. This means that, in steady state, the negative-sequence frame in the decoupling network does not rotate with  $\theta_{1-} = -\omega_1 t - \phi_{vn}$ , but rather with  $-\theta_{1+} = -\omega_1 t - \phi_{vp}$ . When representing the negative-sequence voltage in such a frame, the space vector has the form  $V_n e^{j(\phi_{vp} - \phi_{vn})}$ . Note that, unless  $\phi_{vp} = \phi_{vn}$ , the signal that serves as input to the  $atan2$  function does not have a  $q$  channel equal to 0. Therefore, the output of the  $atan2$  function in Fig. 2 is, in steady state, equal to  $\phi_{vp} - \phi_{vn}$ . The  $\theta_{PLL-}$ , then, can be obtained by subtracting  $\theta_{PLL+}$  from the output of the  $atan2$  function. This method is the DDSRF-PLL shown in [2] but with additional blocks to calculate  $\theta_{PLL-}$  and  $V_n$ . This method is called indirect tracking since it only tracks directly  $\theta_{1+}$ , while  $\theta_{1-}$  is derived from the signals in the network.

### III. LTP MODEL OF METHOD 1

In this article, the  $dq$  frame rotating with  $\theta_{1+}$  is called ideal positive-sequence  $dq$  frame whereas the non-ideal one rotates at  $\theta_{PLL+}$ . Similarly is defined for the negative sequence.


**FIGURE 3.**  $V_{dq+}^{\text{dec}}$  as a function of  $V_{dq+}^{\text{dec}}$ .

**FIGURE 4.**  $V_{dq+}^{\text{dec}}$  as a function of positive  $dq$  frame signals.

### A. STEP 1: SOLVING THE DECOUPLING NETWORK

The first step is to analyse the transformations named A-F in Fig. 1. For the transformation A:

$$\begin{aligned} \overrightarrow{V_{\text{nonideal}dq+}} &= \overrightarrow{V_{\text{ideal}dq+}} e^{-j\Delta\theta_{1+}} \approx \overrightarrow{V_{\text{ideal}dq+}} (1 - j\Delta\theta_{1+}) \\ &= \overrightarrow{V_{\text{ideal}dq+}} - jV_p \Delta\theta_{1+} - jV_n e^{-j(2\omega_1 t + \phi_{vp} + \phi_{vn})} \Delta\theta_{1+}. \end{aligned} \quad (2)$$

The input to transformation B is  $\overrightarrow{V_{dq+}^{\text{fil}}}$ , which in steady-state is approximately  $V_n$ . Thus, the output of this transformation in steady-state should be  $V_n e^{-j(\omega_1 t + \phi_{vn})}$ . Therefore:

$$\begin{aligned} \overrightarrow{V_{\text{Bout}}} &= \overrightarrow{V_{dq-}^{\text{fil}}} e^{j\theta_{1-}} e^{j\Delta\theta_{1-}} \approx \overrightarrow{V_{dq-}^{\text{fil}}} e^{j\theta_{1-}} \\ &\quad + jV_n e^{-j(\omega_1 t + \phi_{vn})} \Delta\theta_{1-}. \end{aligned} \quad (3)$$

The transformations C, D, E and F can be found similarly. Taking into account all transformations, it can be found:

$$\begin{aligned} \overrightarrow{V_{dq+}^{\text{dec}}} &= \overrightarrow{G_{dq+}(s)} (\overrightarrow{V_{\text{ideal}dq+}} - jV_p \Delta\theta_{1+} \\ &\quad - jV_n e^{-j(\phi_{vp} + \phi_{vn})} \Delta\theta_{1-} e^{-j2\omega_1 t}) \end{aligned} \quad (4)$$

$$\begin{aligned} \overrightarrow{V_{dq-}^{\text{dec}}} &= \overrightarrow{G_{dq+}^*(s)} (\overrightarrow{V_{\text{ideal}dq-}} - jV_n \Delta\theta_{1-} \\ &\quad - jV_p e^{+j(\phi_{vp} + \phi_{vn})} \Delta\theta_{1+} e^{+j2\omega_1 t}) \end{aligned} \quad (5)$$

where

$$\overrightarrow{G_{dq+}(s)} = \frac{1 - F(s + j2\omega_1)}{1 - F(s)F(s + j2\omega_1)}. \quad (6)$$

Additionally, if it is defined  $H_{\text{PLL}+}(s) = \left[ K_{p\text{PLL}+} + \frac{K_{i\text{PLL}+}}{s} \right] \frac{1}{s}$  (and similarly for the negative sequence) from the schematic in Fig. 1, it is straightforward to find:<sup>2</sup>

$$\Delta\theta_{1+} = V_{dq+}^{\text{dec}} H_{\text{PLL}+}(s) = \frac{1}{2j} (\overrightarrow{V_{dq+}^{\text{dec}}} - \overrightarrow{V_{dq+}^{\text{dec}*}}) H_{\text{PLL}+}(s) \quad (7)$$

<sup>2</sup>The normalization block is ignored for now but it is included later.

$$\Delta\theta_{1-} = V_{dq-}^{\text{dec}} H_{\text{PLL}-}(s) = \frac{1}{2j} (\overrightarrow{V_{dq-}^{\text{dec}}} - \overrightarrow{V_{dq-}^{\text{dec}*}}) H_{\text{PLL}-}(s). \quad (8)$$

Combining (7) and (8) with (4) and (5) is not simple since both (4) and (5) depend on both  $\Delta\theta_{1+}$  and  $\Delta\theta_{1-}$ .

### B. STEP 2: EXPRESSING $V_{dq+}^{\text{dec}}$ AS A FUNCTION OF $\Delta\theta_{1+}$ ONLY

Substituting (8) into (4) leads to the schematic in Fig. 3. An auxiliary term  $\overrightarrow{x_1}$  can be defined such that:

$$\overrightarrow{x_1} = \overrightarrow{G_{dq+}^*(s)} (\overrightarrow{V_{\text{ideal}dq-}} - jV_p e^{+j(\phi_{vp} + \phi_{vn})} \Delta\theta_{1+} e^{+j2\omega_1 t}) \quad (9)$$

This means that:

$$\overrightarrow{V_{dq-}^{\text{dec}}} = \overrightarrow{x_1} - \overrightarrow{G_{dq+}^*(s)} jV_n \Delta\theta_{1-}. \quad (10)$$

Using (10) and its conjugate in (8) and operating leads to:

$$\Delta\theta_{1-} = (\overrightarrow{x_1} - \overrightarrow{x_1}^*) \frac{H_{\text{PLL}-}(s)}{2j(1 + V_n H_{\text{PLL}-}(s) G_{\text{re}}(s))}. \quad (11)$$

Using (9) and (11), Fig. 3 can be transformed into Fig. 4.<sup>3</sup> The transfer functions that appear in Fig. 4 in the negative  $dq$  frame can be easily taken into the positive  $dq$  frame with a frequency shift [28]. This leads to the following equation:

$$\begin{aligned} \overrightarrow{V_{dq+}^{\text{dec}}} &= \overrightarrow{V_{\text{ideal}dq+}} \overrightarrow{T_1} + \Delta\theta_{1+} \overrightarrow{T_2} \\ &\quad + \overrightarrow{V_{\text{ideal}dq+}^*} e^{-j4\omega_1 t} \overrightarrow{T_3} + \Delta\theta_{1+} e^{-j4\omega_1 t} \overrightarrow{T_4} \end{aligned} \quad (12)$$

where  $\overrightarrow{T_1} - \overrightarrow{T_4}$  are defined in Appendix B.

### C. STEP 3: FINAL EXPRESSION FOR $\Delta\theta_{1+}$

The expression (12) can be plugged into (7), and after operating, the following expression is obtained:

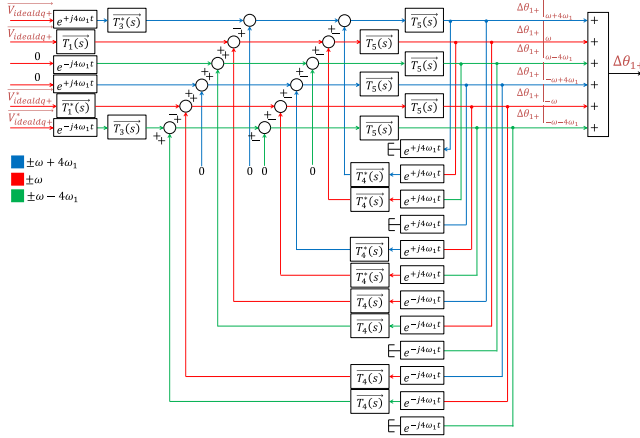
$$\begin{aligned} \Delta\theta_{1+} &= \overrightarrow{T_5} [\overrightarrow{V_{\text{ideal}dq+}} \overrightarrow{T_1} - \overrightarrow{V_{\text{ideal}dq+}^*} \overrightarrow{T_1}^* + \overrightarrow{V_{\text{ideal}dq+}^*} e^{-j4\omega_1 t} \overrightarrow{T_3} \\ &\quad - \overrightarrow{V_{\text{ideal}dq+}} e^{+j4\omega_1 t} \overrightarrow{T_3}^* + \Delta\theta_{1+} e^{-j4\omega_1 t} \overrightarrow{T_4} \\ &\quad - \Delta\theta_{1+} e^{+j4\omega_1 t} \overrightarrow{T_4}^*]. \end{aligned} \quad (13)$$

where  $\overrightarrow{T_5}$  is defined in Appendix B. The expression (13) clearly reveals the LTP dynamics of the PLL. If the voltage (in the positive  $dq$  frame) has a component at a frequency  $\omega$ , then the  $\Delta\theta_{1+}$  will have the components<sup>4</sup>  $\pm\omega$ , which in turn will create in a feedback loop through  $\overrightarrow{T_4}$  and  $\overrightarrow{T_4}^*$  the frequencies  $\pm\omega \pm 4\omega_1$ . Note that this is a theoretically infinite procedure, in which  $\pm\omega \pm 4\omega_1$  will create also a new frequencies at  $\pm\omega \pm 8\omega_1$ ; etc. In Fig. 5 the main frequency paths (i.e. at the frequencies  $\pm\omega$ ,  $\pm\omega \pm 4\omega_1$ ) are shown.

If other frequency components are neglected (this is further discussed in Section IX), then it is possible to solve the equations for each frequency path shown in Fig. 5. The direct path

<sup>3</sup>In Fig. 4,  $\Delta\theta_{1+}^*$  appears although  $\Delta\theta_{1+}^* = \Delta\theta_{1+}$  since  $\Delta\theta_{1+}$  is not a vector.

<sup>4</sup>As an example, if  $\Delta\theta_{1+}$  has the component  $A \cos(\omega t)$  this can only be represented in vector form as  $\frac{A}{2} e^{j\omega t} + \frac{A}{2} e^{-j\omega t}$ .



**FIGURE 5.** Schematic of equation (13), which clearly reveals the LTP dynamics of the PLL.

(at frequency  $\omega$ ) leads to:

$$\Delta\theta_{1+}|_{\omega} = \frac{c_1 + d_1}{1 + a_1 + b_1} \overrightarrow{V_{\text{ideal}dq+}} \overrightarrow{TF_{\text{PLL}1+}(s)} \quad (14)$$

with:

$$\begin{aligned} a_1 &= \overrightarrow{T_4^*(s + j4\omega_1)} \overrightarrow{T_5(s + j4\omega_1)} \overrightarrow{T_4} \overrightarrow{T_5} \\ b_1 &= \overrightarrow{T_4(s - j4\omega_1)} \overrightarrow{T_5(s - j4\omega_1)} \overrightarrow{T_4^*} \overrightarrow{T_5^*} \\ c_1 &= \overrightarrow{T_1} \overrightarrow{T_5} \\ d_1 &= -\overrightarrow{T_3^*(s + j4\omega_1)} \overrightarrow{T_5(s + j4\omega_1)} \overrightarrow{T_4} \overrightarrow{T_5^*}. \end{aligned} \quad (15)$$

Solving the conjugate frequencies leads to:

$$\Delta\theta_{1+}|_{-\omega} = \frac{c_2 + d_2}{1 + a_2 + b_2} \overrightarrow{V_{\text{ideal}dq+}^*} \overrightarrow{TF_{\text{PLL}2+}(s)} \quad (16)$$

with:

$$\begin{aligned} a_2 &= a_1; \quad b_2 = b_1; \quad c_2 = -\overrightarrow{T_1^*} \overrightarrow{T_5^*} \\ d_2 &= -\overrightarrow{T_3(s - j4\omega_1)} \overrightarrow{T_5(s - j4\omega_1)} \overrightarrow{T_4^*} \overrightarrow{T_5^*}. \end{aligned} \quad (17)$$

The other frequencies in  $\Delta\theta_{1+}$  can be solved now, leading to the final model of the DDSRF-PLL for the positive-sequence phase-angle as shown in (18).

$$\begin{aligned} \Delta\theta_{1+} &\approx \overrightarrow{V_{\text{ideal}dq+}} \overrightarrow{TF_{\text{PLL}1+}} + \overrightarrow{V_{\text{ideal}dq+}^*} \overrightarrow{TF_{\text{PLL}2+}} \\ &+ \overrightarrow{V_{\text{ideal}dq+}} e^{j4\omega_1 t} \overrightarrow{TF_{\text{PLL}3+}} + (\overrightarrow{V_{\text{ideal}dq+}} e^{j4\omega_1 t})^* \overrightarrow{TF_{\text{PLL}4+}} \\ &+ \overrightarrow{V_{\text{ideal}dq+}} e^{-j4\omega_1 t} \overrightarrow{TF_{\text{PLL}5+}} + (\overrightarrow{V_{\text{ideal}dq+}} e^{-j4\omega_1 t})^* \overrightarrow{TF_{\text{PLL}6+}} \end{aligned} \quad (18)$$

(18) uses an approximate sign, since in reality there are an infinite number of PLL couplings. In here, only the first round of couplings is shown. The question as to how many couplings need to be included is answered in Section IX-F.

#### D. STEP 4: NEGATIVE-SEQUENCE PHASE-ANGLE MODEL

The final model for the negative-sequence phase-angle can be similarly derived and it is shown in (19).

$$\begin{aligned} \Delta\theta_{1-} &\approx \overrightarrow{V_{\text{ideal}dq-}} \overrightarrow{TF_{\text{PLL}1-}} + \overrightarrow{V_{\text{ideal}dq-}^*} \overrightarrow{TF_{\text{PLL}2-}} \\ &+ \overrightarrow{V_{\text{ideal}dq-}} e^{-j4\omega_1 t} \overrightarrow{TF_{\text{PLL}3-}} + (\overrightarrow{V_{\text{ideal}dq-}} e^{-j4\omega_1 t})^* \overrightarrow{TF_{\text{PLL}4-}} \\ &+ \overrightarrow{V_{\text{ideal}dq-}} e^{j4\omega_1 t} \overrightarrow{TF_{\text{PLL}5-}} + (\overrightarrow{V_{\text{ideal}dq-}} e^{j4\omega_1 t})^* \overrightarrow{TF_{\text{PLL}6-}} \end{aligned} \quad (19)$$

It can be shown that  $\overrightarrow{TF_{\text{PLL}2+}(s)} = \overrightarrow{TF_{\text{PLL}1+}^*(s)}$ ,  $\overrightarrow{TF_{\text{PLL}4+}(s)} = \overrightarrow{TF_{\text{PLL}3+}^*(s)}$  and  $\overrightarrow{TF_{\text{PLL}6+}(s)} = \overrightarrow{TF_{\text{PLL}5+}^*(s)}$ . The same applies for the complex transfer functions in (19).

#### E. STEP 5: INCLUDING NORMALIZATION

For including the normalization block shown in Fig. 1, it is necessary to linearise (1) through a Taylor series expansion (around the steady-state operating point  $(V_n, 0)$ ). The result is:

$$V_{q-}^{\text{norm}} \approx \frac{V_{q-}^{\text{dec}}}{V_n}. \quad (20)$$

Thus, for including the normalization block, the only step is to substitute  $H_{\text{PLL}-}(s)$  by  $\frac{V_{\text{nom}} H_{\text{PLL}-}(s)}{V_n}$  in all equations.

#### IV. LTI MODEL OF METHOD 1

For developing the LTI model, the time-varying coefficients are ignored. In method 1, this means that (4) and (5) become:

$$\overrightarrow{V_{dq+}^{\text{dec}}} = \overrightarrow{G_{dq+}(s)} (\overrightarrow{V_{\text{ideal}dq+}} - jV_p \Delta\theta_{1+}) \quad (21)$$

$$\overrightarrow{V_{dq-}^{\text{dec}}} = \overrightarrow{G_{dq+}^*(s)} (\overrightarrow{V_{\text{ideal}dq-}} - jV_n \Delta\theta_{1-}) \quad (22)$$

Combining these expressions with (7) and (8) is straightforward, and leads to the following model:

$$\Delta\theta_{1+} = \overrightarrow{V_{\text{ideal}dq+}} \overrightarrow{TF_{\text{PLL}1+LTI}} + \overrightarrow{V_{\text{ideal}dq+}^*} \overrightarrow{TF_{\text{PLL}2+LTI}} \quad (23)$$

$$\Delta\theta_{1-} = \overrightarrow{V_{\text{ideal}dq-}} \overrightarrow{TF_{\text{PLL}1-LTI}} + \overrightarrow{V_{\text{ideal}dq-}^*} \overrightarrow{TF_{\text{PLL}2-LTI}}$$

$$\overrightarrow{TF_{\text{PLL}1+LTI}} = \frac{H_{\text{PLL}+}(s) \overrightarrow{G_{dq+}(s)}}{2j(1 + V_p H_{\text{PLL}+}(s) G_{\text{re}}(s))} \quad (24)$$

$$\overrightarrow{TF_{\text{PLL}1-LTI}} = \frac{H_{\text{PLL}-}(s) \overrightarrow{G_{dq+}^*(s)}}{2j(1 + V_n H_{\text{PLL}-}(s) G_{\text{re}}(s))}$$

where  $\overrightarrow{TF_{\text{PLL}2+LTI}} = \overrightarrow{TF_{\text{PLL}1+LTI}^*}$ ,  $\overrightarrow{TF_{\text{PLL}2-LTI}} = \overrightarrow{TF_{\text{PLL}1-LTI}^*}$ . For including the normalization block, the same steps as explained for the LTP model (Section III-E) should be taken.



## V. LTP MODEL OF METHOD 2

### A. STEP 1: SOLVING THE DECOUPLING NETWORK

In a similar process as it is done with the method 1, the following equations can be obtained for the method 2:

$$\begin{aligned} \overrightarrow{V_{dq+}^{\text{dec}}} &= \overrightarrow{G_{dq+}(s)}(\overrightarrow{V_{\text{idealdq+}}} - jV_p \Delta\theta_{1+}) \\ &\quad + jV_n e^{-j(\phi_{vp} + \phi_{vn})} \Delta\theta_{1+} e^{-j2\omega_1 t} \end{aligned} \quad (25)$$

$$\begin{aligned} \overrightarrow{V_{dq-}^{\text{dec}}} &= \overrightarrow{G_{dq+}^*(s)} e^{+j(\phi_{vp} - \phi_{vn})} (\overrightarrow{V_{\text{idealdq-}}} + jV_n \Delta\theta_{1+}) \\ &\quad - jV_p e^{+j(\phi_{vp} + \phi_{vn})} \Delta\theta_{1+} e^{+j2\omega_1 t} \end{aligned} \quad (26)$$

In this case,  $\Delta\theta_{1-}$  does not appear in (25) nor in (26), since the decoupling network only uses  $\theta_{\text{PLL}+}$ . In (26) there is a new term,  $e^{+j(\phi_{vp} - \phi_{vn})}$ , that does not appear in the equations of M1.

This term is here because, in the M2, the  $\overrightarrow{V_{dq-}^{\text{dec}}}$  is a signal that ideally rotates with  $-\theta_{1+} = -\omega_1 t - \phi_{vp}$ , instead of rotating with  $\theta_{1-} = -\omega_1 t - \phi_{vn}$  as in method 1.

### B. STEP 2: FINAL EXPRESSION FOR $\Delta\theta_{1+}$

The expression (25) can be used in (7). Operating leads to:

$$\begin{aligned} \Delta\theta_{1+} &= \overrightarrow{T_6} [\overrightarrow{V_{\text{idealdq+}}} \overrightarrow{G_{dq+}(s)} - \overrightarrow{V_{\text{idealdq+}}^*} \overrightarrow{G_{dq+}^*(s)}] \\ &\quad + \Delta\theta_{1+} e^{+j2\omega_1 t} \overrightarrow{T_7} - \Delta\theta_{1+} e^{-j2\omega_1 t} \overrightarrow{T_7^*}. \end{aligned} \quad (27)$$

where  $\overrightarrow{T_6}$  and  $\overrightarrow{T_7}$  are defined in Appendix C. Similarly as with method 1, the expression (27) reveals LTP dynamics, although this time they are dependent on  $\pm 2\omega_1$  instead of  $\pm 4\omega_1$ . Performing similar operations and schematics as with method 1, the following LTP model can be obtained:

$$\begin{aligned} \Delta\theta_{1+} &\approx \overrightarrow{V_{\text{idealdq+}}} \overrightarrow{TF_{\text{PLL}1+}} + \overrightarrow{V_{\text{idealdq+}}^*} \overrightarrow{TF_{\text{PLL}2+}} \\ &\quad + \overrightarrow{V_{\text{idealdq+}}} e^{j2\omega_1 t} \overrightarrow{TF_{\text{PLL}3+}} + (\overrightarrow{V_{\text{idealdq+}}} e^{j2\omega_1 t})^* \overrightarrow{TF_{\text{PLL}4+}} \\ &\quad + \overrightarrow{V_{\text{idealdq+}}} e^{-j2\omega_1 t} \overrightarrow{TF_{\text{PLL}5+}} + (\overrightarrow{V_{\text{idealdq+}}} e^{-j2\omega_1 t})^* \overrightarrow{TF_{\text{PLL}6+}} \end{aligned} \quad (28)$$

### C. STEP 3: NEGATIVE-SEQUENCE PHASE-ANGLE MODEL

Here, it is necessary to linearise  $\text{atan}\left(\frac{V_{q-}^{\text{fil}}}{V_{d-}^{\text{fil}}}\right)$  around the point  $(\Im\{V_n e^{+j(\phi_{vp} - \phi_{vn})}\}, \Re\{V_n e^{+j(\phi_{vp} - \phi_{vn})}\})$ . This results in:

$$\begin{aligned} \text{atan}\left(\frac{V_{q-}^{\text{fil}}}{V_{d-}^{\text{fil}}}\right) &\approx \phi_{vp} - \phi_{vn} + \frac{(e^{+j(\phi_{vp} - \phi_{vn})} + e^{-j(\phi_{vp} - \phi_{vn})})}{2V_n} V_{q-}^{\text{fil}} \\ &\quad - \frac{(e^{+j(\phi_{vp} - \phi_{vn})} - e^{-j(\phi_{vp} - \phi_{vn})})}{j2V_n} V_{d-}^{\text{fil}}. \end{aligned} \quad (29)$$

Since  $\theta_{\text{PLL}-} = -\theta_{\text{PLL}+} + \text{atan}\left(\frac{V_{q-}^{\text{fil}}}{V_{d-}^{\text{fil}}}\right)$ , and also  $\theta_{\text{PLL}+} = \omega_1 t + \phi_{vp} + \Delta\theta_{1+}$  and  $\theta_{\text{PLL}-} = -\omega_1 t - \phi_{vn} + \Delta\theta_{1-}$ , then:

$$\begin{aligned} \Delta\theta_{1-} &= -\Delta\theta_{1+} + \frac{(e^{+j(\phi_{vp} - \phi_{vn})} + e^{-j(\phi_{vp} - \phi_{vn})})}{2V_n} V_{q-}^{\text{fil}} \\ &\quad - \frac{(e^{+j(\phi_{vp} - \phi_{vn})} - e^{-j(\phi_{vp} - \phi_{vn})})}{j2V_n} V_{d-}^{\text{fil}}. \end{aligned} \quad (30)$$

Knowing that  $\overrightarrow{V_{dq-}^{\text{fil}}} = F(s) \overrightarrow{V_{dq-}^{\text{dec}}}$ , the terms  $V_{d-}^{\text{fil}}$  and  $V_{q-}^{\text{fil}}$  can be derived from (26), while  $\Delta\theta_{1+}$  is shown in (28). The final LTP model for  $\Delta\theta_{1-}$  for method 2 is:

$$\begin{aligned} \Delta\theta_{1-} &\approx \overrightarrow{V_{\text{idealdq-}}} \overrightarrow{TF_{\text{PLL}1-}} + \overrightarrow{V_{\text{idealdq-}}^*} \overrightarrow{TF_{\text{PLL}2-}} \\ &\quad + \overrightarrow{V_{\text{idealdq-}}} e^{-j2\omega_1 t} \overrightarrow{TF_{\text{PLL}3-}} + (\overrightarrow{V_{\text{idealdq-}}} e^{-j2\omega_1 t})^* \overrightarrow{TF_{\text{PLL}4-}} \\ &\quad + \overrightarrow{V_{\text{idealdq-}}} e^{+j2\omega_1 t} \overrightarrow{TF_{\text{PLL}5-}} + (\overrightarrow{V_{\text{idealdq-}}} e^{+j2\omega_1 t})^* \overrightarrow{TF_{\text{PLL}6-}} \end{aligned} \quad (31)$$

## VI. LTI MODEL OF METHOD 2

For developing the LTI model, the time-varying coefficients are ignored. Thus, (25) and (26) become:

$$\overrightarrow{V_{dq+}^{\text{dec}}} = \overrightarrow{G_{dq+}(s)}(\overrightarrow{V_{\text{idealdq+}}} - jV_p \Delta\theta_{1+}) \quad (32)$$

$$\overrightarrow{V_{dq-}^{\text{dec}}} = \overrightarrow{G_{dq+}^*(s)} e^{+j(\phi_{vp} - \phi_{vn})} (\overrightarrow{V_{\text{idealdq-}}} + jV_n \Delta\theta_{1+}) \quad (33)$$

Since (32) is the same as (21), the LTI model of  $\Delta\theta_{1+}$  is the same in method 2 as in method 1. For  $\Delta\theta_{1-}$ , knowing that  $\overrightarrow{V_{dq-}^{\text{fil}}} = F(s) \overrightarrow{V_{dq-}^{\text{dec}}}$ , it is necessary to use (33) in (30) and, ignoring oscillating terms, the LTI model results in:

$$\Delta\theta_{1-} = \overrightarrow{V_{\text{idealdq-}}} \overrightarrow{TF_{\text{PLL}1\text{-LTI}}} + \overrightarrow{V_{\text{idealdq-}}^*} \overrightarrow{TF_{\text{PLL}2\text{-LTI}}} \quad (34)$$

where  $\overrightarrow{TF_{\text{PLL}2\text{-LTI}}} = \overrightarrow{TF_{\text{PLL}1\text{-LTI}}^*}$  and

$$\overrightarrow{TF_{\text{PLL}1\text{-LTI}}} = \frac{1}{2jV_n} (F(s) \overrightarrow{G_{dq+}^*(s)}). \quad (35)$$

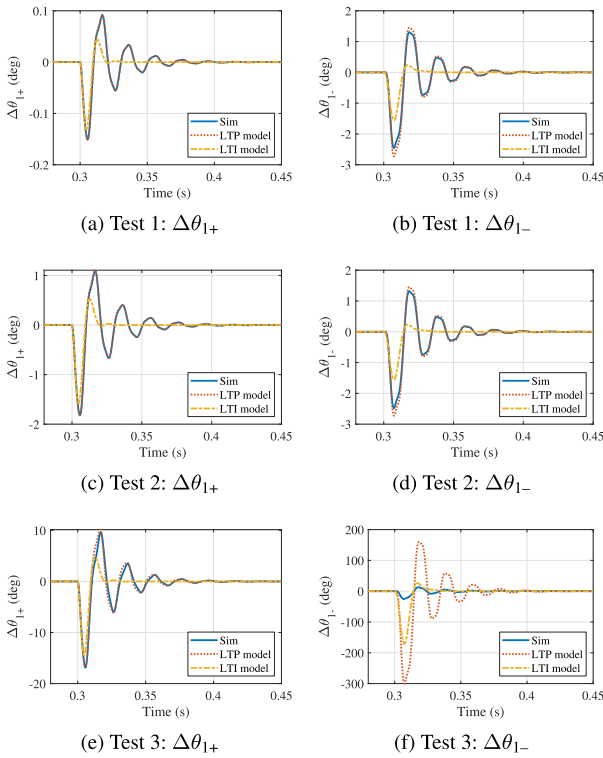
## VII. VALIDATION OF LTI AND LTP MODELS

### A. TIME DOMAIN VALIDATION

The models are validated with time-domain simulations. The PLL parameters in Appendix A are used, and also  $K = 1/\sqrt{2}$ . Three tests are performed:

- *Test 1:* From a steady-state operating point with  $V_n = 5\%$ ,  $V_n$  is increased 10% of its value (small-signal perturbation).
- *Test 2:* From a steady-state operating point with  $V_n = 60\%$ ,  $V_n$  is increased 10% of its value (small-signal perturbation).
- *Test 3:* From a steady-state operating point with  $V_n = 5\%$ ,  $V_n$  is increased until  $V_n = 60\%$  (large-signal perturbation).

The results for the DDSRF-PLL method 1 are shown in Fig. 6. It can be seen that, for small-signal perturbations (Test 1 and Test 2), the LTI model fails to predict the oscillations that appear in the phase angles. The LTI model predicts a much cleaner and stable response than what the simulations show. The LTP model, however, perfectly predicts the trend for small-signal perturbations. In the Test 3, where there is a large-signal perturbation, the LTI model again is inaccurate. For the LTP model, the positive-sequence phase-angle transient is perfectly predicted, but not the negative-sequence one.



**FIGURE 6.** Validation of LTI and LTP models for method 1 in the time domain.

This test shows that, even if periodic, the LTP model is still small-signal.

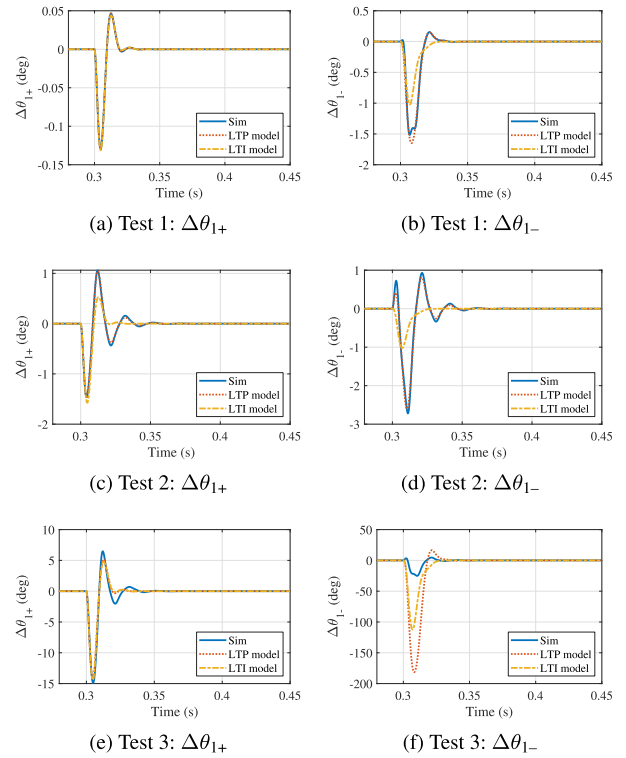
Fig. 7 analyses the DDSRF-PLL method 2. Test 1 shows that the LTI model can predict the transient of a small-signal perturbation if  $V_n$  is low (at least for  $\Delta\theta_{1+}$ ). However, if  $V_n$  is increased (Test 2), the LTI model is no longer small-signal accurate. Just like in method 1, the LTI model underestimates the oscillations. In contrast, the LTP model follows the transient after a small-signal perturbation no matter the  $V_n$  level. In Test 3, both models are inaccurate since both models are linearised.

### B. FREQUENCY DOMAIN VALIDATION

Different frequency sweeps were performed for different  $V_n$  levels. The results are shown in Fig. 8 (method 1). Certain transfer functions are not shown since they are conjugates of the transfer functions displayed. Fig. 8 shows that the LTP model is perfectly accurate for different levels of voltage imbalance, while the LTI model does not have a perfect overlap with the frequency scan. The model for method 2 is also validated (Fig. 9). Both figures use the parameters in Appendix A, and also  $K = 1/\sqrt{2}$ .

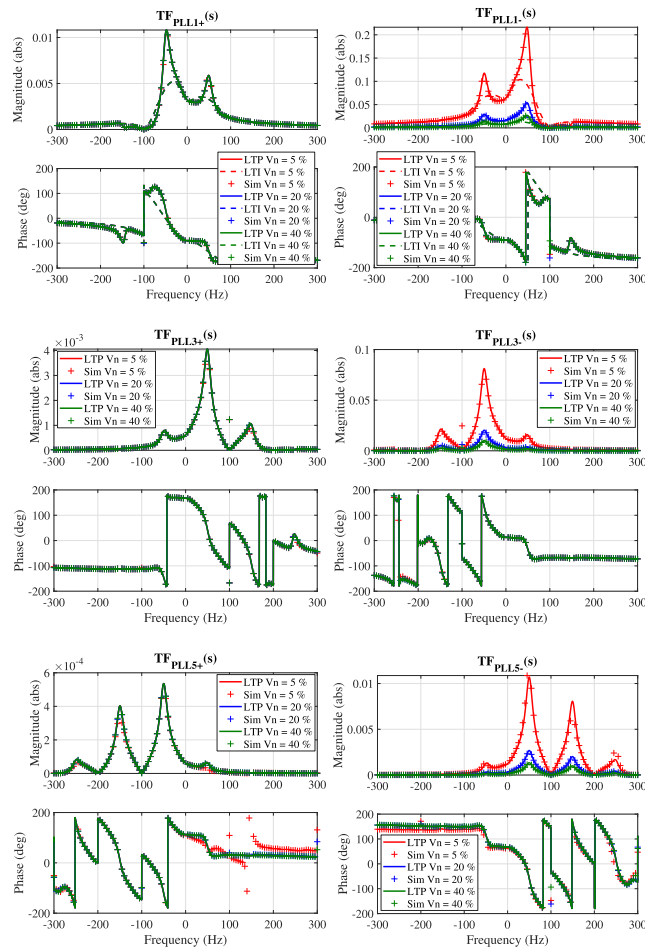
### VIII. DIFFERENCES BETWEEN THE LTI AND LTP MODELS

In (18) and (23), it appears as if the only difference between the LTI and LTP model is that the LTP model predicts the couplings, while the LTI model does not. However, this is

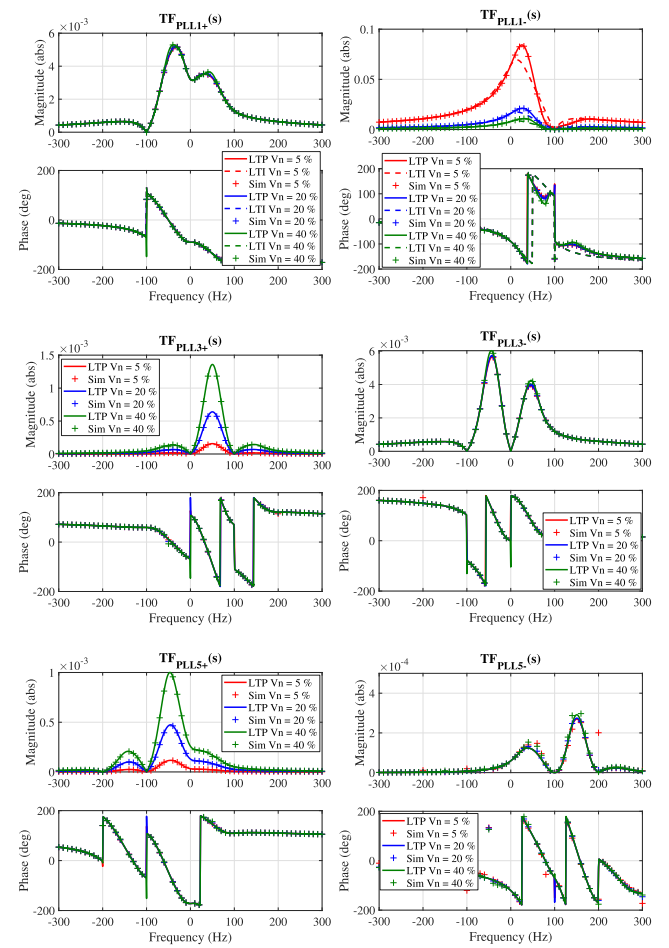


**FIGURE 7.** Validation of LTI and LTP models for method 2 in the time domain.

not the main difference. For illustration purposes, the time-domain Test 1 results for method 1 are analysed in detail. Fig. 10 shows the decomposition of the time-domain response of the LTP model, according to the contribution to the total response by each of its transfer functions. The black dotted line in Fig. 10 corresponds to the LTP line shown in Fig. 6 a). As it can be seen, the coupling transfer functions contribute to the response; however, the biggest contribution comes from  $\overrightarrow{TF_{PLL1+}}$  and  $\overrightarrow{TF_{PLL2+}}$ , and in fact, the total response of the LTP model (summing the responses of all transfer functions) is relatively similar to the response that the model would give if only  $\overrightarrow{TF_{PLL1+}}$  and  $\overrightarrow{TF_{PLL2+}}$  would be considered. Further, it is important to notice that the response given by  $\overrightarrow{TF_{PLL1+}}$  and  $\overrightarrow{TF_{PLL2+}}$  is not at all similar to such given by  $\overrightarrow{TF_{PLL1+LTI}}$  and  $\overrightarrow{TF_{PLL2+LTI}}$ , which is shown in Fig. 6 a) (under the label LTI model). This is due to the fact that  $\overrightarrow{TF_{PLL1+}}$  and  $\overrightarrow{TF_{PLL2+}}$  are very different transfer functions than  $\overrightarrow{TF_{PLL1+LTI}}$  and  $\overrightarrow{TF_{PLL2+LTI}}$ . This can be seen in the fact that the expression for  $\overrightarrow{TF_{PLL1+}}$  (shown in (14)) is very different from the expression for  $\overrightarrow{TF_{PLL1+LTI}}$  (shown in (24)). The transfer functions are different, and therefore they have different poles and stability properties. Therefore, the main difference between the LTI and LTP models is not that the LTP model predicts the couplings but that, through  $\overrightarrow{TF_{PLL1+}}$  and  $\overrightarrow{TF_{PLL2+}}$ , it is able to predict important oscillatory behaviour that appears in the PLL in the presence of imbalance.



**FIGURE 8.** Validation of LTI and LTP models for method 1 in the frequency domain.



**FIGURE 9.** Validation of LTI and LTP models for method 2 in the frequency domain.

## IX. STABILITY ANALYSIS

### A. STABILITY ANALYSIS WITH THE LTI MODEL

The LTI and LTP models have two subsystems: one for the positive sequence and one for the negative sequence. In method 1, the LTI model for the positive sequence can be written as (36) and for the negative sequence as (37).

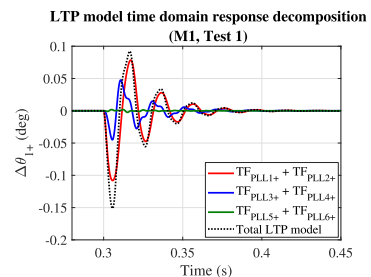
$$\begin{bmatrix} \Delta\theta_{1+} \\ \Delta\theta_{1+}^* \end{bmatrix} = [M_{LTI1+}]_{2 \times 2} \begin{bmatrix} V_{\text{idealdq}+} \\ V_{\text{idealdq}+}^* \end{bmatrix} \quad (36)$$

$$\begin{bmatrix} \Delta\theta_{1-} \\ \Delta\theta_{1-}^* \end{bmatrix} = [M_{LTI1-}]_{2 \times 2} \begin{bmatrix} V_{\text{idealdq}-} \\ V_{\text{idealdq}-}^* \end{bmatrix} \quad (37)$$

$M_{LTI1+}$  is a  $2 \times 2$  complex transfer function matrix derived from (23).  $M_{LTI1+}$  is shown in (38).  $M_{LTI1-}$  can be found analogously.

$$M_{LTI1+} = \begin{bmatrix} \overrightarrow{TF_{PLL1+LTI}(s)} & \overrightarrow{TF_{PLL2+LTI}(s)} \\ \overrightarrow{TF_{PLL2+LTI}^*(s)} & \overrightarrow{TF_{PLL1+LTI}^*(s)} \end{bmatrix} \quad (38)$$

According to multivariable stability theory, in order to evaluate the stability with a MIMO LTI model, the MIMO poles



**FIGURE 10.** Decomposition of LTP model response (M1, Test 1).

of  $M_{LTI1+}$  and  $M_{LTI1-}$  need to be found, and all should lay in the Left Half Plane (LHP) [29].<sup>5</sup>

<sup>5</sup>In a MIMO LTI system, there are two main ways of calculating the poles. The first one is to calculate the Smith-McMillan form of the transfer function matrix, a canonical form in which the poles are directly accessible [29]. Another option is to plot together all the poles of each individual transfer function in the matrix. This option gives the correct placement of the poles of the MIMO system, although the multiplicities of the poles are unknown. The second option is used in this article since the computations are faster, and since the multiplicities of the poles are not relevant (i.e. it is not important to know the number of poles in the RHP, but only whether there is any pole there).



## B. STABILITY ANALYSIS WITH THE LTP MODEL

In the LTP model, the positive and negative sequence sub-systems are shown in (39) and (40), respectively.  $M_{LTP1+}$  is a complex transfer function matrix derived from (18). The resulting matrix is (41) shown at the bottom of this page.  $M_{LTP1-}$  can be found analogously.

$$\begin{bmatrix} \Delta\theta_{1+} \\ \Delta\theta_{1+}^* \\ \Delta\theta_{1+}e^{+j4\omega_1 t} \\ \Delta\theta_{1+}^*e^{-j4\omega_1 t} \\ \Delta\theta_{1+}e^{-j4\omega_1 t} \\ \Delta\theta_{1+}^*e^{+j4\omega_1 t} \end{bmatrix} = [M_{LTP1+}]_{6 \times 6} \begin{bmatrix} \overrightarrow{V_{idealdq+}} \\ \overrightarrow{V_{idealdq+}^*} \\ \overrightarrow{V_{idealdq+}}e^{+j4\omega_1 t} \\ \overrightarrow{V_{idealdq+}^*}e^{-j4\omega_1 t} \\ \overrightarrow{V_{idealdq+}}e^{-j4\omega_1 t} \\ \overrightarrow{V_{idealdq+}^*}e^{+j4\omega_1 t} \end{bmatrix} \quad (39)$$

$$\begin{bmatrix} \Delta\theta_{1-} \\ \Delta\theta_{1-}^* \\ \Delta\theta_{1-}e^{-j4\omega_1 t} \\ \Delta\theta_{1-}^*e^{+j4\omega_1 t} \\ \Delta\theta_{1-}e^{+j4\omega_1 t} \\ \Delta\theta_{1-}^*e^{-j4\omega_1 t} \end{bmatrix} = [M_{LTP1-}]_{6 \times 6} \begin{bmatrix} \overrightarrow{V_{idealdq-}} \\ \overrightarrow{V_{idealdq-}^*} \\ \overrightarrow{V_{idealdq-}}e^{-j4\omega_1 t} \\ \overrightarrow{V_{idealdq-}^*}e^{+j4\omega_1 t} \\ \overrightarrow{V_{idealdq-}}e^{+j4\omega_1 t} \\ \overrightarrow{V_{idealdq-}^*}e^{-j4\omega_1 t} \end{bmatrix} \quad (40)$$

The  $M_{LTP1+}$  and  $M_{LTP1-}$  shown in (39) and (40) are the HTF matrices of the system, and are 6 x 6. In reality, when an LTP system is represented in the frequency domain, the HTF matrix has an infinite order [14]. In here, the HTF matrix is 6 x 6 because only the first round of couplings is considered.  $M_{LTP1+}$  could be 2 x 2 if the couplings are ignored (considering  $\overrightarrow{TF_{PLL3+}} - \overrightarrow{TF_{PLL6+}}$  negligible). Alternatively,  $M_{LTP1+}$  could be higher order than 6 x 6 if the second or more rounds of couplings are considered. A question arises as to what is the appropriate order to model. The issue of model truncation is explained subsequently.

Formally, the poles of an LTP system are the locations in the complex  $s$ -plane where the HTF is not analytic [30]. According to [30], the LTP poles appear in different strips. The horizontal strip confined by  $y = \omega_p/2$  and  $y = -\omega_p/2$  (with  $\omega_p$  being the pumping frequency of the LTP system; i.e.  $4\omega_1$  in M1 and  $2\omega_1$  in M2) is called the fundamental strip. The poles that appear in this strip are then reflected to other strips (i.e. complementary strips) in such a way that only the imaginary part of the poles change, but not the real part. This is illustrated in Fig. 11. This means that there are infinite LTP poles, but since the  $x$ -axis value of the poles does not change when comparing different strips, it is not necessary to look at all the poles of the system in order to assess stability, but only

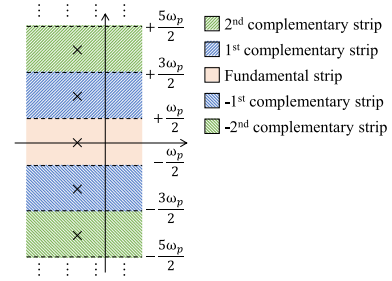


FIGURE 11. Infinite pole repetition in an LTP system [30].

whether the poles in the fundamental strip fall entirely in the LHP or not.

In this article, it is considered that a certain HTF order is enough as long as the poles in the fundamental strip do not significantly change. In practice, this means that the conclusions of the stability study (the stability limit) remain unchanged if the order is increased. As it will be shown later, the  $M_{LTP1+}$  2 x 2 HTF matrix predicts the same poles in the fundamental strip (and thus, the same stability limits) as the  $M_{LTP1+}$  6 x 6 HTF matrix. Since the LTP model is approximated as an LTI MIMO model of a certain order, the poles can be determined in the same way as with MIMO LTI theory [31].<sup>6</sup>

For method 2, the LTI and LTP MIMO matrices can be found similarly, leading to  $M_{LTI2+}$ ,  $M_{LTI2-}$ ,  $M_{LTP2+}$  and  $M_{LTP2-}$ . In M2, however, it is only necessary to check the poles of the positive-sequence sub-system, since only  $\theta_{PLL+}$  is fed back.

## C. STABILITY RESULTS: METHOD 1

In this section, the parameter  $K$  that defines the cut-off frequency of the low-pass filter  $F(s) = \frac{\omega_f}{s + \omega_f}$  (with  $\omega_f = K\omega_1$ ) is changed. According to simulations, the DDSRF-PLL method 1 becomes unstable for  $K$  values higher than  $K_{lim}$ , being:

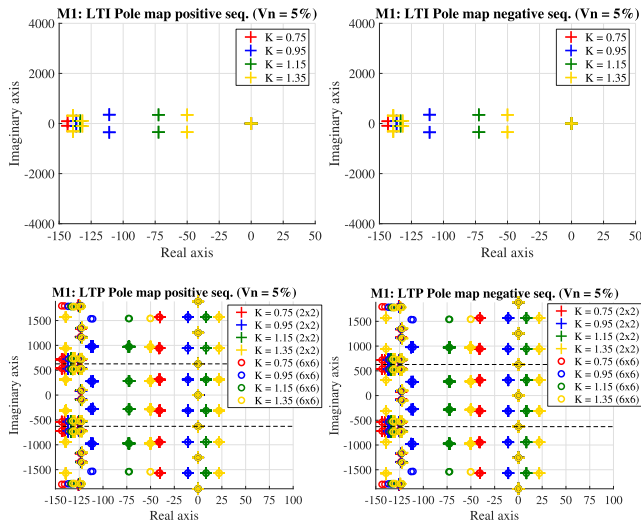
- $K_{lim} = 1.05$  for  $V_n = 5\%$ .
- $K_{lim} = 1.05$  for  $V_n = 40\%$ .

In Fig. 12 the pole maps<sup>7</sup> from  $M_{LTI1+}$  and  $M_{LTI1-}$  are shown for different  $K$  when  $V_n = 5\%$ . As it can be seen, the LTI method predicts incorrectly the stability boundary, since it predicts that, for  $K = 1.15$  and  $K = 1.35$ , the system is still stable. The  $K$  has to be increased until  $K = 2.45$  in order to predict instability (not shown in the figure). The pole maps of  $M_{LTP1+}$  and  $M_{LTP1-}$  are also shown in Fig. 12, and they

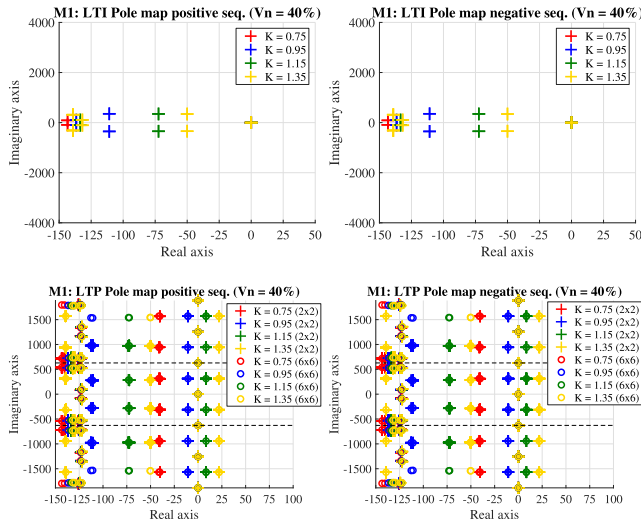
<sup>6</sup>This means that the same method for pole determination will be used for the LTP system as for the LTI system, which is to plot the poles of each individual transfer function in the matrix. This method is also used in [32].

<sup>7</sup>Only the poles close to the  $x = 0$  line are shown.

$$\begin{bmatrix} \overrightarrow{TF_{PLL1+}(s)} & \overrightarrow{TF_{PLL2+}(s)} & \overrightarrow{TF_{PLL3+}(s)} & \overrightarrow{TF_{PLL4+}(s)} & \overrightarrow{TF_{PLL5+}(s)} & \overrightarrow{TF_{PLL6+}(s)} \\ \overrightarrow{TF_{PLL2+}^*(s)} & \overrightarrow{TF_{PLL1+}^*(s)} & \overrightarrow{TF_{PLL4+}^*(s)} & \overrightarrow{TF_{PLL3+}^*(s)} & \overrightarrow{TF_{PLL6+}^*(s)} & \overrightarrow{TF_{PLL5+}^*(s)} \\ \overrightarrow{TF_{PLL5+}(s - j4\omega_1)} & \overrightarrow{TF_{PLL4+}(s - j4\omega_1)} & \overrightarrow{TF_{PLL1+}(s - j4\omega_1)} & 0 & 0 & \overrightarrow{TF_{PLL2+}(s - j4\omega_1)} \\ \overrightarrow{TF_{PLL4+}^*(s + j4\omega_1)} & \overrightarrow{TF_{PLL5+}^*(s + j4\omega_1)} & 0 & \overrightarrow{TF_{PLL1+}^*(s + j4\omega_1)} & \overrightarrow{TF_{PLL2+}^*(s + j4\omega_1)} & 0 \\ \overrightarrow{TF_{PLL3+}(s + j4\omega_1)} & \overrightarrow{TF_{PLL6+}(s + j4\omega_1)} & 0 & \overrightarrow{TF_{PLL2+}(s + j4\omega_1)} & \overrightarrow{TF_{PLL1+}(s + j4\omega_1)} & 0 \\ \overrightarrow{TF_{PLL6+}^*(s - j4\omega_1)} & \overrightarrow{TF_{PLL3+}^*(s - j4\omega_1)} & \overrightarrow{TF_{PLL2+}^*(s - j4\omega_1)} & 0 & 0 & \overrightarrow{TF_{PLL1+}^*(s - j4\omega_1)} \end{bmatrix} \quad (41)$$



**FIGURE 12.** Stability results for method 1 ( $V_n = 5\%$ ). The LTP pole map shows the fundamental strip, and two complementary strips ( $y$ -axis:  $[-\frac{3\omega_p}{2}, \frac{3\omega_p}{2}]$  with  $\omega_p = 4\omega_1$ ).



**FIGURE 13.** Stability results for method 1 ( $V_n = 40\%$ ). The LTP pole map shows the fundamental strip, and two complementary strips ( $y$ -axis:  $[-\frac{3\omega_p}{2}, \frac{3\omega_p}{2}]$  with  $\omega_p = 4\omega_1$ ).

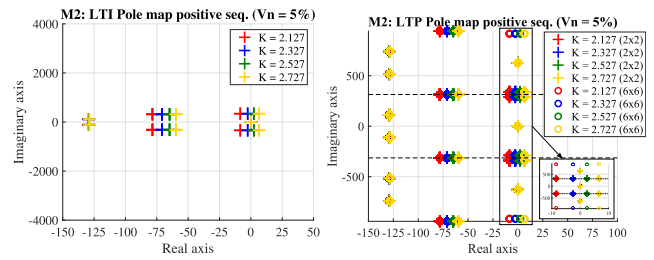
correctly predict the stability boundary. Note that, no matter that the  $2 \times 2$  or  $6 \times 6$  LTP model is used, the stability limit is predicted accurately. In Fig. 13 the pole maps are shown for  $V_n = 40\%$ . Again, the stability prediction of the LTI model is incorrect, whereas the LTP models are accurate.

#### D. STABILITY RESULTS: METHOD 2

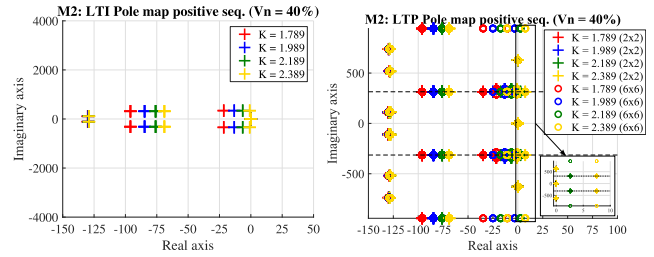
Here,  $K$  is also changed. According to simulations, the method 2 becomes unstable for  $K$  values higher than  $K_{lim}$ , being:

- $K_{lim} = 2.427$  for  $V_n = 5\%$ .
- $K_{lim} = 2.089$  for  $V_n = 40\%$ .

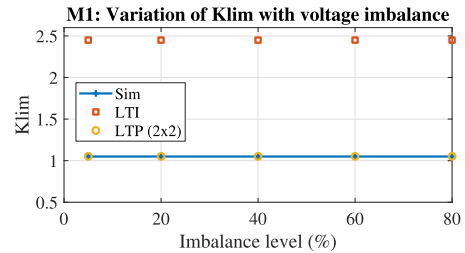
In Fig. 14 the pole map from  $M_{LTI2+}$  is shown for different  $K$  when  $V_n = 5\%$ . Note that, in method 2, it is only necessary to check the poles of the positive-sequence sub-system, since



**FIGURE 14.** Stability results for method 2 ( $V_n = 5\%$ ). The LTP pole map shows the fundamental strip, and two complementary strips ( $y$ -axis:  $[-\frac{3\omega_p}{2}, \frac{3\omega_p}{2}]$  with  $\omega_p = 2\omega_1$ ).



**FIGURE 15.** Stability results for method 2 ( $V_n = 40\%$ ). The LTP pole map shows the fundamental strip, and two complementary strips ( $y$ -axis:  $[-\frac{3\omega_p}{2}, \frac{3\omega_p}{2}]$  with  $\omega_p = 2\omega_1$ ).



**FIGURE 16.** Method 1: variation of  $K_{lim}$  with voltage imbalance given by simulations, the LTI model and the LTP ( $2 \times 2$ ) model.

only  $\theta_{PLL+}$  is used in the decoupling network. As it can be seen, the LTI method predicts quite accurately the stability boundary, since it predicts that, for the two  $K$  values higher than  $K_{lim}$ , the system is unstable. In contrast, when  $V_n = 40\%$ , the LTI model becomes inaccurate (Fig. 15). The LTP models ( $2 \times 2$  or  $6 \times 6$ ) are accurate no matter the  $V_n$  level.

#### E. EXPLANATION OF THE STABILITY RESULTS

The results in relation to method 1 are explained first. In M1, it is important to look at the expressions (4) and (5). The periodic terms in (5) depend on  $V_p$  and, thus, the periodic terms are important even if  $V_n$  is low. Since the LTI model ignores all periodic terms, the stability of the network is wrongly predicted by the LTI model (see again Fig. 12 and Fig. 13). In contrast, the LTP model considers the periodic terms, and predicts stability accurately for all  $V_n$  levels. This is clearly shown in Fig. 16.

Thus, the periodic terms explain the inaccuracy of the LTI model shown in Fig. 16. Other issues to explain in this figure

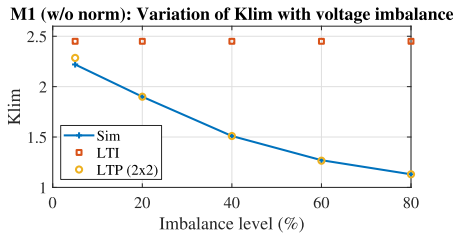


FIGURE 17. Method 1 (w/o normalization): variation of  $K_{lim}$  with voltage imbalance given by simulations, the LTI model and the LTP (2x2) model.

are: a) why the stability limit is independent on  $V_n$  (i.e. why  $K_{lim}$  is fixed at 1.05); and b) why the prediction of the LTI model is constant. The first issue is due to the normalization block. Including the normalization block makes the dynamics to be independent on  $V_n$ . In fact, the pole maps of the PLL do not change when  $V_n$  increases (compare the pole maps shown in Fig. 12 and Fig. 13). In contrast, if the normalization block is excluded, Fig. 17 is obtained. It is seen here that, in this case,  $K_{lim}$  depends on  $V_n$ . Here, note that the LTI model is also always inaccurate, although it is more inaccurate when  $V_n$  is high.

The second issue is why the stability prediction of the LTI model for method 1 is independent of  $V_n$ . The  $\overrightarrow{TF_{PLL1+LTI}}$  in (24) does not depend on  $V_n$  and thus its poles (and the poles of  $M_{LTI1+}$ ) do not change with  $V_n$ . With respect to  $\overrightarrow{TF_{PLL1-LTI}}$ , when the normalization block is included, then  $\overrightarrow{TF_{PLL1-LTI}}$  becomes:

$$\overrightarrow{TF_{PLL1-LTI}} = \frac{V_{nom}}{V_n} \frac{H_{PLL-}(s) \overrightarrow{G_{dq+}^*}(s)}{2j(1 + V_{nom}H_{PLL-}(s)G_{re}(s))}. \quad (42)$$

$V_n$  appears in (42) but only as a gain that cannot modify the poles of the system. Consequently, the LTI model in Fig. 16 always predicts a constant stability limit.

With respect to method 2, only  $\theta_{PLL+}$  is used in the decoupling network. Therefore, the relevant periodic terms in order to determine the stability of the network are those which appear in (25) but not the ones which appear in (26). It can be seen that the periodic terms in (25) are low when  $V_n$  is low, and therefore, in the low- $V_n$  cases the LTI model is accurate for stability calculations (see Fig. 14). When  $V_n$  increases, however, the periodic terms become relevant and again the LTI model fails to predict the stability boundary (see Fig. 15).

This is summarized in Fig. 18. In here,  $K_{lim}$  reduces with  $V_n$ , and the LTP model perfectly predicts the trend. The LTI model, however, is accurate only when  $V_n$  is low. Also, the LTI model predicts a constant  $K_{lim} = 2.45$ , independent of  $V_n$ . Of course, this is the case for the LTI model, since in M2 only the  $M_{LTI1+}$  matters for stability and the  $\overrightarrow{TF_{PLL1+LTI}}$  does not depend on  $V_n$ .

### F. LTP MODEL ORDER TRUNCATION

Sections IX-C and IX-D show, both in M1 and M2, that the LTP poles in the fundamental strip do not significantly change

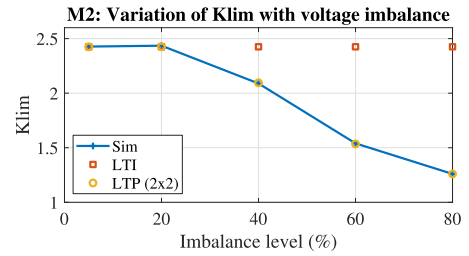


FIGURE 18. Method 2: variation of  $K_{lim}$  with voltage imbalance given by simulations, the LTI model and the LTP (2x2) model.

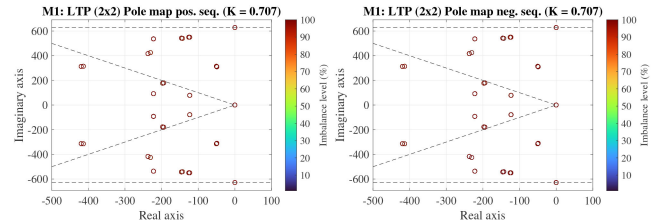


FIGURE 19. Method 1: pole map at different imbalance levels ( $K = 1/\sqrt{2}$ ). The pole map shows the fundamental strip ( $y$ -axis:  $[-\frac{\omega_p}{2}, \frac{\omega_p}{2}]$  with  $\omega_p = 2\omega_1$ ) and the  $\zeta = 1/\sqrt{2}$  damping diagonals.

when they are calculated with the 2 x 2 or 6 x 6 model. With respect to the other strips, the 6x6 model calculates more poles there, which is expected since the 6 x 6 model has a higher order. However, since the poles in the complementary strips are only reflections of the poles in the fundamental strip, only the poles in the fundamental strip are relevant for stability studies. In fact, Fig. 16 – 18 show that the 2 x 2 model calculates the stability boundaries accurately; while Fig. 10 shows that the  $\overrightarrow{TF_{PLL1+}}$  and  $\overrightarrow{TF_{PLL2+}}$  alone predict the most relevant oscillations in the PLL response.

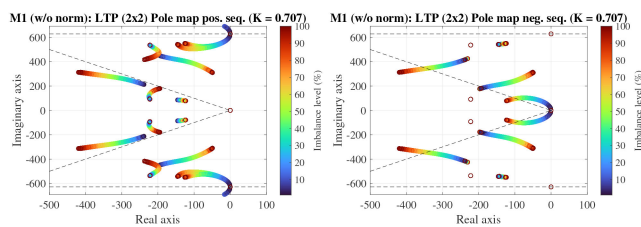
### X. EXAMPLE OF APPLICATION OF THE LTP MODEL

The LTP model has been used to check the stability boundary. However, it can also provide further information, as for example, the stability margin of the PLL. An example of such usage is shown in this section.

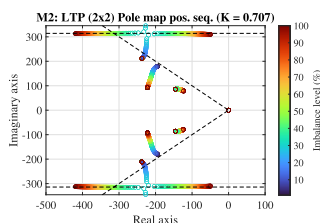
The positive- and negative-sequence pole maps of method 1 when  $K = 1/\sqrt{2}$  are shown in Fig. 19. First of all, it is noticeable that the pole map of method 1 (M1) does not change with  $V_n$ . This is due to the normalization block. If the normalization block is bypassed (in such a way that  $v_{dq-}^{dec}$  is directly fed into the PI), the polemaps vary with  $V_n$ , as shown in Fig. 20.

Fig. 21 shows the eigenvalue plot of method 2, which moves closer to  $x = 0$  as the imbalance level increases. If ensuring a minimum level of damping as  $\zeta = 1/\sqrt{2}$  is a requirement for the PLL design, it can be seen here that this parameter selection would only meet the requirement for low imbalance levels (in particular, for  $V_n < 15\%$ ).

Another example would be to compare PLL types. In recent literature, the comparison of the response of different PLL



**FIGURE 20. Method 1 (w/o norm): pole map at different imbalance levels ( $K = 1/\sqrt{2}$ ). The pole map shows the fundamental strip ( $y$ -axis:  $[-\frac{\omega_p}{2}, \frac{\omega_p}{2}]$ ) with  $\omega_p = 2\omega_1$  and the  $\zeta = 1/\sqrt{2}$  damping diagonals.**



**FIGURE 21. Method 2: pole map at different imbalance levels ( $K = 1/\sqrt{2}$ ). The pole map shows the fundamental strip ( $y$ -axis:  $[-\frac{\omega_p}{2}, \frac{\omega_p}{2}]$ ) with  $\omega_p = \omega_1$  and the  $\zeta = 1/\sqrt{2}$  damping diagonals.**

**TABLE 1. Model Selection Recommendation Depending on the PLL Type and Voltage Imbalance Level**

Imb. level	Direct tracking of negative-sequence					Indirect tracking or no tracking of negative-sequence				
	5%	20%	40%	60%	80%	5%	20%	40%	60%	80%
LTI						x	x			
LTP	x	x	x	x	x			x	x	x

topologies against faults has typically been done by comparing time domain waveforms, which leads to a comparison difficult to quantify. The presented LTP model can potentially be used as an additional tool to time domain simulations to compare different PLL topologies in a more quantitative way.

**XI. GUIDELINES FOR LTI VS LTP MODEL SELECTION IN THE PRESENCE OF IMBALANCE**

This article shows that the LTI model might provide inaccurate stability predictions depending on the  $V_n$  level and the way in which the negative-sequence voltage is tracked, while the LTP model calculates the stability boundary more accurately no matter the imbalance level nor the PLL method. Based on the results of this article, some conclusions can be generalized as basic guidelines for LTI vs LTP model selection for studying stability in PLLs. These conclusions are shown in Table 1.

Table 1 shows that, whenever the PLL indirectly tracks the negative-sequence voltage, for low levels of imbalance the LTI model can be used. In this article it has been shown that, for the DDSRF-PLL method 2, the LTI model gives results with high accuracy until  $V_n = 20\%$  approximately, while for  $V_n = 40\%$  and above the LTP model accuracy is much higher (see Fig. 18). It is clear that, for this specific PLL and for the exact parameter selection, the model-validity boundary

for choosing the LTI model vs the LTP model is between  $V_n = 20\%$  and  $V_n = 40\%$ . For other types of PLLs and parameters, this boundary might differ. It is worth to note that an imbalance level in between 20 – 40% is a relatively high imbalance level that encompasses, to the knowledge of the authors, the imbalance levels that appear in normal operation conditions, and even some mild faulty conditions. It seems reasonable to assume, then, that for other types of PLLs, accurate results can be obtained with the LTI model if the analysis is performed in normal operating conditions, while if the analysis is performed for faulty conditions, the LTP model is probably necessary; although further investigations with other PLL topologies are recommended. In the case in which the negative-sequence is not tracked (which is M2 without the extra blocks to calculate the negative sequence characteristics), this boundary of  $V_n = 20 - 40\%$  will be applicable, since in method 2 the stability depends only on tracking the positive-sequence phase angle. In the case that the negative-sequence voltage is directly tracked, it is always necessary to use an LTP model, since the periodicity in the state variables is brought forward by both  $V_p$  and  $V_n$ .

It could be argued, then, that the LTP model should always be used, especially if the model-validity boundary between using the LTI and LTP model is unknown. However, this ignores one main drawback of the LTP method, which is its higher computational effort with respect to the LTI model.<sup>8</sup> Therefore, the final selection can be performed taking into account the computational efficiency as well, although always taking into account the accuracy considerations shown in Table 1.

**XII. CONCLUSION**

The LTI and LTP models of two different DDSRF-PLL methods in the presence of voltage imbalance have been derived. The article shows that, when the PLL directly tracks the negative-sequence phase-angle (method 1), the LTP terms depend on both  $V_p$  and  $V_n$ , separately. Thus, even if the voltage imbalance is low, the LTP dynamics are relevant and influence the stability of the PLL (due to  $V_p$ ). The main consequence is that the LTI model cannot predict correctly the stability limit at any imbalance level. In contrast, when the PLL indirectly tracks the negative-sequence phase-angle (method 2), the LTP terms depend on  $V_n$  only. Thus, when  $V_n$  is low enough (with respect to  $V_p$ ) the LTI model predicts correctly the stability boundaries, although when  $V_n$  is increased, the LTP model is needed. Some guidelines as to how to pick between LTI and LTP models for stability analysis of DDSRF-PLLs in the presence of imbalance have been given.

Further, the article shows with time domain simulations that, in order to capture the most relevant oscillations that

<sup>8</sup>It is worth to note, however, that the computational effort of the LTP model highly depends on the type of LTP model used. This article uses a closed-loop HTF, although other LTP modelling alternatives are possible, like using open-loop HTFs (e.g. [11]) or using the time domain state-space approach (e.g. [13]), which are more computationally efficient than the closed-loop HTF approach.



occur in the PLL in the presence of  $V_n$ , it is not necessary to include the coupling terms in the LTP model. In fact, it is shown that a  $2 \times 2$  HTF matrix is enough for predicting the stability limit, although the order can always be increased to enhance accuracy.

## APPENDIX A DESIGN OF DDSRF-PLL CONSTANTS

This Appendix discusses the DDSRF-PLL constants. The first design choice is the cut-off frequency of:  $F(s) = \frac{\omega_f}{s + \omega_f}$ . This frequency is usually set as  $\omega_f = K\omega_1$ , where  $K$  is the parameter changed in Section IX to test the LTI and LTP methods. The second design choice is the constants in the PI regulators. In this article, the same strategy as in [2] is followed, which consists in ignoring the decoupling network and analysing the SRF-PLL separately. In this case, the SRF-PLL is a second-order small-signal model with two poles whose locations depend on  $K_{pPLL}$  and  $K_{iPLL}$ . The damping and frequency of the poles can be imposed (in this article,  $\omega_c = 2\pi 30$  rad/s and  $\xi = 1/\sqrt{2}$ ) as:

$$K_{iPLL} = \frac{\omega_c^2}{V_{nom}}; K_{pPLL} = \frac{2\xi\omega_c}{V_{nom}} \quad (43)$$

where  $V_{nom}$  is the nominal converter voltage (in this article,  $V_{nom} = 110\sqrt{2}$  V). These PI values are used in the positive-sequence SRF-PLL in both methods. In the negative-sequence SRF in method 1, these constants are used too, since the negative-sequence voltage is normalized and scaled to  $V_{nom}$ .

## APPENDIX B EXTRA EQUATIONS FOR LTP MODEL OF METHOD 1

A generic term  $\vec{T}$  is defined as  $\vec{T} = \vec{T}_0(s + j2\omega_1)$  (with  $\vec{T}_0$  defined in (11)) and:

$$\begin{aligned} \vec{T}_1 &= \vec{G}_{dq+}(s) \left( 1 - jV_n \vec{G}_{dq+}^*(s + j2\omega_1) \vec{T} \right) \\ \vec{T}_2 &= -jV_p \vec{T}_1 \\ \vec{T}_3 &= jV_n e^{-2j(\phi_{vp} + \phi_{vn})} \vec{G}_{dq+}(s) \vec{G}_{dq+}(s + j2\omega_1) \vec{T} \\ \vec{T}_4 &= jV_p \vec{T}_3 \\ \vec{T}_5 &= \frac{H_{PLL+}(s)}{2j - H_{PLL+}(s)(\vec{T}_2 - \vec{T}_2^*)} \end{aligned} \quad (44)$$

## APPENDIX C EXTRA EQUATIONS FOR LTP MODEL OF METHOD 2

$$\begin{aligned} \vec{T}_6 &= \frac{H_{PLL+}(s)}{2j(1 + V_p H_{PLL+}(s) G_{re}(s))} \\ \vec{T}_7 &= jV_n e^{j(\phi_{vp} + \phi_{vn})} \vec{G}_{dq+}^*(s) \end{aligned} \quad (45)$$

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